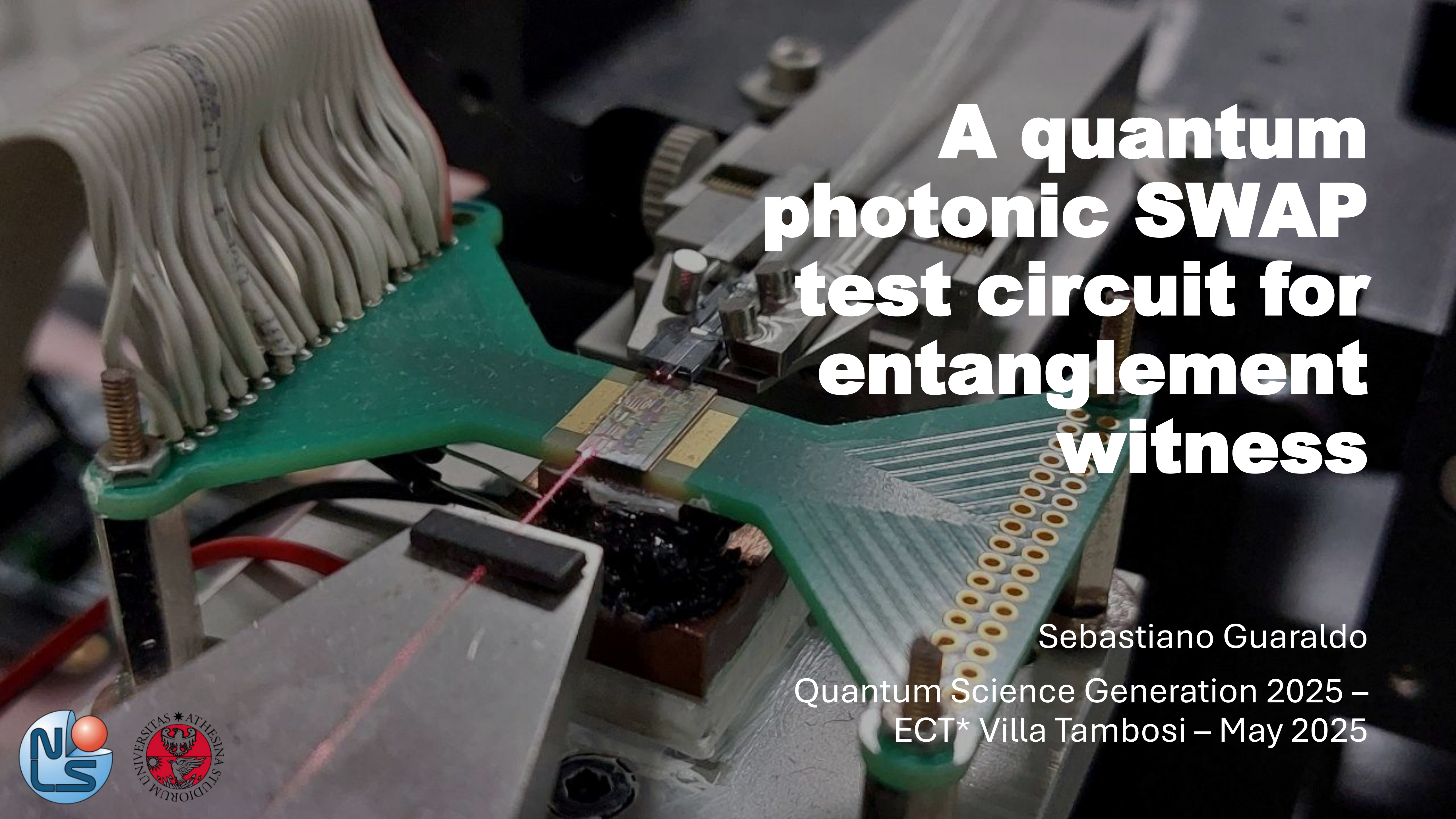


A quantum photonic SWAP test circuit for entanglement witness

Sebastiano Guaraldo

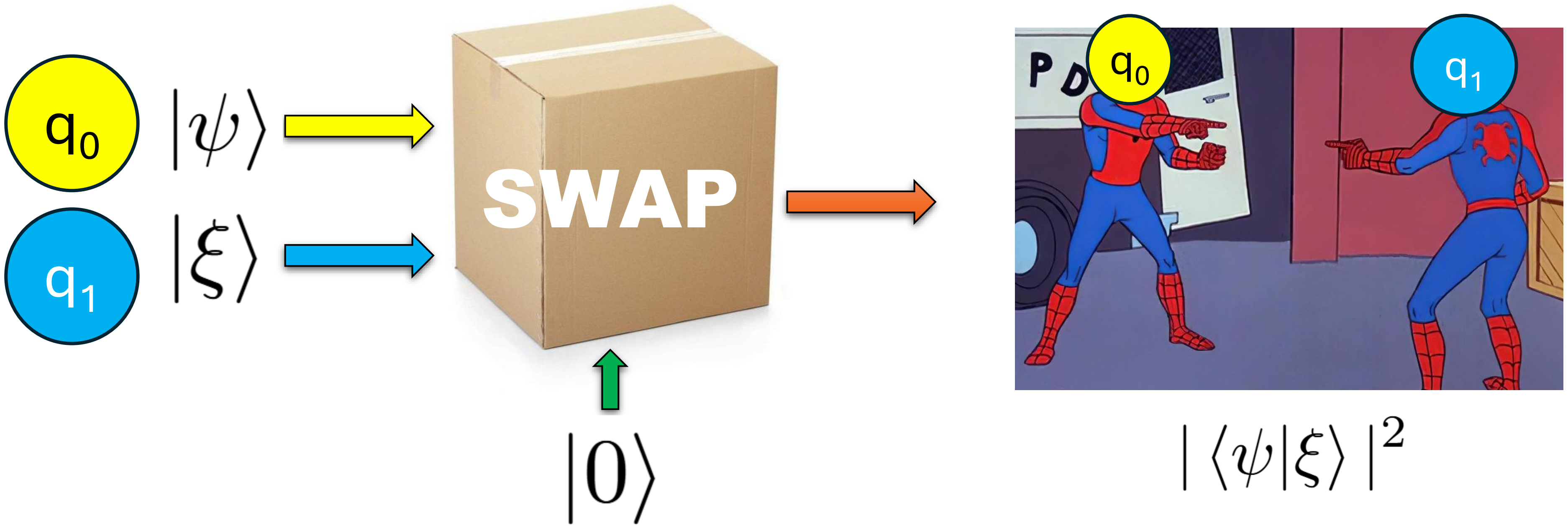
Quantum Science Generation 2025 – ECT* Villa Tambosi – May 2025

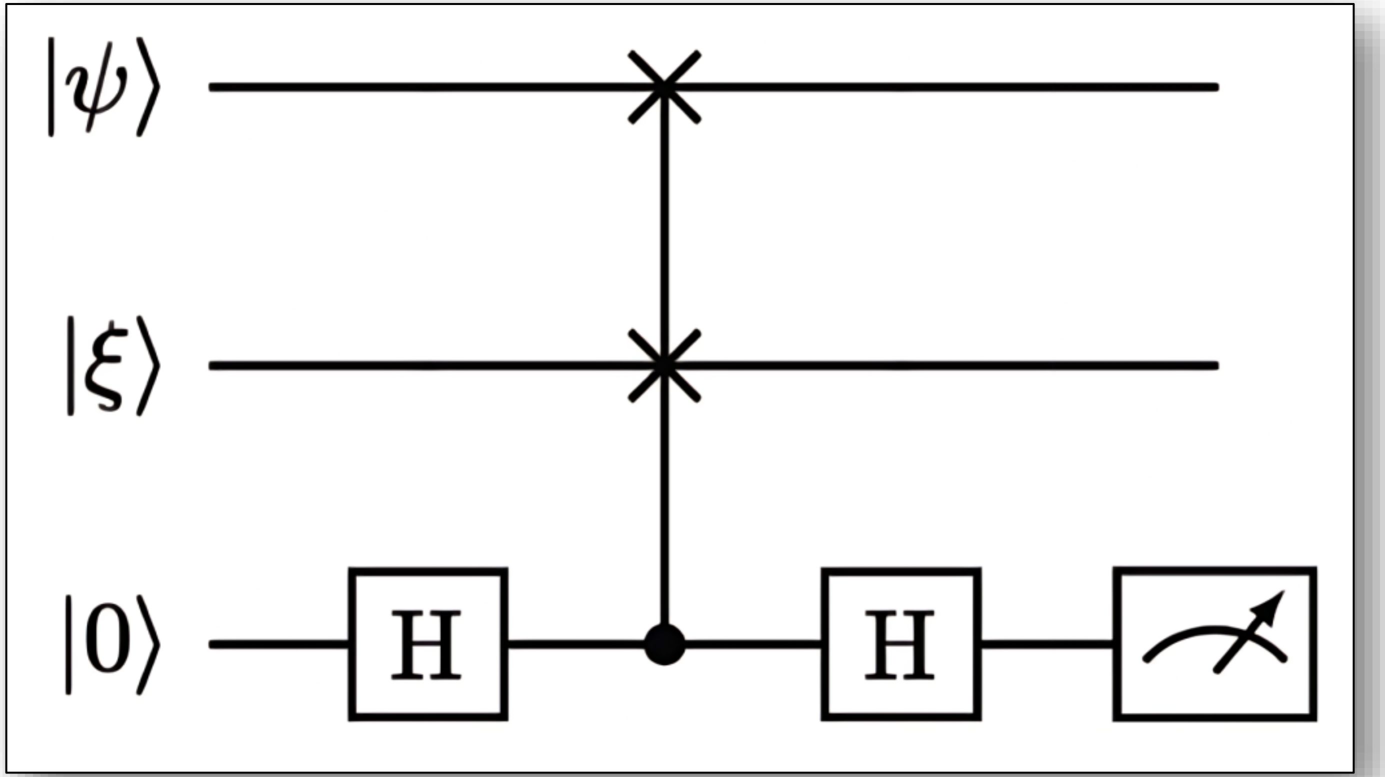
Entanglement

- **Entanglement** is a **fundamental resource** for quantum technologies
- In general, **detecting** entanglement is **difficult.**
- **SWAP test** circuits are an efficient alternative to traditional methods.

SWAP test

The SWAP test is a quantum algorithm designed to assess the **overlap** (quantified by the squared scalar product) **between two input quantum states** $|\psi\rangle$ and $|\xi\rangle$.





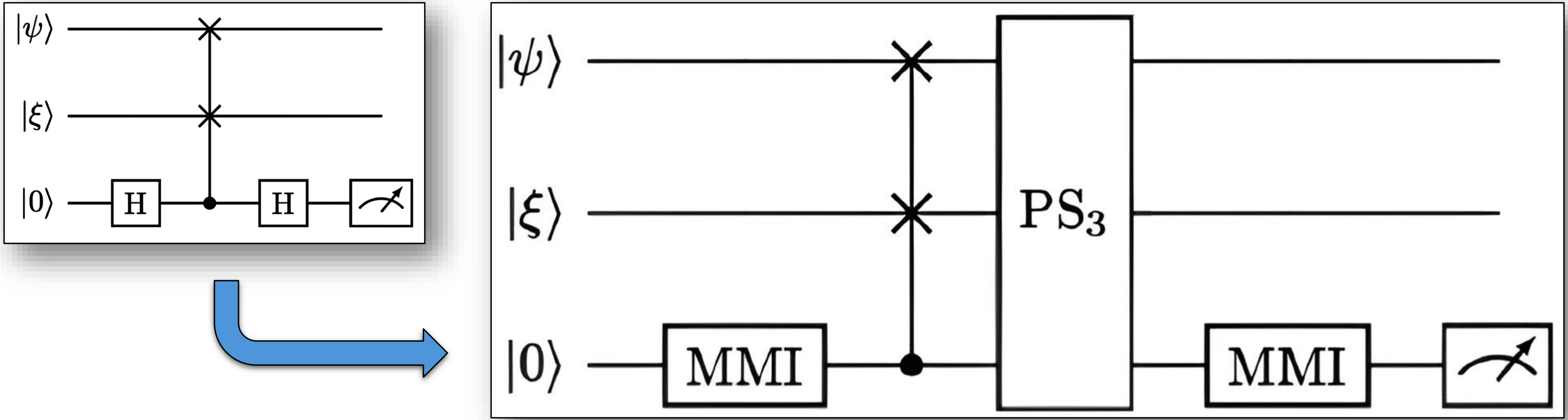
$$|\langle\psi|\xi\rangle|^2 = 2P(0) - 1 = 1 - 2P(1)$$

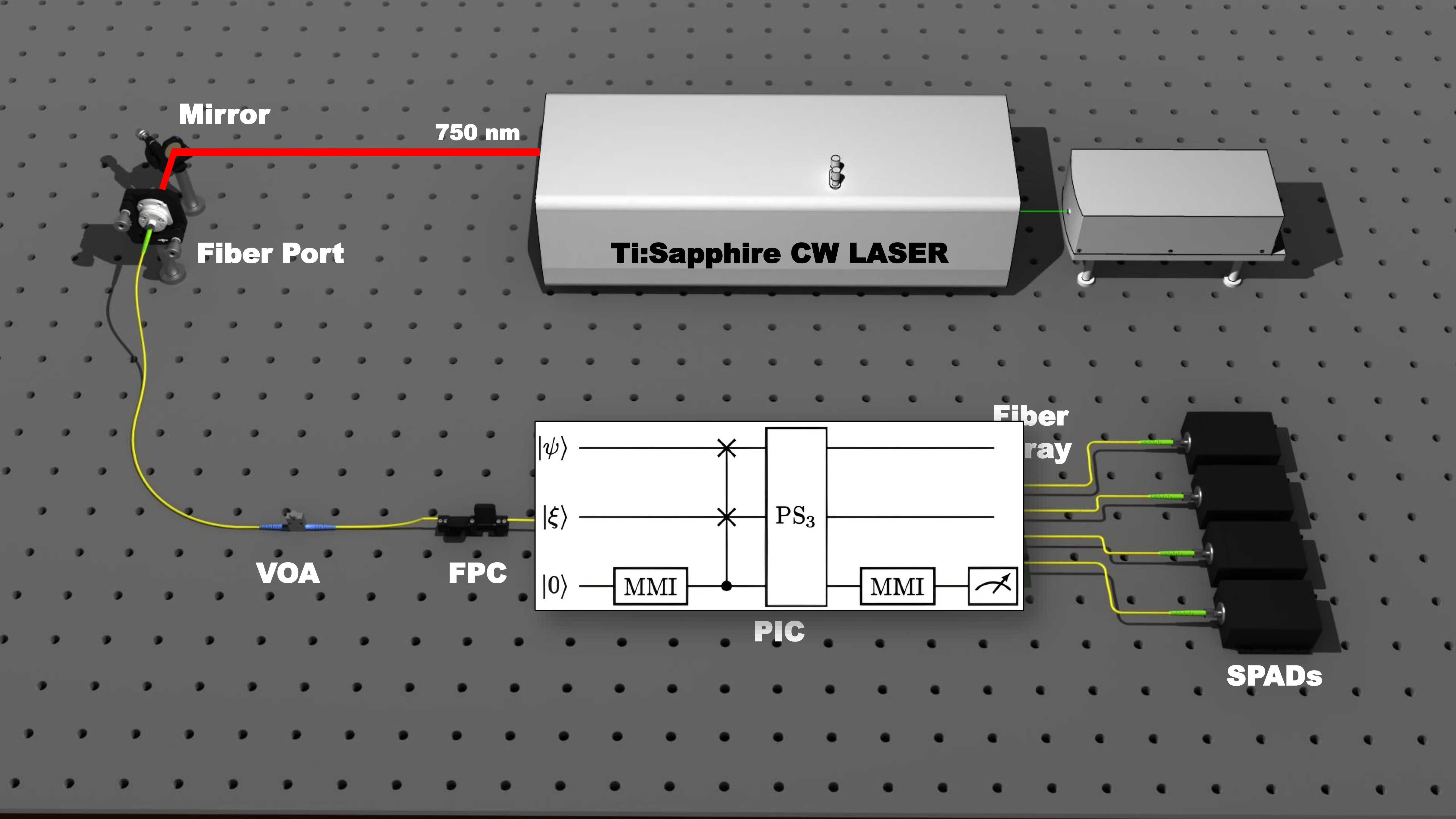
Computation

Sampling

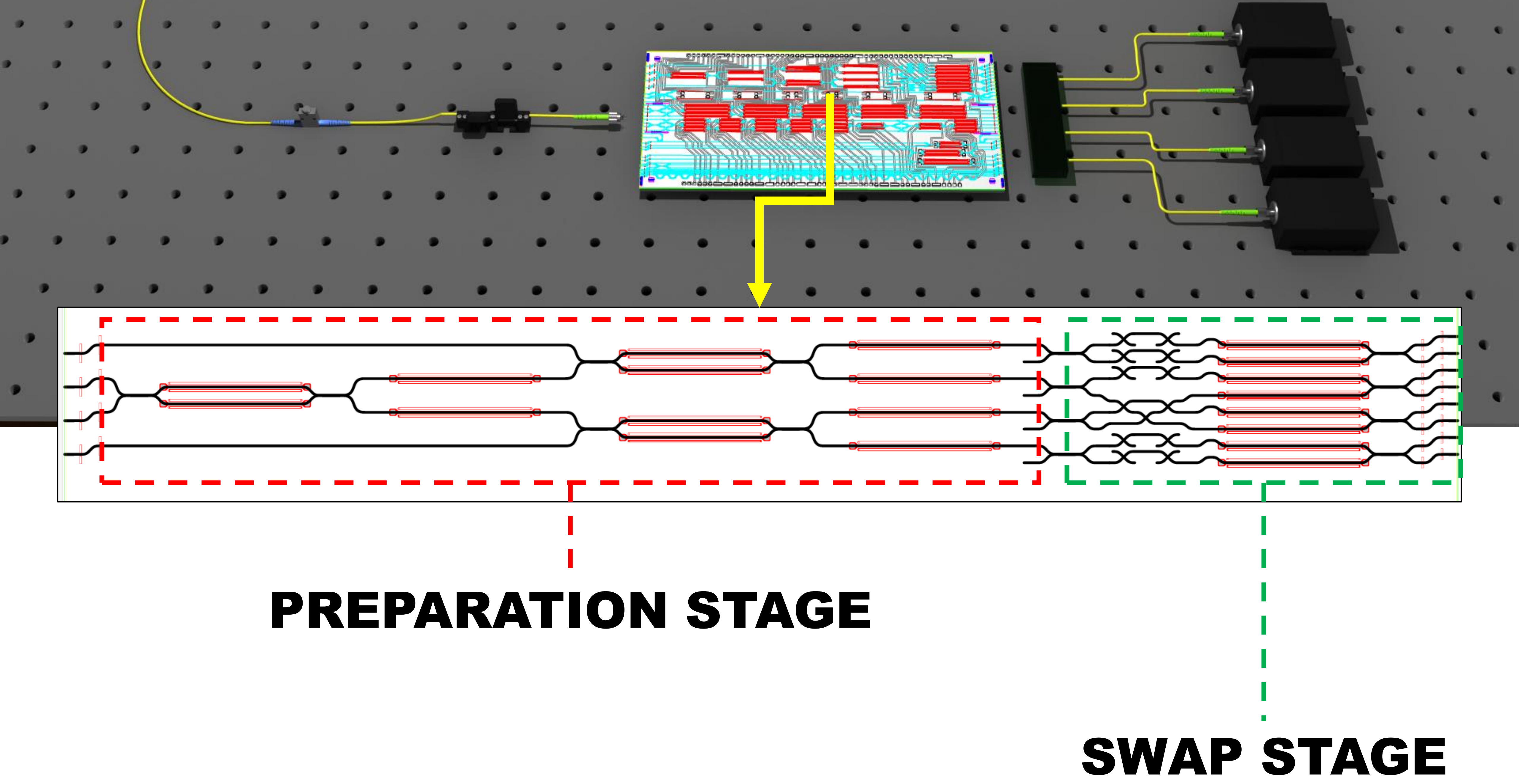
Linear photonic SWAP circuit

The SWAP test algorithm can be implemented on a **Photonic Integrated Circuit (PIC)** [1]





Mirror
750 nm
Ti:Sapphire CW LASER
Fiber Port
Fiber
ray
$|\psi\rangle$
$|\xi\rangle$
$|0\rangle$
MMI
PS_3
MMI
VOA
FPC
PIC
SPADs

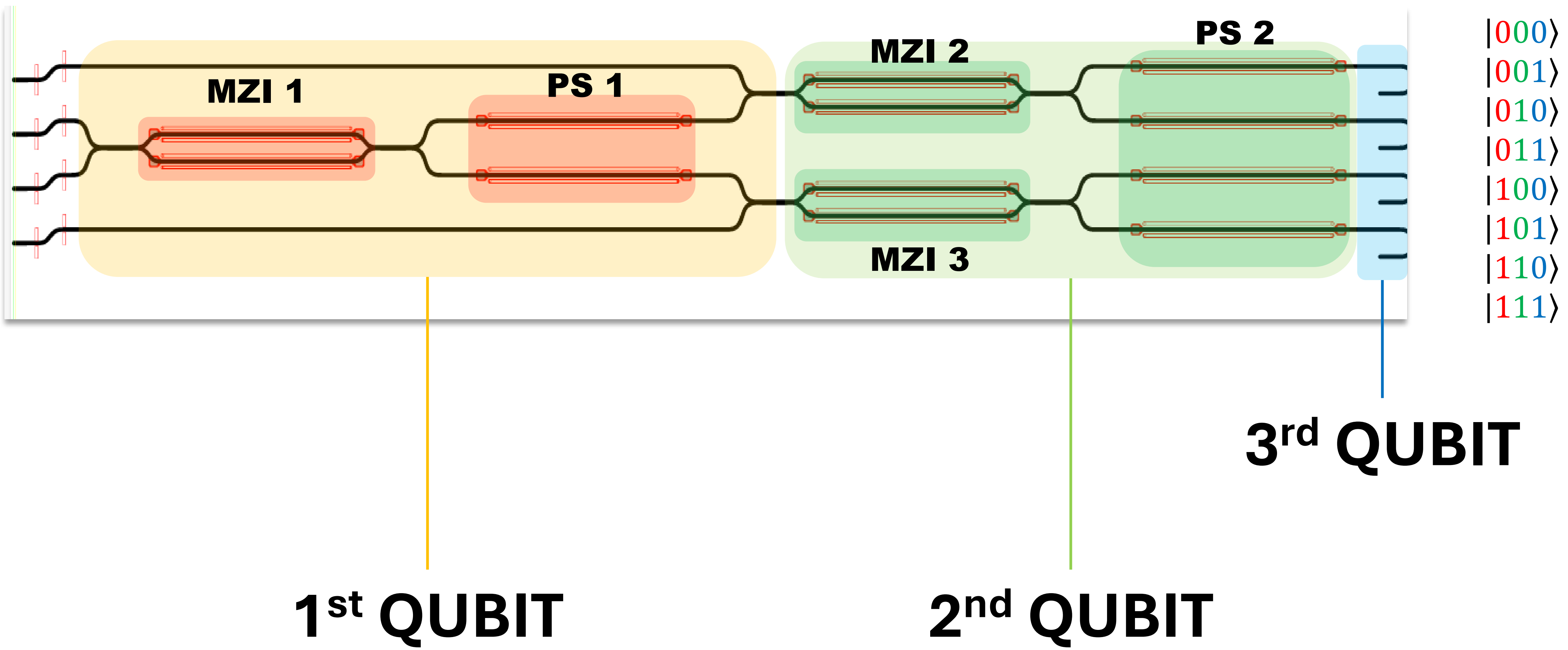

PREPARATION STAGE
SWAP STAGE

Preparation stage

To encode states in the PIC, **path-encoded single photons** are used, where the state is determined by the waveguide through which a single photon travels

Since the input qubits are three, 2^3 paths are necessary. The **encoding** is achieved by **converting the waveguide numbers to binary.**

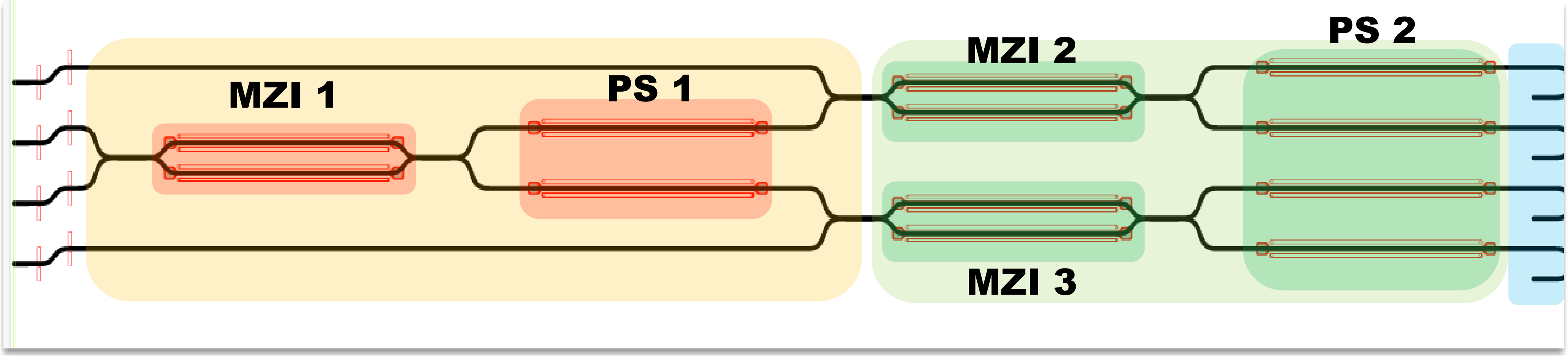
$$
\begin{aligned}
|0\rangle &\rightarrow |000\rangle \equiv |0\rangle \otimes |0\rangle \otimes |0\rangle\,, & |1\rangle &\rightarrow |001\rangle \equiv |0\rangle \otimes |0\rangle \otimes |1\rangle\,,\\
|2\rangle &\rightarrow |010\rangle \equiv |0\rangle \otimes |1\rangle \otimes |0\rangle\,, & |3\rangle &\rightarrow |011\rangle \equiv |0\rangle \otimes |1\rangle \otimes |1\rangle\,,\\
|4\rangle &\rightarrow |100\rangle \equiv |1\rangle \otimes |0\rangle \otimes |0\rangle\,, & |5\rangle &\rightarrow |101\rangle \equiv |1\rangle \otimes |0\rangle \otimes |1\rangle\,,\\
|6\rangle &\rightarrow |110\rangle \equiv |1\rangle \otimes |1\rangle \otimes |0\rangle\,, & |7\rangle &\rightarrow |111\rangle \equiv |1\rangle \otimes |1\rangle \otimes |1\rangle
\end{aligned}
$$



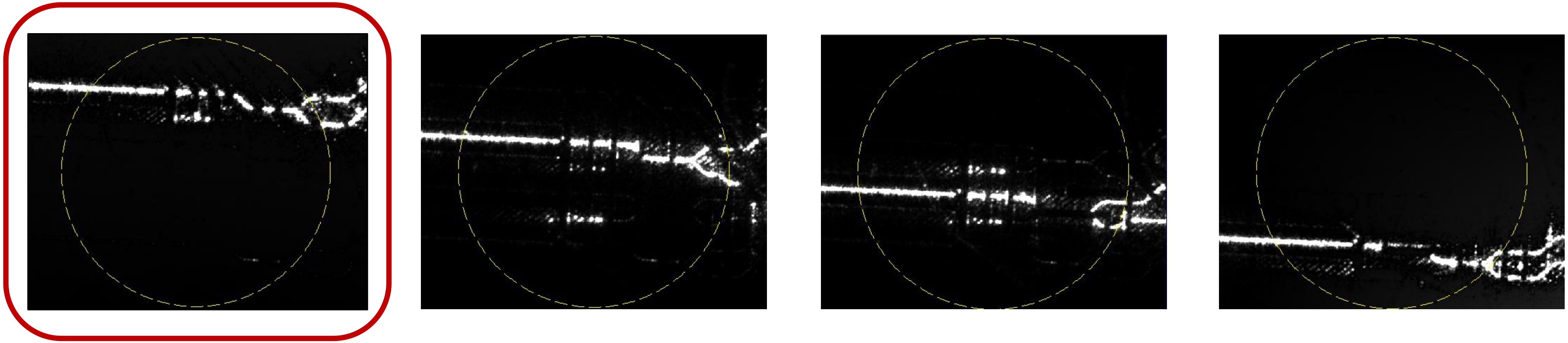
MZI: Mach-Zehnder Interferometer

PS: Phase Shifter

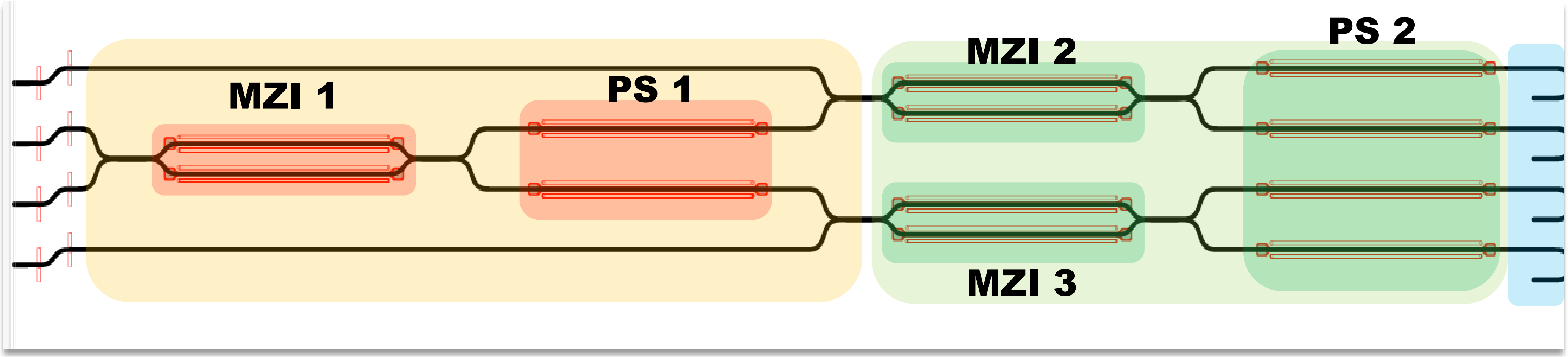
Example: basis states



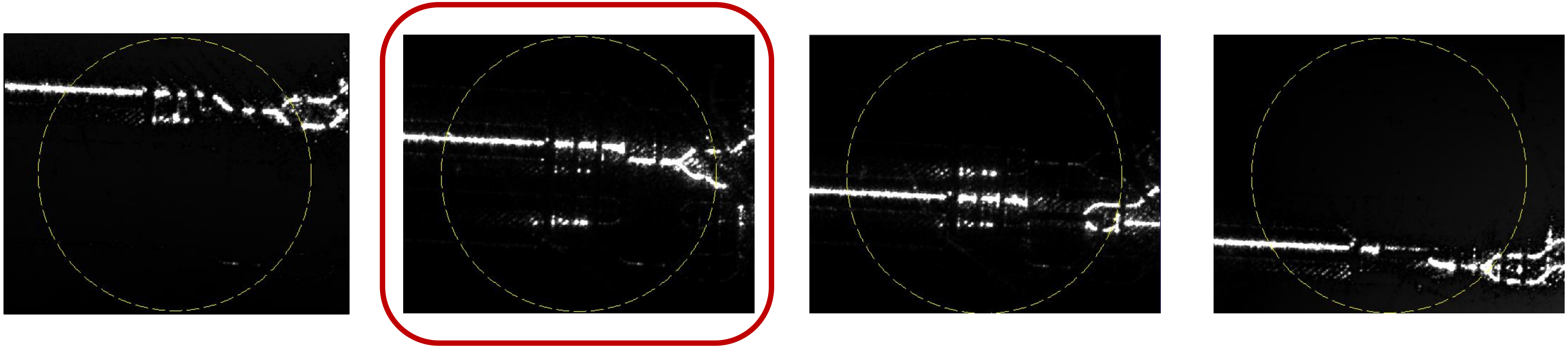
$|000\rangle$
$|001\rangle$
$|010\rangle$
$|011\rangle$
$|100\rangle$
$|101\rangle$
$|110\rangle$
$|111\rangle$



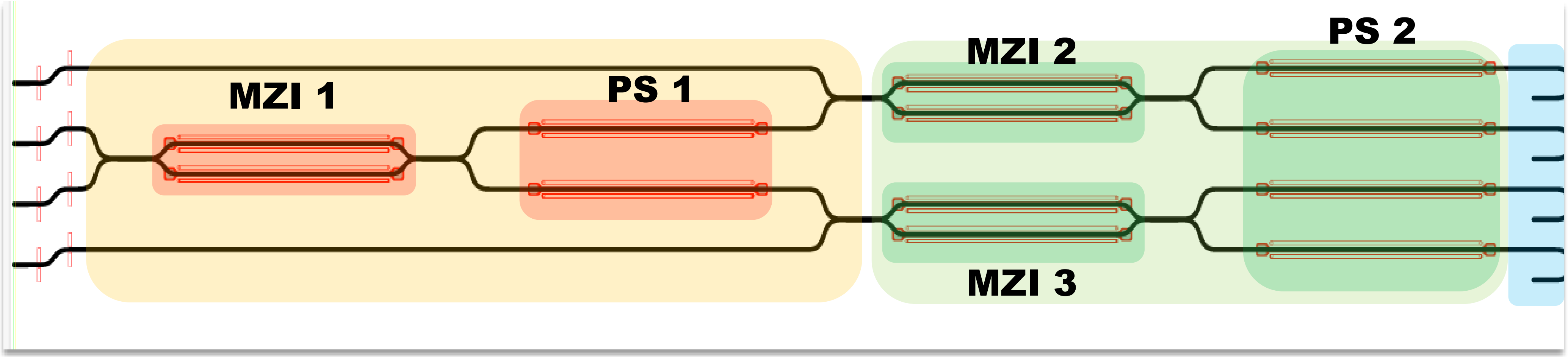
Example: basis states



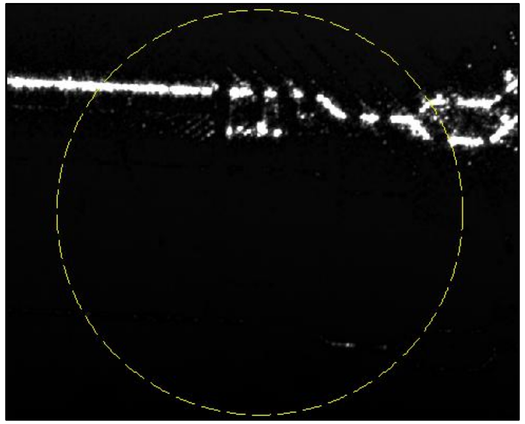
$|000\rangle$
$|001\rangle$
$|010\rangle$
$|011\rangle$
$|100\rangle$
$|101\rangle$
$|110\rangle$
$|111\rangle$

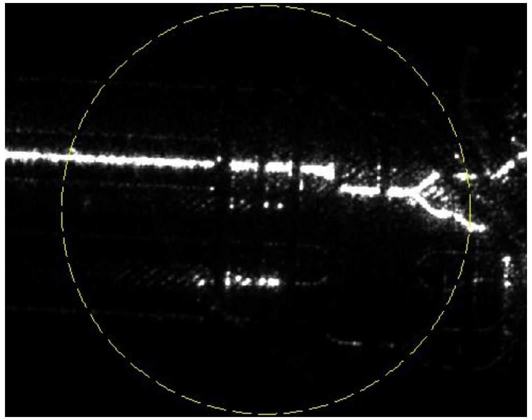


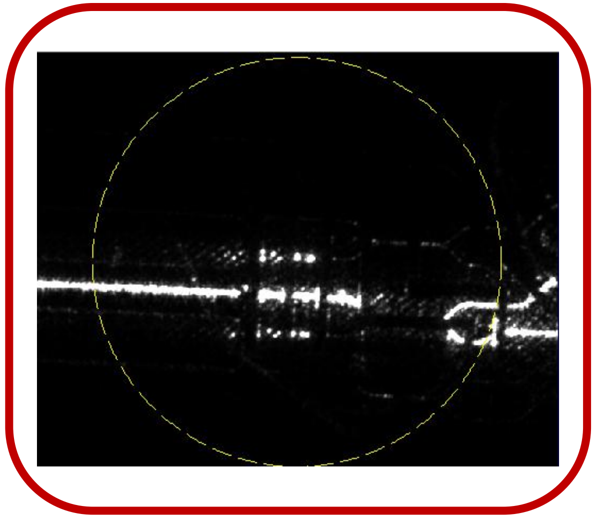
Example: basis states

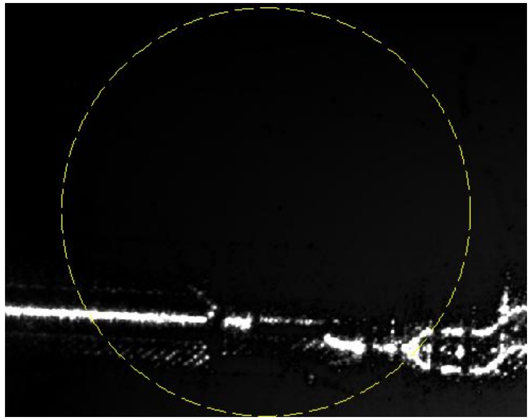


$|000\rangle$
$|001\rangle$
$|010\rangle$
$|011\rangle$
$|100\rangle$
$|101\rangle$
$|110\rangle$
$|111\rangle$

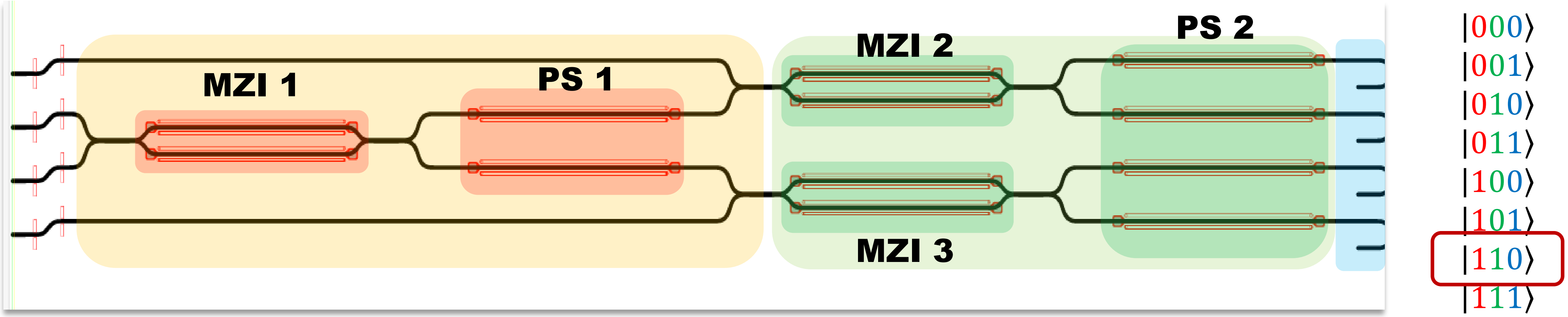


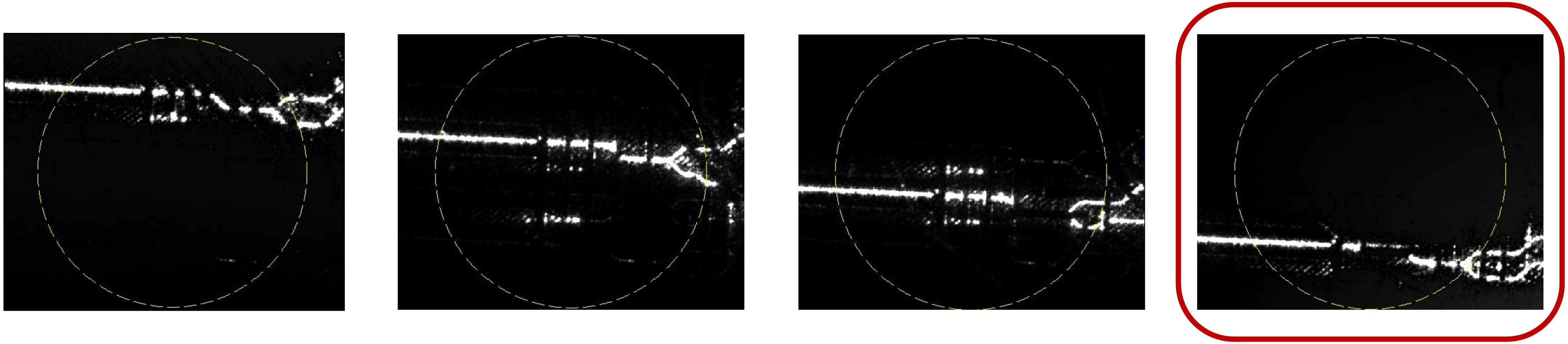




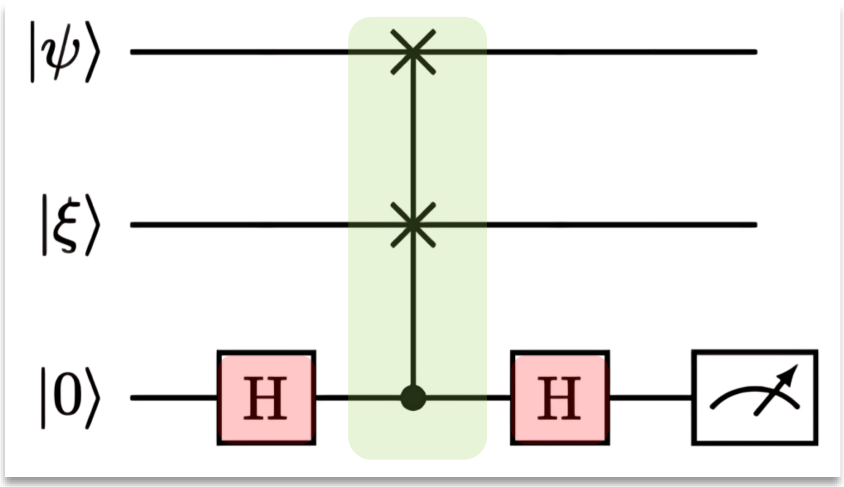


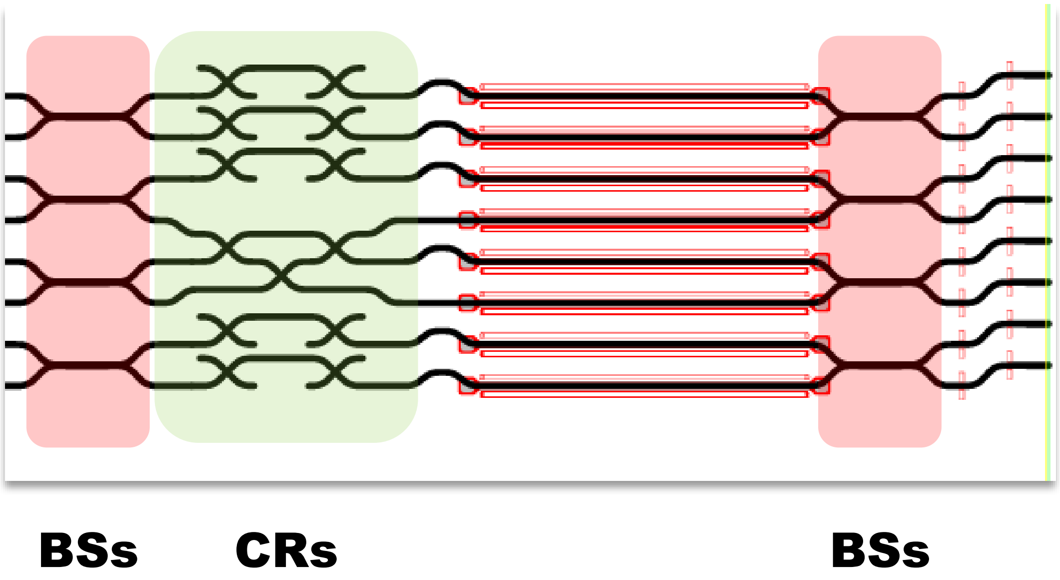
Example: basis states





SWAP test stage

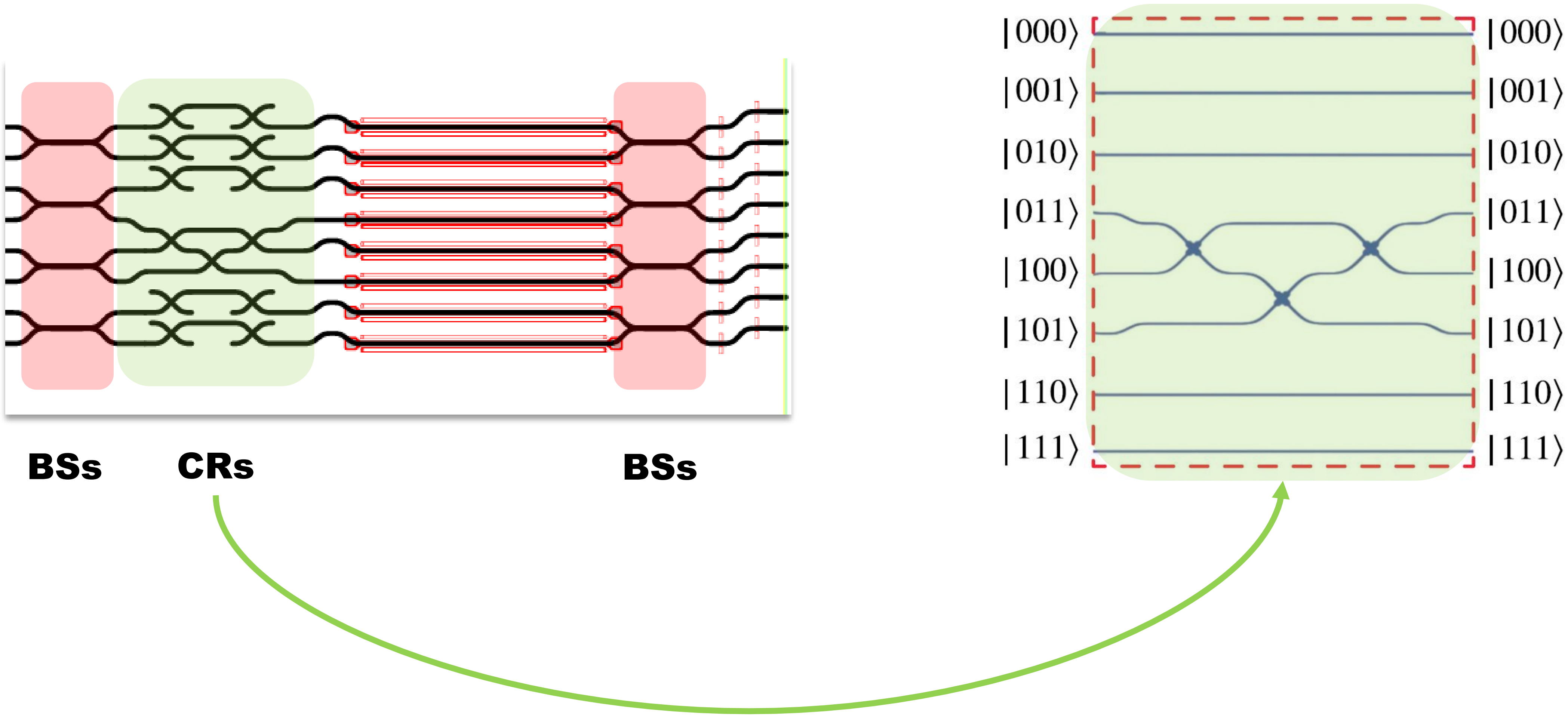




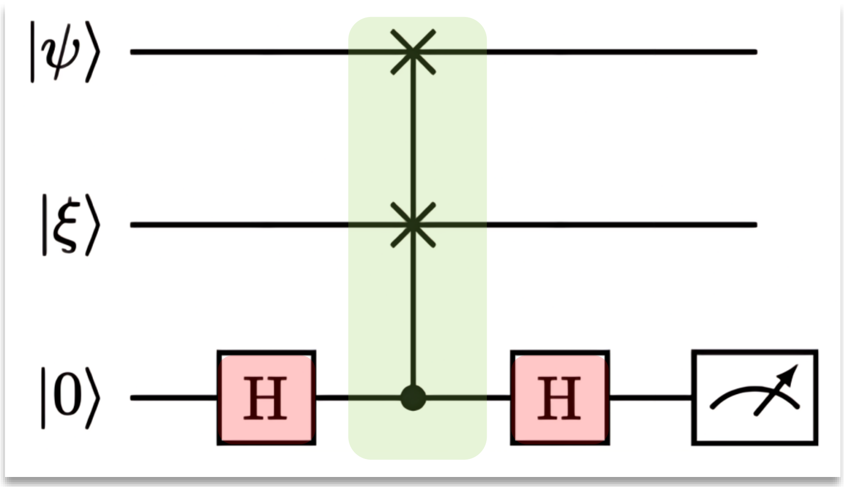
BS: Beam Splitter

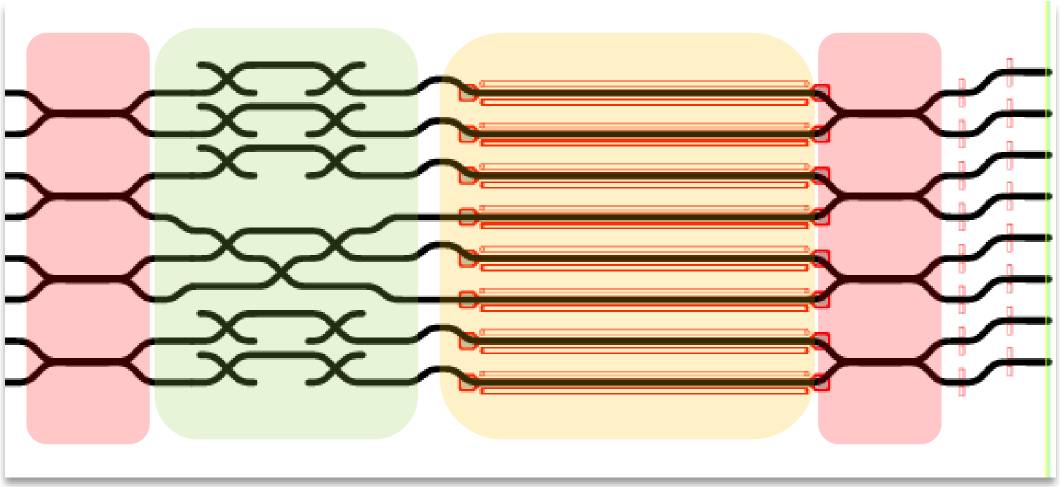
CR: Waveguide Crossing

SWAP test stage



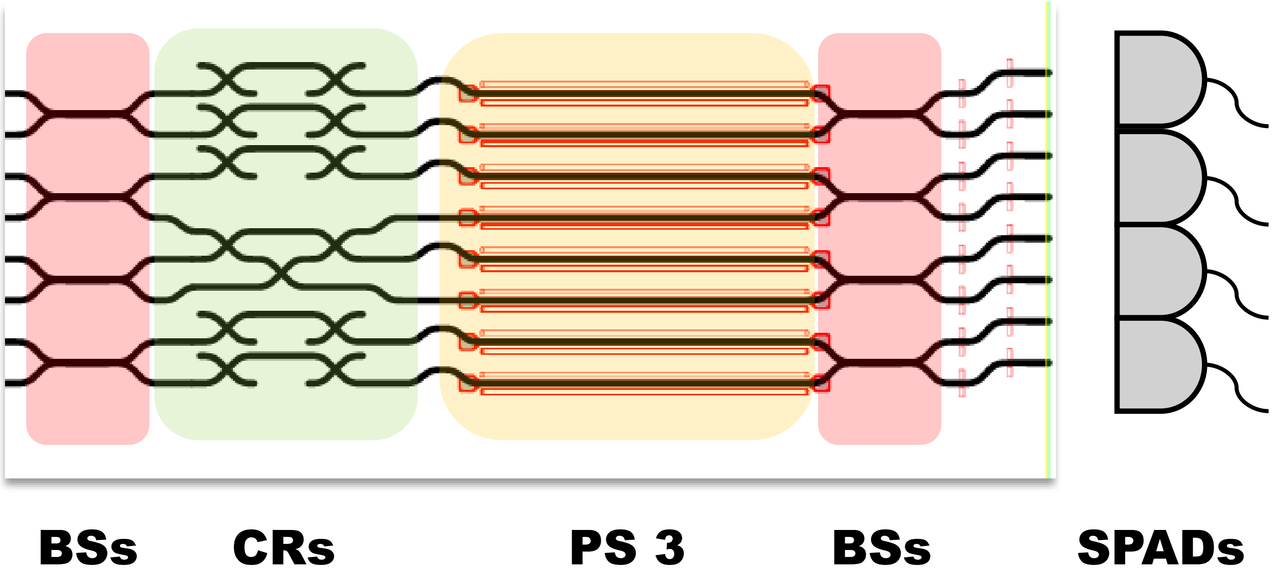
SWAP test stage





BSs CRs PS 3 BSs

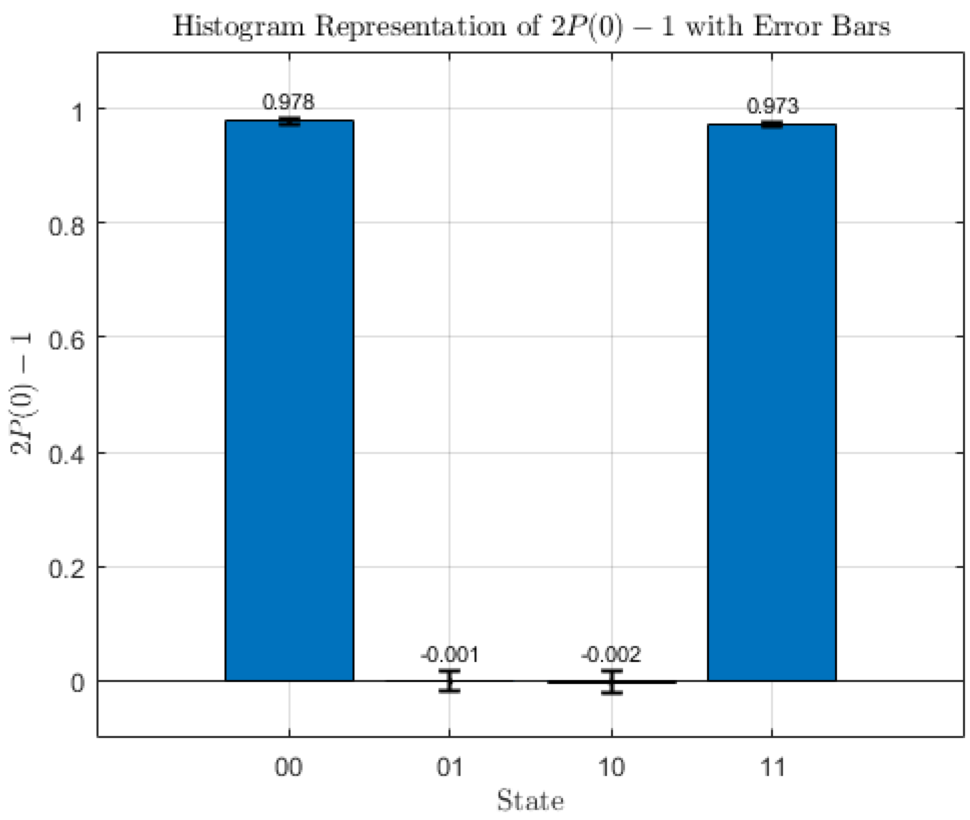
SWAP test stage

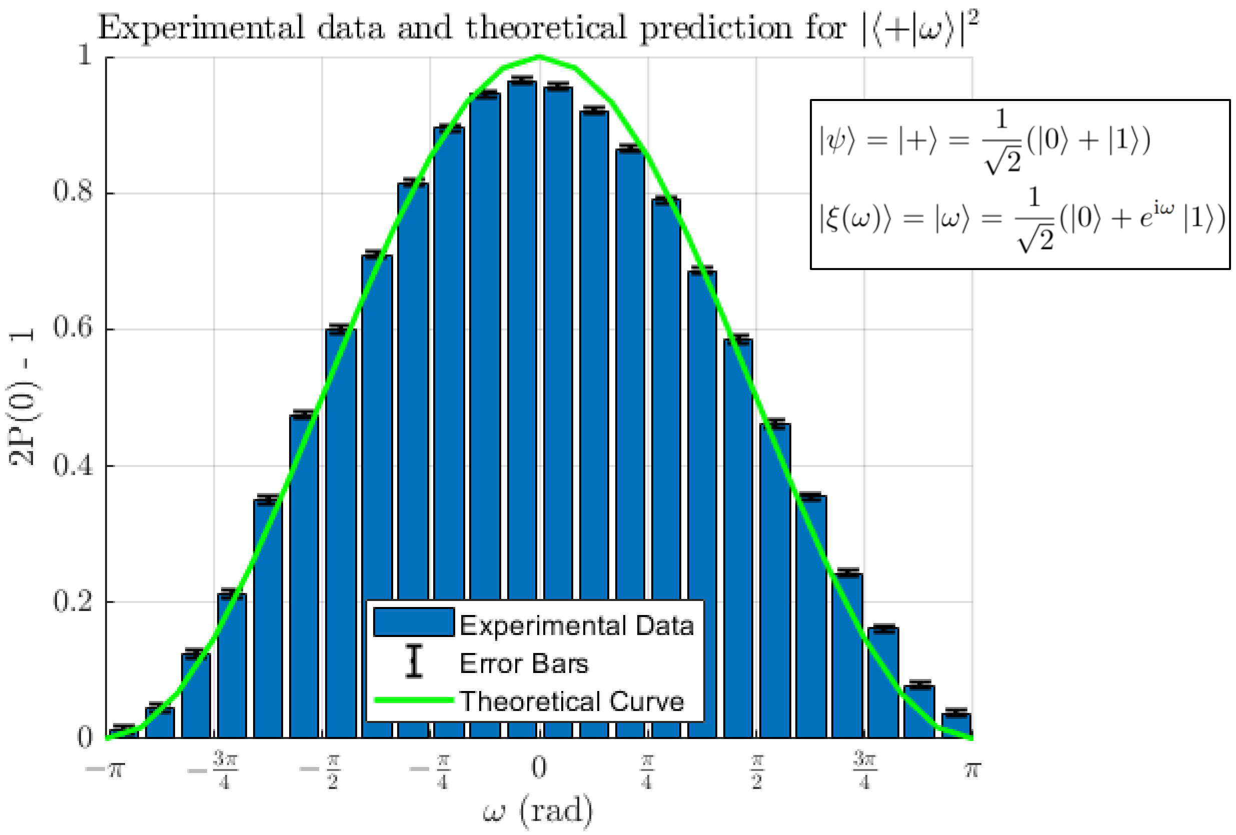


$$P(x) = \frac{1}{2}(1 - (-)^x |\langle\psi|\xi\rangle|^2)$$

$$P(x) = \frac{N_x}{N_0 + N_1}$$

Some results for separable states





Advantages

1. **Only linear** optical integrated elements are used
2. **No** need for **non-linearity, heralding or post-selection** to achieve multi-qubit gates
3. **Simplicity** and **robustness**
4. **Minimum** number of **qubits** and **gates**
5. **The mapping** to the **gate-based language** is direct
6. **Room temperature** operation

Disadvantages

1. **Scalability**
2. **Destructive** measurement

P(1) as en entanglement witness

Let $|\phi\rangle \in \mathbb{C}^2 \otimes \mathbb{C}^2$ be a pure state of the form

$$|\phi >= \alpha|00 > +\beta|11 > +\gamma|10 > +\delta|01 >$$
$$\alpha, \beta, \gamma, \delta \in \mathbb{C}, \ |\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$$

In the SWAP test circuit, initially fed with the state$|\phi\rangle \otimes |0\rangle$, the probability of obtaining the state 1 on the ancilla is given by

$$P(1) = \frac{1}{2}|\gamma - \delta|^2$$

Then

$$P(1) > \frac{1}{2} \Longrightarrow |\phi\rangle \text{ is } \textbf{entangled}$$

$$P(1) > \frac{1}{2} \not\Longleftarrow |\phi\rangle \text{ is } \textbf{entangled}$$

Example: Bell states

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \quad \rightarrow \quad \alpha = \beta = 0,\ \delta = -\gamma = \frac{1}{\sqrt{2}} \quad \rightarrow \quad P(1) = 1$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \quad \rightarrow \quad \alpha = \beta = 0,\ \delta = \gamma = \frac{1}{\sqrt{2}} \quad \rightarrow \quad P(1) = 0$$

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \quad \rightarrow \quad \alpha = \beta = \frac{1}{\sqrt{2}},\ \delta = \gamma = 0 \quad \rightarrow \quad P(1) = 0$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \quad \rightarrow \quad \alpha = -\beta = \frac{1}{\sqrt{2}},\ \delta = \gamma = 0 \quad \rightarrow \quad P(1) = 0$$

Example: Bell states

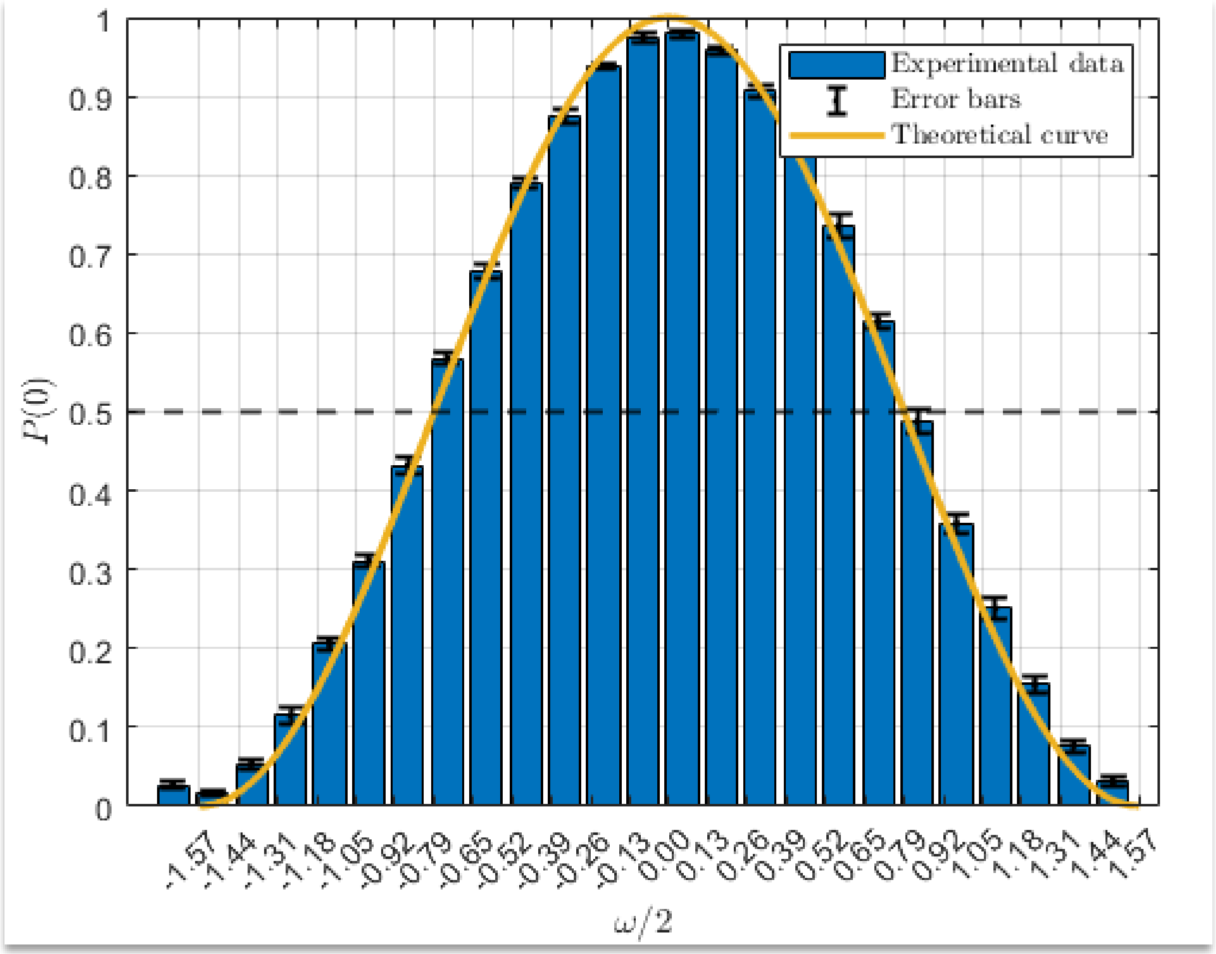
$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \quad \rightarrow \quad \alpha = \beta = 0,\ \delta = -\gamma = \frac{1}{\sqrt{2}} \quad \rightarrow \quad P(1) = 1$$

	$\frac{1}{\sqrt{2}}(\|00\rangle \pm \|11\rangle)$	$\frac{1}{\sqrt{2}}(\|01\rangle + \|10\rangle)$	$\frac{1}{\sqrt{2}}(\|01\rangle - \|10\rangle)$
Theoretical P(1)	0%	0%	100%
Model 2σ C.I. P(1)	[0, 1.38] %	[0, 5.14]%	[95.49,100]%
Experimental P(1)	(1.3 ± 0.2)%	(3.1 ± 0.4)%	(98.0 ± 0.2)%

Let us consider the category of states:

$$|\Psi(\omega)\rangle = \frac{1}{\sqrt{2}}(|01\rangle + e^{i\omega}|10\rangle)$$

$$P(0) = 1 - P(1) = \cos^2\left(\frac{\omega}{2}\right)$$



Mixed states

Noisy enviroment, non-idealities, ... $\longrightarrow$ **Mixed states!**

For example:

$$\rho = p \left|\Phi\right\rangle \left\langle\Phi\right| + \frac{1-p}{4}\mathbb{1}$$

In our case, we can use the **PPT criterion:** ρ is separable iff its partial transpose has no negative eigenvalues (2x2 and 2x3 systems).

The **threshold** for entanglement **moves**:

$$P(1) > \frac{1}{2}$$

In our case, we can use the **PPT criterion:** ρ is separable iff its partial transpose has no negative eigenvalues (2x2 and 2x3 systems).

The **threshold** for entanglement **moves**:

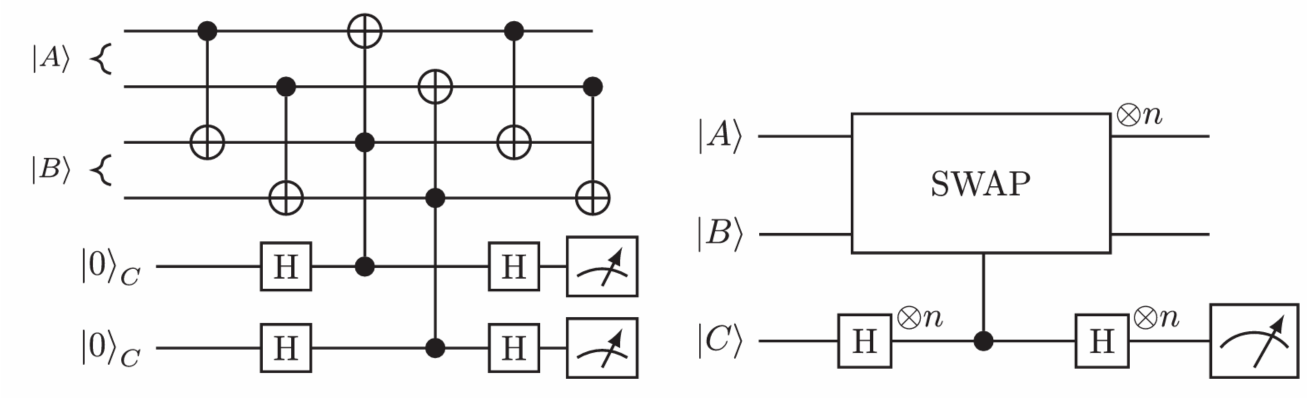
$$P(1) > \frac{1}{2} \quad \Rightarrow \quad P(1) > \frac{2}{3}$$

Summary

- The **SWAP test algorithm** allows to compute the overlap between two quantum states
- It can be implemented in a simple and robust way on a **photonic chip** based on **linear optics** and working at **room temperature**
- The SWAP test can be used to **witness** the presence of **entanglement** both for pure and mixed states (more precisely, Werner states).

Future perspectives

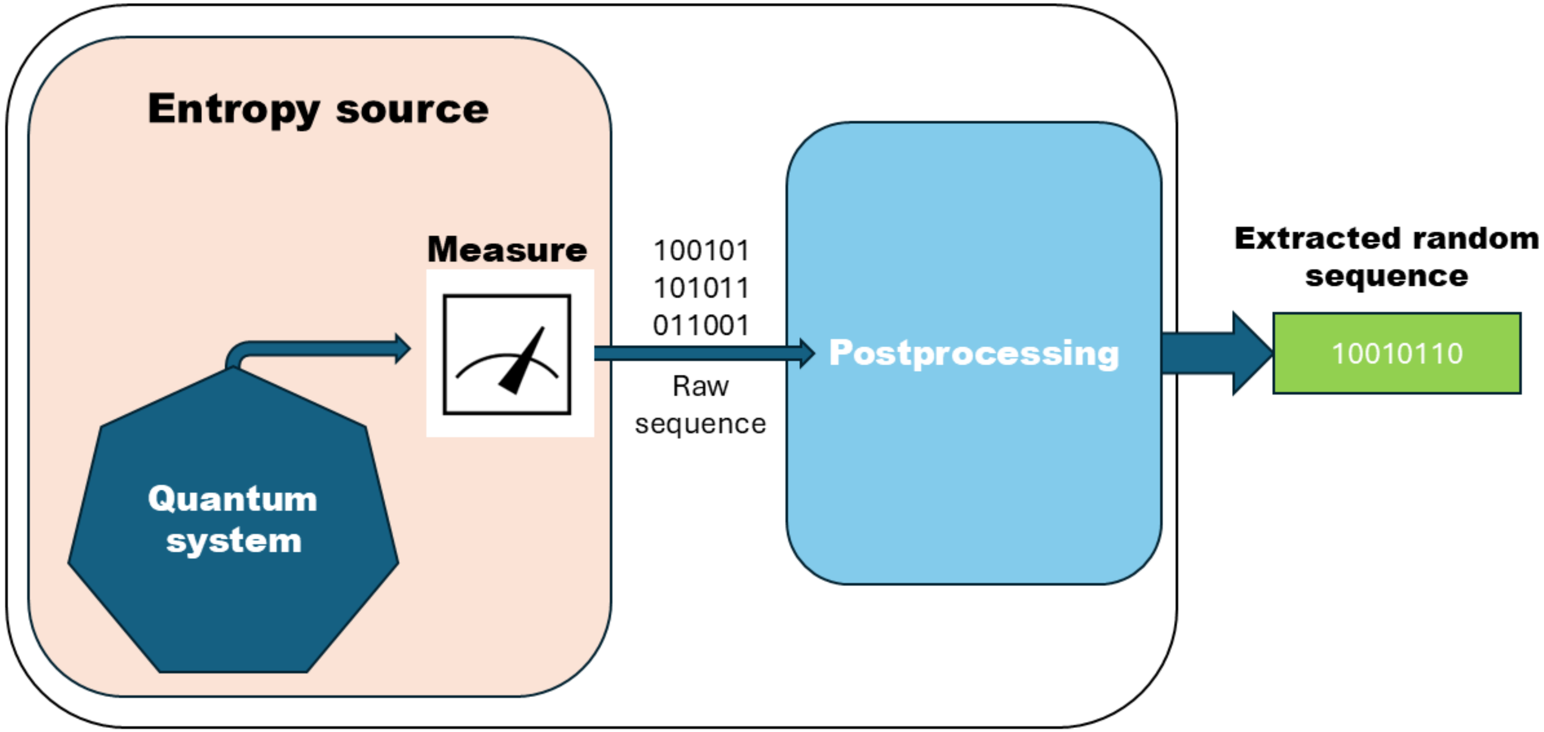
- The SWAP test can also be adapted to an **efficient and quantitative test** for entanglement of pure states. This is possible through a **parallelized SWAP** [2]



- Recently, other theoretical studies further investigated the possibility to exploit the SWAP test for **entanglement of generic mixed states** and to **compute multipartite entanglement measures** [3], [4], [5],[6]

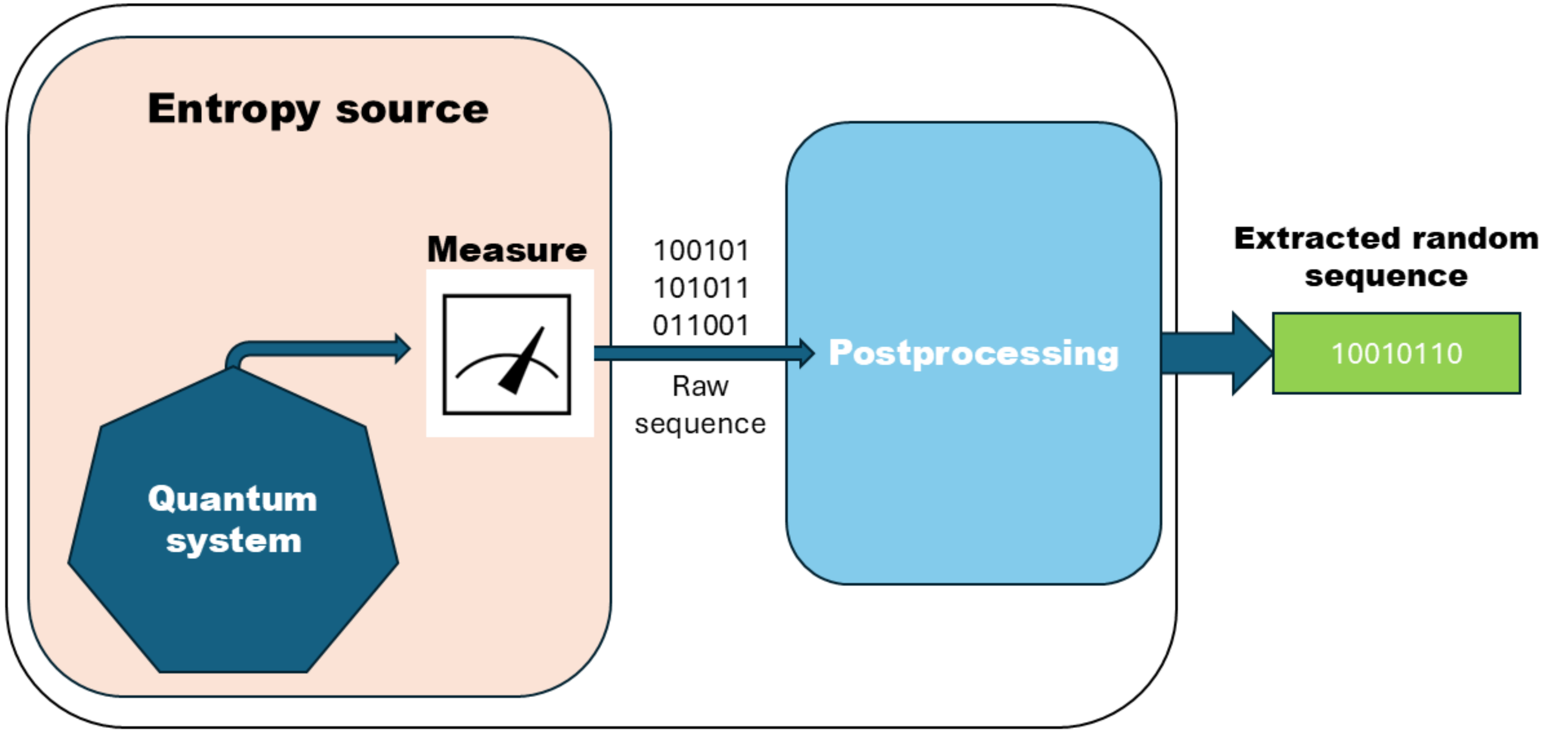
Future perspectives

- The entanglement witness characteristic of the SWAP test can be exploited in the **certification of the min-entropy of a QRNG**



Future perspectives

- The entanglement witness characteristic of the SWAP test can be exploited in the **certification of the min-entropy of a QRNG**



If $\mathbb{P}(1) = 1/2 + \epsilon/2$

$$H_{\min} \geq -\log_2\left(\frac{1+\sqrt{1-\epsilon^2}}{2}\right)$$

Dott. Alessio Baldazzi

Dott. Matteo Sanna

Dott. Nicolò Leone

Prof. Lorenzo Pavesi

Prof. Stefano Azzini

Prof.ssa Sonia Mazzucchi

THANK YOU!

References

[1] Baldazzi, Alessio, et al. "A linear photonic swap test circuit for quantum kernel estimation." *Quantum Science and Technology* 9.4 (2024): 045053.

[2] Foulds, S., Kendon, V., & Spiller, T. (2021). The controlled SWAP test for determining quantum entanglement. *Quantum Science and Technology*, *6*(3), 035002.

[3] Beckey, Jacob L., et al. "Computable and operationally meaningful multipartite entanglement measures." *Physical Review Letters* 127.14 (2021): 140501.

[4] Zhang, Rui-Qi, et al. "The controlled SWAP test for entanglement of mixed quantum states." *Europhysics Letters* 146.1 (2024): 18001.

[5] Foulds, Steph, Oliver Prove, and Viv Kendon. "Generalising concentratable entanglement for practical applications: mixed, qudit, and optical states." *arXiv preprint arXiv:2112.04333* (2021).

[6] Wang, Yitian, and Qing-yu Cai. "Efficient swap test method for enhanced multipartite entanglement quantification." *Physics Letters A* 541 (2025): 130416.