

Muon Capture Helping New Physics Studies



TECHNISCHE
UNIVERSITÄT
DARMSTADT

Lotta Jokiniemi

Herzberg Fellow

Institut für Kernphysik – Theoriezentrum
TU Darmstadt



Arthur B. McDonald
Canadian Astroparticle Physics Research Institute



LOEWE

Exzellente Forschung für
Hessens Zukunft

Collaborators



TECHNISCHE
UNIVERSITÄT
DARMSTADT



D. Araujo Najera, M. Gennari, P. Navrátil

K. Kravvaris



J. Kotila



Caltech

R. Plestid



TECHNISCHE
UNIVERSITÄT
DARMSTADT

M. Drissi

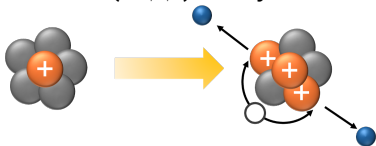
Introduction

Ab initio Studies on Ordinary Muon Capture

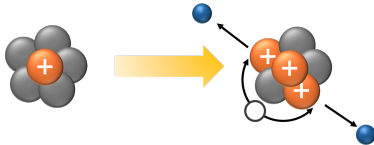
Radiative Muon Capture as Background of $\mu \rightarrow e$ Conversion

Summary

Neutrinoless double-beta ($0\nu\beta\beta$) decay



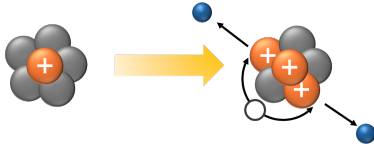
Neutrinoless double-beta ($0\nu\beta\beta$) decay



- **Lepton-number violation**

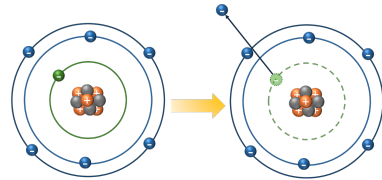
Muon Capture and Beyond Standard Model (BSM) Physics

Neutrinoless double-beta ($0\nu\beta\beta$) decay

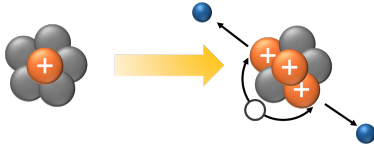


■ **Lepton-number violation**

Muon-to-Electron Conversion in Nuclei

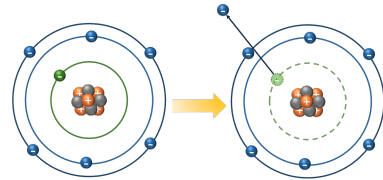


Neutrinoless double-beta ($0\nu\beta\beta$) decay



■ **Lepton-number violation**

Muon-to-Electron Conversion in Nuclei



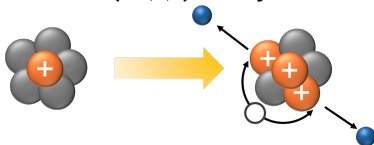
■ **Lepton-flavor violation**

Muon Capture and Beyond Standard Model (BSM) Physics



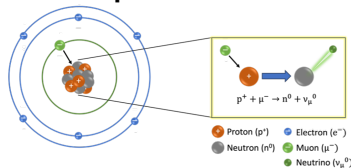
TECHNISCHE
UNIVERSITÄT
DARMSTADT

Neutrinoless double-beta decay ($0\nu\beta\beta$) decay

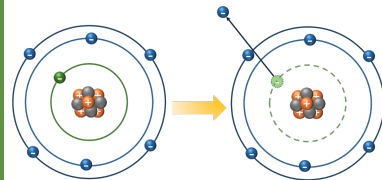


■ **Lepton-number violation**

Muon Capture on Nuclei

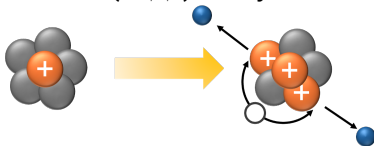


Muon-to-Electron Conversion in Nuclei



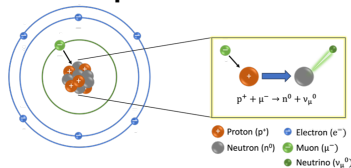
■ **Lepton-flavor violation**

Neutrinoless double-beta decay ($0\nu\beta\beta$) decay



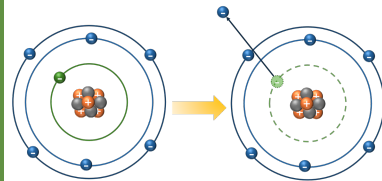
■ **Lepton-number violation**

Muon Capture on Nuclei



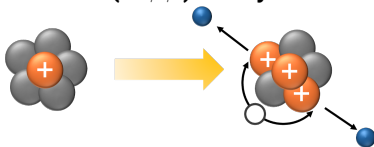
■ **Probe for $0\nu\beta\beta$ decay**

Muon-to-Electron Conversion in Nuclei



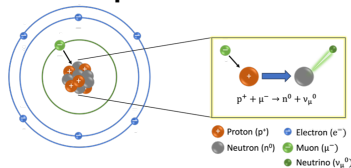
■ **Lepton-flavor violation**

Neutrinoless double-beta decay ($0\nu\beta\beta$) decay



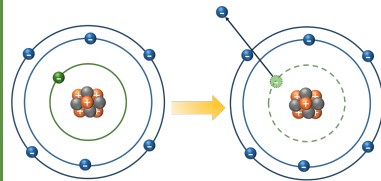
- **Lepton-number violation**

Muon Capture on Nuclei



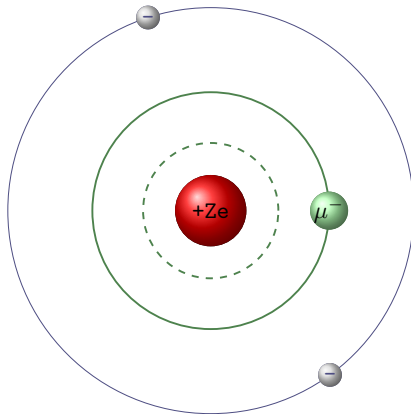
- **Probe for $0\nu\beta\beta$ decay**
- **Background for muon-to-electron conversion**

Muon-to-Electron Conversion in Nuclei



- **Lepton-flavor violation**

- Starting point: a **muonic atom**

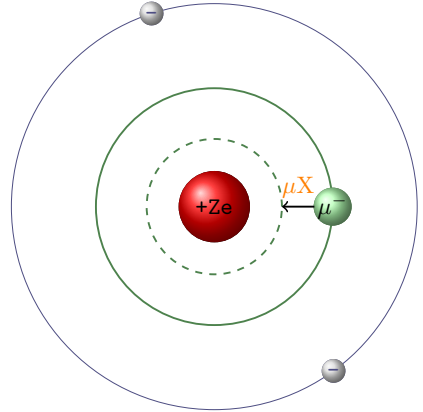


Muon Capture on Nuclei



TECHNISCHE
UNIVERSITÄT
DARMSTADT

- Starting point: a **muonic atom**
- The muon quickly transitions to the lowest, **$1s_{1/2}$ orbit**

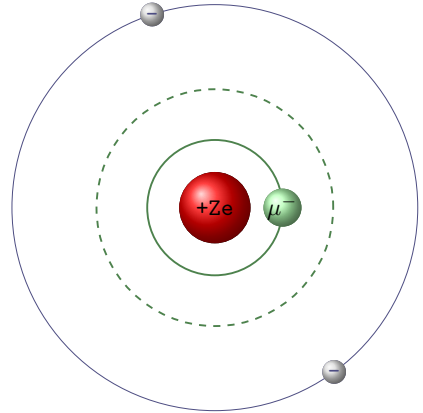


Muon Capture on Nuclei

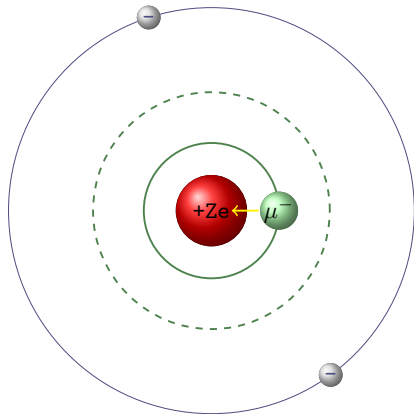
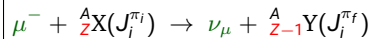


TECHNISCHE
UNIVERSITÄT
DARMSTADT

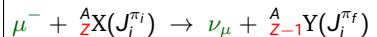
- Starting point: a **muonic atom**
- The muon quickly transitions to the lowest, **$1s_{1/2}$ orbit**
- From there, the muon can be **captured by the nucleus**



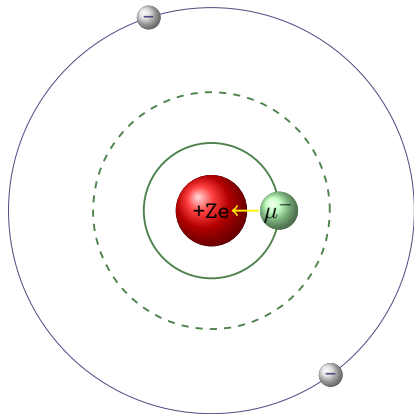
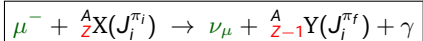
- Starting point: a **muonic atom**
- The muon quickly transitions to the lowest, **$1s_{1/2}$ orbit**
- From there, the muon can be **captured by the nucleus**
 - ▣ **Ordinary muon capture (OMC):**



- Starting point: a **muonic atom**
- The muon quickly transitions to the lowest, **$1s_{1/2}$ orbit**
- From there, the muon can be **captured by the nucleus**
 - ▣ **Ordinary muon capture (OMC):**



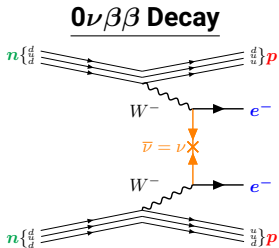
- ▣ **Radiative muon capture (RMC):**



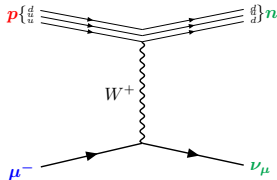
$0\nu\beta\beta$ Decay vs Ordinary Muon Capture



TECHNISCHE
UNIVERSITÄT
DARMSTADT



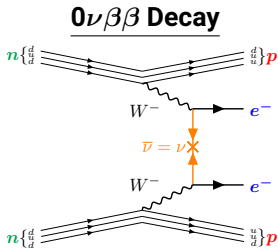
Ordinary Muon Capture



$0\nu\beta\beta$ Decay vs Ordinary Muon Capture

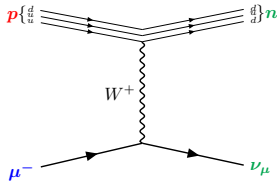


TECHNISCHE
UNIVERSITÄT
DARMSTADT



■ $q \approx 1/|r_1 - r_2| \approx 100 - 200 \text{ MeV}$

Ordinary Muon Capture

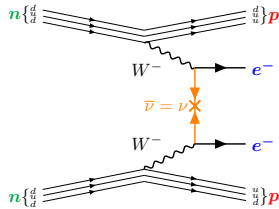


$0\nu\beta\beta$ Decay vs Ordinary Muon Capture



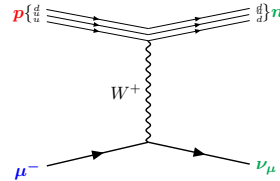
TECHNISCHE
UNIVERSITÄT
DARMSTADT

$0\nu\beta\beta$ Decay



■ $q \approx 1/|r_1 - r_2| \approx 100 - 200 \text{ MeV}$

Ordinary Muon Capture



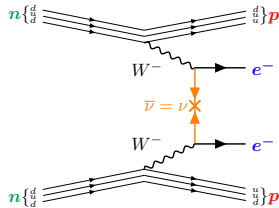
■ $q \approx m_\mu + E_i - E_f \approx 100 \text{ MeV}$

$0\nu\beta\beta$ Decay vs Ordinary Muon Capture



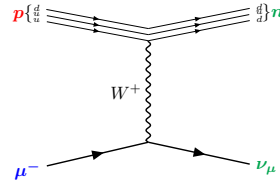
TECHNISCHE
UNIVERSITÄT
DARMSTADT

$0\nu\beta\beta$ Decay



■ $q \approx 1/|r_1 - r_2| \approx 100 - 200 \text{ MeV}$

Ordinary Muon Capture



■ $q \approx m_\mu + E_i - E_f \approx 100 \text{ MeV}$

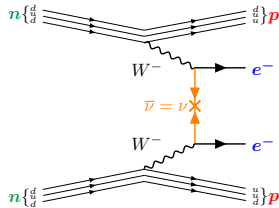
$$j^{\alpha\dagger} = \bar{\Psi} [g_V(q^2)\gamma^\alpha + ig_M(q^2)\frac{\sigma^{\alpha\beta}}{2M}q_\beta - g_A(q^2)\gamma^\alpha\gamma_5 - g_P(q^2)q^\alpha\gamma_5] \Psi$$

$0\nu\beta\beta$ Decay vs Ordinary Muon Capture



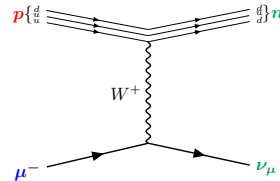
TECHNISCHE
UNIVERSITÄT
DARMSTADT

$0\nu\beta\beta$ Decay



- $q \approx 1/|r_1 - r_2| \approx 100 - 200 \text{ MeV}$
- **No experimental signal so far**

Ordinary Muon Capture



- $q \approx m_\mu + E_i - E_f \approx 100 \text{ MeV}$
- **Has been measured**

$$j^{\alpha\dagger} = \bar{\Psi} [g_V(q^2)\gamma^\alpha + ig_M(q^2)\frac{\sigma^{\alpha\beta}}{2M}q_\beta - g_A(q^2)\gamma^\alpha\gamma_5 - g_P(q^2)q^\alpha\gamma_5] \Psi$$

MONUMENT (a.k.a OMC4DBD) Experiment at PSI



TECHNISCHE
UNIVERSITÄT
DARMSTADT

Eur. Phys. J. C (2024) 84:1188
<https://doi.org/10.1140/epjc/s10052-024-13470-6>

THE EUROPEAN
PHYSICAL JOURNAL C



Regular Article

Experimental Physics

The MONUMENT experiment: ordinary muon capture studies for $0\nu\beta\beta$ decay

G. R. Araujo¹, D. Bajpai², L. Baudis¹, V. Belov^{4,5}, E. Bossio^{5,6,a}, T. E. Cocolios⁷, H. Ejiri⁸, M. Fomina⁴, K. Gusev^{4,5}, I. H. Hashim⁹, M. Helnes⁷, S. Kazartsev⁴, A. Knecht¹⁰, E. Mondragón^{5,b}, Z. W. Ng⁹, I. Ostrovskiy^{2,3}, F. Othman⁹, N. Rumyantseva^{4,5}, S. Schönert⁵, M. Schwarz⁵, E. Shevchik⁴, M. Shirchenko⁴, Yu. Shitov¹¹, E. O. Sushenok⁴, J. Suhonen^{12,13}, S. M. Vogiatzi¹⁰, C. Wiesinger⁵, I. Zhitnikov⁴, D. Zinatulina⁴



MONUMENT (a.k.a OMC4DBD) Experiment at PSI



TECHNISCHE
UNIVERSITÄT
DARMSTADT

Eur. Phys. J. C (2024) 84:1188
<https://doi.org/10.1140/epjc/s10052-024-13470-6>

THE EUROPEAN
PHYSICAL JOURNAL C



Regular Article

Experimental Physics

The MONUMENT experiment: ordinary muon capture studies for $0\nu\beta\beta$ decay



G. R. Araujo¹, D. Bajpai², L. Baudis¹, V. Belov^{4,5}, E. Bossio^{5,6,a}, T. E. Cocolios⁷, H. Ejiri⁸, M. Fomina⁴, K. Gusev^{4,5}, I. H. Hashim⁹, M. Helnes⁷, S. Kazartsev⁴, A. Knecht¹⁰, E. Mondragón^{5,b}, Z. W. Ng⁹, I. Ostrovskiy^{2,3}, F. Othman⁹, N. Rumyantseva^{4,5}, S. Schönert⁵, M. Schwarz⁵, E. Shevchik⁴, M. Shirchenko⁴, Yu. Shitov¹¹, E. O. Sushenok⁴, J. Suhonen^{12,13}, S. M. Vogiatzi¹⁰, C. Wiesinger⁵, I. Zhitnikov⁴, D. Zinatulina⁴



MONUMENT (a.k.a OMC4DBD) Experiment at PSI



TECHNISCHE
UNIVERSITÄT
DARMSTADT

Eur. Phys. J. C (2024) 84:1188
<https://doi.org/10.1140/epjc/s10052-024-13470-6>

THE EUROPEAN
PHYSICAL JOURNAL C



Regular Article

Experimental Physics

The MONUMENT experiment: ordinary muon capture studies for $0\nu\beta\beta$ decay



G. R. Araújo¹, D. Bajpai², L. Baudis¹, V. Belov^{4,5}, E. Bossio^{5,6,a}, T. E. Cocolios⁷, H. Ejiri⁸, M. Fomina⁴, K. Gusev^{4,5}, I. H. Hashim⁹, M. Helnes⁷, S. Kazartsev⁴, A. Knecht¹⁰, E. Mondragón^{5,b}, Z. W. Ng⁹, I. Ostrovskiy^{2,3}, F. Othman⁹, N. Rumyantseva^{4,5}, S. Schönert⁵, M. Schwarz⁵, E. Shevchik⁴, M. Shirchenko⁴, Yu. Shitov¹¹, E. O. Sushenok⁴, J. Suhonen^{12,13}, S. M. Vogiatzi¹⁰, C. Wiesinger⁵, I. Zhitnikov⁴, D. Zinatulina⁴

■ Aim 1: Measure OMC in $\beta\beta$ -decay nuclei



MONUMENT (a.k.a OMC4DBD) Experiment at PSI



TECHNISCHE
UNIVERSITÄT
DARMSTADT

Eur. Phys. J. C (2024) 84:1188
<https://doi.org/10.1140/epjc/s10052-024-13470-6>

THE EUROPEAN
PHYSICAL JOURNAL C



Regular Article

Experimental Physics

The MONUMENT experiment: ordinary muon capture studies for $0\nu\beta\beta$ decay



G. R. Araujo¹, D. Bajpai², L. Baudis¹, V. Belov^{4,5}, E. Bossio^{5,6,a}, T. E. Cocolios⁷, H. Ejiri⁸, M. Fomina⁴, K. Gusev^{4,5}, I. H. Hashim⁹, M. Helnes⁷, S. Kazartsev⁴, A. Knecht¹⁰, E. Mondragón^{5,b}, Z. W. Ng⁹, I. Ostrovskiy^{2,3}, F. Othman⁹, N. Rumyantseva^{4,5}, S. Schönert⁵, M. Schwarz⁵, E. Shevchik⁴, M. Shirchenko⁴, Yu. Shitov¹¹, E. O. Sushenok⁴, J. Suhonen^{12,13}, S. M. Vogiatzi¹⁰, C. Wiesinger⁵, I. Zhitnikov⁴, D. Zinatulina⁴

- **Aim 1:** Measure OMC in $\beta\beta$ -decay nuclei
- **Aim 2:** Measure OMC in light nuclei accessible by *ab initio* nuclear theory



Introduction

Ab initio Studies on Ordinary Muon Capture

Radiative Muon Capture as Background of $\mu \rightarrow e$ Conversion

Summary

Ordinary muon capture rate

$$W(J_i \rightarrow J_f) = \frac{1}{2J_i + 1} \left(1 - \frac{q}{m_\mu + AM} \right) q^2 \sum_{\kappa, u} |g_V M_V(\kappa, u) + g_A M_A(\kappa, u) + g_M M_M(\kappa, u) + g_P M_P(\kappa, u)|^2$$

PHYSICAL REVIEW

VOLUME 118, NUMBER 2

APRIL 15, 1960

Theory of Allowed and Forbidden Transitions in Muon Capture Reactions*

MASATO MORITA

Columbia University, New York, New York

AND

AKIHIKO FUJII†

Brookhaven National Laboratory, Upton, Long Island, New York

(Received November 9, 1959)

- Rates written in terms of reduced one-body matrix elements:

$$\langle \Psi_f | \sum_{s=1}^A \hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s) | \Psi_i \rangle = \frac{1}{\sqrt{2u+1}} \sum_{pn} \langle n | \hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s) | p \rangle \langle \Psi_f | [a_n^\dagger \tilde{a}_p]_u | \Psi_i \rangle$$

$\mathcal{M}[kwux]$	$\hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s)$
$\mathcal{M}[0 w u]$	$j_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{0wu}^{M_f-M_i}(\hat{\mathbf{r}}_s) \delta_{wu}$
$\mathcal{M}[1 w u]$	$j_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{1wu}^{M_f-M_i}(\hat{\mathbf{r}}_s, \boldsymbol{\sigma}_s)$
$\mathcal{M}[0 w u \pm]$	$[j_w(qr_s) G_{-1}(r_s) \mp \frac{1}{q} j_{w\mp 1}(qr_s) \frac{d}{dr_s} G_{-1}(r_s)] \mathcal{Y}_{0wu}^{M_f-M_i}(\hat{\mathbf{r}}_s) \delta_{wu}$
$\mathcal{M}[1 w u \pm]$	$[j_w(qr_s) G_{-1}(r_s) \mp \frac{1}{q} j_{w\mp 1}(qr_s) \frac{d}{dr_s} G_{-1}(r_s)] \mathcal{Y}_{1wu}^{M_f-M_i}(\hat{\mathbf{r}}_s, \boldsymbol{\sigma}_s)$
$\mathcal{M}[0 w u p]$	$ij_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{0wu}^{M_f-M_i}(\hat{\mathbf{r}}_s) \boldsymbol{\sigma}_s \cdot \mathbf{p}_s \delta_{wu}$
$\mathcal{M}[1 w u p]$	$ij_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{1wu}^{M_f-M_i}(\hat{\mathbf{r}}_s, \mathbf{p}_s)$

Morita, Fujii, *Phys. Rev.* **118**, 606 (1960)

- Rates written in terms of reduced one-body matrix elements:

$$\langle \Psi_f | \sum_{s=1}^A \hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s) | \Psi_i \rangle = \frac{1}{\sqrt{2u+1}} \sum_{pn} \langle n | \hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s) | p \rangle \langle \Psi_f | [a_n^\dagger \tilde{a}_p]_u | \Psi_i \rangle$$

s.p. NME

$\mathcal{M}[kwux]$	$\hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s)$
$\mathcal{M}[0 w u]$	$j_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{0wu}^{M_f-M_i}(\hat{\mathbf{r}}_s) \delta_{wu}$
$\mathcal{M}[1 w u]$	$j_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{1wu}^{M_f-M_i}(\hat{\mathbf{r}}_s, \boldsymbol{\sigma}_s)$
$\mathcal{M}[0 w u \pm]$	$[j_w(qr_s) G_{-1}(r_s) \mp \frac{1}{q} j_{w\mp 1}(qr_s) \frac{d}{dr_s} G_{-1}(r_s)] \mathcal{Y}_{0wu}^{M_f-M_i}(\hat{\mathbf{r}}_s) \delta_{wu}$
$\mathcal{M}[1 w u \pm]$	$[j_w(qr_s) G_{-1}(r_s) \mp \frac{1}{q} j_{w\mp 1}(qr_s) \frac{d}{dr_s} G_{-1}(r_s)] \mathcal{Y}_{1wu}^{M_f-M_i}(\hat{\mathbf{r}}_s, \boldsymbol{\sigma}_s)$
$\mathcal{M}[0 w u p]$	$ij_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{0wu}^{M_f-M_i}(\hat{\mathbf{r}}_s) \boldsymbol{\sigma}_s \cdot \mathbf{p}_s \delta_{wu}$
$\mathcal{M}[1 w u p]$	$ij_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{1wu}^{M_f-M_i}(\hat{\mathbf{r}}_s, \mathbf{p}_s)$

Morita, Fujii, *Phys. Rev.* **118**, 606 (1960)

- Rates written in terms of reduced one-body matrix elements:

$$\langle \Psi_f | \sum_{s=1}^A \hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s) | \Psi_i \rangle = \frac{1}{\sqrt{2u+1}} \sum_{pn} \langle n | \hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s) | p \rangle \langle \Psi_f | [a_n^\dagger \tilde{a}_p]_u | \Psi_i \rangle$$

s.p. NME OBTD

$\mathcal{M}[kwux]$	$\hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s)$
$\mathcal{M}[0 w u]$	$j_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{0wu}^{M_f-M_i}(\hat{\mathbf{r}}_s) \delta_{wu}$
$\mathcal{M}[1 w u]$	$j_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{1wu}^{M_f-M_i}(\hat{\mathbf{r}}_s, \boldsymbol{\sigma}_s)$
$\mathcal{M}[0 w u \pm]$	$[j_w(qr_s) G_{-1}(r_s) \mp \frac{1}{q} j_{w\mp 1}(qr_s) \frac{d}{dr_s} G_{-1}(r_s)] \mathcal{Y}_{0wu}^{M_f-M_i}(\hat{\mathbf{r}}_s) \delta_{wu}$
$\mathcal{M}[1 w u \pm]$	$[j_w(qr_s) G_{-1}(r_s) \mp \frac{1}{q} j_{w\mp 1}(qr_s) \frac{d}{dr_s} G_{-1}(r_s)] \mathcal{Y}_{1wu}^{M_f-M_i}(\hat{\mathbf{r}}_s, \boldsymbol{\sigma}_s)$
$\mathcal{M}[0 w u p]$	$ij_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{0wu}^{M_f-M_i}(\hat{\mathbf{r}}_s) \boldsymbol{\sigma}_s \cdot \mathbf{p}_s \delta_{wu}$
$\mathcal{M}[1 w u p]$	$ij_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{1wu}^{M_f-M_i}(\hat{\mathbf{r}}_s, \mathbf{p}_s)$

Morita, Fujii, *Phys. Rev.* **118**, 606 (1960)

- Rates written in terms of reduced one-body matrix elements:

$k = 0, 1$:
spin coupling
of μ and ν_μ

$$\langle \Psi_f | \sum_{s=1}^A \hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s) | \Psi_i \rangle = \frac{1}{\sqrt{2u+1}} \sum_{pn} \langle n | \hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s) | p \rangle \langle \Psi_f | [a_n^\dagger \tilde{a}_p]_u | \Psi_i \rangle$$

s.p. NME OBTD

$\mathcal{M}[kwux]$	$\hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s)$
$\mathcal{M}[0 w u]$	$j_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{0wu}^{M_f-M_i}(\hat{\mathbf{r}}_s) \delta_{wu}$
$\mathcal{M}[1 w u]$	$j_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{1wu}^{M_f-M_i}(\hat{\mathbf{r}}_s, \boldsymbol{\sigma}_s)$
$\mathcal{M}[0 w u \pm]$	$[j_w(qr_s) G_{-1}(r_s) \mp \frac{1}{q} j_{w\mp 1}(qr_s) \frac{d}{dr_s} G_{-1}(r_s)] \mathcal{Y}_{0wu}^{M_f-M_i}(\hat{\mathbf{r}}_s) \delta_{wu}$
$\mathcal{M}[1 w u \pm]$	$[j_w(qr_s) G_{-1}(r_s) \mp \frac{1}{q} j_{w\mp 1}(qr_s) \frac{d}{dr_s} G_{-1}(r_s)] \mathcal{Y}_{1wu}^{M_f-M_i}(\hat{\mathbf{r}}_s, \boldsymbol{\sigma}_s)$
$\mathcal{M}[0 w u p]$	$ij_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{0wu}^{M_f-M_i}(\hat{\mathbf{r}}_s) \boldsymbol{\sigma}_s \cdot \mathbf{p}_s \delta_{wu}$
$\mathcal{M}[1 w u p]$	$ij_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{1wu}^{M_f-M_i}(\hat{\mathbf{r}}_s, \mathbf{p}_s)$

Morita, Fujii, *Phys. Rev.* **118**, 606 (1960)

- Rates written in terms of reduced one-body matrix elements:

$k = 0, 1$:
spin coupling
of μ and ν_μ

w : parity change

$$\langle \Psi_f | \sum_{s=1}^A \hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s) | \Psi_i \rangle = \frac{1}{\sqrt{2u+1}} \sum_{pn} \langle n | \hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s) | p \rangle \langle \Psi_f | [a_n^\dagger \tilde{a}_p]_u | \Psi_i \rangle$$

s.p. NME OBTD

$\mathcal{M}[kwux]$	$\hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s)$
$\mathcal{M}[0 w u]$	$j_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{0wu}^{M_f-M_i}(\hat{\mathbf{r}}_s) \delta_{wu}$
$\mathcal{M}[1 w u]$	$j_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{1wu}^{M_f-M_i}(\hat{\mathbf{r}}_s, \boldsymbol{\sigma}_s)$
$\mathcal{M}[0 w u \pm]$	$[j_w(qr_s) G_{-1}(r_s) \mp \frac{1}{q} j_{w\mp 1}(qr_s) \frac{d}{dr_s} G_{-1}(r_s)] \mathcal{Y}_{0wu}^{M_f-M_i}(\hat{\mathbf{r}}_s) \delta_{wu}$
$\mathcal{M}[1 w u \pm]$	$[j_w(qr_s) G_{-1}(r_s) \mp \frac{1}{q} j_{w\mp 1}(qr_s) \frac{d}{dr_s} G_{-1}(r_s)] \mathcal{Y}_{1wu}^{M_f-M_i}(\hat{\mathbf{r}}_s, \boldsymbol{\sigma}_s)$
$\mathcal{M}[0 w u p]$	$ij_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{0wu}^{M_f-M_i}(\hat{\mathbf{r}}_s) \boldsymbol{\sigma}_s \cdot \mathbf{p}_s \delta_{wu}$
$\mathcal{M}[1 w u p]$	$ij_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{1wu}^{M_f-M_i}(\hat{\mathbf{r}}_s, \mathbf{p}_s)$

Morita, Fujii, Phys. Rev. **118**, 606 (1960)

- Rates written in terms of reduced one-body matrix elements:

$\langle \Psi_f | | \sum_{s=1}^A \hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s) | | \Psi_i \rangle = \frac{1}{\sqrt{2u+1}} \sum_{pn} \langle n | | \hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s) | | p \rangle \langle \Psi_f | | [a_n^\dagger \tilde{a}_p]_u | | \Psi_i \rangle$

$k = 0, 1$:
 spin coupling
 of μ and ν_μ
 w : parity change
 u : tensor rank:
 $|J_i - J_f| \leq u \leq J_i + J_f$

$\mathcal{M}[kwux]$	$\hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s)$
$\mathcal{M}[0 w u]$	$j_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{0wu}^{M_f - M_i}(\hat{\mathbf{r}}_s) \delta_{wu}$
$\mathcal{M}[1 w u]$	$j_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{1wu}^{M_f - M_i}(\hat{\mathbf{r}}_s, \boldsymbol{\sigma}_s)$
$\mathcal{M}[0 w u \pm]$	$[j_w(qr_s) G_{-1}(r_s) \mp \frac{1}{q} j_{w \mp 1}(qr_s) \frac{d}{dr_s} G_{-1}(r_s)] \mathcal{Y}_{0wu}^{M_f - M_i}(\hat{\mathbf{r}}_s) \delta_{wu}$
$\mathcal{M}[1 w u \pm]$	$[j_w(qr_s) G_{-1}(r_s) \mp \frac{1}{q} j_{w \mp 1}(qr_s) \frac{d}{dr_s} G_{-1}(r_s)] \mathcal{Y}_{1wu}^{M_f - M_i}(\hat{\mathbf{r}}_s, \boldsymbol{\sigma}_s)$
$\mathcal{M}[0 w u p]$	$ij_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{0wu}^{M_f - M_i}(\hat{\mathbf{r}}_s) \boldsymbol{\sigma}_s \cdot \mathbf{p}_s \delta_{wu}$
$\mathcal{M}[1 w u p]$	$ij_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{1wu}^{M_f - M_i}(\hat{\mathbf{r}}_s, \mathbf{p}_s)$

Morita, Fujii, Phys. Rev. **118**, 606 (1960)

- Rates written in terms of reduced one-body matrix elements:

$$\langle \Psi_f | \sum_{s=1}^A \hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s) | \Psi_i \rangle = \frac{1}{\sqrt{2u+1}} \sum_{pn} \langle n | \hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s) | p \rangle \langle \Psi_f | [a_n^\dagger \tilde{a}_p]_u | \Psi_i \rangle$$

s.p. NME
OBTD

$k = 0, 1$:
spin coupling
of μ and ν_μ

w : parity change

u : tensor rank:

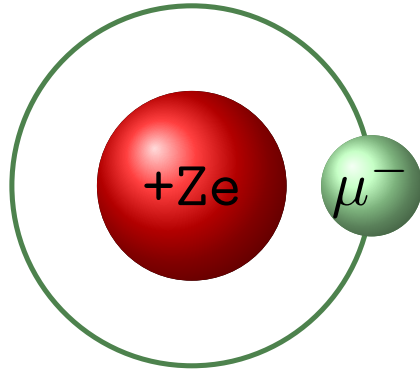
$|J_i - J_f| \leq u \leq J_i + J_f$

$\mathcal{M}[kwux]$	$\hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s)$
$\mathcal{M}[0 w u]$	$j_w(qr_s) \mathbf{G}_{-1}(r_s) \mathcal{Y}_{0wu}^{M_f-M_i}(\hat{\mathbf{r}}_s) \delta_{wu}$
$\mathcal{M}[1 w u]$	$j_w(qr_s) \mathbf{G}_{-1}(r_s) \mathcal{Y}_{1wu}^{M_f-M_i}(\hat{\mathbf{r}}_s, \boldsymbol{\sigma}_s)$
$\mathcal{M}[0 w u \pm]$	$[j_w(qr_s) \mathbf{G}_{-1}(r_s) \mp \frac{1}{q} j_{w\mp 1}(qr_s) \frac{d}{dr_s} \mathbf{G}_{-1}(r_s)] \mathcal{Y}_{0wu}^{M_f-M_i}(\hat{\mathbf{r}}_s) \delta_{wu}$
$\mathcal{M}[1 w u \pm]$	$[j_w(qr_s) \mathbf{G}_{-1}(r_s) \mp \frac{1}{q} j_{w\mp 1}(qr_s) \frac{d}{dr_s} \mathbf{G}_{-1}(r_s)] \mathcal{Y}_{1wu}^{M_f-M_i}(\hat{\mathbf{r}}_s, \boldsymbol{\sigma}_s)$
$\mathcal{M}[0 w u p]$	$ij_w(qr_s) \mathbf{G}_{-1}(r_s) \mathcal{Y}_{0wu}^{M_f-M_i}(\hat{\mathbf{r}}_s) \boldsymbol{\sigma}_s \cdot \mathbf{p}_s \delta_{wu}$
$\mathcal{M}[1 w u p]$	$ij_w(qr_s) \mathbf{G}_{-1}(r_s) \mathcal{Y}_{1wu}^{M_f-M_i}(\hat{\mathbf{r}}_s, \mathbf{p}_s)$

Morita, Fujii, Phys. Rev. **118**, 606 (1960)

■ Muon wave function

$$\psi_{\mu}(\kappa, \mu; \mathbf{r}) = \begin{bmatrix} -iF_{\kappa}(r)\chi_{-\kappa\mu} \\ G_{\kappa}(r)\chi_{\kappa\mu} \end{bmatrix}$$

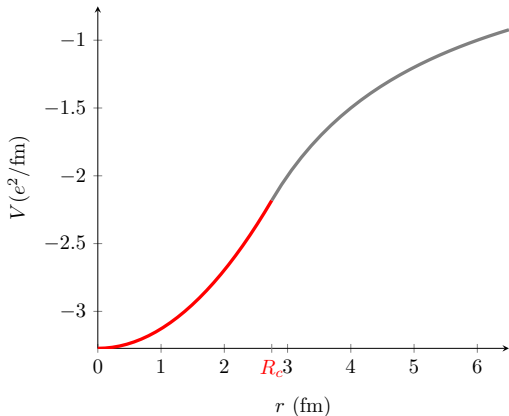


- Muon wave function

$$\psi_{\mu}(\kappa, \mu; \mathbf{r}) = \begin{bmatrix} -iF_{\kappa}(r)\chi_{-\kappa\mu} \\ G_{\kappa}(r)\chi_{\kappa\mu} \end{bmatrix}$$

- from Dirac equations with potential

$$V(r) = \begin{cases} -\frac{Ze^2}{2R_c} \left[3 - \left(\frac{r}{R_c} \right)^2 \right], & \text{if } r \leq R_c \\ -\frac{Ze^2}{r}, & \text{if } r > R_c \end{cases}$$



■ Muon wave function

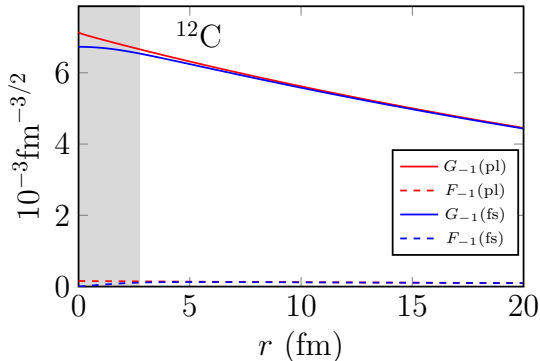
$$\psi_{\mu}(\kappa, \mu; \mathbf{r}) = \begin{bmatrix} -iF_{\kappa}(r)\chi_{-\kappa\mu} \\ G_{\kappa}(r)\chi_{\kappa\mu} \end{bmatrix}$$

■ from Dirac equations with potential

$$V(r) = \begin{cases} -\frac{Ze^2}{2R_c} \left[3 - \left(\frac{r}{R_c} \right)^2 \right], & \text{if } r \leq R_c \\ -\frac{Ze^2}{r}, & \text{if } r > R_c \end{cases}$$

■ Capture rates written in terms of reduced matrix elements

$$\langle \Psi_f || \sum_{s=1}^A \mathbf{G}_{-1}(\mathbf{r}_s) \hat{O}_s(\mathbf{r}_s, \mathbf{p}_s) \tau_-^s || \Psi_i \rangle$$



LJ, P. Navrátil, J. Kotila, K. Kravvaris, *Phys. Rev. C* 109, 065501 (2024)

Nuclear Forces from Chiral Effective Field Theory



TECHNISCHE
UNIVERSITÄT
DARMSTADT

- We use Nucleon-Nucleon (NN) and 3-Nucleon (3N) forces based on Chiral Effective Field Theory:

	NN	3N	4N
LO $(Q/\Lambda_\chi)^0$			
NLO $(Q/\Lambda_\chi)^2$			
NNLO $(Q/\Lambda_\chi)^3$			
N ³ LO $(Q/\Lambda_\chi)^4$			

H. Hergert, *Front. Phys.* 8 (2020)

Nuclear Forces from Chiral Effective Field Theory



TECHNISCHE
UNIVERSITÄT
DARMSTADT

- We use Nucleon-Nucleon (NN) and 3-Nucleon (3N) forces based on Chiral Effective Field Theory:

- ▣ $NN(N^3LO) + 3N(N^2LO, \text{Inl})$

Entem, Machleidt, Phys. Rev. C 68, 041001 (2003) (NN)

Somà et al., Phys. Rev. C 101, 014318 (2020) (3N)

	NN	3N	4N
LO $(Q/\Lambda_\chi)^0$			
NLO $(Q/\Lambda_\chi)^2$			
NNLO $(Q/\Lambda_\chi)^3$			
N³LO $(Q/\Lambda_\chi)^4$			

H. Hergert, Front. Phys. 8 (2020)

- We use Nucleon-Nucleon (NN) and 3-Nucleon (3N) forces based on Chiral Effective Field Theory:

- $NN(N^3LO)+3N(N^2LO, \text{Inl})$

Entem, Machleidt, Phys. Rev. C 68, 041001 (2003) (NN)

Somà et al., Phys. Rev. C 101, 014318 (2020) (3N)

- $NN(N^4LO)+3N(N^2LO, \text{Inl})$

Entem, Machleidt, Nosyk, Phys. Rev. C 96, 024004 (2017) (NN)

Gysbers et al., Nature Phys. 15, 428 (2019) (3N)

	NN	3N	4N
LO $(Q/\Lambda_\chi)^0$			
NLO $(Q/\Lambda_\chi)^2$			
NNLO $(Q/\Lambda_\chi)^3$			
N ³ LO $(Q/\Lambda_\chi)^4$			

H. Hergert, Front. Phys. 8 (2020)

Nuclear Forces from Chiral Effective Field Theory



TECHNISCHE
UNIVERSITÄT
DARMSTADT

■ We use Nucleon-Nucleon (NN) and 3-Nucleon (3N) forces based on Chiral Effective Field Theory:

▣ NN($N^3\text{LO}$)+3N($N^2\text{LO}$, Inl)

Entem, Machleidt, Phys. Rev. C 68, 041001 (2003) (NN)

Somà et al., Phys. Rev. C 101, 014318 (2020) (3N)

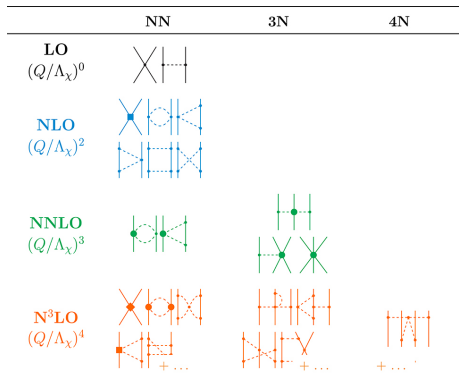
▣ NN($N^4\text{LO}$)+3N($N^2\text{LO}$, Inl)

Entem, Machleidt, Nosyk, Phys. Rev. C 96, 024004 (2017) (NN)

Gysbers et al., Nature Phys. 15, 428 (2019) (3N)

▣ NN($N^4\text{LO}$)+3N($N^2\text{LO}$, Inl , E_7)

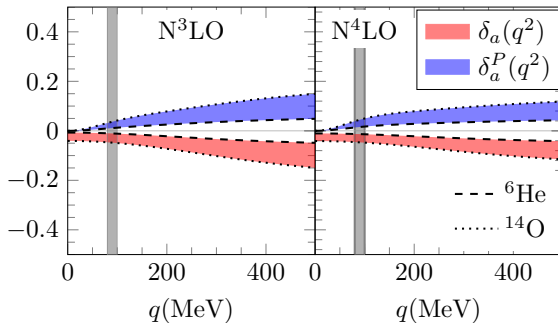
Girlanda, Kievsky, Viviani, Phys. Rev. C 84, 014001 (2011) (E_7)



H. Hergert, Front. Phys. 8 (2020)

- One-body (1b) axial-vector currents given by

$$\mathbf{J}_{i,1b}^3 = \frac{\tau_i^3}{2} \left(g_A \boldsymbol{\sigma}_i - \frac{g_P}{2m_N} \mathbf{q} \cdot \boldsymbol{\sigma}_i \right)$$



- One-body (1b) axial-vector currents given by

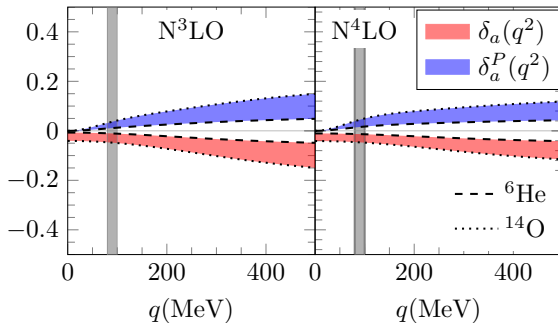
$$\mathbf{J}_{i,1b}^3 = \frac{\tau_i^3}{2} \left(g_A \boldsymbol{\sigma}_i - \frac{g_P}{2m_N} \mathbf{q} \cdot \boldsymbol{\sigma}_i \right)$$

- Approximate 2BCs by normal-ordering w.r.t. spin-isospin-symmetric reference state:

Hoferichter, Menéndez, Schwenk, *Phys. Rev. D* **102**,074018 (2020)

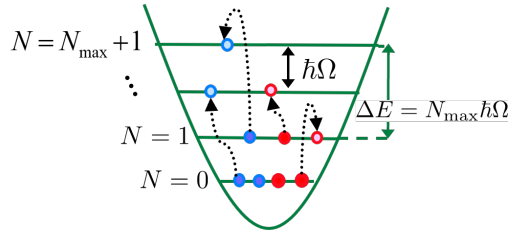
$$\mathbf{J}_{i,2b}^{\text{eff}} = \sum_j (1 - P_{ij}) \mathbf{J}_{ij}^3$$

$$\rightarrow \mathbf{J}_{i,2b}^{\text{eff}} = g_A \frac{\tau_i^3}{2} \left[\delta a(\mathbf{q}^2) \boldsymbol{\sigma}_i + \frac{\delta a^P(\mathbf{q}^2)}{q^2} (\mathbf{q} \cdot \boldsymbol{\sigma}_i) \mathbf{q} \right]$$



- Solve the nuclear many-body problem

$$H^{(A)}\Psi^{(A)}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A) = E^{(A)}\Psi^{(A)}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A)$$

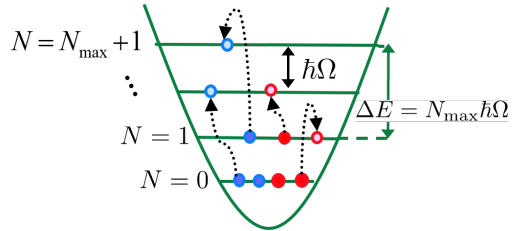


- Solve the nuclear many-body problem

$$H^{(A)}\Psi^{(A)}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A) = E^{(A)}\Psi^{(A)}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A)$$

- **Two- (NN)** and **3-body (3N)** forces from χ EFT:

$$H^{(A)} = \sum_{i=1}^A \frac{p_i^2}{2m} + \sum_{i<j=1}^A v_{ij}^{\text{NN}}(\mathbf{r}_i - \mathbf{r}_j) + \sum_{i<j<k=1}^A v_{ijk}^{\text{3N}}$$



- Solve the nuclear many-body problem

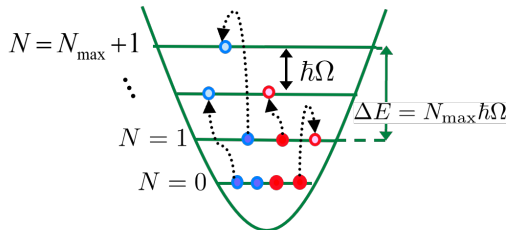
$$H^{(A)}\Psi^{(A)}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A) = E^{(A)}\Psi^{(A)}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A)$$

- **Two- (NN)** and **3-body (3N)** forces from χ EFT:

$$H^{(A)} = \sum_{i=1}^A \frac{p_i^2}{2m} + \sum_{i<j=1}^A v_{ij}^{\text{NN}}(\mathbf{r}_i - \mathbf{r}_j) + \sum_{i<j<k=1}^A v_{ijk}^{\text{3N}}$$

- Expansion in **Harmonic Oscillator (HO)** basis:

$$\Psi^{(A)} = \sum_{N=0}^{N_{\max}} \sum_j c_{Nj} \Phi_{Nj}^{\text{HO}}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A)$$



Dependency on Harmonic-Oscillator Frequency

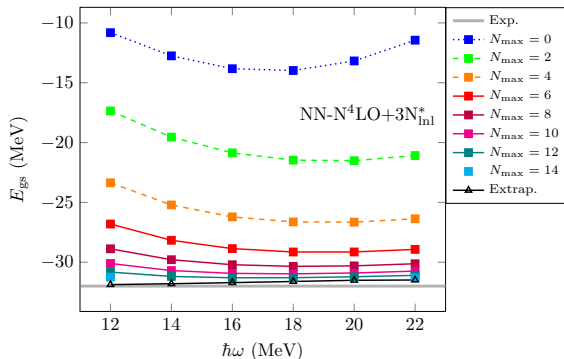


TECHNISCHE
UNIVERSITÄT
DARMSTADT

Ground-state energy of ${}^6\text{Li}$

$$\psi^{(A)} = \sum_{N=0}^{N_{\max}} \sum_j = c_{Nj} \phi_{Nj}^{\text{HO}}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A)$$

- The expansion depends on the HO frequency ($\hbar\omega$) because of the $N_{\max}$ truncation
 - ▢ Increasing $N_{\max}$ leads towards converged results

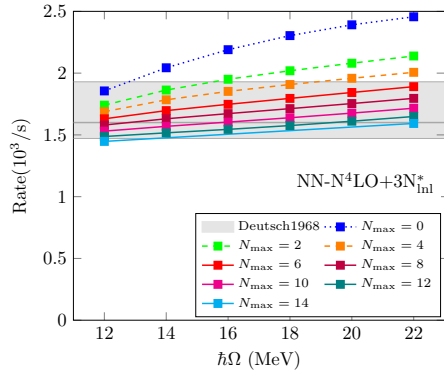
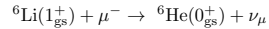
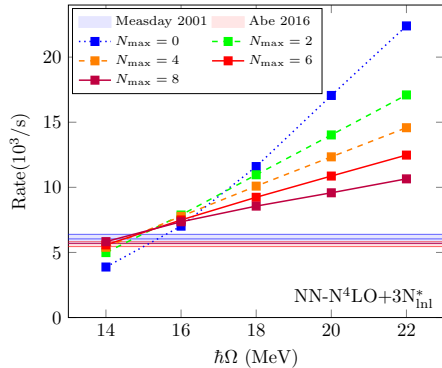
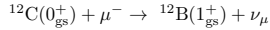


LJ, P. Navrátil, J. Kotila, K. Kravvaris, *Phys. Rev. C* 109, 065501 (2024)

Frequency-Dependence of Muon-Capture Rates



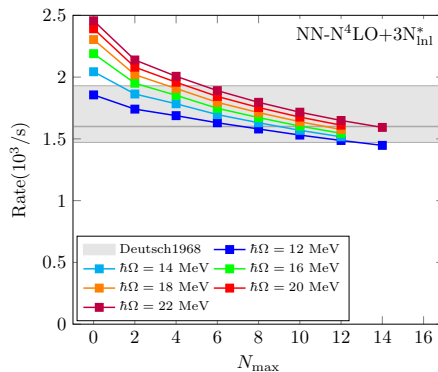
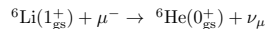
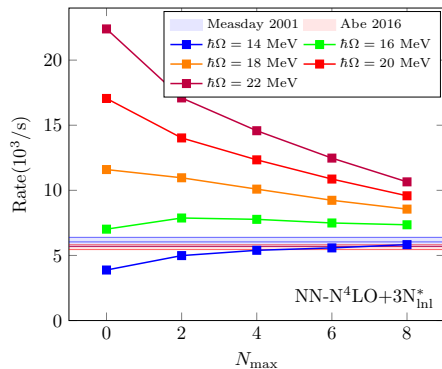
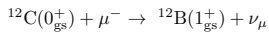
TECHNISCHE
UNIVERSITÄT
DARMSTADT



Frequency-Dependence of Muon-Capture Rates



TECHNISCHE
UNIVERSITÄT
DARMSTADT

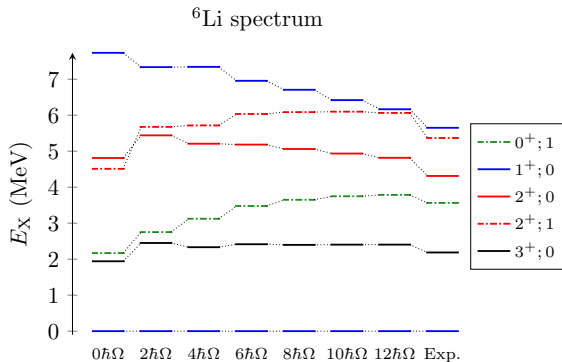


[LJ, P. Navrátil, J. Kotila, K. Kravvaris, Phys. Rev. C 109, 065501 \(2024\)](#)

Energy Spectrum of ${}^6\text{Li}$



TECHNISCHE
UNIVERSITÄT
DARMSTADT



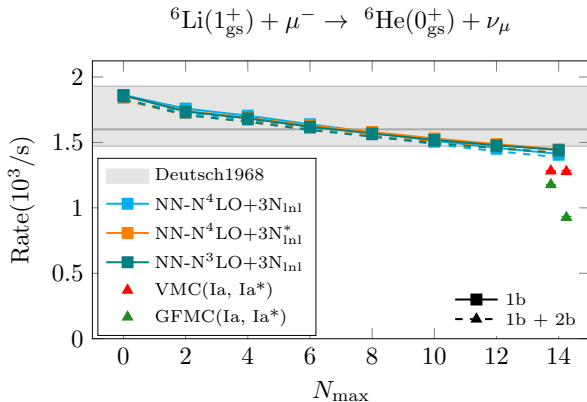
LJ, P. Navrátil, J. Kotila, K. Kravvaris, Phys. Rev. C 109, 065501 (2024)

Muon Capture to the Ground State of ${}^6\text{He}$



TECHNISCHE
UNIVERSITÄT
DARMSTADT

- NCSM slightly underestimating the experiment



LJ, P. Navrátil, J. Kotila, K. Kravvaris, *Phys. Rev. C* 109, 065501 (2024)

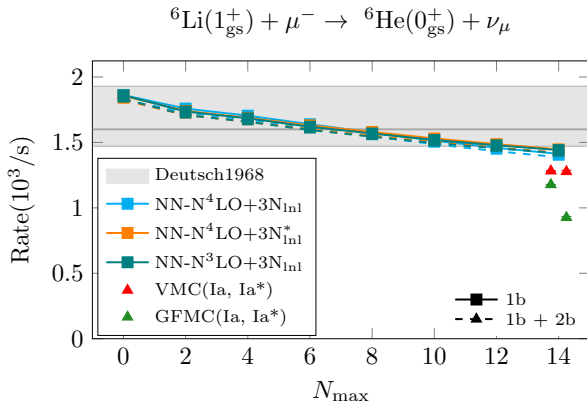
Muon Capture to the Ground State of ${}^6\text{He}$



TECHNISCHE
UNIVERSITÄT
DARMSTADT

- NCSM slightly underestimating the experiment
- Results consistent with other *ab initio* methods: **Variational Monte Carlo** and **Green's Function Monte Carlo**

G.B. King et al., Phys. Rev. C. 105, L042501 (2022)



L.J. P. Navrátil, J. Kotila, K. Kravvaris, Phys. Rev. C 109, 065501 (2024)

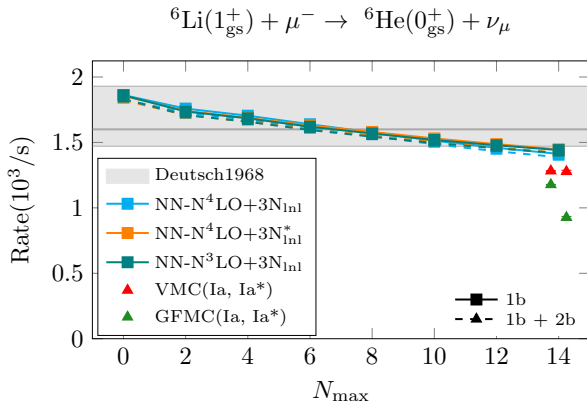
Muon Capture to the Ground State of ${}^6\text{He}$



TECHNISCHE
UNIVERSITÄT
DARMSTADT

- NCSM slightly underestimating the experiment
- Results consistent with other *ab initio* methods: **Variational Monte Carlo** and **Green's Function Monte Carlo**
- Slow convergence due to clustering not captured by the NCSM?

G.B. King et al., Phys. Rev. C. 105, L042501 (2022)

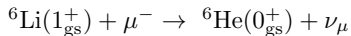


L.J. P. Navrátil, J. Kotila, K. Kravvaris, Phys. Rev. C 109, 065501 (2024)

Muon Capture to the Ground State of ${}^6\text{He}$



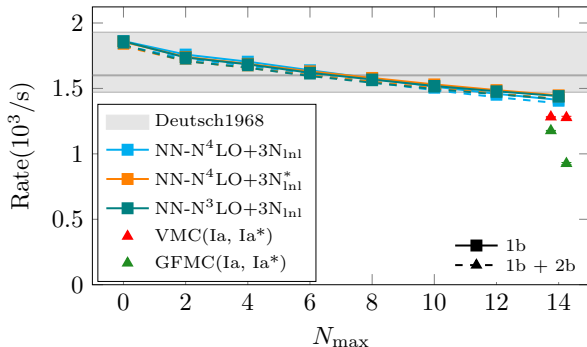
TECHNISCHE
UNIVERSITÄT
DARMSTADT



- NCSM slightly underestimating the experiment
- Results consistent with other *ab initio* methods: **Variational Monte Carlo** and **Green's Function Monte Carlo**

G.B. King et al., Phys. Rev. C. 105, L042501 (2022)

- Slow convergence due to clustering not captured by the NCSM?
 - ▣ NCSM with continuum (NCSMC) might give better results?



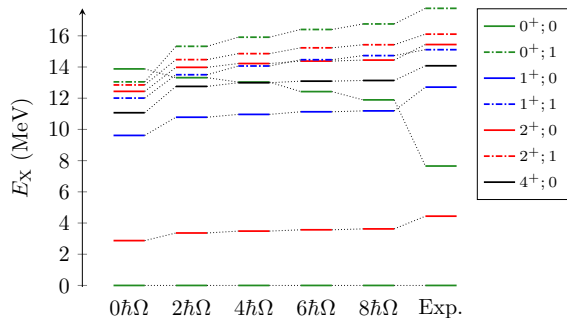
L.J. P. Navrátil, J. Kotila, K. Kravvaris, Phys. Rev. C 109, 065501 (2024)

Energy Spectra of ^{12}C and ^{12}B

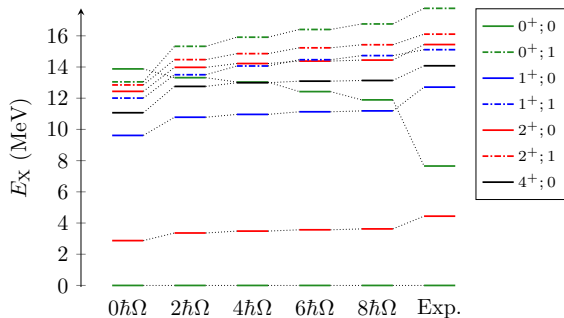


TECHNISCHE
UNIVERSITÄT
DARMSTADT

^{12}C spectrum



^{12}C spectrum



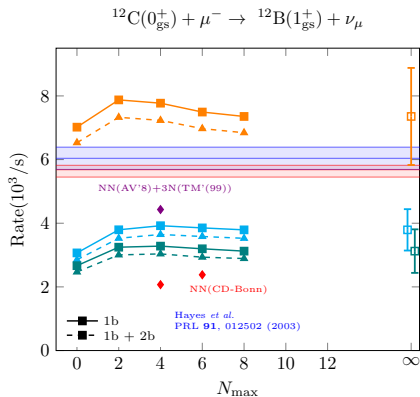
LJ, P. Navrátil, J. Kotila, K. Kravvaris, *Phys. Rev. C* 109, 065501 (2024)

Muon Capture to the Ground State of ^{12}B



TECHNISCHE
UNIVERSITÄT
DARMSTADT

■ Significant interaction dependence



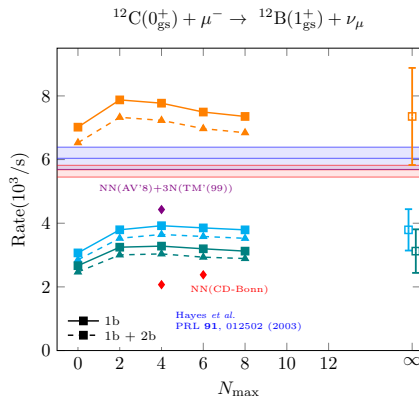
Muon Capture to the Ground State of ^{12}B



TECHNISCHE
UNIVERSITÄT
DARMSTADT

■ Significant interaction dependence

- ▣ The $\text{NN} - \text{N}^4\text{LO} + 3\text{N}_{\text{Inl}}^*$ interaction with the additional spin-orbit term most consistent with experiment



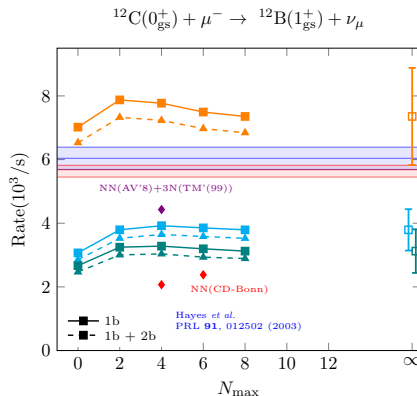
Muon Capture to the Ground State of ^{12}B



TECHNISCHE
UNIVERSITÄT
DARMSTADT

- Significant interaction dependence
 - ▣ The $\text{NN} - \text{N}^4\text{LO} + 3\text{N}_{\text{Inl}}^*$ interaction with the additional spin-orbit term most consistent with experiment
- The results are comparable with earlier NCSM calculations with phenomenological interactions

Hayes et al., Phys. Rev. Lett. 91, 012501 (2003)



L.J. P. Navrátil, J. Kotila, K. Kravvaris, Phys. Rev. C 109, 065501 (2024)

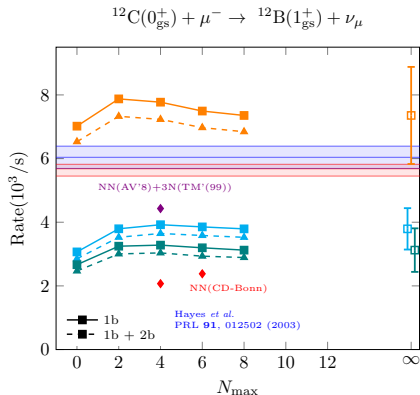
Muon Capture to the Ground State of ^{12}B



TECHNISCHE
UNIVERSITÄT
DARMSTADT

- Significant interaction dependence
 - ▣ The $\text{NN} - \text{N}^4\text{LO} + 3\text{N}_{\text{Inl}}^*$ interaction with the additional spin-orbit term most consistent with experiment
- The results are comparable with earlier NCSM calculations with phenomenological interactions
- **3-body forces** essential to reproduce the measured rate

Hayes et al., Phys. Rev. Lett. 91, 012501 (2003)

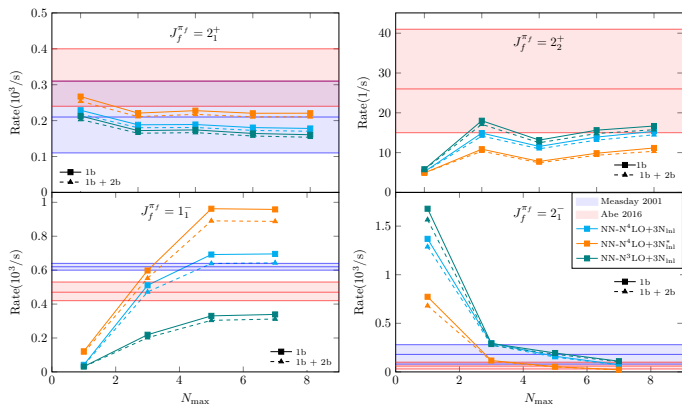


L.J. P. Navrátil, J. Kotila, K. Kravvaris, Phys. Rev. C 109, 065501 (2024)

Muon Capture to Excited States of ^{12}B



TECHNISCHE
UNIVERSITÄT
DARMSTADT

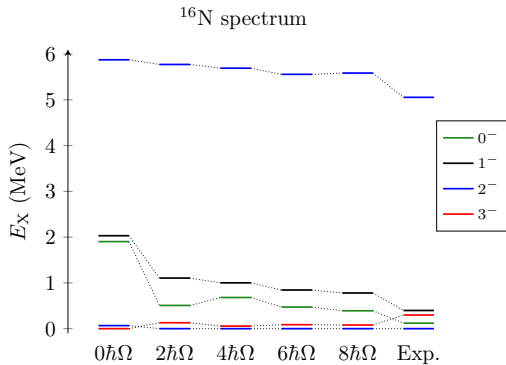


LJ, P. Navrátil, J. Kotila, K. Kravvaris, *Phys. Rev. C* 109, 065501 (2024)

Energy Spectrum of ^{16}N



TECHNISCHE
UNIVERSITÄT
DARMSTADT

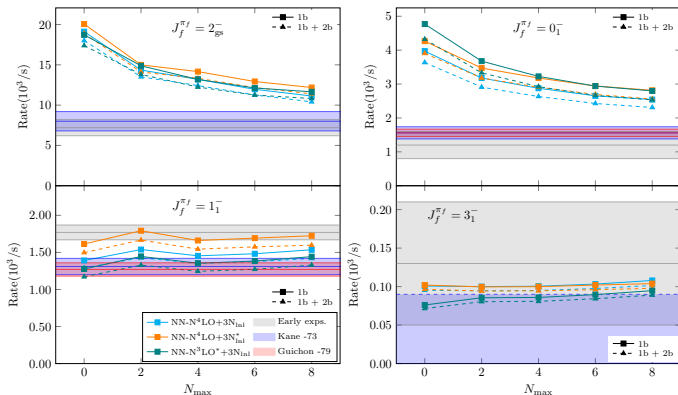


L.J., P. Navrátil, J. Kotila, K. Kravvaris, Phys. Rev. C 109, 065501 (2024)

Muon Capture on ^{16}N

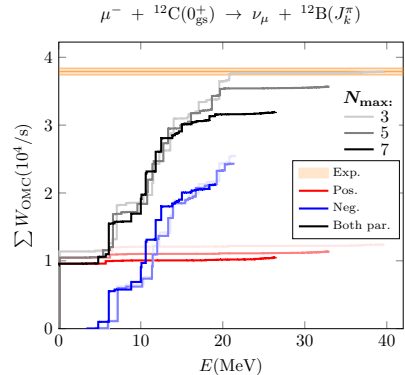


TECHNISCHE
UNIVERSITÄT
DARMSTADT



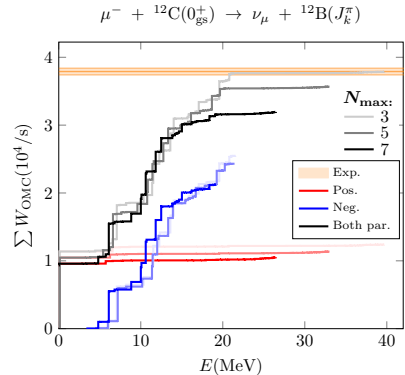
LJ, P. Navrátil, J. Kotila, K. Kravvaris, *Phys. Rev. C* 109, 065501 (2024)

- Rates obtained by summing over ~ 50 final states of each parity



LJ, P. Navrátil, J. Kotila, K. Kravvaris, Phys. Rev. C 109, 065501 (2024)

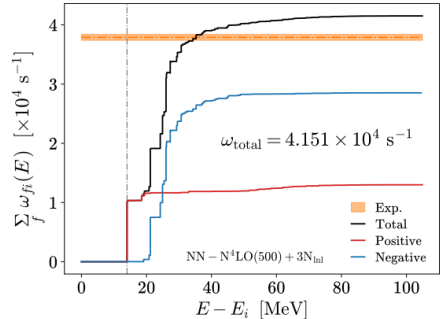
- Rates obtained by summing over ~ 50 final states of each parity
- Summing up the rates up to ~ 20 MeV, we capture $\sim 85\%$ of the total rate in both ^{12}B and ^{16}N



LJ, P. Navrátil, J. Kotila, K. Kravvaris, *Phys. Rev. C* 109, 065501 (2024)

- Rates obtained by summing over ~ 50 final states of each parity
- Summing up the rates up to ~ 20 MeV, we capture $\sim 85\%$ of the total rate in both ^{12}B and ^{16}N
- Better estimation with the **Lanczos strength function method** ongoing

D. Araujo Najera, M. Gennari, L.J. M. Drissi, P. Navrátil, work in progress



D. Araujo Najera, M. Gennari, L.J. M. Drissi, P. Navrátil, work in progress



Introduction

Ab initio Studies on Ordinary Muon Capture

Radiative Muon Capture as Background of $\mu \rightarrow e$ Conversion

Summary

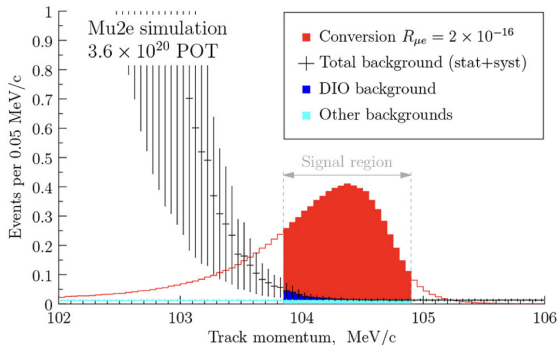
Radiative Muon Capture as Background of $\mu \rightarrow e$ Conversion



TECHNISCHE
UNIVERSITÄT
DARMSTADT

- RMC the main background for $\mu^- \rightarrow e^+$ searches:

$$\mu^- + {}^A_Z X(J_i^{\pi_i}) \rightarrow \nu_\mu + {}^A_{Z-1} Y(J_f^{\pi_f}) + \gamma ; \gamma \rightarrow e^+ e^-$$



R. H. Bernstein, Front. Phys. 7 (2019)

Radiative Muon Capture as Background of $\mu \rightarrow e$ Conversion



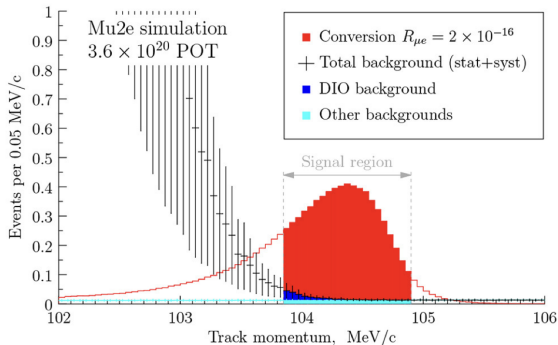
TECHNISCHE
UNIVERSITÄT
DARMSTADT

- RMC the main background for $\mu^- \rightarrow e^+$ searches:

$$\mu^- + {}^A_Z X(J_i^{\pi_i}) \rightarrow \nu_\mu + {}^A_{Z-1} Y(J_f^{\pi_f}) + \gamma; \gamma \rightarrow e^+ e^-$$

- Measured branching ratios of RMC $\sim 10^{-5}$

D. S. Armstrong et al., Phys. Rev. C 46, 1094 (1993)



R. H. Bernstein, Front. Phys. 7 (2019)

Radiative Muon Capture as Background of $\mu \rightarrow e$ Conversion



TECHNISCHE
UNIVERSITÄT
DARMSTADT

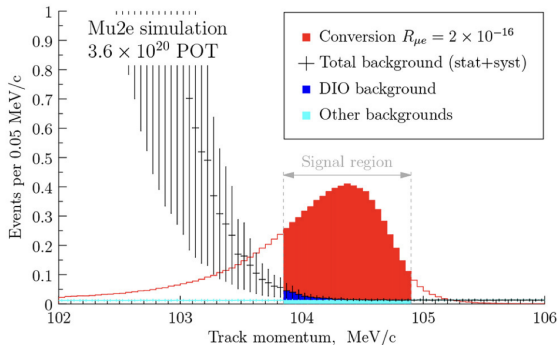
- RMC the main background for $\mu^- \rightarrow e^+$ searches:

$$\mu^- + {}^A_Z X(J_i^{\pi_i}) \rightarrow \nu_\mu + {}^A_{Z-1} Y(J_f^{\pi_f}) + \gamma; \gamma \rightarrow e^+ e^-$$

- Measured branching ratios of RMC $\sim 10^{-5}$

D. S. Armstrong et al., Phys. Rev. C 46, 1094 (1993)

- 0.1 – 1% probability for $\gamma \rightarrow e^+ e^-$



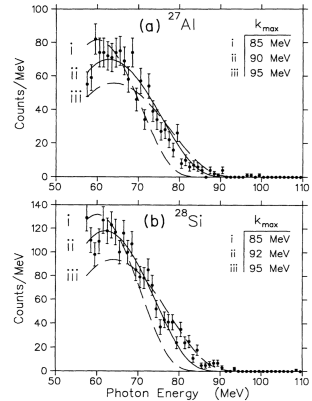
R. H. Bernstein, Front. Phys. 7 (2019)

RMC Spectra and Closure Approximation



TECHNISCHE
UNIVERSITÄT
DARMSTADT

- RMC data has large uncertainties near the endpoint

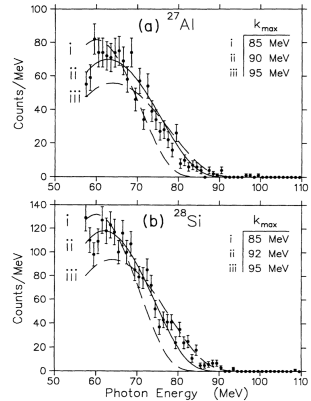


D. S. Armstrong et al., Phys. Rev. C 46, 1094 (1993)

RMC Spectra and Closure Approximation

- RMC data has large uncertainties near the endpoint
- RMC photon spectrum commonly parametrized in the **"closure approximation"** *H. Primakoff, Rev. Mod. Phys. 31, 802 (1959)*

$$\frac{dW_{\text{RMC}}}{dk} \propto x(1-x^2)[1-2x(1-x)], \text{ where } x = k/k_{\text{max}}$$



D. S. Armstrong et al., Phys. Rev. C 46, 1094 (1993)

RMC Spectra and Closure Approximation

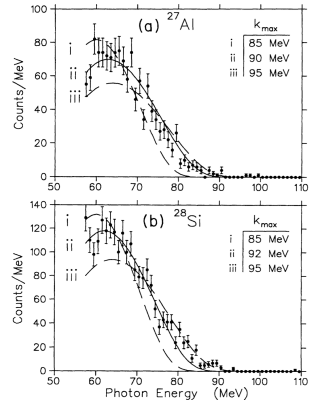


TECHNISCHE
UNIVERSITÄT
DARMSTADT

- RMC data has large uncertainties near the endpoint
- RMC photon spectrum commonly parametrized in the **"closure approximation"** *H. Primakoff, Rev. Mod. Phys. 31, 802 (1959)*

$$\frac{dW_{\text{RMC}}}{dk} \propto x(1-x^2)[1-2x(1-x)], \text{ where } x = k/k_{\text{max}}$$

- This does not provide a reliable extrapolation to the endpoint



D. S. Armstrong et al., Phys. Rev. C 46, 1094 (1993)

RMC Spectra and Closure Approximation

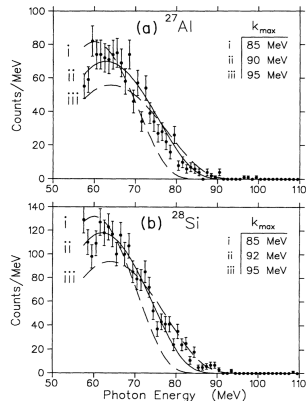


TECHNISCHE
UNIVERSITÄT
DARMSTADT

- RMC data has large uncertainties near the endpoint
- RMC photon spectrum commonly parametrized in the **"closure approximation"** *H. Primakoff, Rev. Mod. Phys. 31, 802 (1959)*

$$\frac{dW_{\text{RMC}}}{dk} \propto x(1-x^2)[1-2x(1-x)], \text{ where } x = k/k_{\text{max}}$$

- This does not provide a reliable extrapolation to the endpoint
- Fitted k_{max} values below the kinematic endpoint
 $E_{\gamma}^{\text{max}} = 101.36 \text{ MeV } (^{27}\text{Al})$



D. S. Armstrong et al., Phys. Rev. C 46, 1094 (1993)

RMC Estimates from Chiral Perturbation Theory + Nuclear Shell Model

$$\mu^- + \frac{A}{Z}X(J_i^{\pi_i}) \rightarrow \nu_\mu + \frac{A}{Z-1}Y(J_f^{\pi_f}) + \gamma$$

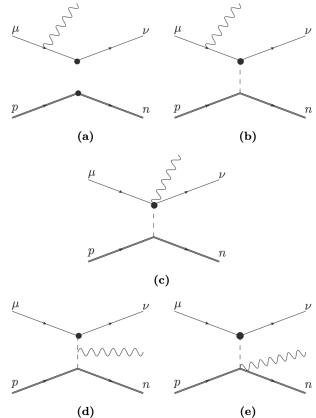
- Differential decay rate:

$$\frac{d\Gamma_B}{d\omega} = \frac{1}{2M} \int_{-1}^1 d\cos\theta \Pi(\cos\theta, \omega) \langle |\mathcal{M}_{B\gamma}|^2 \rangle$$

with

$$\Pi(\cos\theta, \omega) = \frac{1}{8} \frac{1}{(2\pi)^3} \frac{|\mathbf{k}_\nu| \omega}{M - \omega(1 - \cos\theta)}$$

- **Aim: Differential decay rate with 10% uncertainty near the endpoint**



RMC Estimates from Chiral Perturbation Theory + Nuclear Shell Model

$$\mu^- + \frac{A}{Z}X(J_i^{\pi_i}) \rightarrow \nu_\mu + \frac{A}{Z-1}Y(J_f^{\pi_f}) + \gamma$$

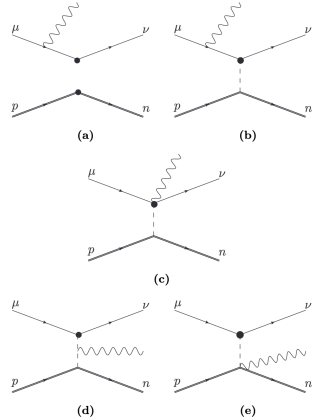
- Differential decay rate:

$$\frac{d\Gamma_B}{d\omega} = \frac{1}{2M} \int_{-1}^1 d\cos\theta \Pi(\cos\theta, \omega) \langle |\mathcal{M}_{B\gamma}|^2 \rangle$$

with

$$\Pi(\cos\theta, \omega) = \frac{1}{8} \frac{1}{(2\pi)^3} \frac{|\mathbf{k}_\nu| \omega}{M - \omega(1 - \cos\theta)}$$

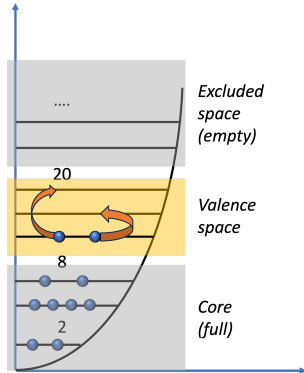
- **Aim: Differential decay rate with 10% uncertainty near the endpoint**



Nuclear Wave Functions from Nuclear Shell Model



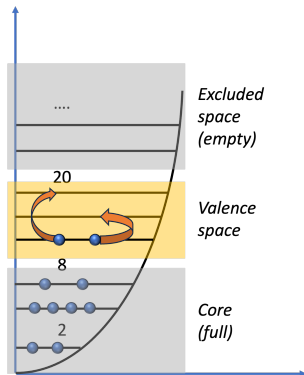
TECHNISCHE
UNIVERSITÄT
DARMSTADT



Nuclear Wave Functions from Nuclear Shell Model

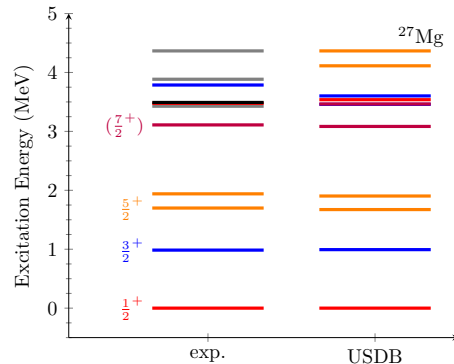


TECHNISCHE
UNIVERSITÄT
DARMSTADT



- ✓ Solve nuclear many-body problem in valence space

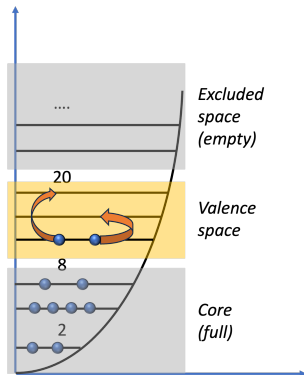
$$H^{\text{eff}} \psi^{(A)} = E^{(A)} \psi^{(A)}$$



Nuclear Wave Functions from Nuclear Shell Model



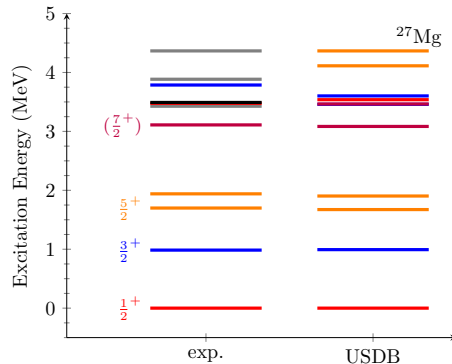
TECHNISCHE
UNIVERSITÄT
DARMSTADT



- ✓ Solve nuclear many-body problem in valence space

$$H^{\text{eff}} \psi^{(A)} = E^{(A)} \psi^{(A)}$$

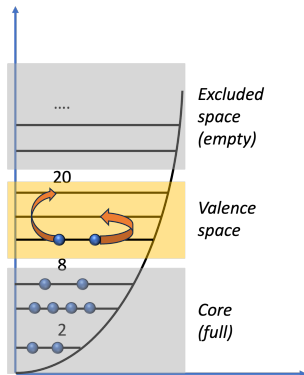
- ✓ Compute the needed nuclear matrix elements



Nuclear Wave Functions from Nuclear Shell Model



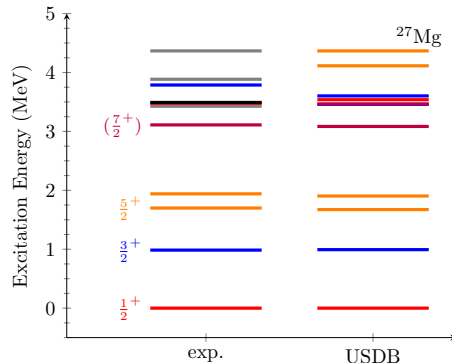
TECHNISCHE
UNIVERSITÄT
DARMSTADT



- ✓ Solve nuclear many-body problem in valence space

$$H^{\text{eff}} \psi^{(A)} = E^{(A)} \psi^{(A)}$$

- ✓ Compute the needed nuclear matrix elements
 - TODO: Compute the differential RMC rate





Introduction

Ab initio Studies on Ordinary Muon Capture

Radiative Muon Capture as Background of $\mu \rightarrow e$ Conversion

Summary

Summary



TECHNISCHE
UNIVERSITÄT
DARMSTADT

- Ordinary muon capture on nuclei can probe $0\nu\beta\beta$ decay
- *Ab initio* predictions for partial OMC rates in good agreement with data
- *Ab initio* total OMC rates look promising
- Radiative muon capture important background for $\mu \rightarrow e$
- Work in progress to estimate differential RMC rate with χ PT and nuclear shell model

