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Structure Characteristics of Light Nuclei Calculated within the Variational Approach

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- **Statement of the problem**
- **Hamiltonian and potentials for the five-particle model of ^{14}C, ^{14}O, and ^{14}N nuclei**
- **Method (variational method in Gaussian representation)**
- **R.m.s. radii and r.m.s. distances between particles**
- **Charge density distributions, and charge formfactors**
- **Pair correlation functions**
- **Momentum distributions**
- **Probability density and two spatial configurations**
- **Formfactors of ^{12}C, ^{16}O, and ^{20}Ne nuclei within the α–cluster model**
- **Conclusions**

B. E. Grinyuk, D. V. Piatnytskyi. Ukr. J. Phys. **61**, No. 8, 674 (2016)
doi: 10.15407/ujpe61.08.0674

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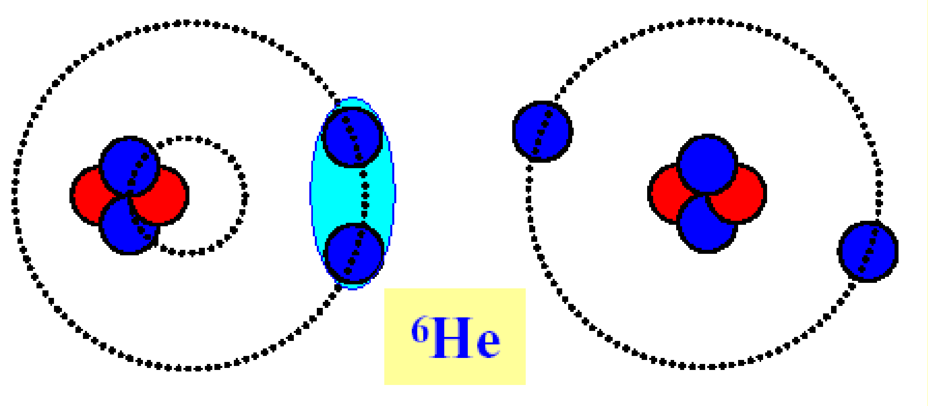
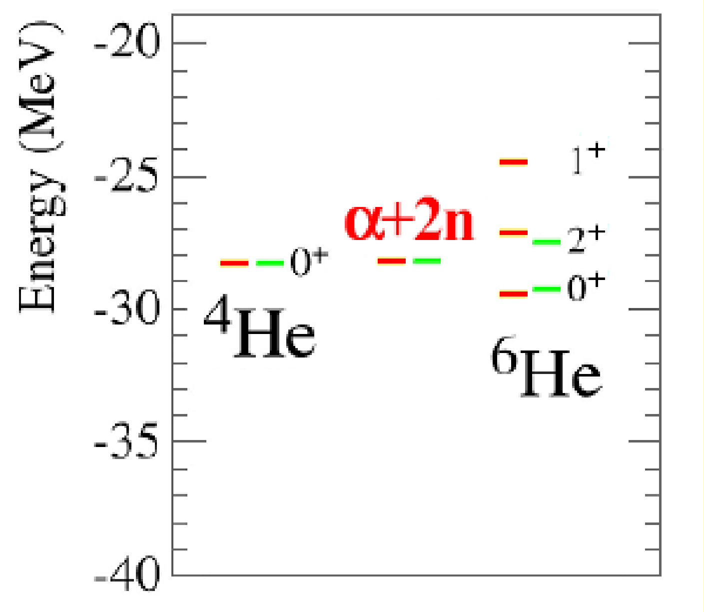
Structure of ^{6}He nucleus

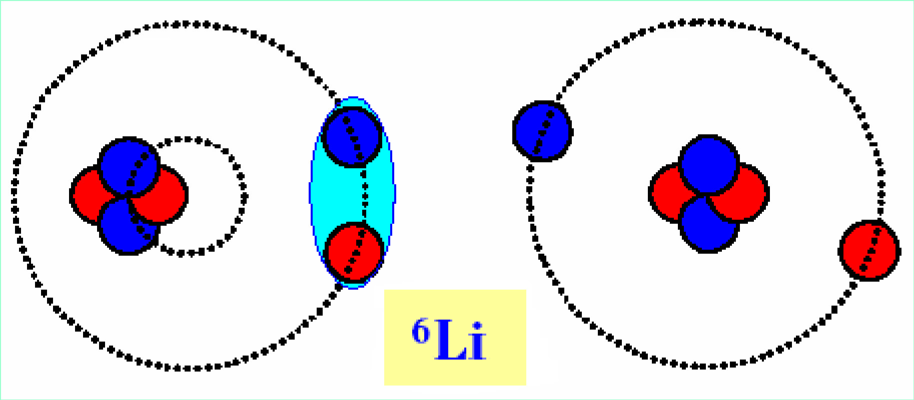


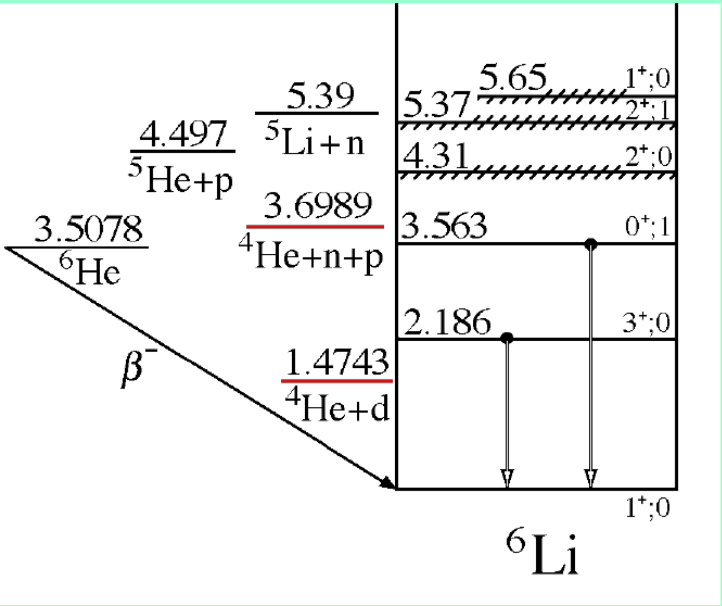


Isotope	Half-life	Spin	Isospin	Core + Valence
He-6	807 ms	0^+	1	$\alpha + 2n$

$E = -0.9734 \pm 0.0010\ MeV$	*G.Audi, A.H.Wapstra*. Nucl.Phys. **A595** (1995) p.409.
$\langle R_{ch}^2 \rangle^{1/2} = 2.054 \pm 0.014\ fm$ $= 2.068 \pm 0.011\ fm$	*Li-Bang Wang*. Phys. Rev. Lett. **93**, 142501 (2004). *P.Mueller, I.A.Sulai, A.C.C.Villari et al.*, Phys. Rev. Lett. **99**, 252501 (2007).
$\langle R_m^2 \rangle^{1/2} = 2.57 \pm 0.10\ fm$ $= 2.59 \pm 0.05\ fm$	*L.V.Chulkov, B.V.Danilin, V.D.Efros, A.A.Korsheninnikov and M.V.Zhukov*, Europhys. Lett. **8** (1989) 245. *B.V.Danilin, S.N.Ershov, and J.S.Vaagen*, Phys. Rev. C **71**, 057301 (2005).

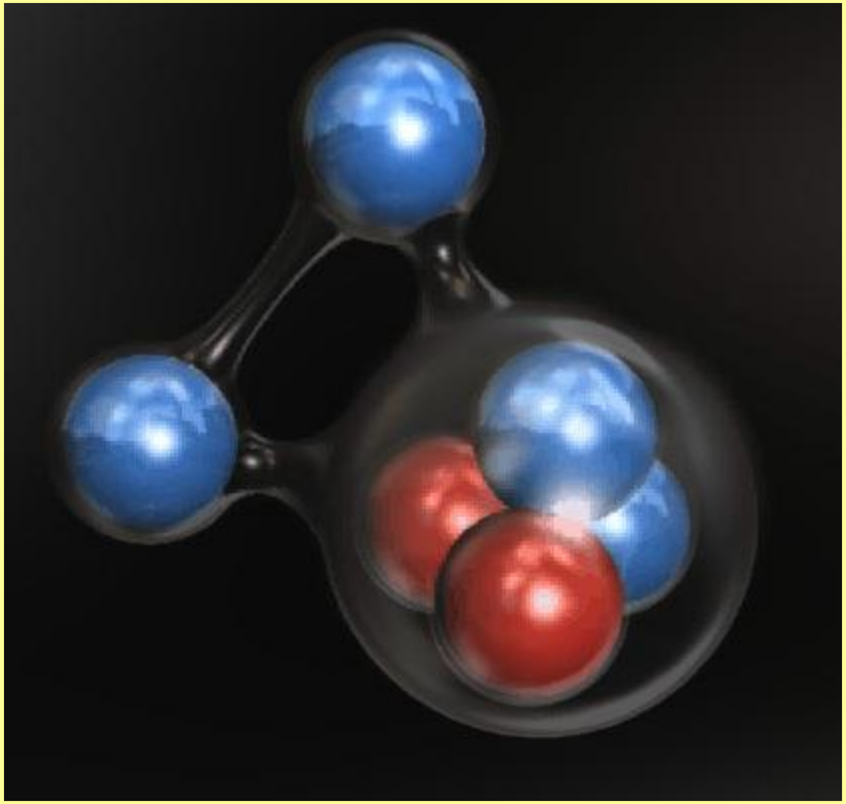
Structure of ^{6}Li nucleus

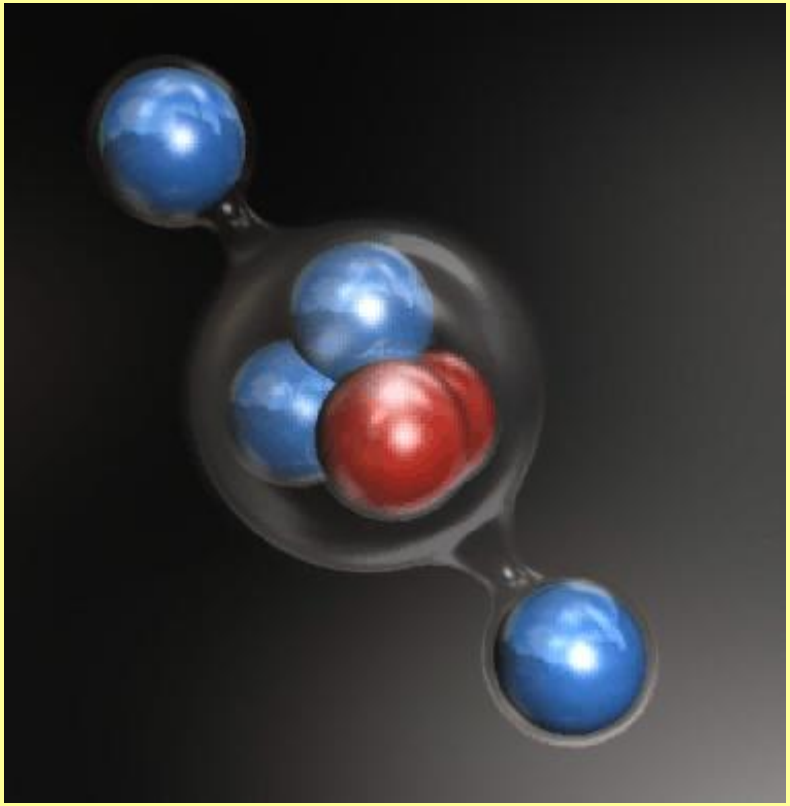




E= - 3.699±0.001 MeV	*G.Audi, A.H.Wapstra*. Nucl.Phys. **A595** (1995) p.409.
$\langle R^2_{ch} \rangle^{1/2}$ = 2.56±0.05 fm = 2.51±0.04 fm	*C.W. de Jager, H. De Vries, and C. De Vries, At. Data Nucl. Data Tables **36**, 495 (1987).* *W. Nörtershäuser, A. Dax, Guido Ewald et al.*, Kluwer Academic Publishers, Netherlands, 2005. **Noertershaeuser-Laser2004.tex**; 12/04/2005; 10 p.
$\langle R^2_{m} \rangle^{1/2}$ = 2.45±0.07 fm	*P. Egelhof et al., Eur. Phys. J. A **15,** 27-33 (2002).*

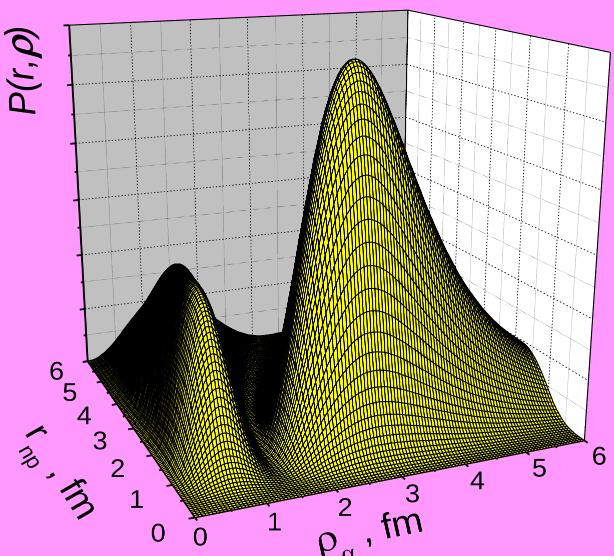
Structure of ^{6}He nucleus



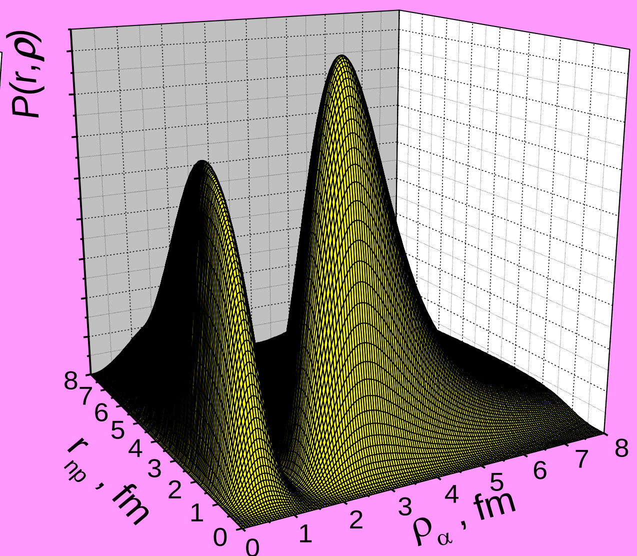


The structure of the wave function

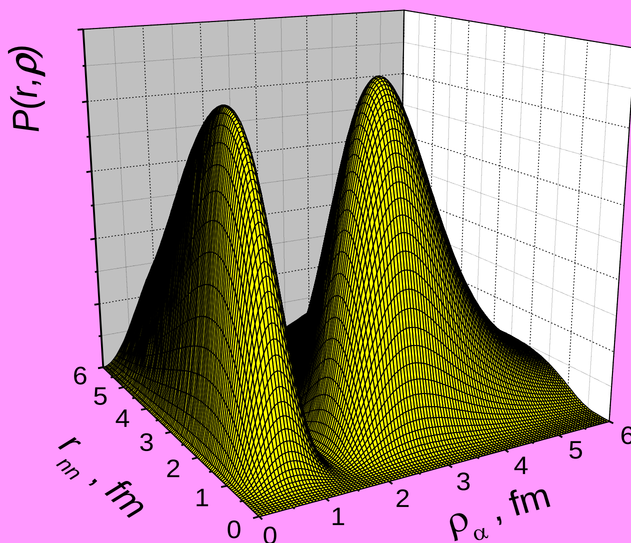
$$P(r,\rho) = r^2\rho^2 \int d\Omega \left|\Phi(\mathbf{r},\boldsymbol{\rho})\right|^2$$



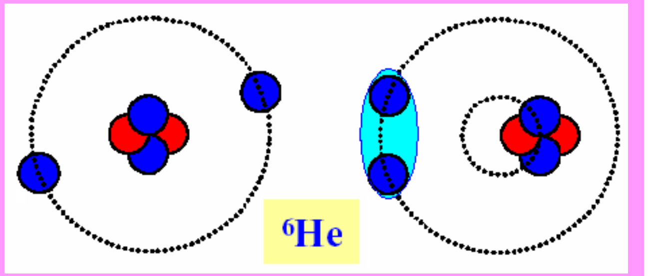
^{6}Li (J=1$^+$)

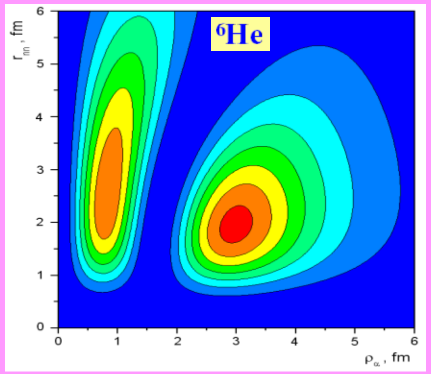


^{6}Li (J=0$^+$)

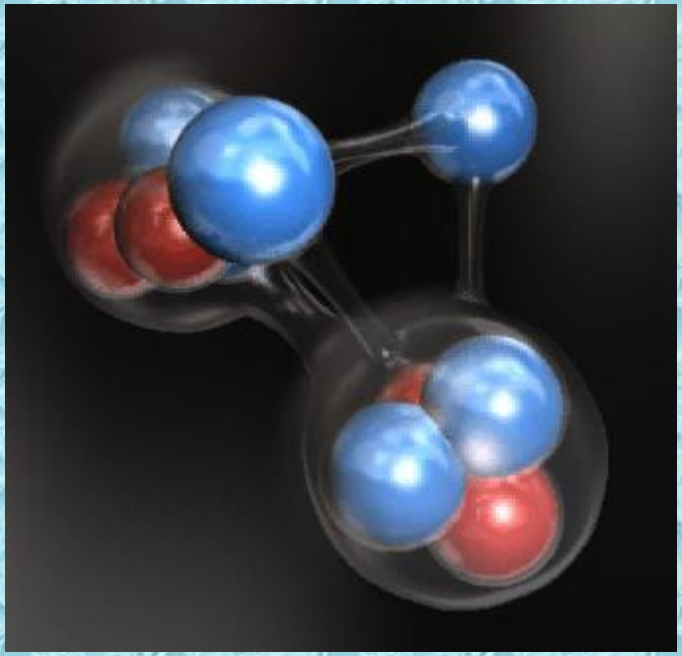


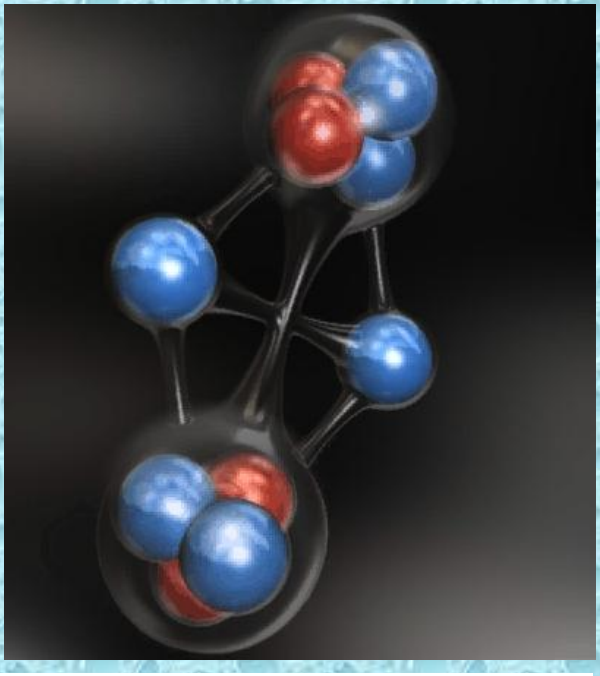
^{6}He

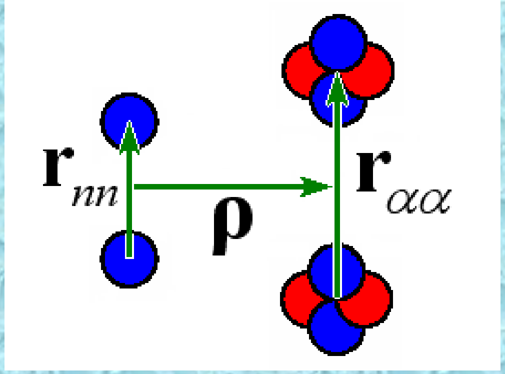




Light cluster nuclei ^{10}C and ^{10}Be







^{10}C: $p + p + \alpha + \alpha$

Half-life 19.29 sec $E = -3.73$ MeV ($J^{\pi} = 0^{+}$, T = 1)

$R_{ch} = 2.42$ fm

^{10}Be: $n + n + \alpha + \alpha$

Half-life 1.387×10^{6} years $E = -8.387$ MeV ($J^{\pi} = 0^{+}$, T = 1)

$R_{ch} = 2.357(21)$ fm

B. E. Grinyuk, I. V. Simenog. Physics of Atomic Nuclei **77**, 415 (2014).
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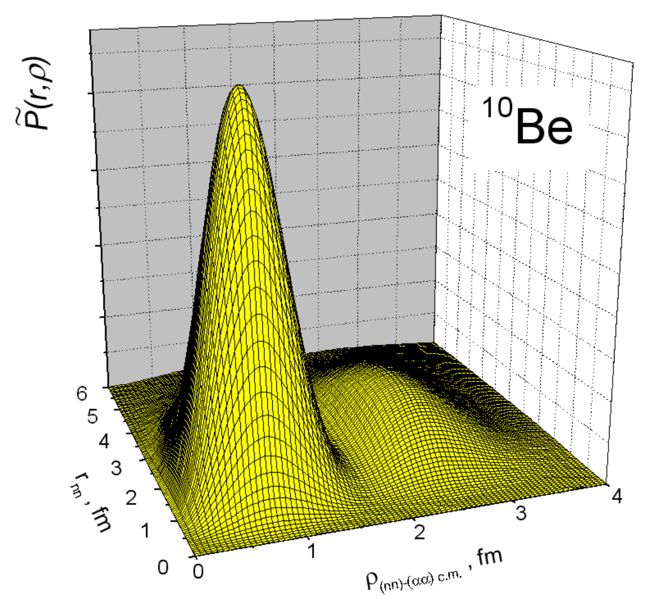
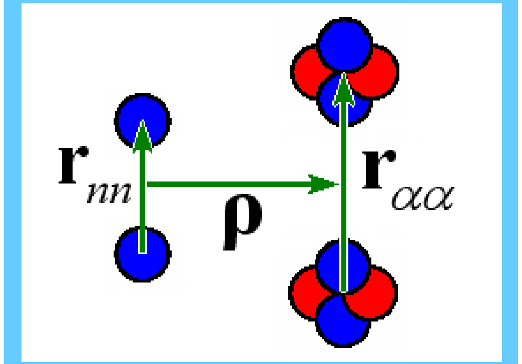
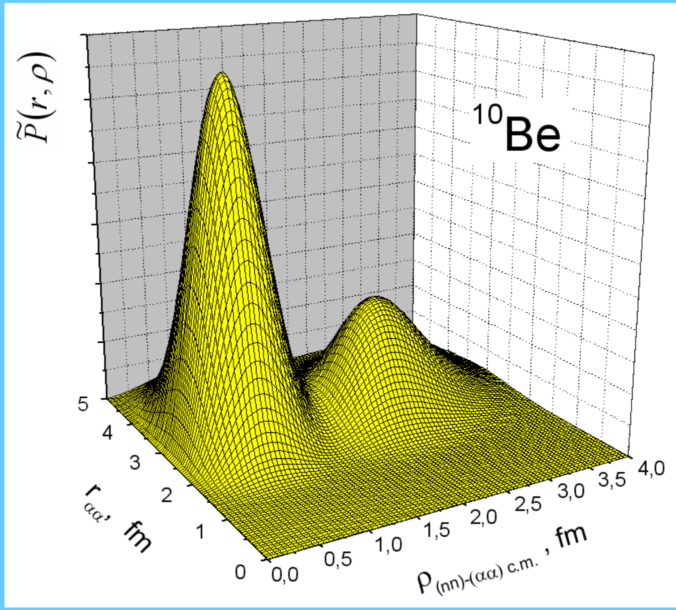
The wave function of ^{10}Be nucleus

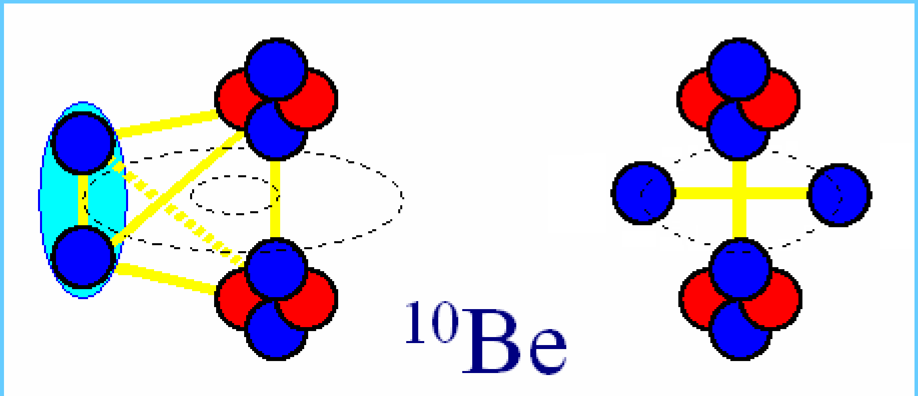
$$\widetilde{P}(r,\rho) = r^2\rho^2 \int d\Omega \int d\mathbf{r}_{\alpha\alpha} \left|\Phi\left(\mathbf{r},\boldsymbol{\rho},\mathbf{r}_{\alpha\alpha}\right)\right|^2$$

$$\widetilde{P}(r,\rho) = r^2\rho^2 \int d\Omega \int d\mathbf{r}_{NN} \left|\Phi\left(\mathbf{r}_{NN},\boldsymbol{\rho},\mathbf{r}\right)\right|^2$$



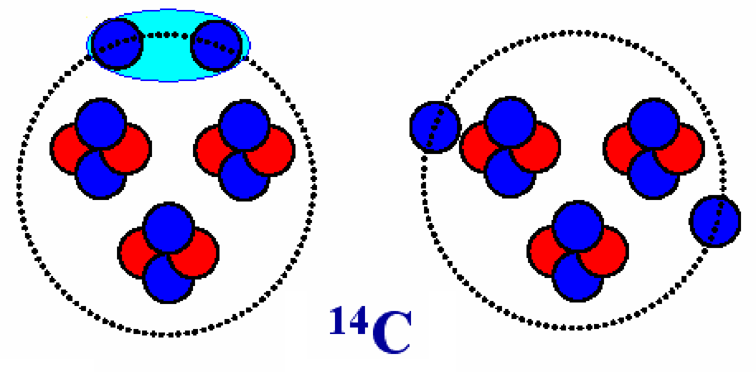


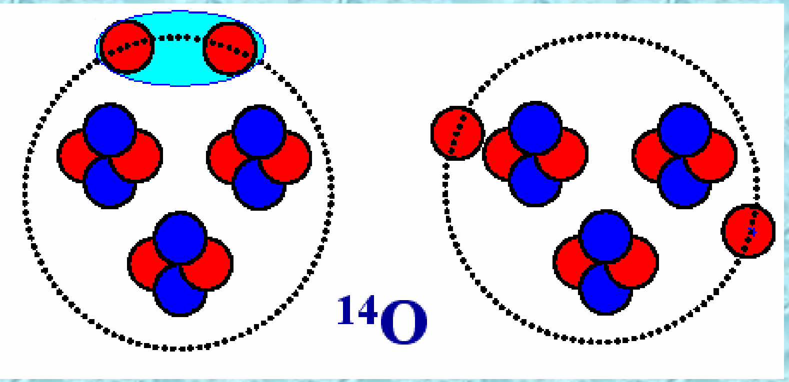




What is the structure of the mirror nuclei ^{14}C and ^{14}O? And that of ^{14}N?

?





Isotope	Half-life	Spin	Isospin	Core + Valence
^{14}C	5730 ± 40 y	0^+	1	$3\alpha + 2n$

Isotope	Half-life	Spin	Isospin	Core + Valence
^{14}O	70.6 s	0^+	1	$3\alpha + 2p$

Hamiltonian for ^{14}O

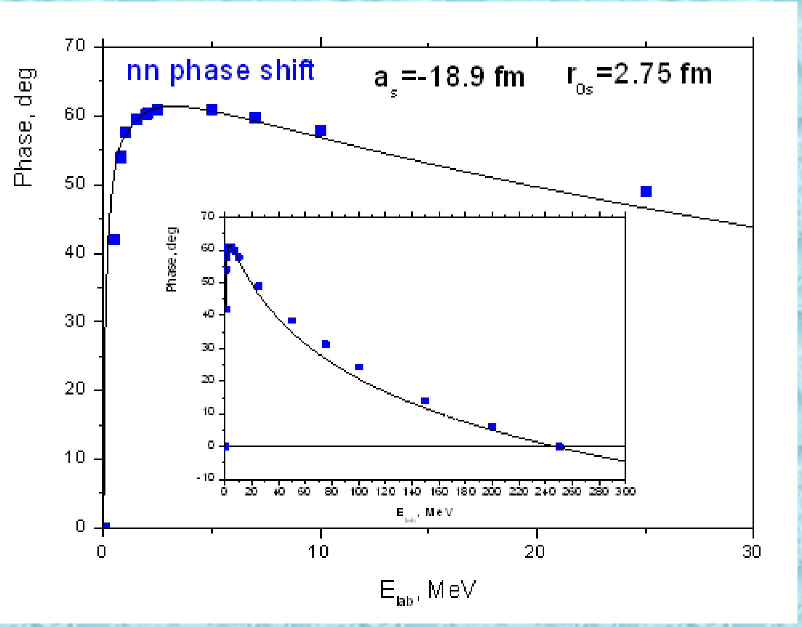
$$\hat{H} = \sum_{i=1}^{2} \frac{\mathbf{p}_i^2}{2m_p} + \sum_{i=3}^{5} \frac{\mathbf{p}_i^2}{2m_\alpha} + U_{pp}(r_{12}) + \sum_{j>i=3}^{5} \hat{U}_{\alpha\alpha}(r_{ij}) +$$

$$+ \sum_{i=1}^{2} \sum_{j=3}^{5} \hat{U}_{p\alpha}(r_{ij}) + \sum_{j>i=1}^{5} \frac{Z_i Z_j e^2}{r_{ij}}.$$

Similar form has the Hamiltonian for ^{14}C (less number of Coulomb terms)

Parameters of the singlet V_{nn} potential (energies in *MeV*, radii in *fm*).

Potential $V_{nn}(r)$:	$a_{nn,s}$	$r_{0nn,s}$
$V_{nn}(r) = \sum_{k=1}^{3} V_{0k} \exp\left(-(r/r_{0k})^2\right)$ $V_{01} = 952.15$, $r_{01} = 0.440$, $V_{02} = -79.39$, $r_{02} = 0.959$, $V_{03} = -37.89$, $r_{03} = 1.657$.	– 18.9	2.75
Experiment:	– 18.9	2.75± ±0.05



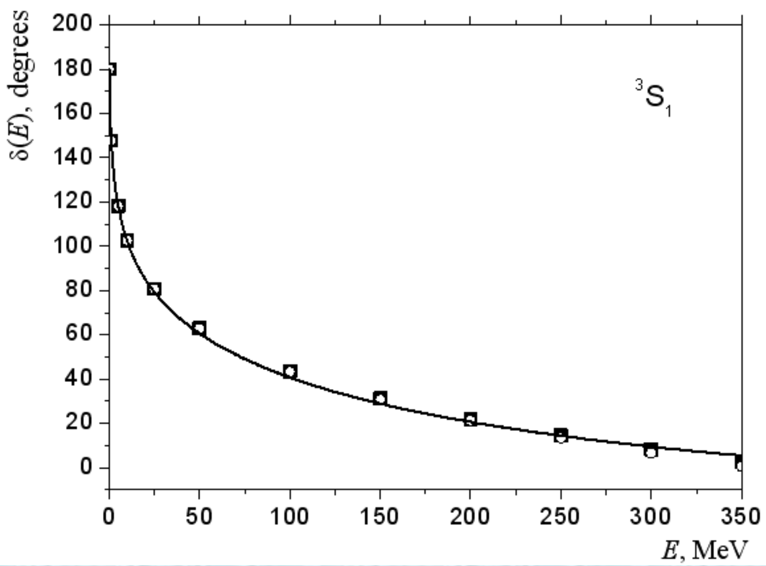
Hamiltonian for ^{14}N

$$\hat{H} = \frac{\mathbf{p}_1^2}{2m_p} + \frac{\mathbf{p}_2^2}{2m_n} + \sum_{i=3}^{5} \frac{\mathbf{p}_i^2}{2m_\alpha} + U_{pn}(r_{12}) + \sum_{j>i=3}^{5} \hat{U}_{\alpha\alpha}(r_{ij}) +$$

$$+ \sum_{j=3}^{5} \hat{U}_{p\alpha}(r_{1j}) + \sum_{j=3}^{5} \hat{U}_{n\alpha}(r_{2j}) + \sum_{j>i=1}^{5} \frac{Z_i Z_j e^2}{r_{ij}}$$

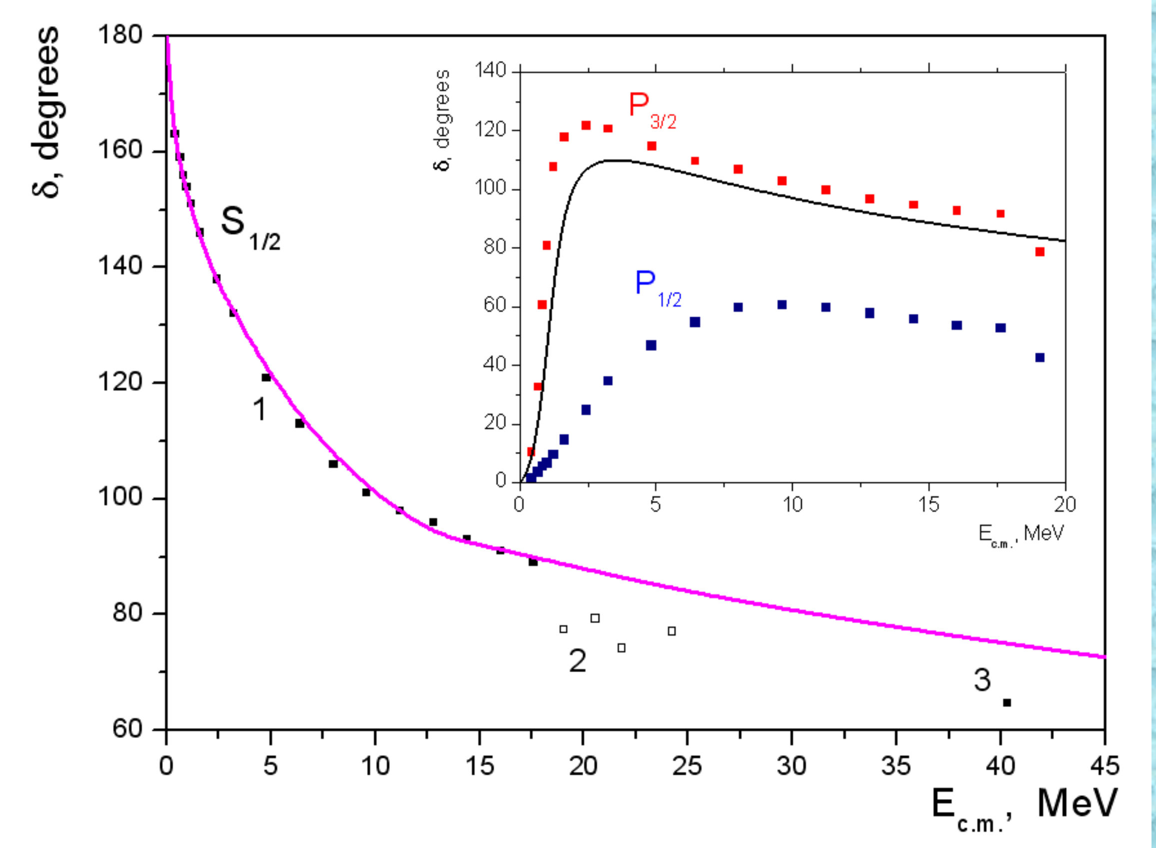
Parameters of the triplet V_{np} potential (energies in MeV, radii in fm).

Potential $V_{np}(r)$ in the triplet state:	$a_{np,t}$	$r_{0np,t}$	ε_d	R_d
$V_{np}(r) = \sum_{k=1}^{2} V_{0k} \exp\left(-(r/r_{0k})^2\right)$ V_{01} = 840.545, r_{01}=0.440, V_{02} = –146.046, r_{02}=1.271.	5.424	1.783	–2.224576	2.140
Experiment:	5.424± ±0.003 [12]	1.760± ±0.005 [12]	–2.224575(9) [13]	2.1402± ±0.0028 [14]



$n\alpha$-interaction potential

$$\hat{V}_{n\alpha} = V(r) + g|u(r)\rangle\langle u(r_1)| \equiv V(r) + gu(r)\int u(r_1)\ldots d\mathbf{r}_1$$

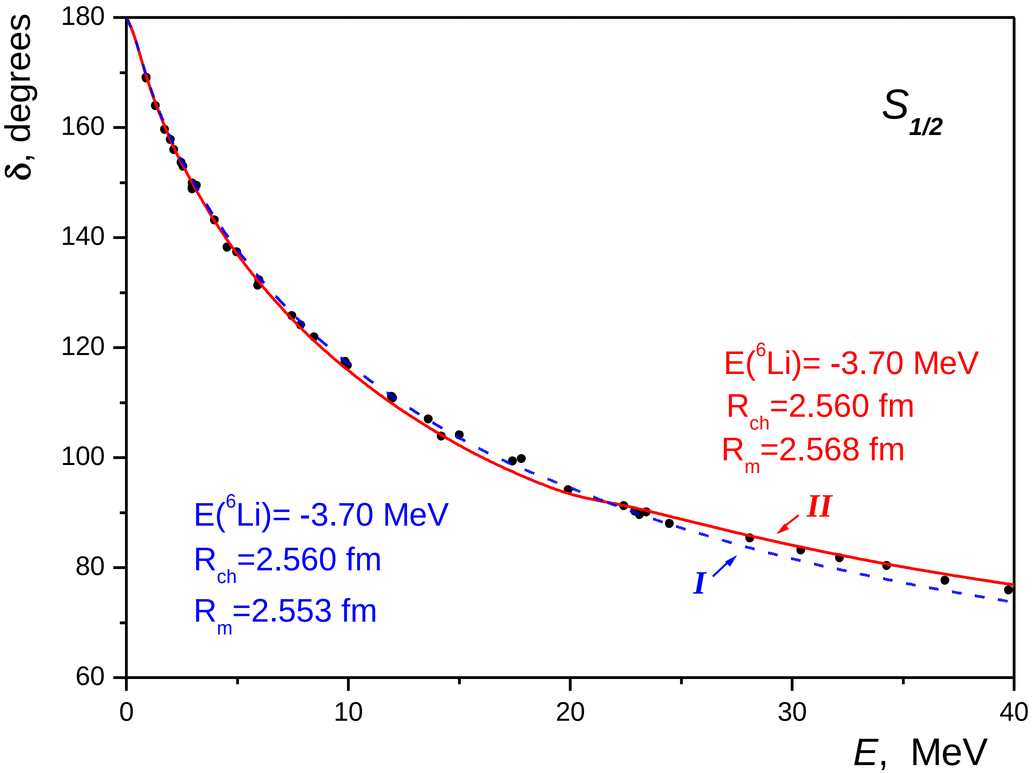


$$V(r) = -50.208\exp\left(-\left(\frac{r}{2.32}\right)^2\right),\quad g = 135.0,\quad u(r) = \pi^{-3/4}\exp\left(-\left(\frac{r}{2.7}\right)^2\right)$$

pα-interaction potential

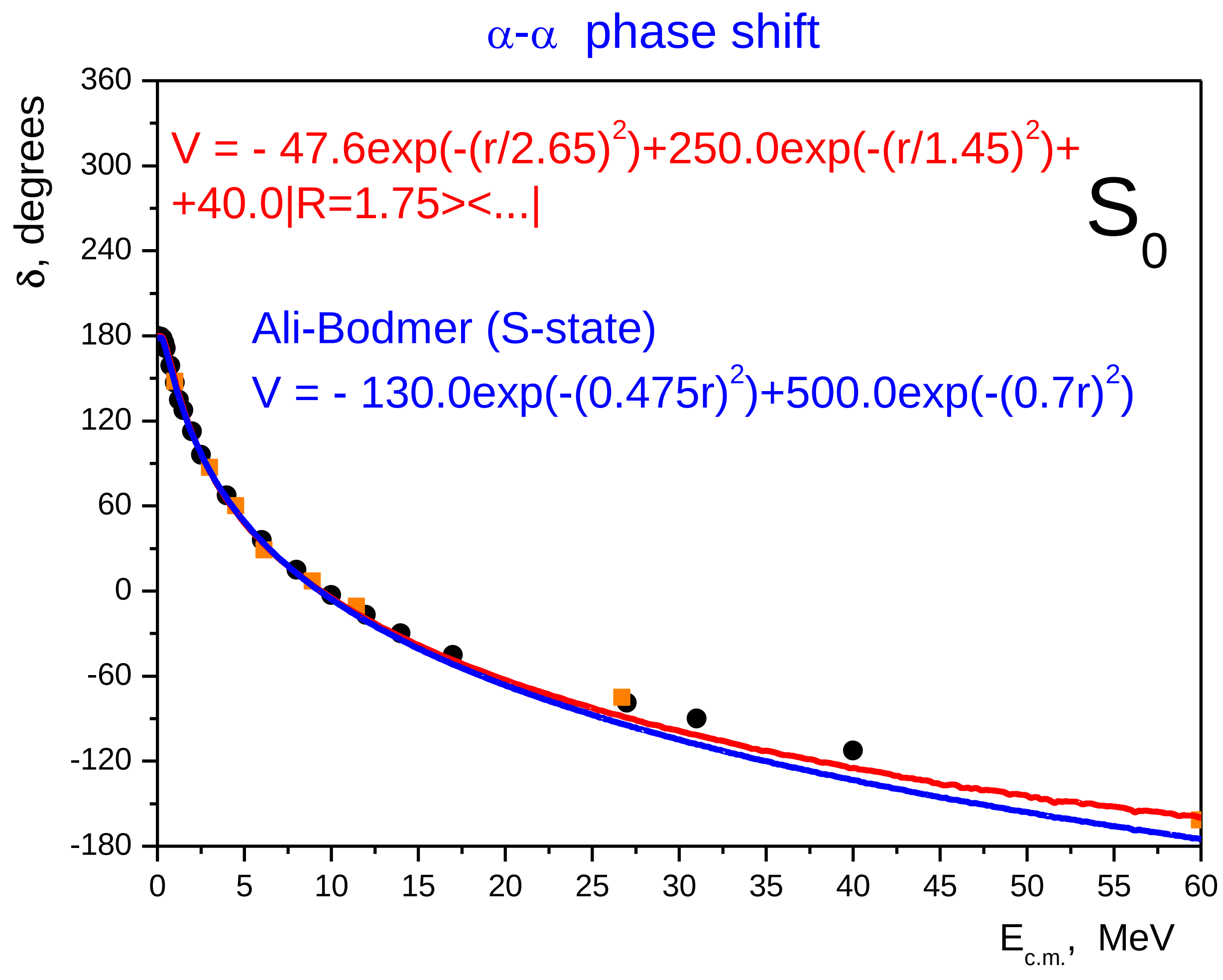
$$\hat{V}_{p\alpha}\Psi(r) = V(r)\Psi(r) + gu(r)\int u(r_1)\Psi(r_1)\,d\mathbf{r}_1 \;.$$

$\hat{V}_{p\alpha}$ potential
(*I*), $V_{p\alpha} \neq V_{n\alpha}$ $V_0(r) = -43.09\exp\left(-(r/2.34)^2\right)$ $g = 135.0$ Mev·fm^{-3}, $u(r) = \pi^{-3/4}\exp\left(-(r/2.67)^2\right)$
(*II*), $V_{p\alpha} = V_{n\alpha}$ $V_0(r) = -45.13\exp\left(-(r/2.37)^2\right)$ $g = 140.0$ Mev·fm^{-3}, $u(r) = \pi^{-3/4}\exp\left(-(r/2.7)^2\right)$



$\alpha\alpha$-interaction potential

$$\hat{V}_{\alpha\alpha} = V(r) + g|u(r)\rangle\langle u(r_1)| \equiv V(r) + gu(r)\int u(r_1)\ldots d\mathbf{r}_1$$



Potentials used for ^{14}C and ^{14}O nuclei

V_{nn}	$952.15\exp\left(-(r/0.44)^2\right)-79.39\exp\left(-(r/0.959)^2\right)-37.89\exp\left(-(r/1.657)^2\right)$
$V_{n\alpha}$	$-43.95\exp\left(-(r/2.25)^2\right)+140.0\left\|\exp\left(-(r/2.79)^2\right)\right\rangle\left\langle\ldots\right\|$
$V_{\alpha\alpha}$	$-46.5\exp\left(-(r/2.55)^2\right)+240.0\exp\left(-(r/1.3)^2\right)+60.0\left\|\exp\left(-(r/1.765)^2\right)\right\rangle\left\langle\ldots\right\|$

V_{pp}	$952.15\exp\left(-(r/0.44)^2\right)-79.39\exp\left(-(r/0.959)^2\right)-37.89\exp\left(-(r/1.657)^2\right)$
$V_{p\alpha}$	$-44.757\exp\left(-(r/2.27)^2\right)+140.0\left\|\exp\left(-(r/2.79)^2\right)\right\rangle\left\langle\ldots\right\|$
$V_{\alpha\alpha}$	$-46.5\exp\left(-(r/2.55)^2\right)+240.0\exp\left(-(r/1.3)^2\right)+60.0\left\|\exp\left(-(r/1.765)^2\right)\right\rangle\left\langle\ldots\right\|$

Nuclei	E	E_{exp}	R_{ch}	$R_{\mathrm{ch,\,exp}}$
^{14}C	-20.398	-20.398	2.500	2.496 [17] 2.503 [12]
^{14}O	-13.845	-13.845	2.415	—

Potentials used for ^{14}N nucleus

V_{np}	$-146.046\exp\left(-(r/1.271)^2\right)+840.545\exp\left(-(r/0.44)^2\right)$
$V_{n\alpha}$	$-43.85\exp\left(-(r/2.25)^2\right)+140.0\left\|\exp\left(-(r/2.79)^2\right)\right\rangle\left\langle\ldots\right\|$
$V_{p\alpha}$	$-44.757\exp\left(-(r/2.27)^2\right)+140.0\left\|\exp\left(-(r/2.79)^2\right)\right\rangle\left\langle\ldots\right\|$
$V_{\alpha\alpha}$	$-43.5\exp\left(-(r/2.746)^2\right)+240.0\exp\left(-(r/1.53)^2\right)+60.0\left\|\exp\left(-(r/1.765)^2\right)\right\rangle\left\langle\ldots\right\|$

Nucleus	E	E_{exp}	R_{ch}	$R_{\mathrm{ch,exp}}$
^{14}N	– 19.772	– 19.772	2.558	2.5582

nα-, *pα*- and *αα*- phase shifts

$$V_0(r,r') \equiv 2\pi r r' \int_{-\pi}^{\pi} \hat{V}(\mathbf{r},\mathbf{r}') P_0(\cos\theta) d(\cos\theta), \qquad \hat{V}(\mathbf{r},\mathbf{r}') \equiv V(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}') + g|u(r)\rangle\langle u(r')|$$

Variable phase approach leads to an equation with singularities:

$$\frac{d}{dr}\delta_0(r) = -\frac{2}{k}\sin(kr+\delta_0(r))\int_0^r dr' V_0(r,r')\sin(kr'+\delta_0(r')) \times$$
$$\times \exp\left(-\int_{r'}^{r} ds \frac{d\delta_0(s)}{ds} ctg(ks+\delta_0(s))\right);$$

F.Calogero. Nuovo Cimento **33**, 352 (1964)

An approach without singularities:

$$u_0''(r) + k^2 u_0(r) - \int_0^\infty V_0(r, r_1) u_0(r_1) dr_1 = 0;$$

$$u_0(r) = c_1(r)\sin(kr) + c_2(r)\cos(kr);$$

$$\begin{cases} c_1'(r) = \dfrac{\cos(kr)}{k} \displaystyle\int_0^\infty V_0(r, r_1)\left(c_1(r_1)\sin(kr_1) + c_2(r_1)\cos(kr_1)\right)dr_1; \\ c_2'(r) = -\dfrac{\sin(kr)}{k} \displaystyle\int_0^\infty V_0(r, r_1)\left(c_1(r_1)\sin(kr_1) + c_2(r_1)\cos(kr_1)\right)dr_1; \end{cases}$$

$$c_1(0) = 1, \quad c_2(0) = 0, \qquad tg(\delta_0) = \lim_{r\to\infty} \frac{c_2(r)}{c_1(r)}.$$

$$\begin{cases} c_1'(r) = \displaystyle\int_0^\infty V_0(r, r_1)\left(c_1(r_1)\cdot r_1 + c_2(r_1)\right)dr_1; \\ c_2'(r) = -r\displaystyle\int_0^\infty V_0(r, r_1)\left(c_1(r_1)\cdot r_1 + c_2(r_1)\right)dr_1; \end{cases}$$

$$u_0(r) = c_1(r)\cdot r + c_2(r);$$

$$c_1(0) = 1, \quad c_2(0) = 0,$$

$$a_0 = \lim_{r\to\infty}\left(-\frac{c_2(r)}{c_1(r)}\right).$$

An approach without singularities:

$$u_l''(r)+\left(k^2-\frac{l(l+1)}{r^2}-\frac{\frac{2\mu}{\hbar^2}Z_1Z_2e^2}{r}\right)u_l(r)-\int_0^\infty V_l(r,r_1)u_l(r_1)dr_1=0$$

$$u_l(r)=C_1(r)F_l(kr,\eta)+C_2(r)G_l(kr,\eta), \qquad \eta\equiv\frac{\mu Z_1Z_2e^2}{\hbar^2 k}$$

$$\begin{cases} C_1'(r)=\dfrac{G_l(kr,\eta)}{k}\displaystyle\int_0^\infty V_l(r,r_1)\left(C_1(r_1)F_l(kr_1,\eta)+C_2(r_1)G_l(kr_1,\eta)\right)dr_1, \\ C_2'(r)=-\dfrac{F_l(kr,\eta)}{k}\displaystyle\int_0^\infty V_l(r,r_1)\left(C_1(r_1)F_l(kr_1,\eta)+C_2(r_1)G_l(kr_1,\eta)\right)dr_1; \end{cases}$$

$C_1(0)\neq 0$ (in particular, $C_1(0)=1$) and $C_2(0)=0$; $\quad tg(\gamma_l(k))=\lim_{r\to\infty}\dfrac{C_2(r)}{C_1(r)}$.

$\delta_l=\gamma_l+\beta_l$, where β_l is the Coulomb phase shift

An approach without singularities:

$$\frac{2\pi\eta}{\exp(2\pi\eta)-1} k\, ctg(\gamma_0(k)) + 2k\eta\, h(\eta) = -\frac{1}{a_p} + \frac{1}{2} r_{0p} k^2 - P_p r_{0p}^3 k^4 + \dots, \quad \text{(for } l=0\text{)}$$

where $h(\eta) \equiv \eta^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2+\eta^2)} - \ln(\eta) - \gamma$, and $\gamma = 0.5772\dots$ is the Euler constant.

$$\tilde{C}_1(r) \equiv C_1(r)\frac{2\pi\eta}{\exp(2\pi\eta)-1}, \qquad \tilde{C}_2(r) \equiv \lim_{k\to 0}\frac{C_2(r)}{k}, \qquad a_p = -\lim_{r\to\infty}\frac{\tilde{C}_2(r)}{\tilde{C}_1(r)}.$$

$$\begin{cases} \tilde{C}_1'(r) = H_1\left(\frac{r}{R}\right)\int_0^{\infty} V_0(r,r_1)\left(\tilde{C}_1(r_1)\, r_1 L_1\left(\frac{r_1}{R}\right) + \tilde{C}_2(r_1) H_1\left(\frac{r_1}{R}\right)\right) dr_1, \\ \tilde{C}_2'(r) = -r\, L_1\left(\frac{r}{R}\right)\int_0^{\infty} V_0(r,r_1)\left(\tilde{C}_1(r_1)\, r_1 L_1\left(\frac{r_1}{R}\right) + \tilde{C}_2(r_1) H_1\left(\frac{r_1}{R}\right)\right) dr_1; \end{cases}$$

$\tilde{C}_1(0) \neq 0$ (in particular, $\tilde{C}_1(0) = 1$) and $\tilde{C}_2(0) = 0$; $\quad \frac{1}{R} \equiv \frac{2\mu Z_1 Z_2 e^2}{\hbar^2}$;

$$L_1(x) \equiv \frac{1}{\sqrt{x}} I_1\left(2\sqrt{x}\right), \quad H_1(x) \equiv 2\sqrt{x} K_1\left(2\sqrt{x}\right),$$

where I_1 and K_1 are the modified Bessel functions.

Variational method

$$\Phi_3 = \hat{S}\sum_{k=1}^{K} D_k \varphi_k \equiv \hat{S}\sum_{k=1}^{K} D_k \exp\left(-a_k(\mathbf{r}_1-\mathbf{r}_2)^2 - b_k(\mathbf{r}_1-\mathbf{r}_3)^2 - c_k(\mathbf{r}_2-\mathbf{r}_3)^2\right)$$

$$\Phi_4 = \hat{S}\sum_{k=1}^{K} D_k \varphi_k \equiv \hat{S}\sum_{k=1}^{K} D_k \exp\left(-a_k r_{12}^2 - b_k r_{13}^2 - c_k r_{14}^2 - d_k r_{23}^2 - e_k r_{24}^2 - f_k r_{34}^2\right)$$

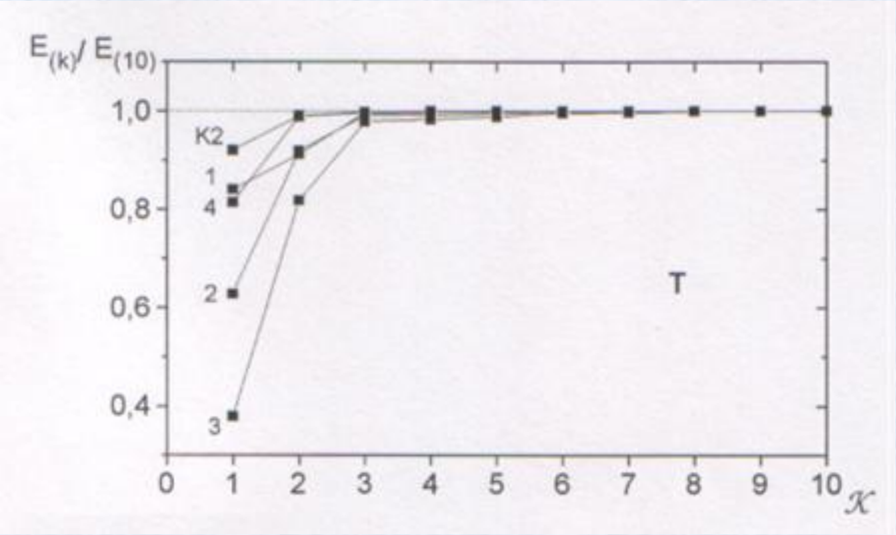
$$\Phi_5 = \hat{S}\sum_{k=1}^{K} D_k \varphi_k \equiv \hat{S}\sum_{k=1}^{K} D_k \exp\left(-\sum_{j>i=1}^{5} a_{k(ij)} r_{ij}^2\right)$$

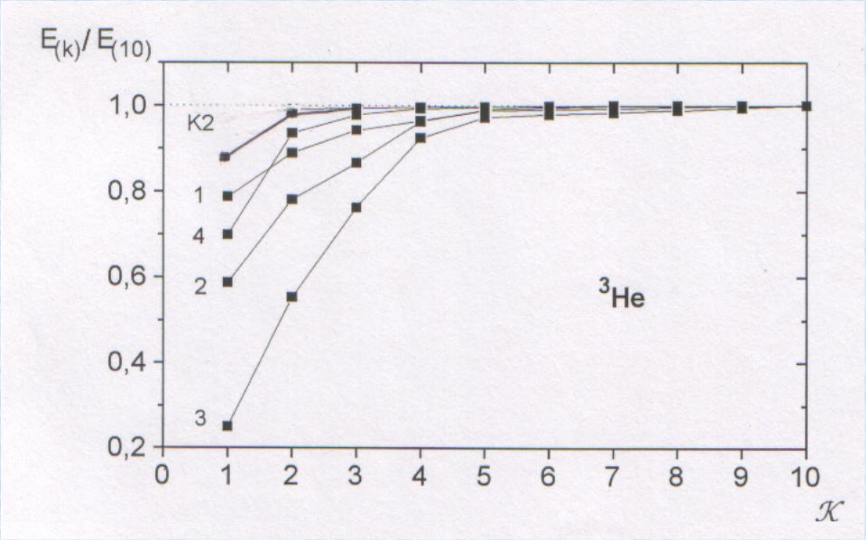
$$\sum_m D_m\left(\langle \hat{S}\varphi_k | \hat{H} | \hat{S}\varphi_m \rangle - E\langle \hat{S}\varphi_k | \hat{S}\varphi_m \rangle\right) = 0, \qquad k, m = 1, 2, \dots, K$$

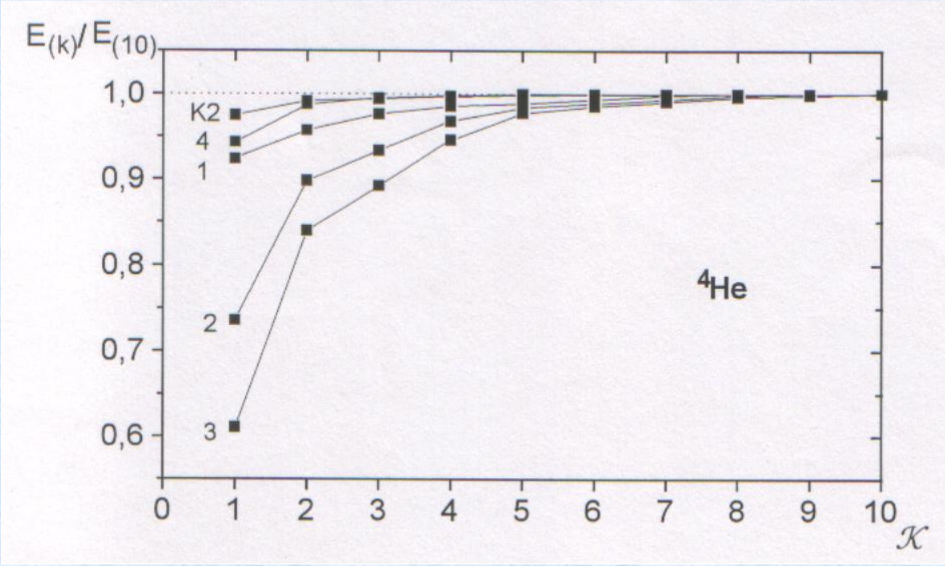
Kukulin V. I. and Krasnopol'sky V. M. **A Stochastic Variational Method for Few-Body Systems // J. Phys. G: Nucl. Phys. - 1977. - Vol. 3, No. 6. - P. 795 - 811.**

Suzuki Y., Varga K. **Stochastic Variational Approach to Quantum Mechanical Few-Body Problems // Springer-Verlag Berlin Heidelberg, 1998.**

Convergence







Root mean square (r.m.s.) radii and r.m.s. relative distances

$$\rho_n(r) = \langle \Phi | \delta(\mathbf{r} - (\mathbf{r}_n - \mathbf{R}_{c.m.})) | \Phi \rangle, \qquad R_n = \left(\int r^2 \rho_n(r) d\mathbf{r} \right)^{\frac{1}{2}};$$

$$\rho_{\alpha c.m.}(r) = \langle \Phi | \delta(\mathbf{r} - (\mathbf{r}_\alpha - \mathbf{R}_{c.m.})) | \Phi \rangle, \qquad R_\alpha = \left(\int r^2 \rho_{\alpha c.m.}(r) d\mathbf{r} \right)^{\frac{1}{2}};$$

$$g_{nn}(r) = \langle \Phi | \delta(\mathbf{r} - (\mathbf{r}_{n_1} - \mathbf{r}_{n_2})) | \Phi \rangle; \qquad g_{n\alpha}(r) = \langle \Phi | \delta(\mathbf{r} - (\mathbf{r}_n - \mathbf{r}_\alpha)) | \Phi \rangle;$$

$$r_{nn} = \left(\int r^2 g_{nn}(r) d\mathbf{r} \right)^{\frac{1}{2}}; \qquad r_{n\alpha} = \left(\int r^2 g_{n\alpha}(r) d\mathbf{r} \right)^{\frac{1}{2}}.$$

$$R_i^2 = \frac{1}{M^2} \left((M - m_i) \sum_{j \neq i} m_j r_{ij}^2 - \sum_{\substack{j<k \\ (j \neq i,\, k \neq i)}} m_j m_k r_{jk}^2 \right)$$

R.m.s. radii and relative distances

^{14}C
$$n_{\rm ch}(r) = \int n_\alpha(|\mathbf{r}-\mathbf{r}'|)\, n_{{\rm ch},{}^4He}(r')\, d\mathbf{r}'$$

$$R^2_{ch} = R^2_\alpha + R^2_{ch}({}^4He)$$

^{14}O
$$n_{\rm ch}(r) = \frac{3}{4}\int n_\alpha(|\mathbf{r}-\mathbf{r}'|)\, n_{{\rm ch},{}^4He}(r')\, d\mathbf{r}' + \frac{1}{4}\int n_p(|\mathbf{r}-\mathbf{r}'|)\, n_{{\rm ch},p}(r')\, d\mathbf{r}'$$

$$R^2_{ch} = \frac{3}{4}\left(R^2_\alpha + R^2_{ch}({}^4He)\right) + \frac{1}{4}\left(R^2_p + R^2_{ch}(p)\right)$$

	r_{NN}	$r_{N\alpha}$	$r_{\alpha\alpha}$	R_N	R_α	R_m	R_{ch}
^{14}C	2.621	2.667	3.189	1.786	1.852	2.493	2.500
^{14}O	2.732	2.750	3.239	1.864	1.882	2.520	2.415

R.m.s. radii and relative distances for ^{14}N

$$n_{\rm ch}(r) = \frac{6}{7}\int n_\alpha(|\mathbf{r}-\mathbf{r}'|)\, n_{{\rm ch},{}^4{\rm He}}(r')\, d\mathbf{r}' + \\ + \frac{1}{7}\int n_p(|\mathbf{r}-\mathbf{r}'|)\, n_{{\rm ch},p}(r')\, d\mathbf{r}'$$

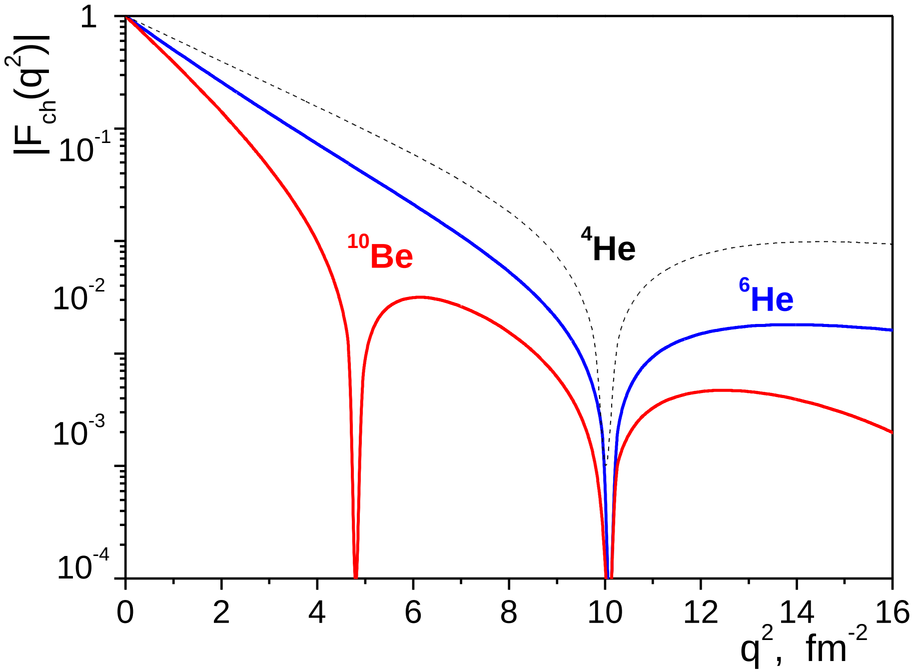
$$R^2_{ch} = \frac{6}{7}\left(R^2_\alpha + R^2_{ch}\left({}^4He\right)\right) + \frac{1}{7}\left(R^2_p + R^2_{ch}(p)\right)$$

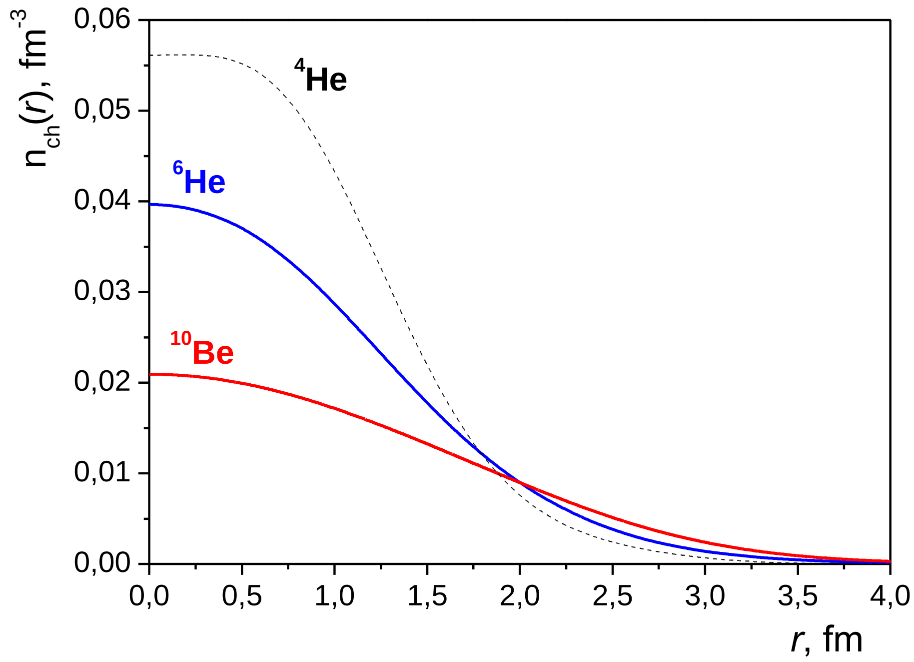
r_{pn}	$r_{p\alpha}$	$r_{n\alpha}$	$r_{\alpha\alpha}$	R_p	R_n	R_α	R_m	$R_{\rm ch}$
2.237	2.692	2.683	3.559	1.598	1.585	2.064	2.556	2.558

Charge density distributions and formfactors of ^{6}He and ^{10}Be nuclei

$$n_{ch,\ ^{10}Be}(r) = \int n_{\alpha\, c.m.}(\mathbf{r}-\mathbf{r}')\, n_{ch,\ ^4He}(r')\, d\mathbf{r}'$$

$$F_{ch}(q) = \int e^{-i(\mathbf{qr})} n_{ch}(r)\, d\mathbf{r}, \qquad F_{ch,\ ^{10}Be}(q) \cong F_{\alpha\, \mathrm{c.m.}}(q) \cdot F_{ch,\ ^4He}(q)$$

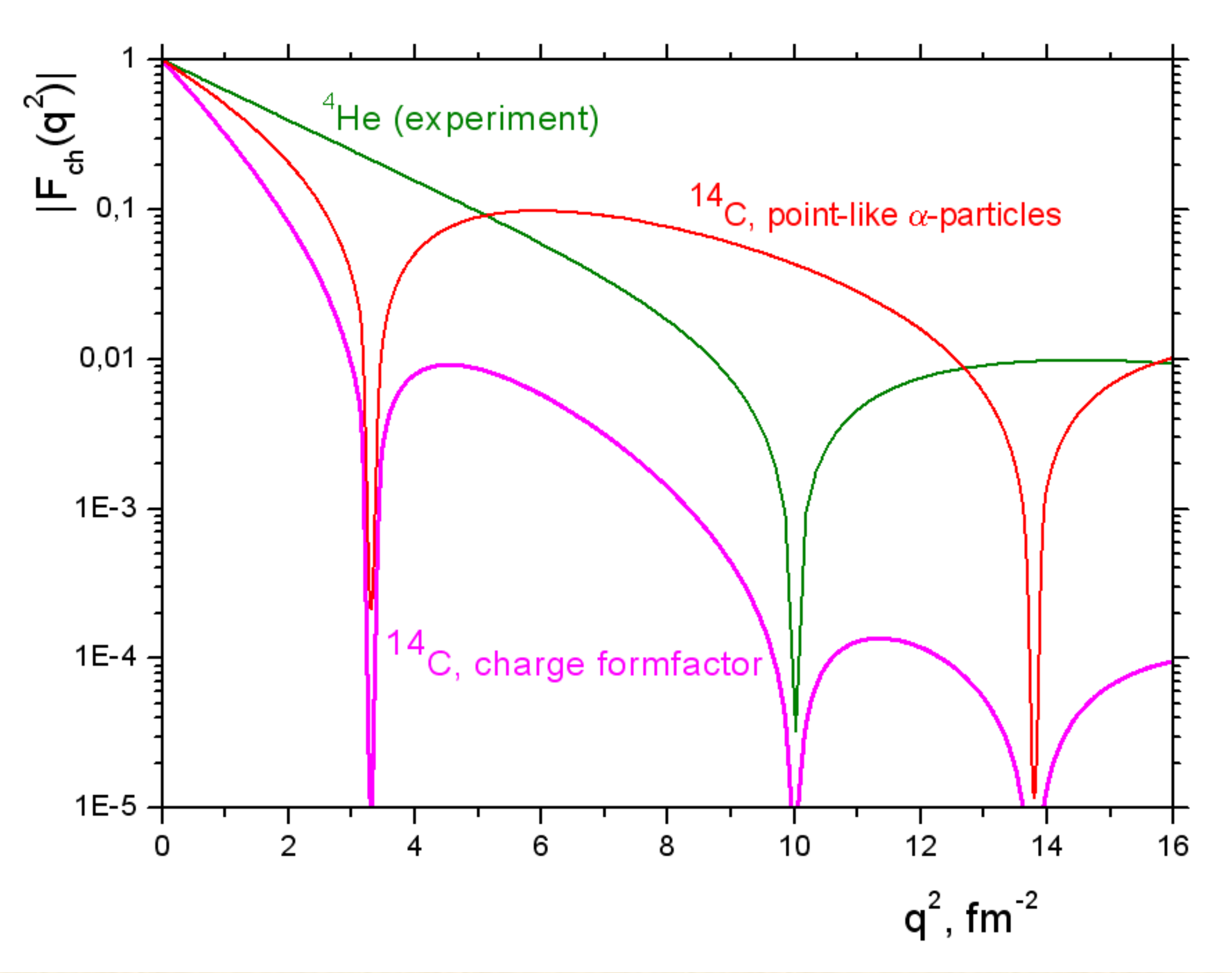




Charge formfactor of ^{14}C nucleus

$$n_{ch,\,^{14}C}(r) = \int n_{\alpha\, c.m.}(\mathbf{r}-\mathbf{r}')\, n_{ch,\,^4He}(r')\, d\mathbf{r}'$$

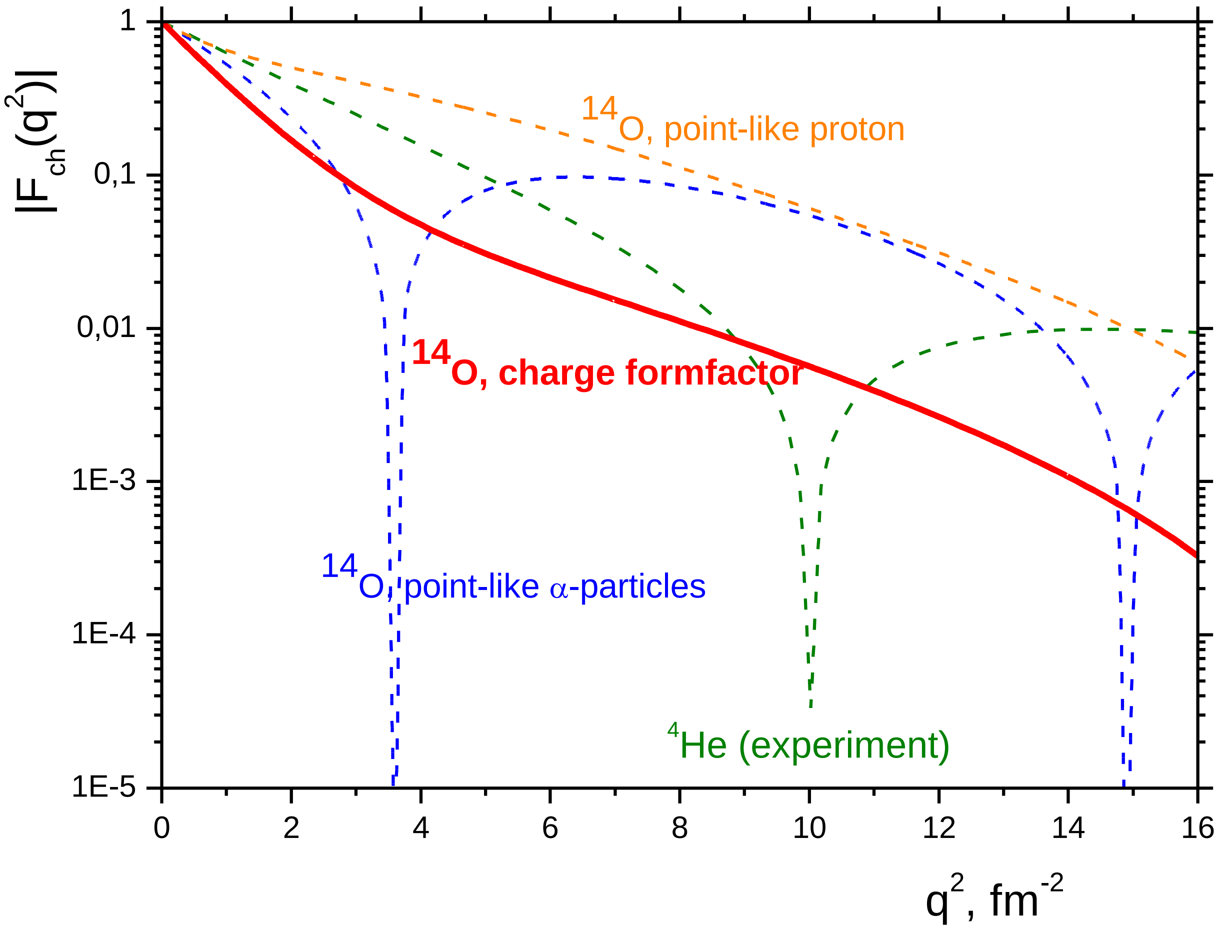
$$F_{ch}(q) = \int e^{-i(\mathbf{qr})} n_{ch}(r)\, d\mathbf{r}, \qquad F_{ch,\,^{14}C}(q) \cong F_{\alpha\, c.m.}(q)\cdot F_{ch,\,^4He}(q)$$



Charge formfactor of ^{14}O nucleus

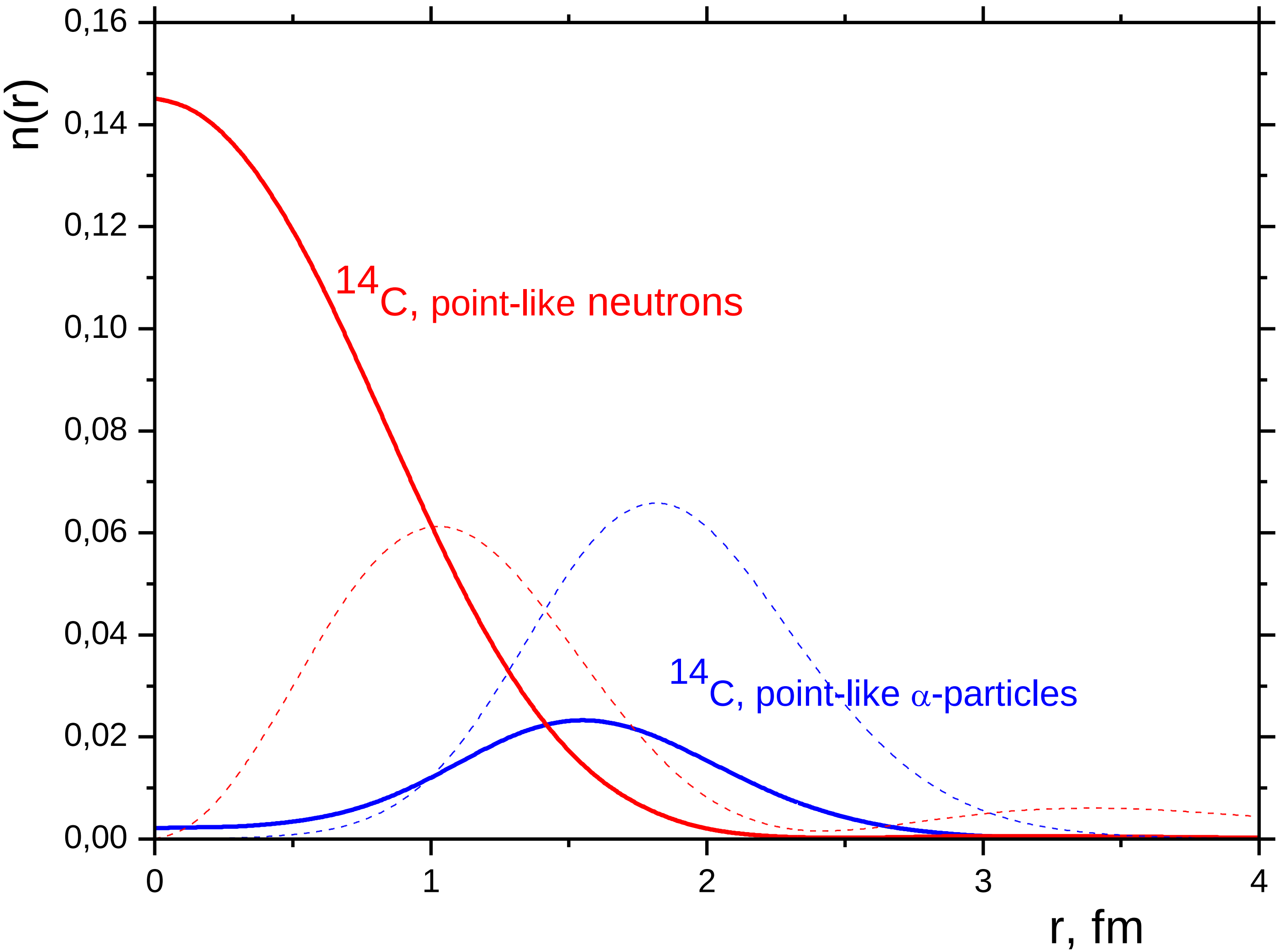
$$n_{\text{ch}}(r) = \frac{3}{4}\int n_\alpha(|\mathbf{r}-\mathbf{r}'|)\, n_{\text{ch},{}^4\text{He}}(r')\, d\mathbf{r}' + \frac{1}{4}\int n_p(|\mathbf{r}-\mathbf{r}'|)\, n_{\text{ch},p}(r')\, d\mathbf{r}'$$

$$F_{\text{ch},{}^{14}\text{O}}(q) = \frac{3}{4}F_{\alpha,{}^{14}\text{O}}(q)\, F_{\text{ch},{}^4\text{He}}(q) + \frac{1}{4}F_{p,{}^{14}\text{O}}(q)\, F_{\text{ch},p}(q)$$



Density distributions in ^{14}C nucleus

$$n_i(r) = \langle \Phi | \, \delta\left(\mathbf{r} - (\mathbf{r}_i - \mathbf{R}_{\mathrm{c.m.}})\right) | \Phi \rangle$$



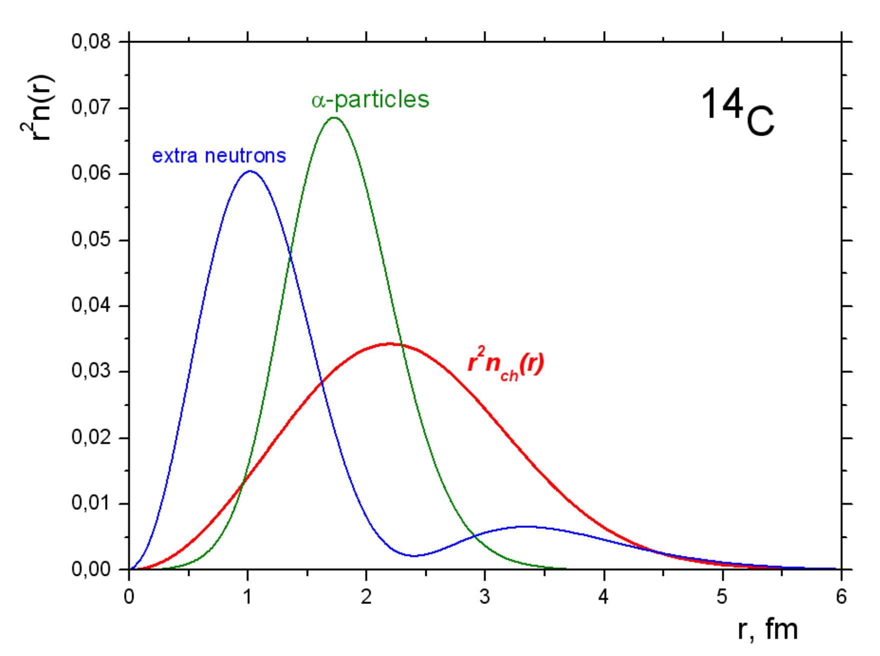
Charge density distribution

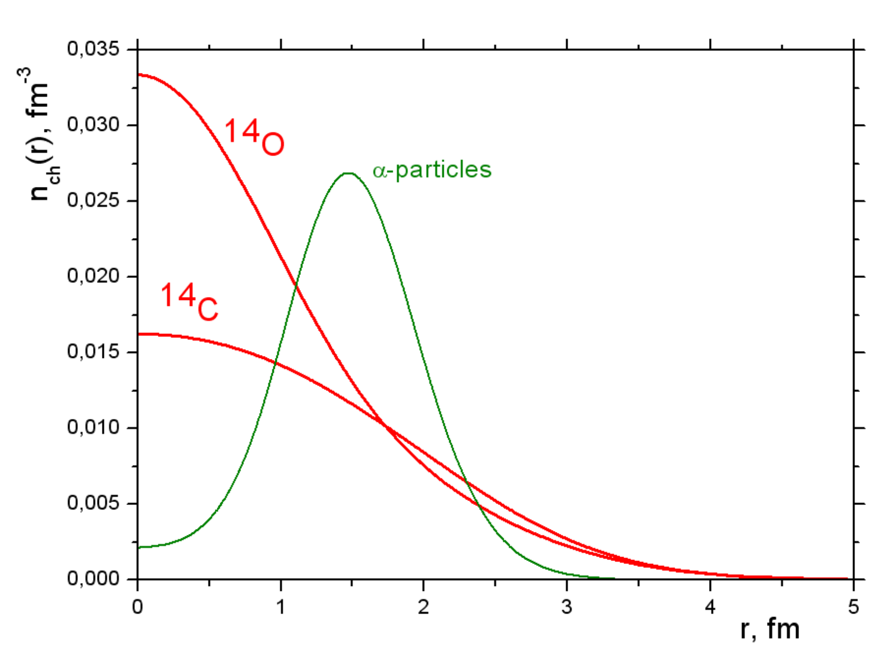
^{14}C:

$$n_{\mathrm{ch}}(r) = \int n_\alpha(|\mathbf{r}-\mathbf{r}'|)\, n_{\mathrm{ch},{}^4\mathrm{He}}(r')\, d\mathbf{r}'$$

^{14}O:

$$n_{\mathrm{ch}}(r) = \frac{3}{4}\int n_\alpha(|\mathbf{r}-\mathbf{r}'|)\, n_{\mathrm{ch},{}^4\mathrm{He}}(r')\, d\mathbf{r}' + \frac{1}{4}\int n_p(|\mathbf{r}-\mathbf{r}'|)\, n_{\mathrm{ch},p}(r')\, d\mathbf{r}'$$

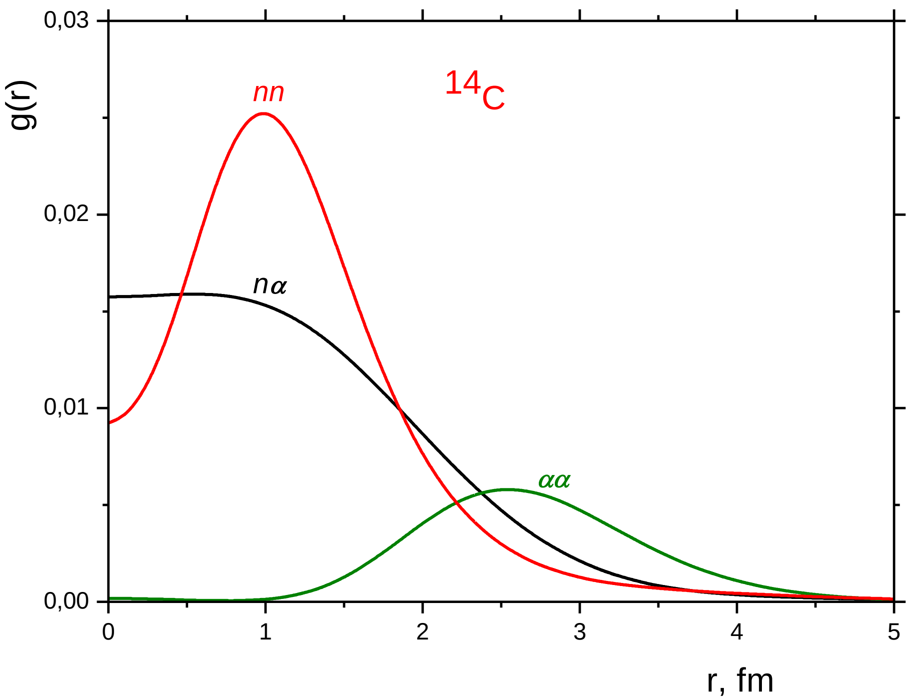


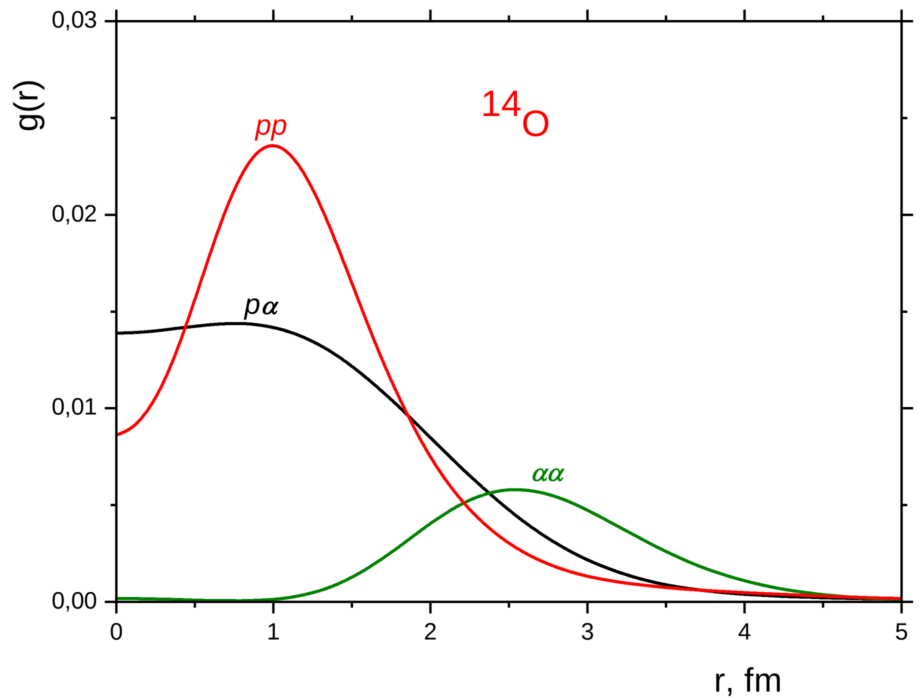


Pair correlation functions

$$g_{nn}(r)=\langle\Phi|\delta(\mathbf{r}-(\mathbf{r}_{n_1}-\mathbf{r}_{n_2}))|\Phi\rangle;\quad g_{n\alpha}(r)=\langle\Phi|\delta(\mathbf{r}-(\mathbf{r}_{n}-\mathbf{r}_{\alpha}))|\Phi\rangle;\quad g_{\alpha\alpha}(r)=\langle\Phi|\delta(\mathbf{r}-(\mathbf{r}_{\alpha_1}-\mathbf{r}_{\alpha_2}))|\Phi\rangle;$$

$$r_{nn}=\left(\int r^2 g_{nn}(r)\,d\mathbf{r}\right)^{\frac{1}{2}};\qquad r_{n\alpha}=\left(\int r^2 g_{n\alpha}(r)\,d\mathbf{r}\right)^{\frac{1}{2}};\qquad r_{\alpha\alpha}=\left(\int r^2 g_{\alpha\alpha}(r)\,d\mathbf{r}\right)^{\frac{1}{2}}.$$





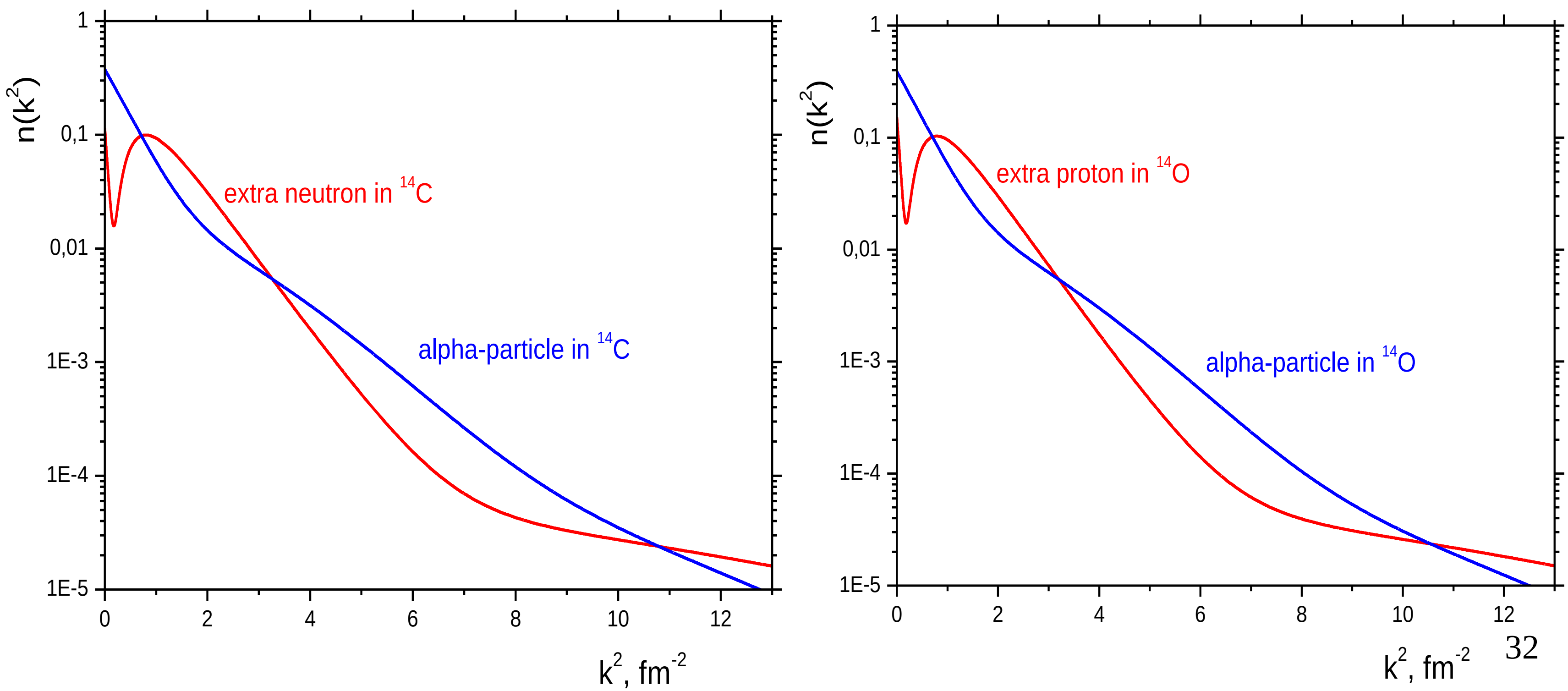
Momentum distributions

$$n_n(k) = \left\langle \tilde{\Phi} \left| \frac{1}{2} \sum_{i=1}^{2} \delta\left(\mathbf{k} - \left(\mathbf{k}_i - \mathbf{K}_{c.m.}\right)\right) \right| \tilde{\Phi} \right\rangle , \quad \left\langle E_{kin} \right\rangle_n = \int \frac{k^2}{2m_n} n_n(k) d\mathbf{k} ,$$

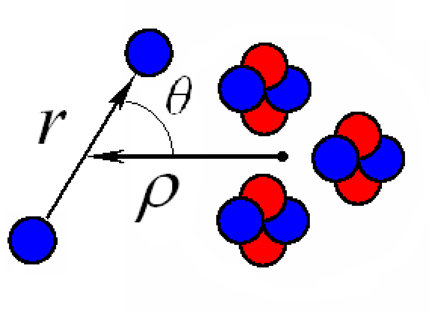
$$n_\alpha(k) = \left\langle \tilde{\Phi} \left| \frac{1}{3} \sum_{i=3}^{5} \delta\left(\mathbf{k} - \left(\mathbf{k}_i - \mathbf{K}_{c.m.}\right)\right) \right| \tilde{\Phi} \right\rangle , \quad \left\langle E_{kin} \right\rangle_\alpha = \int \frac{k^2}{2m_\alpha} n_\alpha(k) d\mathbf{k}$$

$$^{14}\mathrm{C}: \quad \left\langle E_{kin,n} \right\rangle \cong 32.66 \text{ MeV}, \quad \left\langle E_{kin,\alpha} \right\rangle \cong 6.83 \text{ MeV}$$

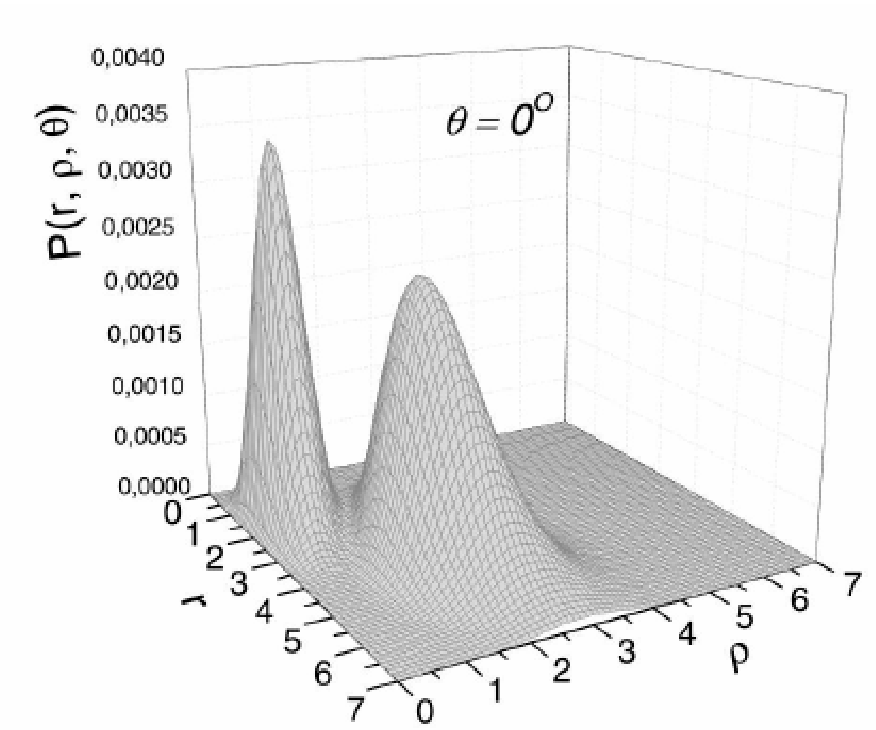
$$^{14}\mathrm{O}: \quad \left\langle E_{kin,n} \right\rangle \cong 31.77 \text{ MeV}, \quad \left\langle E_{kin,\alpha} \right\rangle \cong 6.62 \text{ MeV}$$

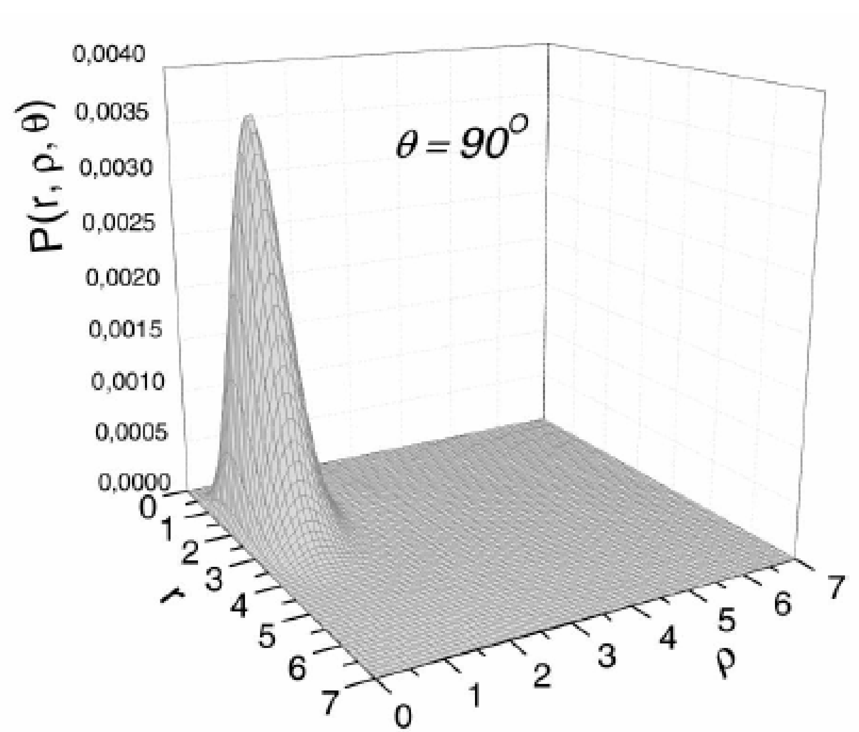


Probability density *P(r,ρ,θ)*

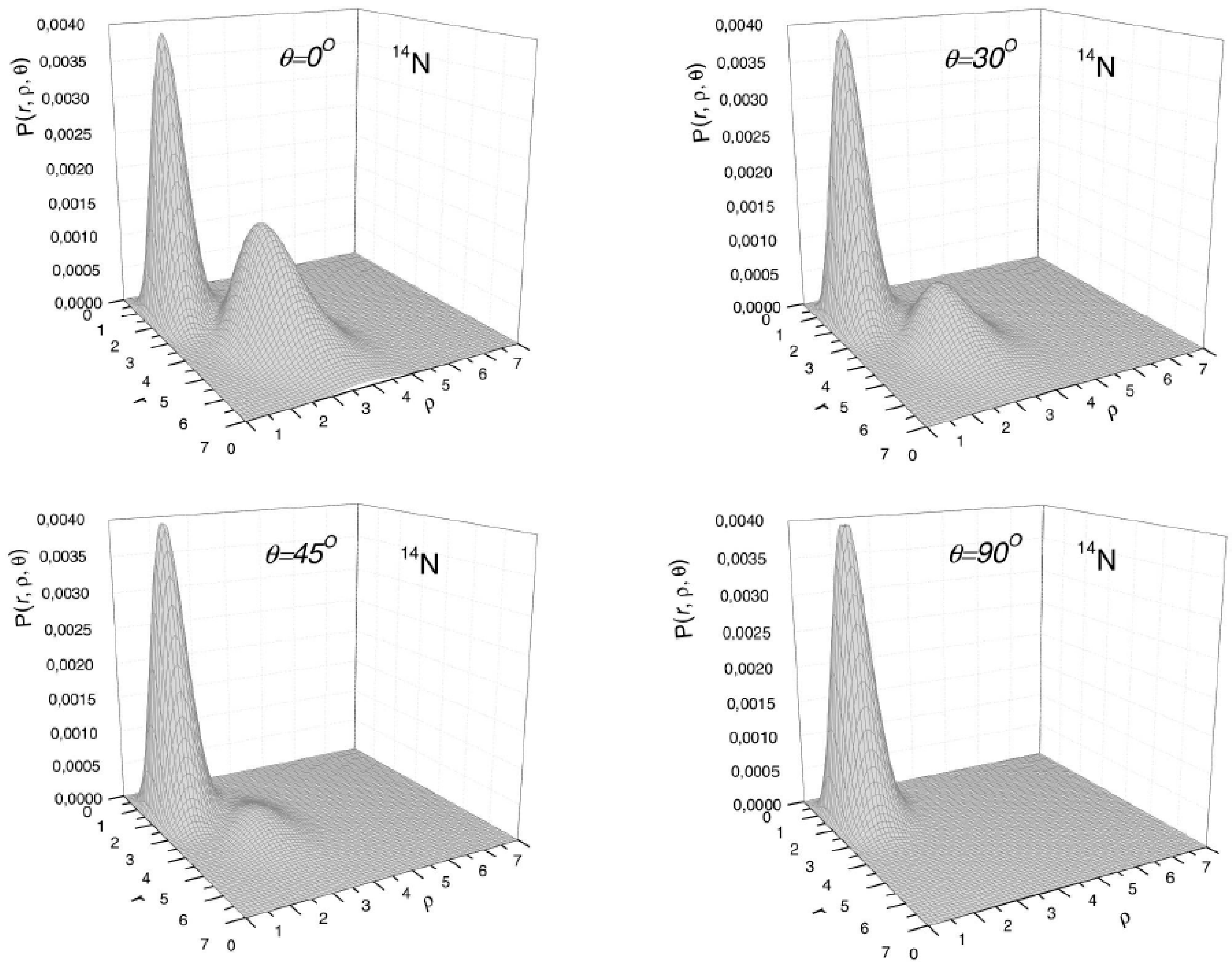


$$P(r,\rho,\theta) \equiv r^2 \rho^2 \langle \Phi | \delta(\mathbf{r} - \mathbf{r}_{nn}) \delta(\boldsymbol{\rho} - \boldsymbol{\rho}_{nn-\alpha\alpha\alpha}) | \Phi \rangle$$



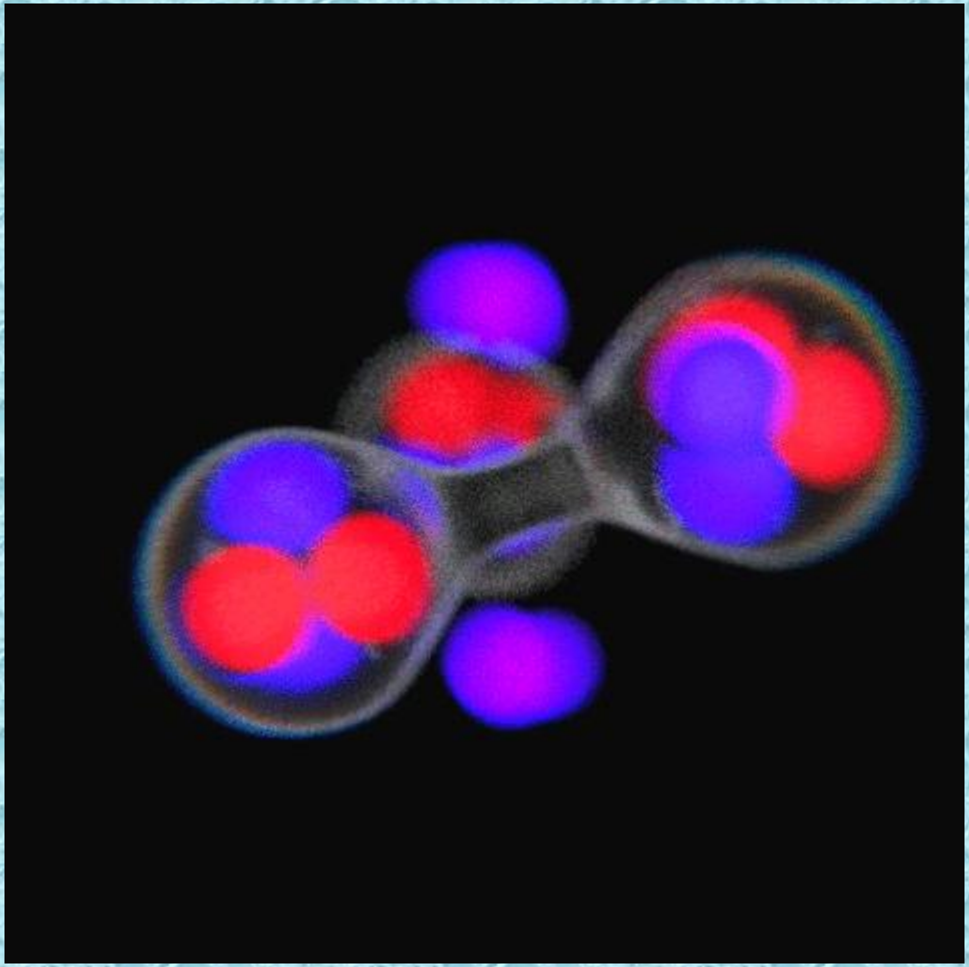


Probability density $P(r,\rho,\theta)$ for ^{14}N

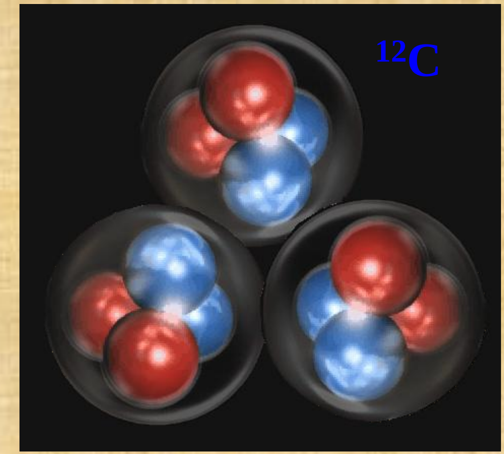


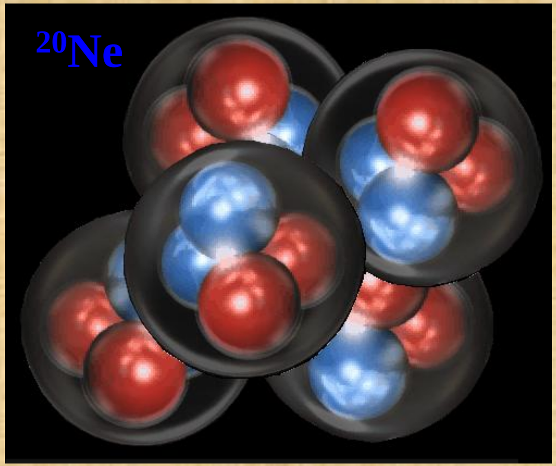
Two configurations in the ground state of ^{14}N nucleus manifesting themselves in the $P(r,\rho,\theta)$ function at different angles θ

A schematic model of ^{14}C or ^{14}O nucleus



Form factors of ^{12}C, ^{16}O, and ^{20}Ne nuclei within the α-cluster model

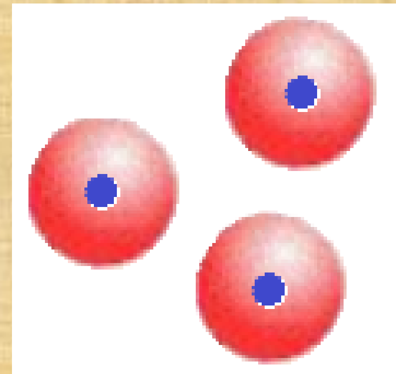


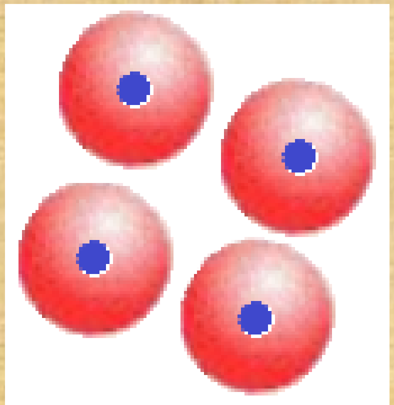


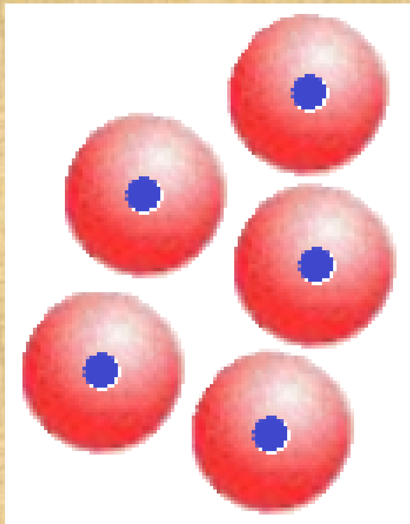
$$E_{g.s.} = -28.29567 \text{ MeV}$$

$$m_\alpha = 3.7273794 \text{ GeV}/c^2$$

$$R_{\alpha,ch} = 1.679 \text{ fm}$$







Statement of the problem

$$\hat{H} = -\frac{\hbar^2}{2m_\alpha}\sum_{k=1}^{N}\Delta_k + \sum_{k>n=1}^{N}\left(\hat{V}_{kn} + \frac{4e^2}{r_{kn}}\right)$$

$$\hat{V}(r) = U(r) + g\,|u(r)\rangle\langle u(\ldots)| = U(r) + g u(r)\int u(r_1)\ldots dr_1$$

Nucleus	E, MeV	R_{ch}, fm
^{12}C	– 7.2748	2.470
^{16}O	– 14.4368	2.706
^{20}Ne	– 19.1668	3.005

A few words about Helm approximation

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Inelastic and Elastic Scattering of 187-Mev Electrons from Selected Even-Even Nuclei*

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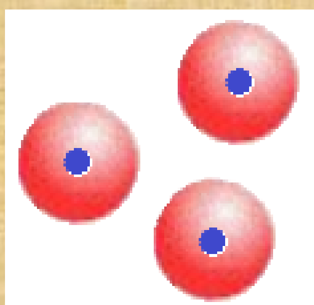
(Received August 27, 1956)

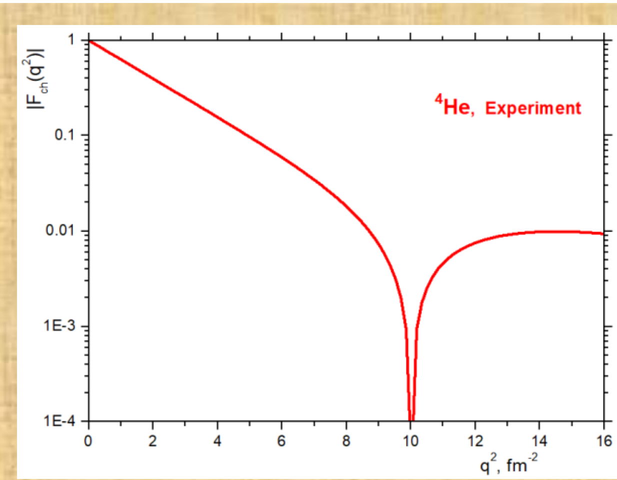
$$n_i\left(r\right) = \left\langle \Phi \right| \delta\left(\mathbf{r} - \left(\mathbf{r}_i - \mathbf{R}_{\mathrm{c.m.}}\right)\right) \left|\Phi\right\rangle$$



$$n_{ch}\left(r\right) = \int n_{\alpha}\left(r'\right) n_{ch,{}^4He}\left(\left|\mathbf{r} - \mathbf{r}'\right|\right) d\mathbf{r}'$$

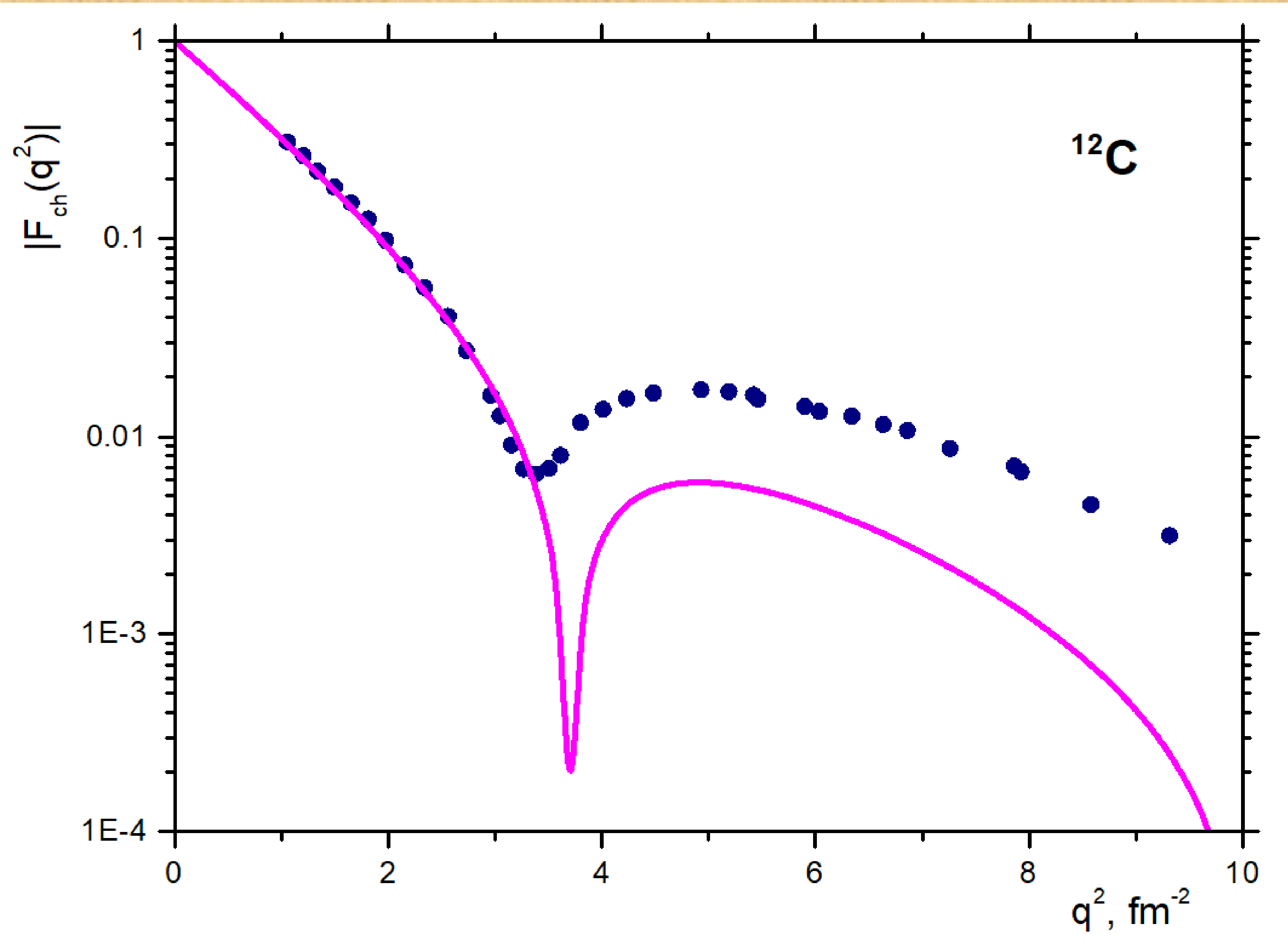
$$F_{ch}\left(q\right) = \int e^{-i(\mathbf{q}\cdot\mathbf{r})} n_{ch}\left(r\right) d\mathbf{r} = F_{\alpha}\left(q\right) \cdot F_{ch,{}^4He}\left(q\right)$$

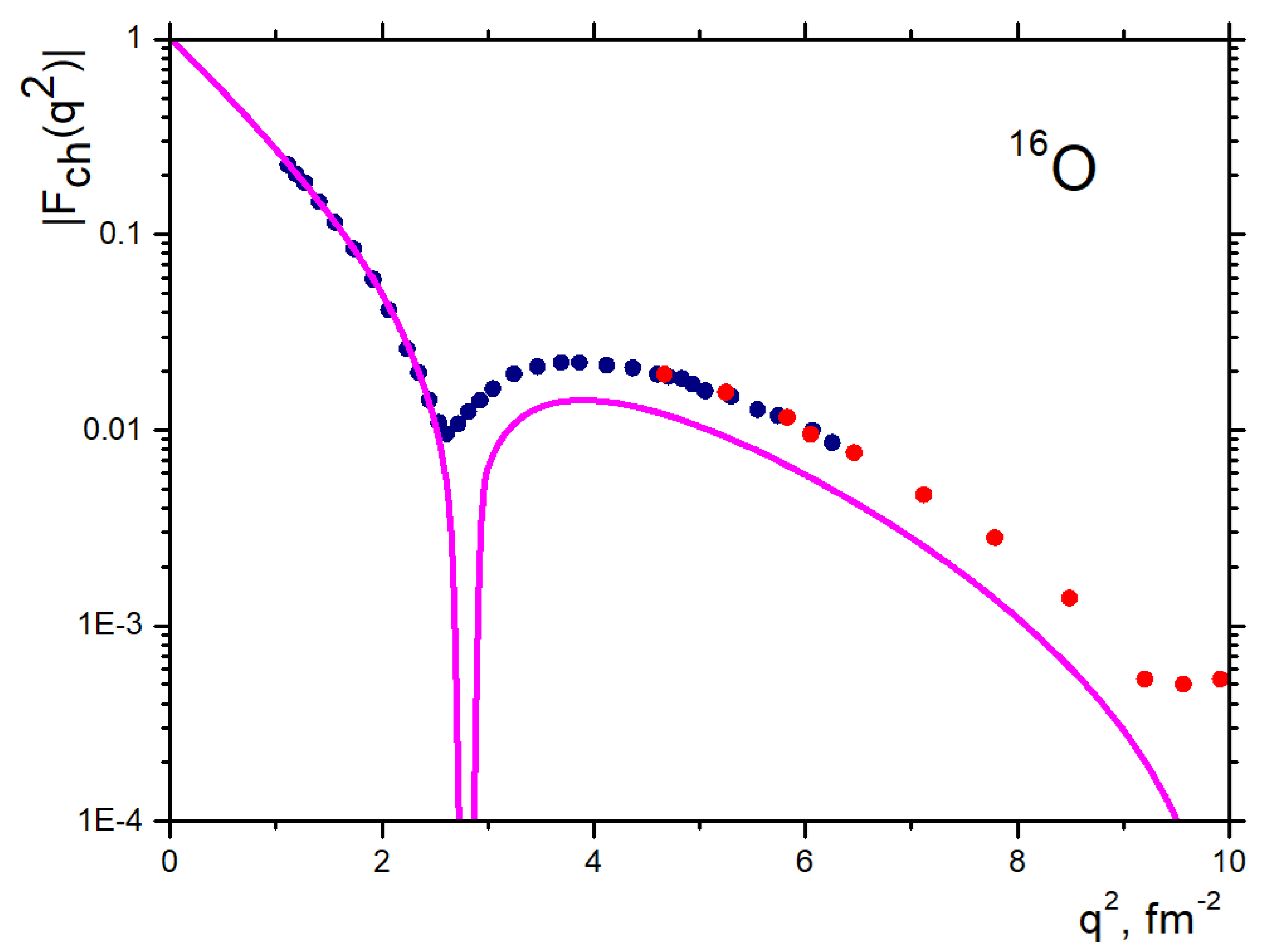
$$R_{ch}^2 = R_{\alpha}^2 + R_{ch,{}^4He}^2$$



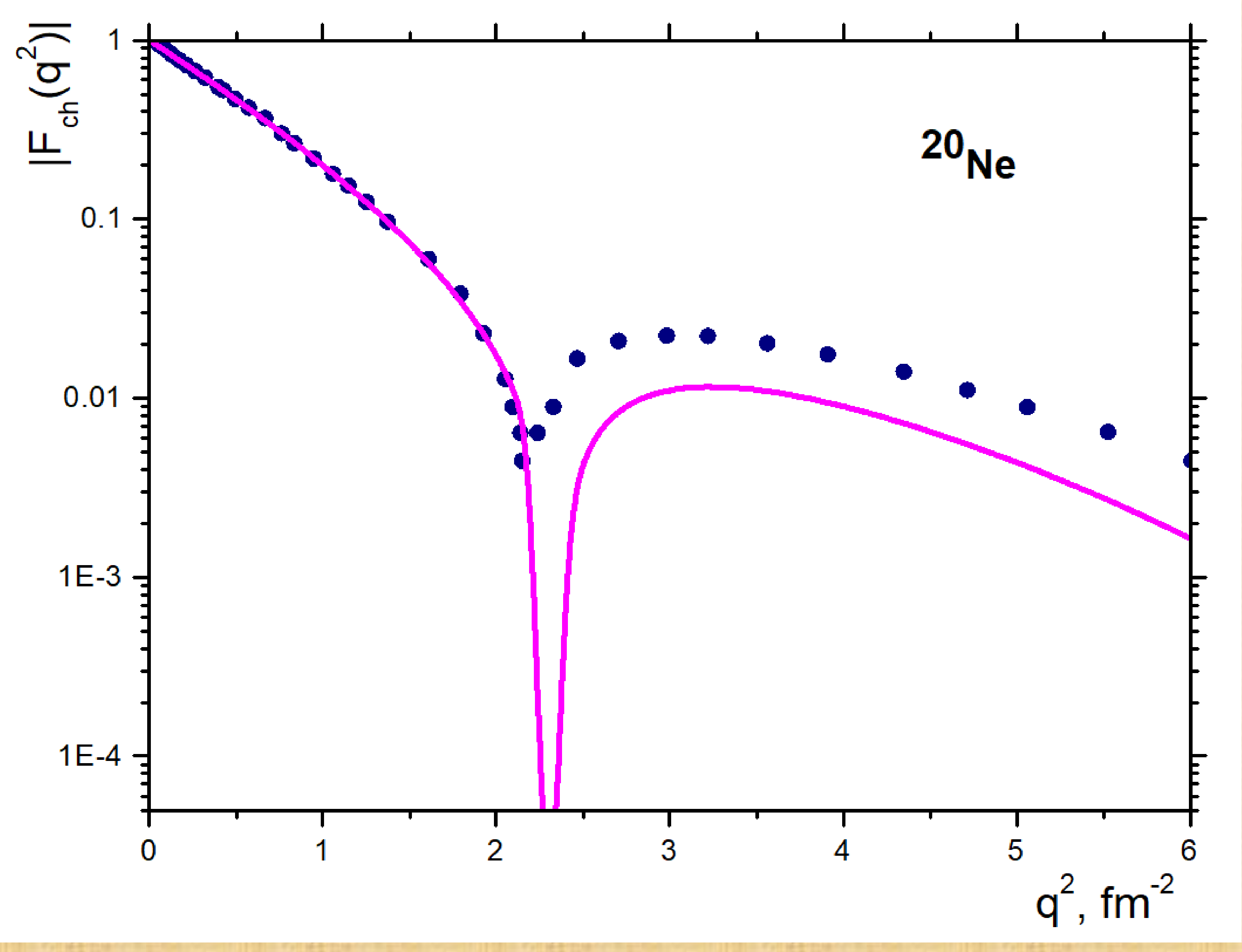
$$R_{ch}^2 = \int r^2 n_{ch}\left(r\right) d\mathbf{r} = -6 \left.\frac{dF_{ch}\left(q\right)}{d\left(q^2\right)}\right|_{q\to 0}$$

Elastic form factors of ^{12}C, ^{16}O, and ^{20}Ne in Helm approximation



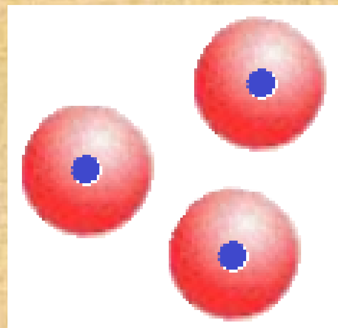


$|F_{ch}(q^2)|$
^{16}O
1
0.1
0.01
1E-3
1E-4
0
2
4
6
8
10
q^2, fm^{-2}

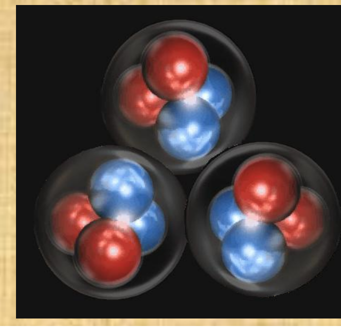


$|F_{ch}(q^2)|$
1
0.1
0.01
1E-3
1E-4
^{20}Ne
0
1
2
3
4
5
6
q^2, fm^{-2}

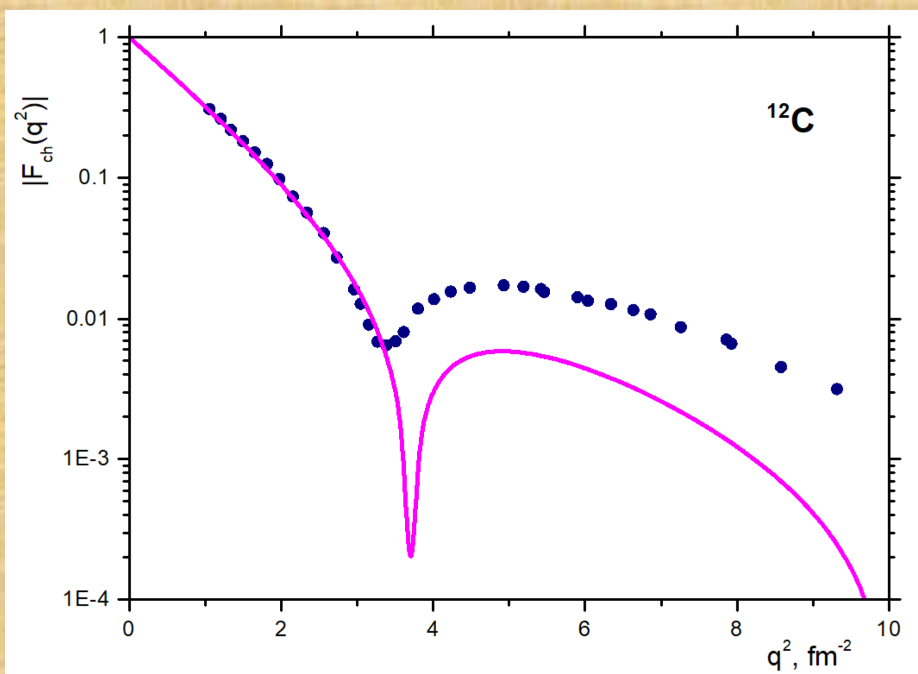
The role of the protons exchange effects and elastic form factor of ^{12}C

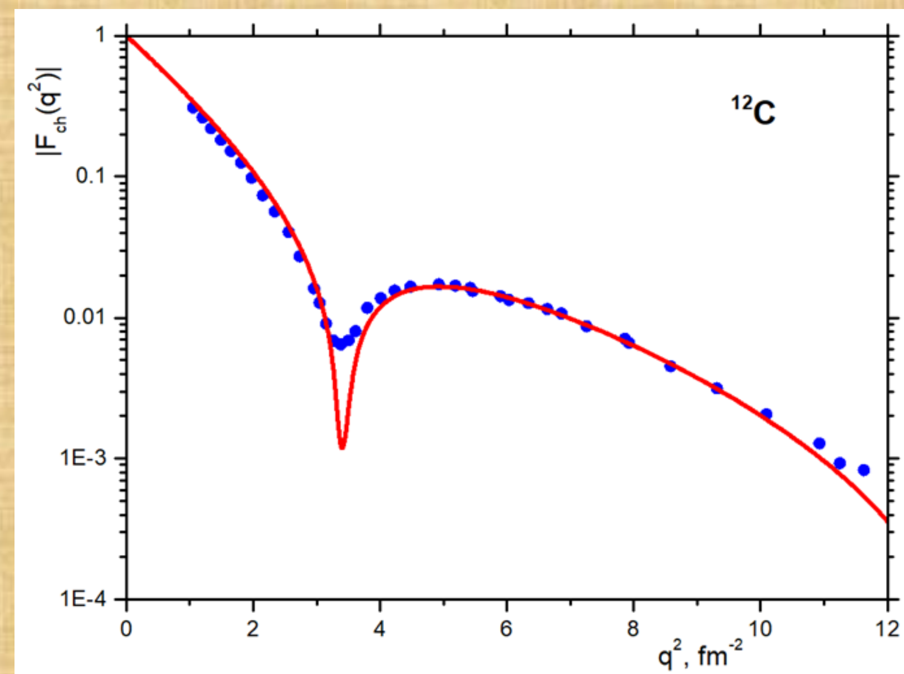


$$\Psi\left(\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3\right)$$

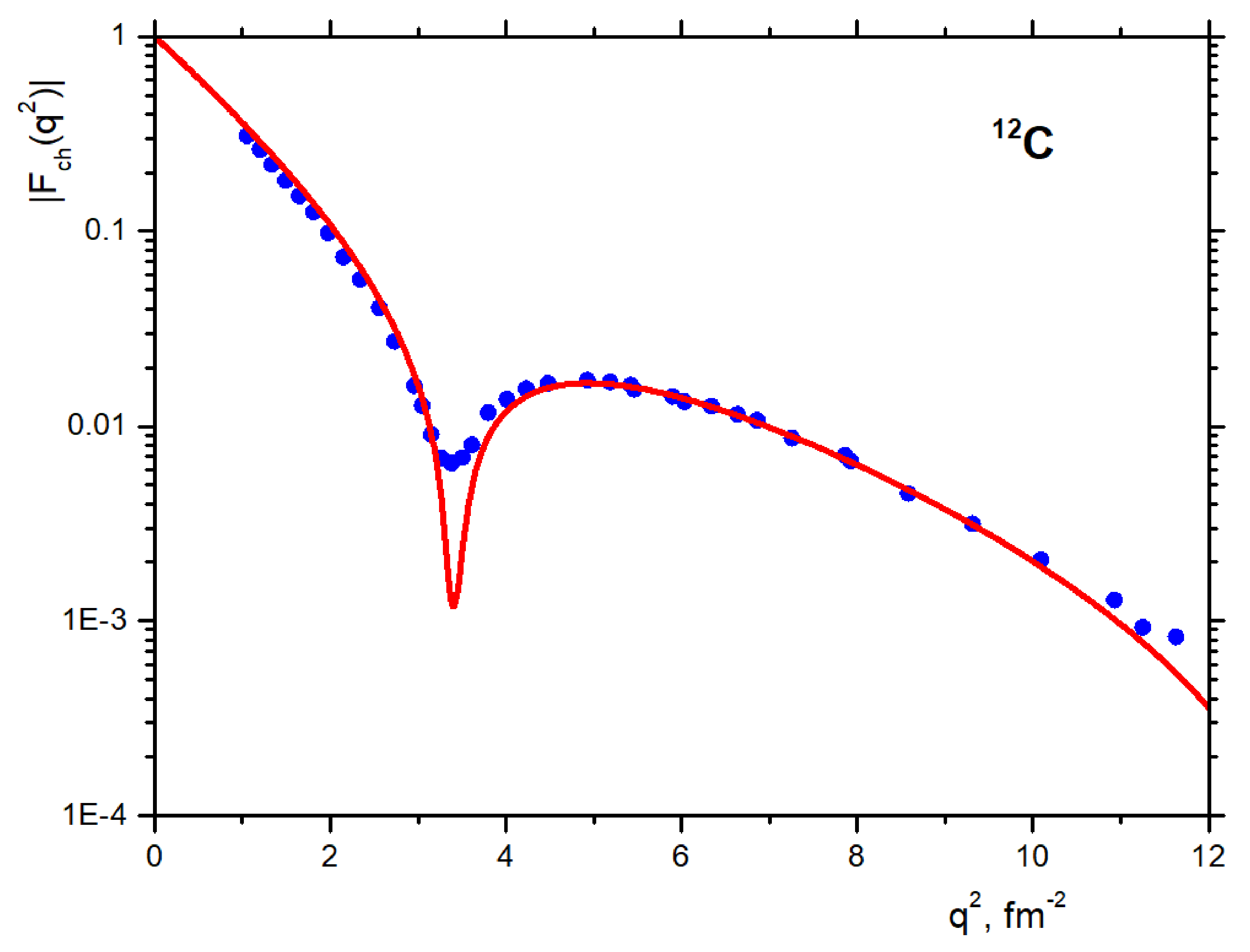


$$\Psi\left(\mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3\right) \times$$
$$\times \hat{A}\, \Phi_{^4He}(1,2,3,4)\, \Phi_{^4He}(5,6,7,8)\, \Phi_{^4He}(9,10,11,12)$$





Form factor of ^{12}C nucleus with an account of the proton exchange



Conclusions

1. *NN*-, *nα*-, *pα*- and *αα*- interaction potentials are proposed in concordance with the energies of ^{14}C and ^{14}O, and with r.m.s. charge radius of ^{14}C
2. Within a five-particle model, the wave functions of ^{14}C, ^{14}N, and ^{14}O nuclei are found in Gaussian representation using the variational method
4. Density distributions of extra nucleons as well as of *α*-particles are found, and r.m.s. radii and r.m.s. relative distances are calculated
5. The charge density distributions and charge form factors are found in Helm approximation
6. The charge radius of ^{14}O nucleus is predicted (2.415 fm)
7. The pair correlation functions are calculated
8. The momentum distributions are found and analyzed
9. Two spatial configurations are revealed in the ground state of all the considered nuclei with two extra nucleons
10. The form factors of ^{12}C, ^{16}O, and ^{20}Ne nuclei are calculated within the Helm approximation
11. The description of form factors with high precision needs some correction of the well-known Helm approximation

THANK YOU !