Design of an analog quantum simulator with superconducting qubits

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Outline

- Introduction
- Physical problem
- Quantum architecture
- Next steps

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QUART&T project

QUantum Architectures for Theory & Technology (QUART&T)

Goal
Develop quantum architectures where theoretical models and phenomena of interest to the INFN can be tested.

Motivation
Simulate quantum many-body systems that have a physical related interest. Examples are nuclear reaction and dynamics, lattice quantum chromodynamics.

Approach
Realize an analog quantum simulator dedicated to a specific problem.

QUART&T collaboration

INFN Units:

- INFN Bologna
- INFN Ferrara
- INFN Firenze
- INFN Lecce
- INFN Milano
- **INFN Milano-Bicocca**
- INFN Gruppo Collegato di Salerno (INFN Napoli)

INFN National Laboratories:

- INFN Laboratori Nazionali di Frascati (LNF)
- INFN Laboratori Nazionali di Legnaro (LNL)

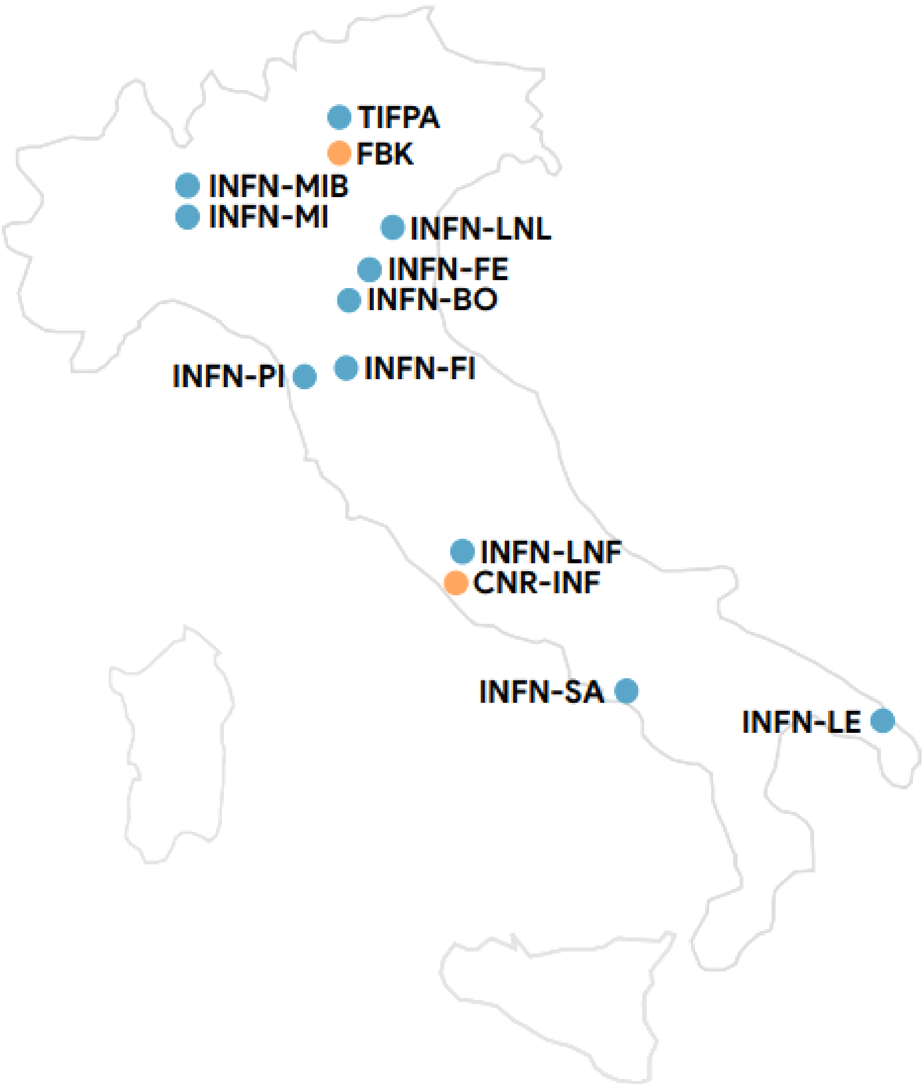
INFN Research Center:

- Trento Institute for Fundamental Physics and Applications (TIFPA)

External Research Centers:

- Fondazione Bruno Kessler (FBK, Trento)
- Istituto di Fotonica e Nanotecnologie (CNR-IFN, Roma)

CSN5 Technological research



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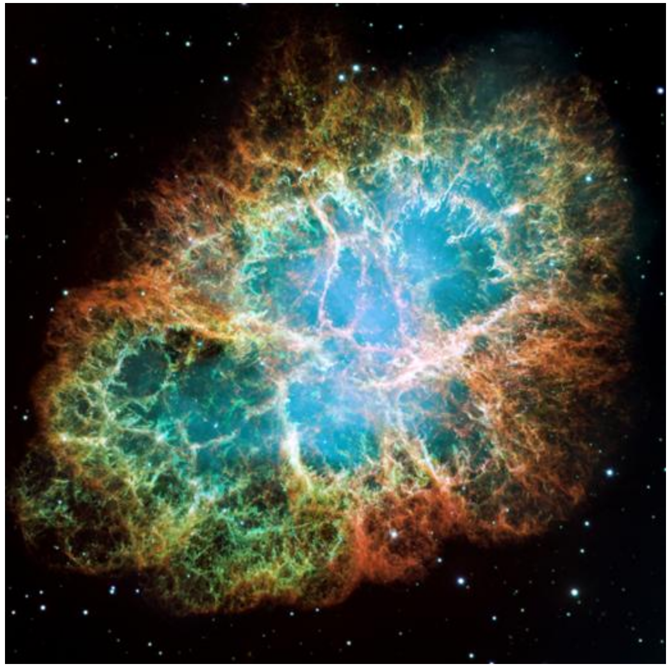
Neutrinos oscillations in dense environments

- In high-density neutrino environments — such as supernovae, neutron star mergers, or the early universe — neutrino flavor oscillations are affected by neutrino-neutrino interactions.



The Crab Nebula, a remnant of a supernovae.

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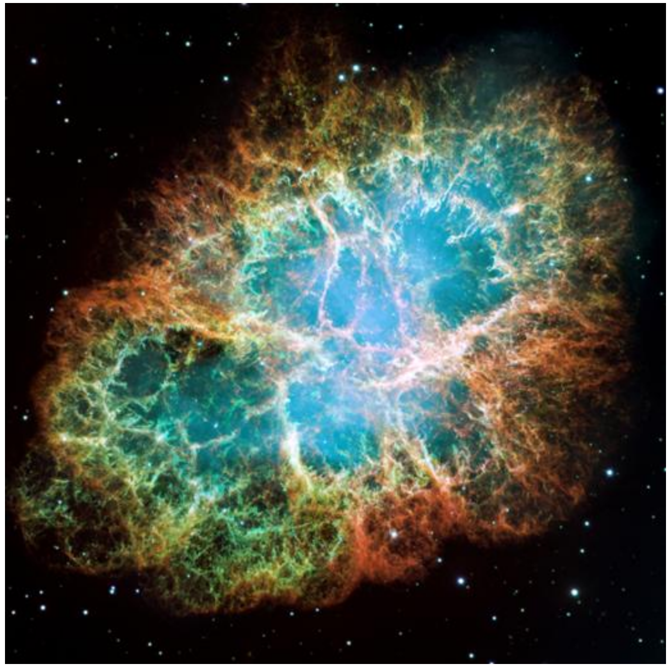
- The Hamiltonian for a **two flavors oscillation** can be written as:

$$H = \sum_{k=1}^{N} \vec{b} \cdot \vec{\sigma}_k + \sum_{p<q}^{N} J_{pq}\, \vec{\sigma}_p \cdot \vec{\sigma}_q \qquad \text{with } \vec{\sigma}_k = (\sigma_k^x, \sigma_k^y, \sigma_k^z)$$

1. Vacuum mixing
2. MSW effect

3. Neutrino–neutrino interaction



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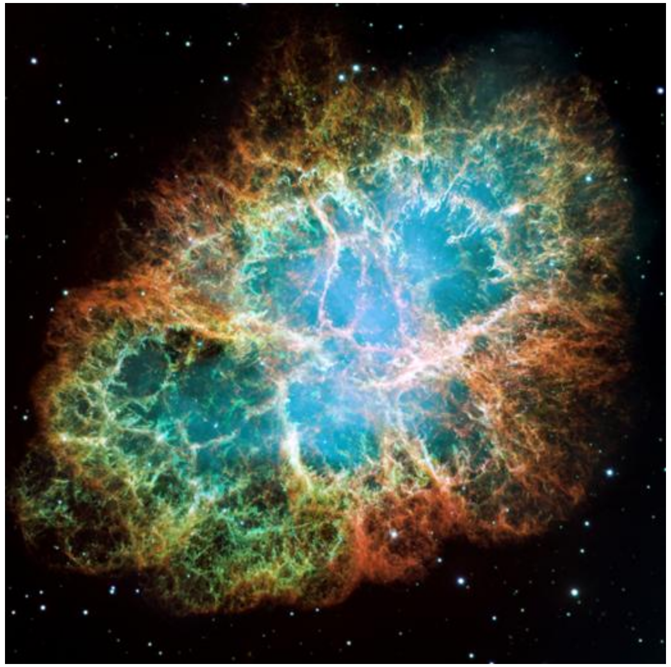
- An **exact solution** is necessary to understand the role of quantum correlations.



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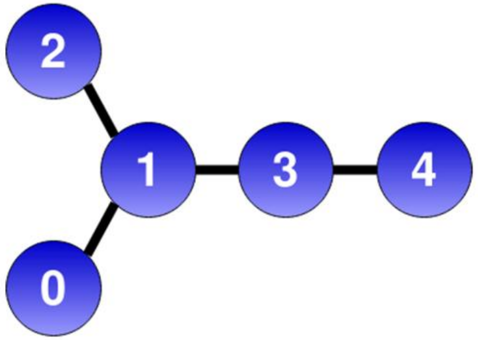
Quantum digital simulations

[Phys.Rev.D 104 (2021) 6, 063009]

- System complexity on a classical computer scales as $\sim e^N$. On a quantum machine $\sim N^k$.

$$e \longrightarrow |0\rangle$$

$$\tau \longrightarrow |1\rangle$$



IBM Quantum Canary Processor Vigo

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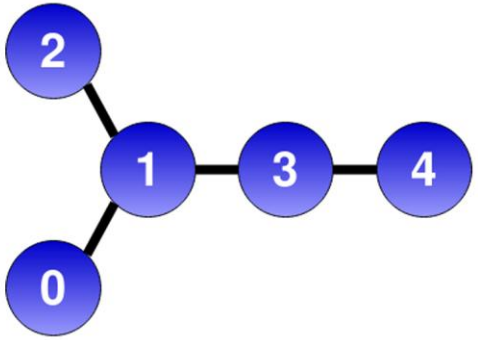
- **First-order Trotter-Suzuki decomposition**:

$$U(t) = \prod_{j=1}^{N} e^{-it\vec{b}\cdot\vec{\sigma}_j} \prod_{p<q}^{N} e^{-itJ_{pq}\vec{\sigma}_p\cdot\vec{\sigma}_q}$$

- **All-to-all connectivity** realized using SWAP gates.



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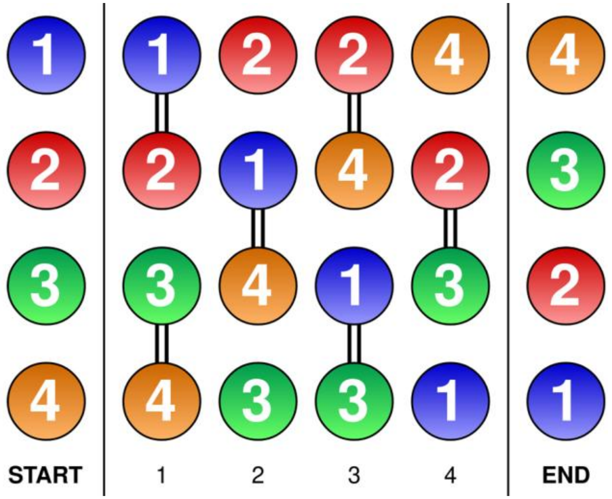


Scheme to realize a single time-step simulation. Each double line is formed by 3 $\binom{N}{2}$ *CNOT + 15* $\binom{N}{2}$ *single qubit gates.*

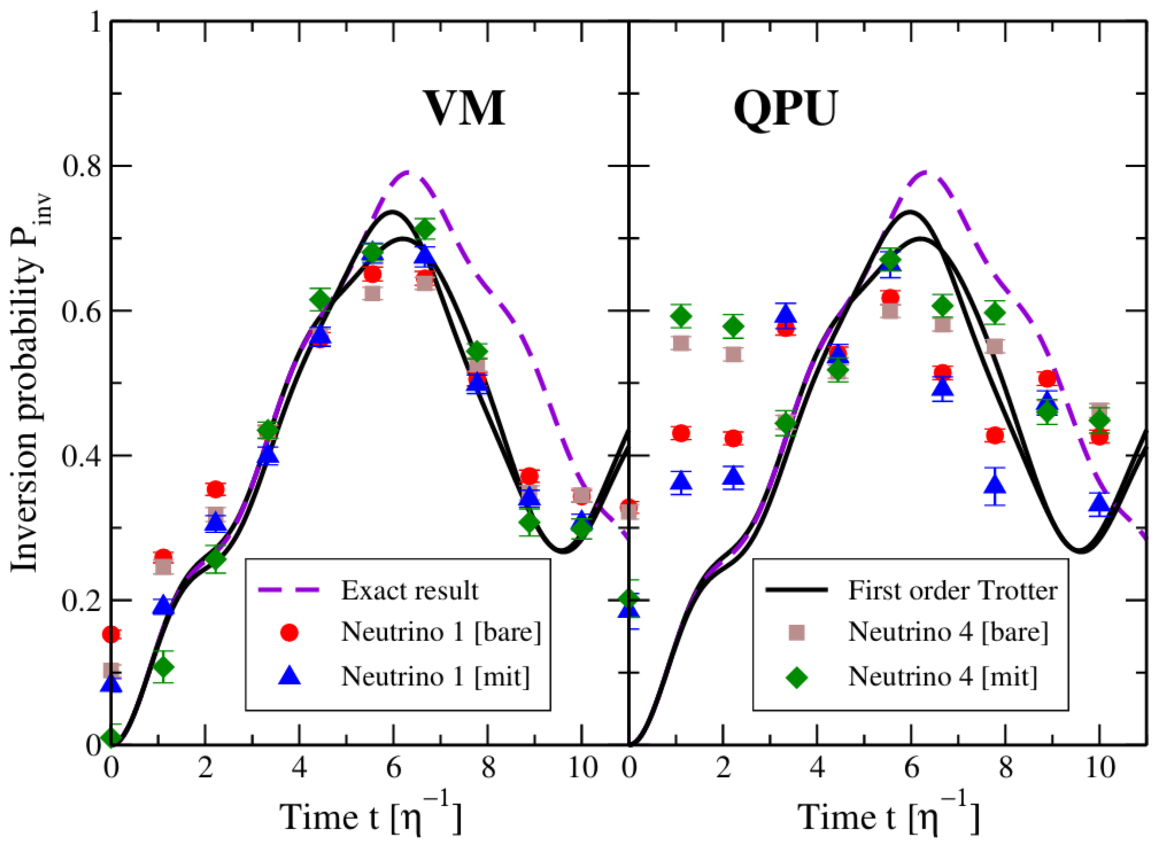
Quantum digital simulation - Limitations

- **The first-order Trotter-Suzuki** decomposition introduce an error of order $O(t^2)$ on the simulation results.

- **N gates** are needed for each simulation step t. This require long run time to reach higher simulated times.

- **Single qubit noise** limits both results accuracy and simulation length.

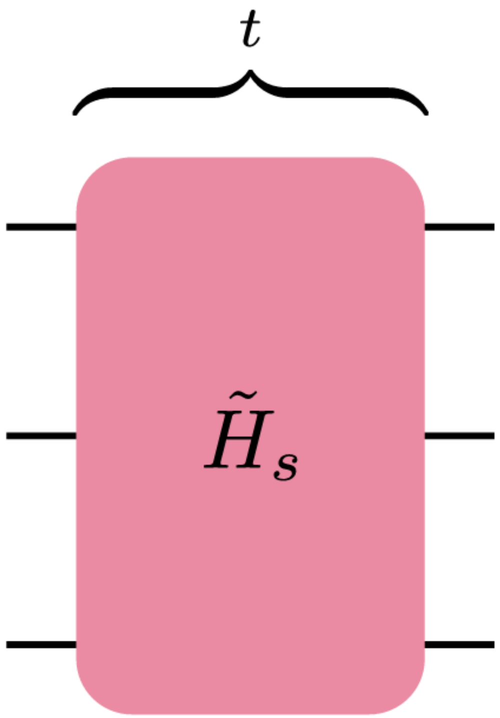
- **SWAP gate errors** increase as single qubit errors occurs.



Flavor inversion probability for neutrino 1 and neutrino 4

Analog approach

- **Analog quantum simulator**: a quantum device whose time evolution emulates the dynamics of a more complex quantum model.



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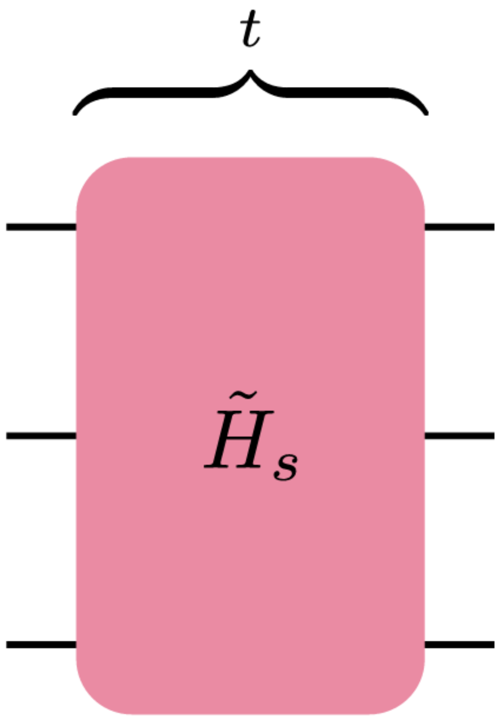
Main advantages:

- Each simulated time step require less operations.
- No approximation needed for the time evolution operator.



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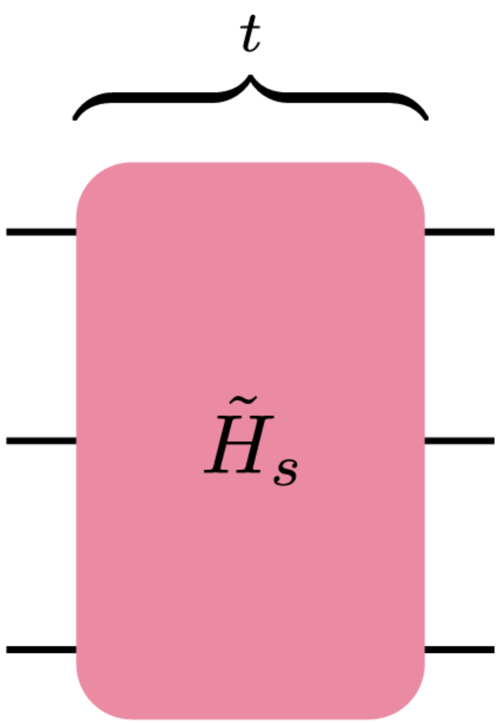

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- **2D superconducting platform** ⇒
  - Highly engineerable
  - Parameter tunability

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Hamiltonian

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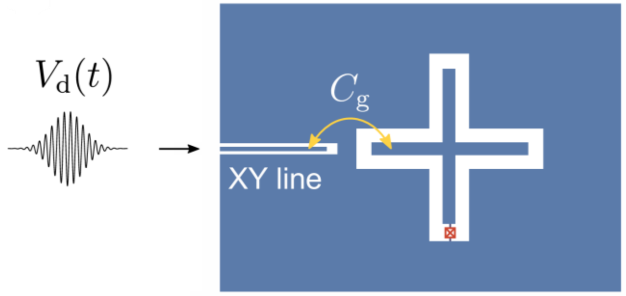
Single qubit gates:

- External drive on single qubits

$$\downarrow$$

$$H \sim \Omega\, V_d(t)\big(I\sigma_x + Q\sigma_y\big) \quad I = \cos\phi \;, Q = \sin\phi$$

- Continuous virtual Z gate → $H \sim \frac{\delta\omega}{2}\sigma_z$



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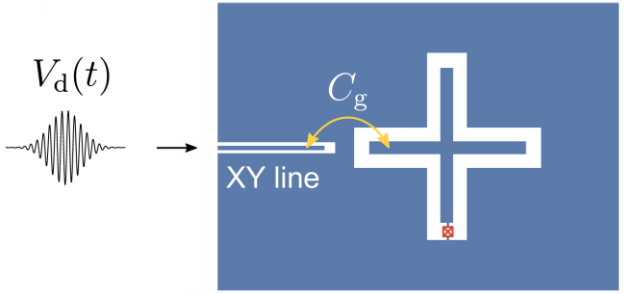
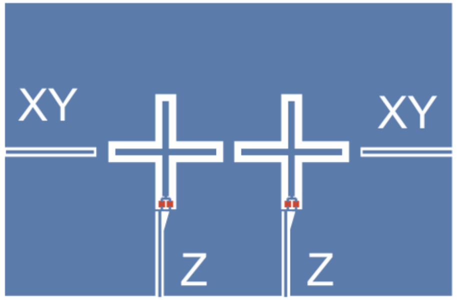


Two qubit interactions:

- Capacitive coupling → $H \sim \frac{g}{2}\big(\sigma_x\sigma_x + \sigma_y\sigma_y\big)$
- Higher states energy shift → $H \sim \frac{\zeta}{4}(\sigma_z\sigma_z)$
- Parametric couplings



Two qubit interaction – Capacitive coupling

- Direct capacitive coupling between transmon qubits is described by:

$$H = -g([\sigma^+ - \sigma^-] \otimes [\sigma^+ - \sigma^-])$$

- Under **Rotating Wave Approximation** (valid if $\omega_{q1}, \omega_{q2} \gg g$) fast oscillating terms disappear leaving:

$$H = g(\sigma^+\sigma^- + \sigma^-\sigma^+) = \boxed{\frac{g}{2}(\sigma_x\sigma_x + \sigma_y\sigma_y)}$$

Interaction we are looking for!

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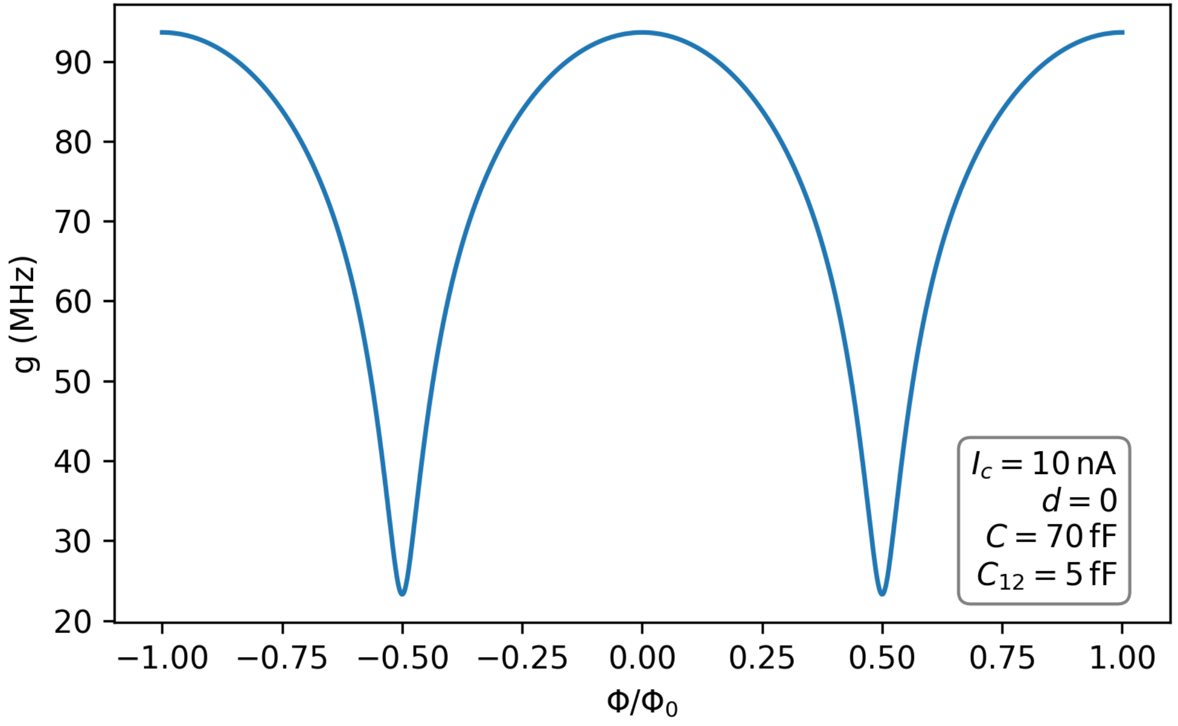
Interaction we are looking for!

- g has a **qubit frequency dependency**:

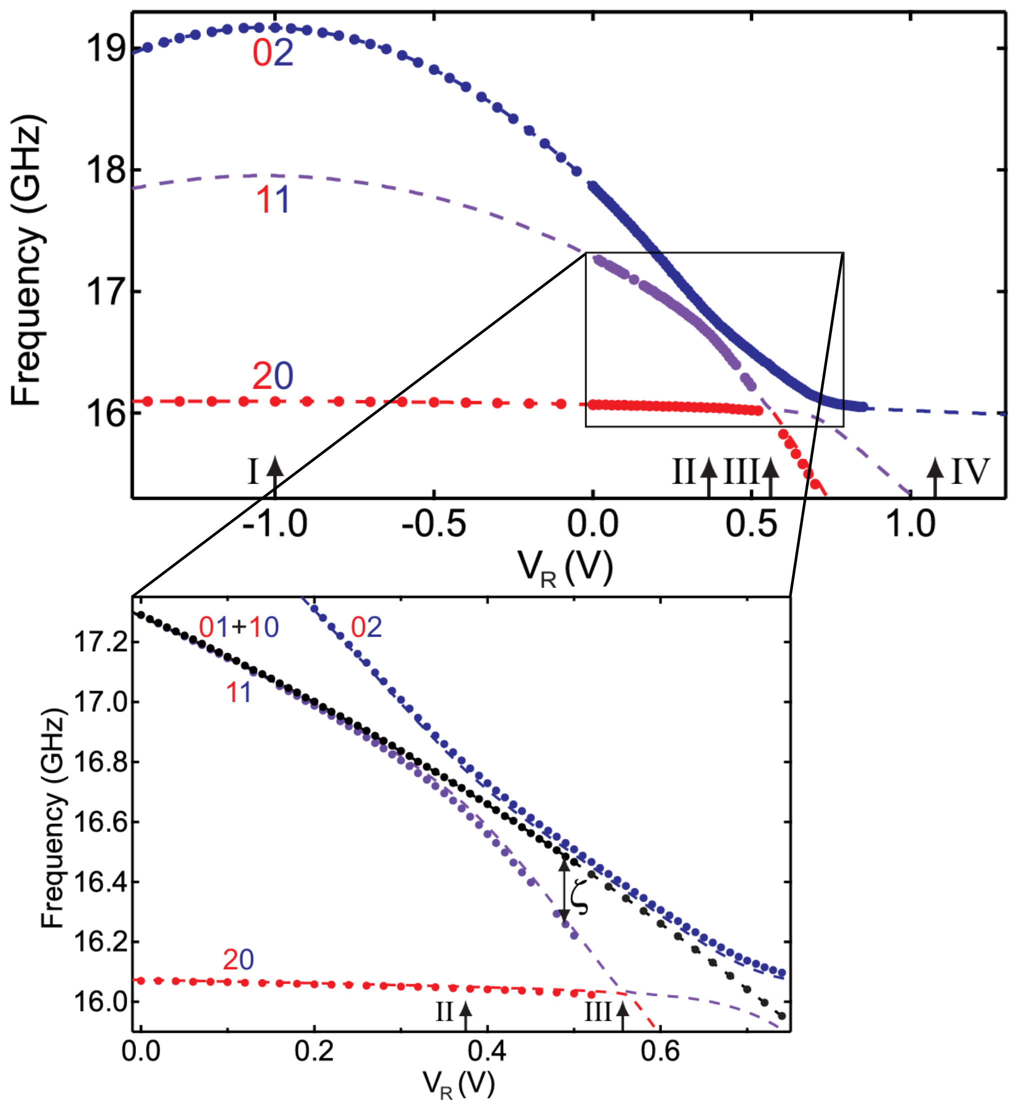
$$g \sim \frac{1}{2}\frac{e^2}{\hbar}\frac{C_{12}}{\sqrt{C_1C_2}}\sqrt{\omega_1\omega_2}$$



Two qubit interaction – ZZ coupling [*Nature* **460**, 240–244 (2009)]

- Idea: use the **energy repulsion** between $|20\rangle$, $|02\rangle$ and $|11\rangle$ to realize an effective ZZ interaction on the computational subspace;

$$\zeta = E_{|11\rangle} - E_{|01\rangle} - E_{|10\rangle}$$



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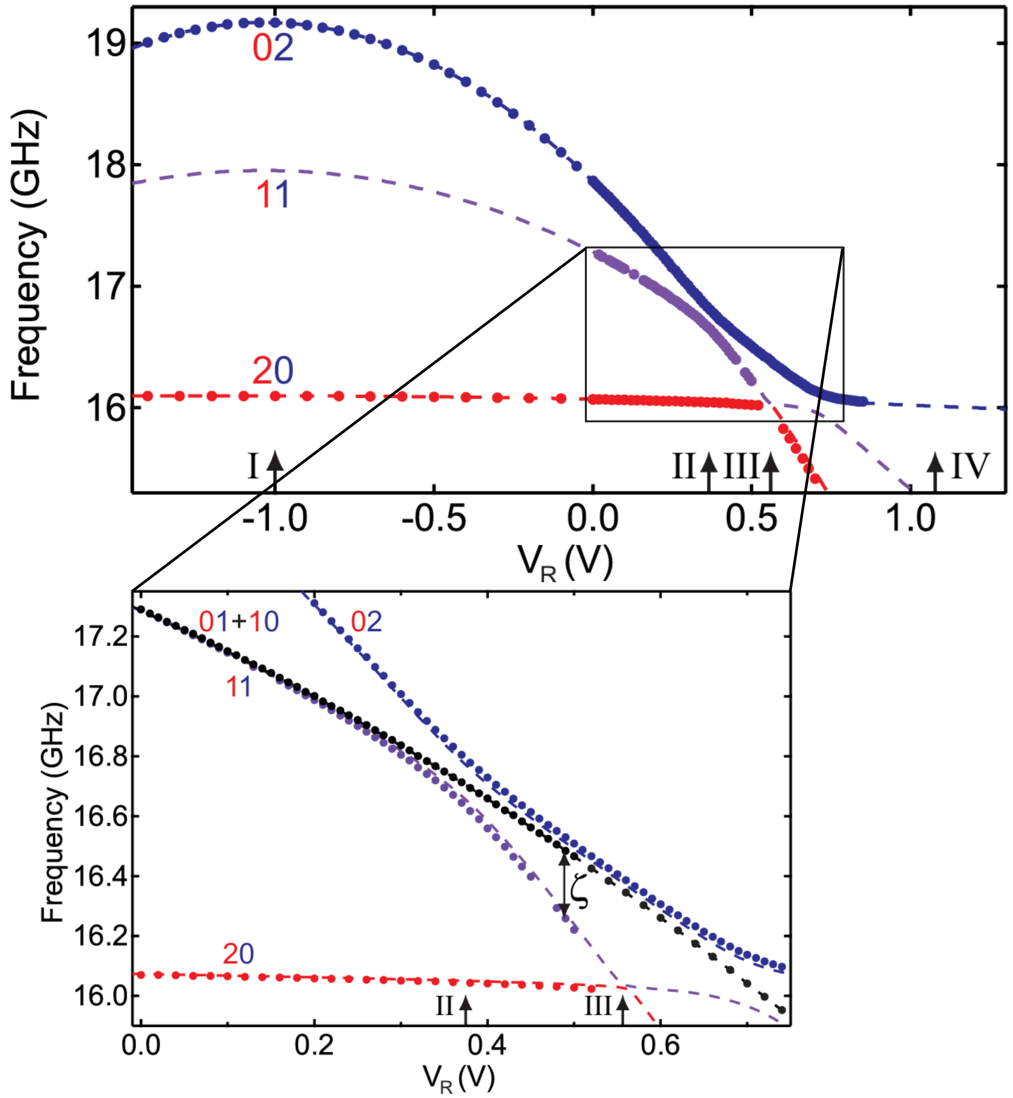
- The interaction hamiltonian on the computational subspace is:

$$H \sim \frac{\zeta}{4}\sigma_z \otimes \sigma_z - A\,\sigma_z \otimes \mathbb{I} - B\,\mathbb{I} \otimes \sigma_z$$

Interaction we are looking for! Can be used for single qubit gates!

- Same type of interaction used in digital architectures to realize a CPHASE gate.



Two qubit interaction – ZZ coupling

How to properly create the ZZ effective interaction? [Appl. Phys. Rev. 1 2019]

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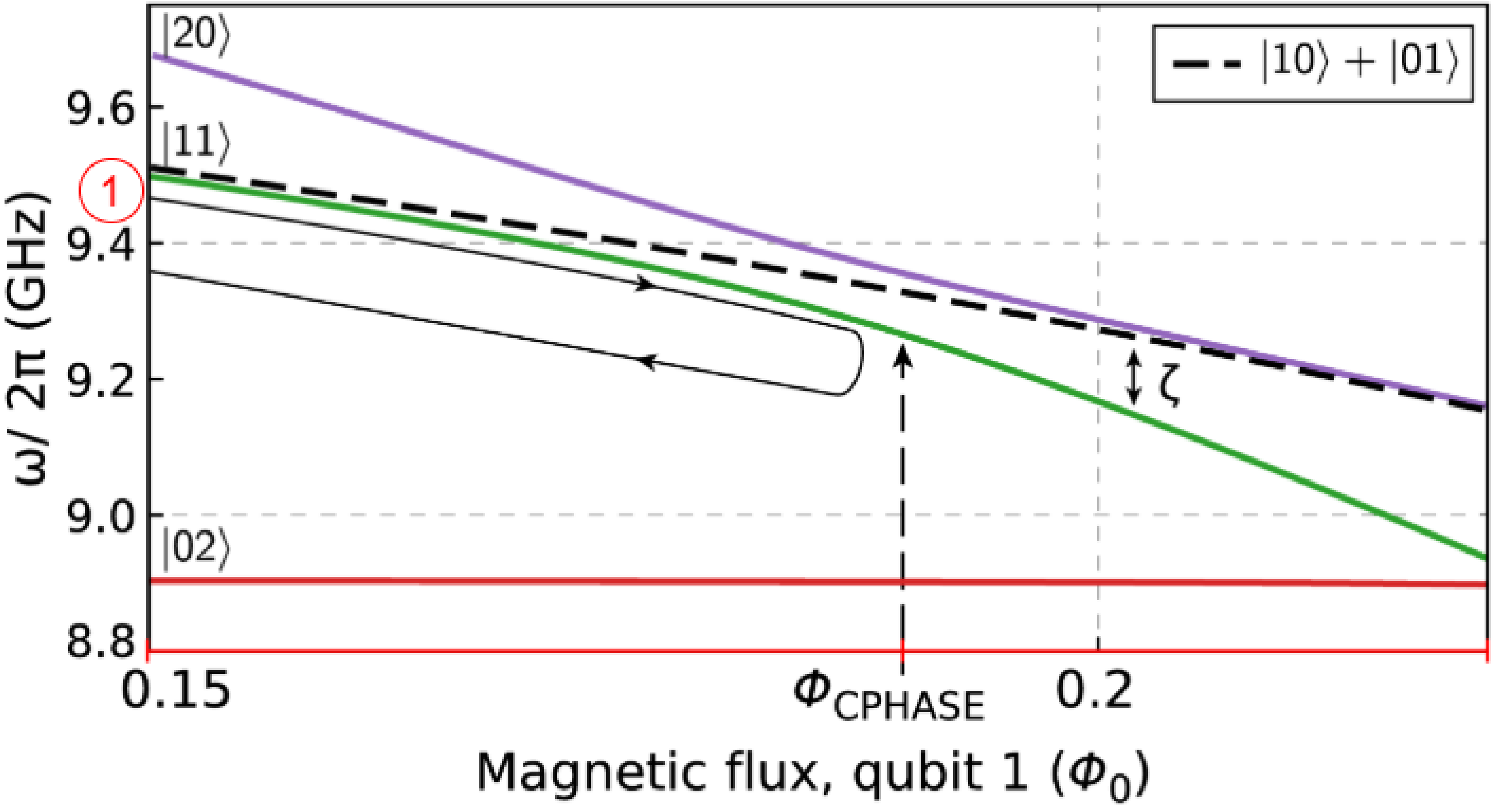

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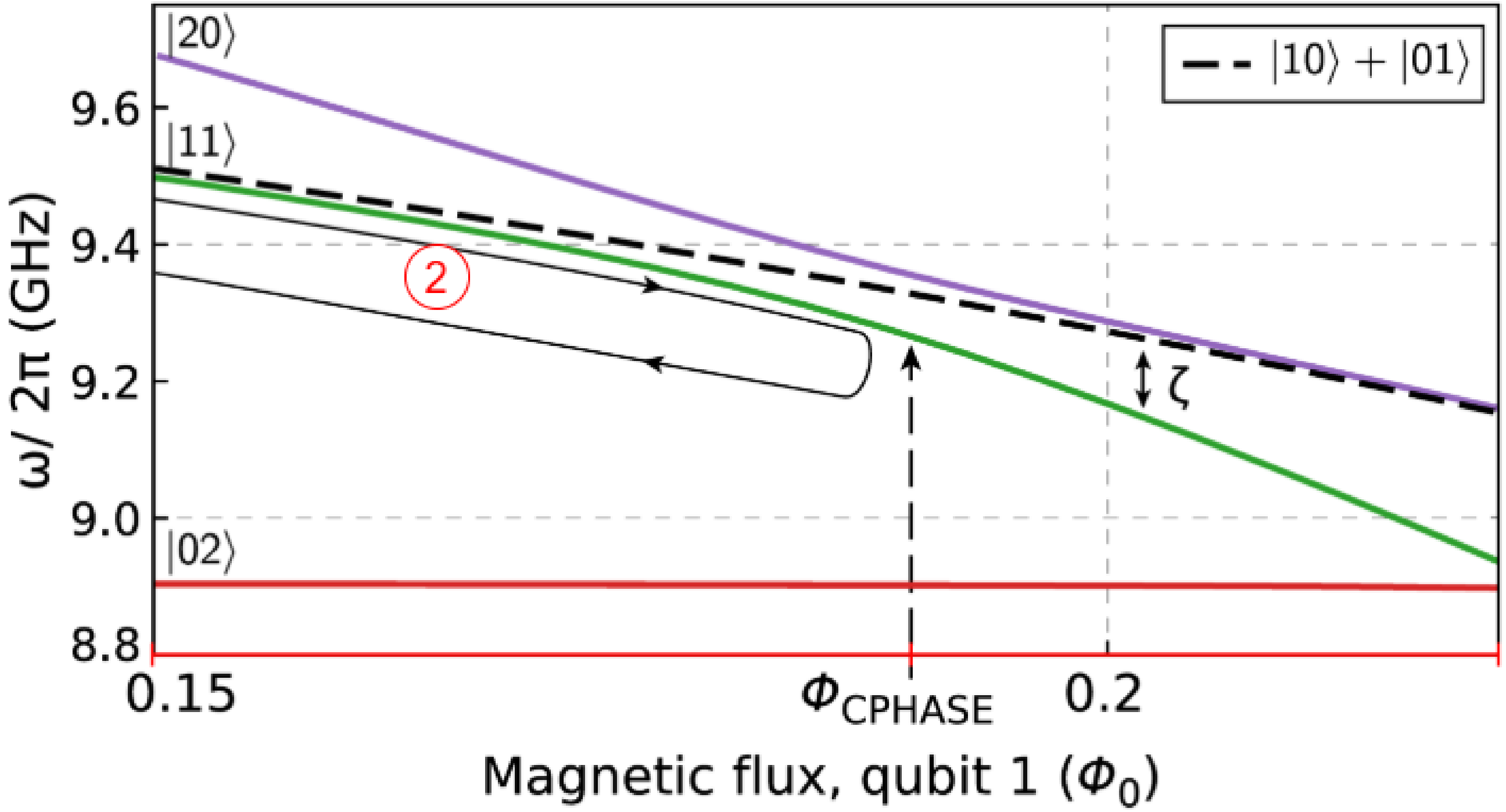

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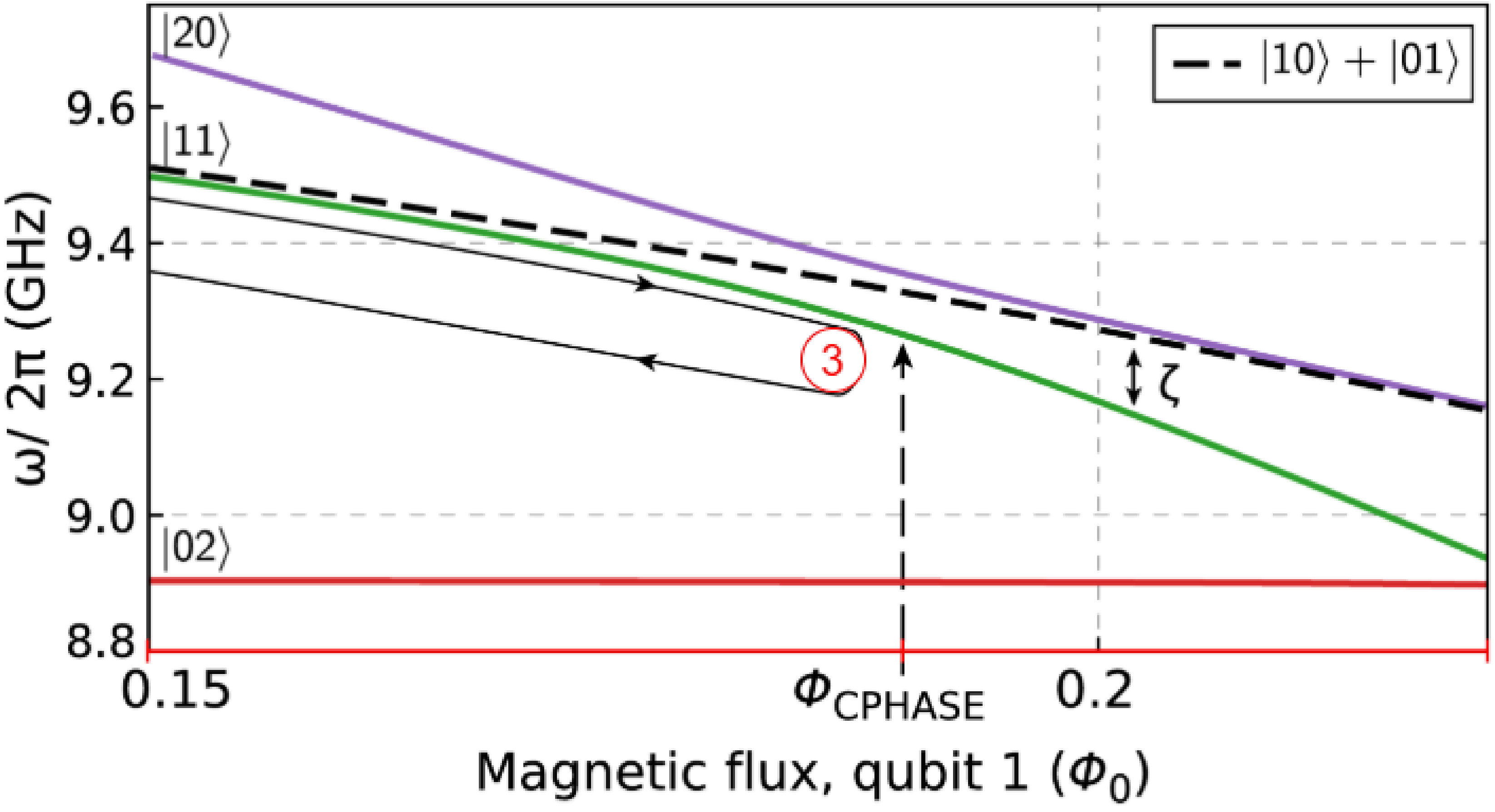

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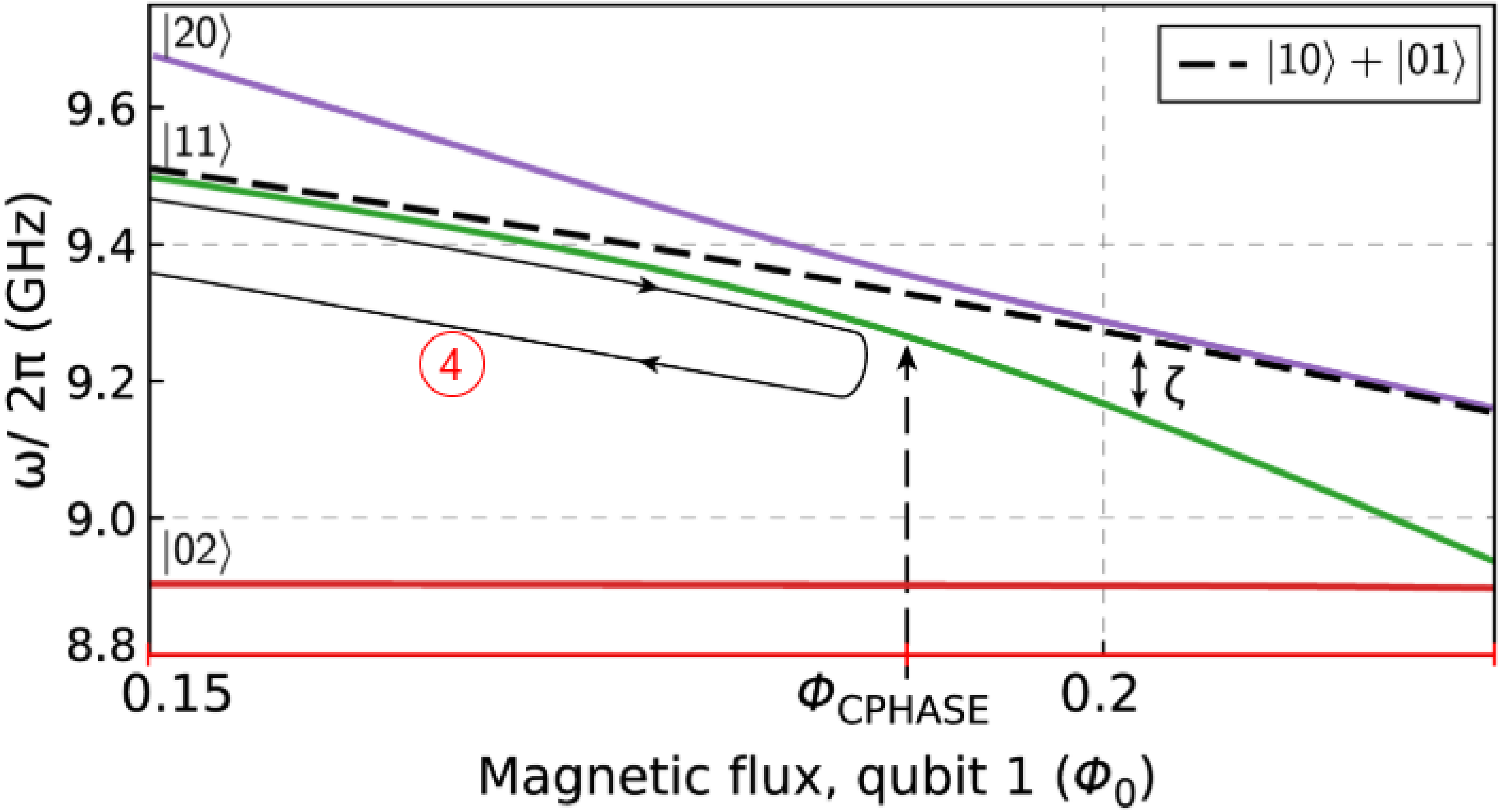

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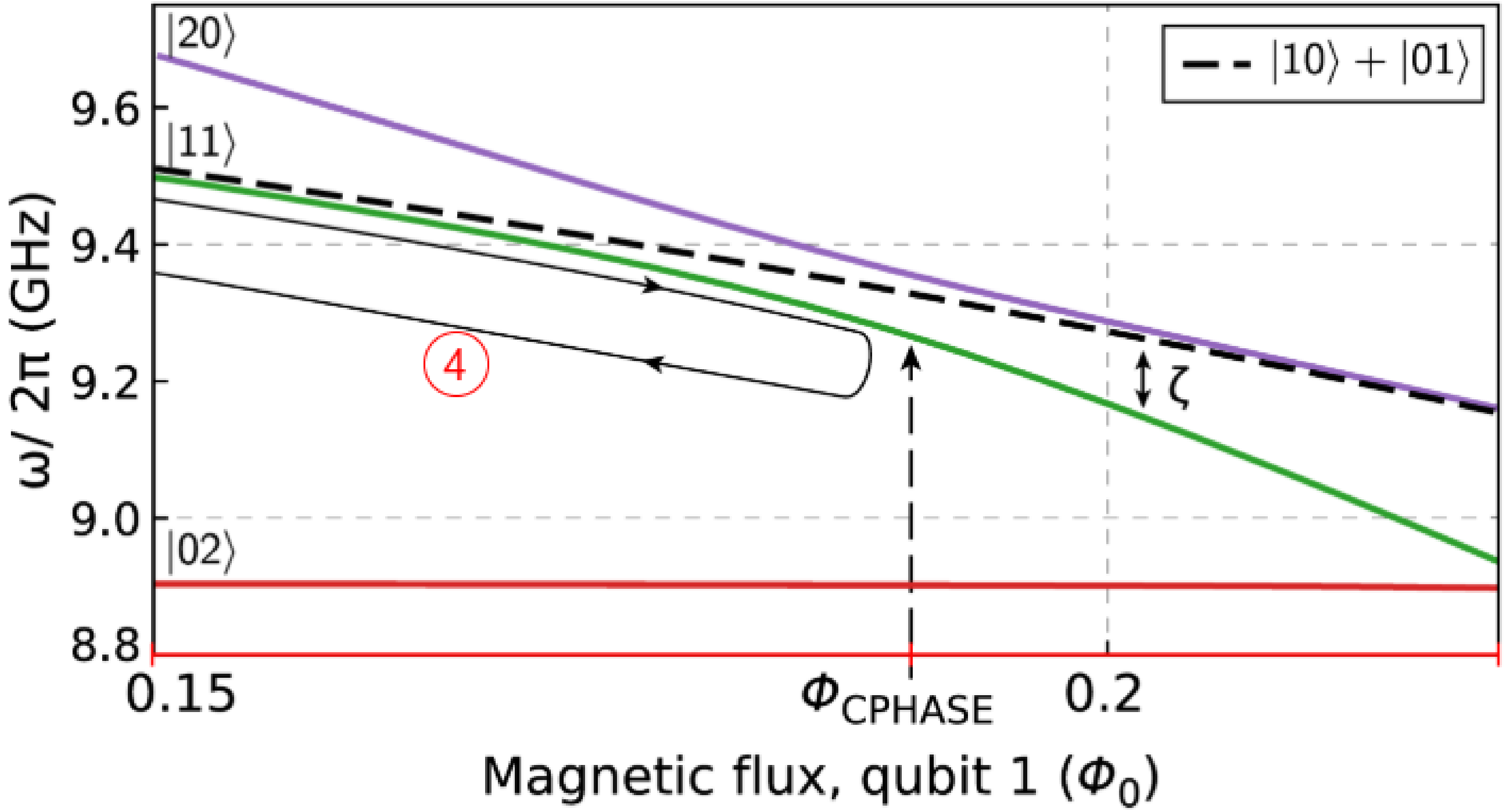


Adiabatic motion is fundamental to **always stay in one dressed state**.

↓

Prevent the system from evolving into $|20\rangle$.

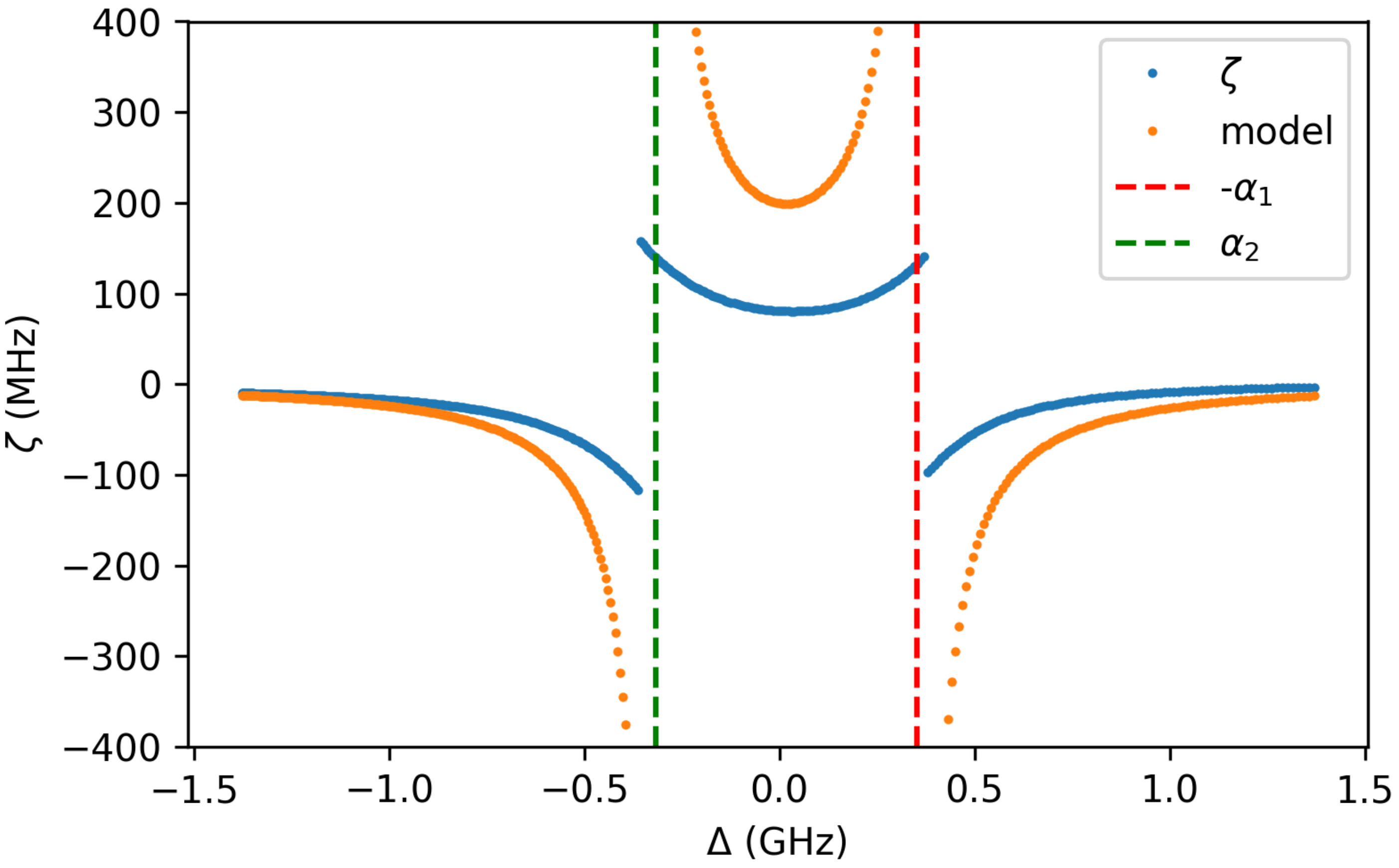
Adiabatic condition: $\left|\frac{d\omega}{dt}\right| \ll 8g^2$

Two qubit interaction – ZZ coupling strength

- There exist an analytical model valid in the dispersive regime ($|\omega_1 - \omega_2| \gg g$):

$$\zeta \approx 2g^2\left(\frac{1}{\Delta - \alpha_2} - \frac{1}{\Delta + \alpha_1}\right)$$

- **Numerical approximations** are needed for resonance regimes.

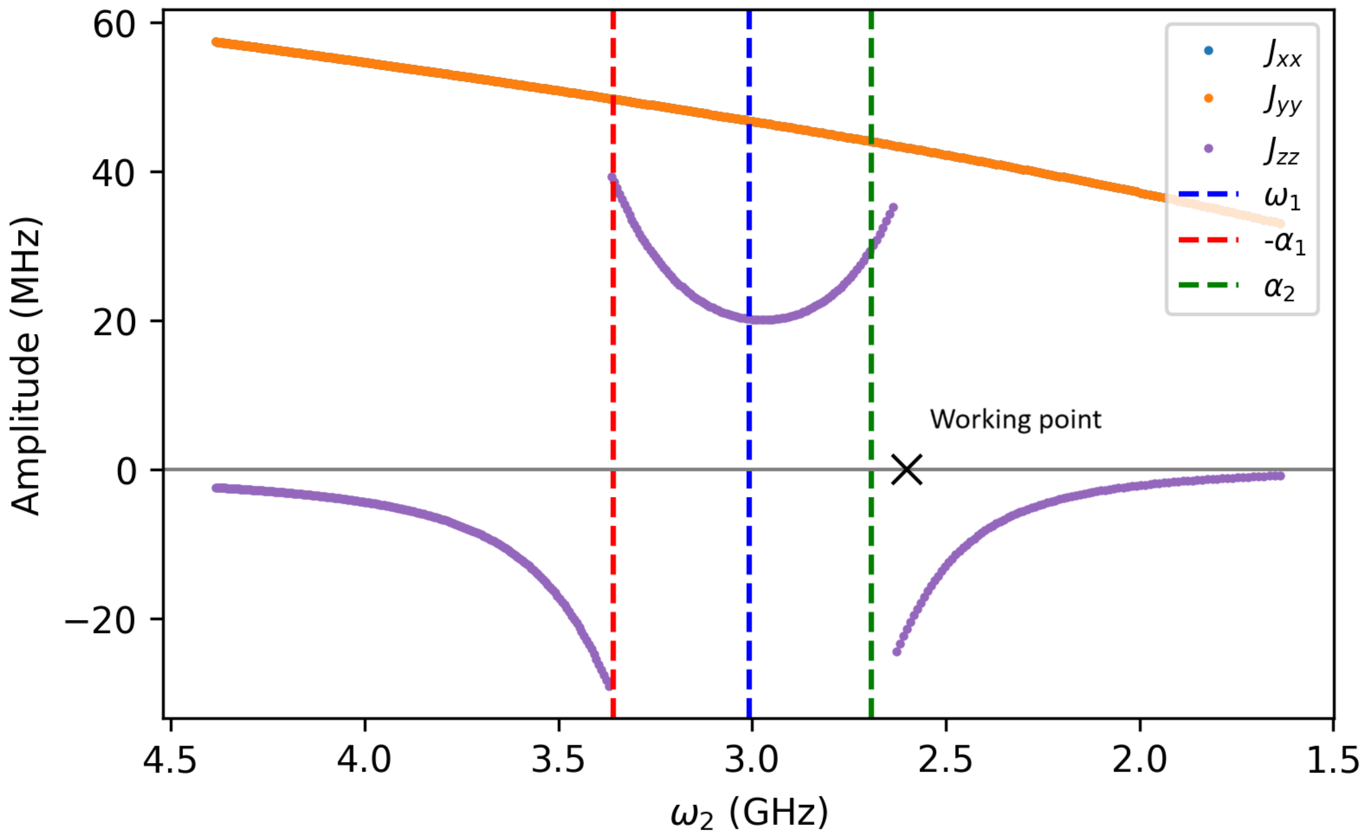


Two qubit interaction – Strength comparison

- At the working point - coupling strength parameters:

$$J_{xx} = 43\ MHz \quad J_{yy} = 43\ MHz \quad J_{zz} = -21\ MHz$$

$$\Delta_{J_{xx}} \sim 20\ MHz \qquad \Delta_{J_{zz}} \sim 65\ MHz$$



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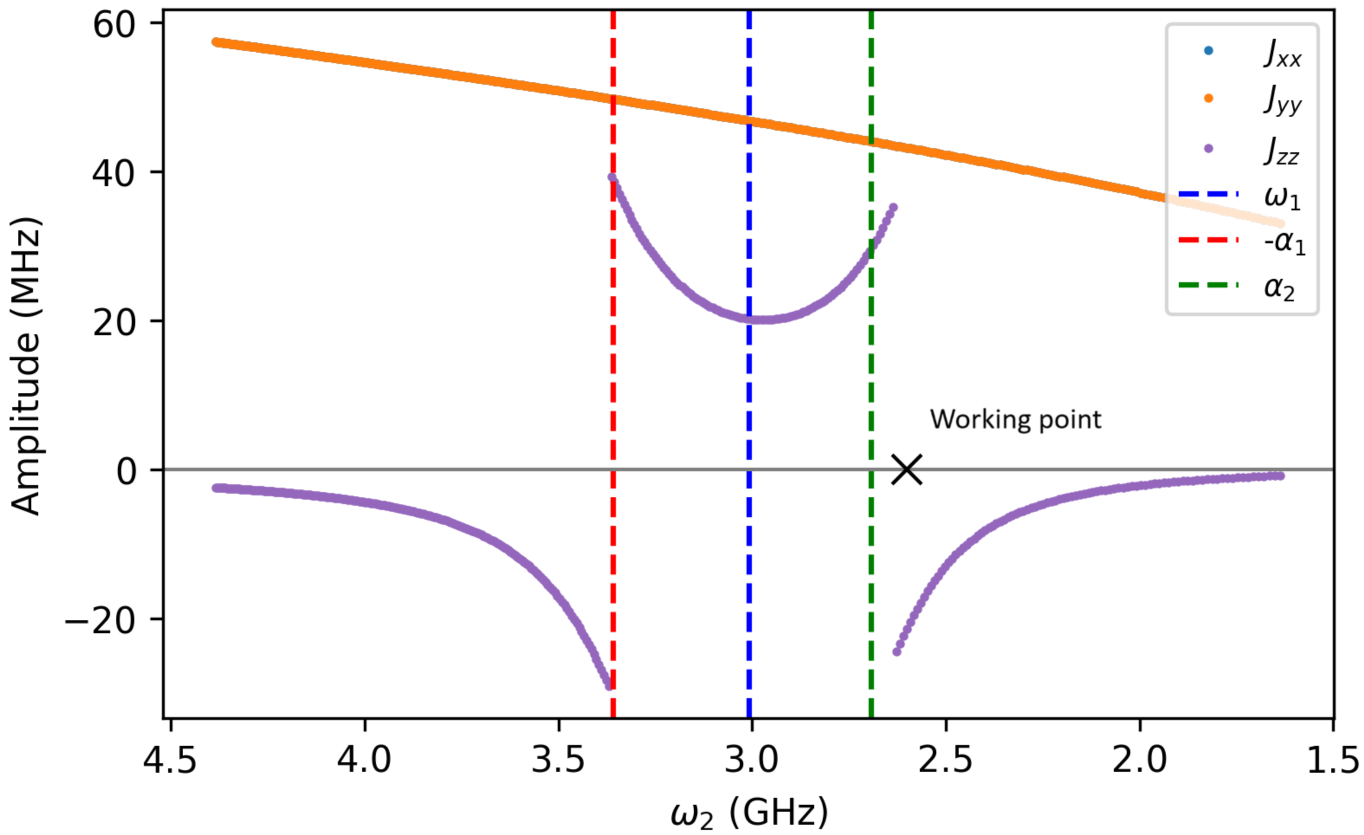
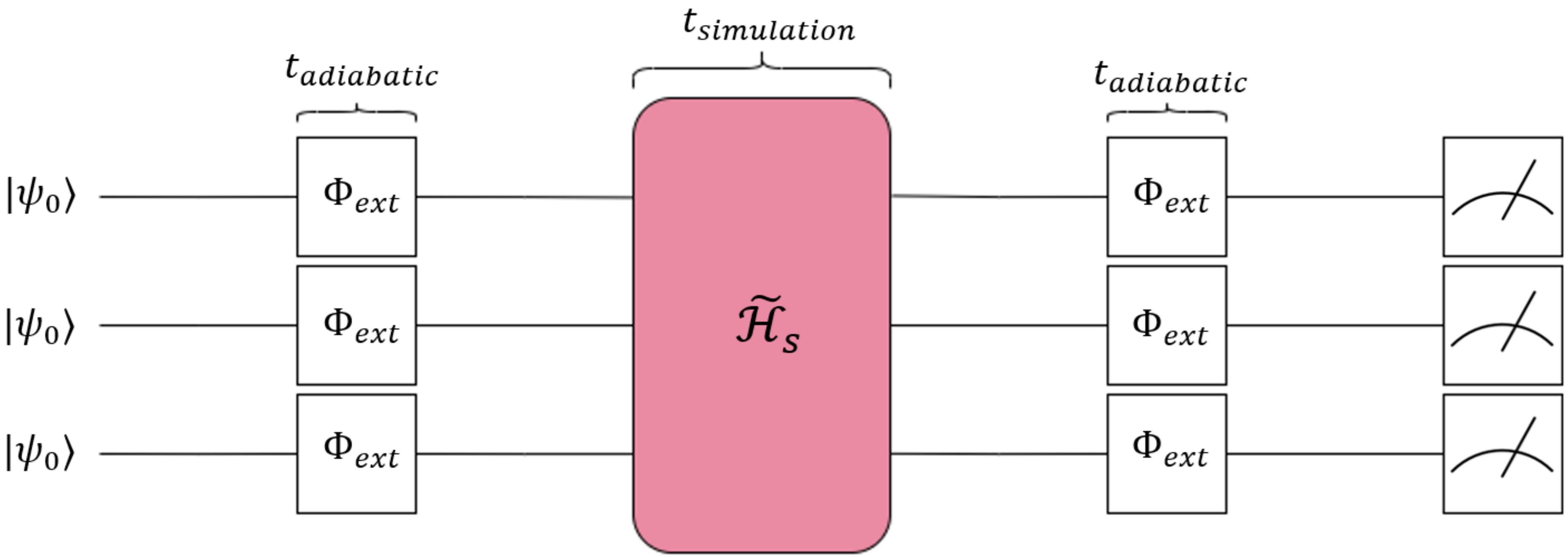


Future developments

- Engineering design parameters to obtain **coupling strength with the same order of magnitude.**
- Interaction between more qubits.

How to use – Simulation setup



1) *Prepare the initial state in a idling point*

2) *Change qubit frequencies to reach the working point*

3) *Let the system evolve*

4) *Goes back to the idling point*

5) *Measure*

- Process the data considering the **unwanted time evolution** caused by moving between idling and working points.
- Single qubit rotations can be turned on sending electromagnetic pulses during H_s time evolution.

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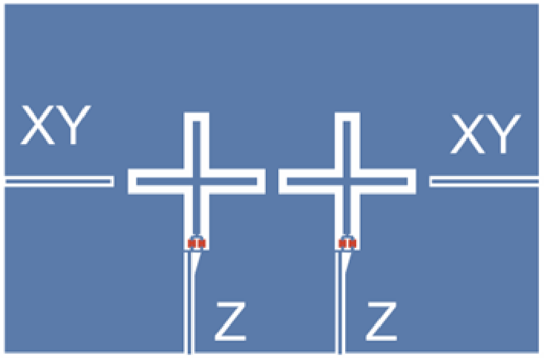
Scaling to more qubits

- **Directly capacitive coupling**:
  - Limited to a low number.
  - Difficult to design precise coupling capacitances C_{ij}.



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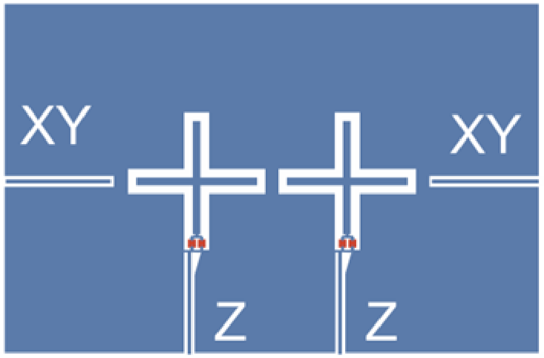
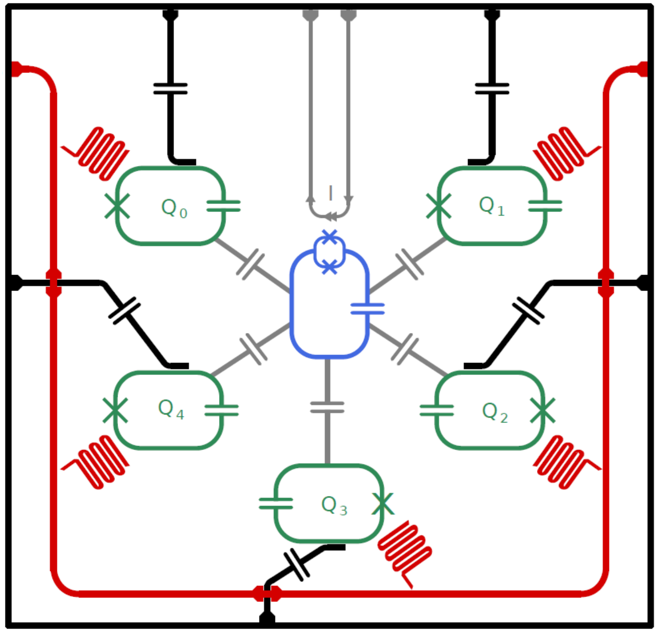


- Use **one «central» resonator** to connect many qubits:
  - Hamiltonian interaction terms do not connect directly data qubits.
  - Frequency overcrowding in the resonator.



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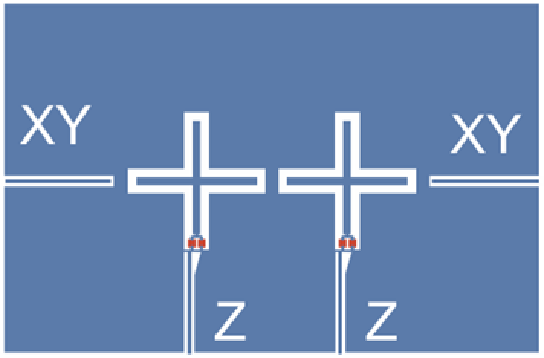

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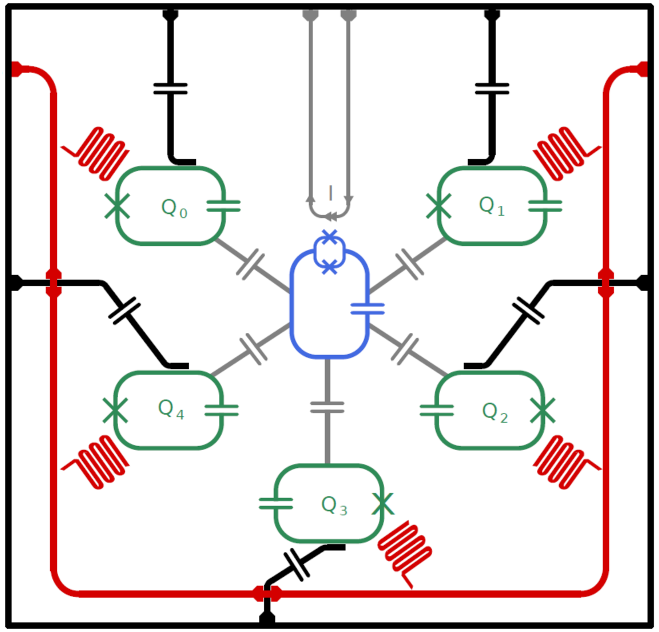


- The presence of many qubits can alter the energy spectrum → **change the influence from higher states** → harder to obtain ZZ interaction.

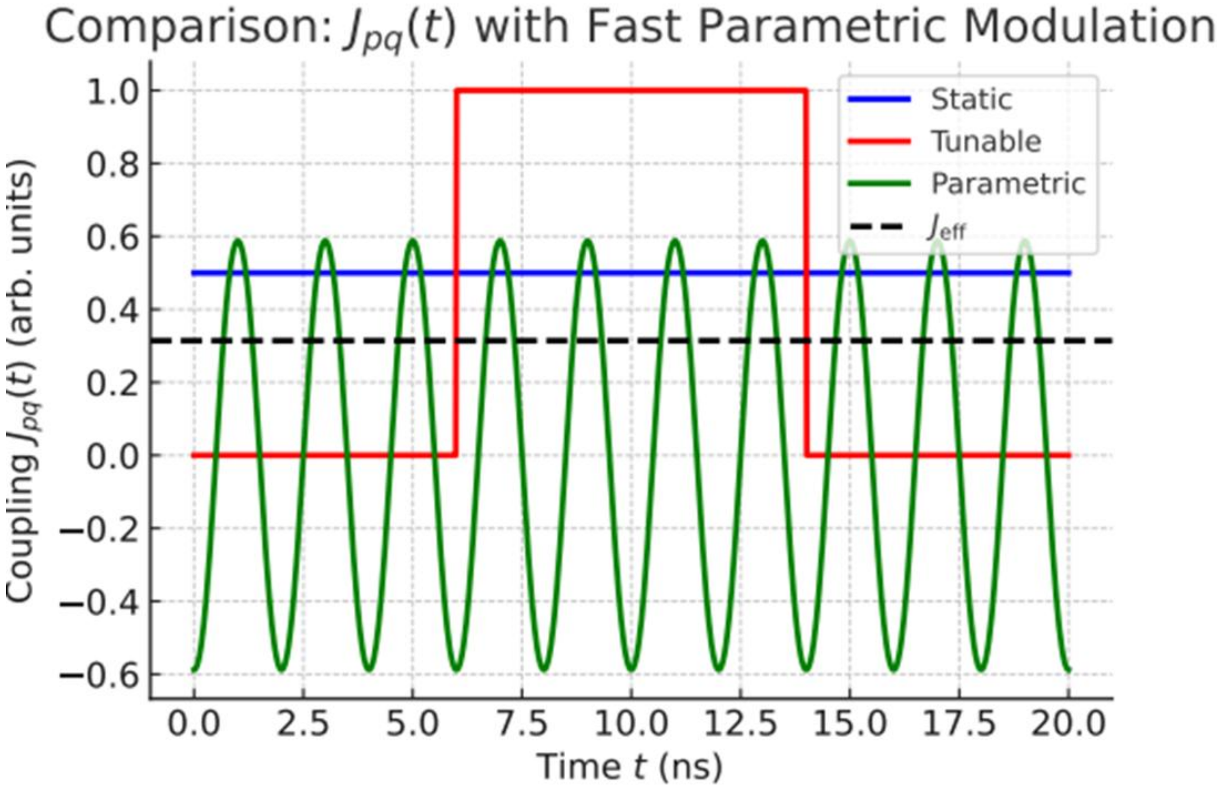
Different approaches – Parametric coupling

- **Key idea**: using an **AC pulse to modulate the qubit frequency**. By rapidly varying the frequency qubit 1 at the detuning $\Delta = \omega_1 - \omega_2$ we can activate a SWAP interaction with qubit 2.

- Time-dependent coupling:

$$H_{int} \sim J(t)\,(\sigma^-\sigma^+ + \sigma^+\sigma^-)$$

- ZZ coupling can be activated similarly. Exploiting higher energy levels.



[10.48550/arXiv.2305.02907]

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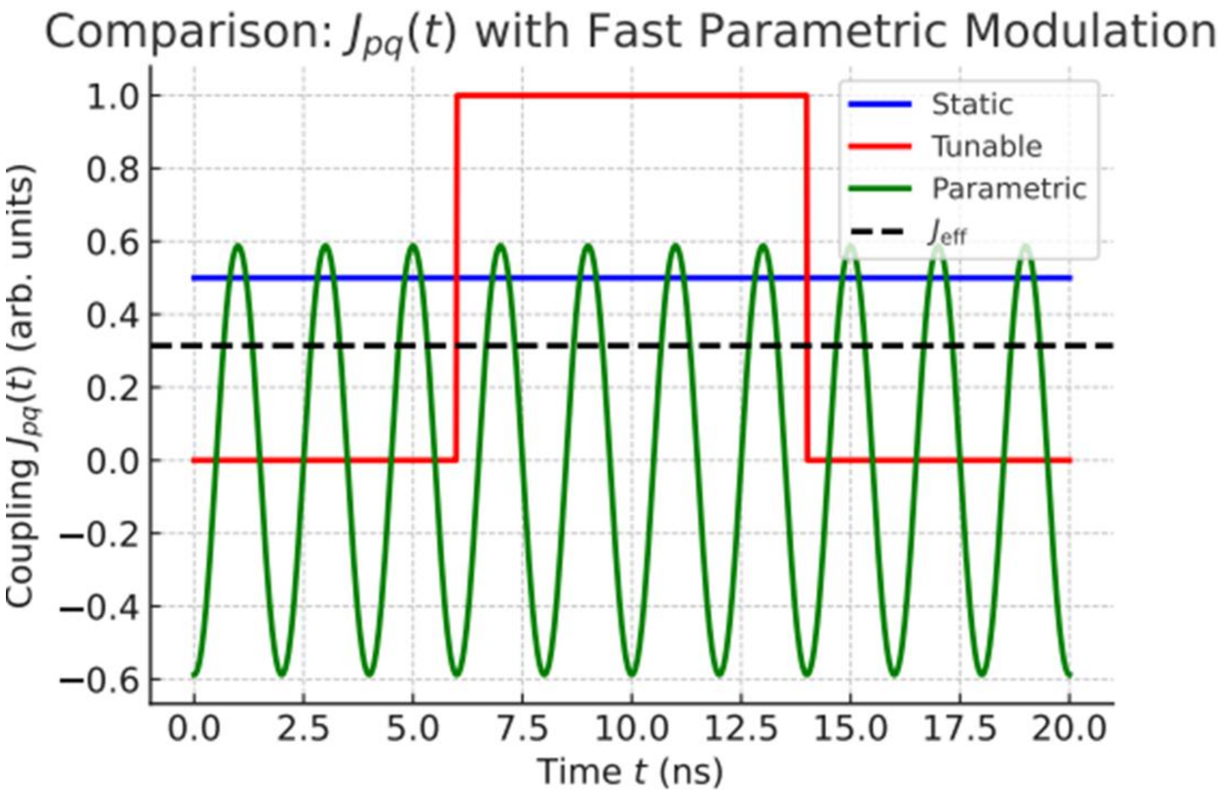
Advantages:

- High control over coupling strength amplitude.
- Qubit frequency independent strength → no frequency overcrowding.



[10.48550/arXiv.2305.02907]

Thank you for your attention!

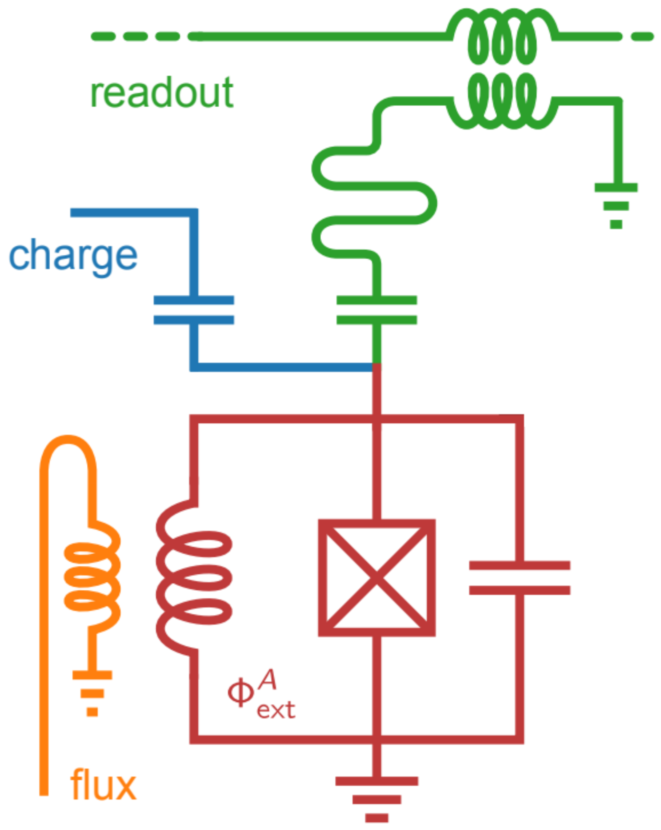
UNIMIB/INFN-MIB group: Matteo Borghesi, Pietro Campana, Lucia Canonica, Rodolfo Carobene, Alessandro Cattaneo, Hervè Corti, Marco Faverzani, Elena Ferri, Sara Gamba, Andrea Giachero, Marco Gobbo, Danilo Labranca, Roberto Moretti, Angelo Nucciotti, Luca Origo

Different approaches – Fluxonium

- The **fluxonium** is superconducting qubit which offers an **high control on anharmonicity**. Typical values are between $0.5 - 1.5\ GHz$ and can be both positive and negative.

- High anharmonicity → High ZZ interaction.

- It is composed of a Josephson Junction, a large inductance (array of JJ), and a capacitor.



[PhysRevLett.129.010502]

Complications:

- Fabrication becomes more complicated;
- Higher flux sensitivity;
- Require more space → harder to scale;
- Read out is more complicated.