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A negative index Josephson metamaterial

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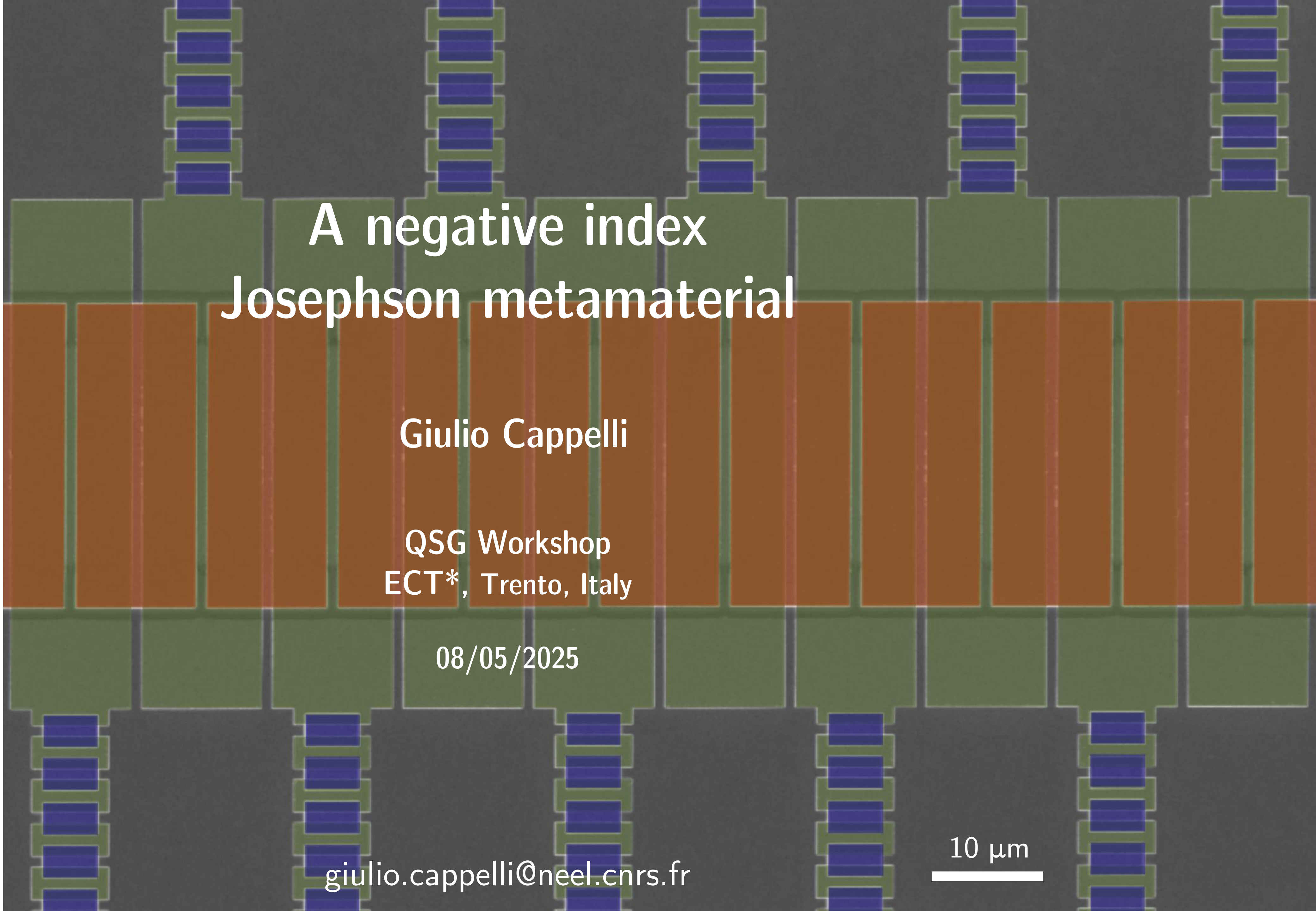
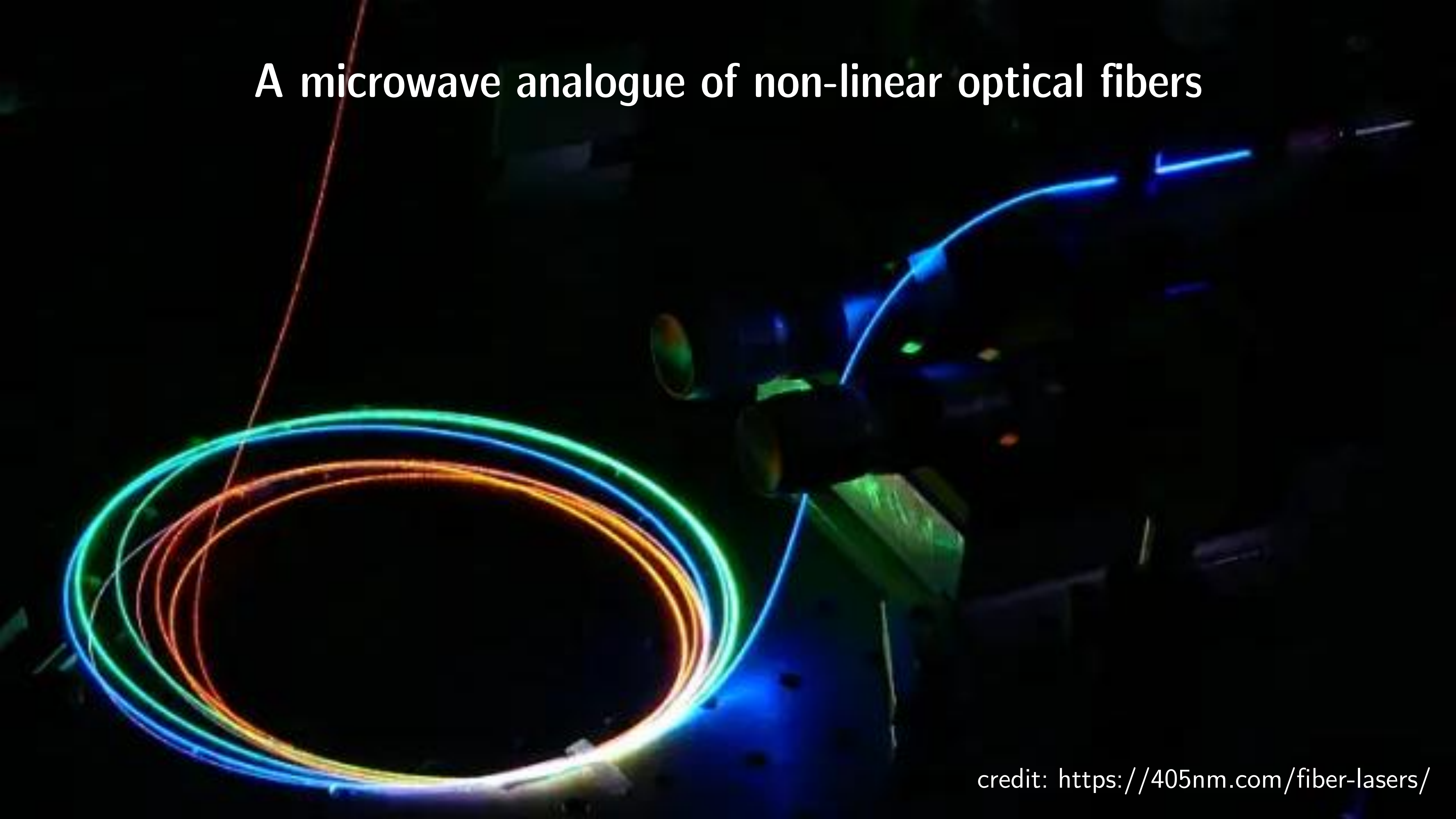
QSG Workshop
ECT*, Trento, Italy

08/05/2025



giulio.cappelli@neel.cnrs.fr

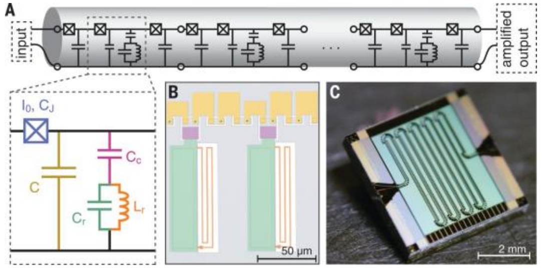
A microwave analogue of non-linear optical fibers



credit: https://405nm.com/fiber-lasers/

Josephson metamaterials: applications

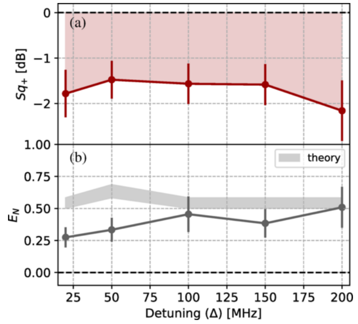
1. Traveling wave parametric amplifiers



For a review see:
→ Esposito *et al.* **Appl. Phys. Lett. (2021)**

→ Macklin *et al.* **Science (2015)**

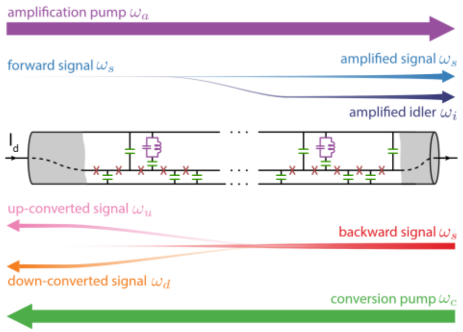
2. Broadband multi-mode entanglement generation



See also:
→ Perelshtein *et al.* **Phys. Rev. Applied (2022)**
→ Qiu *et al.* **Nat. Phys. (2023)**

→ Esposito *et al.* **Phys. Rev. Lett (2022)**

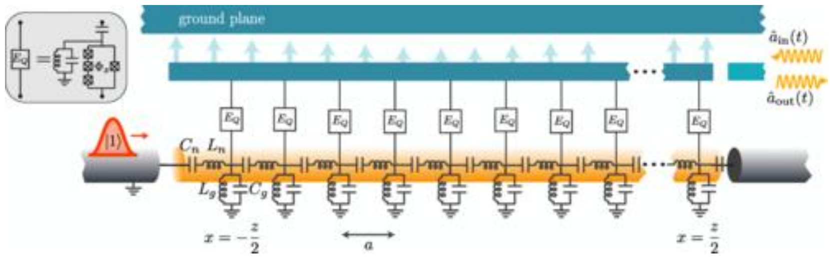
3. Non-reciprocal traveling wave devices



See also:
→ Praquin *et al.* **arXiv:2406,19751 (2024)**
→ Ranadive *et al.* **arXiv:2406,19752 (2024)**

→ Malnou *et al.* **arXiv:2406,19476 (2024)**

4. Traveling wave single-photon counters



→ Grimsmo *et al.* **Phys. Rev. Applied (2021)**

Outline

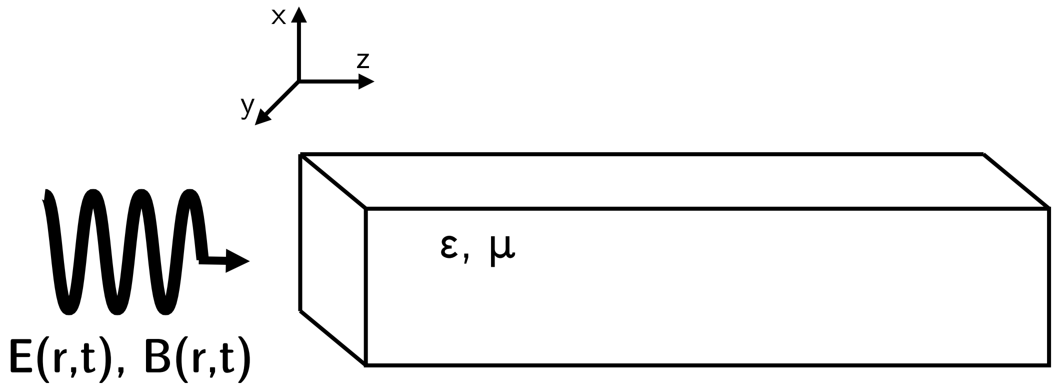
Negative refractive index

Traveling wave parametric interactions in a negative index Josephson metamaterial

Quantum optics with a negative index Josephson metamaterial

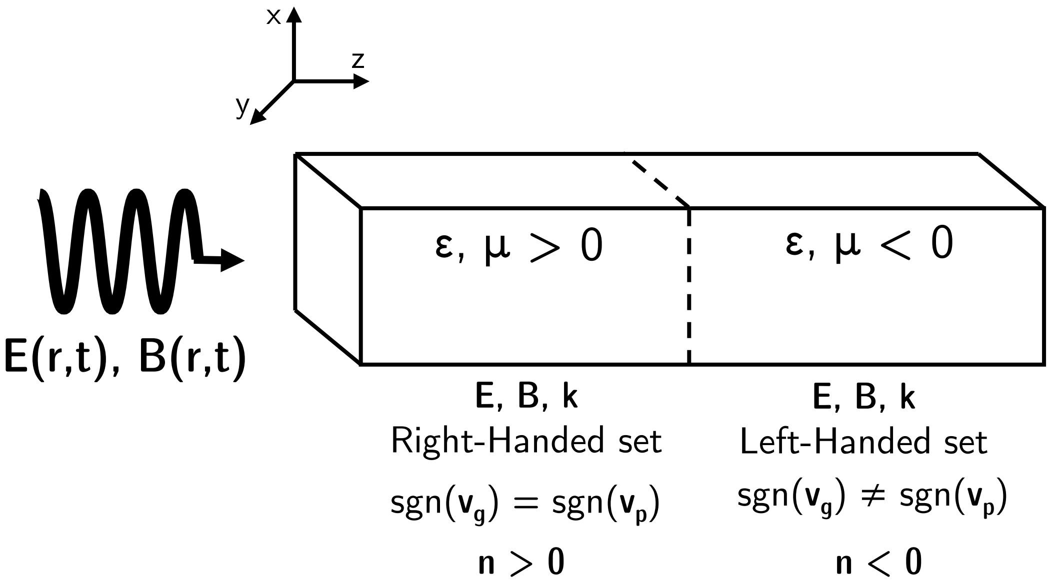
Negative index materials

→V. G. Veselago, **Sov.Phys.Usp. 10 509 (1968)**



Negative index materials

→V. G. Veselago, **Sov.Phys.Usp. 10 509 (1968)**



For a review see: J. B. Pendry and David R. Smith **Physics Today 57 (6), 37–43 (2004)**

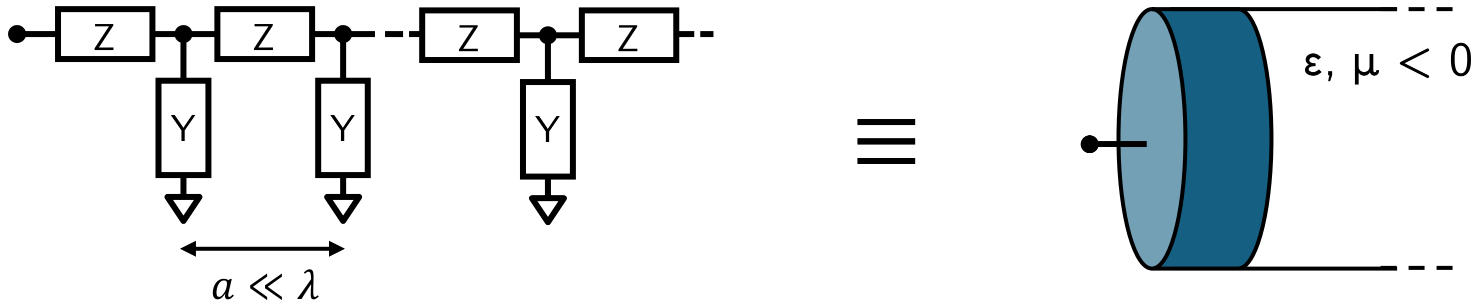
First realization: R. A. Shelby, D. R. Smith, and S. Schultz, **Science 292, 77-79 (2001)**

How do we build such a material ?

Metamaterials in the microwave

$a \sim 10\ \mu m$

$\lambda \sim 1\ mm$



→ G. V. Eleftheriades, A. K. Iyer and P. C. Kremer, ***IEEE Transactions on Microwave Theory and Techniques*** **50, 12, 2702-2712, (2002)**

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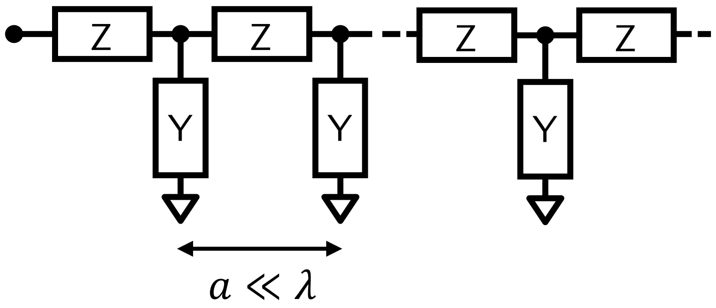
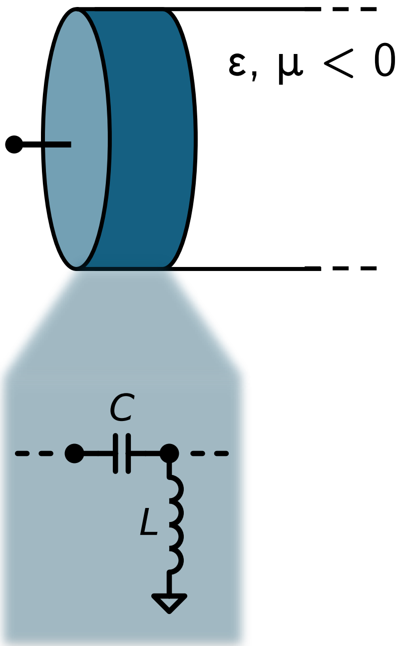


$\equiv$



$Y = i\omega\varepsilon$

$Z = i\omega\mu$

$Y = 1/i\omega L$

$Z = 1/i\omega C$

$\Rightarrow$

$\varepsilon = -1/\omega^2 L$

$\mu = -1/\omega^2 C$

→ G. V. Eleftheriades, A. K. Iyer and P. C. Kremer, ***IEEE Transactions on Microwave Theory and Techniques*** **50, 12, 2702-2712, (2002)**

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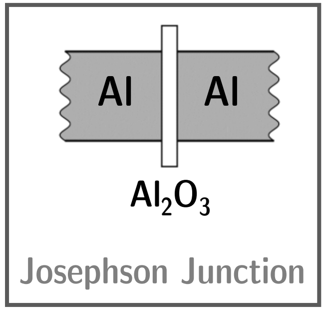
Josephson junction as a nonlinear inductor



Phase across the junction $\phi = \int V dt$

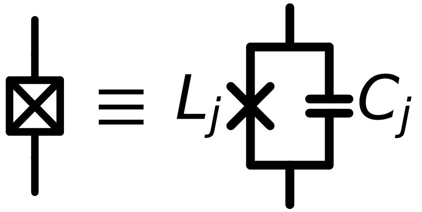
$$\mathrm{I}_{\mathrm{JJ}}(\phi) = \mathrm{I}_{\mathrm{c}} \sin\frac{\phi}{\varphi_0} \approx \mathrm{I}_{\mathrm{c}}\frac{\phi}{\varphi_0} - \mathrm{I}_{\mathrm{c}}\frac{1}{6}\left(\frac{\phi}{\varphi_0}\right)^3 + \cdots$$

$\mathrm{L}_{\mathrm{J}} = \varphi_0 / \mathrm{I}_{\mathrm{C}}$

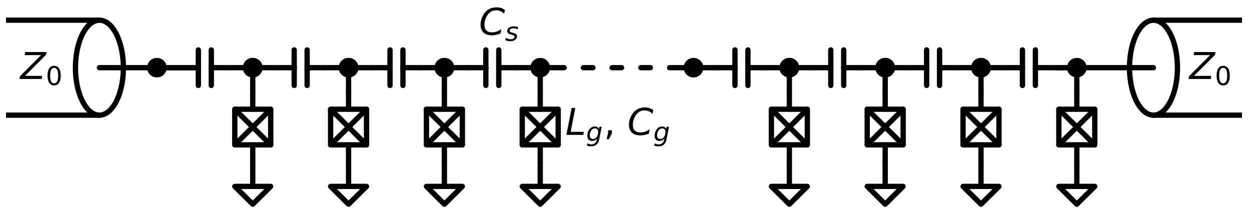
4WM interactions

$$P = \epsilon_0[\chi^{(1)}E + \boldsymbol{\chi}^{(\mathbf{3})} E^3 + \cdots]$$

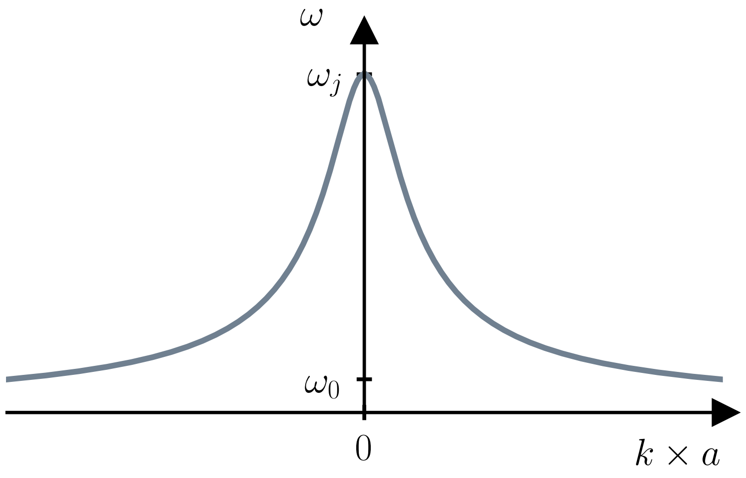
Circuit representation



Left-handed Josephson metamaterial



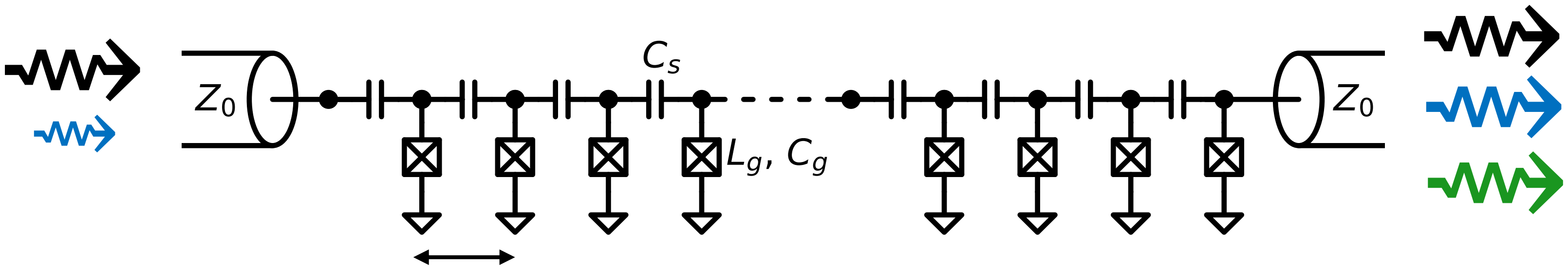
$a \ll \lambda \Rightarrow f \gg \omega_0$



$$\underbrace{\frac{1}{\omega_j^2}\frac{\partial^2\phi(x,t)}{\partial t^2} - \frac{a^2}{\omega_0^2}\frac{\partial^4\phi(x,t)}{\partial x^2\partial t^2} + \phi(x,t)}_{\text{Linear dispersion}} \underbrace{- \frac{\phi^3(x,t)}{6I_0^2L_g^2}}_{\text{Non-linear term}} = 0$$

$$\omega_0 = \frac{1}{\sqrt{L_g C_s}} \qquad \omega_J = \frac{1}{\sqrt{L_g C_g}}$$

Traveling wave parametric amplification



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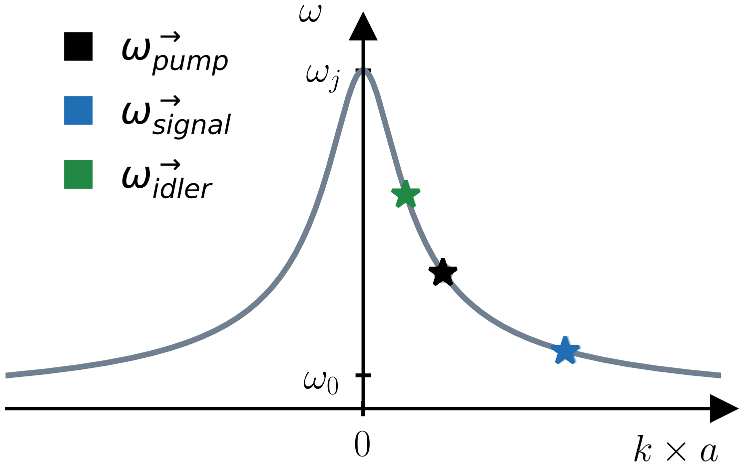
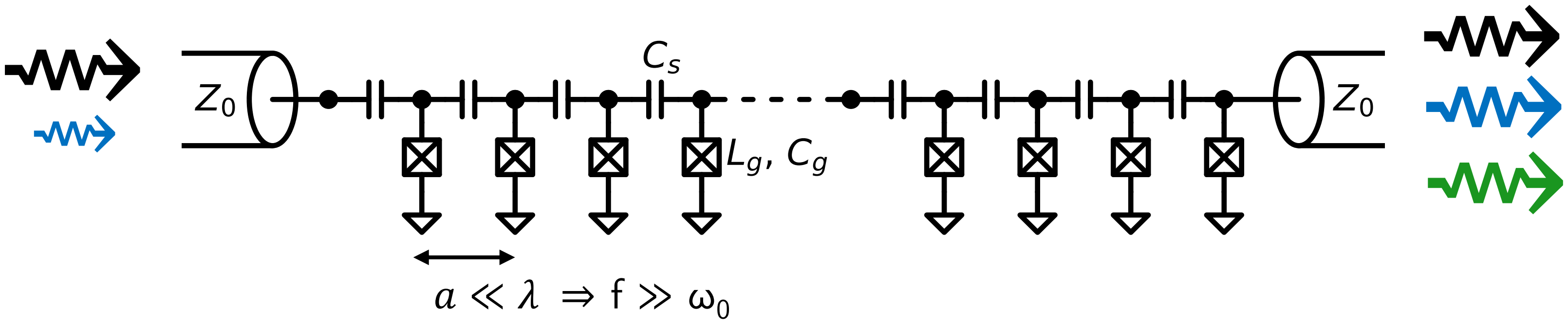
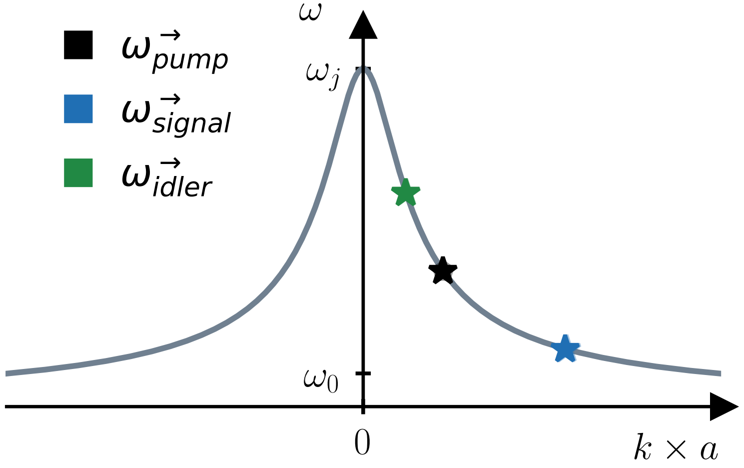


4WM amplification

Energy conservation: $\Delta\omega = 2\omega_p - \omega_s - \omega_i = 0$

Traveling wave parametric amplification: phase matching





4WM amplification

Energy conservation: $\Delta\omega = 2\omega_p - \omega_s - \omega_i = 0$

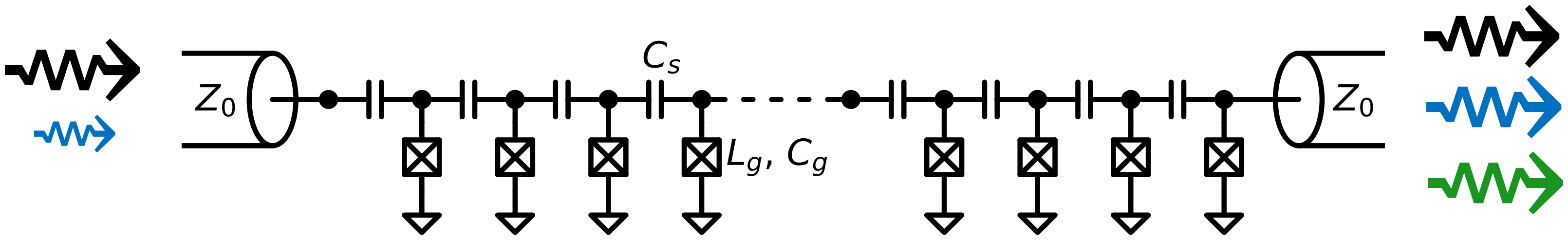
Phase matching condition: $\Delta k = \Delta k_L + \Delta k_{NL} = 0$

$$\Delta k_L = 2k_p - k_s - k_i$$

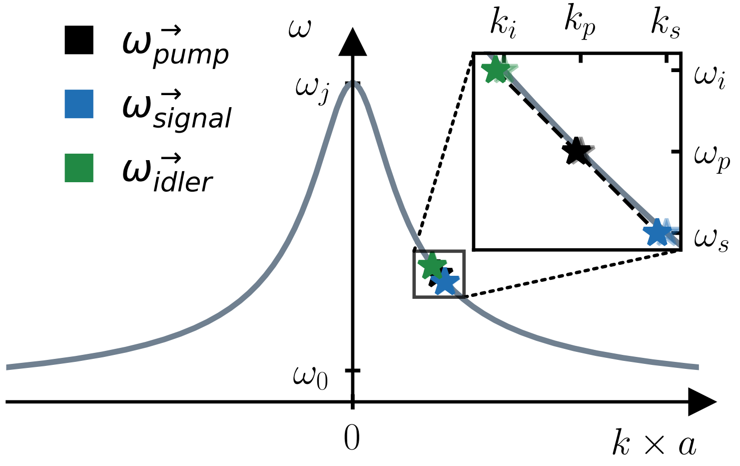
$$\Delta k_{NL} = 2\alpha_{pp} - \underbrace{\alpha_{sp} - \alpha_{ip}}_{\text{cross-Kerr}}$$

self-Kerr ($2\alpha_{pp}$) cross-Kerr

Self-phase matched parametric amplification



$a \ll \lambda \Rightarrow f \gg \omega_0$



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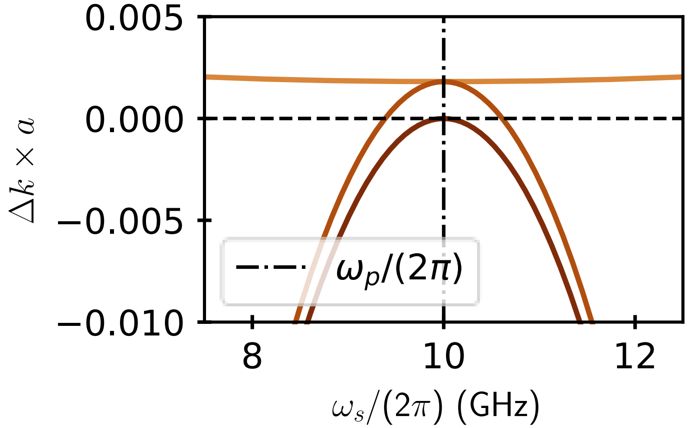
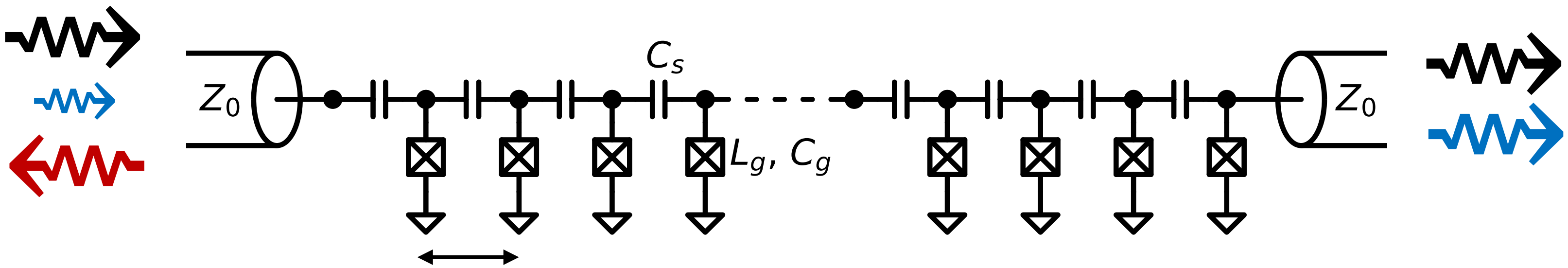
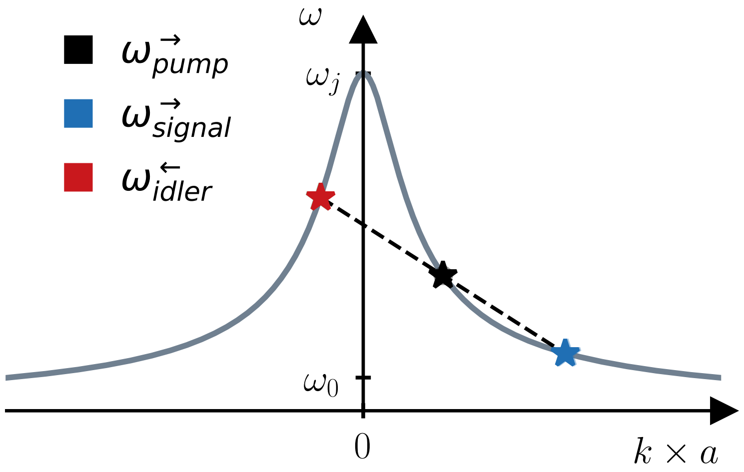


→ Kow, C.. *et al.* **arXiv:2201.04660 (2022)**

Parametric amplification of counter-propagating waves



$a \ll \lambda \Rightarrow f \gg \omega_0$



4WM amplification

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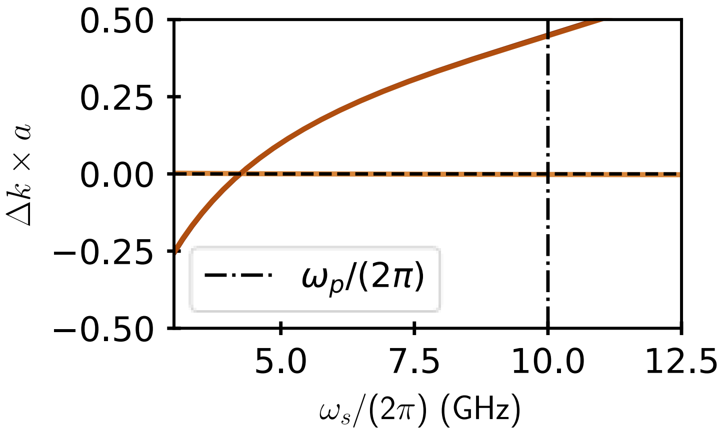
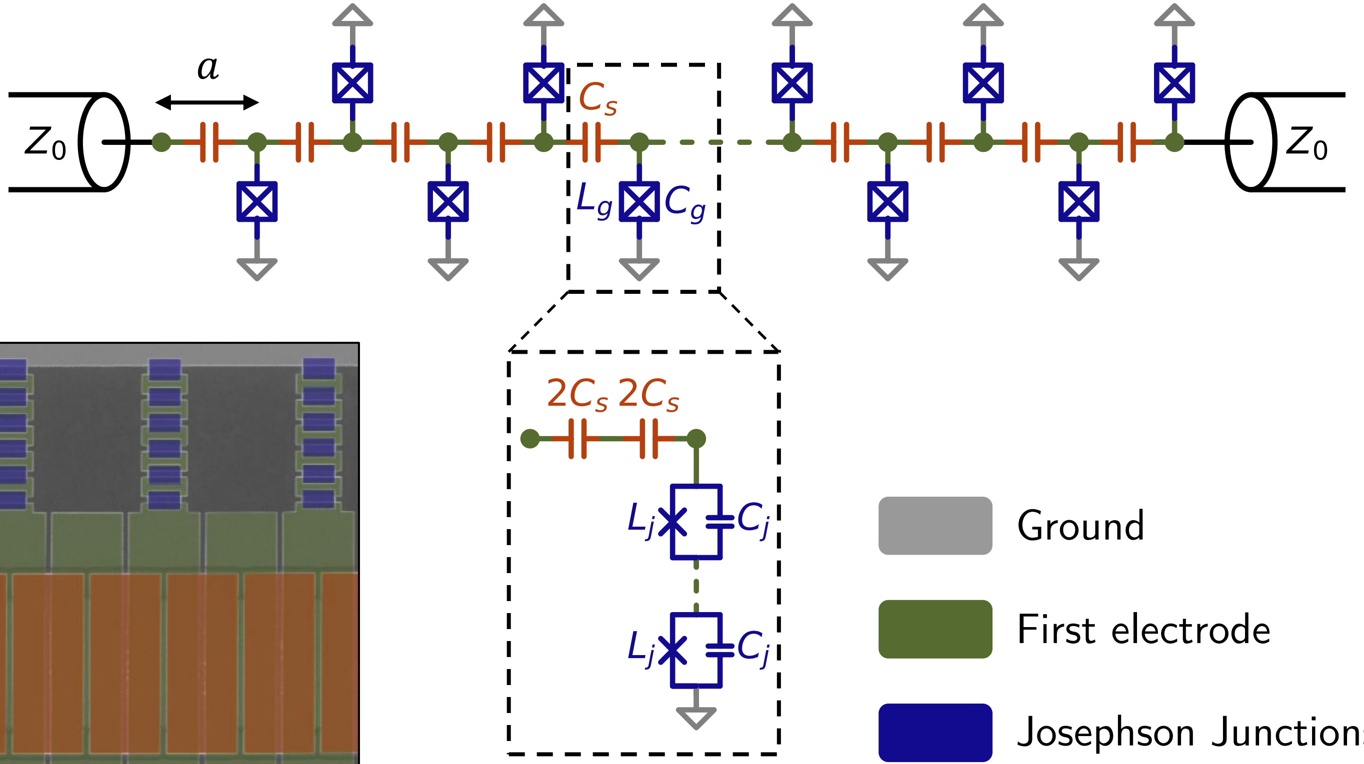
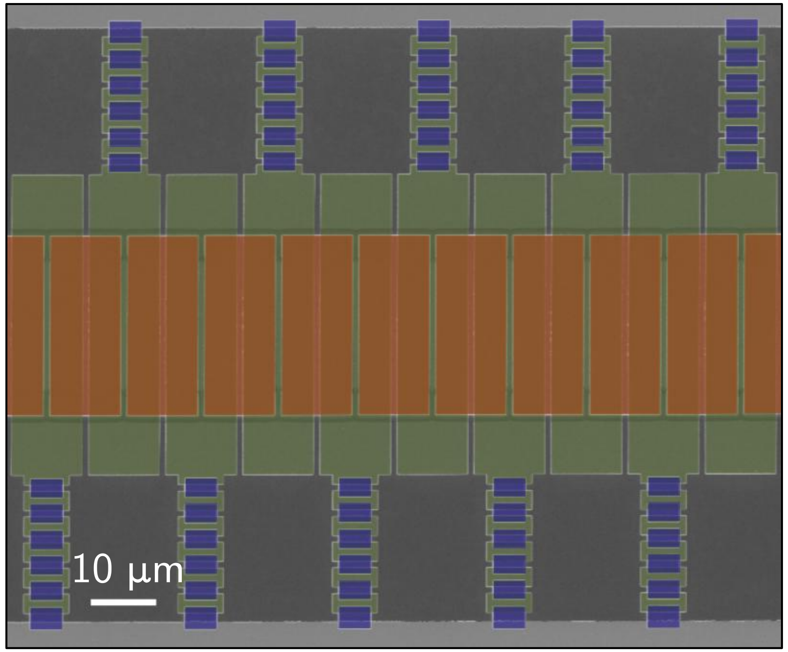


Left-handed Josephson metamaterial: the device



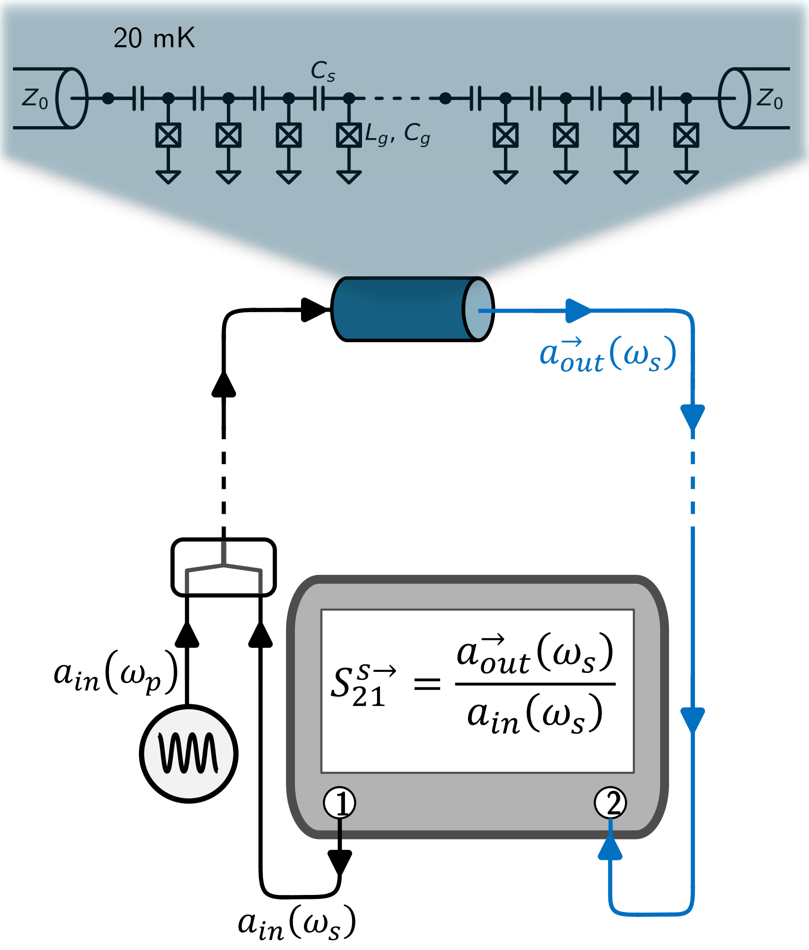


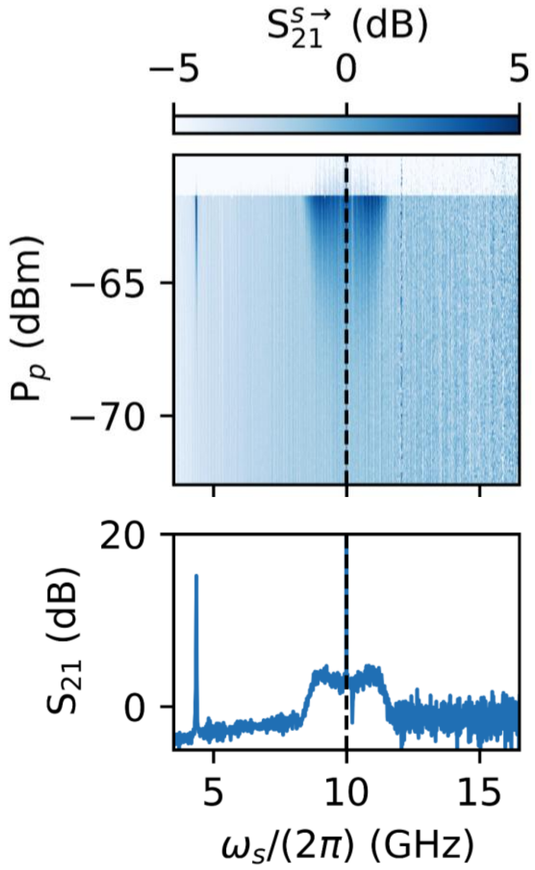
- Ground
- First electrode
- Josephson Junctions
- Capacitors

Parameters:

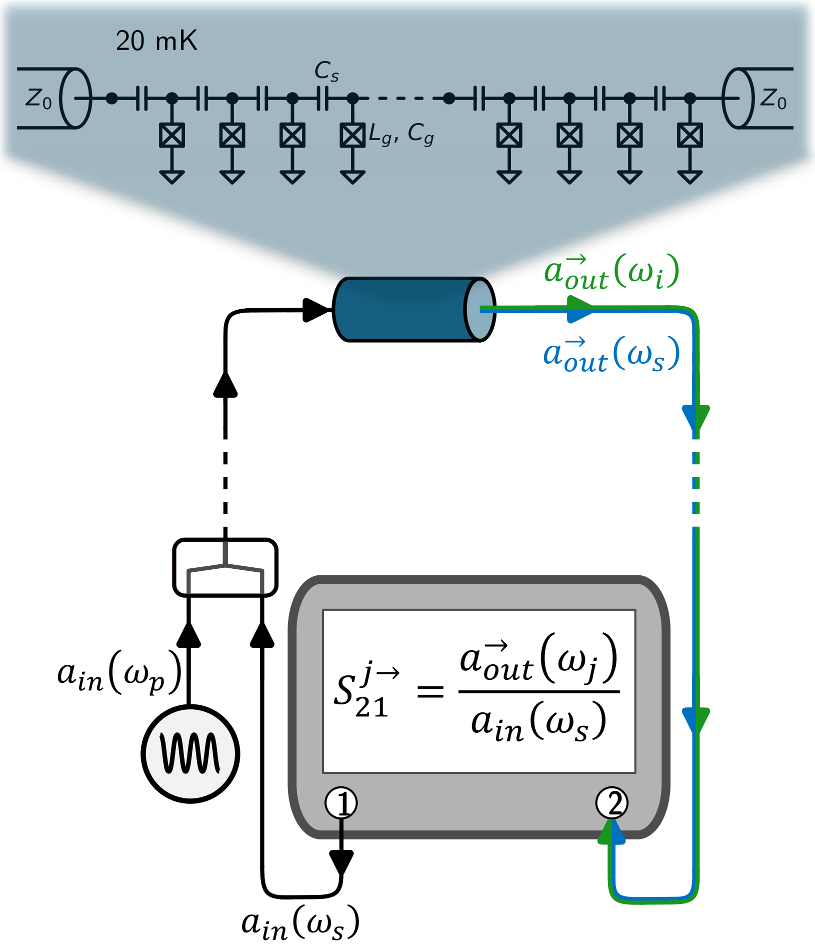
600 unit cells
$L_g = 4.8$ nH
$C_g = 8.4$ fF
$C_s = 871$ fF
$Z = 75\ \Omega$
$f_0 = 2.5$ GHz
$f_J = 25$ GHz

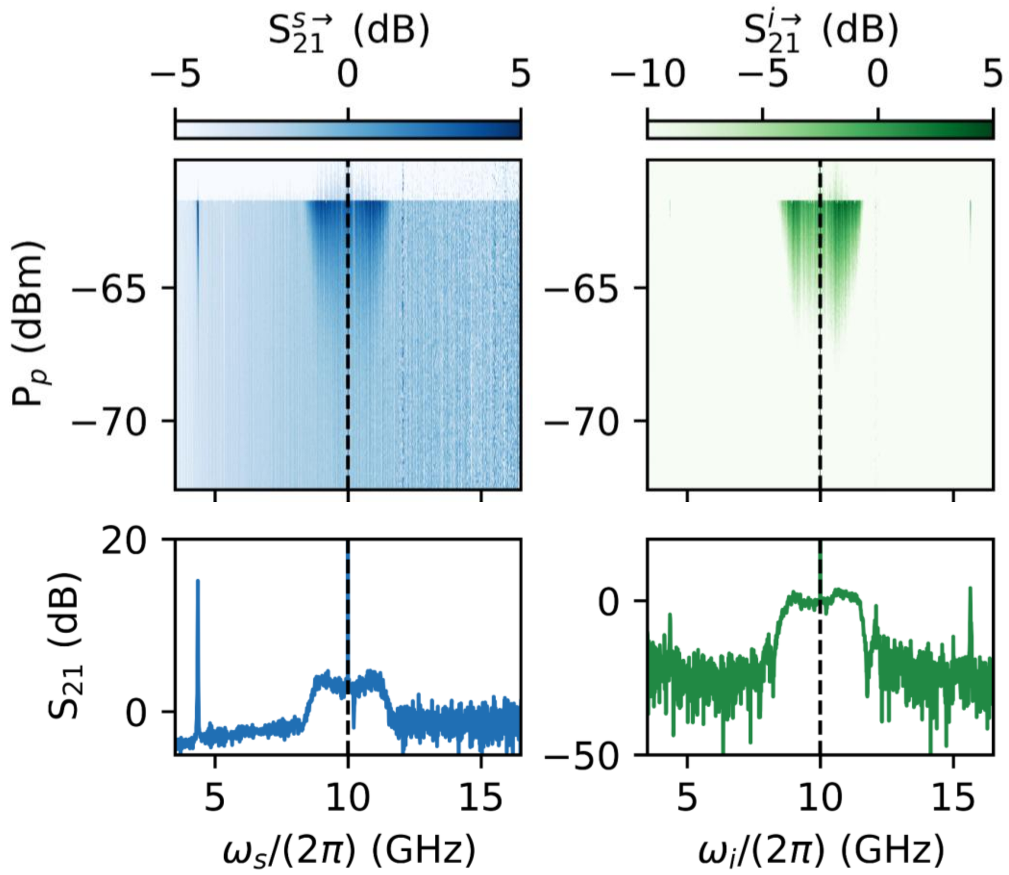
Left-handed Josephson metamaterial: amplification





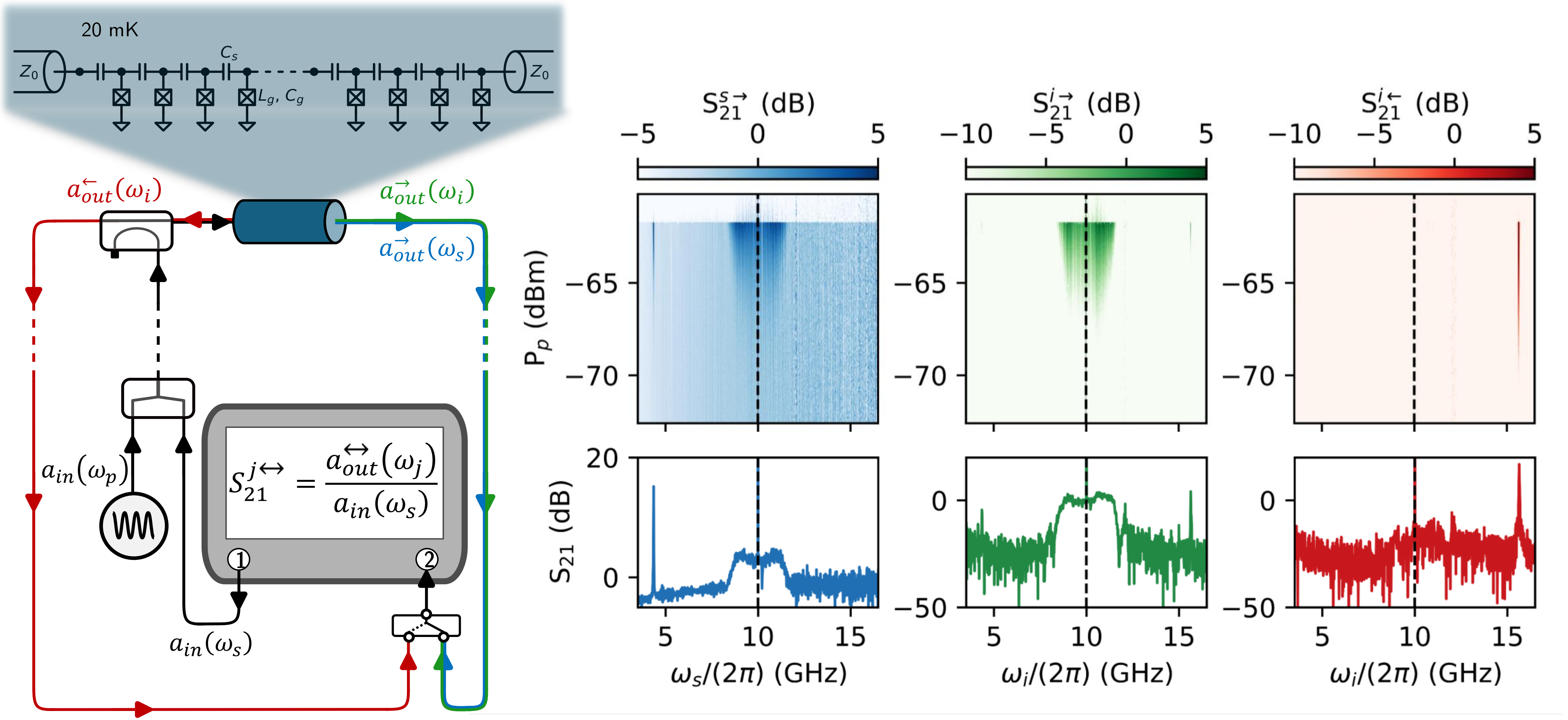
Left-handed Josephson metamaterial: amplification





$\omega_i = 2\omega_p - \omega_s$

Left-handed Josephson metamaterial: amplification



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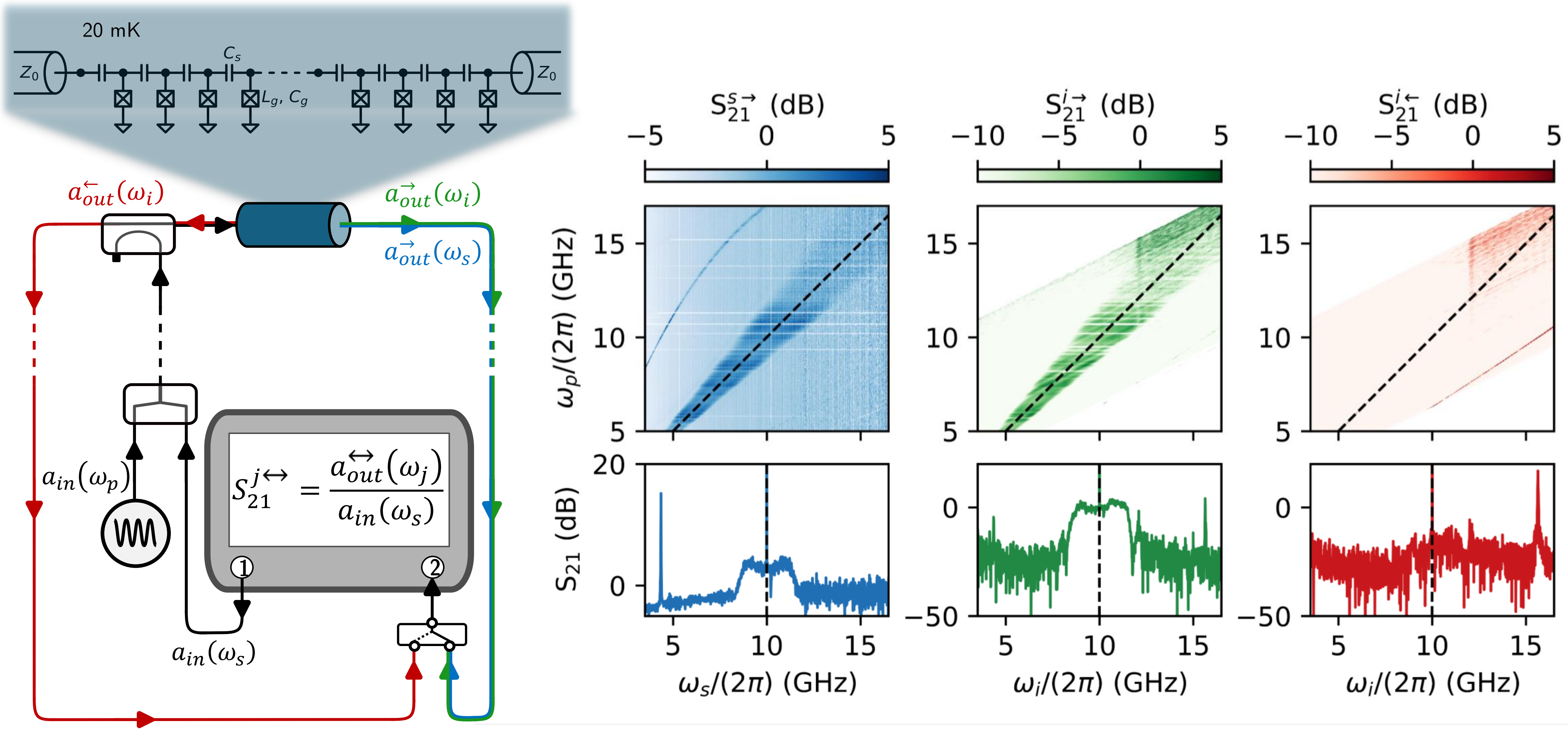
Left-handed Josephson metamaterial: amplification



$\omega_i = 2\omega_p - \omega_s$

Outline

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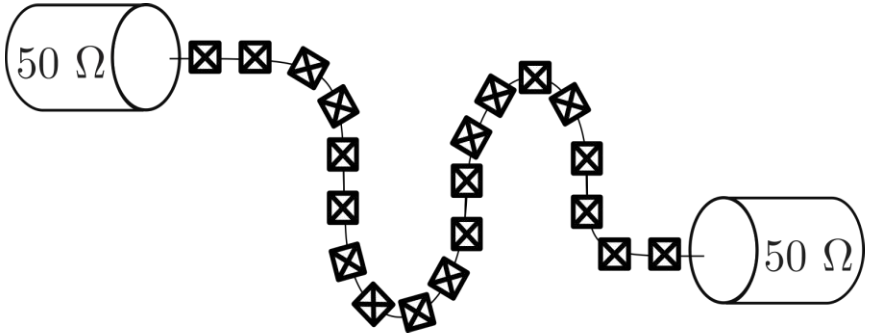
Quantum optics with a negative index Josephson metamaterial

Josephson metamaterials: not only amplification...



$$\mathsf{U}_{\mathsf{JJ}}(\phi) = \mathsf{E}_{\mathsf{J}}\left(1 - \cos\frac{\phi}{\varphi_0}\right) \approx \frac{1}{2}\mathsf{E}_{\mathsf{J}}\left(\frac{\phi}{\varphi_0}\right)^2 + \frac{1}{24}\mathsf{E}_{\mathsf{J}}\left(\frac{\phi}{\varphi_0}\right)^4 + \cdots$$

$\downarrow$ $\hat{\phi}_j \propto (\hat{a}_j + \hat{a}_j^\dagger)$ + RWA

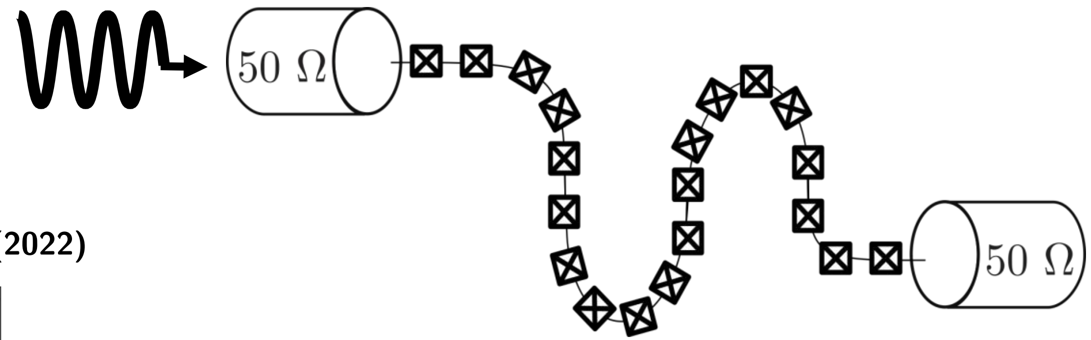
4WM interactions

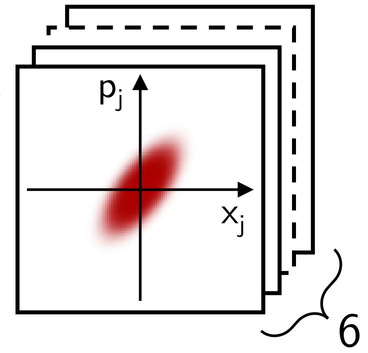
$$\widehat{H}_{int} \propto K\hat{a}_i^\dagger \hat{a}_s^\dagger \hat{a}_p \hat{a}_p + \text{h.c.}$$

$\Downarrow$ $\hat{a}_p \rightarrow \alpha_p$ coherent state

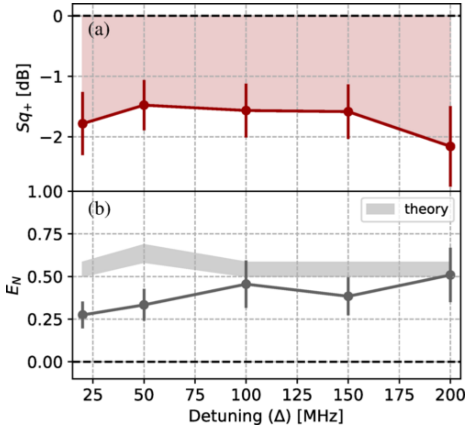
$$\boxed{\widehat{H}_{int} \propto K\alpha_p^2 \hat{a}_i^\dagger \hat{a}_s^\dagger + \text{h.c.}}$$

Two-mode vacuum squeezing





First demonstration in JTWPAs:
Esposito *et al.* **Phys. Rev. Lett (2022)**



See also:
Perelshtein *et al.* **Phys. Rev. Applied (2022)**
Qiu *et al.* **Nat. Phys. (2023)**

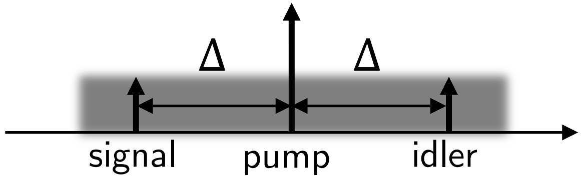
$$\boxed{\hat{H}_{int} \propto K\alpha_p^2 \hat{a}_i^\dagger \hat{a}_s^\dagger + \text{h.c.}}$$

$$\Downarrow$$

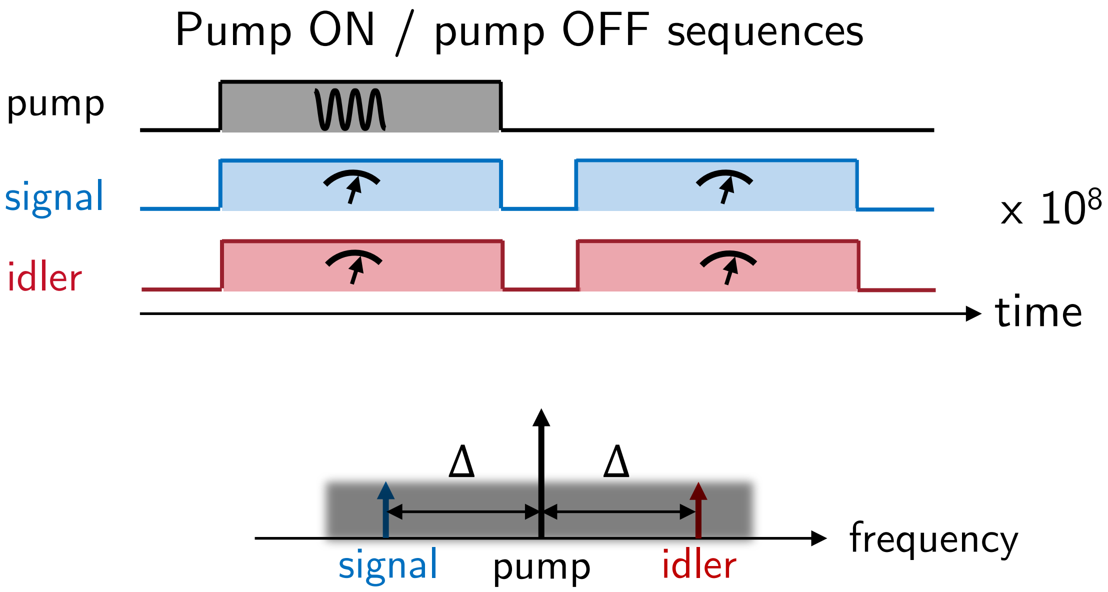
Two-mode squeezing operator

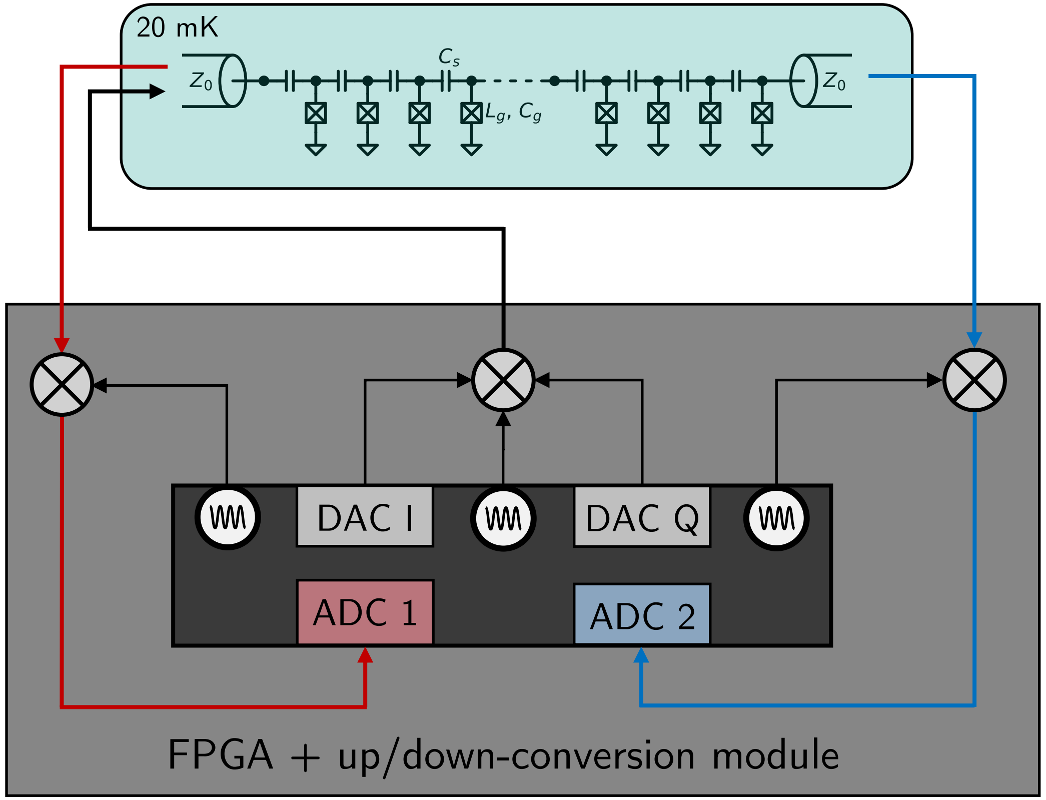
$$\hat{S} = \exp\{ r e^{i\theta} \hat{a}_i^\dagger \hat{a}_s^\dagger - \text{h.c.}\}$$

$$\begin{cases} \hat{x}_j = \dfrac{\hat{a}_j^\dagger + \hat{a}_j}{2} \\ \hat{p}_j = i\dfrac{\hat{a}_j^\dagger - \hat{a}_j}{2} \end{cases}$$



Measuring vacuum squeezing in a negative index material





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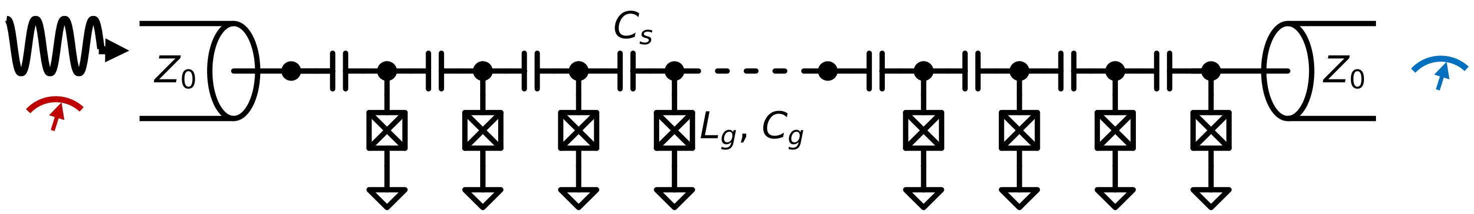
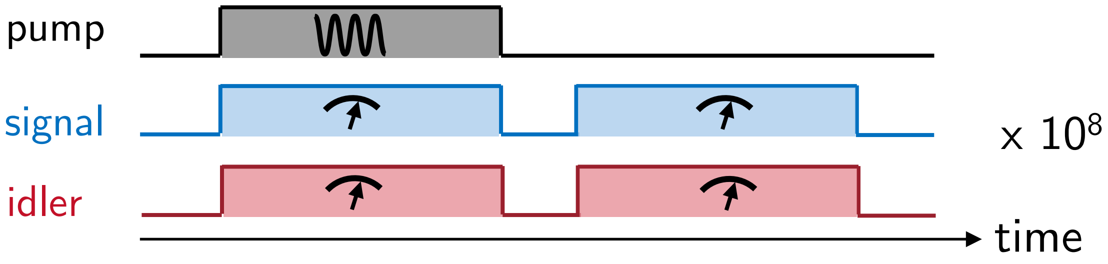
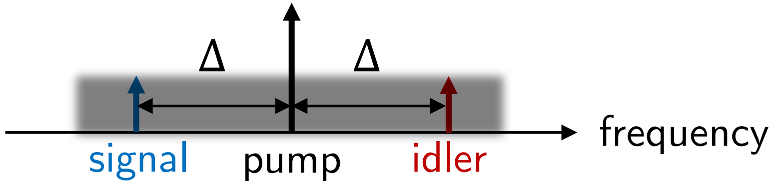
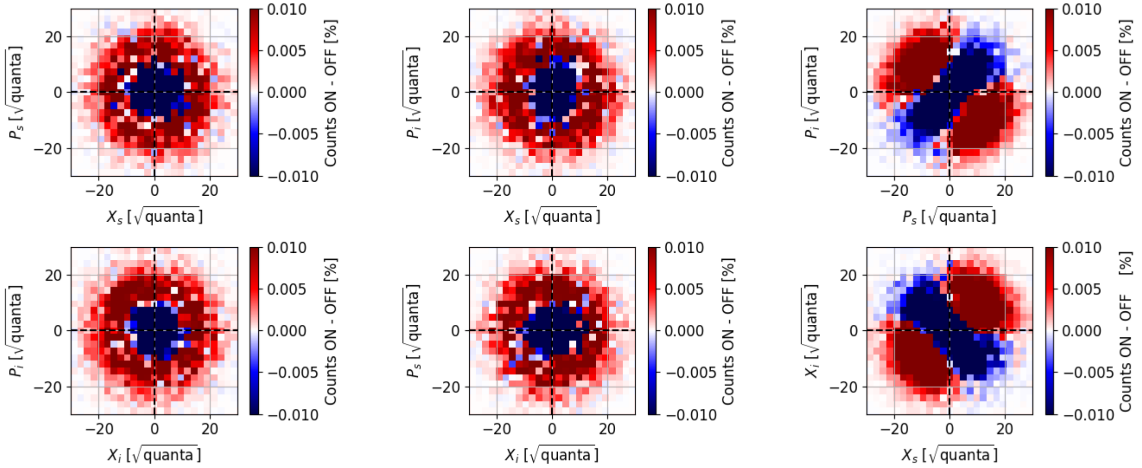


Pump ON / pump OFF sequences





Phase-space histogram distribution

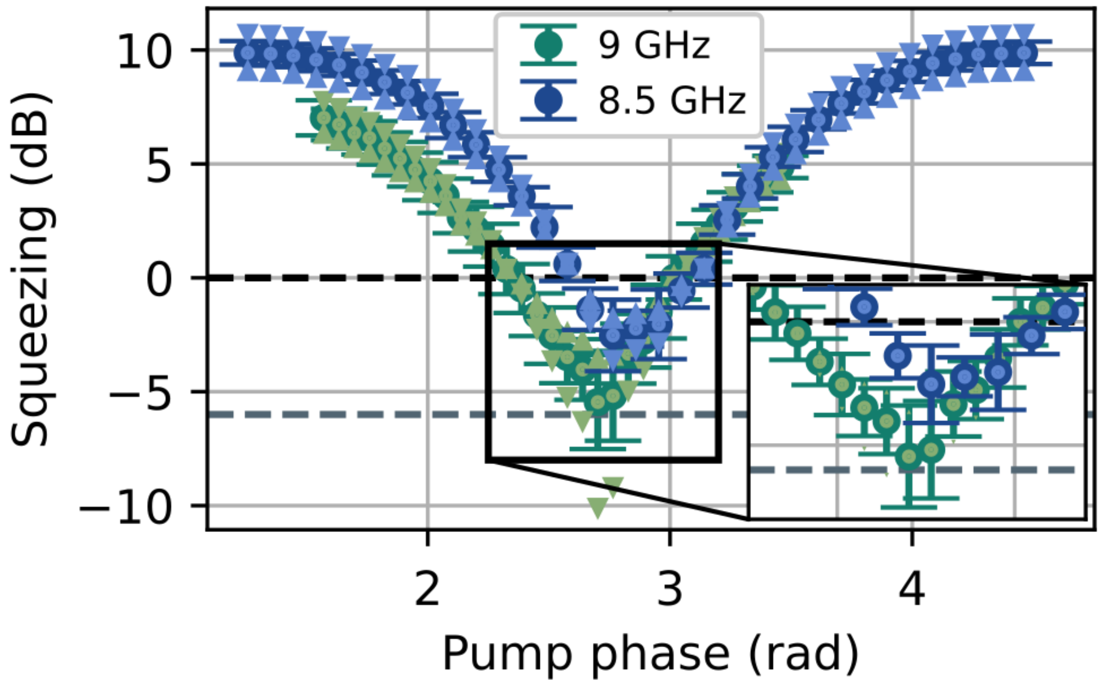


Squeezing between two counter-propagating modes

Collective quadratures:

$$\hat{x}_+ = \hat{x}_s + \hat{x}_i$$
$$\hat{p}_+ = \hat{p}_s + \hat{p}_i$$

$$S_{q+} = 10\log\left(\frac{\langle \hat{x}_+^2 \rangle}{0.5}\right)$$



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Collective quadratures:

$$\hat{x}_+ = \hat{x}_s + \hat{x}_i$$
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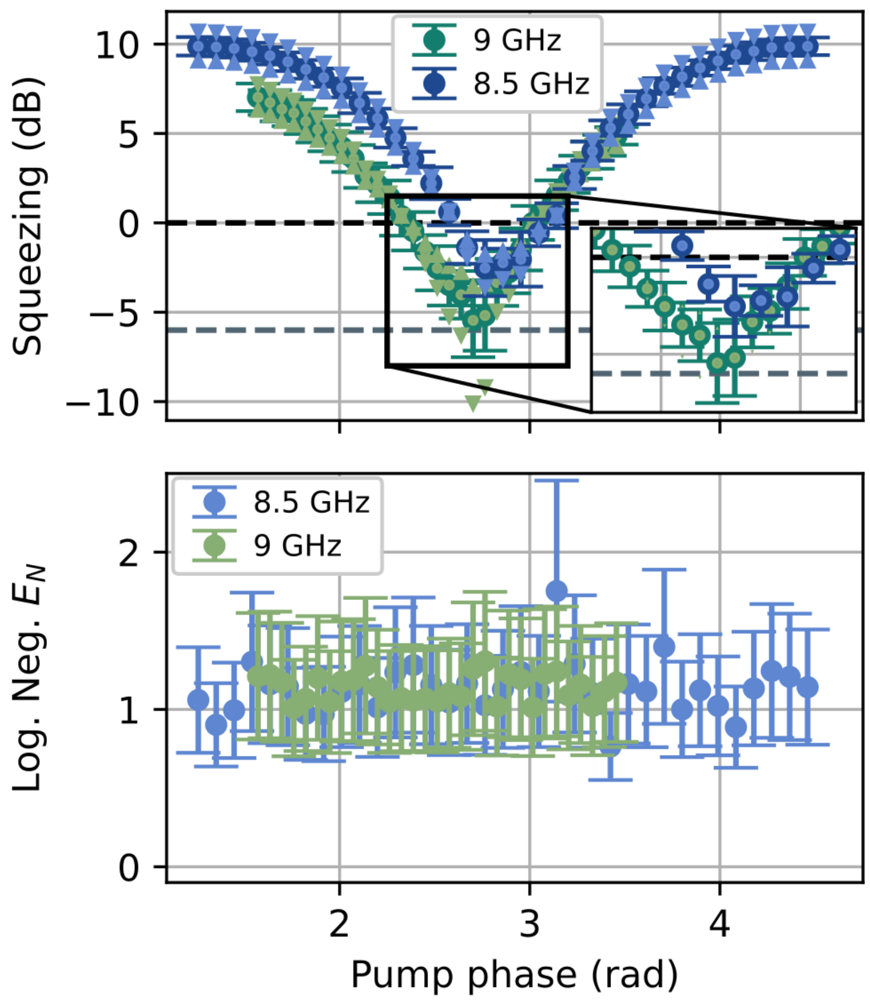
$$S_{q+} = 10\log\left(\frac{\langle \hat{x}_+^2 \rangle}{0.5}\right)$$

$E_N > 0 \quad \Longrightarrow \quad$ Entangled state

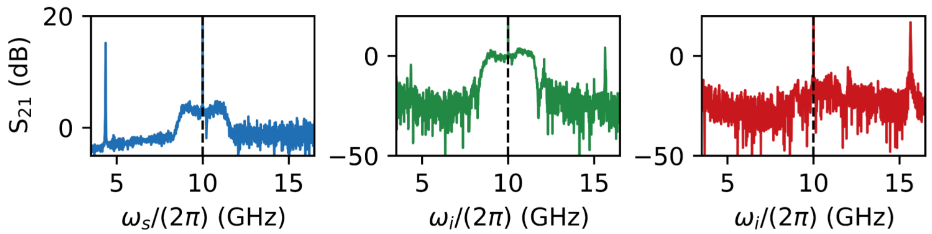
P. Horodecki, **Phys. Lett. A 232, 5, 333 (1997)** – PPT criterion



Summary

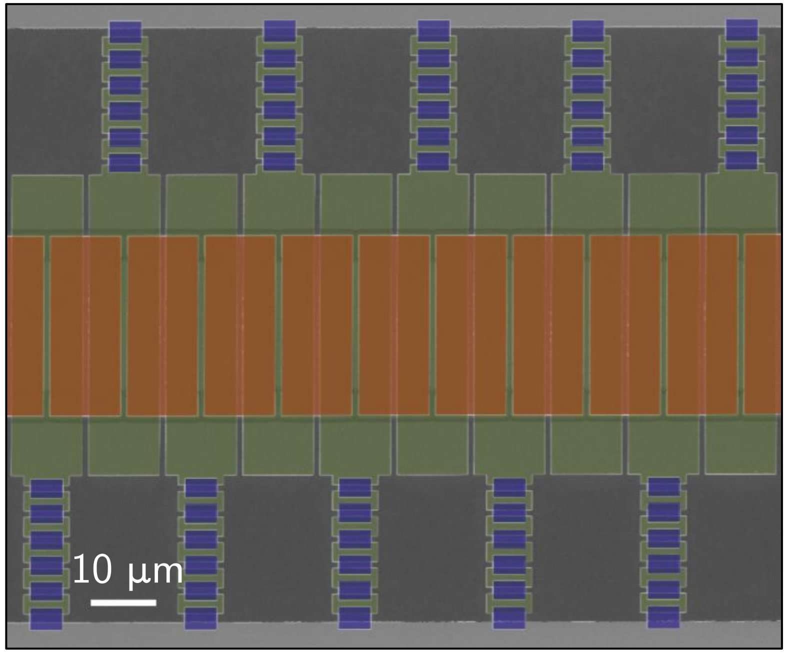
First realization of a Josephson metamaterial showing negative refractive index behavior

- Traveling wave parametric interactions



- Squeezing and entanglement generation between two counter-propagating modes

S_{q+} = -6 dB $\quad$ $2\Delta \approx 10$ GHz $\quad$ $E_N = 1.1$



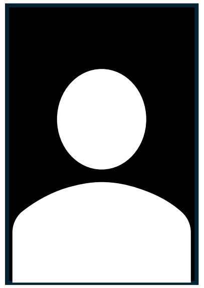
The TWPA team

Nicolas Roch

Gwenael Le Gal

Giulio Cappelli

Bekim Fazliji



Sébastien Moretti

+ SILENT WAVES

↓

Alexis Coissard

The TWPA team

SQC
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Nicolas Roch

Gwenael Le Gal

Giulio Cappelli

Bekim Fazliji

Sébastien Moretti

+ SILENT WAVES → Alexis Coissard

Thank you!