TECHNISCHE UNIVERSITÄT DRESDEN

INSTITUTE OF NUCLEAR AND PARTICLE PHYSICS

Towards precise constraints on Vector-like Leptons from Flavour (violating) observables at one-loop

CLFV Workshop 2025 - ECT*

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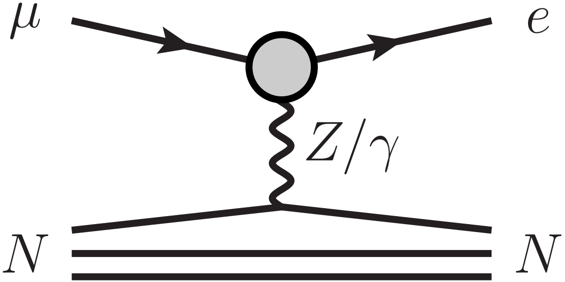
TU Dresden, Institut für Kern- und Teilchenphysik

Trento, IT, 15.04.2025

DISCLAIMER:

"Improve the theoretical rate calculations [...] to the accuracy required by upcoming experiments"



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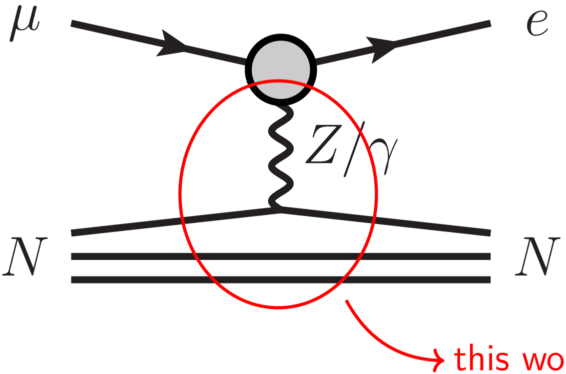

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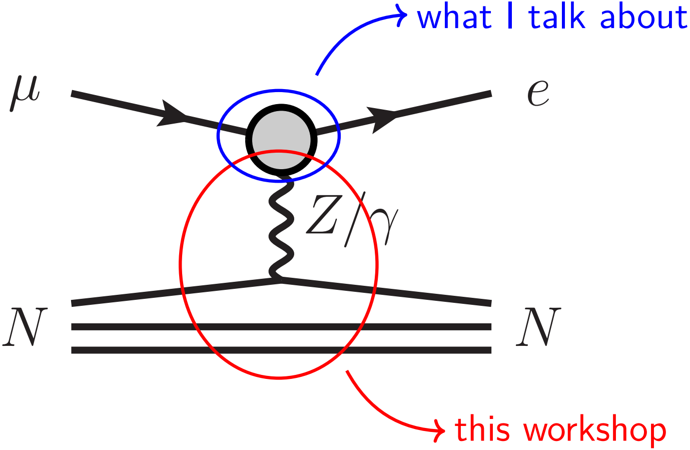

Outline

1. Vector-like Lepton Models
2. Tree-level Phenomenology
   - $g-2$ and Muon–Higgs coupling
   - Lepton Flavour Violation
3. VLL at one-loop
   - Why go beyond tree-level?
   - On-shell renormalization
   - Impact on correlations
4. Outlook

Why vector-like leptons?

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1990s *extra dimensions*, Kaluza-Klein modes, hierarchy problem and Dark Matter

2000s after Brookhaven $(g-2)_\mu$ experiment: -

2010s after Higgs discovery: only 3 chiral families, but VLL evade $h \to \gamma\gamma$ bounds
$\hookrightarrow$ renewed interest particularly in context of $(g-2)_\mu$ and lepton masses

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In our group:

- $(g-2)_\mu$ $\leftrightarrow$ chiral symmetry breaking $\leftrightarrow$ modifications of Higgs mechanism
- more recently LFV: correlations between $\mu \to e$, $\mu \to e\gamma$ and $\mu \to 3e$
- *precision BSM* already in context of SUSY, 2HDM, ... but not VLL
  $\hookrightarrow$ radiative corrections, renormalization, mixing at one-loop

$\Longrightarrow$ **rich phenomenology and interesting technical challenges**

Quantum numbers and representations

VLF: F_L and F_R *same* rep. under $SU(3) \times SU(2) \times U(1) \implies \mathcal{L} \supset M_F \bar{F}_L F_R + h.c.$

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VLL: tree-level couplings to SM leptons l_{Li} or e_{Ri} and Higgs Φ (or $\tilde{\Phi} = \epsilon \Phi^*$)
$\implies$ singlet, doublet or triplet with

Name	N	E	L	$L_{3/2}$	N^α	E^α
SU$(2)_L \times U(1)_Y$	$(\mathbf{1}, 0)$	$(\mathbf{1}, -1)$	$(\mathbf{2}, -\frac{1}{2})$	$(\mathbf{2}, -\frac{3}{2})$	$(\mathbf{3}, 0)$	$(\mathbf{3}, -1)$

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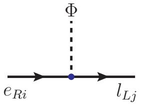
Combinations that break SM *chiral symmetry* $l_{Li} \to e^{i\alpha_i} l_{Li}$ and $e_{Ri} \to e^{i\beta_i} e_{Ri}$

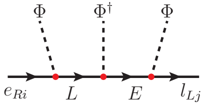
$$L \oplus E, \quad L \oplus N, \quad L_{3/2} \oplus E, \quad L \oplus E^a, \quad L \oplus N^a, \quad L_{3/2} \oplus E^a$$

Example: $L \oplus E$

$$\mathcal{L} \supset -\bar{l}_{Li} y^e_{ij} e_{Rj} \Phi - \bar{l}_{Li} \lambda^i_E \mathsf{P_R} E \Phi - \bar{L} \lambda^i_L \mathsf{P_R} e_{Ri} \Phi - \bar{L}(\lambda \mathsf{P_R} + \bar{\lambda} \mathsf{P_L}) E \Phi + h.c.$$



$\sim y^e_{ji}$



$$\sim \frac{\lambda^i_L M_L \bar{\lambda}^* \lambda^j_E M_E}{M^2_L M^2_E}$$

Mass Basis

$$\mathscr{M}^- = \begin{array}{c|ccc} & e_{Rj} & E_R & L_R^- \\ \hline \bar{e}_{Li} & y^e_{ij}v & \lambda^i_E v & 0 \\ \bar{L}_L^- & \lambda^j_L v & \lambda v & M_L \\ \bar{E}_L & 0 & M_E & \bar{\lambda} v \end{array} \qquad \Longrightarrow \qquad U_L^{-\dagger}\mathscr{M}^- U_R^- = \mathsf{diag}(m_i)$$

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- L_R^- and E_L have **different** eigenvalues under t^3

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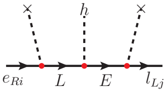
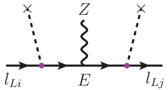
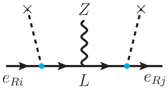
$\longrightarrow$ (LFV) deviation from tree-level SM Higgs and gauge couplings



$$Y^h_{ij} \approx \frac{m_i}{v}\delta_{ij} + 2\frac{\lambda^i_L \bar{\lambda}^* \lambda^j_E}{M_E M_L} v^2 + \mathcal{O}(v^4)$$



$$g^{Ze}_{L,ij} \approx (s_W^2 - \tfrac{1}{2})\delta_{ij} + \frac{1}{2}\frac{\lambda^{i*}_E \lambda^j_E}{M_E^2} v^2 + \mathcal{O}(v^4)$$



$$g^{Ze}_{R,ij} \approx \underbrace{s_W^2 \delta_{ij}}_{\text{SM}} - \underbrace{\frac{1}{2}\frac{\lambda^i_L \lambda^{j*}_L}{M_L^2} v^2}_{\text{VLL}} + \mathcal{O}(v^4)$$

$\Longrightarrow$ $\mathcal{O}(100\%)$ effects in lepton–Higgs coupling easily possible!

Searches and EW precision constraints

EWPM at Z-pole [Phys.Rept. 427 (2006) 257-454]

$$g^Z_{L\mu\mu} = -0.2689 \pm 0.0011$$
$$g^Z_{R\mu\mu} = +0.2323 \pm 0.0013$$

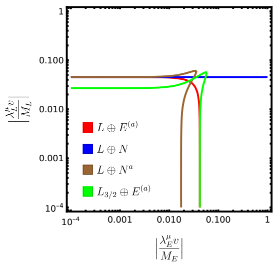
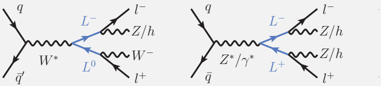


Figure: 1σ Z-pole exclusion contours

$$\implies \left|\frac{\lambda^\mu_i v}{M_i}\right| \lesssim 0.03$$



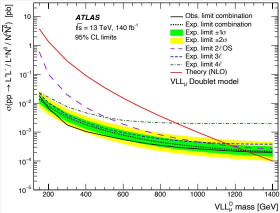
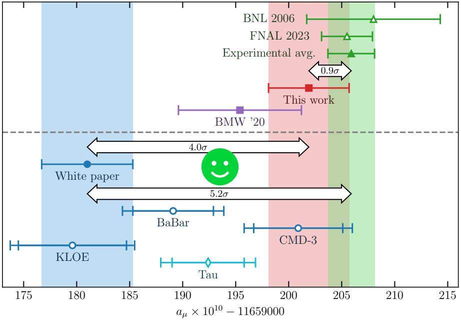


Figure: ATLAS exclusion limit for VLL coupling to muons reprinted from [2411.07143]

Exclusion limits for muon (electron) VLL

$$M_L > 1270\ (1220)\ \text{GeV}$$
$$M_E > 400\ (320)\ \text{GeV}$$

Status Muon g-2



$$\boxed{\Delta a_\mu^{2023} = (24.9 \pm 4.8) \times 10^{-10}}$$

Figure: Comparison between a_μ^{SM} and a_μ^{Exp} for different determinations of $\Delta a_\mu^{\text{HVP}}$. reprinted from [2407.10913]

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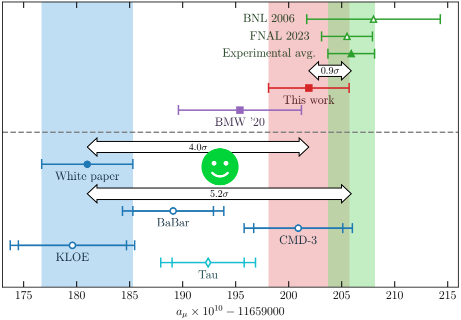

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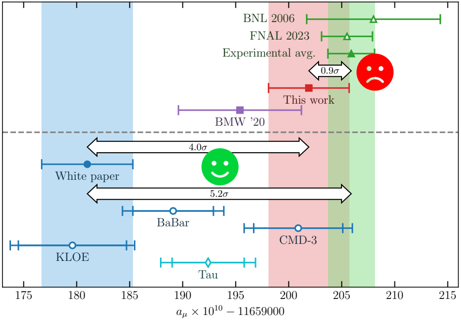

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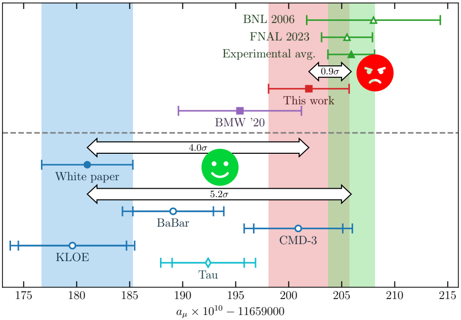

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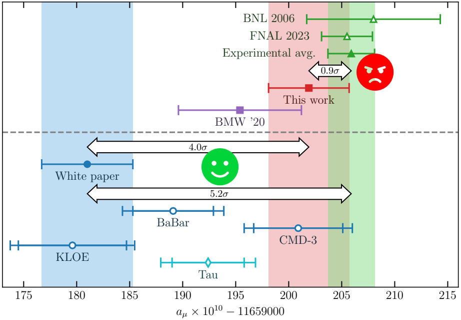

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- Dark Matter
- Neutrino masses
- flavour structure
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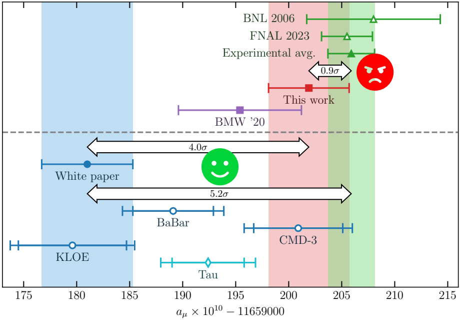

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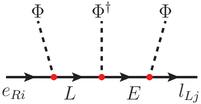
$\Longrightarrow$ **$\boldsymbol{\Delta a_\mu \neq 0}$ or $\boldsymbol{\Delta a_\mu = 0}$ can both give important insight into BSM!**

$g-2$ and Muon–Higgs coupling



$$\Rightarrow \quad \underbrace{\frac{\lambda_L^i \bar{\lambda}^* \lambda_E^j}{M_E M_L}}_{C_{e\Phi}} \quad \bar{l}_{Lj} e_{Ri} \Phi(\Phi^\dagger \Phi)$$

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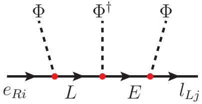

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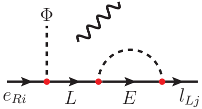

$$\Rightarrow \underbrace{\frac{e}{16\pi^2} \frac{\lambda_L^i \bar{\lambda}^* \lambda_E^j}{M_E M_L}}_{C_{e\gamma}} \bar{l}_{Lj} \sigma^{\alpha\beta} e_{Ri} F_{\alpha\beta} \Phi$$

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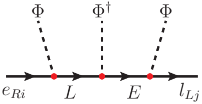

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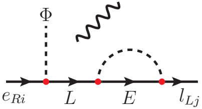

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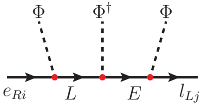
- AMM and Higgs coupling:

$$\Delta a_i = \frac{4 m_i v}{e} \operatorname{Re} C_{e\gamma}^{ij}, \qquad y_{ij}^{\text{eff}} = \frac{m_i}{v} \delta_{ij} + 2v^2 C_{e\Phi}^{ij}$$

- model-dependent correlation: $C_{e\Phi}^{ij} \sim 16\pi^2 C_{e\gamma}^{ij}$

$g-2$ and Muon–Higgs coupling



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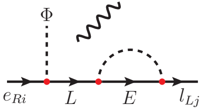

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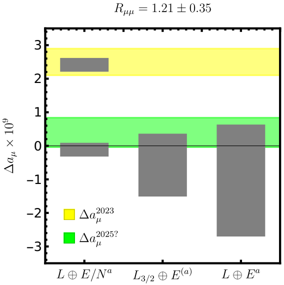

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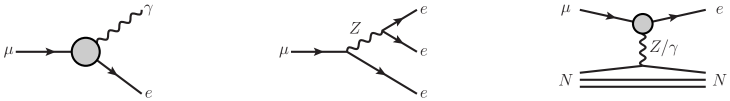
$$\overset{\text{no CPV}}{\Longrightarrow} \quad \boxed{R_{\mu\mu} = \left| \frac{y_{\mu\mu}^{\text{eff}}}{y_\mu^{\text{SM}}} \right|^2 \approx \left| 1 - 0.87 \frac{\Delta a_\mu}{\mathcal{Q} \times 10^{-9}} \right|^2}$$

- CPV → additional term $\sim d_\mu^2$, [Phys.Rev.Lett. 129 (2022) 22]

Model	$\mathcal{Q}$
$L \oplus E/N^a$	1
$L \oplus N$	-
$L_{3/2} \oplus E^{(a)}$	5
$L \oplus E^a$	9

loop ind.	$\sim 16\pi^2$

Lepton Flavour Violation



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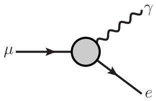
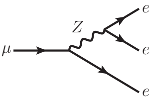
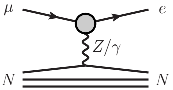






In general:

① LFV + **large** Δa_μ suggests

→ chirality flipping dipole operator

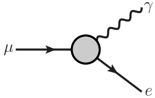
$$\mathcal{L} \supset L_{\underset{ij}{e\gamma}} \, \bar{e}_{Li} \sigma^{\alpha\beta} e_{Rj} F_{\alpha\beta} + h.c.$$

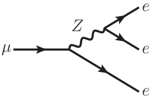
Dipole dominance

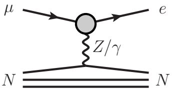
$$\text{BR}(\mu \to 3e) \sim \frac{1}{150} \times \text{BR}(\mu \to e\gamma)$$

$$\text{BR}(\mu \to e) \sim \frac{1}{300} \times \text{BR}(\mu \to e\gamma)$$

Lepton Flavour Violation







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② LFV + **small** Δa_μ suggests

chirality preserving 4-fermion operators

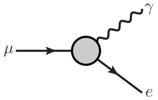
$$\mathcal{L} \supset \underset{ij}{L^{V,LL}_{eq}} (\bar{e}_{Li} \gamma^\mu e_{Lj})(\bar{q}_L \gamma^\mu q_L) + \ldots$$

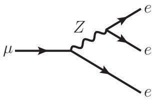
$\Rightarrow \mu \to e\gamma$ suppressed

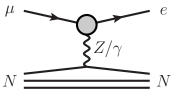
$$\text{BR}(\mu \to 3e) \sim 0.05 \times \text{BR}(\mu \to e)$$

$$\text{BR}(\mu \to e\gamma) \sim 10^{-4} \times \text{BR}(\mu \to e)$$

Lepton Flavour Violation







In general:

① LFV + **large** Δa_μ suggests

→ chirality flipping dipole operator

$$\mathcal{L} \supset \underset{ij}{L_{e\gamma}}\, \bar{e}_{Li}\sigma^{\alpha\beta}e_{Rj}F_{\alpha\beta} + h.c.$$

Dipole dominance

$$\mathrm{BR}(\mu \to 3e) \sim \frac{1}{150} \times \mathrm{BR}(\mu \to e\gamma)$$

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② LFV + **small** Δa_μ suggests

chirality preserving 4-fermion operators

$$\mathcal{L} \supset \underset{ij}{L^{V,LL}_{eq}}(\bar{e}_{Li}\gamma^\mu e_{Lj})(\bar{q}_L\gamma^\mu q_L) + \ldots$$

$\Rightarrow \mu \to e\gamma$ suppressed

$$\mathrm{BR}(\mu \to 3e) \sim 0.05 \times \mathrm{BR}(\mu \to e)$$

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current: $\mathrm{BR}(\mu \to e\gamma) < 4.2 \times 10^{-13}$, $\mathrm{BR}(\mu \to 3e) < 10^{-12}$, $\mathrm{BR}(\mu \to e)_{\mathrm{Au}} < 7 \times 10^{-13}$

future: $\mathrm{BR}(\mu \to e\gamma) \sim 6.0 \times 10^{-14}$, $\mathrm{BR}(\mu \to 3e) \sim 10^{-16}$, $\mathrm{BR}(\mu \to e)_{\mathrm{Al}} \sim 6 \times 10^{-16}$

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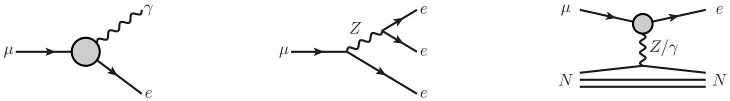

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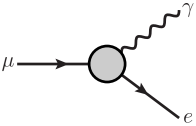
⇒ *which scenarios are realized in the vector-like lepton models?*

LFV with chiral enhancement: $\mu \to e\gamma$

In general: flavour violation quantified by $\theta_{ij} = L_{\substack{e\gamma\\ ij}} / L_{\substack{e\gamma\\ \mu\mu}}$

$$\mathrm{BR}(\mu \to e\gamma) = \frac{m_\mu^3}{4\pi\Gamma_\mu}\left(|L_{\substack{e\gamma\\ 12}}|^2 + |L_{\substack{e\gamma\\ 21}}|^2\right)$$

$$\overset{\text{no CPV}}{\approx} \underbrace{4.2 \times 10^{-13}}_{\text{current bound}} \left(\frac{\Delta a_\mu}{5 \cdot 10^{-10}}\right)^2 \left[\left(\frac{\theta_{\mu e}}{10^{-4}}\right)^2 + \left(\frac{\theta_{e\mu}}{10^{-4}}\right)^2\right]$$



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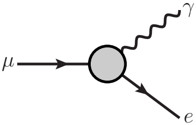


For VLL:

$$L^{e\gamma}_{ij} = \frac{\kappa_D e \bar{\lambda}^*}{64\pi^2 v}\left(\frac{\lambda^i_E v}{M_E}\right)\left(\frac{\lambda^j_L v}{M_L}\right)$$

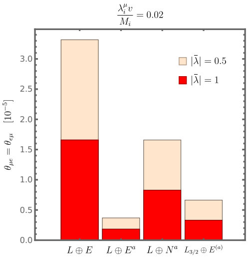
$$\Longrightarrow \quad \theta_{\mu e} = \frac{\lambda^e_L}{\lambda^\mu_L}, \quad \theta_{e\mu} = \frac{\lambda^e_E}{\lambda^\mu_E}$$

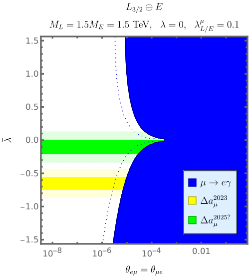
- flavour blind: $\theta_{ij} \sim 1$

  $\Delta a_\mu \lesssim 3.5 \times 10^{-14}$
- naive scaling $\theta_{ij} \sim \sqrt{\frac{m_e}{m_\mu}}$

  $\Delta a_\mu \lesssim 5 \times 10^{-13}$

Model	$L \oplus E$	$L \oplus N$	$L \oplus E^a$	$L \oplus N^a$	$L_{3/2} \oplus E^{(a)}$
κ_D	-1	0	-9	-2	∓5





LFV with chiral enhancement: $\mu \to e$ conversion

Approximate conversion rate [Phys.Rev.D 66 (2002) 096002]

$$\Gamma(\mu \to e) \approx \frac{m_\mu^3}{4v^2} \left\{ \left| v \underset{21}{L_{e\gamma}} D + \frac{2m_\mu}{v} g^{Ze}_{R12} V \right|^2 + \left| v \underset{12}{L_{e\gamma}} D + \frac{2m_\mu}{v} g^{Ze}_{L12} V \right|^2 \right\}$$

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Dipole dominance?

$$1 \overset{!}{\gg} \left|\frac{2m_\mu}{v} g^{Ze}_{L/R12} / v\underset{ij}{L_{e\gamma}}\right| \sim \frac{3}{\bar{\lambda}} \frac{\kappa_Z}{\kappa_D}$$

→ only in the models $L/L_{3/2} \oplus E^a$

Model	$\lvert\kappa_Z/\kappa_D\rvert$
$L \oplus E/N^a$	0.5
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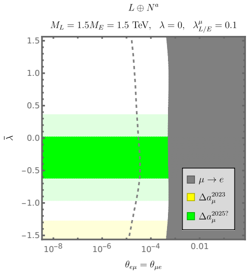
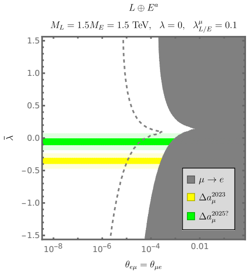
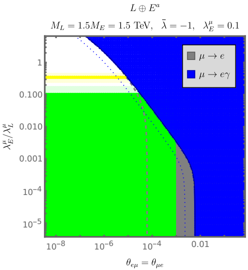


(a) No dipole dominance



(b) dipole dominance

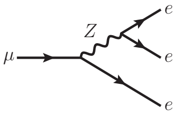


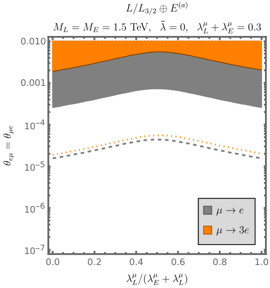
(c) $\mu \to e$ vs $\mu \to e\gamma$

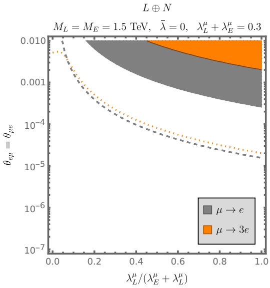
LFV without chiral enhancement

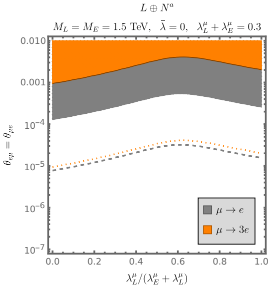
→ leading chiral enhancement vanishes if $\lambda_E^i \bar{\lambda} \lambda_L^j M_E M_L = 0$ (e.g. $\bar{\lambda} = 0$)

$$\mathrm{BR}(\mu \to e) \approx \underbrace{\frac{128\pi^3}{s_W^4}\frac{\Gamma_\mu}{\Gamma_{\text{capt}}}}_{\mathcal{O}(10)} |V|^2\, \mathrm{BR}(\mu \to 3e)$$







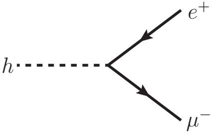


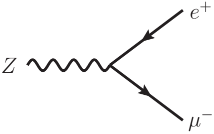
Precision vs collider constraints

- flavour observables $R_{\mu\mu}$ and Δa_μ

$\Longrightarrow$ complementary constraints

- similar correlation for flavour violating observables?





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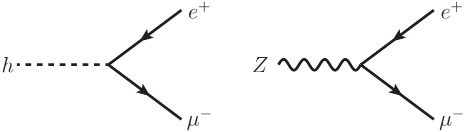

with chiral enhancement

$$\Gamma(h \to \mu e) \approx \frac{M_h}{16\pi}\left(\left|Y^h_{\mu e}\right|^2 + \left|Y^h_{e\mu}\right|^2\right)$$

$\to$ Correlation between dipole coefficient and Higgs coupling

$$Y^h_{ij} = \frac{128\pi^2 v}{\mathcal{Q} e} L_{\underset{ij}{e\gamma}}$$

$$\Longrightarrow \frac{\text{BR}(h \to \mu e)}{\text{BR}(\mu \to e\gamma)} \approx \frac{1}{\mathcal{Q}^2}$$

but: $\text{BR}(h \to \mu e) < 4.4 \times 10^{-5}$

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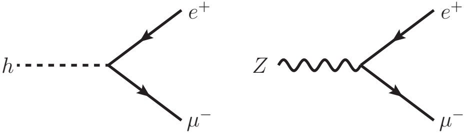

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without chiral enhancement

$$\Gamma(Z \to \mu e) \approx \frac{M_Z}{8\pi}\left(\left|g^{Ze}_{L12}\right|^2 + \left|g^{Ze}_{R12}\right|^2\right)$$

directly correlated with $\mu \to e$ or $\mu \to 3e$

$$\frac{\text{BR}(Z \to \mu e)}{\text{BR}(\mu \to e)} \sim \mathcal{O}(0.1)$$

bound: BR$(Z \to \mu e) < 2.6 \times 10^{-7}$

→ stronger than Higgs but again not competitive

Why go beyond tree-level?

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- (?)

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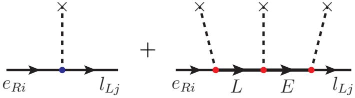
SM Lepton **mass** at tree-level



$$+ \quad \ldots \quad \sim \quad y_{ij} v + \frac{\lambda_L^i \bar{\lambda}^* \lambda_E^j}{M_E M_L} v^3 + \mathcal{O}(v^5)$$

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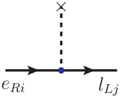
SM Lepton **mass** at tree-level vs one-loop



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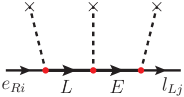

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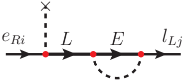

$$+ \quad \ldots \quad \sim \quad \frac{\lambda_L^i \bar{\lambda}^* \lambda_E^j}{16\pi^2} v \; + \; \mathcal{O}(v^3)$$

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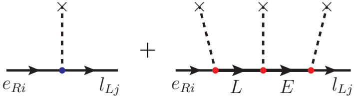

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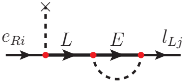

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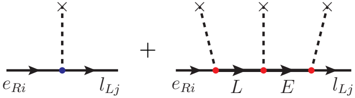

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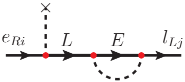

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⇒ what about other observables?

- e.g. muon-Higgs coupling: LHC at $\mathcal{O}(10\%)$ vs tree-level correction $\mathcal{O}(100\%)$
- effect on electroweak precision and LFV constraints?

On-shell scheme: motivation

$\Longrightarrow$ calculation of lepton – Higgs/ Z coupling at one-loop requires *practicable* renormalization scheme

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Major issues

- obtaining correct SM lepton masses require solving SVD for y_i^e at tree-level. one-loop mass shift Δm_i makes this tedious / less efficient
- SM-VLL couplings induces off-diagonal mass corrections Δm_{ij} and mixing at external legs $\Longrightarrow$ non-trivial LSZ normalization $\mathcal{Z}_{ij}$
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Solution: $\Longrightarrow$ on-shell scheme [JHEP 10 (2024) 170]

$\equiv i\Sigma_{ba}(p)$:

ren. cond. for $p^2 \to m_a^2$

$$\underbrace{\widetilde{\mathrm{Re}}\,\Sigma_{ba}(p)u_a(p) = 0,}_{\text{removes mixing and } \Delta m} \qquad \underbrace{\frac{1}{\not{p}-m_a}\widetilde{\mathrm{Re}}\,\Sigma_{aa}u(p) = 0}_{\text{enforces } \mathcal{Z}=\mathbb{1}}$$

On-shell mass renormalization

$\Longrightarrow$ OS conditions fix 5 of in total 13 ren. consts.

$$\delta m_{aa} = \left[U_L^{-\dagger} \begin{pmatrix} \delta(y^e v) & \delta(\lambda_E^i v) & 0 \\ \delta(\lambda_L^i) & \delta(\lambda v) & \delta M_L \\ 0 & \delta M_E & \delta(\bar{\lambda} v) \end{pmatrix} U_R^- \right]_{aa}$$

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$\Longrightarrow$ tedious to disentangle but easy to solve (linear equation...)

On-shell mass renormalization

$\Longrightarrow$ OS conditions fix 5 of in total 13 ren. consts.

fixed at tree-level

$$\delta m_{aa} = \left[U_L^{-\dagger} \begin{pmatrix} \delta(y^e v) & \delta(\lambda^i_E v) & 0 \\ \delta(\lambda^i_L) & \delta(\lambda v) & \delta M_L \\ 0 & \delta M_E & \delta(\bar{\lambda} v) \end{pmatrix} U_R^- \right]_{aa}$$

fixed by OS

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Compare to:

- **QED:** 1 fundamental parameter m fixed by OS condition
- **SM quark sector:** fundamental parameters $y_{ij} \leftrightarrow m_i$ and V_{CKM}
- **2HDM:** 4 scalar masses + mixing angle α (+...)

$\hookrightarrow$ *explicit mixing angles* $\Longrightarrow$ reparametrization in terms of m_i

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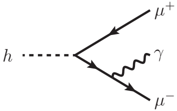
for VLL: 5×5 mixing matrix = better avoid explicit parametrization

Muon–Higgs coupling at one-loop

Up to one-loop order

$$y_{ij}^{\text{eff}} = Y_{ij}^{h}\Big(1 - \alpha B\Big) + \Gamma_{ij}^{1\ell}(p_h^2, p_{\mu^+}^2, p_{\mu^-}^2) + \delta\Gamma_{ij}^{ct}$$

Muon–Higgs coupling at one-loop



real radiation

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real radiation
(IR div.)

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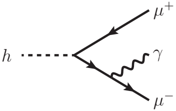
genuine 1ℓ diagrams
(UV + IR div.)



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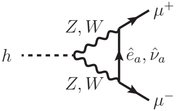

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counterterm contribution
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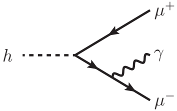
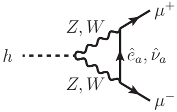
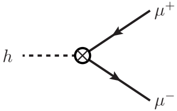


real radiation



genuine one-loop



counterterm

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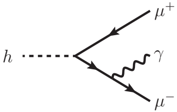
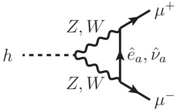
Important checks

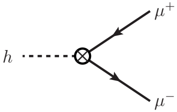
- cancellation of UV divergences ✓
- IR finite after inclusion of real radiation ✓
- cancellation of residual renormalization scale dependence ✓



real radiation



genuine one-loop



counterterm

Muon–Higgs coupling at one-loop

Impact on correlation: *SMEFT* → coefficients at one-loop

$$y_\mu^{\text{SMEFT}} = y_\mu + \underbrace{\mathcal{O}\left(\frac{\lambda_i^3}{16\pi^2}\right)}_{1\ell}, \qquad C_{e\Phi}^{\mu\mu} = \frac{\lambda_i^3 v^2}{M^2}\Bigg[\underbrace{\ \ 1\ \ }_{\text{tree-level}} + \underbrace{\mathcal{O}\left(\frac{\lambda_i^2}{16\pi^2}\right)}_{1\ell}\ \Bigg]$$

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$\Longrightarrow$ (very) large correction to y_μ^{SMEFT} cancels between y_μ^{eff} and m_μ

$$\begin{aligned} \frac{m_\mu}{v} &= y_\mu^{\text{SMEFT}} + C_{e\Phi}^{\mu\mu} v^2 \\ y_{\mu\mu}^{\text{eff}} &= y_\mu^{\text{SMEFT}} + {\color{red}3} C_{e\Phi}^{\mu\mu} v^2 \end{aligned} \quad \Longrightarrow \quad \boxed{\frac{y_{\mu\mu}^{\text{eff}}}{y_\mu^{\text{SM}}} = 1 - 0.87\,\frac{\Delta a_\mu}{10^{-9}}\Big(1 + \mathcal{O}(1\ell)\Big)}$$

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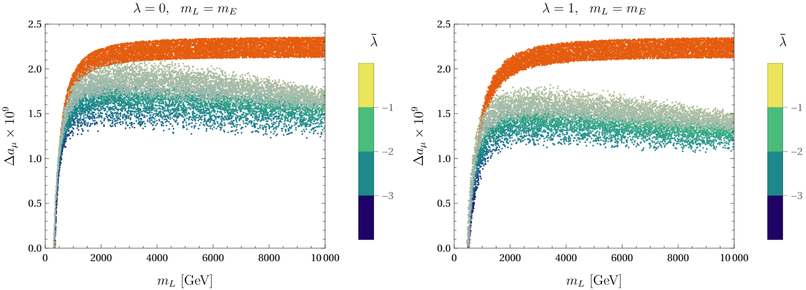
BUT: remaining one-loop corrections are still significant



Summary & Outlook

- VLL have strong impact on Higgs and flavour (violating) physics and upcoming experiments will extensively probe the interesting parameter regions.
- $\Delta a_\mu \to 0$ makes some room to address other questions like origin of lepton masses, LFV and CPV.
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- **Next:** application to EW precision and LFV observables → precise predictions for upcoming experiments

Thank you!

Backup

Backup - Current and Future Experimental Bounds

Collider constraints

Higgs	Z
$\text{BR}(h \to \mu e) < 4.4 \times 10^{-5}$ $\text{BR}(h \to \tau e) < 2.0 \times 10^{-3}$ $\text{BR}(h \to \tau\mu) < 1.5 \times 10^{-3}$	$\text{BR}(Z \to \mu e) < 2.6 \times 10^{-7}$ $\text{BR}(Z \to \tau e) < 5.0 \times 10^{-6}$ $\text{BR}(Z \to \tau\mu) < 6.5 \times 10^{-6}$

LFV decays

$\ell \to \ell'\gamma$	$\ell \to 3\ell'$
$\text{BR}(\mu \to e\gamma) < 4.2 \times 10^{-13}$ $\text{BR}(\tau \to \ell\gamma) \lesssim 4 \times 10^{-8}$	$\text{BR}(\mu \to 3e) < 1.0 \times 10^{-12}$ $\text{BR}(\tau \to 3\ell) \lesssim 2 \times 10^{-8}$

$\mu \to e$ conversion

$\Gamma(\mu^- \text{Au} \to e^- \text{Au})/\Gamma^{\text{Au}}_{\text{capt}} < 7 \times 10^{-13}$

$\Gamma(\mu^- \text{Ti} \to e^- \text{Ti})/\Gamma^{\text{Ti}}_{\text{capt}} < 4.3 \times 10^{-12}$

Future sensitivities:

$\Gamma(\mu^- \text{Al} \to e^- \text{Al})/\Gamma^{\text{Au}}_{\text{capt}} \sim 6 \times 10^{-16}$
$\text{BR}(\mu \to e\gamma) \sim 6 \times 10^{-14}$
$\text{BR}(\mu \to 3e) \sim 10^{-16}$

On-shell scheme: set-up

Example: $L \oplus E$ multiplets with same QN $\rightarrow \varepsilon_{Ra} = (e_{Ri}, E_R)$ and $\ell_{La} = (l_{Li}, L_L)$

$$\mathcal{L} \supset -M_E^a \overline{E}_L \varepsilon_{Ra} - M_L^a \overline{\ell}_{La} L_R - \overline{\ell}_{La} Y_{ab} \varepsilon_{Rb} \Phi - \bar{\lambda} \overline{L}_R E_R \Phi, \qquad Y = \begin{pmatrix} y^e & \lambda_E \\ \lambda_L & \lambda \end{pmatrix}$$

Parameter transformation: $M_i^a \rightarrow M_i^a + \delta M_i^a, \quad Y_{ab} \rightarrow Y_{ab} + \delta Y_{ab}, \quad \bar{\lambda} \rightarrow \lambda + \delta\lambda$

$\Rightarrow$ *Redundancy:* $\ell_L \rightarrow V_L \ell_L$ and $\varepsilon_R \rightarrow V_R \varepsilon_R$. $V_{L/R}$ can be used to make *gauge-basis* field renormalization hermitian **or** set *bare* $M_i^a = \delta_{a4} M_i$ and $y_{ij}^e = y_i^e \delta_{ij}$

Field transformation: multiplets under broken gauge group

$$\begin{pmatrix} \varepsilon_R \\ L_R^- \end{pmatrix} \rightarrow U_R^- Z_R^{\frac{1}{2}} \hat{e}_R, \qquad \begin{pmatrix} \ell_L^- \\ E_L \end{pmatrix} \rightarrow U_L^- Z_L^{\frac{1}{2}} \hat{e}_R$$

- unitary 5×5 $U_{L/R}^-$ diagonalize *renormalized* mass matrix

$\Longrightarrow$ off-diagonal mass (Yukawa) renormalization constants

$$\delta m_{ab} = U_L^{-\dagger} \begin{pmatrix} \delta(Yv) & \delta M_L^a \\ \delta M_E^a & \delta(\bar{\lambda} v) \end{pmatrix} U_R^- \neq \delta m_a \delta_{ab}$$

Compared to 2HDM

$$\begin{pmatrix} H \\ h \end{pmatrix} = \mathcal{R}_\alpha \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}$$

2×2 mass diagonalization matrix R_α
$\Longrightarrow$ explicit parametrization in terms α
$\Longrightarrow$ some of the fundamental parameters can traded for pole masses *and* α

On-shell scheme: renormalization constants

$$\underbrace{\widetilde{\mathrm{Re}}\,\Sigma_{ba}(p)u_a(p)=0,}_{\text{50 equations}}\qquad \underbrace{\frac{1}{\not{p}-m_a}\widetilde{\mathrm{Re}}\,\Sigma_{aa}u(p)=0,}_{\text{5 equations}}\qquad \text{but: 50 } Z_{ab} \text{ and } 2+11 \text{ parameters}$$

$\Longrightarrow$ OS conditions fix $Z_{L/R}$ and $\delta m_{aa} \equiv \boldsymbol{\delta m}$, but not off-diagonal δm_{ab}

$\Longrightarrow$ similar problem to tree-level m_i vs y_i^e, but now $U_{L/R}$ are already known
$\hookrightarrow$ choose 5 params. (Dirac masses and y_i^e), fix rest by external conditions.

$$\boldsymbol{\delta m} = \boldsymbol{\kappa} \begin{pmatrix} \delta(y^e v) \\ \delta M_L \\ \delta M_E \end{pmatrix} + \boldsymbol{\delta c}$$

fixed by OS

remaining consts. (fixed e.g. in $\overline{\text{MS}}$)

determined from $U_{L/R}$

fundamental ren. const.

$\Longrightarrow$ fundamental ren. consts. are given by $\boldsymbol{\kappa}^{-1}(\boldsymbol{\delta m} - \boldsymbol{\delta c})$ and can be obtained numerically or perturbatively.

Note

- m and δm of leptons with different charge are not independent
  $\hookrightarrow$ OS cond. on all leptons fixes more parameters (e.g. also λ or $\bar{\lambda}$) [JHEP 10 (2024) 170]
- alternatively: leave masses of some leptons (e.g. doubly charged or heavy neutrinos) off-shell and compute one-loop mass shift (preferable e.g. in the triplet models)

On-shell Renormalization Constants: Gauge Sector

$$\widetilde{\mathrm{Re}}\,\Sigma_h(M_h^2) = 0, \qquad \lim_{p^2 \to M_h^2} \frac{1}{p^2 - M_h^2} \widetilde{\mathrm{Re}}\,\Sigma'_h(M_h^2) = 0, \qquad \text{and}$$

$$\widetilde{\mathrm{Re}}\,\Sigma^{\mu\nu}_{VV'}(q)\epsilon_\nu(q)\Big|_{q^2 = M^2_{V'}} = 0, \qquad \lim_{q^2 \to M_V^2} \frac{1}{q^2 - M_V^2} \Sigma^{\mu\nu}_{VV}(q)\epsilon_\nu(q) = 0$$

The vector two-point function has the general covariant decomposition

$$-i\Sigma^{\mu\nu}_{VV'}(q) = -i\Sigma^T_{VV'}(q^2)\left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) - i\Sigma^L_{VV'}(q^2)\frac{q^\mu q^\nu}{q^2}$$

Inserting this into the on-shell conditions gives the following renormalization constants at one-loop order

$$\begin{aligned} \delta Z_h &= -\widetilde{\mathrm{Re}}\,\Sigma'_h(M_h^2) & \qquad \delta M_h^2 &= \widetilde{\mathrm{Re}}\,\Sigma_h(M_h^2) \\ \delta Z_{VV} &= -\widetilde{\mathrm{Re}}\,\Sigma^{T\prime}_{VV}(M_V^2), & \qquad \delta M_V^2 &= \widetilde{\mathrm{Re}}\,\Sigma^T_{VV}(M_V^2) \\ \delta Z_{AZ} &= -\frac{2}{M_Z^2}\widetilde{\mathrm{Re}}\,\Sigma^T_{AZ}(M_Z^2) & \qquad \delta Z_{ZA} &= \frac{2}{M_Z^2}\widetilde{\mathrm{Re}}\,\Sigma^T_{AZ}(0) \end{aligned}$$

The charge and vev renormalization constants are given by

$$\frac{\delta e}{e} = -\frac{1}{2}\left(\delta Z_{AA} + \frac{s_W}{c_W}\delta Z_{ZA}\right), \qquad \frac{\delta v}{v} = \frac{\delta M_W^2}{2M_W^2} + \frac{c_W^2}{s_W^2}\left(\frac{\delta M_Z^2}{2M_Z^2} - \frac{\delta M_W^2}{2M_W^2}\right) - \frac{\delta e}{e}$$

On-shell Renormalization Constants: Lepton Sector

$$\Sigma_{ab}(p) = \Sigma^{R}_{ab}(p^2)\not{p}\mathsf{P_R} + \Sigma^{L}_{ab}(p^2)\not{p}\mathsf{P_L} + \Sigma^{SR}_{ab}(p^2)\mathsf{P_R} + \Sigma^{SL}_{ab}(p^2)\mathsf{P_L}.$$

$$\text{for } a = b \begin{cases} (\delta m)_{aa} &= \frac{1}{2}\,\widetilde{\mathrm{Re}}\left[m_a\Sigma^{R}_{aa} + m_a\Sigma^{L}_{aa} + \Sigma^{SR}_{aa} + \Sigma^{SL}_{aa}\right]_{p^2=m_a^2} \\ (\delta Z_R)_{aa} &= -\widetilde{\mathrm{Re}}\left[\Sigma^{R}_{aa} + m_a\left(\Sigma^{SR'}_{aa} + \Sigma^{SL'}_{aa}\right) + m_a^2\left(\Sigma^{R'}_{aa} + \Sigma^{L'}_{aa}\right)\right]_{p^2=m_a^2} \\ (\delta Z_L)_{aa} &= -\widetilde{\mathrm{Re}}\left[\Sigma^{L}_{aa} + m_a\left(\Sigma^{SR'}_{aa} + \Sigma^{SL'}_{aa}\right) + m_a^2\left(\Sigma^{R'}_{aa} + \Sigma^{L'}_{aa}\right)\right]_{p^2=m_a^2} \end{cases}$$

$$\text{for } a \neq b \begin{cases} (\delta Z_R)_{ab} &= \frac{2}{m_a^2 - m_b^2}\Big[m_b^2\,\widetilde{\mathrm{Re}}\,\Sigma^{R}_{ab} + m_a m_b\,\widetilde{\mathrm{Re}}\,\Sigma^{L}_{ab} + m_a\,\widetilde{\mathrm{Re}}\,\Sigma^{SR}_{ab} \\ &\qquad + m_b\,\widetilde{\mathrm{Re}}\,\Sigma^{SL}_{ab} - m_a(\delta m)_{ab} - m_b(\delta m^\dagger)_{ab}\Big]_{p^2=m_b^2} \\ (\delta Z_L)_{ab} &= \frac{2}{m_a^2 - m_b^2}\Big[m_b^2\,\widetilde{\mathrm{Re}}\,\Sigma^{L}_{ab} + m_a m_b\,\widetilde{\mathrm{Re}}\,\Sigma^{R}_{ab} + m_a\,\widetilde{\mathrm{Re}}\,\Sigma^{SL}_{ab} \\ &\qquad + m_b\,\widetilde{\mathrm{Re}}\,\Sigma^{SR}_{ab} - m_a(\delta m^\dagger)_{ab} - m_b(\delta m)_{ab}\Big]_{p^2=m_b^2} \end{cases}.$$

Muon–Higgs coupling at one-loop

$\Longrightarrow$ remaining large loop effects e.g. from corrections to δv

$$\frac{\delta v}{v}\bigg|_{\text{VLL}} \sim \frac{1}{16\pi^2}\sum_i |\lambda_i|^2 \ln\left(\frac{M}{m_h}\right)$$

$+$ enhancement from mixing

$$y^{\text{eff}}_{\mu\mu}\bigg|_{\delta v} \simeq C^{\mu\mu}_{e\Phi} v^2 \times 6\,\tfrac{\delta v}{v}$$

Leading correction $(L \oplus E)$

$$\frac{y^{\text{eff}}_{\mu\mu}}{y^{\text{SM}}_{\mu}}\bigg|_{1\ell} \simeq \frac{\lambda^{\mu}_{E}\bar{\lambda}\lambda^{\mu}_{L} v^3}{16\pi^2 M^2 m_\mu}\left[\sum_i \lambda_i^2 - 12\bar{\lambda}\lambda\right]\ln\left(\frac{M}{m_h}\right)$$