Lepton flavour change in nuclei ECT* Workshop April 14-17 2025

Higgs-mediated lepton flavor violation

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ENERGY

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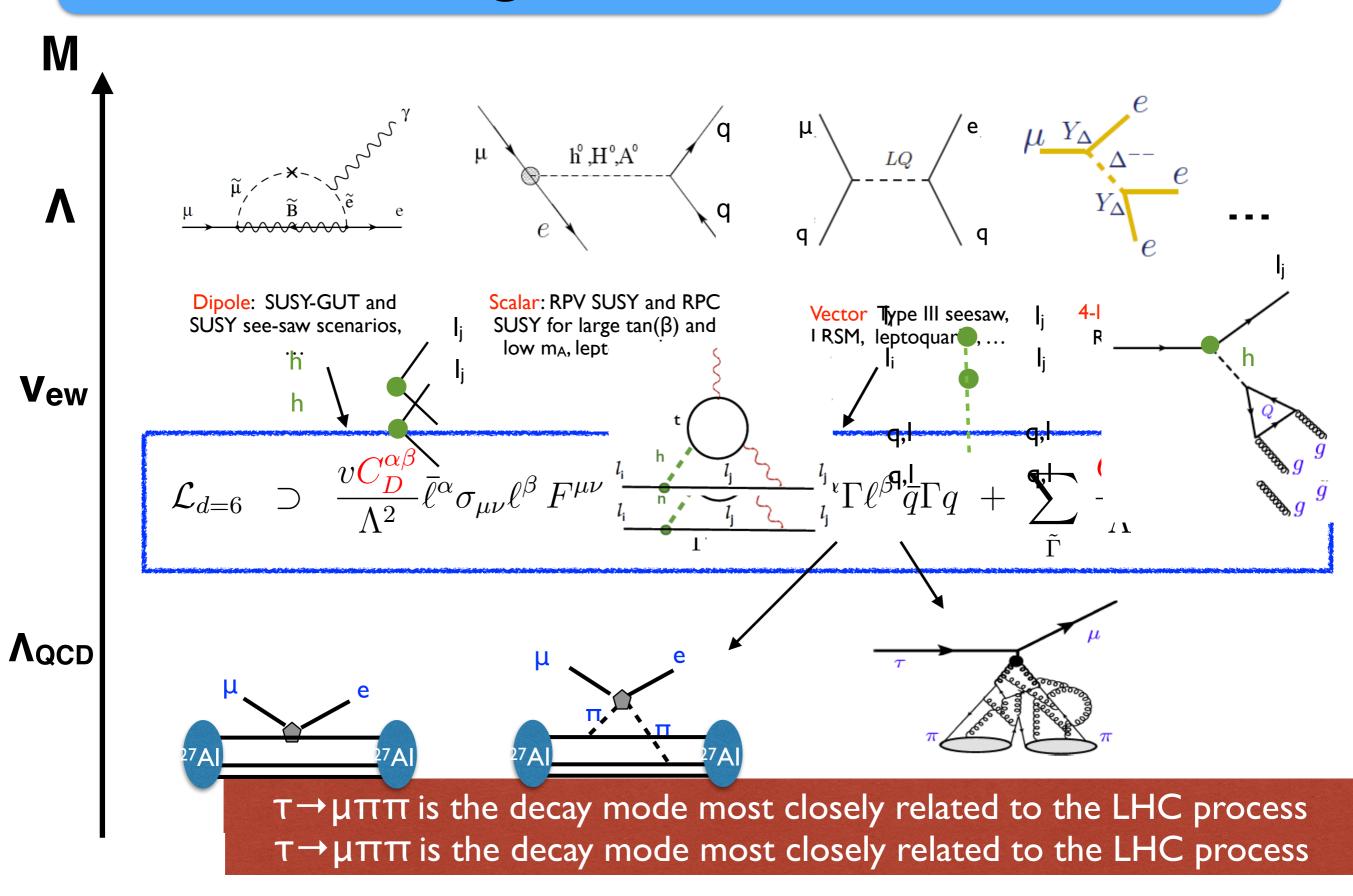
Outline

- Introduction: LFV and new physics
- Use Higgs-mediated LFV to illustrate:
 - Interplay of UV and nuclear physics in μ -to-e conversion
 - Model-diagnosing power of multiple probes

Vast literature — apologies in advance for incomplete referencing

	Connecting scales with EFT: LFV		
Μ			
٨	Motivated by neutrino mass, expect (hope?) that at some scale between Planck and weak scale, there exist new LFV and possibly LNV violating particles and interactions.		
	At low energy, they leave behind local operators		
Vew			
Λ _{QCD}			

Connecting scales with EFT: LFV



CLFV phenomenology

$$\mathcal{L}_{\rm LFV} \supset \frac{v C_D^{\alpha\beta}}{\Lambda^2} \bar{\ell}^{\alpha} \sigma_{\mu\nu} \ell^{\beta} + \sum_{\tilde{\Gamma}} \frac{C_{\tilde{\Gamma}}^{\alpha\beta}}{\Lambda^2} \bar{\ell}^{\alpha} \tilde{\Gamma} \ell^{\beta} \bar{\ell} \tilde{\Gamma} \ell + \sum_{\Gamma} \frac{C_{\Gamma}^{\alpha\beta}}{\Lambda^2} \bar{\ell}^{\alpha} \Gamma \ell^{\beta} \bar{q} \Gamma q + \frac{1}{F_{\alpha\beta}^{\Gamma}} \partial_{\mu} a \, \bar{\ell}^{\alpha} \Gamma^{\mu} \ell^{\beta}$$

→ multiple CLFV measurements needed to extract the underlying physics

• New physics mass scale through any process

 $BR_{\alpha \rightarrow \beta} \sim (v_{ew}/\Lambda)^4 * |(C_n)^{\alpha \beta}|^2$

μ-e sector:	Λ/√C ~ 10⁴-5 TeV	(Muon decays)
τ-μ(e) sector:	$\Lambda/\sqrt{C} \sim 10^2 \text{ TeV}$	(Tau decays)

CLFV phenomenology

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→ multiple CLFV measurements needed to extract the underlying physics

- New physics mass scale through any process
- Relative strength of operators ($[C_D]^{e\mu}vs [C_S]^{e\mu}...$) through $\mu \rightarrow 3e vs \mu \rightarrow e\gamma$ vs $\mu \rightarrow e$ conversion (and similarly for $\tau \rightarrow \mu, e$) \Rightarrow Mediators, mechanism
- Flavor structure of couplings ($[C_D]^{e\mu}$ vs $[C_D]^{\tau\mu}...$) through $\mu \rightarrow e vs \tau \rightarrow \mu$ vs $\tau \rightarrow e \Rightarrow$ Sources of flavor breaking

CLFV phenomenology

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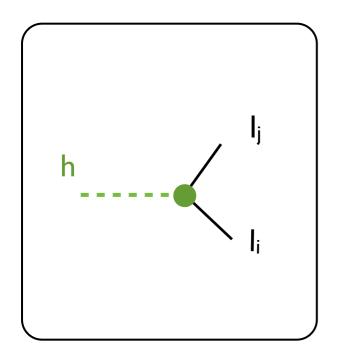
An example: Higgs-mediated LFV

• Simplest framework: LFV Yukawa couplings of the Higgs

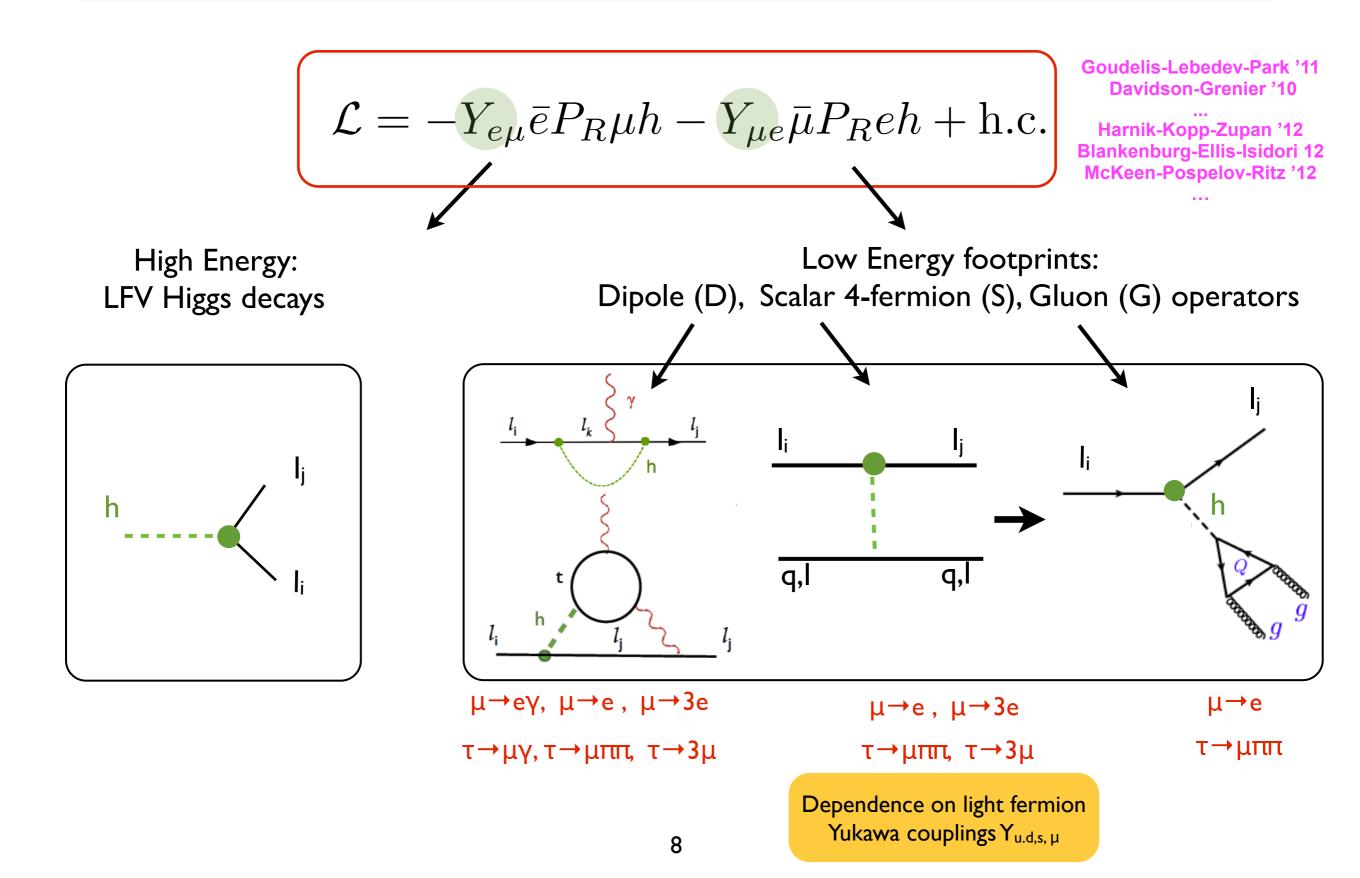
- Achieved in the SM-EFT through a single dim-6 operator that decouples lepton mass matrix from O(h) couplings
- Good starting point if new physics is heavy, arises in many UV models

$\mathcal{L} = -Y_{e\mu}\bar{e}P_R\mu h - Y_{\mu e}\bar{\mu}P_Reh + h.c.$ $\mathbf{L} = -Y_{e\mu}\bar{e}P_R\mu h - Y_{\mu e}\bar{\mu}P_Reh + h.c.$ High Energy:

LFV Higgs decays



LFV Higgs couplings

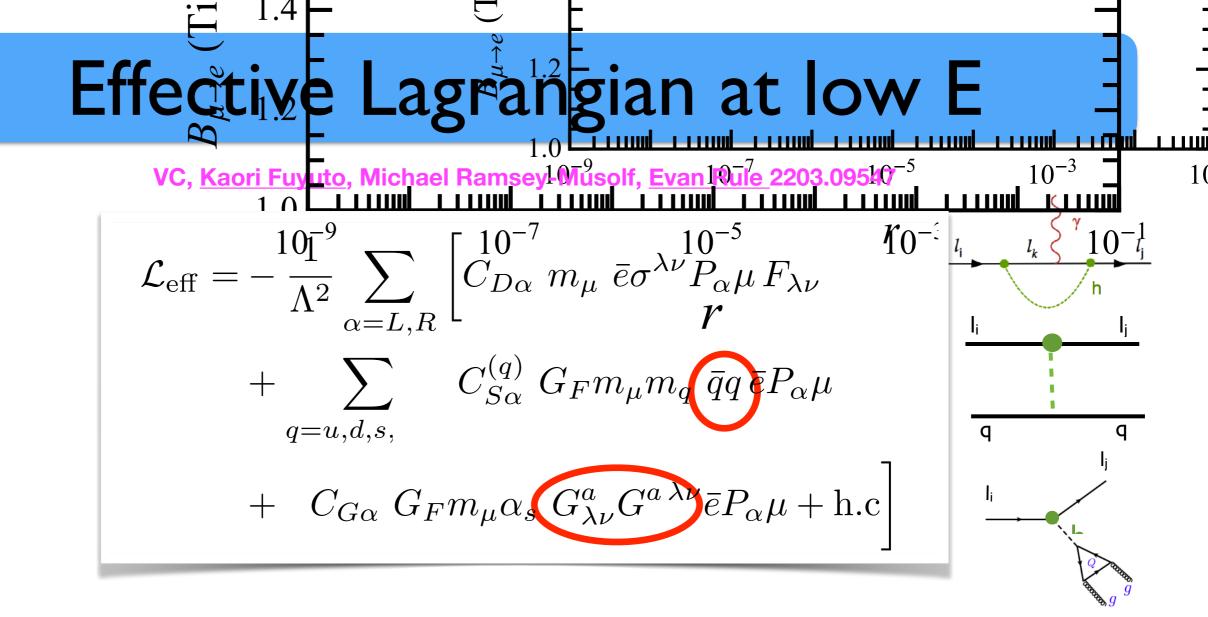


- Matching conditions for minimal Higgsmediated LFV: scalar and gluon
- For C_D, see refs. in previous slide

$$\frac{1}{\Lambda^2} G_F m_{\mu} v C_{SR}^{(q)} = -\frac{1}{m_h^2} Y_{e\mu},$$

$$\frac{1}{\Lambda^2} G_F m_{\mu} v C_{SL}^{(q)} = -\frac{1}{m_h^2} Y_{\mu e},$$

$$C_{G\alpha} = -\frac{1}{2} - \frac{1}{2} (12\pi) \sum_{Q=c,b,t} C_{S\alpha}^{(Q)}$$



- Matching conditions for minimal Higgsmediated LFV: scalar and gluon
- For C_D, see refs. in previous slide

 $\frac{1}{\Lambda^2} G_F m_{\mu} v C_{SR}^{(q)} = -\frac{1}{m_h^2} Y_{e\mu},$ $\frac{1}{\Lambda^2} G_F m_{\mu} v C_{SL}^{(q)} = -\frac{1}{m_h^2} Y_{\mu e},$ $C_{G\alpha} = -\frac{1}{M} (12\pi) \sum_{Q=c,b,t} C_{S\alpha}^{(Q)}$

Matching to ChPT / ChEFT (1)

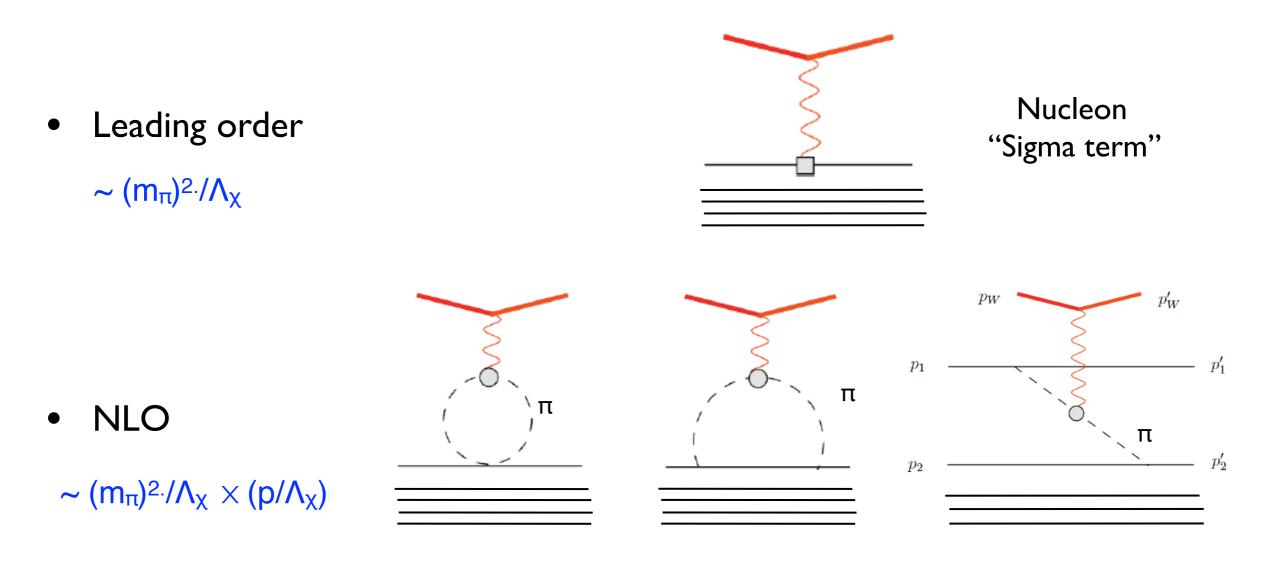
• Chiral power counting for the scalar density (expand in p/Λ_X) with $\Lambda_X \sim \text{GeV}$

 $p \sim p_N, m_{\pi}, q_{ext} (\sim m_{\mu})$

• Leading couplings controlled by

Matching to ChPT / ChEFT (1)

• Chiral power counting for the scalar density (expand in p/ Λ_X) with $\Lambda_X \sim \text{GeV}$



Scalar form factor of the nucleon

Two-nucleon operator (see Bira van Kolck's talk)

Matching to ChPT / ChEFT (2)

• One-nucleon scalar operators

S

Crivellin-Hoferichter-Procura 1404.7134 Korber-Nogga-deVries 1704.01150

u,d

$$\begin{cases} \langle N(\mathbf{k}') | \, C_{S\alpha}^{(u)} m_u \bar{u}u + C_{S\alpha}^{(d)} m_d \bar{d}d \, | N(\mathbf{k}) \rangle \to \ \bar{N}' \, J_{ud,\alpha}^{(1)}(\mathbf{q}) \, N \\ J_{ud,\alpha}^{(1)}(\mathbf{q}) = \left[\sigma_{\pi N} - \frac{3m_{\pi}^3 g_A^2}{64\pi f_{\pi}^2} \, F(\mathbf{q}^2/m_{\pi}^2) \right] C_{S\alpha}^{(0)} - \frac{\delta m_N}{4} \tau_3 \, C_{S\alpha}^{(1)} \\ \langle N(\mathbf{k}') | \, C_{S\alpha}^{(s)} m_s \bar{s}s \, | N(\mathbf{k}) \rangle \to \ \bar{N}' \, J_{s,\alpha}^{(1)}(\mathbf{q}) \, N \qquad J_{s,\alpha}^{(1)}(\mathbf{q}) = \left(\sigma_s - \dot{\sigma}_s \mathbf{q}^2 \right) \, C_{S\alpha}^{(s)} \end{cases}$$

Hadronic input

$$\sigma_{\pi N} = \frac{1}{2} \langle N | (m_u + m_d) (\bar{u}u + \bar{d}d) | N \rangle$$

$$\sigma_s = \langle N | m_s \bar{s}s | N \rangle$$

$$\epsilon = \frac{m_d - m_u}{m_d + m_u}$$

$$\delta m_N = (m_n - m_p)_{\text{strong}} .$$

Isoscalar / isovector combinations of Wilson Coefficients

$$C_{S\alpha}^{(0)} = \frac{C_{S\alpha}^{(u)}(1-\epsilon) + C_{S\alpha}^{(d)}(1+\epsilon)}{2}$$
$$C_{S\alpha}^{(1)} = C_{S\alpha}^{(u)}\left(1-\frac{1}{\epsilon}\right) + C_{S\alpha}^{(d)}\left(1+\frac{1}{\epsilon}\right)$$

Matching to ChPT / ChEFT (2)

One-nucleon scalar operators

S

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u, d

$$\frac{\langle N(\mathbf{k}') | C_{S\alpha}^{(u)} m_u \bar{u}u + C_{S\alpha}^{(d)} m_d \bar{d}d | N(\mathbf{k}) \rangle \rightarrow \bar{N}' J_{ud,\alpha}^{(1)}(\mathbf{q}) N}{J_{ud,\alpha}^{(1)}(\mathbf{q}) = \left[\sigma_{\pi N} - \frac{3m_{\pi}^3 g_A^2}{64\pi f_{\pi}^2} F(\mathbf{q}^2/m_{\pi}^2) \right] C_{S\alpha}^{(0)} - \frac{\delta m_N}{4} \tau_3 C_{S\alpha}^{(1)}} \sqrt{\langle N(\mathbf{k}') | C_{S\alpha}^{(s)} m_s \bar{s}s | N(\mathbf{k}) \rangle} \rightarrow \bar{N}' J_{\alpha}^{(1)}(\mathbf{q}) N \qquad J_{\alpha}^{(1)}(\mathbf{q}) = \left(\sigma_s - \dot{\sigma}_s \mathbf{q}^2 \right) C_{S\alpha}^{(s)}}$$

$$\langle N(\mathbf{k}') | C_{S\alpha}^{(s)} m_s \bar{s}s | N(\mathbf{k}) \rangle \rightarrow \bar{N}' J_{s,\alpha}^{(1)}(\mathbf{q}) N \qquad J_{s,\alpha}^{(1)}(\mathbf{q}) = \left(\sigma_s - \dot{\sigma}_s \mathbf{q}^2 \right) C_{S\alpha}^{(s)}$$

Hadronic input

$$\sigma_{\pi N} = \frac{1}{2} \langle N | (m_u + m_d) (\bar{u}u + \bar{d}d) | N \rangle$$

$$\sigma_s = \langle N | m_s \bar{s}s | N \rangle$$

$$\epsilon = \frac{m_d - m_u}{m_d + m_u}$$

$$\delta m_N = (m_n - m_p)_{\text{strong}} .$$

From lattice QCD & dispersion relations:

$$\sigma_{\pi N} = 59.1(3.5) \text{ MeV}$$
 (L
 $\sigma_s = 41(9) \text{MeV}$ (L
 $\dot{\sigma}_s = 0.3(2) \text{ GeV}^{-1}$ (L
 $\epsilon = 0.365(23)$ (L
 $\delta m_N = 2.32(17) \text{ MeV}$ (L

Matching to ChPT / ChEFT (3)

• Two-nucleon scalar operator

VC-Graesser-Ovanesyan 1205.2695 Korber-Nogga-deVries 1704.01150

$$\langle N(\mathbf{k}_{1}')N(\mathbf{k}_{2}')| C_{S\alpha}^{(u)}m_{u}\bar{u}u + C_{S\alpha}^{(d)}m_{d}\bar{d}d | N(\mathbf{k}_{1})N(\mathbf{k}_{2}) \rangle \rightarrow \bar{N}_{1}'\bar{N}_{2}' J_{ud,\alpha}^{(2)}(\mathbf{q}_{1},\mathbf{q}_{2}) N_{1}N_{2}$$

$$J_{ud,\alpha}^{(2)}(\mathbf{q}_{1},\mathbf{q}_{2}) = -\frac{g_{A}^{2}m_{\pi}^{2}}{4f_{\pi}^{2}} \frac{\sigma_{1} \cdot \mathbf{q}_{1} \sigma_{2} \cdot \mathbf{q}_{2}}{(\mathbf{q}_{1}^{2}+m_{\pi}^{2})(\mathbf{q}_{2}^{2}+m_{\pi}^{2})} \tau_{1} \cdot \tau_{2} C_{S\alpha}^{(0)}$$

• Crude estimate: reduce to a single-nucleon operator by averaging over the second nucleon in a Fermi-gas model

For ²⁷Al

$$\sigma_{\pi N} \to \sigma_{\pi N} - \frac{3g_A^2 m_\pi^2 k_F}{64\pi f_\pi^2} f_{eff}^{SI}$$
 ~10% negative shift

 $f_{eff}^{SI} = 0.43^{+0.03}_{-0.22}$
 $\sigma_{\pi N} \to \sigma_{\pi N} - \frac{3g_A^2 m_\pi^2 k_F}{64\pi f_\pi^2} f_{eff}^{SI}$
 ~10% negative shift

 Nuclear shell model implies smaller results (~1/2), but no way to estimate the uncertainty → need first-principles nuclear calculation

$$B_{\mu \to e} = \left(\frac{v}{\Lambda}\right)^4 \frac{1}{\kappa_{\text{capt}}} \left(\left|\tau^{(+1)}\right|^2 + \left|\tau^{(-1)}\right|^2\right)$$

Two electron angular momentum states

• Amplitude receives dipole and scalar / gluon contribution

$$\tau^{(-1)} = (C_{DL} + C_{DR}) \tau_D^{(-1)} + \tau_S^{(-1)}$$

 Dipole amplitude is controlled by nuclear charge distribution, which also determines muon and electron wave functions (key input is the proton density^{**})

$$\tau_D^{(-1)} = \frac{1}{m_\mu^{3/2}} \int dr \ r^2(-E(r)) \left(g_{-1}^{(e)} f_{-1}^{(\mu)} + f_{-1}^{(e)} g_{-1}^{(\mu)} \right)$$

** Modulo effect discussed by J. Dobaczewski

$$B_{\mu \to e} = \left(\frac{v}{\Lambda}\right)^4 \frac{1}{\kappa_{\text{capt}}} \left(\left|\tau^{(+1)}\right|^2 + \left|\tau^{(-1)}\right|^2\right)$$

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$$\tau^{(-1)} = (C_{DL} + C_{DR}) \tau_D^{(-1)} + \tau_S^{(-1)}$$

• Scalar amplitude: Wilson coeff. X hadronic input X overlap integrals

$$\tau_{S}^{(-1)} = \frac{1}{2} G_F m_{\mu}^2 \sum_{N=p,n} \left[\left(C_{NL}^{\rho} + C_{NR}^{\rho} \right) \tau_{\rho_N}^{(-1)} + \left(C_{NL}^f + C_{NR}^f \right) \tau_{f_N}^{(-1)} \right]$$

ρ_N, f_N : nucleon densities and their derivatives

$$\tau_{\rho_N(f_N)}^{(-1)} = \frac{1}{m_{\mu}^{5/2}} \int dr \ r^2 \left(g_{-1}^{(e)} g_{-1}^{(\mu)} - f_{-1}^{(e)} f_{-1}^{(\mu)} \right) \rho_N(f_N),$$

$$B_{\mu \to e} = \left(\frac{v}{\Lambda}\right)^4 \frac{1}{\kappa_{\text{capt}}} \left(\left|\tau^{(+1)}\right|^2 + \left|\tau^{(-1)}\right|^2\right)$$

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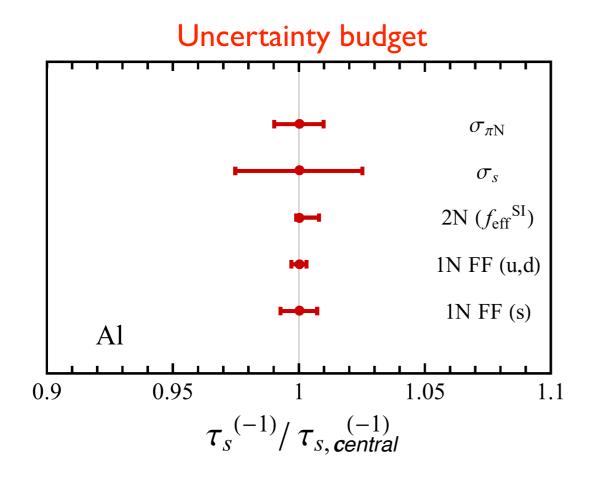
- Scalar amplitude: Wilson coeff. X hadronic input X overlap integrals
- Size of NLO corrections to amplitudes induced by light quarks:
 - Roughly -5% from momentum-dep. in the nucleon form factor
 - Roughly -10% from two-body operator

$$B_{\mu \to e} = \left(\frac{v}{\Lambda}\right)^4 \frac{1}{\kappa_{\text{capt}}} \left(\left|\tau^{(+1)}\right|^2 + \left|\tau^{(-1)}\right|^2\right)$$

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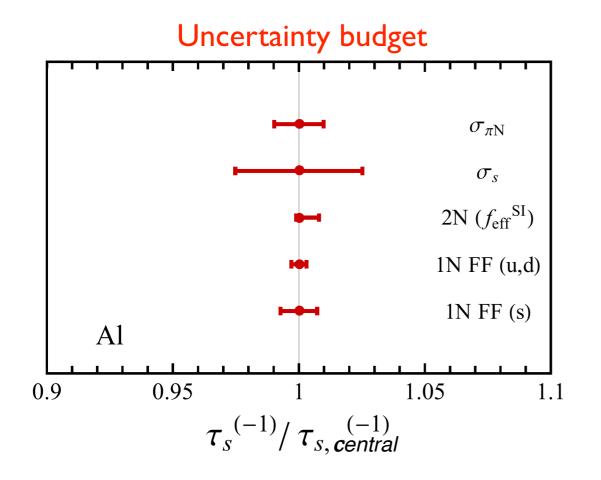
- Largest hadronic uncertainty arises at LO (from sigma terms)
- LO uncertainty from neutron density not a big problem when info from pionic atoms is available (not for ⁴⁴Ti)
- The quite uncertain NLO NN term does note have huge impact for this choice of short-distance physics

$$B_{\mu \to e} = \left(\frac{v}{\Lambda}\right)^4 \frac{1}{\kappa_{\text{capt}}} \left(\left|\tau^{(+1)}\right|^2 + \left|\tau^{(-1)}\right|^2\right)$$

Two electron angular momentum states

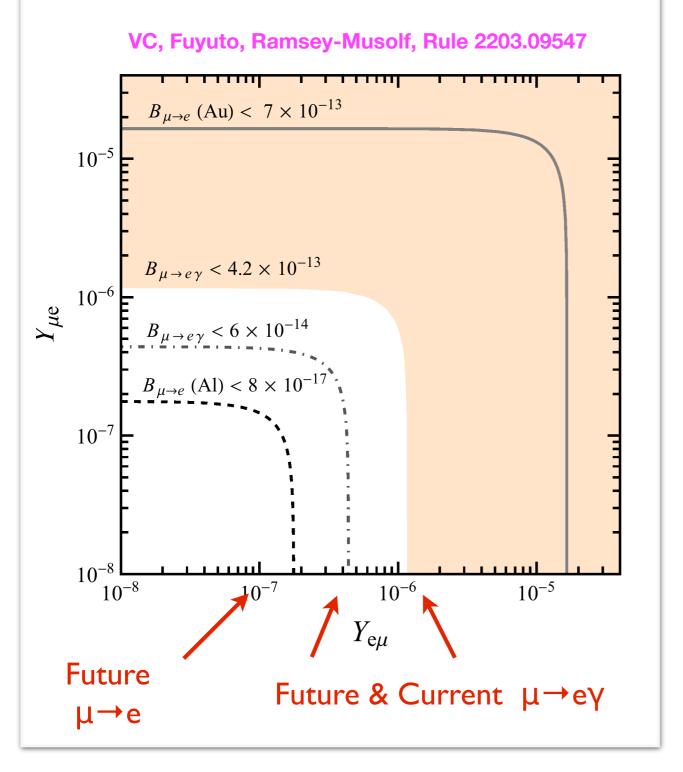
• Amplitude receives dipole and scalar / gluon contribution

$$\tau^{(-1)} = (C_{DL} + C_{DR}) \tau_D^{(-1)} + \tau_S^{(-1)}$$



- Impact of NLO corrections on rate depends on short-distance physics.
- Largest (-20%) when light-quarks dominate
- Typically similar or larger to LO error
 ⇒ phenomenologically relevant

Pattern of LFV μ decays



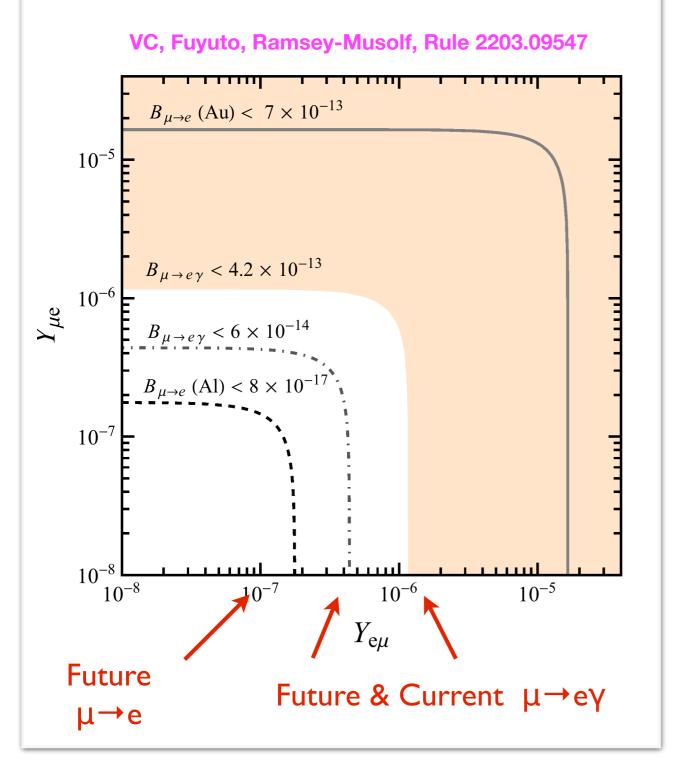
* Diagonal couplings set to SM value

- $\mu \rightarrow e\gamma$ is currently probing $|Y_{\mu e}| \sim 10^{-6}$, which corresponds to BR($h \rightarrow \mu e$) < 10⁻⁹
- Upcoming µ→e conversion experiments will probe |Y_{µe} |~ 10⁻⁷
- Correlated signals in µ→e transitions provide opportunity to test hypothesis of Higgs-mediated LFV

BR(µ→e,AI) / BR(µ→eγ) = 8.7(3) 10⁻³ ** BR(µ→e,Ti) / BR(µ→e,AI) = 1.5(1)

VC, Fuyuto, Ramsey-Musolf, Rule 2203.09547 (See also Crivellin et al. 1404.7134)

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BR(µ→e,Al) / BR(µ→eγ) = 8.7(3) 10⁻³ ** BR(µ→e,Ti) / BR(µ→e,Al) = 1.5(1)

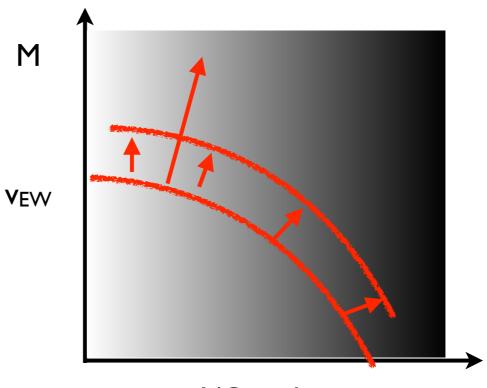
VC, Fuyuto, Ramsey-Musolf, Rule 2203.09547 (See also Crivellin et al. 1404.7134)

• Further scrutiny of uncertainties is desirable

Conclusions & Outlook

- Charged LFV processes probe a broad spectrum of new physics
 - Discovery tools: clean, very high scale reach
 - Model-diagnosing tools: mediators, sources of flavor breaking

- Higgs-mediated LFV
 - Illustrated the interplay of UV physics with hadronic / nuclear aspects of µ-to-e conversion
 - If a discovery is made, µ-to-e processes can be used to test the hypothesis of Higgs-mediated LFV



I/Coupling

Conclusions & Outlook

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 - Discovery tools: clean, very high scale reach
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Exciting experimental prospects

- * 4 (I-2) orders of magnitude improvement in μ (τ) decays
- ★ LHC & EIC will be competitive in τ - μ and τ -e transitions (h → $\tau\mu$, e→ τ)
- **\star** Muon processes have unmatched sensitivity in probing μ -e transitions



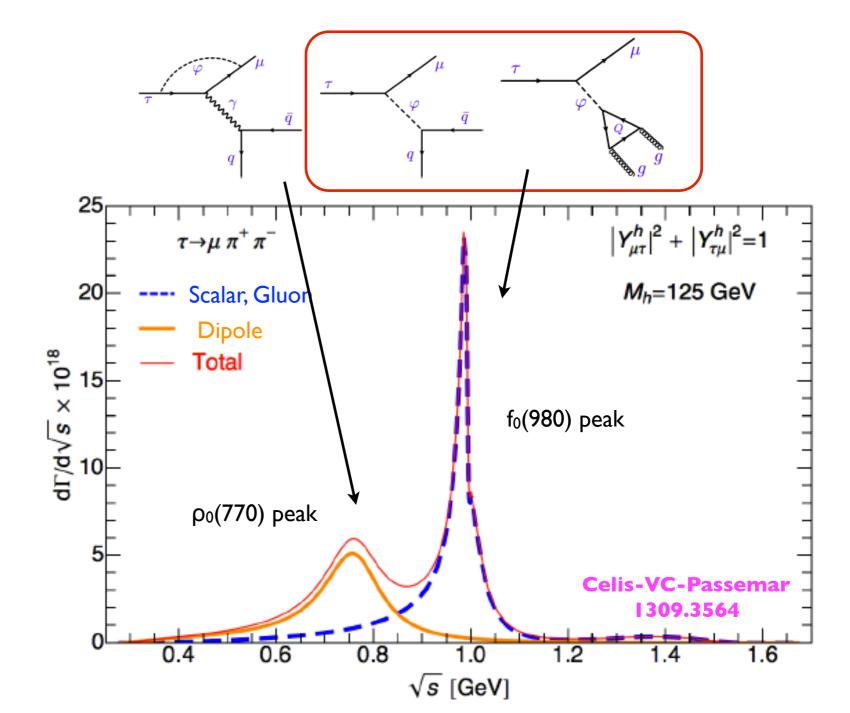
Pattern of LFV τ decays

Radiative mode dominates, followed
 by ππ and 3 lepton

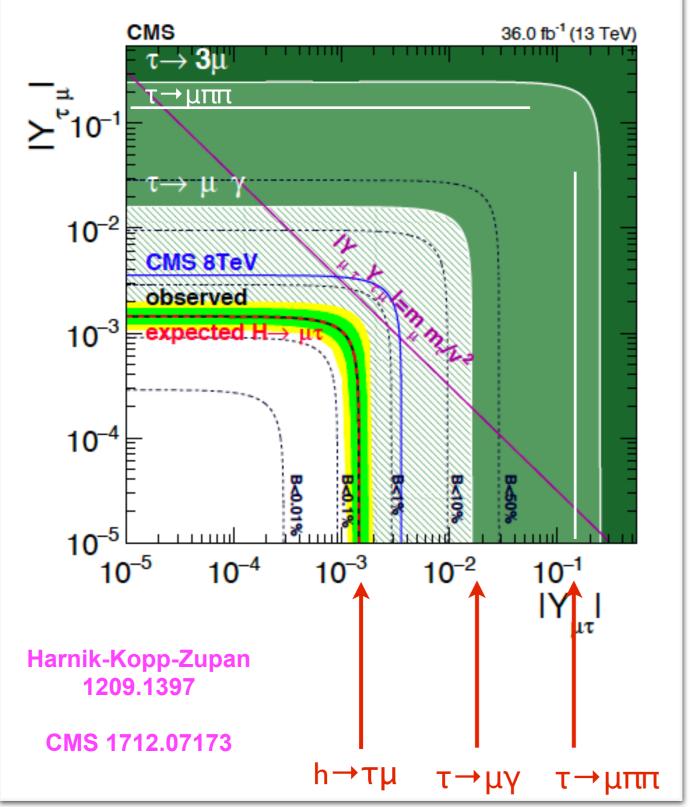
$B(\tau \rightarrow \mu \pi^+ \pi^-) / B(\tau \rightarrow \mu \gamma) = 0.7(1) \times 10^{-2}$

 τ→μππ controlled by Higgs-specific combination of D, S, G → unique signature in ππ spectrum

Plot assumes SM values for $Y_{u.d,s}$, but strength of the f₀(980) peak depends on light quark Yukawas

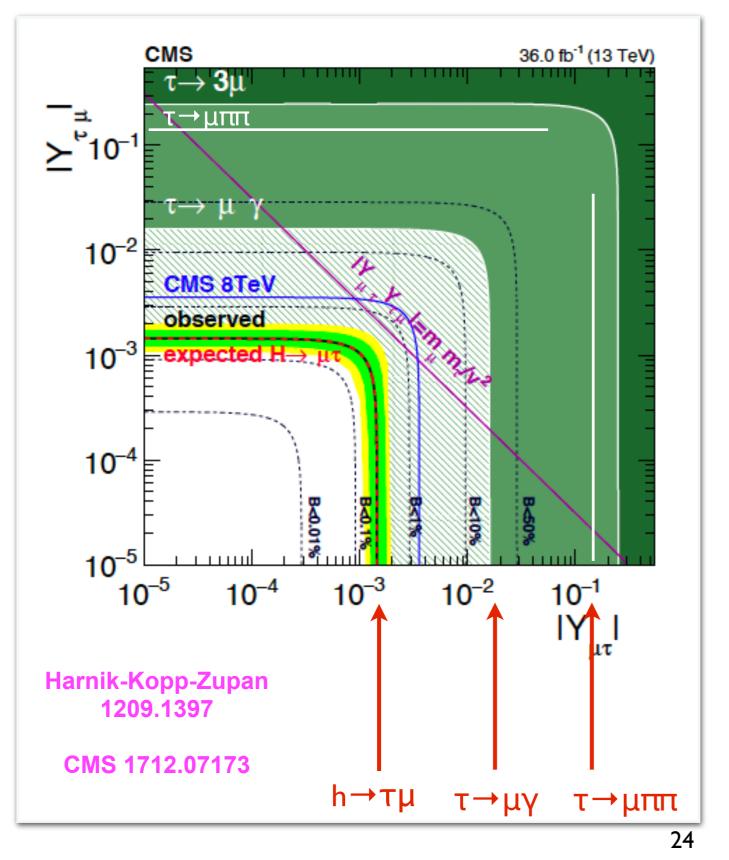


τ - μ sector: h vs τ decays



- Assuming SM values for $Y_{u.d,s}$, current tau BRs (~10⁻⁽⁷⁻⁸⁾) imply $Y_{\tau\mu,\tau e} < 0.01-0.1$, which translates into BR(h $\rightarrow\mu\tau$) < 0.1
- LHC (CMS) limit BR(h→μτ)
 <0.25% (95%CL) is stronger:
 |Υ_{τμ,μτ}| < 0.0011

τ - μ sector: h vs τ decays

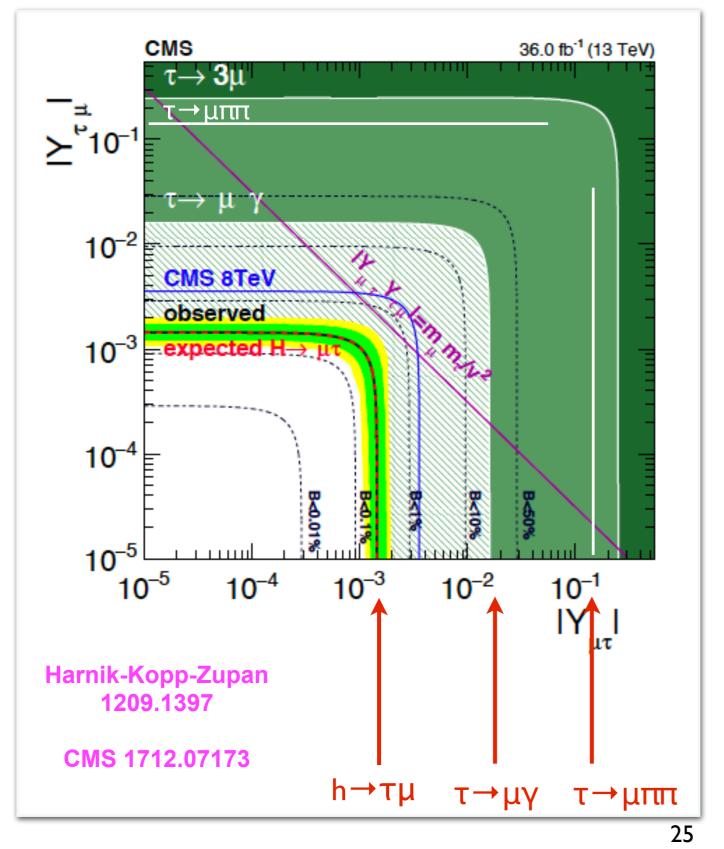


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- LHC (CMS) limit BR(h→μτ)
 <0.25% (95%CL) is stronger:
 |Υ_{τμ,μτ}| < 0.0011
- If use SM values for Y_{u.d,s}, CMS bound implies

$$\begin{split} B(\tau \!\rightarrow\! \mu \gamma) &< 4 \times 10^{-10} \\ B(\tau \!\rightarrow\! \mu \pi^{+} \pi^{-}) &< 2.7 \times 10^{-12} \\ B(\tau \!\rightarrow\! \mu \pi^{0} \pi^{0}) &< 0.8 \times 10^{-12} \end{split}$$

Challenging target for next generation

τ - μ sector: h vs τ decays



- Assuming SM values for $Y_{u.d,s}$, current tau BRs (~10⁻⁽⁷⁻⁸⁾) imply $Y_{\tau\mu,\tau e} < 0.01-0.1$, which translates into BR(h $\rightarrow\mu\tau$) < 0.1
- LHC (CMS) limit BR(h→μτ)
 <0.25% (95%CL) is stronger:
 |Υ_{τμ,μτ}| < 0.00143

If use Y_{u.d,s} ~ Y_b,
 CMS bound implies

 $B(\tau \rightarrow \mu \gamma) < 4.0 \times 10^{-10}$ $B(\tau \rightarrow \mu \pi^{+} \pi^{-}) < 5.5 \times 10^{-9}$ $B(\tau \rightarrow \mu \pi^{0} \pi^{0}) < 2.7 \times 10^{-9}$

Within reach of next generation

Matching for gluon operator

• Similar to scalar density, one- and two-body terms

$$\langle N(\mathbf{k}') | C_{G\alpha} \alpha_s G^a_{\lambda\nu} G^{a\,\lambda\nu} | N(\mathbf{k}) \rangle \rightarrow \bar{N}' J^{(1)}_{G,\alpha}(\mathbf{q}) N$$
$$\langle N(\mathbf{k}'_1) N(\mathbf{k}'_2) | C_{G\alpha} \alpha_s G^a_{\lambda\nu} G^{a\,\lambda\nu} | N(\mathbf{k}_1) N(\mathbf{k}_2) \rangle \rightarrow \bar{N}'_1 \bar{N}'_2 J^{(2)}_{G,\alpha}(\mathbf{q}_1, \mathbf{q}_2) N_1 N_2$$

$$J_{G,\alpha}^{(1)}(\mathbf{q}) = -\frac{8\pi}{9} C_{G\alpha} \left(m_N - \left[\sigma_{\pi N} - \frac{3m_\pi^3 g_A^2}{64\pi f_\pi^2} F(\mathbf{q}^2/m_\pi^2) \right] + \frac{\delta m_N}{2} \tau_3 - \left(\sigma_s - \dot{\sigma}_s \mathbf{q}^2 \right) \right)$$

$$J_{G,\alpha}^{(2)}(\mathbf{q}_1, \mathbf{q}_2) = -\frac{8\pi}{9} C_{G\alpha} \frac{g_A^2 m_\pi^2}{4f_\pi^2} \frac{\sigma_1 \cdot \mathbf{q}_1 \, \sigma_2 \cdot \mathbf{q}_2}{(\mathbf{q}_1^2 + m_\pi^2)(\mathbf{q}_2^2 + m_\pi^2)} \tau_1 \cdot \tau_2 \,.$$