Timescales for thermalization and many-body quantum chaos

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Quantum Science Generation 2025

ECT* Trento

05/05/2025

Jonathan Torres-Herrera

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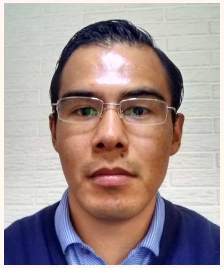

Isaías Vallejo

Adway Kumar Das

David Zarate

PRA **108**, 062201 (2023); PRB **110**, 075138 (2024); PRR **7**, 013181 (2025)

Timescales for thermalization and many-body quantum chaos

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- **Many-body quantum chaos**
- **Dynamical manifestation of many-body quantum chaos**
- **Thermalization time**

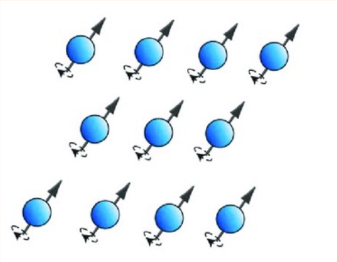
PRA **108**, 062201 (2023); PRB **110**, 075138 (2024); PRR **7**, 013181 (2025)

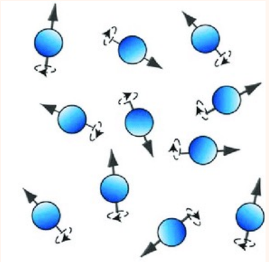
Coherent Evolution in Experiments

NMR

Solid state NMR: nuclear positions are fixed;
They are collectively addressed with magnetic pulses;
Very slow relaxation

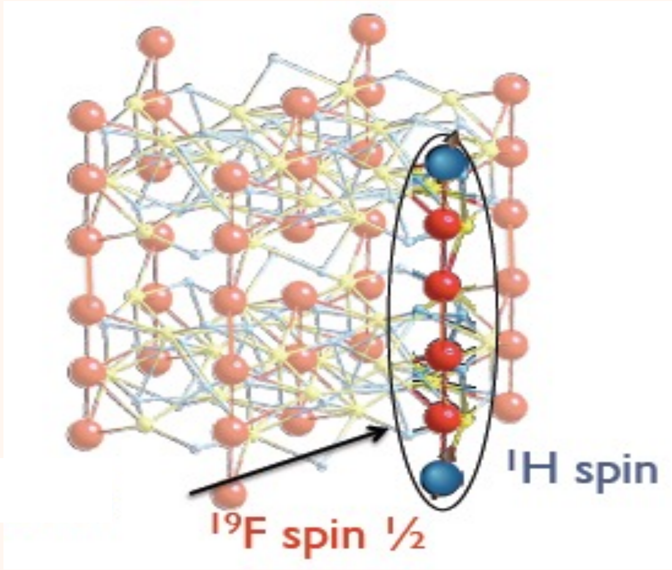
Cory (Waterloo)
Cappellaro (MIT)
Ramanathan (Dartmouth)





Fluorine spins-1/2 are arranged in linear chains.

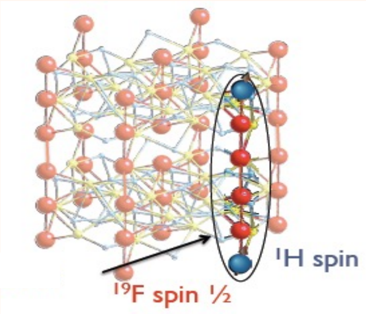
1D Spin-1/2 models



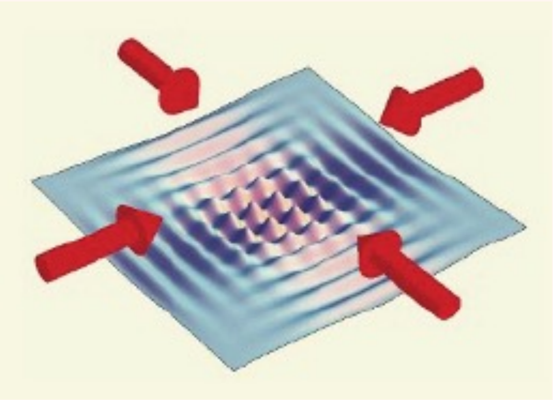
- Coherent evolution
- Pre-thermalization
- Many-body localization

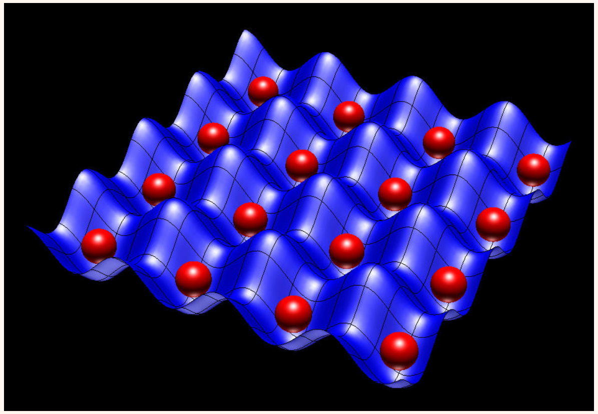
Coherent Evolution in Experiments

NMR



Optical Lattices





Bloch (Max Planck)
Esslinger (ETH)

Greiner (Harvard)
Weiss (Penn Sate)

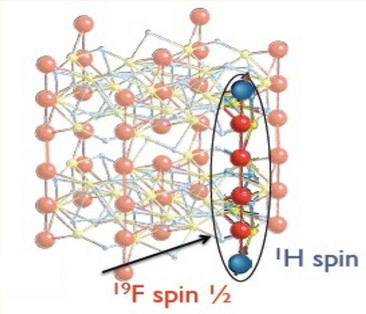
- Coherent evolution
- Pre-thermalization
- Many-body localization

- highly controllable systems – interactions, level of disorder, 1,2,3D
- quasi-isolated -- study evolution for very long time

Spin models (s=1/2 Heisenberg/Ising)
Bose-Hubbard model

Coherent Evolution in Experiments

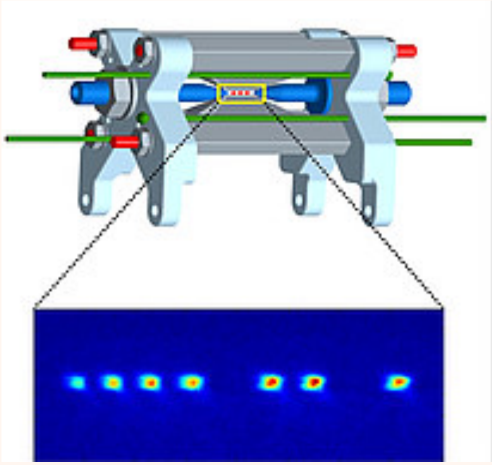
NMR



Ion Traps

Ions trapped via electric and magnetic fields.
Laser used to induce couplings.
Isolated from an external environment.

Long coherence times.
Long-range couplings.

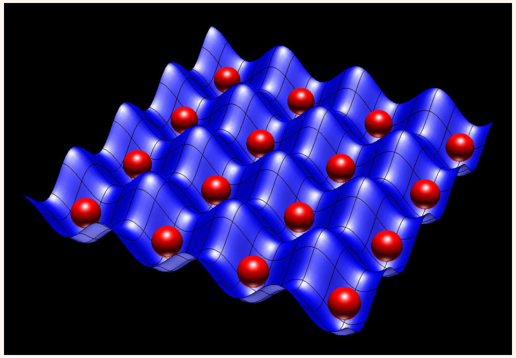


Blatt (Innsbrück)

Monroe (Maryland)

- Coherent evolution
- Pre-thermalization
- Many-body localization

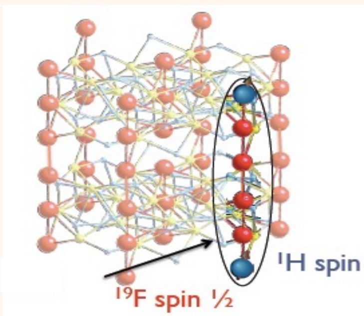
Optical Lattices



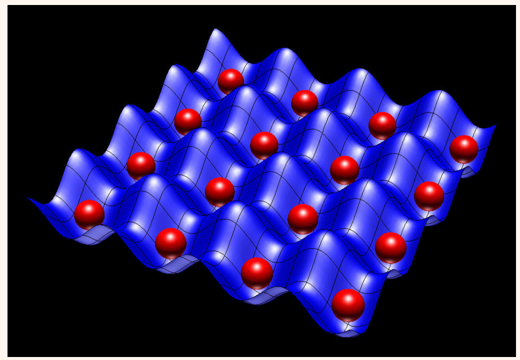
Spin-1/2 models

Complex Strongly Interacting Quantum Systems

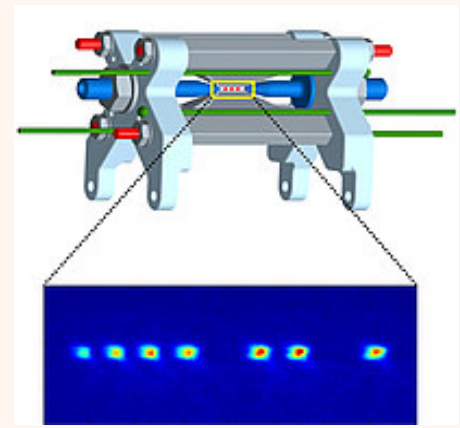
NMR



Optical Lattices

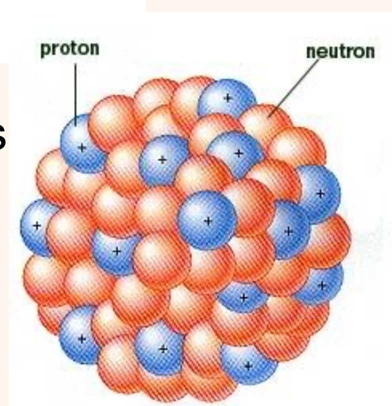


Ion Traps



FONDAZIONE BRUNO KESSLER

ECT*
EUROPEAN CENTRE FOR THEORETICAL STUDIES IN NUCLEAR PHYSICS AND RELATED AREAS



The research focus at the centre is **Nuclear Physics** in a broad sense and related areas.

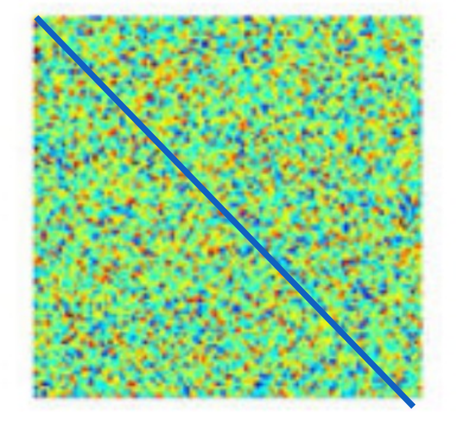
HISTORY

ECT* originated from the combined efforts of the European scientific community of **nuclear physicists…** according to the recommendations of the “community meeting” of October 1992, held at Orsay (France).

Full Random Matrices

- Matrices filled with random numbers: GOE (real and symmetric)

$$\langle H_{ij}^2 \rangle = \begin{cases} 1, & i = j \\ 1/2, & i \neq j. \end{cases}$$



RANDOM MATRICES: Wigner (1950s)
Study statistically the spectra of heavy nuclei
(atoms, molecules, quantum dots)

Full Random Matrices

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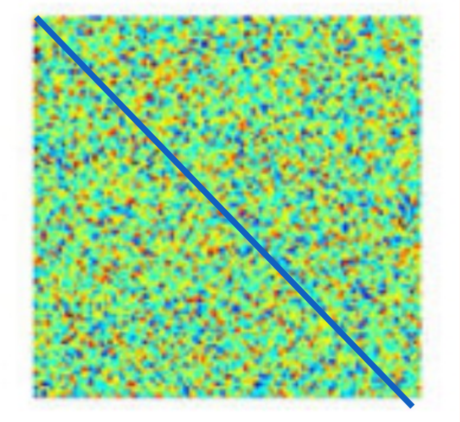


$$H|\alpha\rangle = E_\alpha |\alpha\rangle$$

Random vectors

$$|\alpha\rangle = \begin{pmatrix} C^{(1)} \\ C^{(2)} \\ C^{(3)} \\ C^{(4)} \\ C^{(5)} \\ \ldots \end{pmatrix}$$

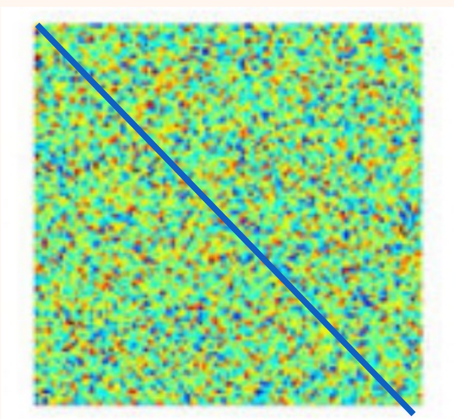
Gaussian
random numbers
(normalization)

Full Random Matrices

- Matrices filled with random numbers: GOE

Level repulsion
Rigid spectrum

$$\langle H_{ij}^2 \rangle = \begin{cases} 1, & i = j \\ 1/2, & i \neq j. \end{cases}$$



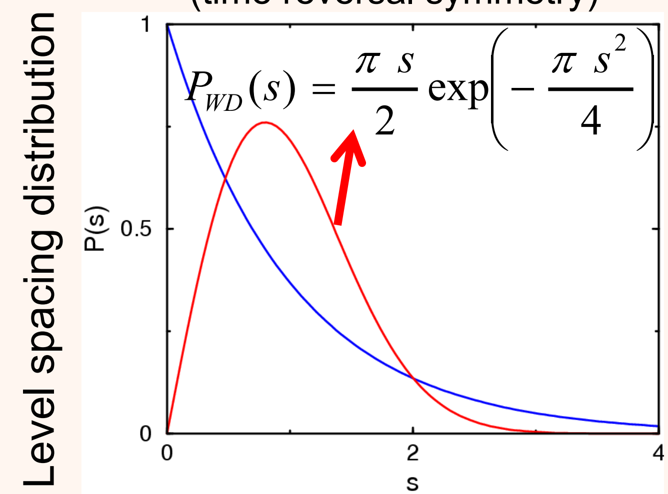
$$H|\alpha\rangle = E_\alpha |\alpha\rangle$$

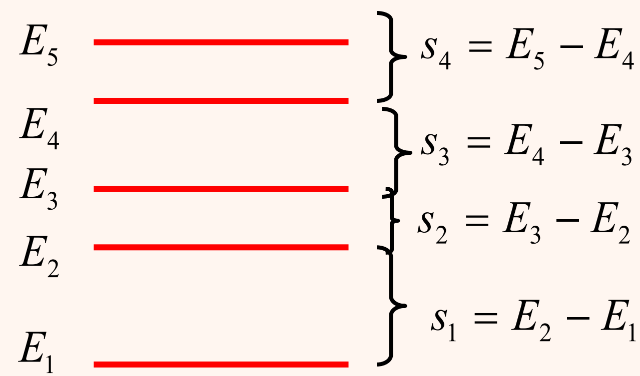
Random vectors

$$|\alpha\rangle = \begin{pmatrix} C^{(1)} \\ C^{(2)} \\ C^{(3)} \\ C^{(4)} \\ C^{(5)} \\ \dots \end{pmatrix}$$

Gaussian
random numbers
(normalization)

Wigner-Dyson distribution
(time reversal symmetry)





Full Random Matrices

- Matrices filled with random numbers: GOE

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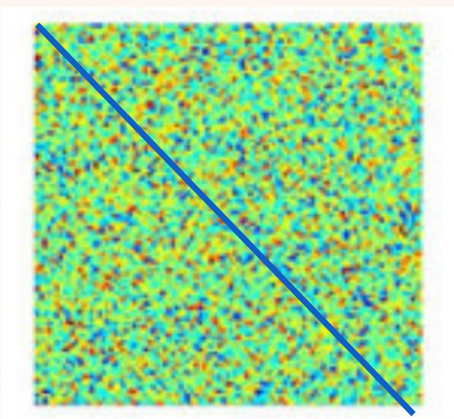


$$H|\alpha\rangle = E_\alpha |\alpha\rangle$$

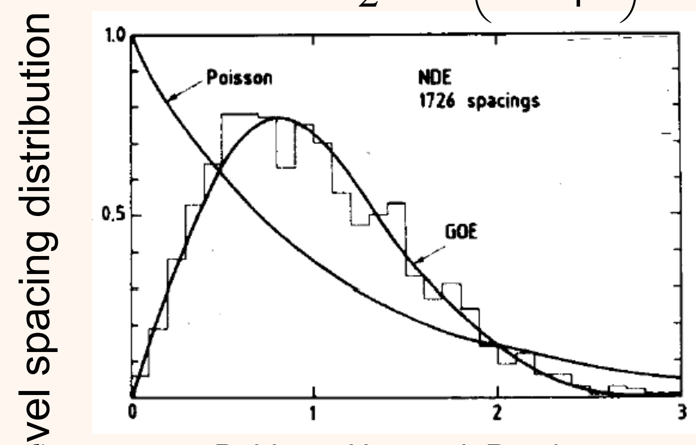
Random vectors

$$|\alpha\rangle = \begin{pmatrix} C^{(1)} \\ C^{(2)} \\ C^{(3)} \\ C^{(4)} \\ C^{(5)} \\ \dots \end{pmatrix}$$

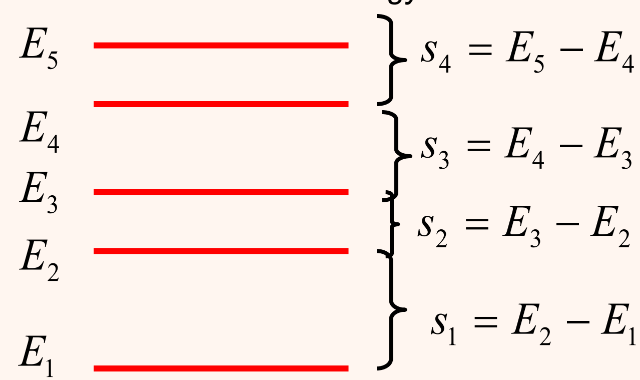
Gaussian random numbers (normalization)

Level repulsion; Rigid spectrum

$$P_{WD}(s) = \frac{\pi s}{2} \exp\left(-\frac{\pi s^2}{4} \right)$$

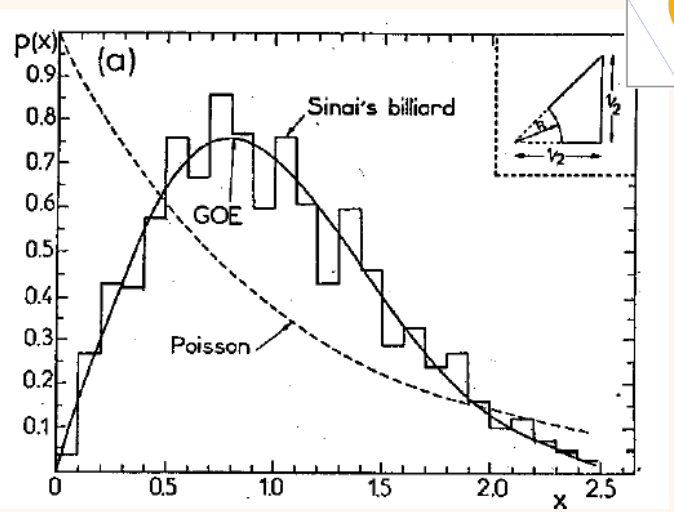


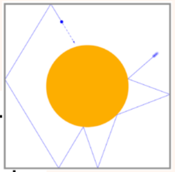
Bohigas, Haq and Pandey (1983)
Nuclear Data for Science and Technology



Level Spacing Distribution and Chaos

The nearest neighbor spacing distribution versus s for the ***quantum* Sinai billiard**. The histogram comprises about 1000 consecutive eigenvalues.





Level Statistics vs Classical chaos

Quantum chaos
=
signatures of classical chaos found in the quantum domain

Correspondence well established for systems with few degrees of freedom

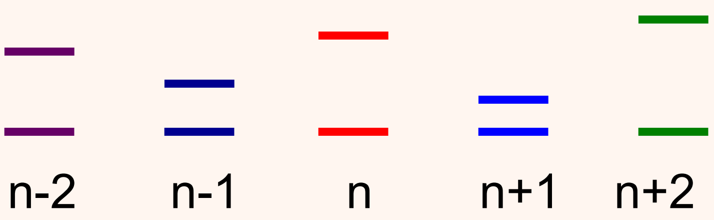
*) G. Casati, F. Valz-Gris, and I. Guarneri, On the connection between quantization of nonintegrable systems and statistical theory of spectra, Lett. Nuovo Cimento 28, 279 (1980).
*) O. Bohigas, M. J. Giannoni, and C. Schmit, Characterization of Chaotic Quantum Spectra and Universality of Level Fluctuation Laws, Phys. Rev. Lett. 52, 1 (1984).

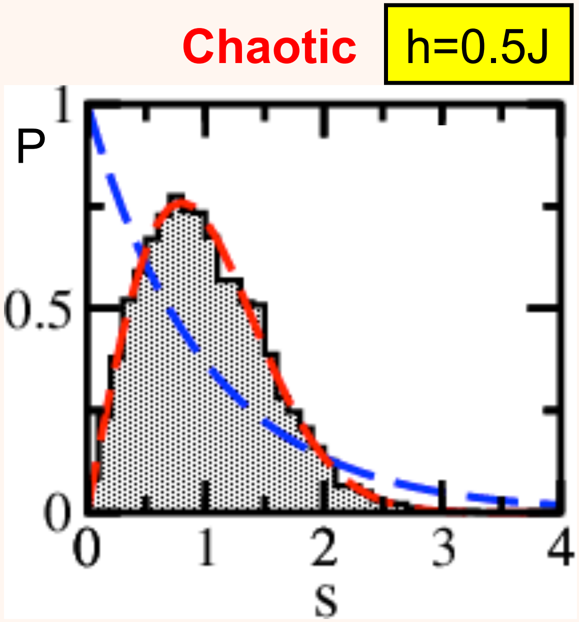
Physical Model

1D spin-1/2 system with nearest-neighbor couplings and onsite disorder

$$H = \sum_{n=1}^{L} \frac{h_n}{2}\sigma_n^z + \sum_{n=1}^{L} \frac{J}{4}\left[\sigma_n^z\sigma_{n+1}^z + \left(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y\right)\right]$$

$$h_n \in [-h, h]$$





Many-body quantum chaos: interaction between particles

Thermalization vs Quantum Chaos

Full Random Matrices vs Physical Models

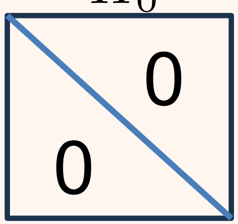
Thermalization

$$\langle O(t)\rangle = \langle \Psi(t)|O|\Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} O_{\beta\alpha} + \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha}$$

$$|\Psi(t)\rangle = \sum_\alpha e^{-iE_\alpha t}|\alpha\rangle \qquad O_{\beta\alpha} = \langle\beta|O|\alpha\rangle$$

$$H|\alpha\rangle = E_\alpha|\alpha\rangle$$

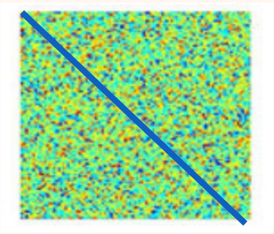
H_0



$$H_0|n\rangle = \epsilon_n|n\rangle$$

$$|\Psi(0)\rangle = |n_0\rangle$$

$H = H_0 + V$



$$|\Psi(t)\rangle = e^{-iHt}|\Psi(0)\rangle$$

Thermalization

$$\langle O(t) \rangle = \langle \Psi(t) | O | \Psi(t) \rangle = \sum_{\alpha \neq \beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} O_{\beta\alpha} + \sum_\alpha | C_\alpha^{ini} |^2 O_{\alpha\alpha}$$

$$\langle |C_\alpha|^2 \rangle = \frac{1}{D} \qquad O_{\beta\alpha} = \langle \beta | O | \alpha \rangle$$

$$H|\alpha\rangle = E_\alpha |\alpha\rangle$$

$$|\alpha\rangle = \begin{pmatrix} C_\alpha^1 \\ C_\alpha^2 \\ C_\alpha^3 \\ C_\alpha^4 \\ C_\alpha^5 \\ \dots \end{pmatrix} \Rightarrow |\Psi(0)\rangle = \begin{pmatrix} C_1^{ini} \\ C_2^{ini} \\ C_3^{ini} \\ C_4^{ini} \\ C_5^{ini} \\ \dots \end{pmatrix}$$

Thermalization

Rigid spectrum

$$\langle O(t)\rangle = \langle \Psi(t)|O|\Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} O_{\beta\alpha} + \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha}$$

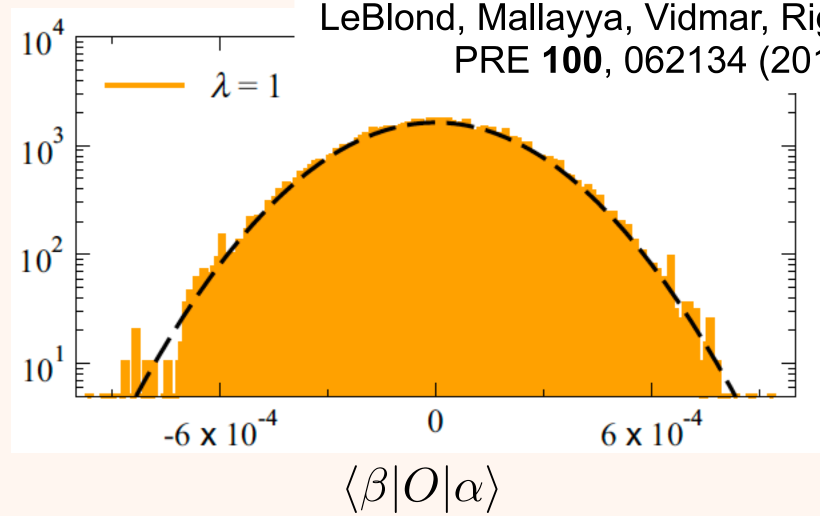
$$\langle |C_\alpha|^2\rangle = \frac{1}{D} \qquad O_{\beta\alpha} = \langle\beta|O|\alpha\rangle$$

$$H|\alpha\rangle = E_\alpha|\alpha\rangle$$

Beugeling, Moessner, Haque
PRE **91**, 012144 (2015)
(Gaussian distribution)

$$|\alpha\rangle = \begin{pmatrix} C_\alpha^1 \\ C_\alpha^2 \\ C_\alpha^3 \\ C_\alpha^4 \\ C_\alpha^5 \\ \dots \end{pmatrix} \Rightarrow |\Psi(0)\rangle = \begin{pmatrix} C_1^{ini} \\ C_2^{ini} \\ C_3^{ini} \\ C_4^{ini} \\ C_5^{ini} \\ \dots \end{pmatrix}$$

<u>Off-diagonal ETH</u>



LeBlond, Mallayya, Vidmar, Rigol
PRE **100**, 062134 (2019)

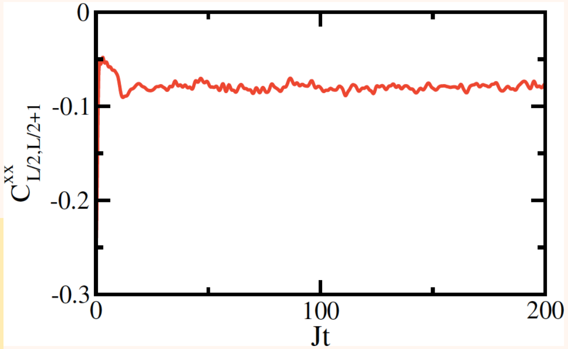
Thermalization

$$\langle O(t)\rangle = \langle \Psi(t)|O|\Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} O_{\beta\alpha} + \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha}$$

$$O_{\alpha\alpha} = \langle\alpha|O|\alpha\rangle$$

<u>Equilibration</u>:

Size of the fluctuations
PRE **88**, 032913 (2013)
SciPostPhys**15**, 244 (2023)



Thermalization

$$\langle O(t)\rangle = \langle \Psi(t)|O|\Psi(t)\rangle = \sum_{\alpha\neq\beta} C_\beta^{ini*} C_\alpha^{ini} e^{i(E_\beta - E_\alpha)t} O_{\beta\alpha} + \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha}$$

$$\langle\alpha|O|\alpha\rangle$$

Infinite time average — Thermodynamic average

$$\overline{\langle O(t)\rangle} \equiv \sum_\alpha |C_\alpha^{ini}|^2 O_{\alpha\alpha} \overset{=?}{\longleftrightarrow} O_{micro} \equiv \frac{1}{\mathcal{N}_{E_0,\Delta E}} \sum_{\substack{\alpha \\ |E_0 - E_\alpha| < \Delta E}} O_{\alpha\alpha}$$

depends on the initial conditions — depends only on the energy

$$|\alpha\rangle = \begin{pmatrix} C_\alpha^1 \\ C_\alpha^2 \\ C_\alpha^3 \\ C_\alpha^4 \\ C_\alpha^5 \\ \dots \end{pmatrix}$$

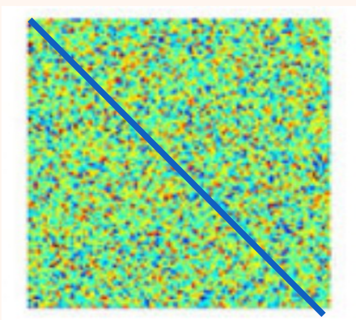
Normalized random vectors

ETH: the expectation values $O_{\alpha\alpha}$ of few-body observables do not fluctuate for eigenstates close in energy

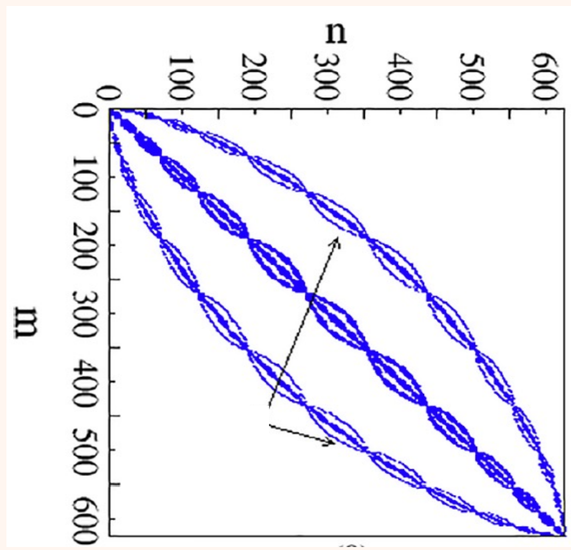
Random Matrices vs Physical Systems

GOE (real and symmetric)

$$\langle H_{ij}^2 \rangle = \begin{cases} 1, & i = j \\ 1/2, & i \neq j. \end{cases}$$



Physical
many-body quantum systems



$$\langle \uparrow \downarrow \uparrow \downarrow | H | \uparrow \downarrow \downarrow \uparrow \rangle = J_{34}$$

$$\langle \downarrow \uparrow \uparrow \downarrow | H | \downarrow \uparrow \downarrow \uparrow \rangle = J_{34}$$

Random Matrices vs Physical Systems

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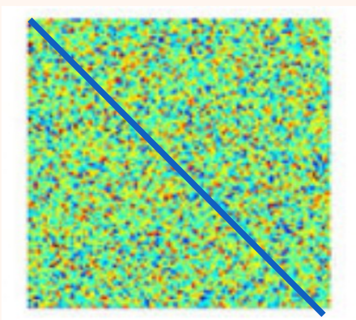

Physical
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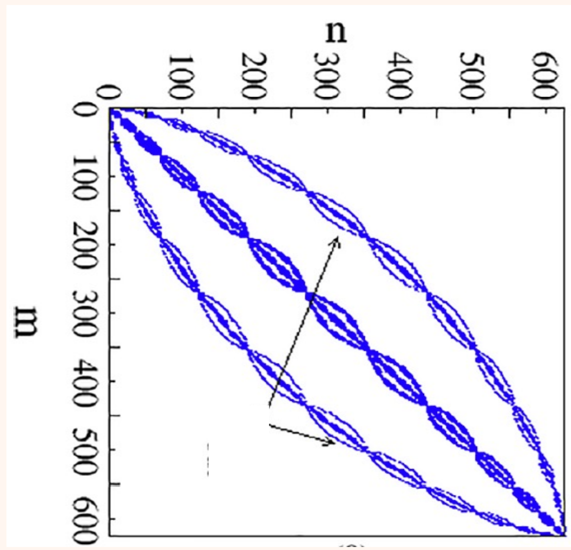

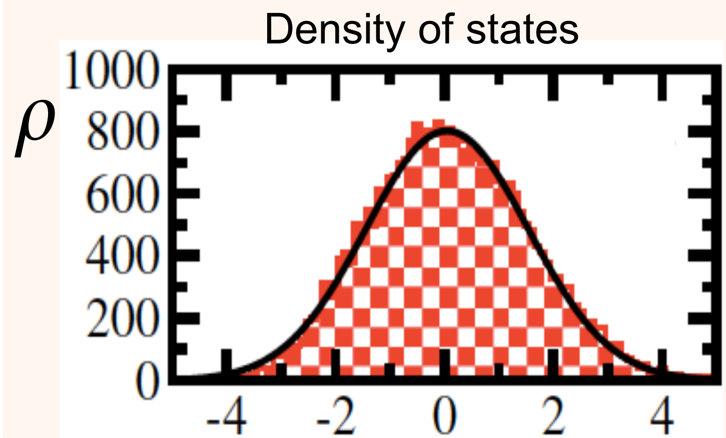

French & Wong, PLB (1970)

E_α

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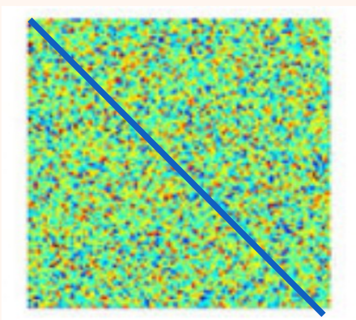
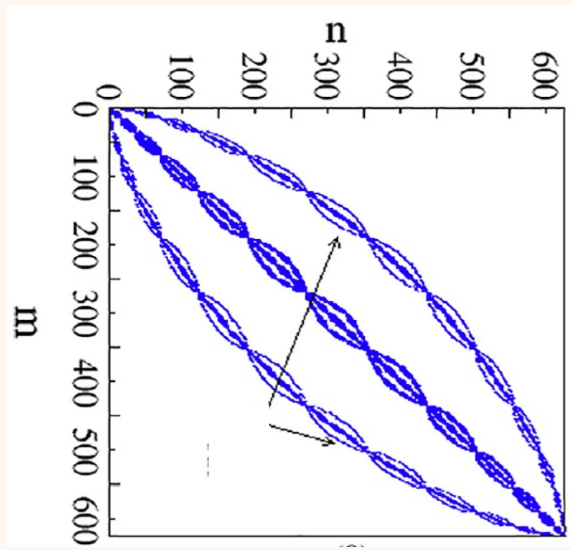
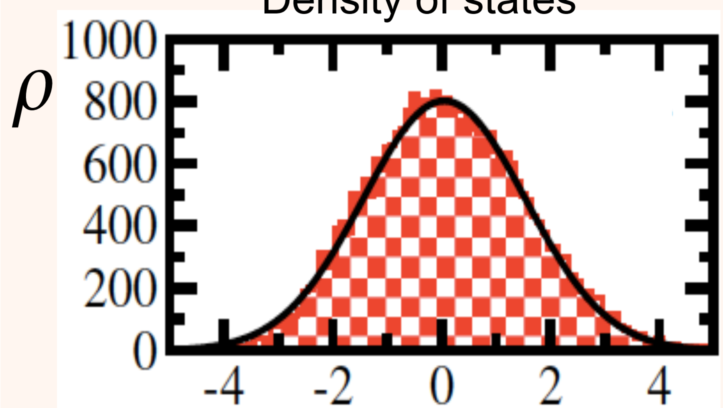


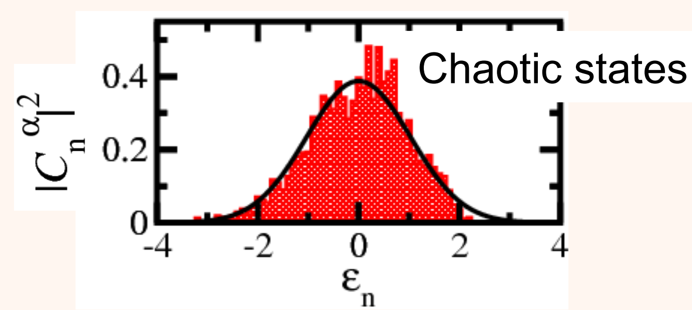
Physical
many-body quantum systems





French & Wong, PLB (1970) E_α

Thermalization in realistic systems happens **away** from the edges of the spectrum and for **few-body observables**



LFS, Borgonovi, Izrailev
PRE **85**, 036209 (2012)

New developments:
deep thermalization; non-abelian ETH.

Timescales and Thermalization

Full Random Matrices vs Physical Models

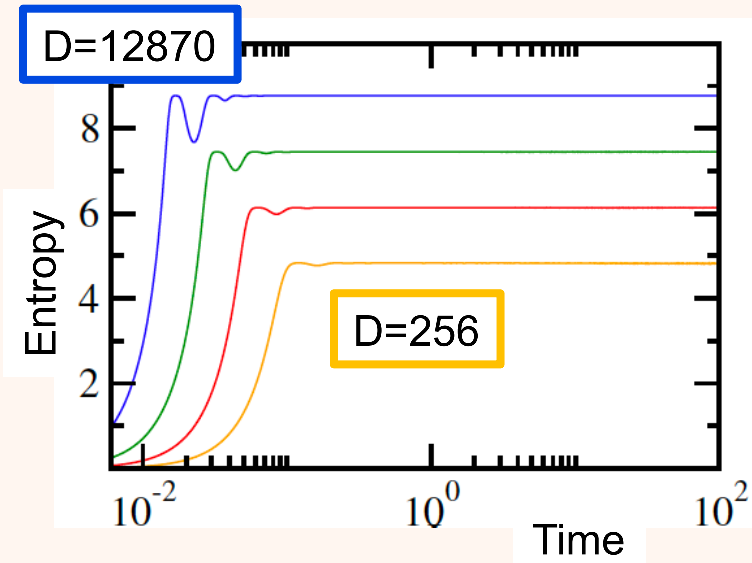
Timescales: GOE

"Participation" entropy: $-\ln(\sum_n |\langle n|\Psi(t)\rangle|^4)$

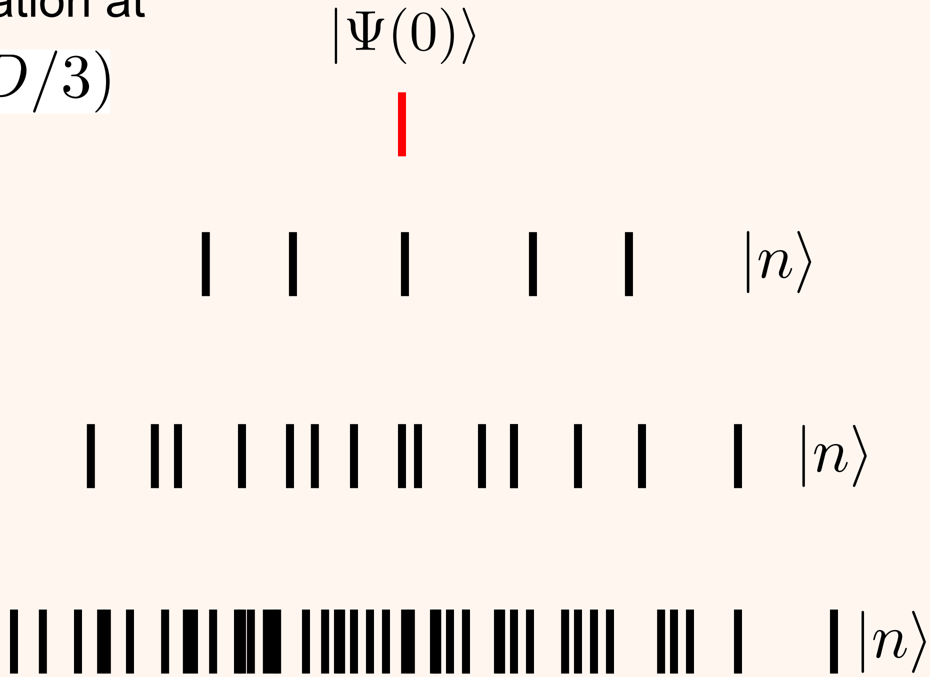
$$H_0|n\rangle = \epsilon_n|n\rangle$$

$$H = H_0 + V$$

GOE model



Saturation at $\ln(D/3)$

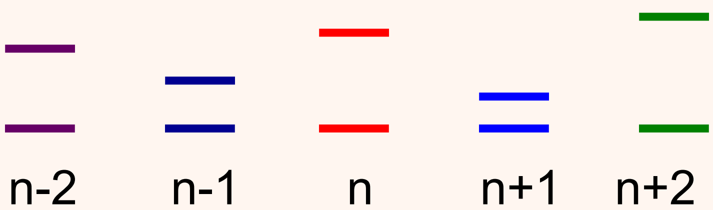


Physical Model

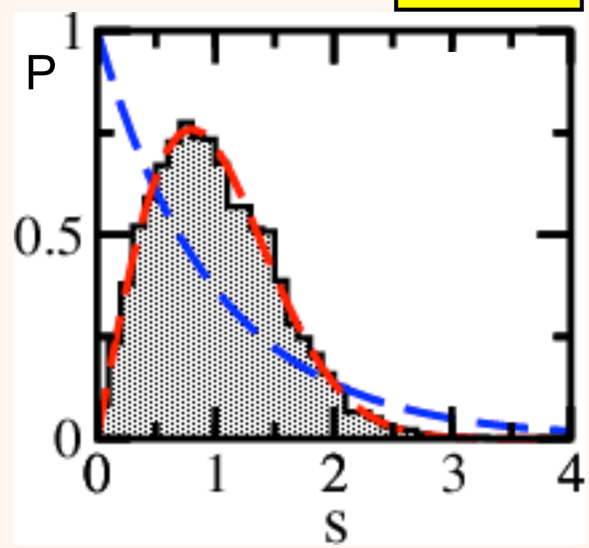
1D spin-1/2 system with nearest-neighbor couplings and onsite disorder

$$H = \sum_{n=1}^{L} \frac{h_n}{2} \sigma_n^z + \sum_{n=1}^{L} \frac{J}{4} \left[\sigma_n^z \sigma_{n+1}^z + \left(\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y \right) \right]$$

$$h_n \in [-h, h]$$



Chaotic h=0.5J



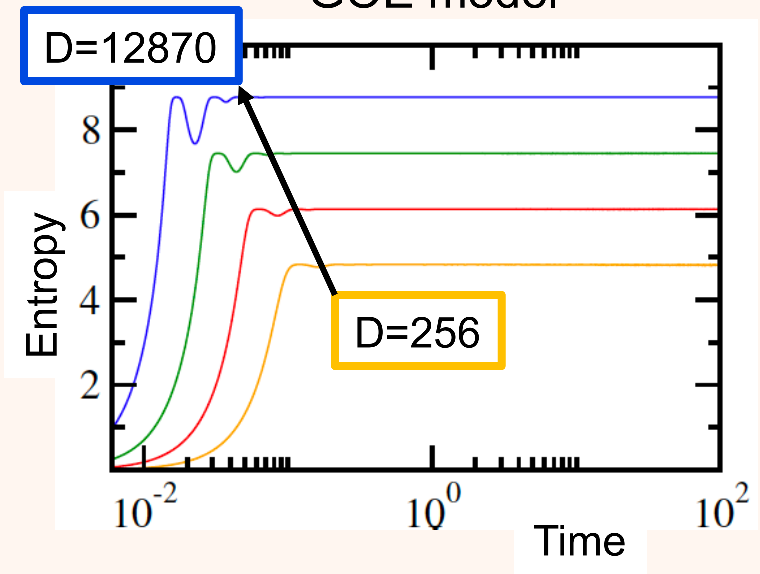
Initial states with energy in the middle of the spectrum

$$\Psi(0) = \uparrow \downarrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \uparrow$$

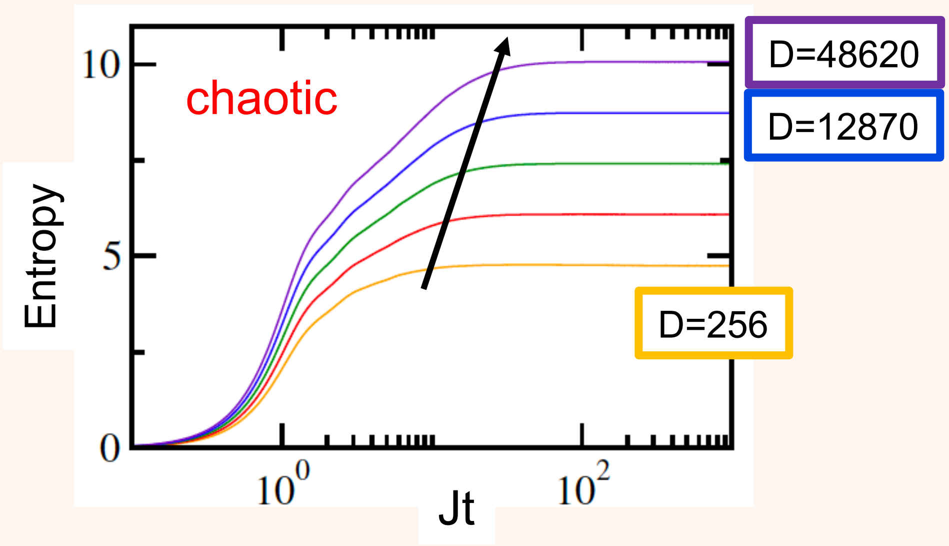
Timescales: GOE vs Physical Model

"Participation" entropy: $-\ln(\sum_n |\langle n|\Psi(t)\rangle|^4)$

GOE model



1D disordered spin-1/2 model



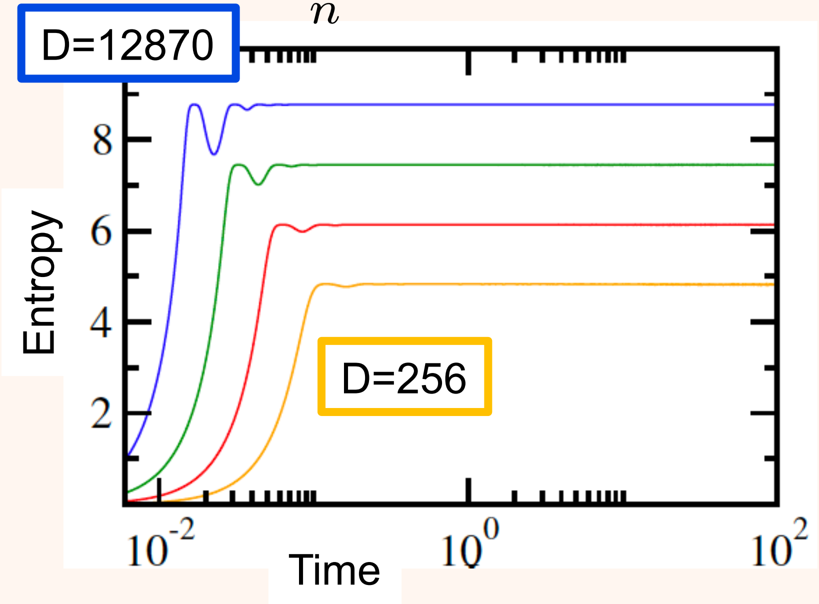
$$t^* \propto L^\gamma$$

PRB **104**, 085117 (2021)

Timescales: GOE

“Participation” Entropy:

$$-\ln\left(\sum_n |\langle n|\Psi(t)\rangle|^4\right)$$



Survival Probability:

$$\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$$

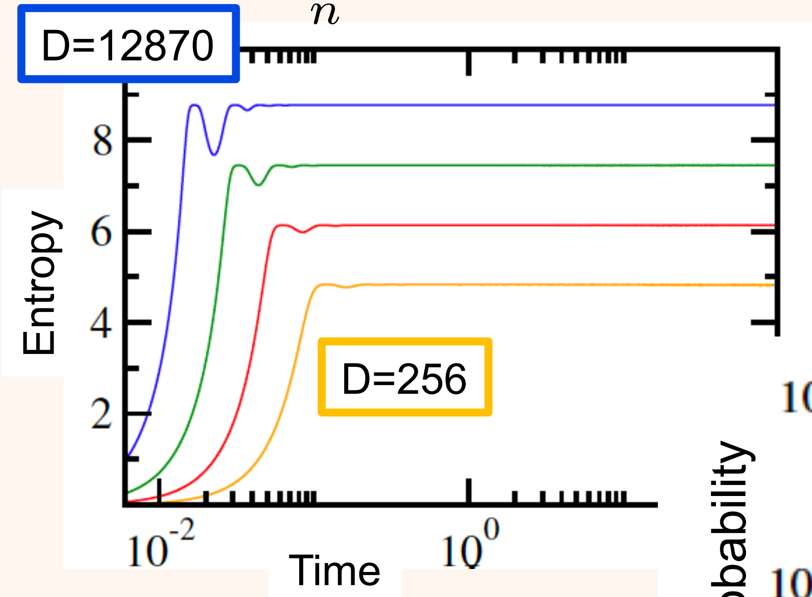
$$\frac{1-\overline{SP}}{D-1}\left[D\frac{\mathcal{J}_1^2(2\Gamma t)}{(\Gamma t)^2} - b_2\left(\frac{\Gamma t}{2D}\right)\right] + \overline{SP}$$

PRB **97**, 060303 (R) (2018)
PRB **99**, 174313 (2019)

Timescales: GOE

"Participation" Entropy:

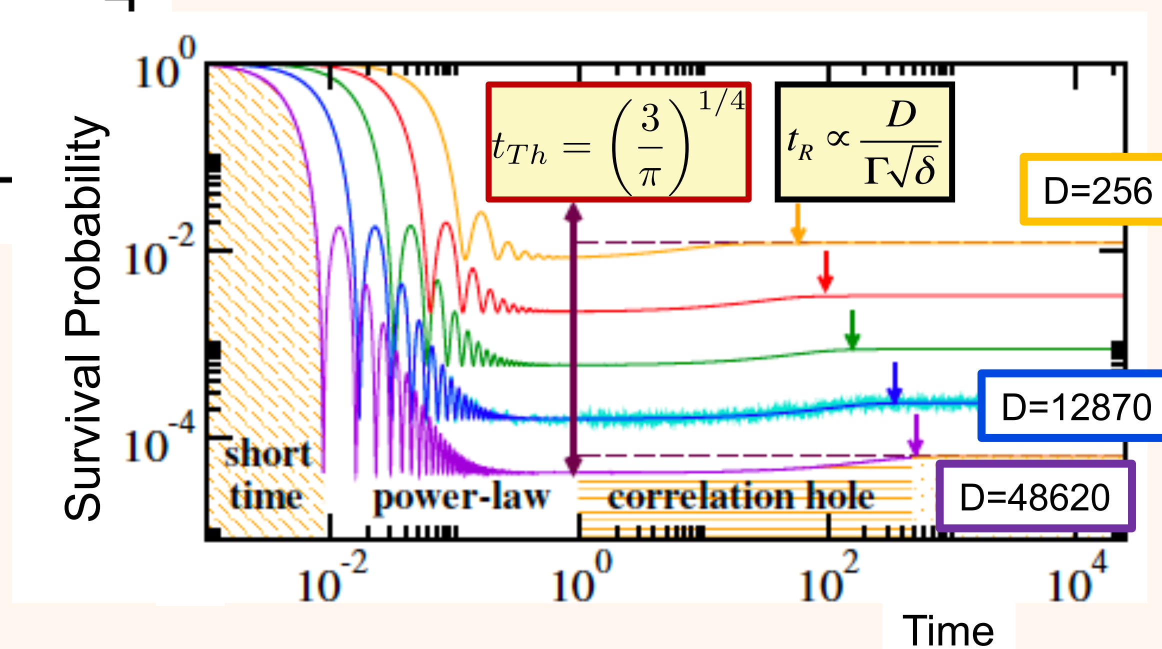
$$-\ln\left(\sum_n |\langle n|\Psi(t)\rangle|^4\right)$$



Survival Probability:

$$\left|\langle\Psi(0)\,|\,\Psi(t)\rangle\right|^2$$

$$\frac{1-\overline{SP}}{D-1}\left[D\frac{\mathcal{J}_1^2(2\Gamma t)}{(\Gamma t)^2}-b_2\left(\frac{\Gamma t}{2D}\right)\right]+\overline{SP}$$



PRB **97**, 060303 (R) (2018)
PRB **99**, 174313 (2019)

Correlation Hole

Dynamical manifestation of quantum chaos

Survival Probability and Spectral Form Factor

Survival Probability: $\left|\langle \Psi(0) \mid \Psi(t) \rangle\right|^2$

$\Psi(0) = \uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow$

$$\langle SP(t)\rangle = \langle \sum_{\alpha,\beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 e^{-i(E_\alpha - E_\beta)t} \rangle$$

$C_\alpha^{ini} = \langle \alpha | \Psi(0) \rangle$

Quench dynamics
(cold atoms, ion traps)

Spectral form factor

$$\mathrm{SFF}(t) = \frac{1}{D^2} \left\langle \sum_{\alpha,\beta} e^{i(E_\alpha - E_\beta)t} \right\rangle$$

Survival Probability and Spectral Form Factor

Survival Probability: $\left|\langle \Psi(0) | \Psi(t) \rangle\right|^2$

$\Psi(0) = \uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow$

$$C_\alpha^{ini} = \langle \alpha | \Psi(0) \rangle$$

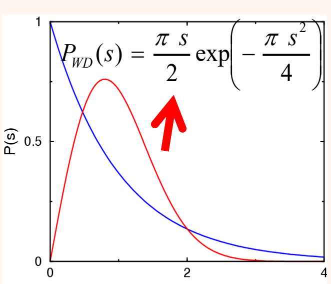
$$\langle SP(t) \rangle = \langle \sum_{\alpha,\beta} |C_\alpha^{ini}|^2 |C_\beta^{ini}|^2 e^{-i(E_\alpha - E_\beta)t} \rangle$$

Quench dynamics
(cold atoms, ion traps)

Spectral form factor

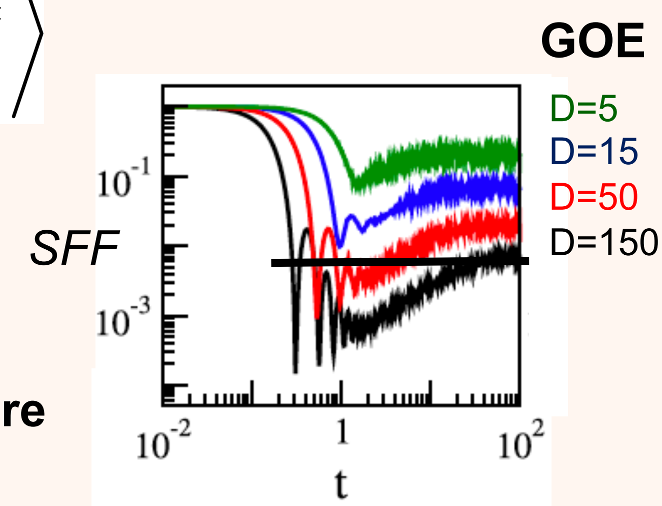
$$\mathrm{SFF}(t) = \frac{1}{D^2} \left\langle \sum_{\alpha,\beta} e^{i(E_\alpha - E_\beta)t} \right\rangle$$

Fourier transform of the
two-point spectral correlation function



Signature of spectral correlations:

Slope-dip-ramp-plateau structure



Correlation Hole

VOLUME 56, NUMBER 23 PHYSICAL REVIEW LETTERS 9 JUNE 1986

Fourier Transform: A Tool to Measure Statistical Level Properties in Very Complex Spectra

Luc Leviandier, Maurice Lombardi, Rémi Jost, and Jean Paul Pique
Laboratoire de Spectrométrie Physique, Université Scientifique et Médicale de Grenoble, 38402 Saint Martin d'Hères, France, and Service National des Champs Intenses, Centre National de la Recherche Scientifique, 38042 Grenoble Cedex, France
(Received 27 November 1985)

VOLUME 58, NUMBER 5 PHYSICAL REVIEW LETTERS 2 FEBRUARY 1987

Chaos and Dynamics on 0.5–300-ps Time Scales in Vibrationally Excited Acetylene: Fourier Transform of Stimulated-Emission Pumping Spectrum

J. P. Pique,[a] Y. Chen, R. W. Field, and J. L. Kinsey
Department of Chemistry and George Harrison Spectroscopy Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 27 October 1986)

Chemical Physics 146 (1990) 21–38
North-Holland

Correlations in anticrossing spectra and scattering theory. Analytical aspects

T. Guhr and H.A. Weidenmüller
Max-Planck-Institut für Kernphysik, 6900 Heidelberg, FRG

Received 12 December 1989

Experimental results of anticrossing spectroscopy in molecules, in particular the correlation hole, are discussed in a theoretical model. The laser measurements are modelled in terms of the scattering matrix formalism originally developed for compound nucleus scattering. Random matrix theory is used in the framework of this model. The correlation hole is analytically derived for small singlet–triplet coupling. In the case of the data on methylglyoxal this limit is realistic if the spectrum is indeed a superposition of several pure sequences as one can conclude from the analysis of the measurements.

PHYSICAL REVIEW E, VOLUME 65, 026214

Signatures of the correlation hole in total and partial cross sections

T. Gorin* and T. H. Seligman
Centro de Ciencias Fisicas, University of Mexico (UNAM), CP 62210 Cuernavaca, Mexico
(Received 3 August 2001; published 24 January 2002)

In a complex scattering system with few open channels, say a quantum dot with leads, the correlation properties of the poles of the scattering matrix are most directly related to the internal dynamics of the system. We may ask how to extract these properties from an analysis of cross sections. In general this is very difficult, if we leave the domain of isolated resonances. We propose to consider the cross correlation function of two different elastic or total cross sections. For these we can show numerically and to some extent also analytically a significant dependence on the correlations between the scattering poles. The difference between uncorrelated and strongly correlated poles is clearly visible, even for strongly overlapping resonances.

J. Phys. A: Math. Theor. 46 (2013) 275303 (12pp) doi:10.1088/1751-8113/46/27/275303

Fidelity under isospectral perturbations: a random matrix study

F Leyvraz[1,2], A García[1], H Kohler[3] and T H Seligman[1,2]

PHYSICAL REVIEW A VOLUME 46, NUMBER 8 15 OCTOBER 1992

Spectral autocorrelation function in the statistical theory of energy levels

Y. Alhassid
*Center for Theoretical Physics, Sloane Physics Laboratory, Yale University, New Haven, Connecti
and the A.W. Wright Nuclear Structure Laboratory, Yale University, New Haven, Connecticut*

R. D. Levine
The Fritz Haber Research Center for Molecular Dynamics, The Hebrew University, Jerusalem 91
(Received 11 October 1991; revised manuscript received 5 May 1992)

VOLUME 67, NUMBER 10 PHYSICAL REVIEW LETTERS 2 SEPTEMBER 1991

Time-Dependent Manifestations of Quantum Chaos

Joshua Wilkie and Paul Brumer
Chemical Physics Theory Group, Department of Chemistry, University of Toronto, Toronto, Ontario, Canada M5S 1A1
(Received 11 April 1991)

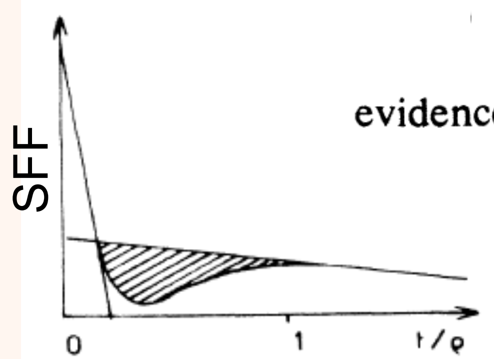
Correlation Hole

VOLUME 56, NUMBER 23 PHYSICAL REVIEW LETTERS 9 JUNE 1986

Fourier Transform: A Tool to Measure Statistical Level Properties in Very Complex Spectra

Luc Leviandier, Maurice Lombardi, Rémi Jost, and Jean Paul Pique
Laboratoire de Spectrométrie Physique, Université Scientifique et Médicale de Grenoble, 38402 Saint Martin d'Hères, France, and Service National des Champs Intenses, Centre National de la Recherche Scientifique, 38042 Grenoble Cedex, France
(Received 27 November 1985)

We show that the Fourier transform of very complex spectra gives a sound measurement of long-range statistical properties of levels even in cases of badly resolved, poorly correlated spectra. Examples of nuclear energy levels, highly excited acetylene vibrational levels, and singlet-triplet anticrossing spectra in methylglyoxal are displayed.



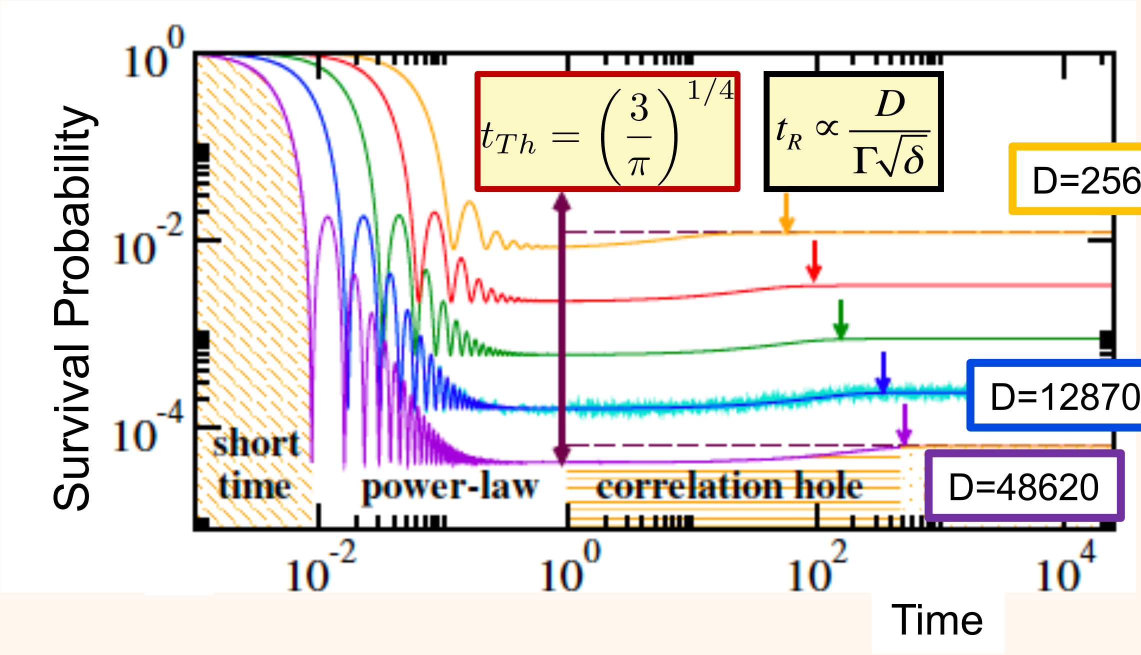
The presence of level correlation is thus evidenced by a "correlation hole"

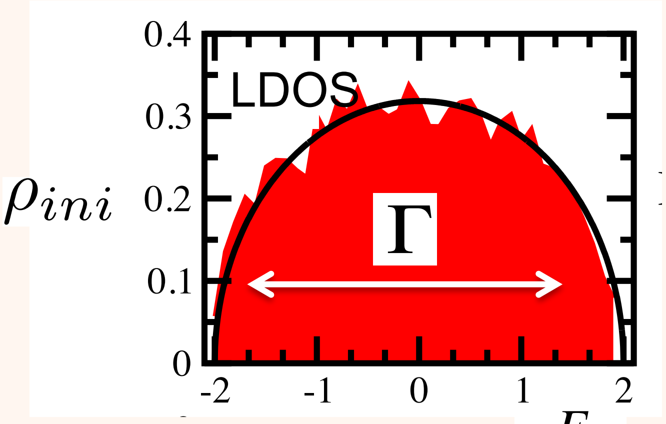
Survival Probability: GOE

$$|\langle \Psi(0) | \Psi(t) \rangle|^2$$

$$\frac{1-\overline{SP}}{D-1}\left[D\frac{\mathcal{J}_1^2(2\Gamma t)}{(\Gamma t)^2} - b_2\left(\frac{\Gamma t}{2D}\right)\right] + \overline{SP}$$

2-level form factor



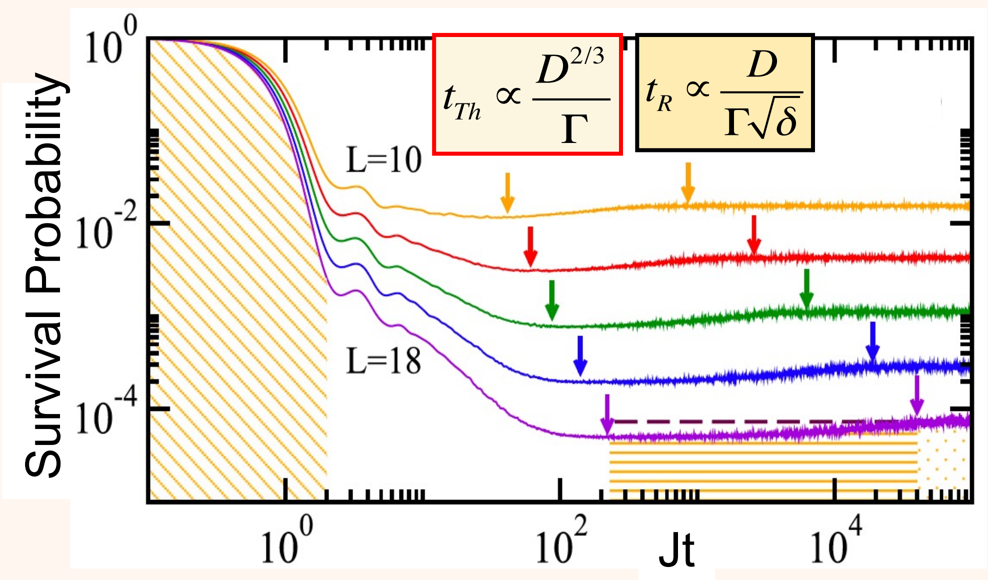


PRB **97**, 060303 (R) (2018)
PRB **99**, 174313 (2019)

Survival Probability: Physical Model

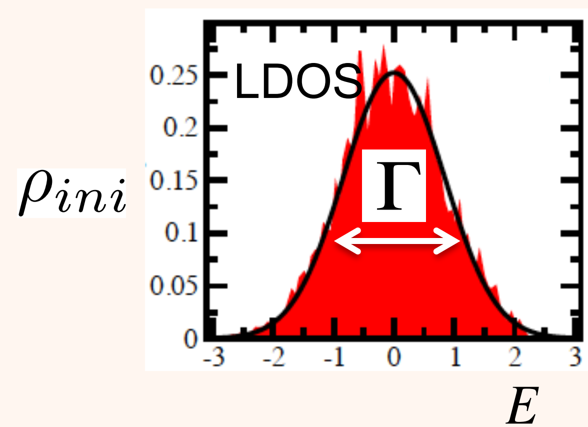
$$\Psi(0) = \uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow$$

$$|\langle \Psi(0)|\Psi(t)\rangle|^2$$



$$\frac{1-\overline{SP}}{(D-1)}\left[\frac{De^{-\Gamma^2 t^2}}{\mathcal{N}^2}\mathcal{F}(t) - b_2\left(\frac{\Gamma t}{\sqrt{2\pi}D}\right)\right] + \overline{SP}$$

$$\mathcal{F}(t) = \left|\text{erf}\left(\frac{E_{\max} + it\Gamma^2}{\sqrt{2}\Gamma}\right) - \text{erf}\left(\frac{E_{\min} + it\Gamma^2}{\sqrt{2}\Gamma}\right)\right|^2$$



$$H = \sum_{n=1}^{L}\frac{h_n}{2}\sigma_n^z + \sum_{n=1}^{L}\frac{J}{4}\left[\sigma_n^z\sigma_{n+1}^z + \left(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y\right)\right]$$

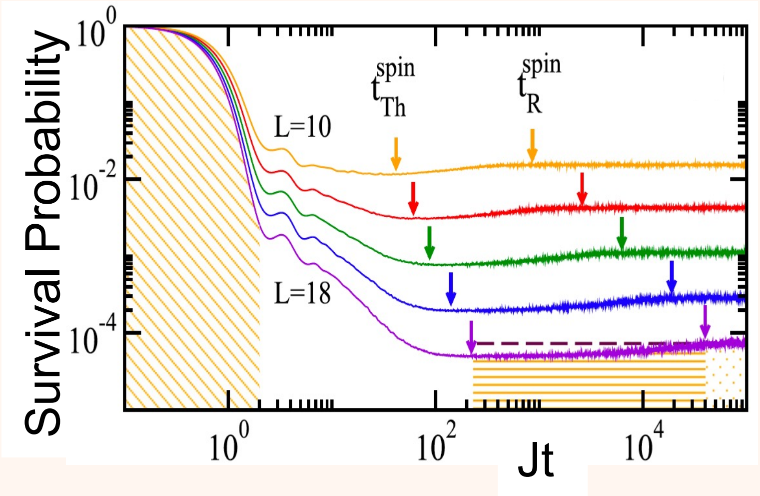
PRB **97**, 060303 (R) (2018)
PRB **99**, 174313 (2019)

Correlation Hole: Advantages/Disadvantages

ADVANTAGES:

Short- and long-range correlation
Emerges despite symmetries

Dynamical quantity



DISADVANTAGES:

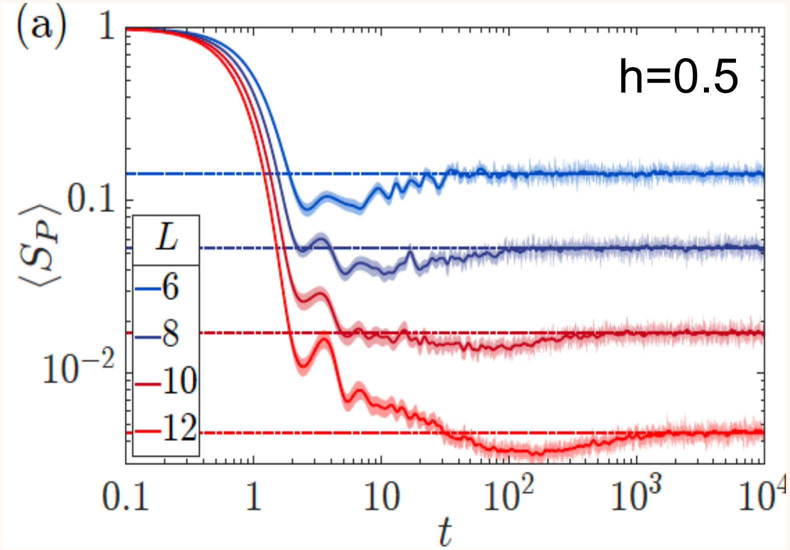
Long-times

Non-local quantity

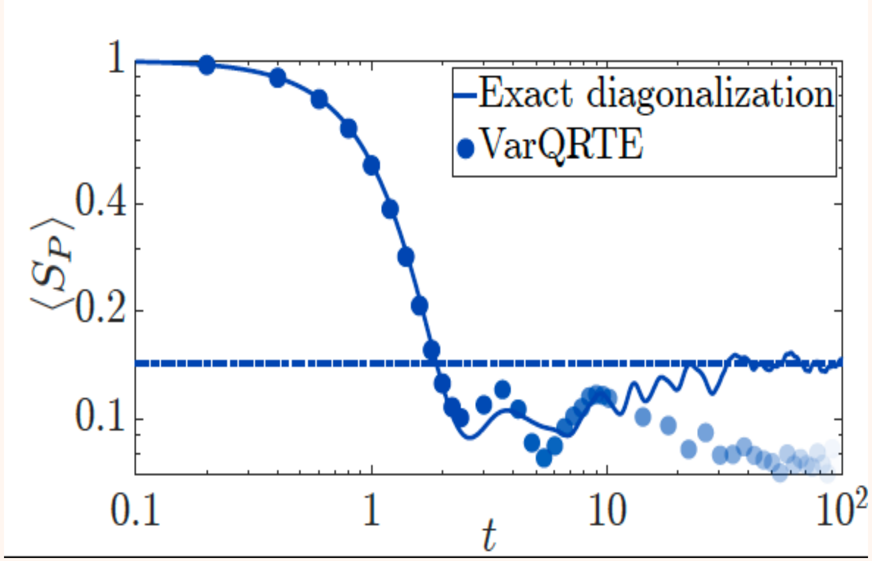
Non-self-averaging

Experimental Detection of the Correlation Hole

Small Systems



$$|\langle \Psi(0)|\Psi(t)\rangle|^2$$

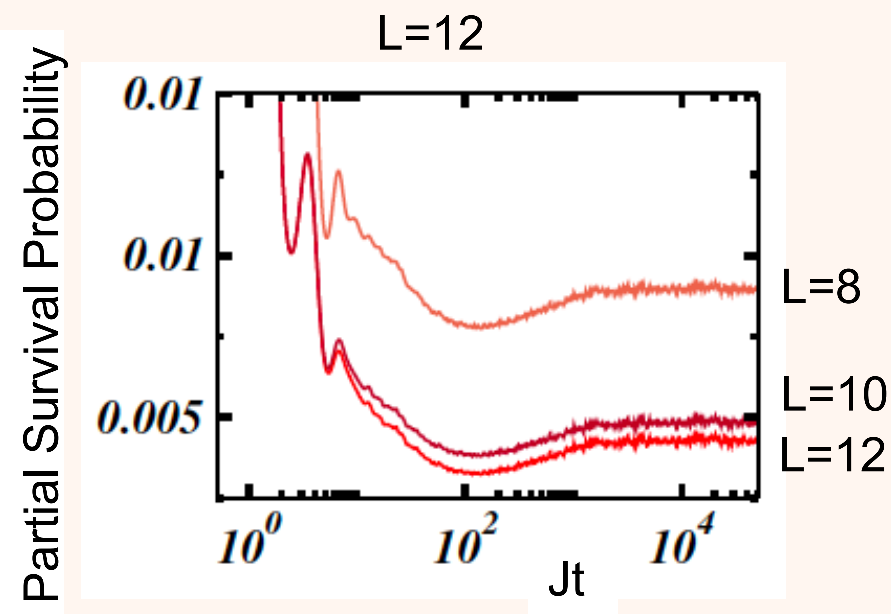


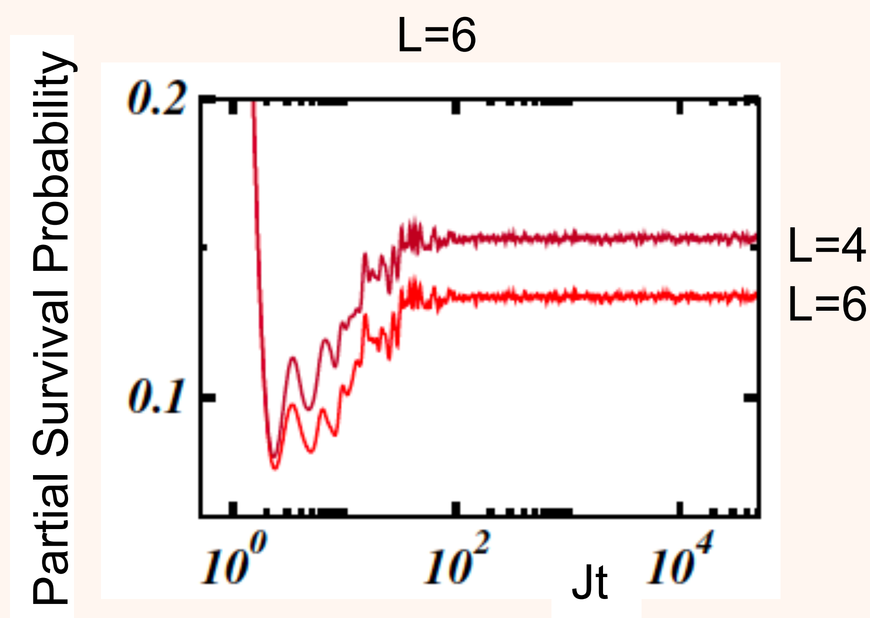
Proposal for many-body quantum chaos detection
PRR **7**, 013181 (2025)

Quasi-local Observable

Partial
Survival Probability

$$S_P^{(2,4)}(t) = \left| \langle \uparrow \boxed{\downarrow\uparrow} \downarrow | e^{-iHt} | \uparrow \boxed{\downarrow\uparrow} \downarrow \rangle \right|^2 + \left| \langle \downarrow \boxed{\downarrow\uparrow} \uparrow | e^{-iHt} | \uparrow \boxed{\downarrow\uparrow} \downarrow \rangle \right|^2$$





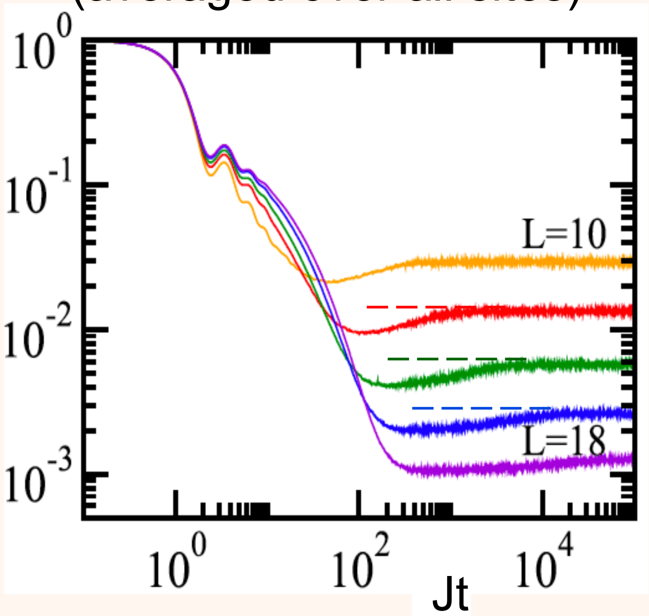
Immanuel Bloch, Monika Aidelsburger
arXiv:2501.16995

Local Observable

Spin Autocorrelation Function:

$$\frac{1}{L}\sum_{k=1}^{L}\langle\Psi(0)|\hat{\sigma}_k^z\hat{\sigma}_k^z(t)|\Psi(0)\rangle$$

Spin Autocorrelation Function
(averaged over all sites)



PRB **97**, 060303 (R) (2018)

PRB **104**, 085117 (2021)

PRR **7**, 013181 (2025)

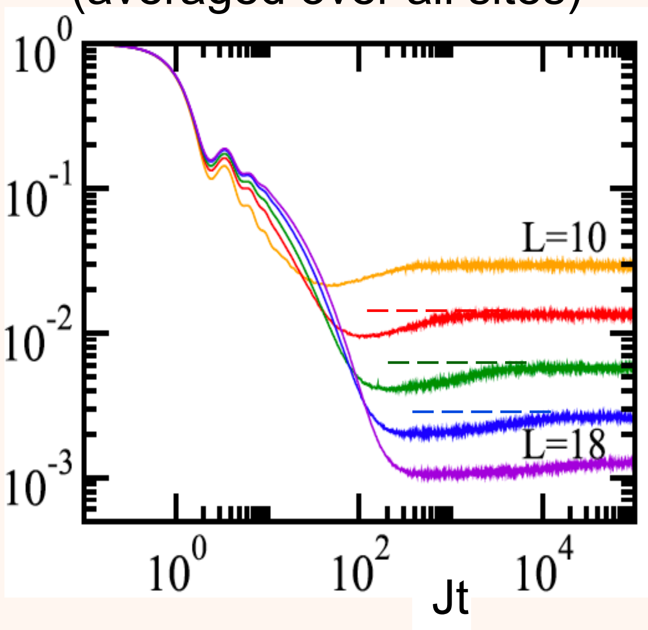
Local Observable

Spin Autocorrelation Function:

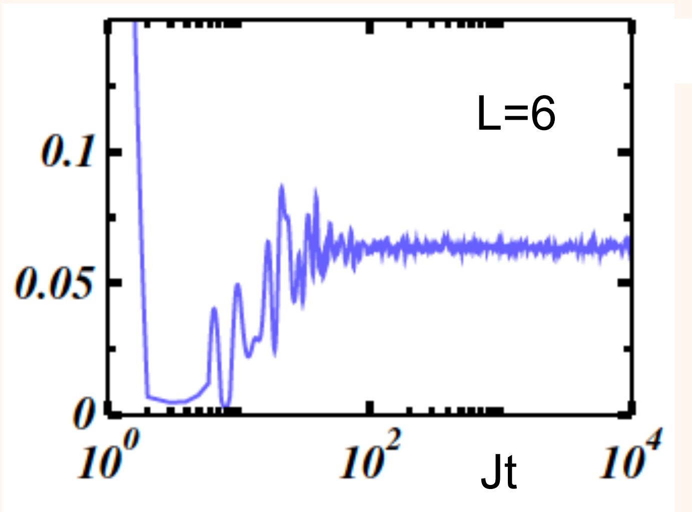
$$\frac{1}{L}\sum_{k=1}^{L}\langle\Psi(0)|\hat{\sigma}_k^z\hat{\sigma}_k^z(t)|\Psi(0)\rangle$$

Detection of many-body quantum chaos with a single site.

Spin Autocorrelation Function (averaged over all sites)



Spin Autocorrelation Function (for a single site)



PRB **97**, 060303 (R) (2018)

PRB **104**, 085117 (2021)

PRR **7**, 013181 (2025)

Self-Averaging

A quantity O is self-averaging when its relative variance goes to zero as the system size increases

$$\mathcal{R}_O(t) = \frac{\sigma_O^2(t)}{\left\langle O(t) \right\rangle^2} = \frac{\left\langle O^2(t) \right\rangle - \left\langle O(t) \right\rangle^2}{\left\langle O(t) \right\rangle^2}$$

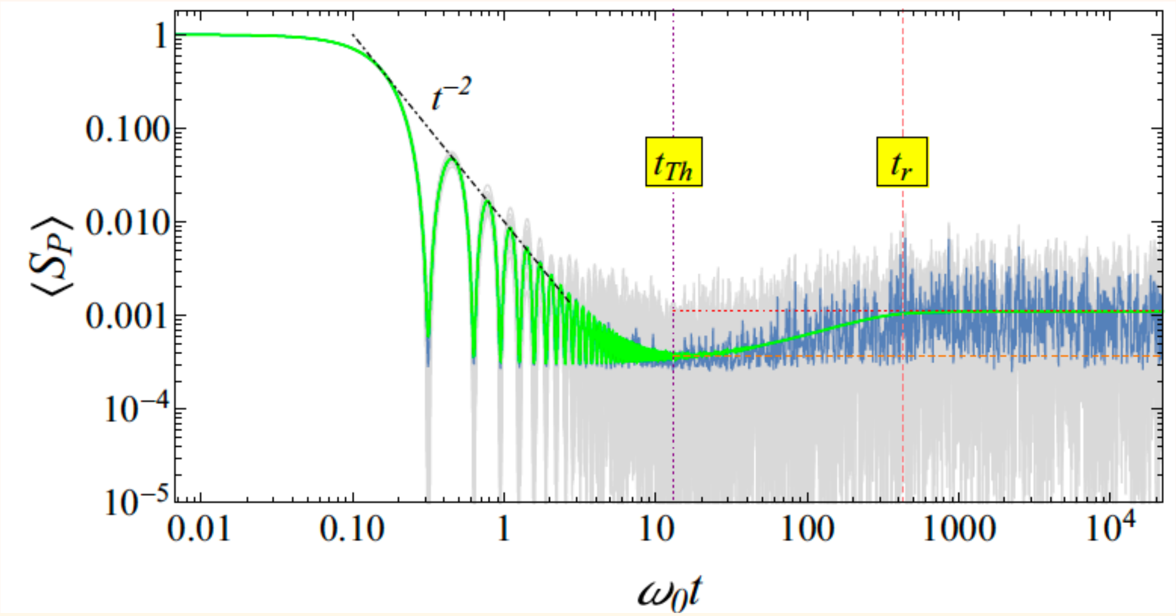
By increasing the system size, one can **reduce** the number of samples used in

- experiments
- statistical analysis.

If the system exhibits self-averaging, its physical properties are independent of the specific realization.

PRR **3,** L032030 (2021)
PRE **102**, 062126 (2020)
PRB **102**, 094310 (2020)
PRB **101**, 174312 (2020)

Lack of Self-Averaging



Dicke model
PRE **100**, 012218 (2019)

N. Argaman, F.-M. Dittes, E. Doron, J. P. Keating, A. Yu. Kitaev, M. Sieber, and U. Smilansky, Correlations in the Actions of Periodic Orbits Derived from Quantum Chaos, Phys. Rev. Lett. 71, 4326 (1993)

B. Eckhardt and J. Main, Semiclassical Form Factor of Matrix Element Fluctuations, Phys. Rev. Lett. 75, 2300 (1995)

R. E. Prange, The Spectral form Factor is Not Self-Averaging, Phys. Rev. Lett. 78, 2280 (1997)

P. Braun and F. Haake, Self-averaging characteristics of spectral fluctuations, J. Phys. A 48, 135101 (2015)

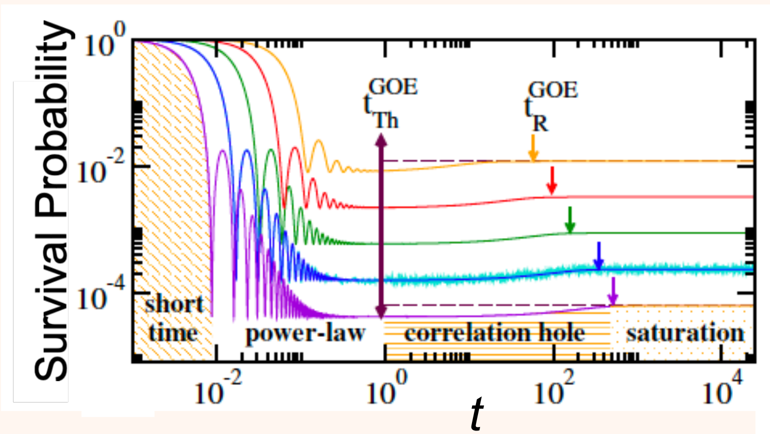
Analytical with GOE:
SP and SFF are nowhere self-averaging

PRB **101**, 174312 (2020)

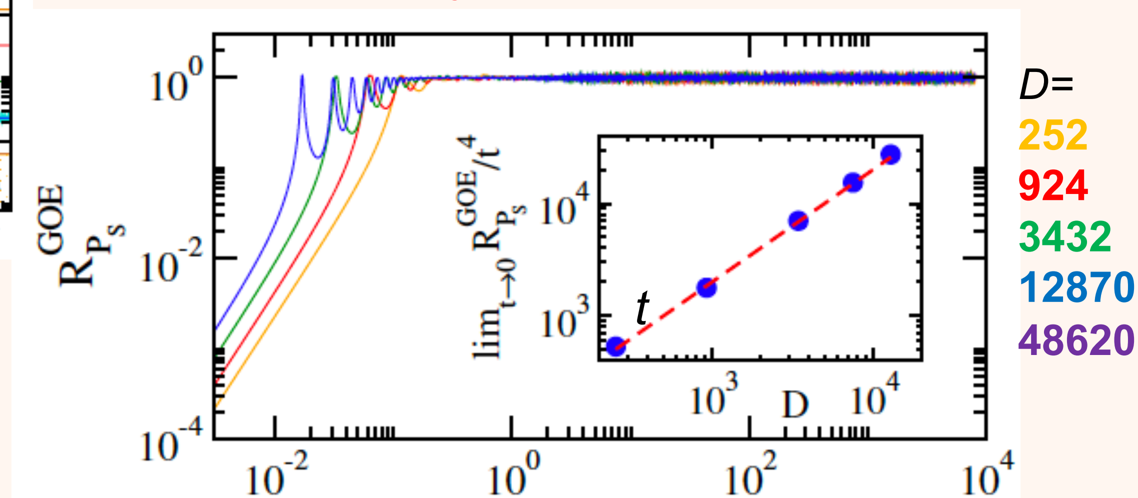
Lack of Self-Averaging: Analytical Results

$$SP(t) = \left| \langle \Psi(0) | e^{-iHt} | \Psi(0) \rangle \right|^2$$

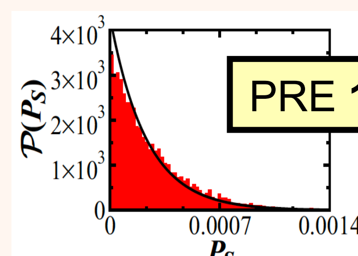
$$\mathcal{R}_{SP}(t) = \frac{\sigma^2_{SP}(t)}{\langle SP(t) \rangle^2} = \frac{\langle SP^2(t) \rangle - \langle SP(t) \rangle^2}{\langle SP(t) \rangle^2}$$



Analytical results: GOE



$$\left\langle \sum_{\alpha \neq \gamma \neq \beta \neq \delta} e^{-i(E_\alpha - E_\beta + E_\gamma - E_\delta)t} \left| c_\alpha^{(0)} \right|^2 \left| c_\beta^{(0)} \right|^2 \left| c_\gamma^{(0)} \right|^2 \left| c_\delta^{(0)} \right|^2 \right\rangle$$



PRE **102**, 062126 (2020)

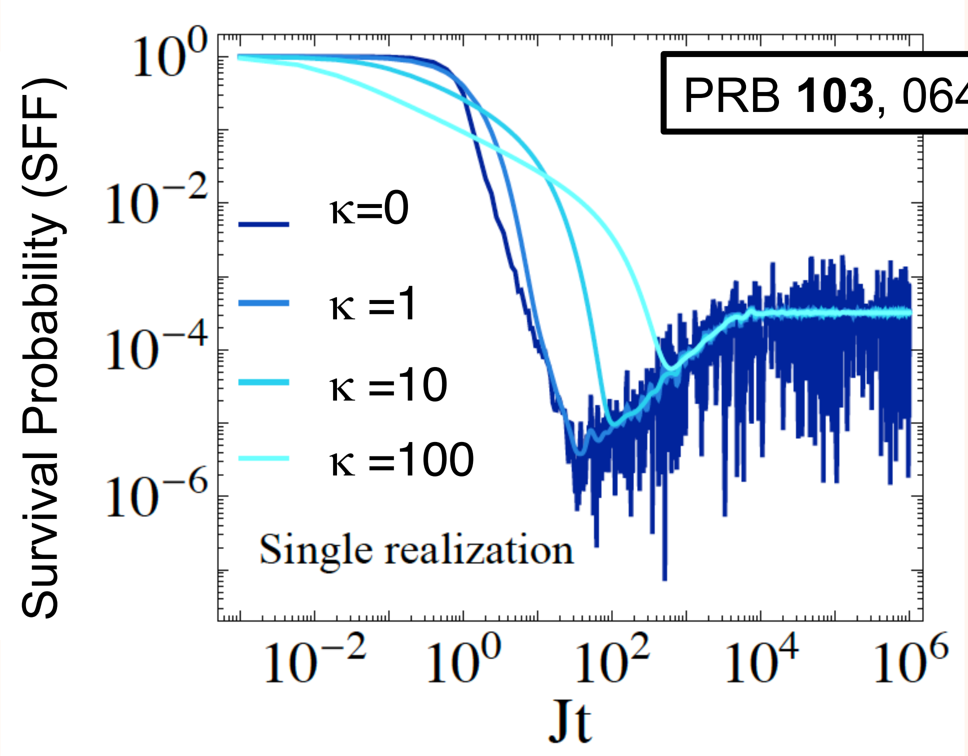
PRB **101**, 174312 (2020)

Avoiding averages with decoherence

$$\frac{d\rho}{dt} = -i[H,\rho] - \kappa[H,[H,\rho]]$$

dephasing strength

Tameshtit and Sipe,
Survival probability and chaos in an open quantum system,
PRA **45**, 8280 (1992)



PRB **103**, 064309 (2021)

Adolfo del Campo

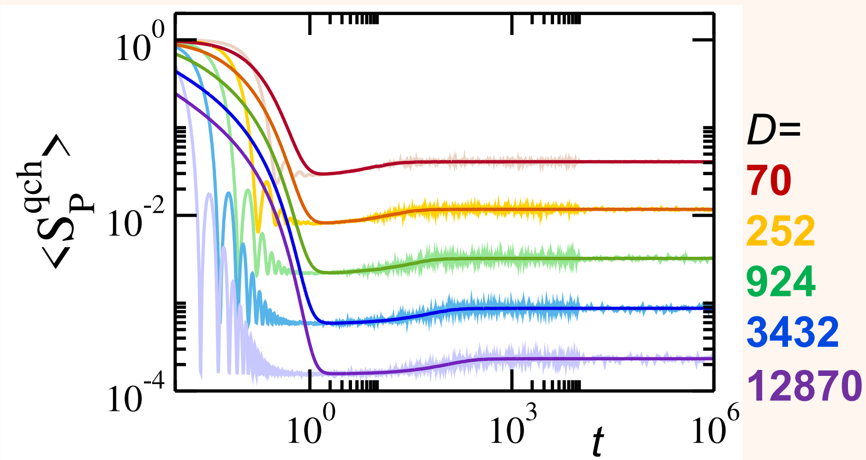
Initial Gibbs states

$$|c_n^{(0)}|^2 = \frac{e^{-\beta E_n}}{\sum_m e^{-\beta E_m}}$$

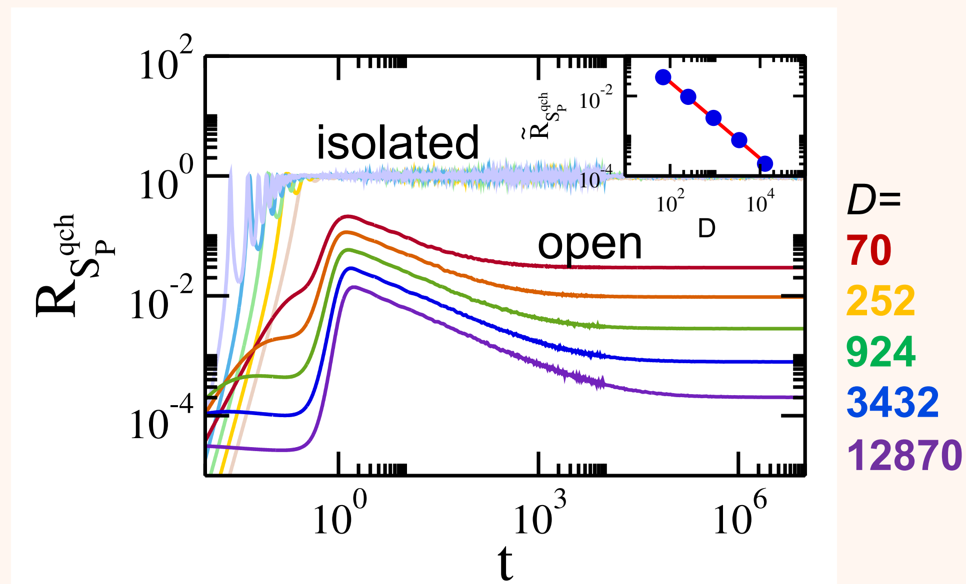
PRA **108**, 062201 (2023)

Self-averaging in GOE matrices

$$\mathcal{R}_{SP}(t) = \frac{\sigma_{SP}^2(t)}{\langle SP(t)\rangle^2} = \frac{\langle SP^2(t)\rangle - \langle SP(t)\rangle^2}{\langle SP(t)\rangle^2}$$



$$\overline{R_{S_P}^{\kappa\neq 0}} = \frac{\sigma^2_{\mathrm{IPR}_0}}{\langle \mathrm{IPR}_0\rangle^2}$$

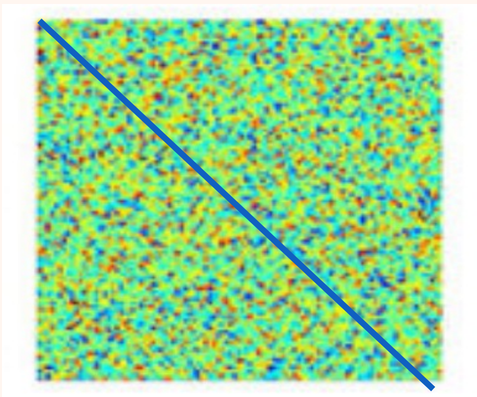


PRA **108**, 062201 (2023) Gibbs

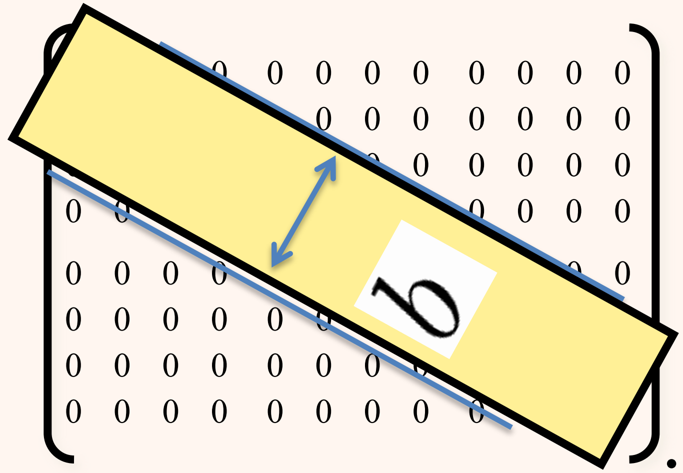
PRB **110**, 075138 (2024) quench

Random matrices models

Full random matrices



Wigner Band Random Matrix



Diagonal elements: integers

$\cdots, -2, -1, 0, 1, 2, \cdots$

Off-diagonal elements

$|v_{mn}| = v$

the signs of the v_{ij} are random

- Random real numbers
- Sparsity

Fyodorov, Casati, Izrailev, Prosen

Power-law banded random matrices

$\langle H_{ij} \rangle = 0$ Gaussian random numbers

$$\langle H_{ij}^2 \rangle = \begin{cases} 1, & i = j \\ (1 + |i-j|^{2\alpha})^{-1}, & i \neq j, \end{cases}$$

$\alpha < 1$ Ergodic (chaotic) phase

Rosenzweig-Porter ensemble

$$\langle H_{ij}^2 \rangle = \begin{cases} 1, & i = j \\ \frac{1}{2N^\gamma}, & i \neq j, \end{cases}$$

$\gamma < 1$ Ergodic (chaotic) phase

Two-Body Random Ensembles

Two-body random ensembles (1970)

French, Wong, Flores, Bohigas, Brody, Mello, Guhr, Weidenmüller, Izrailev, Flambaum, Kota, Zelevinsky, Horoi, Volya, Alhassid, Prosen, Seligman, …

$$H = \sum_k \varepsilon_k a_k^\dagger a_k + \lambda \sum_{k \le l, p \le q} \langle pq | V | kl \rangle a_p^\dagger a_q^\dagger a_l a_k,$$

Embedded Random Ensembles

SYK model

Sachdev & Ye
PRL 70,3339 (1993)

We consider the ensemble of Hamiltonians

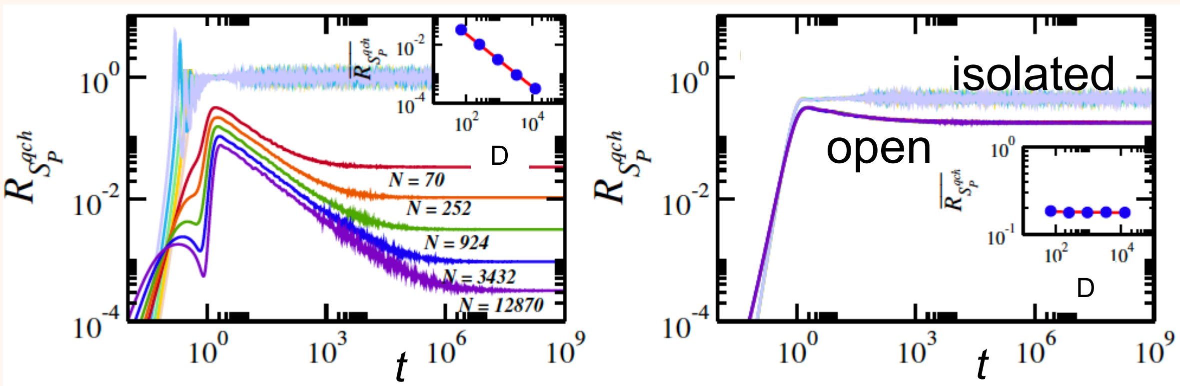
$$\mathcal{H} = \frac{1}{\sqrt{NM}} \sum_{i>j} J_{ij} \hat{S}_i \cdot \hat{S}_j, \tag{2}$$

where the sum over i, j extends over $N \to \infty$ sites, the exchange constants J_{ij} are mutually uncorrelated and selected with probability $P(J_{ij}) \sim \exp[-J_{ij}^2/(2J^2)]$, the $\hat{S}$ are the spin operators of the group SU(M), and the states on each site belong to a representation labeled by the integer n_b [$n_b = 2S$ for SU(2); more generally n_b is the

Self-averaging in power-law banded random matrices

$$\mathcal{R}_{SP}(t) = \frac{\sigma^2_{SP}(t)}{\langle SP(t)\rangle^2} = \frac{\langle SP^2(t)\rangle - \langle SP(t)\rangle^2}{\langle SP(t)\rangle^2}$$

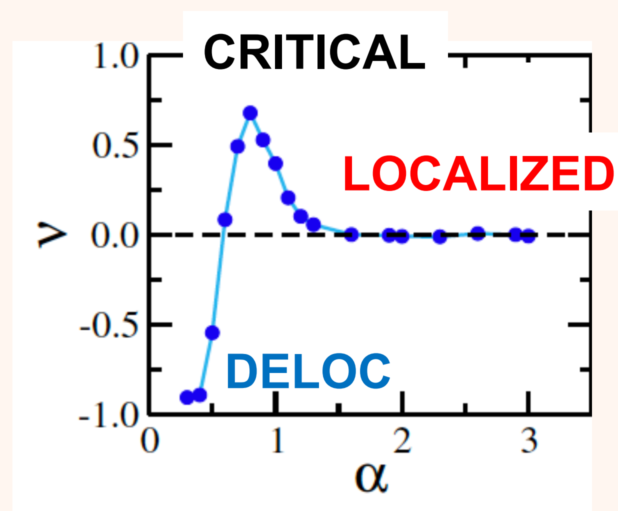
PRB **110**, 075138 (2024)



$$\overline{R^{\kappa\neq 0}_{S_P^{qch}}} \propto D^{\nu}$$

Power-law banded random matrices:
LOCALIZATION for $\alpha>1$

$$\langle H_{ij}^2\rangle = \begin{cases} 1, & i=j, \\ \left(1+|i-j|^{2\alpha}\right)^{-1}, & i\neq j, \end{cases}$$



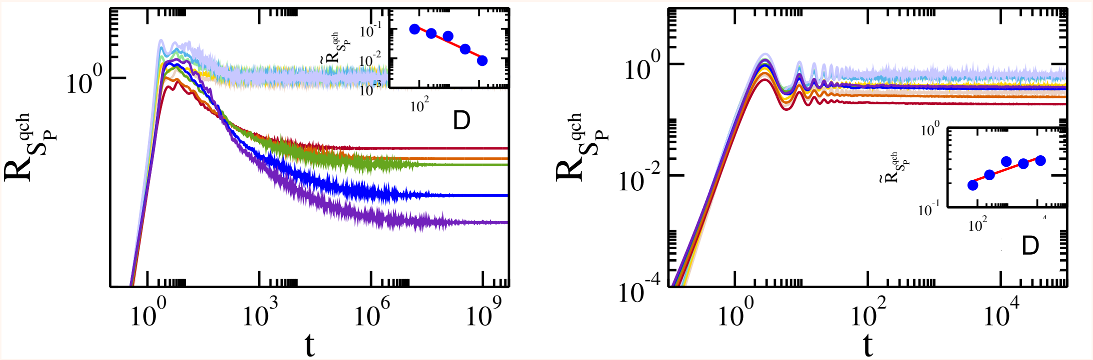
GOE — deloc — critical — localized — tridiagonal → α

0 — 1 — ∞

Self-averaging in open physical systems

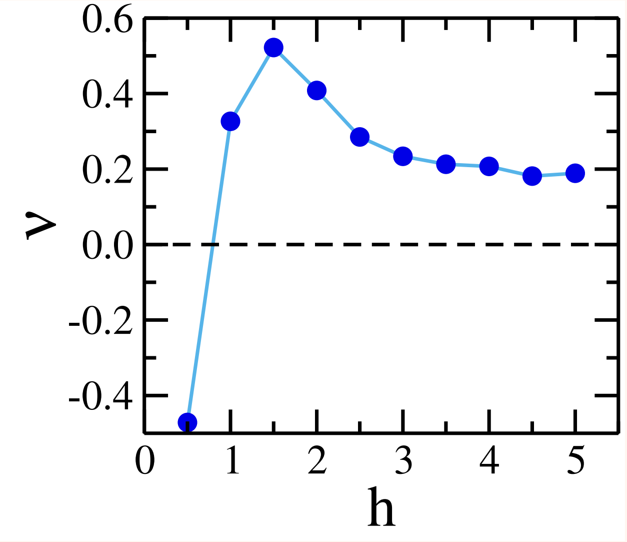
$$\mathcal{R}_{SP}(t) = \frac{\sigma^2_{SP}(t)}{\langle SP(t)\rangle^2} = \frac{\langle SP^2(t)\rangle - \langle SP(t)\rangle^2}{\langle SP(t)\rangle^2}$$

CHAOTIC (h=0.5) **NON-CHAOTIC** (h=5)



L=
8
10
12
14
16

$$\overline{R^{\kappa\neq 0}_{S_P^{qch}}} \propto D^{\nu}$$



$$H = \sum_{n=1}^{L} \frac{h_n}{2}\sigma_n^z + \sum_{n=1}^{L} \frac{J}{4}\left[\sigma_n^z\sigma_{n+1}^z + \left(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y\right)\right]$$

$$\overline{R^{\kappa\neq 0}_{S_P}} = \frac{\sigma^2_{\mathrm{IPR}_0}}{\langle \mathrm{IPR}_0\rangle^2}$$

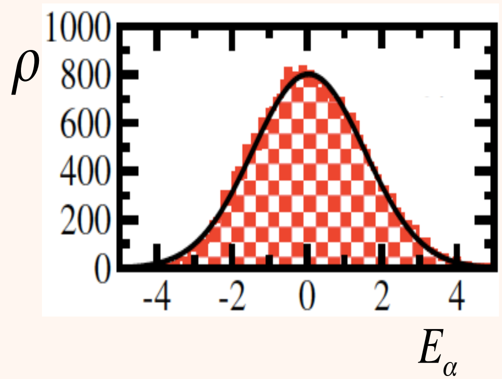
PRB **110**, 075138 (2024)

Lack of self-averaging vs initial state

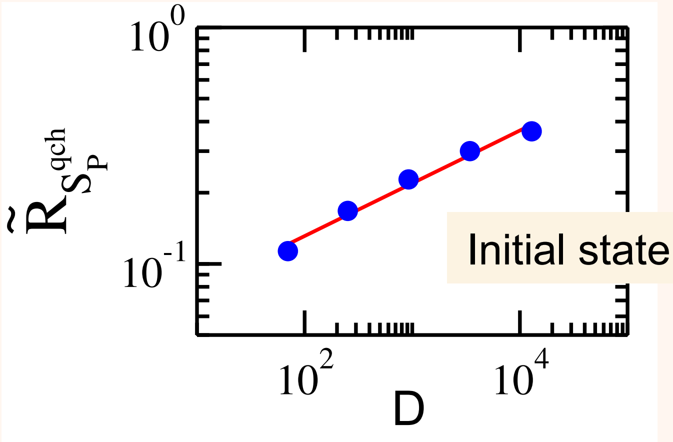
$$\mathcal{R}_{SP}(t) = \frac{\sigma^2_{SP}(t)}{\langle SP(t)\rangle^2} = \frac{\langle SP^2(t)\rangle - \langle SP(t)\rangle^2}{\langle SP(t)\rangle^2}$$

$$\overline{R^{\kappa\neq 0}_{S_P^{qch}}} \propto D^{\nu}$$

Density of states



CHAOS (h=0.5)



PRB **110**, 075138 (2024)

Summary

- The time to reach thermal equilibrium in a chaotic system depends on the quantity, model and initial state.

- Polynomial increase with *L*.
  Quantities with **correlation hole:** Exponentially long time in *L* to equilibrate.

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  It could be **detected experimentally** (quench: SP, spin autocorrelation function).

PRR **7**, 013181 (2025)

Summary

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- Polynomial increase with *L*.
  Quantities with **correlation hole:** Exponentially long time in *L* to equilibrate.

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  It could be **detected experimentally** (quench: SP, spin autocorrelation function).
  PRR **7**, 013181 (2025)

- Lack of **self-averaging**: PRB **110**, 075138 (2024)
  Avoided by **opening** the system to a dephasing environment (**chaotic** systems).

- Opening the system **reduces fluctuations**.

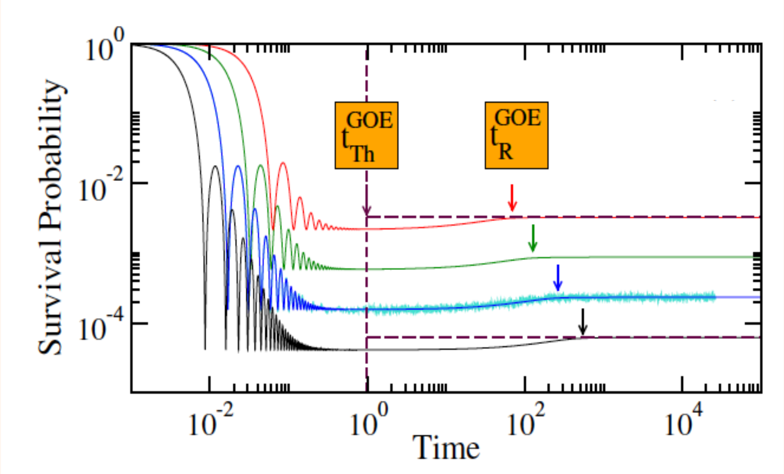
Additional Slides

Correlation Hole and System Size

Survival probability for GOE matrices:

$$\overline{SP} = 3/D$$

$$SP_{min} = 2/D$$



Relative depth of the correlation hole:

$$\kappa = \frac{\langle \overline{O} \rangle - \langle O \rangle_{\min}}{\langle \overline{O} \rangle}$$

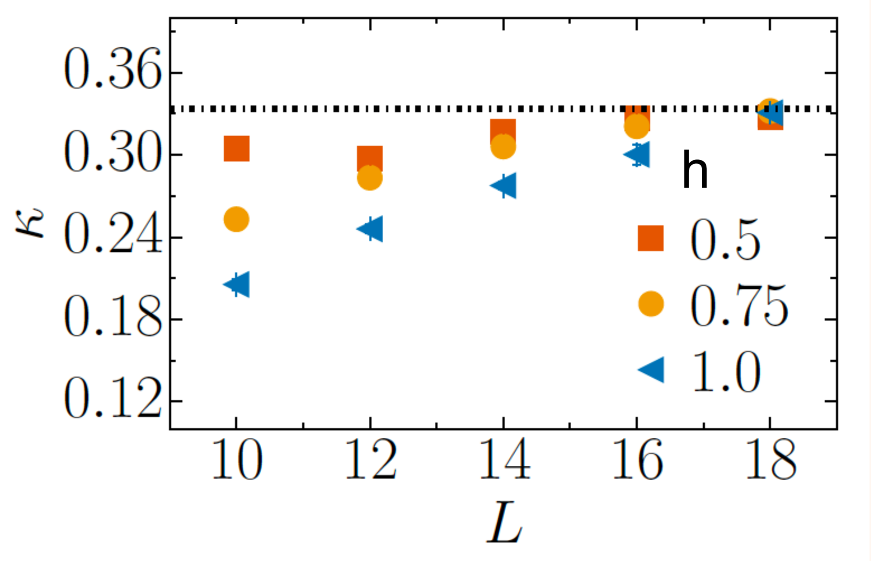
$$\kappa = 1/3$$

Lezama, Torres, Bernal, Bar Lev & LFS, PRB **104**, 085117(2021)

Correlation Hole for the Survival Probability

Relative depth of the correlation hole:

$$\kappa = \frac{\langle\overline{O}\rangle - \langle O\rangle_{\min}}{\langle\overline{O}\rangle}$$



Survival probability for realistic chaotic systems:

$$\kappa = 1/3$$

$$h \leq J = 1$$

Lezama, Torres, Bernal, Bar Lev & LFS, PRB **104**, 085117(2021)

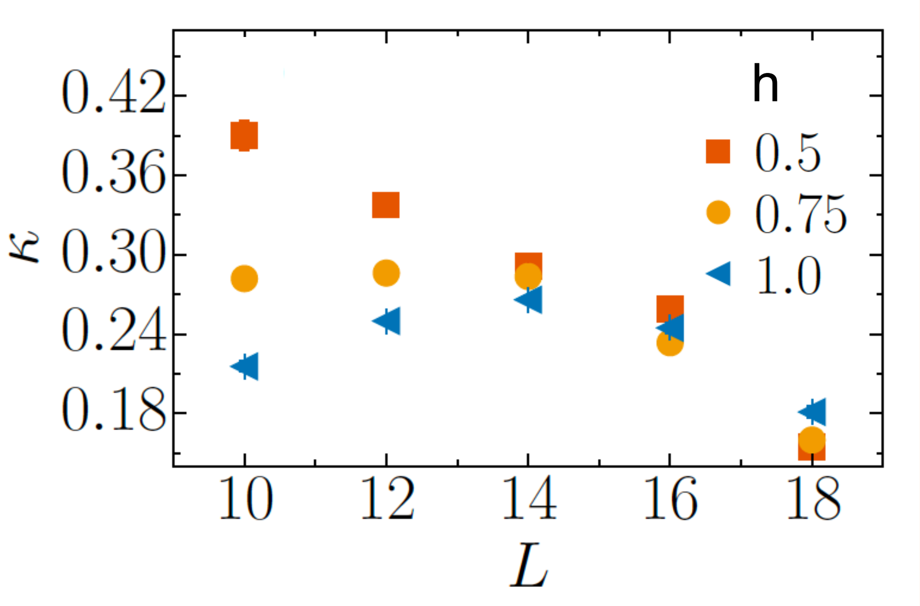
Correlation Hole for the Spin Autocorrelation Function

Relative depth of the correlation hole:

$$\kappa = \frac{\langle \overline{O} \rangle - \langle O \rangle_{\min}}{\langle \overline{O} \rangle}$$

Spin autocorrelation function for realistic chaotic systems:

$$h \leq J = 1$$



Lezama, Torres, Bernal, Bar Lev & LFS, PRB **104**, 085117(2021)

What is Many-Body Quantum Chaos?

1D Spin-1/2 Systems

Integrable system:

XXZ model

Poisson

$$H = \sum_{n=1}^{L-1} \frac{J}{4} (\Delta \sigma_n^z \sigma_{n+1}^z + \sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y)$$

Ann. Phys. **529**, 1600284 (2017)

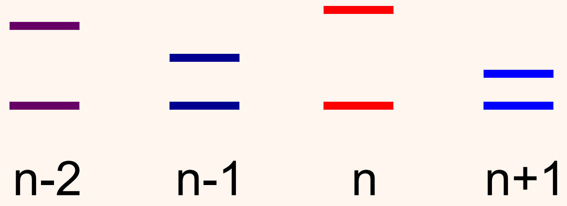
WIGNER-DYSON DISTRIBUTION

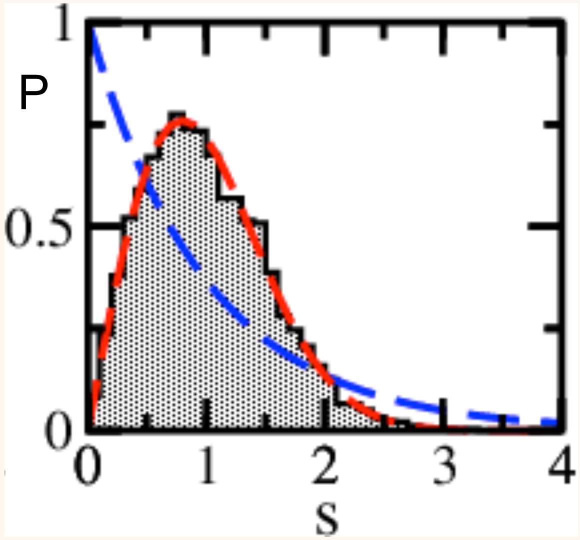
Wigner-Dyson via disorder, further couplings…

$$H = \sum_{n=1}^{L} \frac{h_n}{2} \sigma_n^z + \sum_{n=1} \frac{J}{4} (\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y + \Delta \sigma_n^z \sigma_{n+1}^z)$$

Random numbers

$h_n \in [-h, h]$



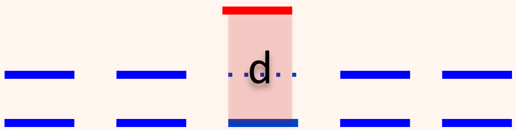


JPA **37**, 4723 (2004); PRA **69**, 042304 (2004); NJP **11,** 043026 (2009)

Speck of Chaos

Wigner-Dyson with a single defect!

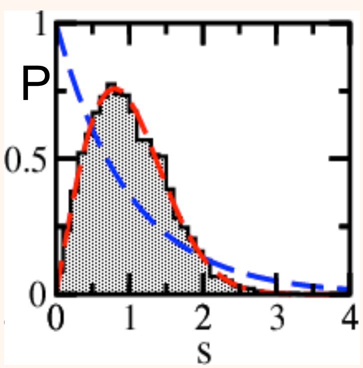
$$H_{one} = \frac{d_{L/2}}{2}\sigma^z_{L/2} + \sum_{n=1}^{L-1}\frac{J}{4}(\sigma^x_n\sigma^x_{n+1} + \sigma^y_n\sigma^y_{n+1} + \Delta\sigma^z_n\sigma^z_{n+1})$$

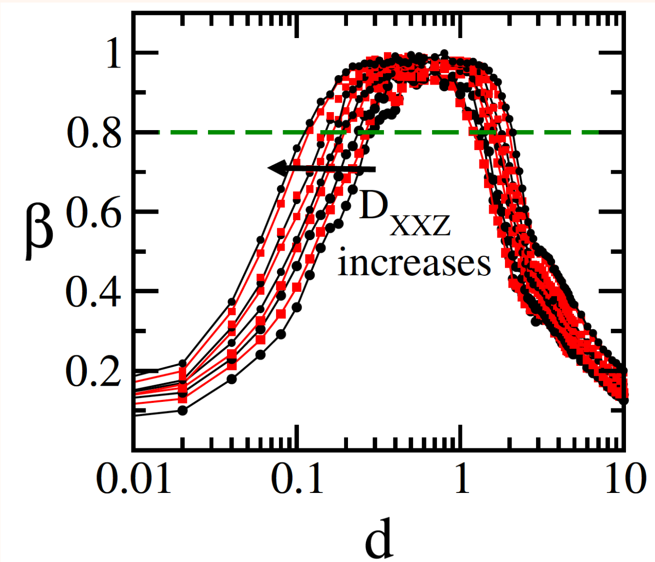


LFS,
JPA **37**, 4723 (2004)

$\beta \sim 1$ **chaos**

$$P_{WD}(s) = \frac{\pi s}{2}\exp\left(-\frac{\pi s^2}{4}\right)$$





(local perturbation)

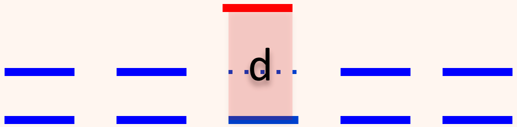
Gubin & LFS
AJP **80**, 246 (2012)

$$P(s) = (\beta+1)bs^{\beta}\exp(-bs^{\beta+1})$$

Speck of Chaos
PRR **2**, 043034 (2020)
LFS, Bernal, Torres

Speck of Chaos

$$H_{one} = \frac{d_{L/2}}{2}\sigma^z_{L/2} + \sum_{n=1}^{L-1}\frac{J}{4}\left(\sigma^x_n\sigma^x_{n+1} + \sigma^y_n\sigma^y_{n+1} + \Delta\sigma^z_n\sigma^z_{n+1}\right)$$



LFS,
JPA **37**, 4723 (2004)

Chaos: Level statistics, chaotic eigenstates,
diagonal and off-diagonal elements of O

Chaos is the mechanism for **thermalization**

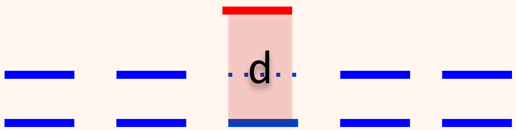
Chaos is the condition for the validity of ETH

Torres & LFS
PRE **89**, 062110 (2014)

Speck of Chaos
PRR **2**, 043034 (2020)
LFS, Bernal, Torres

Speck of Chaos

$$H_{one} = \frac{d_{L/2}}{2}\sigma^z_{L/2} + \sum_{n=1}^{L-1}\frac{J}{4}\left(\sigma^x_n\sigma^x_{n+1} + \sigma^y_n\sigma^y_{n+1} + \Delta\sigma^z_n\sigma^z_{n+1}\right)$$



LFS,
JPA **37**, 4723 (2004)

Chaos: Level statistics, chaotic eigenstates, diagonal and off-diagonal elements of O
Chaos is the mechanism for **thermalization**
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Torres & LFS
PRE **89**, 062110 (2014)

Ballistic quantum transport

Brenes, Mascarenhas, Rigol & Goold
PRB **98**, 235128 (2018)

For instance, a small local perturbation suffices to make a system "chaotic" according to the LSD [level spacing distribution], despite transport remaining that of an integrable model (ballistic).

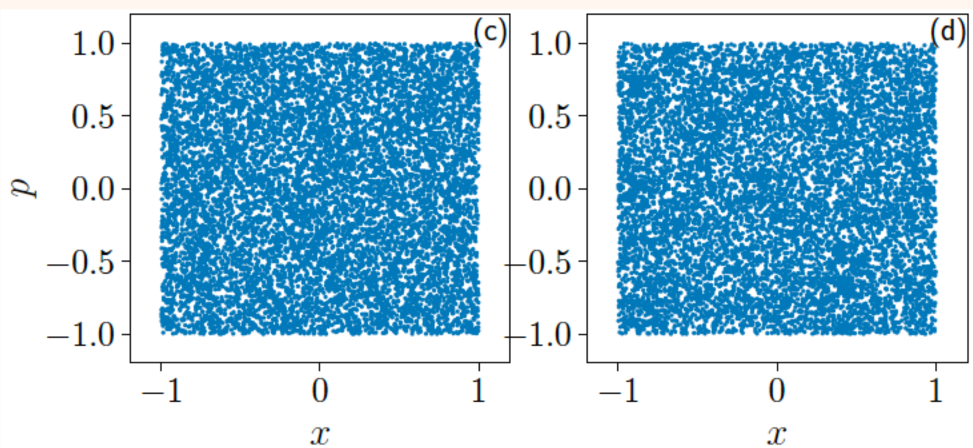
M. Znidaric
PRL**125**, 180605 (2020)

Wigner-Dyson vs Ergodicity

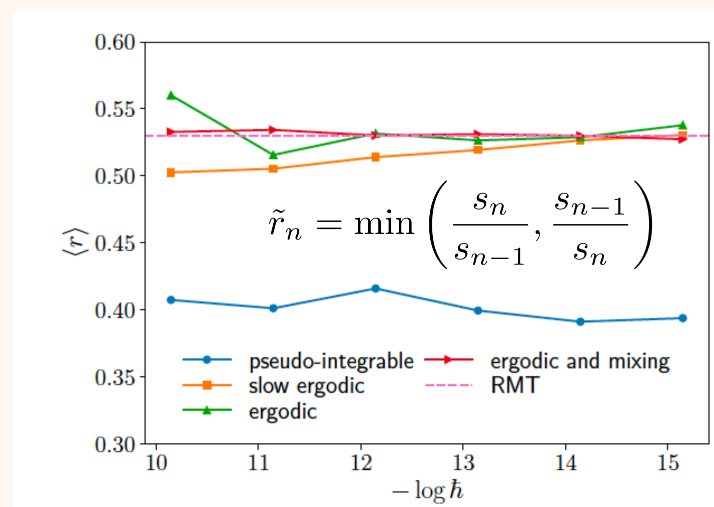
Statistical and dynamical properties of the quantum triangle map
Jiaozi Wang, Giuliano Benenti, Giulio Casati, Wen-ge Wang
JPA **55**, 234002 (2022)

We study the statistical and dynamical properties of the **quantum triangle map**, whose classical counterpart can exhibit **ergodic** and mixing dynamics, but is **never chaotic**. Numerical results show that ergodicity is a sufficient condition for spectrum and eigenfunctions to **follow the prediction of Random Matrix Theory**, even though the underlying classical dynamics is not chaotic.

Classical map is ergodic,
but Lyapunov exponent = zero



Quantum map shows
Wigner-Dyson distribution



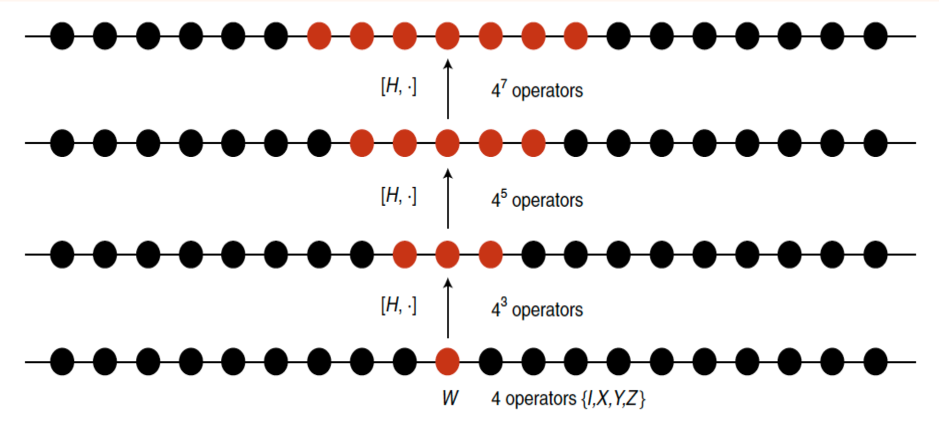
Spread of quantum information: OTOC

$$C(t) = \langle |[W(t), V(0)]|^2 \rangle$$

$$W(t) = \sigma_k^x(t);\ V = \sigma_k^x$$

$$W(t) = e^{iHt} W e^{-iHt} \qquad H = \ldots \sigma_{k-1}^z \sigma_k^z + \sigma_k^z \sigma_{k+1}^z \cdots$$

W(t) spreads over many sites



(nested commutators)

$$W(t) = \sum_{\ell=0}^{\infty} \frac{(it)^\ell}{\ell!} [H, \ldots [H, W], \ldots]$$

OTOC measures this growth

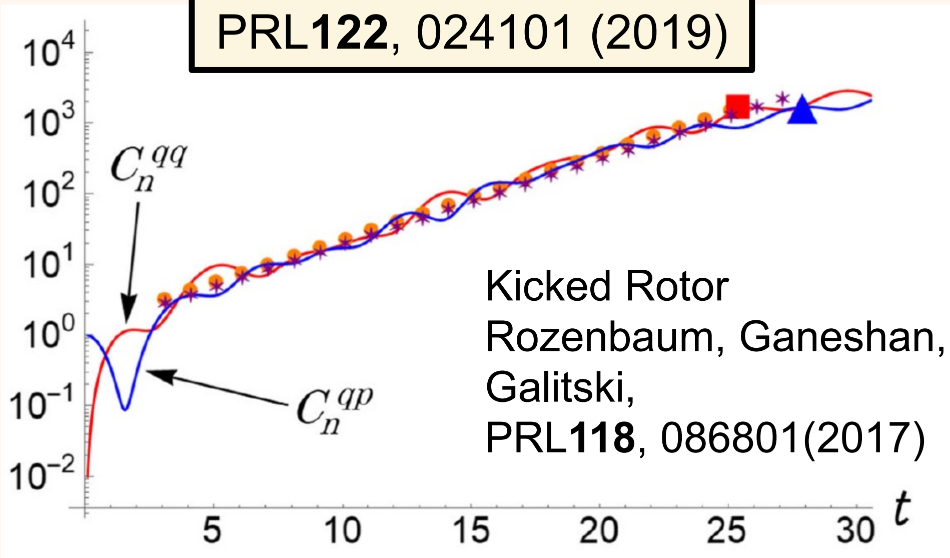
Out-of-time-ordered four-point correlator: OTOC

$$C(t) = \langle |[W(t), V(0)]|^2 \rangle$$

Spread of information

Exponential growth due to **chaos**

Dicke Model
Chávez et al,
PRL**122**, 024101 (2019)



Kicked Rotor
Rozenbaum, Ganeshan, Galitski,
PRL**118**, 086801(2017)

REGULAR system

Exponential growth at **critical** points

PRB 98, 134303 (2018)
PRL123,160401 (2019)
PRE 101, 010202(R) (2020)
PRL124, 140602 (2020)
JHEP11, (2020) 068

What is many-body quantum chaos?

Level statistics as in random matrix theory.

Diffusion.

Exponential growth of the OTOC.

Adiabatic Complexity.

Fidelity susceptibility: How sensitive a system's eigenstates are to small changes in the Hamiltonian.