Poor man's Majorana modes in interacting devices

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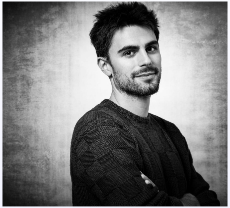
University of Pisa

6 May 2025

Work in collaboration with:

Phys. Rev. B 111, 174501 (2025); arXiv:2411.11981



Lorenzo Maffi - Padova

Maximiliam Nitsch - Lund

Virgil Baran - Bucharest

Rubén Souto - CSIC at Madrid

Martin Leijnse - Lund

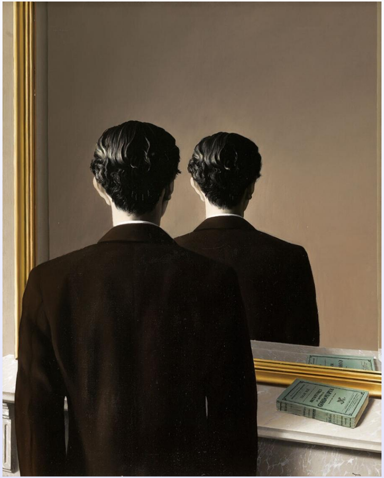
Jens Paaske - Copenhagen

Outlook

1. The search for Majorana modes
2. Poor man's Majorana modes
3. Floating devices
4. The poor man's Majorana tetron

The search for Majorana modes in superconductor devices

Considerable efforts have been devoted to the quest for Majorana modes.



- Majoranas are localized ‘real’ modes:

$$\{\gamma_i, \gamma_j\} = 2\delta_{ij}$$

- Zero energy $\rightarrow$ Degeneracies
- Non-Abelian anyons $\rightarrow$ Non-locality
- Simplest topological Qubits

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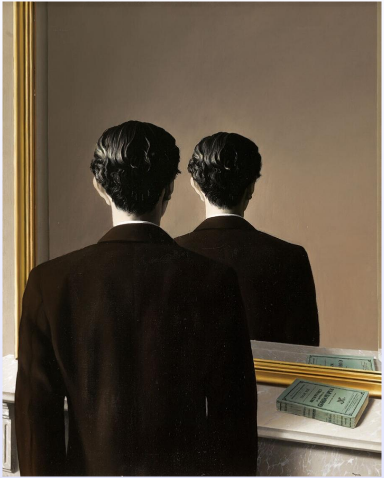

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- Main recipes: **Hybrid SC - Semi devices**
- Main observations: **transport**

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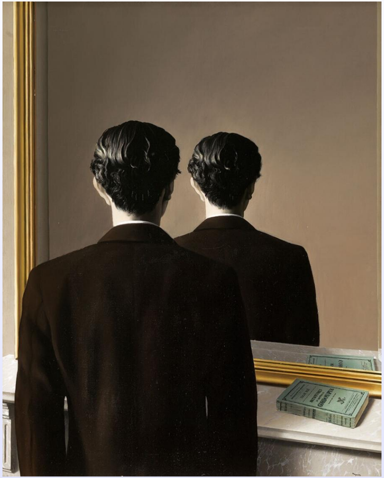

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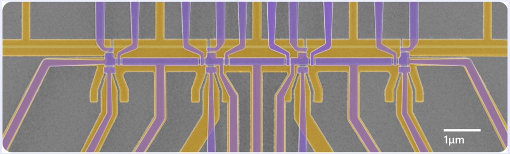
- Simplest topological Qubits
- Main recipes: **Hybrid SC - Semi devices**
- Main observations: **transport**
- Transport in two-terminal devices <u>do not reveal topology</u> and non-locality

Recent approaches for the study of Majorana modes

Topological superconductors

- Topological protection
- Microscopic approach
- Long proximitized nanowires
- Localization of the Majoranas is not optimal
- Many competing subgap states



Microsoft, Nature 2025

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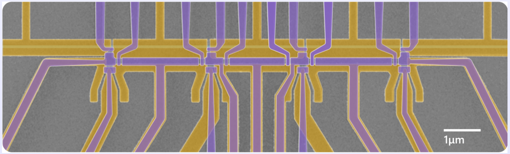
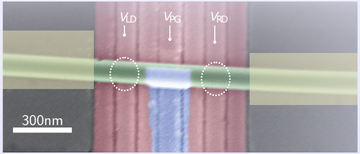


Microsoft, Nature 2025

Poor man's Majorana modes

- **No** topological protection
- Mesoscopic approach via fine-tuning
- Hybrid quantum dot - superconductors
- Trade off between robustness and controllability
- Easier to realize



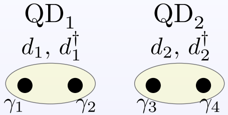
QuTech [Dvir et al., Nature 2023]

Poor man's Majorana modes: minimal Kitaev chain

Leijnse and Flensberg PRB (2012), Liu et al. PRL (2022)

Two quantum dots:

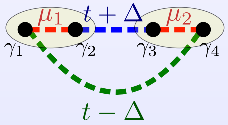
$$H_{\mathrm{K}} = -t\left(d_1^\dagger d_2 + d_2^\dagger d_1\right) - \Delta\left(d_2^\dagger d_1^\dagger + d_1 d_2\right) + \sum_{a=1,2} \mu_a d_a^\dagger d_a$$



4 Majorana operators:

$$\gamma_{2a-1} = d_a + d_a^\dagger, \quad \gamma_{2a} = i\left(d_a - d_a^\dagger\right), \quad \gamma = \gamma^\dagger, \quad \{\gamma_a, \gamma_b\} = 2\delta_{a,b}$$

$$H_{\mathrm{K}} = \frac{(t+\Delta)}{2} i\gamma_3\gamma_2 + \frac{(t-\Delta)}{2} i\gamma_4\gamma_1 + \sum_{a=L,R} \frac{\mu_a}{2} i\gamma_{2a}\gamma_{2a-1}$$



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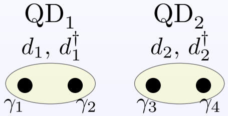

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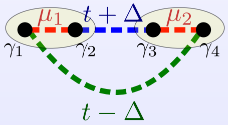
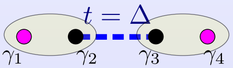


Sweet spot for Poor Man's Majoranas: $\mu_a = 0\,,\ \Delta = t$

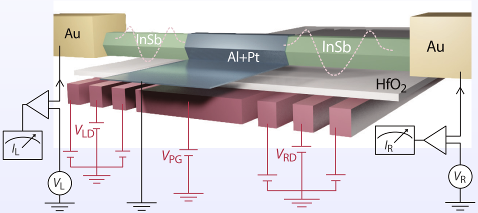


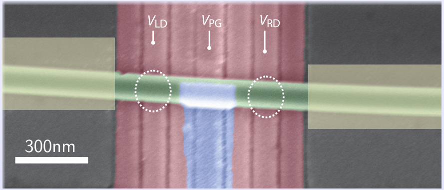
Poor man's Majorana modes: experimental platforms

QuTech - Delft

Experimental realizations are based on superconductor - semiconductor heterostructures

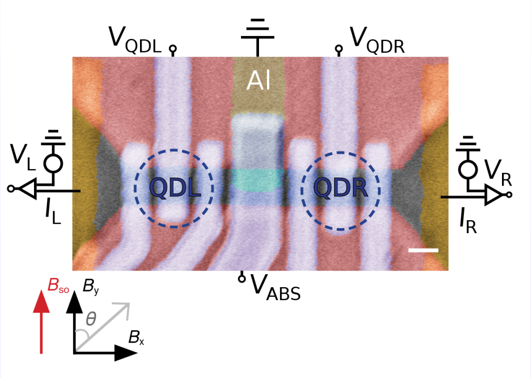
Nanowires + SC





Dvir et al., Nature 2023

2D semiconductors + SC

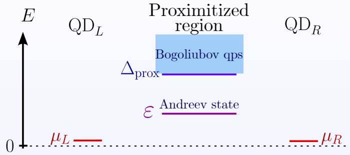


ten Haaf et al., Nature 2024

How to tune Δ and t?

Elastic cotunneling and crossed Andreev reflection

- Hybrid systems with spin-orbit coupling
- Tuning relies on an Andreev state
- Second-order coherent processes



Liu et al., PRL 2022

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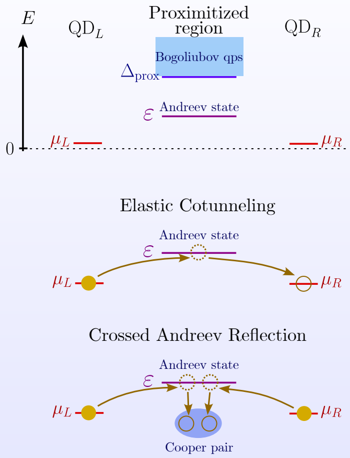

- $t \to t_{\text{COT}}$ **Elastic cotunneling** $d_R^\dagger d_L$
- $\Delta \to \Delta_{\text{CAR}}$: **Crossed Andreev reflection** $d_L d_R$

Liu et al., PRL 2022

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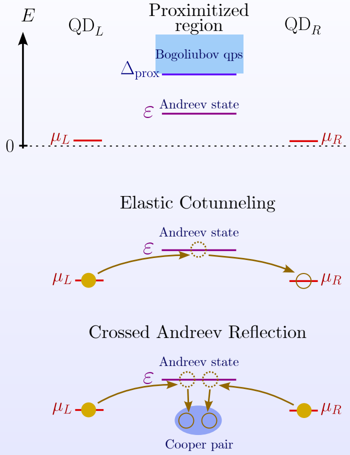

- $t \to t_{\text{COT}}$ **Elastic cotunneling** $d_R^\dagger d_L$
- $\Delta \to \Delta_{\text{CAR}}$: **Crossed Andreev reflection** $d_L d_R$
- Gate voltages
  $\to$ Energy of the virtual Andreev state ε
  $\to$ Ratio $t_{\text{COT}}/\Delta_{\text{CAR}}$
  $\to$ Dot energy levels μ_a

Liu et al., PRL 2022

What about floating devices?

- For grounded SC the system exchanges Cooper pair with the environment
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N includes Cooper pairs, Andreev and Bogoliubov quasiparticles.

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- **Charging energy:**

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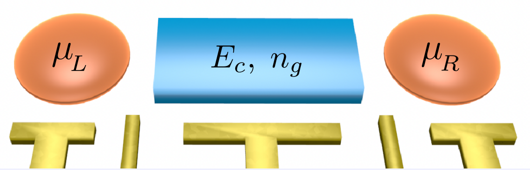
  N includes Cooper pairs, Andreev and Bogoliubov quasiparticles.
- **Main Applications:**
  - Electron teleportation mediated by Majoranas (Fu, PRB 2010)
  - Standard tool to definition topological qubits (tetrons)

$$4 \text{ Majoranas: states } |n_1 n_2\rangle \in \left\{ \frac{|01\rangle \pm |10\rangle}{\sqrt{2}} ; \frac{|00\rangle \pm |11\rangle}{\sqrt{2}} \right\}$$

  - Integration with transmons for readout or control
  - **Topological Kondo effect**

Are poor man's Majorana modes compatible with charging energies?

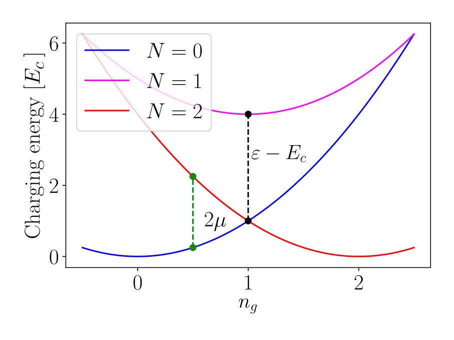
Souto, Baran, Nitsch, Maffi, Paaske, Leijnse, MB, PRB (2025)



Effective model:

$$H = E_c(2\hat{N}_c - n_g)^2 + \sum_{a=1,2} \mu_a d_a^\dagger d_a$$

$$- t_{\mathrm{COT}}\left(d_L^\dagger d_R + \mathrm{H.c.}\right) - \Delta_{\mathrm{CAR}}\left(\mathrm{e}^{-i\hat{\phi}} d_R^\dagger d_L^\dagger + \mathrm{H.c.}\right)$$



- Ground states: $N = 2N_c = 0, 2$
- Andreev state: $N = 1$

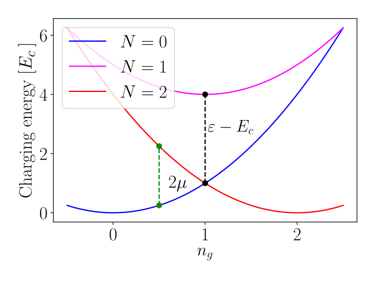
Poor man's Majorana modes with a floating SC island

Odd sector: $|0,1,N_c\rangle,\ |1,0,N_c\rangle$

$$H_o = \begin{bmatrix} \mu_L & t_{\mathrm{COT}} \\ t_{\mathrm{COT}} & \mu_R \end{bmatrix} + E_c(2N_c - n_g)^2$$

Even sector: $|0,0,N_c\rangle,\ |1,1,N_c-1\rangle$

$$H_e = \begin{bmatrix} E_c(2N_c - n_g)^2 & \Delta_{\mathrm{CAR}} \\ \Delta_{\mathrm{CAR}} & E_c(2N_c - 2 - n_g)^2 + \mu_L + \mu_R \end{bmatrix}$$



Sweet spot (fine tuning) conditions:

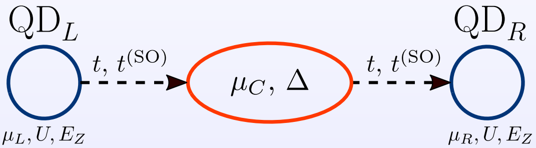
- Well-defined CAR processes: $\mu_L + \mu_R = 4E_c\,(2N_c - 1 - n_g) \equiv \delta E_c$
- Even-odd degeneracy: $|\Delta_{\mathrm{CAR}}| = |t_{\mathrm{COT}}| + \delta E_c/2$
- At the sweet spot, the ground states match the Majorana states:

$$|01, N_c = 1\rangle - |10, N_c = 1\rangle\,, \qquad |00, N_c = 1\rangle - |11, N_c = 0\rangle$$

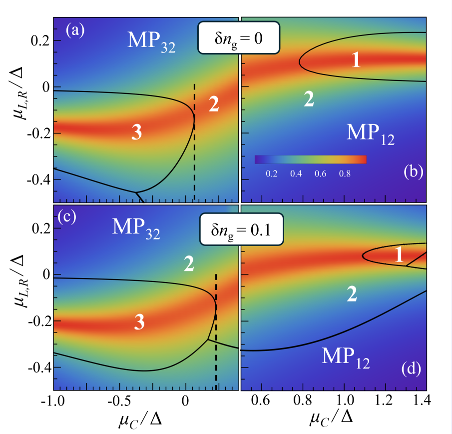
Detuning based on microscopic parameters

Majorana polarization

Microscopic models beyond perturbation theory for realistic estimates



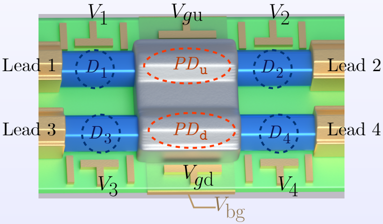
- Majorana Polarization:
  $\mathrm{MP} = 1$ for localized Majoranas
  $\mathrm{MP} = 0$ for delocalized Majoranas
- Data for $E_c = 0.2\Delta$
  $(t = 0.5\Delta;\ t^{(\mathrm{SO})} = 0.1\Delta;\ U = 5\Delta)$
- $\mathrm{MP} \sim 0.98$ at the sweet spot

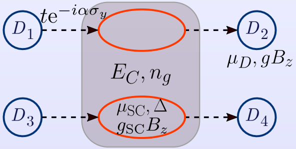


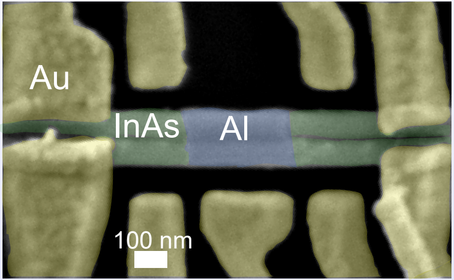
The poor man's Majorana tetron

Nitsch, Maffi, Baran, Souto, Paaske, Leijnse, MB, arXiv:2411.11981

- Two-Majorana systems are not sufficient to probe non-local Majorana properties
- Four-Majorana systems can define effective non-local spin-$1/2$ degrees of freedom







Vekris et al., NanoLetters 2022

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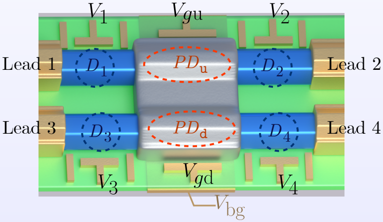

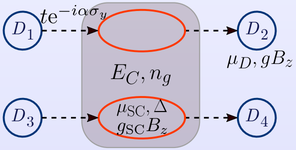

Can we use poor man's Majoranas to build a tetron?

- Non-interacting Majorana modes cannot describe this interacting system
- Unavoidable deviations from Majorana states with even N_{tot}

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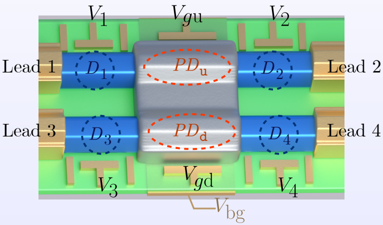

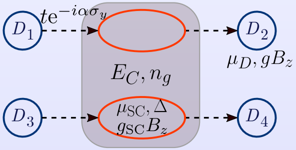

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- Regime with two-fold degeneracy of odd states

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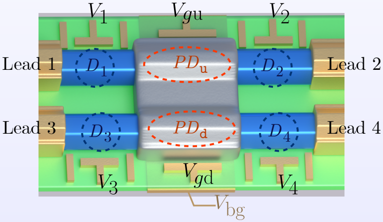

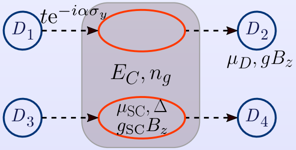

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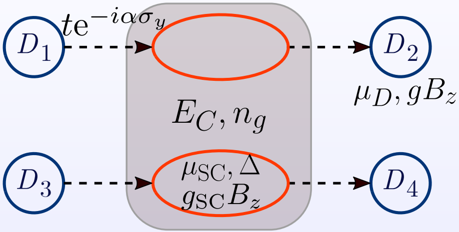
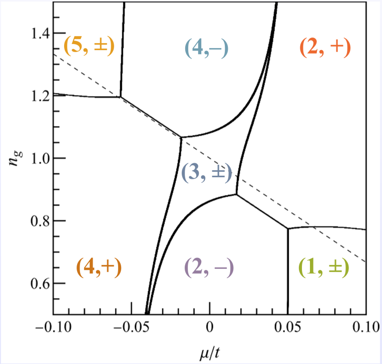
- Possibility of approaching the **topological Kondo effect**

Poor man's tetron: degenerate regime

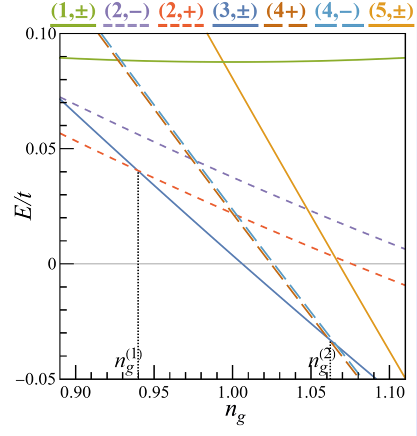


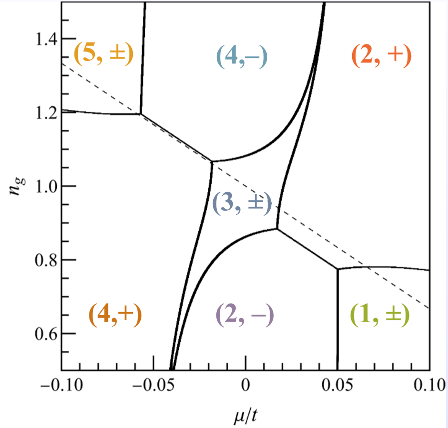
Sectors $(N_{\text{tot}}, P_{\text{d}})$



- Sweet spot condition for CAR: $\mu = 2E_c(1 - n_g)$
- CAR and cotunneling defined in each nanowire separately
- Conservation of the fermionic parity P_τ in each nanowire

Poor man's tetron and the topological Kondo effect





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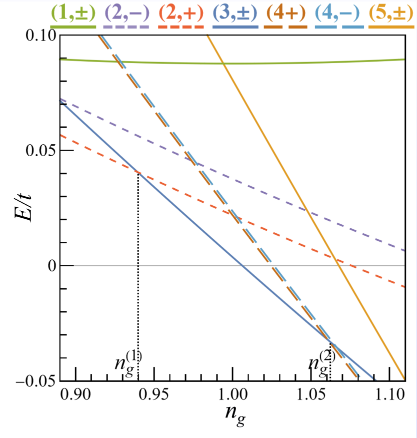

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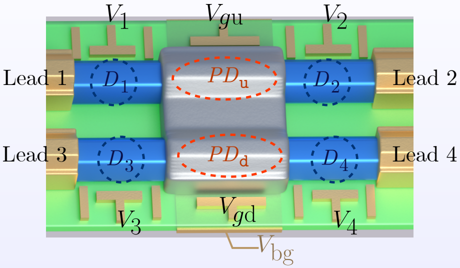
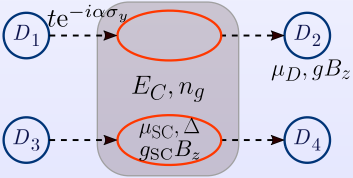
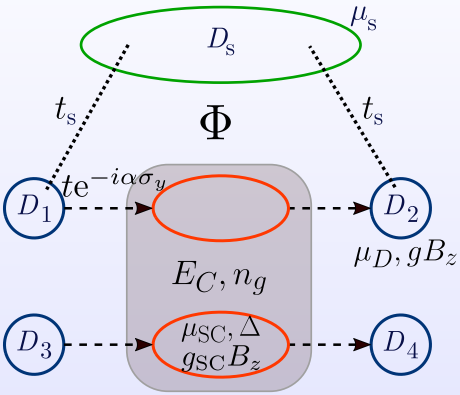




- The **degenerate GSs** in $(3, \pm)$ display Majorana correlations
- Non-local spin 1/2 degree of freedom
- The poor man's tetron behaves as a **quantum impurity** coupled to M leads
- Fine-tuning to approach the **topological Kondo regime**:
  - Fractional conductance $G_{ij} = \frac{2}{M}\frac{e^2}{h}$
  - Non-Fermi liquid
  - Observation of Majorana non-locality

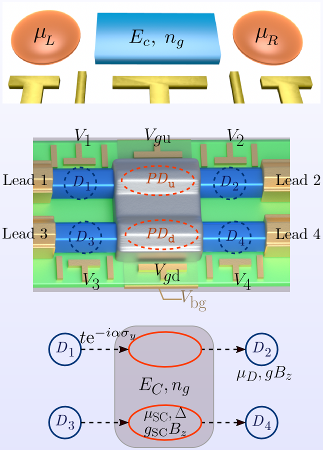
Read-out protocols for the poor man's tetron

Fermionic parity can be read as in recent reflectometry experiments by Microsoft



Conclusions

Souto, Baran, Nitsch, Maffi, Paaske, Leijnse, MB, Phys. Rev. B 111, 174501 (2025), arXiv:2411.11981



- Poor man's Majorana devices: controllable but non-protected
- They can be integrated with floating SC islands: charging energy
- 2-dot scenario: high Majorana polarization
- 4-dot scenario: definition of a non-local spin 1/2

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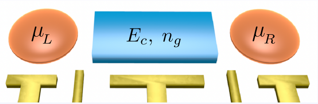
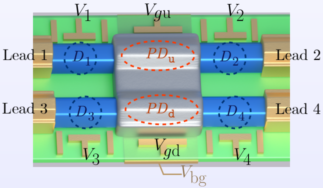
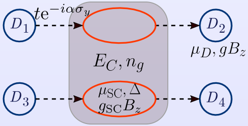
Souto, Baran, Nitsch, Maffi, Paaske, Leijnse, MB, Phys. Rev. B 111, 174501 (2025), arXiv:2411.11981







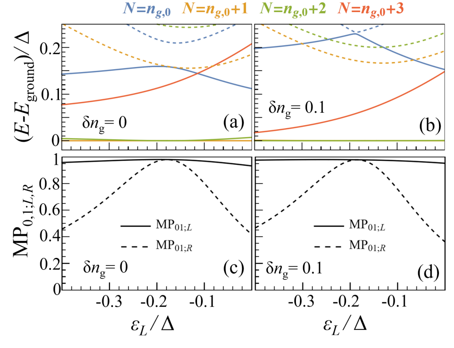
- Poor man's Majorana devices:
  controllable but non-protected
- They can be integrated with floating SC islands:
  charging energy
- 2-dot scenario: high Majorana polarization
- 4-dot scenario: definition of a non-local spin 1/2

Applications:

- Exotic transport properties:
  topological Kondo, Cooper pair splitters, ...
- Study of critical points
- Building blocks for artificial quantum matter
- Integration with readout schemes:
  reflectometry, transmons,...

Microscopic model and deviations away from the sweet spot

$E_c = 0.2\Delta,\ U = 5\Delta,\ E_Z = 1.5\Delta,\ t = 0.5\Delta,\ t^{(\mathrm{SO})} = 0.1\Delta$



$$H_I = \sum_{\sigma=\uparrow\downarrow} \varepsilon_c c^\dagger_\sigma c_\sigma + (\Delta c^\dagger_\uparrow c^\dagger_\downarrow \mathrm{e}^{-i\hat{\phi}} + \mathrm{H.c.}) + \mathrm{E_c}\left(\hat{\mathrm{N}}_\mathrm{I} - \mathrm{n_g}\right)^2$$

$$H_T = \sum_{\sigma\nu} \left(t_\nu d^\dagger_{\nu\sigma} c_\sigma + s_\nu s_\sigma t^{\mathrm{SO}}_\nu d^\dagger_{\nu\sigma} c_{\bar{\sigma}} + \mathrm{H.c.}\right)$$

$$H_d = \sum_{\nu,\sigma} \varepsilon_{\nu\sigma} d^\dagger_{\nu\sigma} d_{\nu\sigma} + U_\nu n_{\nu\uparrow} n_{\nu\downarrow}$$