Lepton flavour change in nuclei
ECT* Workshop
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Introduction to Chiral Perturbation Theory

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Outline

- Prologue:
    - Connecting scales using EFTs: from BSM to nuclear physics

- EFTs for low-energy QCD and nuclear physics
    - Chiral perturbation theory for mesons and baryons (π, N)
    - Nuclear EFT (Lecture by B. van Kolck)

- (My talk: Higgs-induced Lepton Flavor Violation)

A note on references

- References to the original papers will be sloppy (e.g. "Weinberg '79") and may be incomplete: apologies!

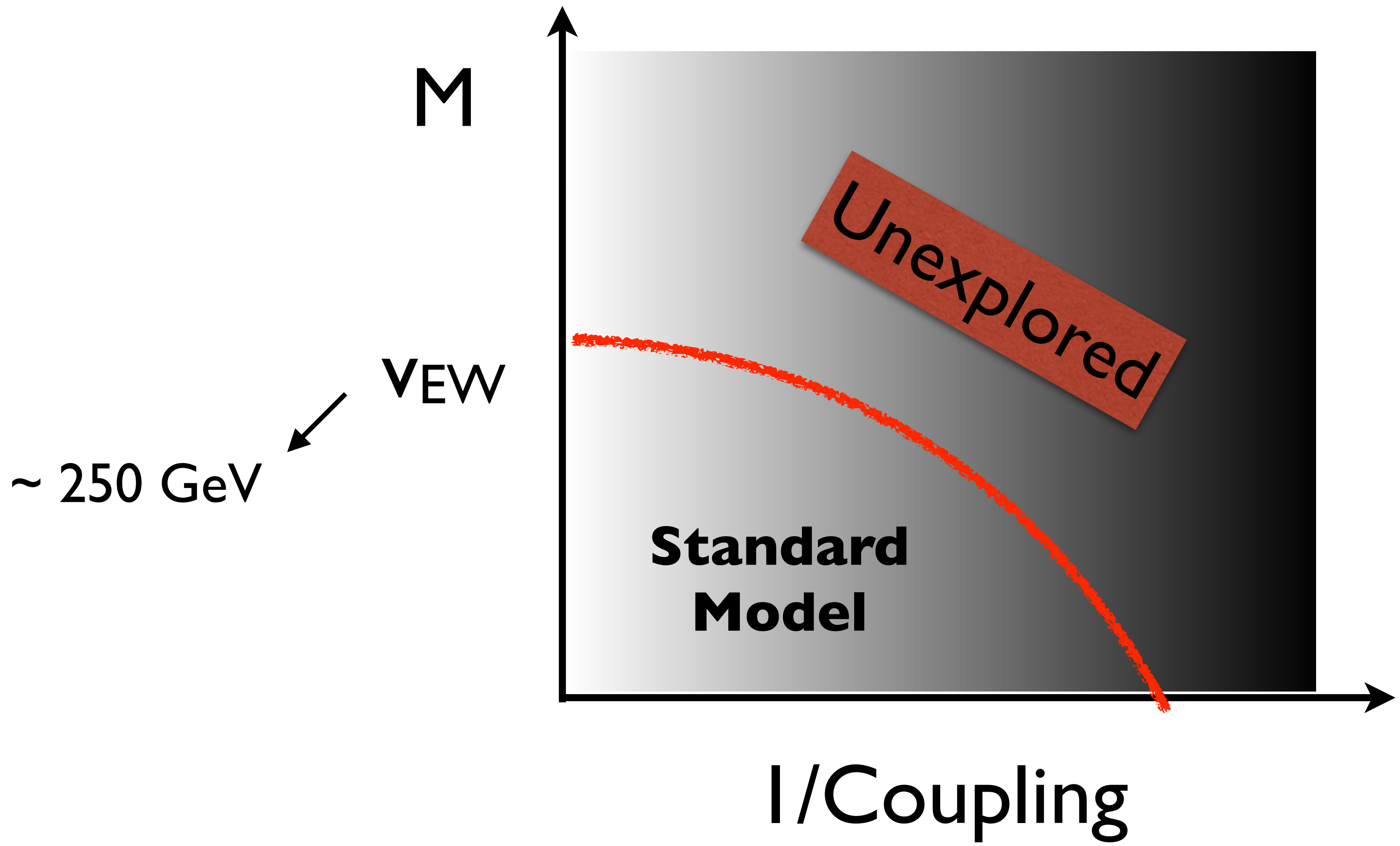
- Here is a selected list of lecture notes on EFT and ChPT:

    - V. Bernard, N. Kaiser, U. Meißner, hep-ph/9501384
    - A. Manohar, hep-ph/9606222
    - G. Ecker, hep-ph/9805500
    - A. Pich, 1804.05664

Connecting scales through EFT: from BSM to nuclear physics

Probing new physics at low energy

- New physics likely needed to address shortcomings of the Standard Model

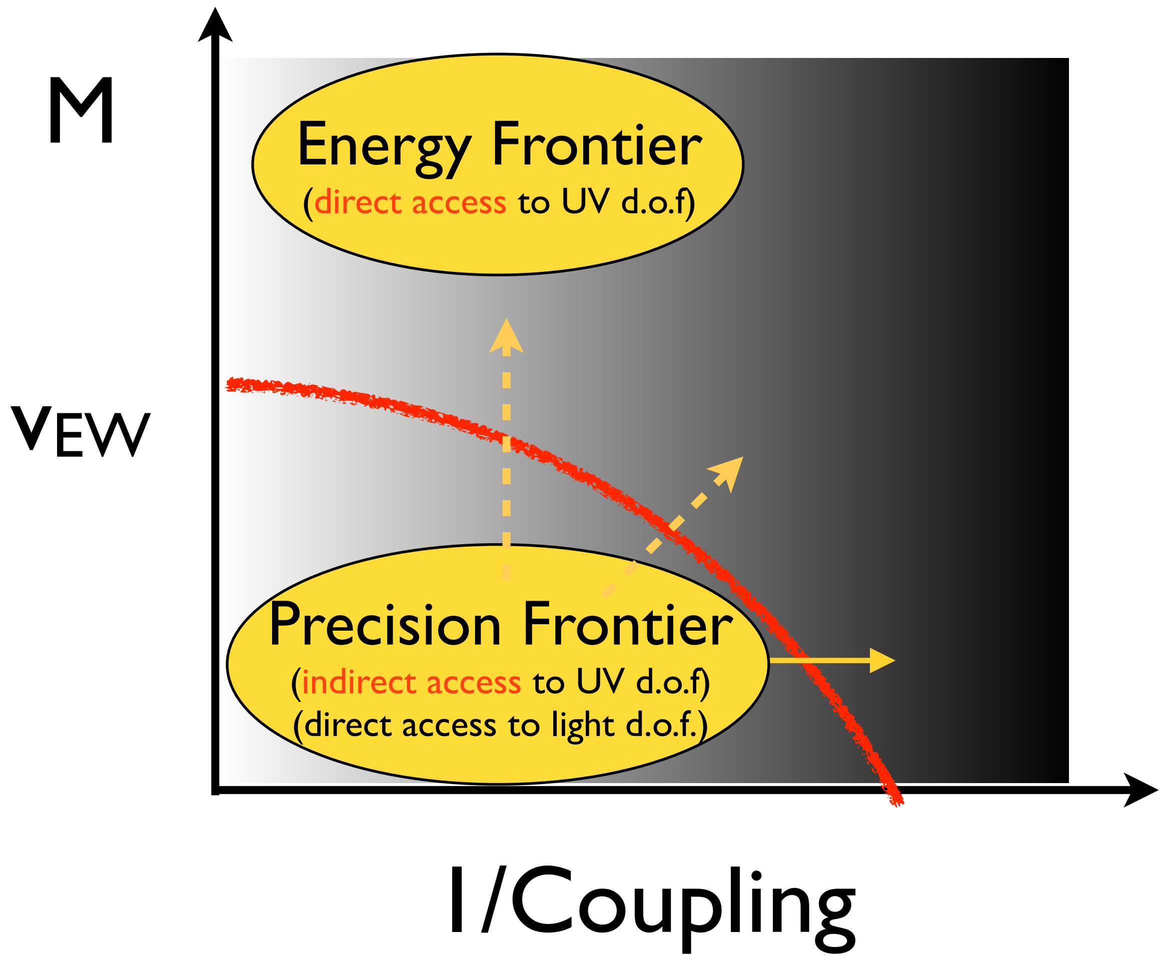


Probing new physics at low energy

- New physics likely needed to address shortcomings of the Standard Model

$$E = mc^2$$

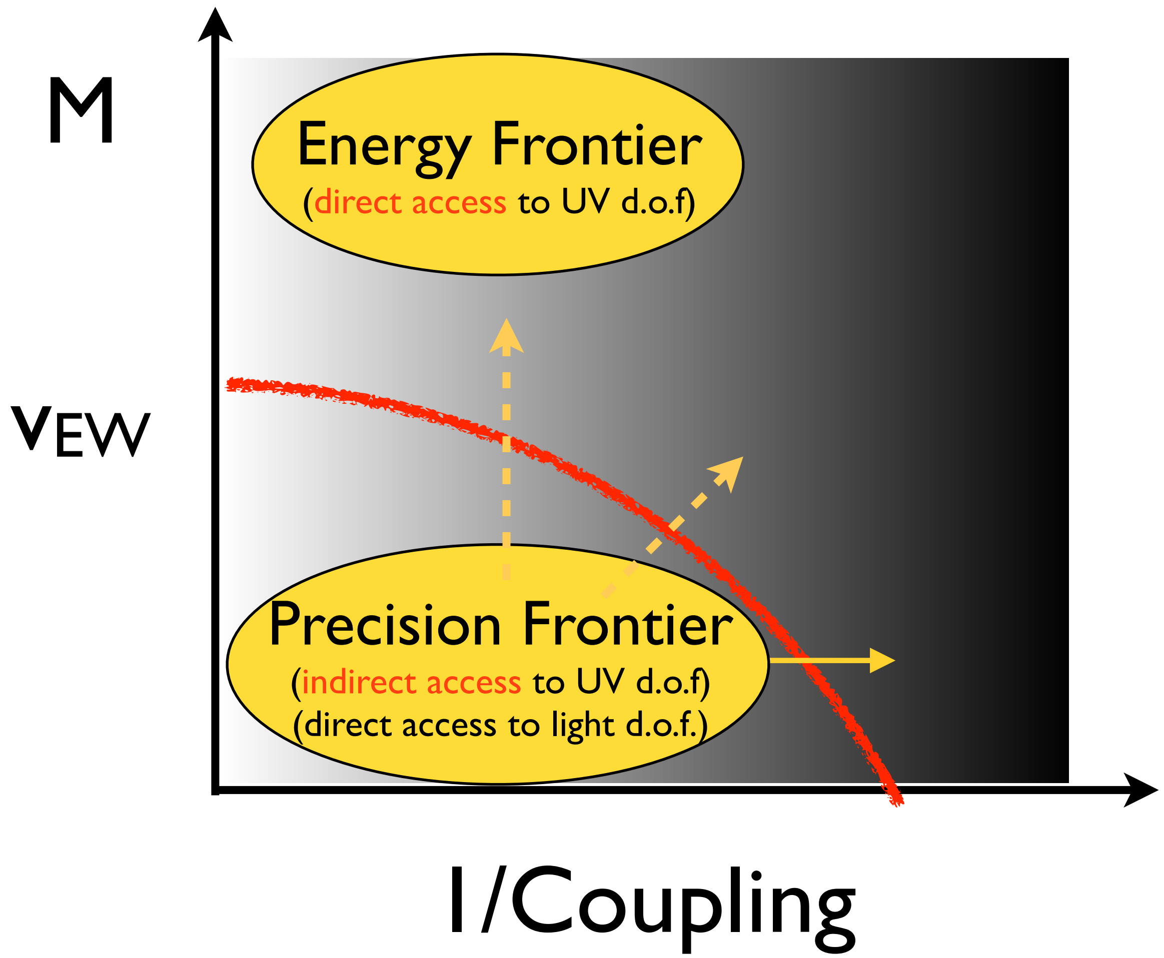
$$\Delta E \, \Delta t \sim h$$



We typically look for new physics with two complementary approaches

Probing new physics at low energy

- New physics likely needed to address shortcomings of the Standard Model



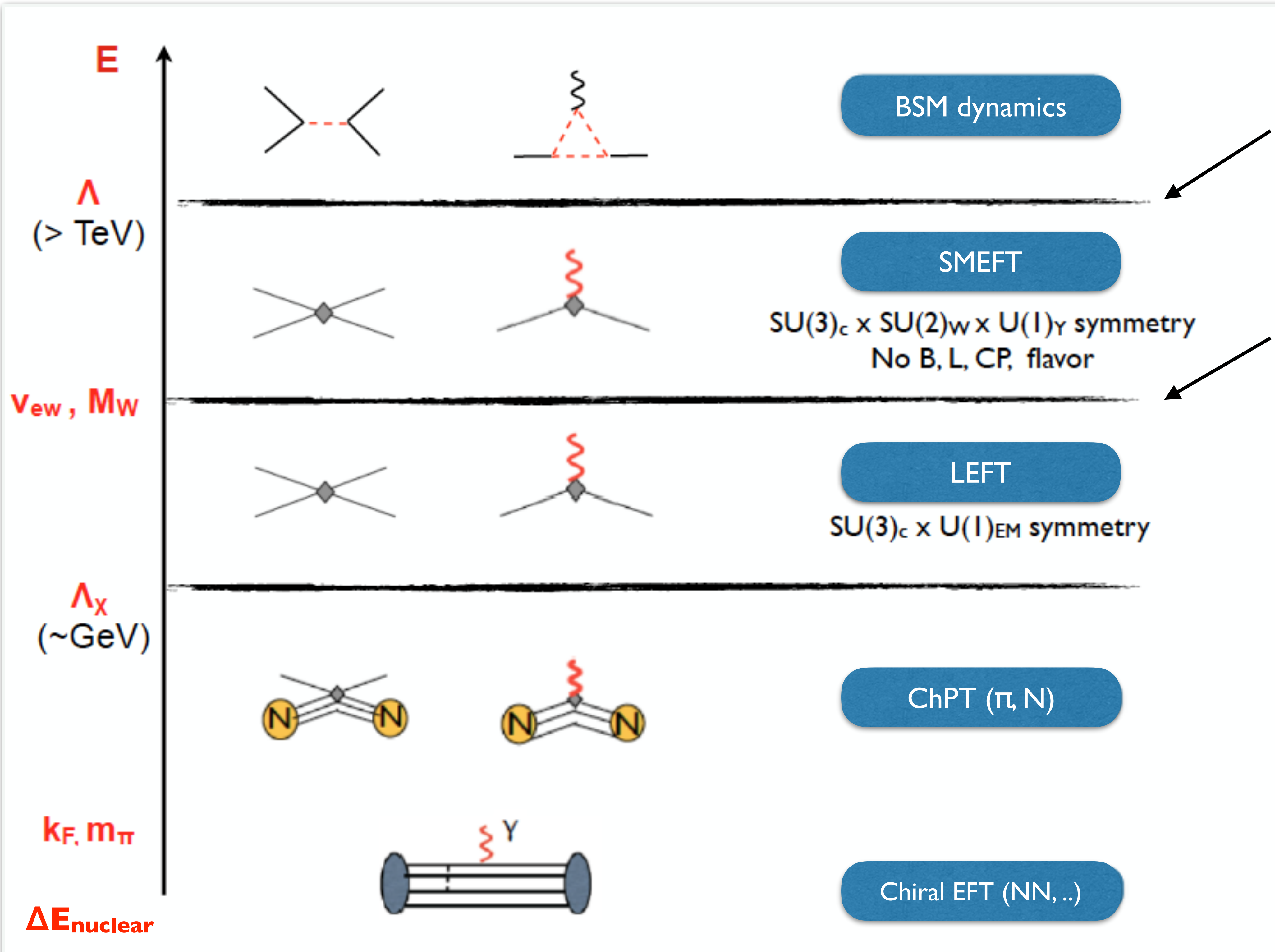
- EWSB mechanism
- Higgs properties
- Direct access to heavy particles
- ...

- L and B non conservation
- CP violation (w/o flavor)
- Flavor violation: quarks, leptons
- Multi-TeV scale force mediators
- Neutrino properties
- Dark sectors
- . . .

Hadronic and nuclear probes
play a prominent role at the Precision Frontier
They involve multi-scale problems!

Connecting scales

To connect UV physics to nuclei, use multiple EFTs

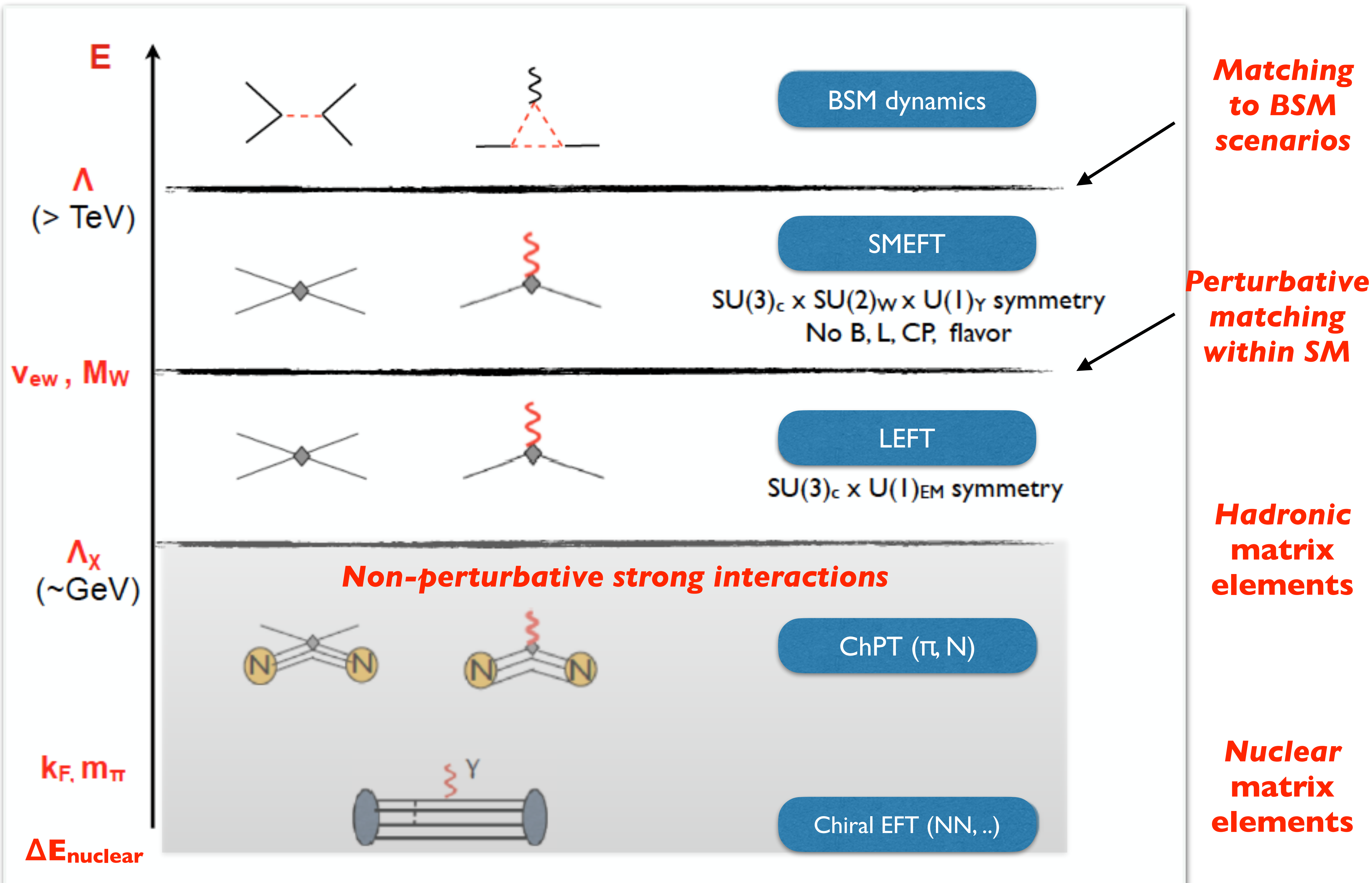


Matching to BSM scenarios

Perturbative matching within SM

Connecting scales

To connect UV physics to nuclei, use multiple EFTs



EFTs for low-energy QCD and nuclear physics

Coming up

- Chiral symmetry and its breaking
- Chiral Perturbation Theory (ChPT) for Goldstone modes (π)
- Heavy Baryon ChPT (N=n,p)
- Electroweak and BSM effects at low energy

Chiral symmetry

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + i\bar{q}_L \gamma^\mu D_\mu q_L + i\bar{q}_R \gamma^\mu D_\mu q_R - \bar{q}_L m_q q_R - \bar{q}_R m_q^\dagger q_L$$

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \qquad q_{L,R} = \left(\frac{1 \mp \gamma_5}{2}\right) q \qquad m_q = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

Chiral symmetry

$$\boxed{\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a} + i\bar{q}_L \gamma^\mu D_\mu q_L + i\bar{q}_R \gamma^\mu D_\mu q_R - \bar{q}_L m_q q_R - \bar{q}_R m_q^\dagger q_L}$$

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- For $m_q = 0$, invariant under independent U(3) transformations of left- and right-handed quarks:

$L,R \in SU(3)$

$$\boxed{\begin{array}{ccc} q_L & \rightarrow & L\, q_L \\ q_R & \rightarrow & R\, q_R \end{array}}$$

$$\boxed{\underbrace{SU(3)_L \times SU(3)_R}_{\text{Chiral group G}} \times [U(1)_V \times U(1)_A]}$$

Chiral group G

Baryon number

Anomalous: not a symmetry

Chiral symmetry

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- Symmetry is broken explicitly by $m_q \neq 0$ (though $m_q/\Lambda_{QCD} \ll 1$)

$$\partial_\mu \left(\bar{q}\gamma^\mu T^a q\right) = \bar{q}\,[T^a, m_q]\, q \qquad \partial_\mu \left(\bar{q}\gamma^\mu \gamma^5 T^a q\right) = \bar{q}\, \{T^a, m_q\}\, i\gamma_5\, q$$

Chiral symmetry

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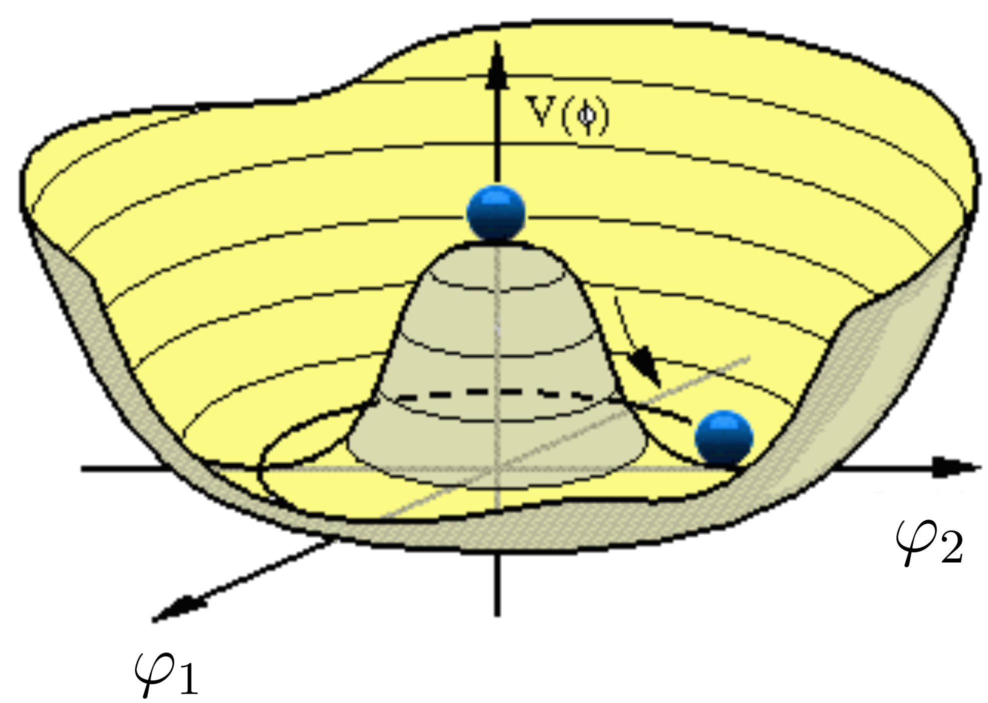
- Symmetry is broken explicitly by $m_q \neq 0$ (though $m_q/\Lambda_{QCD} \ll 1$)

- Evidence of $SU(3)_V$ in hadron spectrum (but no parity doublets): what's going on?

Spontaneous Symmetry Breaking

- Action is invariant under symmetry group, but ground state is not
- For $V \to \infty$: degenerate, physically (but not unitarily) equivalent ground states
- Continuous symmetry: manifold of equivalent classical minima ($\to$ vacua)
- Excitations along the valley of minima $\to$ massless states in the spectrum (Goldstone Bosons)



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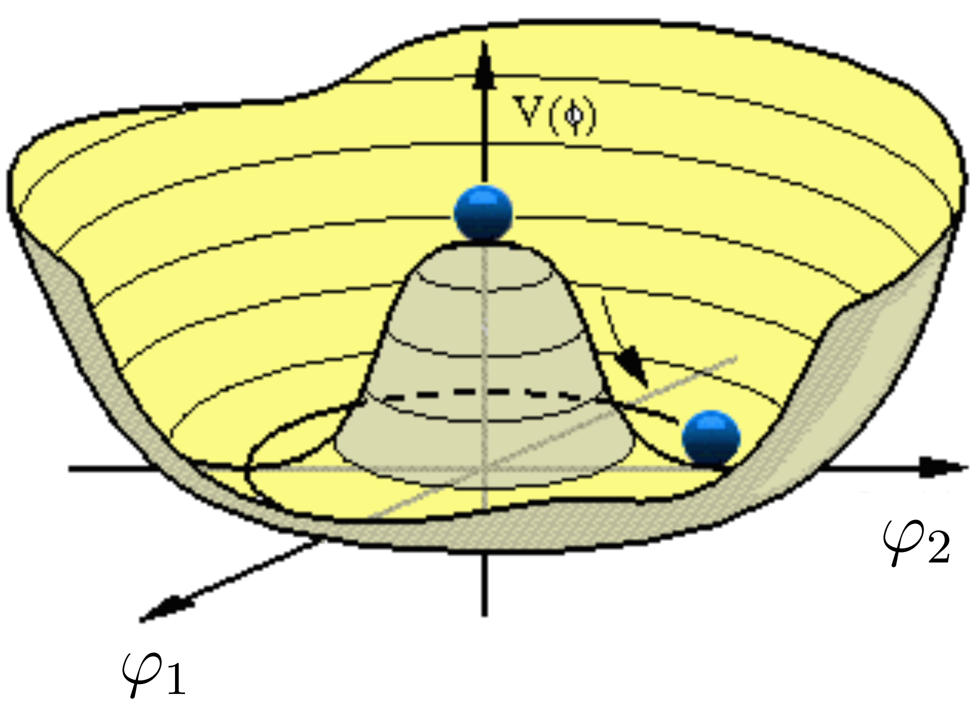
$$V(\varphi_1,\varphi_2) = \lambda(\vec{\varphi}\cdot\vec{\varphi} - v^2)^2 \qquad \phi(x) = \frac{\varphi_1(x) + i\varphi_2(x)}{\sqrt{2}}$$

Symmetry transformation

$$\boxed{\phi \to e^{i\epsilon}\phi}$$

SSB pattern

$$\boxed{O(2) \sim U(1) \to 1}$$



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$$\phi(x) = \frac{\varphi_1(x) + i\varphi_2(x)}{\sqrt{2}} = e^{i\pi(x)}\,(v + \rho(x))$$

Symmetry transformation

$$\boxed{\begin{aligned} &\phi \to e^{i\epsilon}\phi \\ &\pi(x) \to \pi(x) + \epsilon \end{aligned}}$$

SSB pattern

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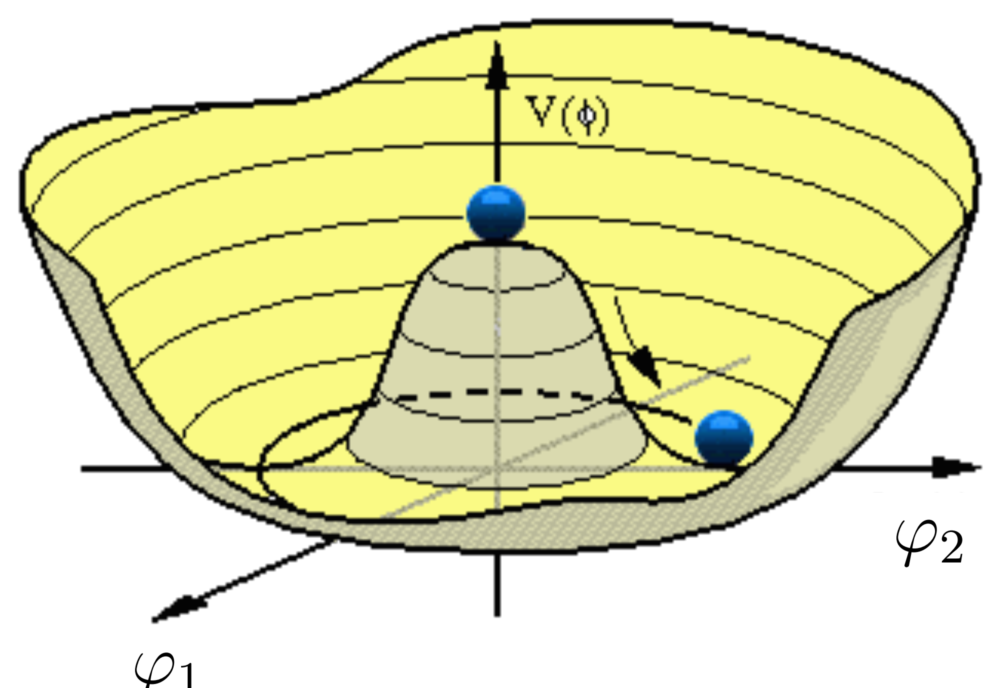

Identify GB through field redefinition — x-dependent parameters of a (broken) symmetry transformation acting on reference vacuum

A more realistic example

Callan, Coleman, Wess, Zumino '69

- SSB pattern G→H: GB fields ~ coordinates of the vacuum manifold G/H

$\phi = (\phi_1, \ldots, \phi_N)$

- Example: O(N) linear sigma model

$$\mathcal{L} = \frac{1}{2}\partial_\mu \boldsymbol{\phi} \cdot \partial^\mu \boldsymbol{\phi} - \lambda(\boldsymbol{\phi}\cdot\boldsymbol{\phi} - v^2)^2$$

Vacuum manifold $\phi_1^2 + \phi_2^2 + \ldots + \phi_N^2 = v^2$

N=3: G=O(3), H=O(2), G/H = S^2

Reference vacuum

$\phi_0 = (0,0,v)$

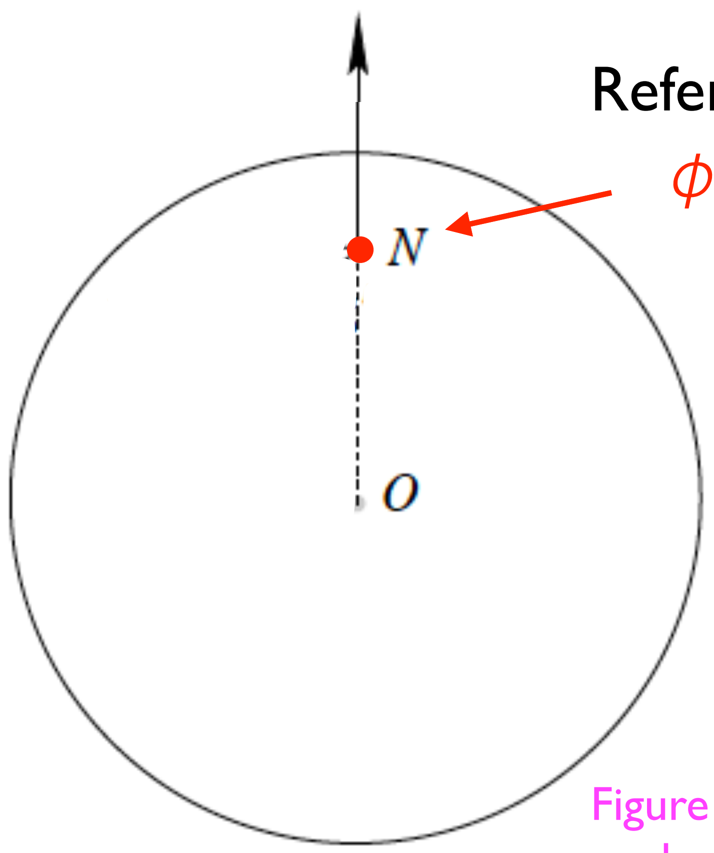

Figure from A. Manohar hep-ph/9606222

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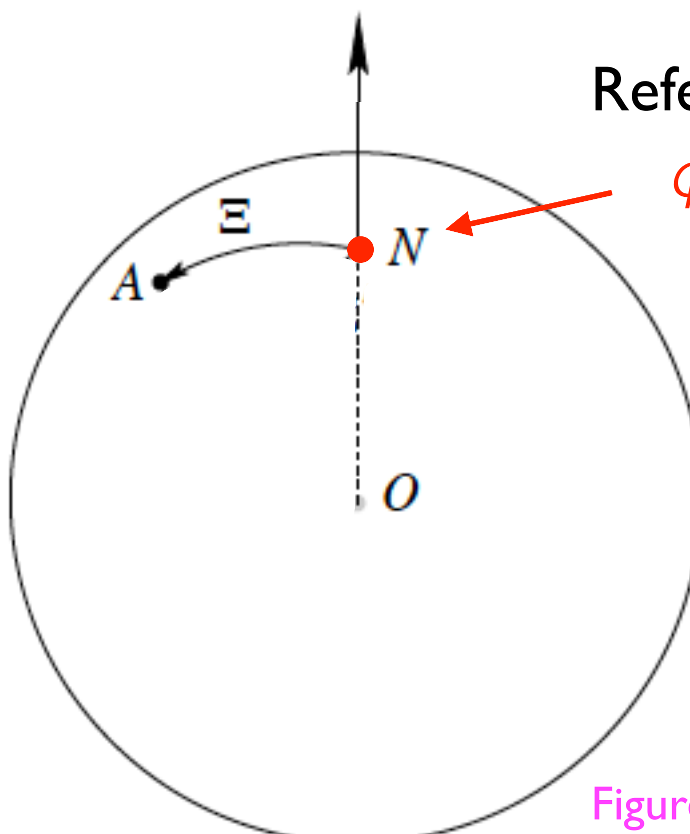
N=3: G=O(3), H=O(2), G/H = S^2

$$\phi_{\rm vac}(x) = \Xi(x)\, \phi_0$$

$$\Xi(x) = e^{i\pi^a(x) A^a} \xrightarrow{N=3} e^{i(\pi^1(x) J^1 + \pi^2(x) J^2)}$$

Goldstone fields broken generators



Reference vacuum

$\phi_0 = (0,0,v)$

Figure from A. Manohar
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Goldstone fields ($\pi^a(x)$)   broken generators (A^a)

$$\boxed{g\, \Xi(x) = \Xi'(x)\, h(g, \Xi(x))}$$

$h \in H$

$g \in G$

Non-linear representation of the group G, linear when restricted to H

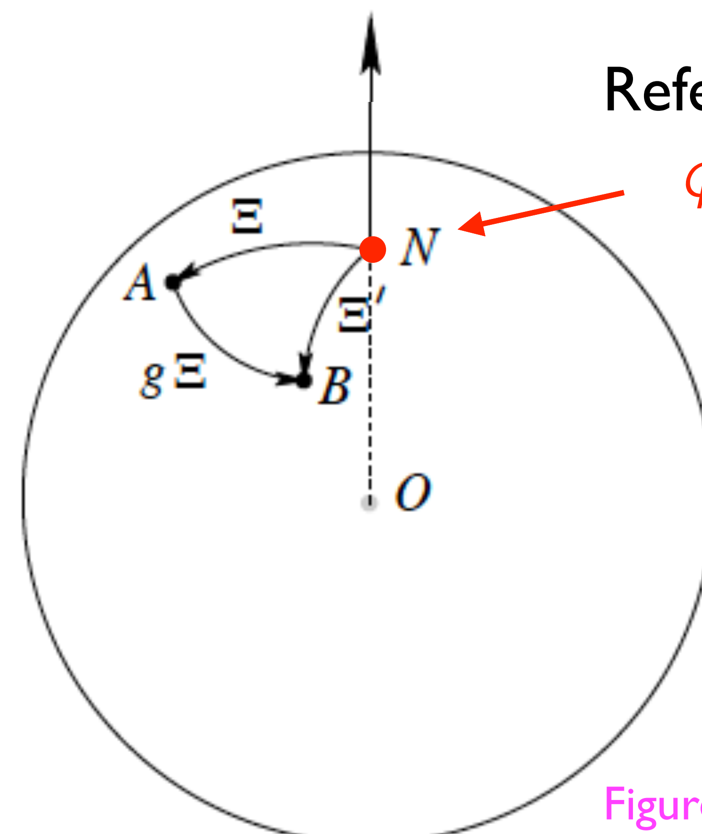


Figure from A. Manohar hep-ph/9606222

QCD: SSB of chiral SU(3)

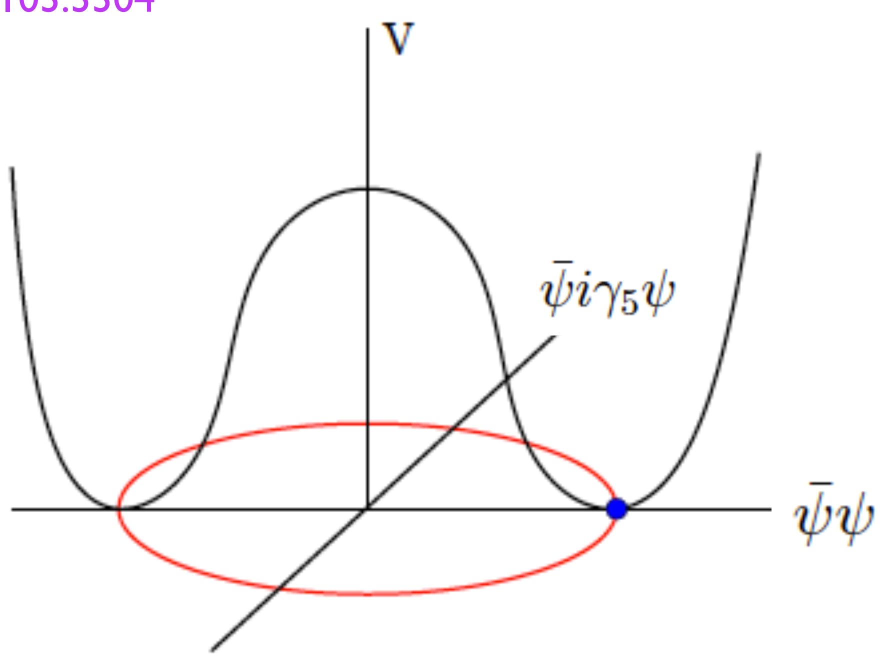
- Empirical & theoretical evidence of breaking pattern

$$\boxed{G = SU(3)_L \times SU(3)_R \;\rightarrow\; H = SU(3)_{V=L+R}}$$

$$\boxed{\begin{array}{lcl} q_L & \rightarrow & L\,q_L \\ q_R & \rightarrow & R\,q_R \end{array}}$$

$$\langle 0|\bar{q}q|0\rangle = \langle 0|\bar{q}_L q_R|0\rangle + \langle 0|\bar{q}_R q_L|0\rangle \neq 0$$

Figure from M. Creutz 1103.3304



- Vector subgroup $SU(3)_V$ (L=R) unbroken and symmetry is approximately manifest in the QCD spectrum [Wigner-Weyl realization]

- Axial generators broken: no parity doublets, but massless pseudoscalars (π,K,η) [Goldstone realization]

Low-energy EFT for GBs

- At very low-E, the only degrees of freedom (d.o.f) are the Goldstone Bosons
- Even though $M_{\pi,K,\eta} \neq 0$ (due to $m_q \neq 0$), π, K, η are still the lightest hadrons
- Use EFT methods to analyze the low-energy dynamics
    - Identify relevant d.o.f. (GBs, n, p) and transformation under the chiral group
    - Write down most general Lagrangian consistent with chiral symmetry
    - Order interactions & amplitudes according to a power counting scheme

Relevant ratio of scales (EFT expansion parameter): p/Λ

'Low scale' $p \sim p_{GB} \sim E_{GB} \sim M_{GB}$

'High scale' Λ: scale of lowest QCD non-GB states ~ O(1 GeV)

Fields and their transformations

- Choice of GB fields (specialize to SU(2)): $u(x) = e^{i(\pi^a(x)/F)\,T^a}$

$$u^2 = U \; = \; e^{i\mathbf{\Phi}/F} \qquad \mathbf{\Phi} = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$$

$$\boxed{\begin{array}{c} u \;\rightarrow\; L\,u\,h^{-1} \;=\; h\,u\,R^\dagger \\ U \longrightarrow L\,U\,R^\dagger \end{array}}$$

$L, R \in SU(2)_{L,R}$ $\qquad$ $h(L, R, \pi) \in SU(2)_V$

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$L, R \in SU(2)_{L,R} \qquad h(L, R, \pi) \in SU(2)_V$

- Matter fields: representations of unbroken subgroup SU(2)$_V$ (isospin)

Chiral covariant derivative

Convenient building blocks to construct invariants

$$N = \begin{pmatrix} p \\ n \end{pmatrix} \qquad N \;\to\; h\,N$$

$$\nabla_\mu N \equiv (\partial_\mu + \Gamma_\mu)N \to \; h\,\nabla_\mu N$$

$$u_\mu \equiv i\left(u\partial_\mu u^\dagger - u^\dagger\partial_\mu u\right) \to h u_\mu h^{-1}$$

$$\Gamma_\mu \equiv \frac{1}{2}\left(u\partial_\mu u^\dagger + u^\dagger\partial_\mu u\right)$$

Effective Lagrangian (π)

- Require invariance of $\mathcal{L}$ under $SU(n)_L x SU(n)_R$ and QCD discrete symmetries
- Organize $\mathcal{L}$ as an expansion in powers of derivatives (low E expansion) and explicit symmetry breaking (quark mass)

$$\mathcal{L}_\pi = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \mathcal{L}_\pi^{(6)} + \ldots$$

$$\boxed{\mathcal{L}_\pi^{(2)} = \frac{F^2}{4} \,\mathrm{Tr}\left[\partial_\mu U \,\partial^\mu U^\dagger\right]}$$

For $m_q=0$, chiral symmetry dictates that GB have derivative interactions:
GBs interact weakly at low energy

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$$\boxed{\mathcal{L}_\pi^{(2)} = \frac{F^2}{4} \mathrm{Tr}\left[\partial_\mu U\, \partial^\mu U^\dagger + 2B\, m_q\, (U + U^\dagger)\right]}$$

- m_q term easiest to derive if assume $m_q \to L\, m_q\, R^\dagger$: $\mathcal{L}_{QCD}$ is 'invariant'

- m_q term gives GB mass terms $M_{PS}^2 \sim B\ m_q \sim p^2$

$$\mathcal{L}_\pi^{(2)} \supset \partial_\mu \pi^- \partial^\mu \pi^+ - \boxed{B\,(m_u + m_d)\,\pi^+ \pi^-}$$

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- Counting rules: $\partial \sim p$ and $m_q \sim p^2$

- At leading order ($O(p^2)$), there appear two 'low energy constants' (LECs) not determined by symmetry: F, B
- Determined by appropriate correlation functions (get their values from experiment or lattice QCD)

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- Noether's currents: identify F with pion decay constant $F = F_\pi$

$$j_R^{\mu a} = \frac{iF^2}{2}\,\mathrm{Tr}\left(T^a\,U\partial_\mu U^\dagger\right)$$

$$j_L^{\mu a} = \frac{iF^2}{2}\,\mathrm{Tr}\left(T^a\,U^\dagger\partial^\mu U\right)$$

$$\boxed{j_A^{\mu a} = j_R^{\mu a} - j_L^{\mu a} = -F\,\partial^\mu\,\pi^a \;+ \ldots}$$

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- Noether's currents: identify F with pion decay constant $F = F_\pi$

- B determined at LO by quark condensate

$$B = -\frac{\langle 0|\bar{q}q|0\rangle}{F^2} = \frac{M_\pi^2}{m_u + m_d}$$

Effective Lagrangian (π)

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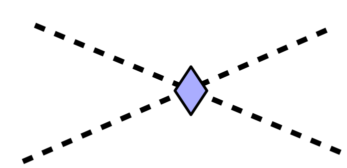
$$\boxed{\mathcal{L}_\pi^{(2)} = \frac{F^2}{4} \mathrm{Tr}\left[\partial_\mu U\, \partial^\mu U^\dagger + 2B\, m_q \left(U + U^\dagger\right)\right]}$$

  - 2n-GB interaction vertices in terms of F, B: ππ scattering, etc.



$$\mathcal{L}_2 \supset \frac{1}{12 f^2} \langle (\Phi \overleftrightarrow{\partial}_\mu \Phi)(\Phi \overleftrightarrow{\partial}^\mu \Phi) \rangle \longrightarrow \boxed{A_{\pi\pi}^{(2)} \sim \frac{p_{\mathrm{ext}}^2}{F^2}}$$

Power counting (1)

How do we compute amplitudes to a given order in p/Λ?

- Higher derivative terms in the effective Lagrangian?
- Loops?
- What sets the scale of the derivative expansion?

Power counting (1)

- Higher derivatives and/or mass insertions in effective Lagrangian:

$$\boxed{\mathcal{L} = \frac{F^2}{4}\left[\mathrm{Tr}\,\partial_\mu U\,\partial^\mu U^\dagger + \frac{1}{\Lambda^2}\mathcal{L}_\pi^{(4)} + \frac{1}{\Lambda^4}\mathcal{L}_\pi^{(6)} + \ldots\right]}$$

$$A_{\pi\pi}^{(2)} \sim \frac{p_{\mathrm{ext}}^2}{F^2} \qquad\qquad A_{\pi\pi}^{(4)} \sim \frac{p_{\mathrm{ext}}^2}{F^2}\,\frac{p_{\mathrm{ext}}^2}{\Lambda^2}$$

$$\left(\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\right)\right)^2$$

$$\mathrm{Tr}\left(\partial_\mu U^\dagger \partial_\nu U\right)\mathrm{Tr}\left(\partial^\mu U^\dagger \partial^\nu U\right)$$

$$\mathrm{Tr}\left(\partial_\mu U^\dagger \partial^\mu U\,(m_q^\dagger U + U^\dagger m_q)\right)$$

$$\mathrm{Tr}\left(m_q U^\dagger m_q U^\dagger\right)$$

$$\ldots$$

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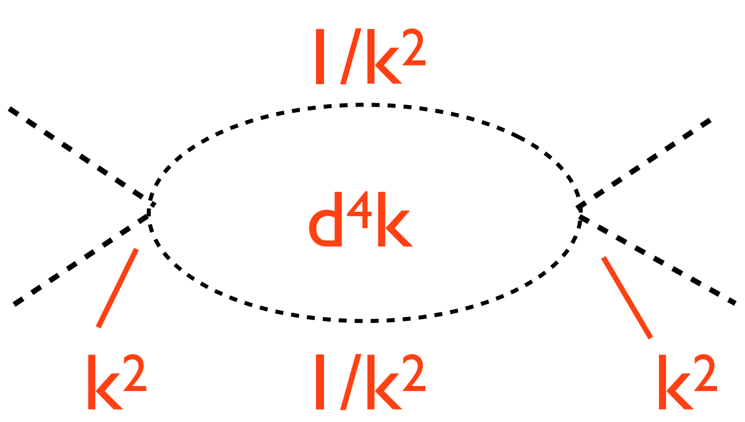
- Higher derivatives and/or mass insertions in effective Lagrangian:

$$\boxed{\mathcal{L} = \frac{F^2}{4}\left[\mathrm{Tr}\,\partial_\mu U\,\partial^\mu U^\dagger + \frac{1}{\Lambda^2}\mathcal{L}_\pi^{(4)} + \frac{1}{\Lambda^4}\mathcal{L}_\pi^{(6)} + \ldots\right]}$$

$$A_{\pi\pi}^{(2)} \sim \frac{p_{\rm ext}^2}{F^2} \qquad\qquad A_{\pi\pi}^{(4)} \sim \frac{p_{\rm ext}^2}{F^2}\,\frac{p_{\rm ext}^2}{\Lambda^2}$$

- What about loops? Start with $\mathcal{L}_\pi^{(2)}$ vertices



$$\boxed{A_{\pi\pi}^{(\rm loop)} \sim \frac{p_{\rm ext}^4}{16\pi^2\ F^4}\ \log\frac{p_{\rm ext}^2}{\mu^2}}$$

Estimate straightforward in mass-independent regulators and subtraction schemes (such as dim reg): amplitude can only contain powers of p, while μ appears only in logs

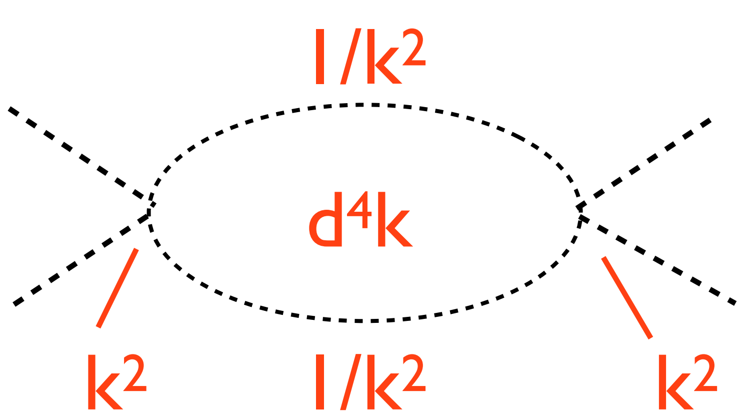
Power counting (1)

- Higher derivatives and/or mass insertions in effective Lagrangian:

$$\boxed{\mathcal{L} = \frac{F^2}{4}\left[\mathrm{Tr}\,\partial_\mu U\,\partial^\mu U^\dagger + \frac{1}{\Lambda^2}\mathcal{L}_\pi^{(4)} + \frac{1}{\Lambda^4}\mathcal{L}_\pi^{(6)} + \ldots\right]}$$

$$A_{\pi\pi}^{(2)} \sim \frac{p_{\rm ext}^2}{F^2} \qquad\qquad A_{\pi\pi}^{(4)} \sim \frac{p_{\rm ext}^2}{F^2}\,\frac{p_{\rm ext}^2}{\Lambda^2}$$

- What about loops? Start with $\mathcal{L}_\pi^{(2)}$ vertices



$$\boxed{A_{\pi\pi}^{(\rm loop)} \sim \frac{p_{\rm ext}^4}{16\pi^2\ F^4}\ \log\frac{p_{\rm ext}^2}{\mu^2}}$$

Dependence on regularization / renormalization scale in loops (μ) canceled by the p^4 low-energy constants: S-matrix elements are μ-independent

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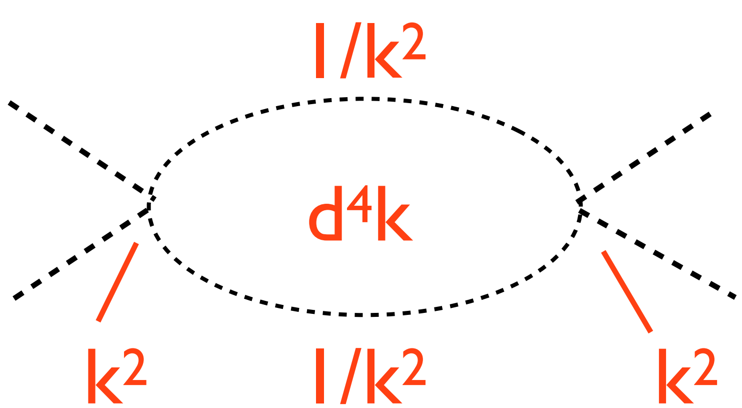

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$$\Lambda_{\rm loop} \sim 4\pi F$$

- In general $\Lambda \leq \Lambda_{\rm loop}$. Naive Dimensional Analysis assumes: $\Lambda \simeq \Lambda_{\rm loop}$
- Correspondence of loops and LECs allows one to assign the following scaling to any operator

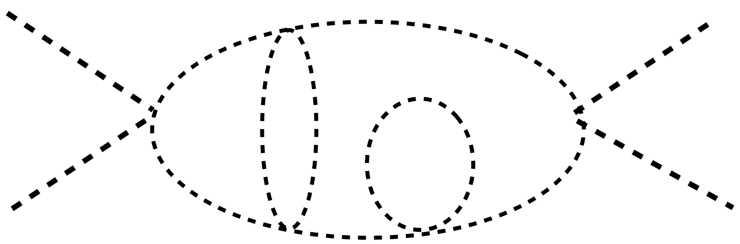
Manohar-Georgi '84

$$\boxed{O(1) \times F^2\Lambda^2 \left(\frac{\partial}{\Lambda}\right)^{n_D} \left(\frac{\pi}{F}\right)^{n_\pi} \left(\frac{N}{F\sqrt{\Lambda}}\right)^{n_N}} \qquad \Lambda = 4\pi F$$

Power counting (2)

Weinberg '79

- Weinberg's general argument



Chiral dimension:
#derivatives + 2 #m_q

#vertices of type i

$$\mathcal{A} \sim \int (d^4p)^L \frac{1}{(p^2)^I} \prod_i \left[p^{d_i}\right]^{V_i} \sim p^\nu$$

$$\boxed{\nu = \sum_i V_i d_i - 2I + 4L = \sum_i V_i (d_i - 2) + 2L + 2}$$

$$L = I - \sum_i V_i + 1$$

$d_i - 2 \geqslant 0$ due to chiral symmetry

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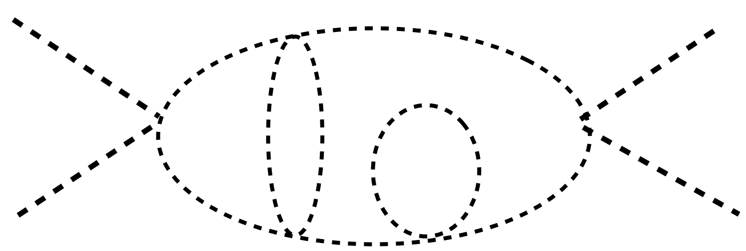

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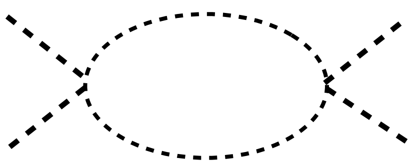
$$L = I - \sum_i V_i + 1$$

d_i -2 $\gtrsim$ 0 due to chiral symmetry

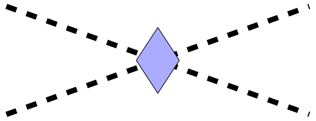
- Loop expansion = low-energy expansion with even powers of ν
- Loop divergences can be reabsorbed by higher order L_{eff}
- EFT is renormalizable (and predictive) to a given order in p/Λ

Power counting (summary)

- To a given order in the expansion of amplitudes in p/Λ:



- Loops: leading IR singularities, perturbative unitarity.
  Higher loops imply higher suppression (this changes in NN EFT)



- “Contact” terms, LECs: UV div.+ finite part, encoding
  short distance (QCD) physics, to be determined from expt.
  or via non-perturbative techniques (LQCD, dispersion relations, …).
  As couplings in any QFT, the LECs satisfy appropriate RGEs.

ChPT with baryons (1)

- Presence of $m_N \sim \Lambda$ spoils manifest power counting as $i\partial_0 N \sim m_N N$
- But nucleons interacting with `soft' pions are nearly on shell

$$\boxed{p^\mu = m_N v^\mu + k^\mu} \qquad \frac{k}{m_N} \ll 1$$

- Propagator takes the form (up to relative corrections ~ k/m_N):

$$i\frac{\not{p} + m_N}{p^2 - m_N^2 + i\epsilon} = i\frac{m_N(1+\not{v}) + \not{k}}{2m_N v\cdot k + k^2 + i\epsilon} \rightarrow \left(\frac{1+\not{v}}{2}\right)\frac{i}{v\cdot k + i\epsilon}$$

Projects on to 'large' particle components of Dirac spinor

Scales as $1/p_{soft}$

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Scales as $1/p_{soft}$

- What Lagrangian generates this?

ChPT with baryons (2)

Georgi '90, Jenkins-Manohar '91

- To get manifest power counting, write Lagrangian in terms of v-dependent fields N_v so that $i\partial_0 N_v \sim k_0 N_v << \Lambda N_v$

$$\boxed{N(x) = e^{-im_N v\cdot x}\Big(N_v(x) + H_v(x)\Big)}$$

$$N_v(x) = e^{im_N v\cdot x}\frac{1+\not{v}}{2}N(x) \qquad H_v(x) = e^{im_N v\cdot x}\frac{1-\not{v}}{2}N(x)$$

$$\boxed{\bar{N}(i\not{\partial} - m_N)N \to \bar{N}_v iv\cdot\partial N_v + \ldots}$$

- Baryon bilinears expressed in terms of $v^\mu, \quad S^\mu = \frac{i}{2}\gamma_5\sigma^{\mu\nu}v_\nu$

$$\boxed{\begin{array}{c} v^\mu = (1,\vec{0}) \\ S^\mu = (0,\vec{\sigma}/2) \end{array}}$$

$$\bar{N}_v\gamma_5 N_v = 0 \qquad \bar{N}_v\gamma_\mu\gamma_5 N_v = 2\bar{N}_v S_\mu N_v$$

$$\bar{N}_v\gamma_\mu N_v = \bar{N}_v v_\mu N_v \qquad \bar{N}_v\sigma_{\mu\nu}N_v = 2\epsilon_{\mu\nu\alpha\beta}v^\alpha\bar{N}_v S^\beta N_v$$

Effective Lagrangian (π,N)

- Use building blocks N, ∇N, u, … and organize according to standard counting rules $\partial \sim p$, $m_q \sim p^2$

$$\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \ldots$$

$$\boxed{\mathcal{L}_{\pi N}^{(1)} = \bar{N}_v\, iv \cdot \nabla\, N_v \;+\; g_A \bar{N}_v\, S \cdot u\, N_v}$$

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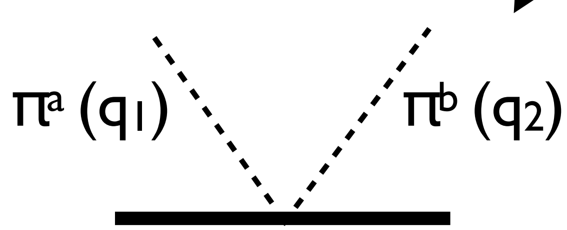
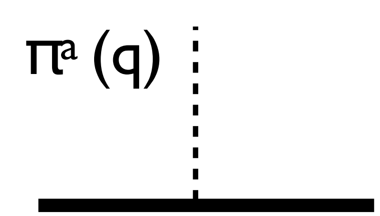


+ 2N4π, …

$$\boxed{\frac{1}{4F^2} v \cdot (q_1 + q_2) \epsilon^{abc} \tau^c}$$



+ 2N3π, …

$$\boxed{\frac{g_A}{F} S \cdot q \tau^a}$$

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$$v^\mu = (1, \vec{0})$$
$$S^\mu = (0, \vec{\sigma}/2)$$

- In higher orders both p/Λ_χ and p/m_N terms appear

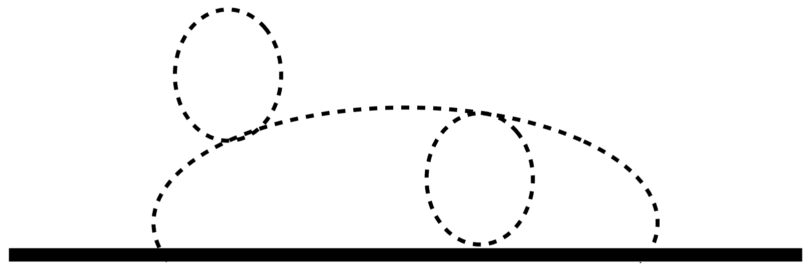
$$\mathcal{L}_{\pi N}^{(2)} : \frac{1}{2m_N} \bar{N}_v \left((v \cdot \nabla)^2 - \nabla \cdot \nabla \right) N_v,$$

$$\bar{N}_v \, (v \cdot u)^2 \, N_v, \quad \bar{N}_v \, u \cdot u \, N_v, \quad \ldots$$

Non-relativistic expansion of kinetic energy

Power counting with nucleons

- Weinberg's general argument for connected amplitudes



$$\mathcal{A} \sim \int (d^4p)^L \frac{1}{(p^2)^{I_\pi}} \frac{1}{(p)^{I_N}} \prod_i \left[p^{d_i}\right]^{V_i} \sim p^\nu$$

$$\boxed{\nu = \sum_i V_i(d_i + n_i/2 - 2) + 2L - E_N/2 + 2}$$

$$L = I_\pi + I_N - \sum_i V_i + 1$$

$$2I_N + E_N = \sum_i V_i n_i$$

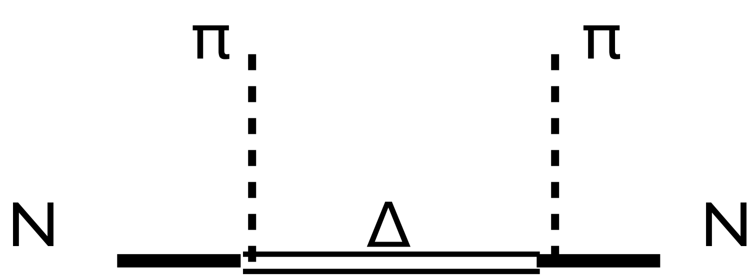
n_i = # of nucleon fields in the vertex

- Loop expansion: low-energy expansion that contains all powers of ν
- Convergence pattern not too natural in certain cases: impact of the Δ?

The role of the Δ(1232)

Jenkins-Manohar '91, Hemmert-Holstein-Kambor '97

- Unnaturally large values of some LECs can be understood in terms of large contributions from the low-lying Δ-baryon:



$$\boxed{\delta \equiv m_\Delta - m_N = 293 \text{ MeV}}$$

LEC~1/δ instead of ~1/Λ

- Can include Δ in the EFT with power counting:

$$\boxed{Q \sim m_\pi \sim \delta \ll \Lambda}$$

Δ-ful

instead of

$$\boxed{Q \sim m_\pi \ll \delta \sim \Lambda}$$

Δ-less

- Improved convergence, LECs have more natural size

Electroweak interactions

Gasser-Leutwyler '84, '85

- Start from QCD with external sources

$$s(x), p(x), l_\mu(x), r_\mu(x)$$

$$\mathcal{L} = \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R - \bar{q}_R (s + ip) q_L - \bar{q}_R (s - ip) q_L$$
$$+ \bar{q}_L \gamma^\mu l_\mu q_L + \bar{q}_R \gamma^\mu r_\mu q_R$$

$$q = \begin{pmatrix} u \\ d \end{pmatrix}$$

External sources describe Standard Model or BSM fields coupled to quark bilinears

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- In the Standard Model (at low E):

$$s + ip = m_q + \text{coupling to Higgs}$$
$$l_\mu = -eQ_L^{\rm em}A_\mu + Q_L^{\rm w}J_\mu^{\rm lept} + Q_L^{\rm w\dagger}J_\mu^{\rm lept\dagger}$$
$$r_\mu = -eQ_R^{\rm em}A_\mu$$

$$Q_L^{\rm em} = Q_R^{\rm em} = \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix} \qquad Q_L^{\rm w} = -2\sqrt{2}G_F \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad J_\mu^{\rm lept} = \bar{e}_L\gamma_\mu\nu_{eL}$$

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$$+ \bar{q}_L \gamma^\mu l_\mu q_L + \bar{q}_R \gamma^\mu r_\mu q_R$$

$$q = \begin{pmatrix} u \\ d \end{pmatrix}$$

- "Spurion" transformation of the sources (so that $\mathcal{L}$ is invariant)

$$\begin{aligned} q_L &\rightarrow L\, q_L \\ q_R &\rightarrow R\, q_R \end{aligned}$$

$$\begin{aligned} (s + ip) &\rightarrow R(s + ip)L^\dagger \\ l_\mu &\rightarrow L l_\mu L^\dagger + i L \partial_\mu L^\dagger \\ r_\mu &\rightarrow R r_\mu R^\dagger + i R \partial_\mu R^\dagger \end{aligned}$$

Invariance under local [L(x), R(x)] chiral transformation

$$Q_L^{\mathrm{em,w}} \rightarrow L Q_L^{\mathrm{em,w}} L^\dagger \qquad Q_R^{\mathrm{em}} \rightarrow R Q_R^{\mathrm{em}} R^\dagger$$

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$$+ \bar{q}_L \gamma^\mu l_\mu q_L + \bar{q}_R \gamma^\mu r_\mu q_R$$

$$q = \begin{pmatrix} u \\ d \end{pmatrix}$$

- Modified EFT building blocks and their transformations (covariant derivatives):

$$D_\mu U = \partial_\mu U - i l_\mu U + i U r_\mu$$

$$D_\mu U \to L \; D_\mu U \; R^\dagger$$

$$\nabla_\mu N = (\partial_\mu + \Gamma_\mu) N$$
$$\Gamma_\mu \equiv \frac{1}{2} \left(u(\partial_\mu - i r_\mu) u^\dagger + u^\dagger (\partial_\mu - i l_\mu) u \right)$$

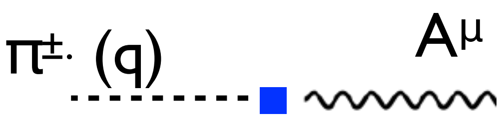
$$\nabla_\mu N \to h \; \nabla_\mu N$$

$$u_\mu \equiv i \left(u(\partial_\mu - i r_\mu) u^\dagger - u^\dagger (\partial_\mu - i l_\mu) u \right)$$
$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u \qquad \chi = 2B(s + ip)$$
$$\mathcal{Q}_L^{\rm em,w} = u^\dagger Q_L^{\rm em,w} u \qquad \mathcal{Q}_R^{\rm em} = u Q_R^{\rm em} u^\dagger$$

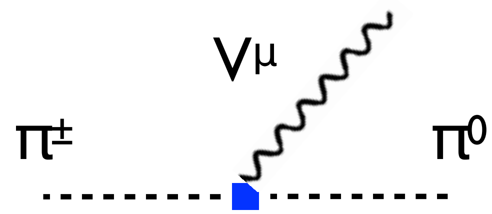
$$O \to h O h^{-1}$$

Examples

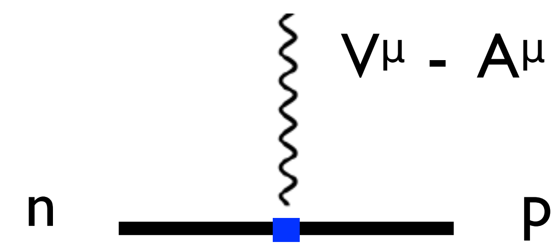
- Weak charged-current interaction vertices (mesons, baryons)



$$\sim F_\pi q^\mu$$



$$\sim (p_1^\mu + p_2^\mu)$$



$$\sim (v^\mu - 2g_A S^\mu)$$

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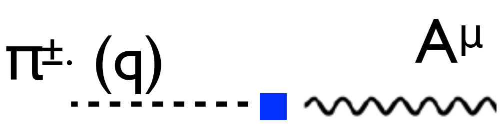

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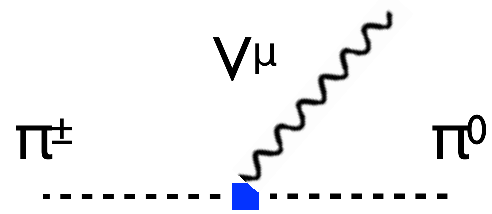

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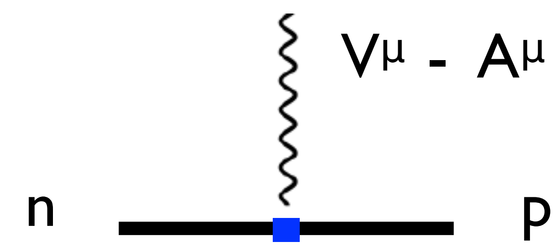
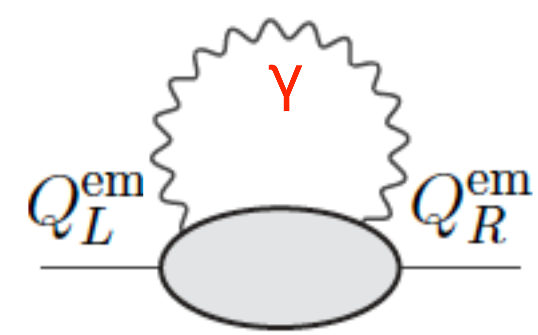


$$\sim (v^\mu - 2g_A S^\mu)$$

- Effects of virtual photons: pion mass splitting



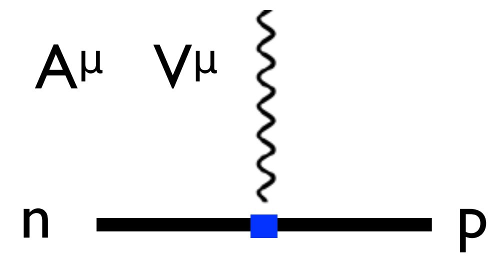
$$\mathcal{L}_\pi^{(e^2)} = e^2 Z F^4 \mathrm{Tr}\left[Q_L^{\rm em} U Q_R^{\rm em} U^\dagger\right] \supset -2e^2 Z F^2 \,\pi^+\pi^- \;+\; \ldots$$

LEC determined in terms of the pion electromagnetic mass splitting

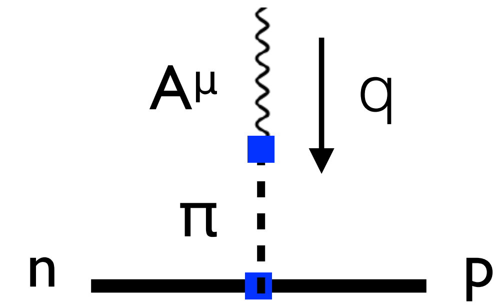
Nucleon weak currents

- "V-A" current relevant for single and double beta decay

- Leading order

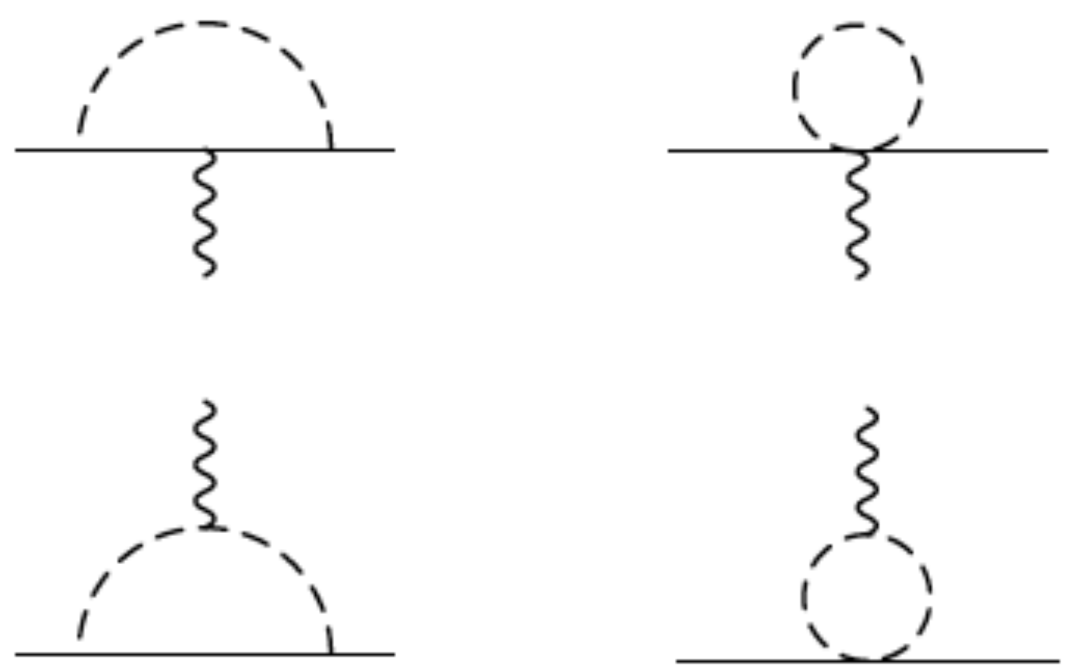


$$\sim (v^\mu - 2g_A S^\mu)$$



$$\sim -2g_A \frac{S \cdot q}{\vec{q}^2 + m_\pi^2} q^\mu$$

- N2LO



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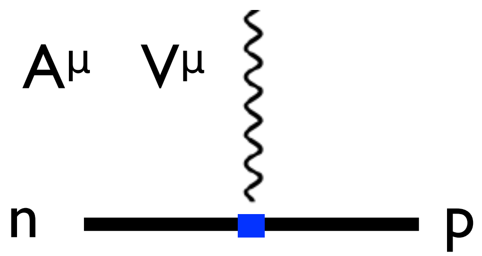

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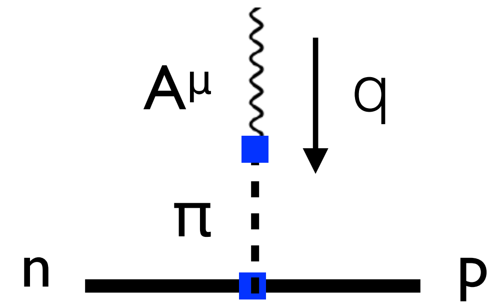


$$\sim -2g_A \frac{S \cdot q}{\vec{q}^2 + m_\pi^2} q^\mu$$

- Including recoil (1/m$_N$) and N2LO effects (form factors):

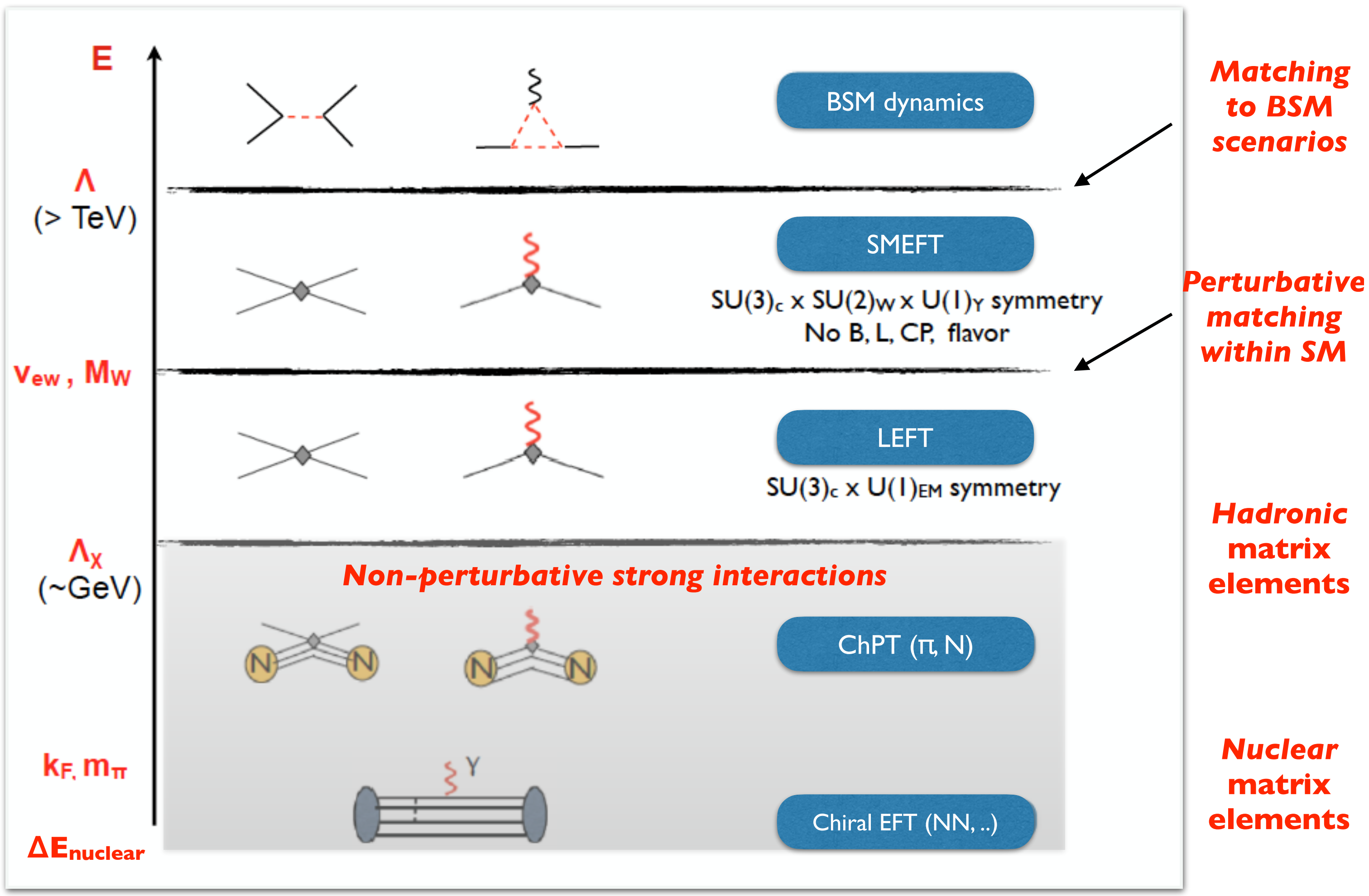
$$J_V^\mu = g_V(\mathbf{q}^2)\left(v^\mu + \frac{p^\mu + p'^\mu}{2m_N}\right) + i g_M(\mathbf{q}^2)\,\epsilon^{\mu\nu\alpha\beta}\,\frac{v_\alpha S_\beta q_\nu}{m_N},$$
$$J_A^\mu = -2g_A(\mathbf{q}^2)\left(S^\mu - \frac{S\cdot(p+p')}{2m_N} v^\mu + \frac{S\cdot q}{\mathbf{q}^2 + m_\pi^2} q^\mu\right).$$

$$g_\alpha(q^2) = g_\alpha(0)\left[1 + \frac{\langle r_\alpha \rangle^2}{6} q^2 + \ldots\right] \qquad g_V(0) = 1 \qquad g_A(0) = g_A \qquad g_M(0) = 1 + \kappa_v = 4.71$$

Beyond quark bilinears

- Can apply similar techniques to obtain the chiral realization of any quark & gluon operator in the LEFT at ~ GeV scale (coming from SM or BSM)
- Identify chiral transformation properties of operator and write down its low-energy realization in terms of GBs and nucleons
- Examples:
  - Non-leptonic weak interactions (four-quark operators, penguins, …)
  - CP-violating operators (EDMs)
  - Lepton-number violation
  - …

Conclusions



- Discussed ChPT as an intermediate step between LEFT and nuclear EFT: from quarks+gluons to pions and nucleons
- Essential step in analysis of EW and BSM interactions of light hadrons and nuclei
- In the following talk I will describe an application to μ→e conversion

Backup

SSB systematics

- SSB G → H : action invariant under G, vacuum under subgroup H

$$\partial_\mu j^\mu_a = 0 \qquad \mathcal{Q}_a = \int_V d^3x\, j^0_a(x) \qquad \frac{d}{dt}\mathcal{Q}_a = 0$$

Unbroken generators:
Wigner-Weyl

$$\mathcal{Q}_a|0\rangle = 0$$

$$U = e^{i\epsilon_a \mathcal{Q}_a}$$

$$U|0\rangle = |0\rangle$$

Degenerate multiplets in the spectrum

Broken generators:
Nambu-Goldstone

$\exists \quad \mathcal{O}$: $\quad v_a \equiv \langle 0|[\mathcal{Q}_a, \mathcal{O}]|0\rangle \neq 0$

$$\Big\downarrow \ \mathcal{Q}_a|0\rangle \neq 0$$

Massless state $|\phi_a\rangle$

$$\langle 0|\mathcal{O}|\phi_a\rangle\, \langle \phi_a|j^0_a(0)|0\rangle \neq 0$$

Massless spin-0 particles in the spectrum (GBs)

Fields and their transformations (1)

Callan, Coleman, Wess, Zumino '69

- SSB pattern G→H: GB fields ~ coordinates of the vacuum manifold G/H

$$\boldsymbol{\phi} = (\phi_1, \ldots, \phi_N)$$

- Example: O(N) linear sigma model

$$\boxed{\mathcal{L} = \frac{1}{2}\partial_\mu \boldsymbol{\phi} \cdot \partial^\mu \boldsymbol{\phi} - \lambda(\boldsymbol{\phi}\cdot\boldsymbol{\phi} - v^2)^2}$$

Vacuum manifold $\phi_1^2 + \phi_2^2 + \ldots + \phi_N^2 = v^2$

N=3: G=O(3), H=O(2), G/H = S^2

$$\phi_{\text{vac}}(x) = \Xi(x)\,\phi_0$$

$$\Xi(x) = e^{i\pi^a(x) A^a} \xrightarrow{N=3} e^{i(\pi^1(x)J^1 + \pi^2(x)J^2)}$$

Goldstone fields broken generators

$$\boxed{g\,\Xi(x) = \Xi'(x)\,h(g, \Xi(x))}$$

$h \in H$

$g \in G$

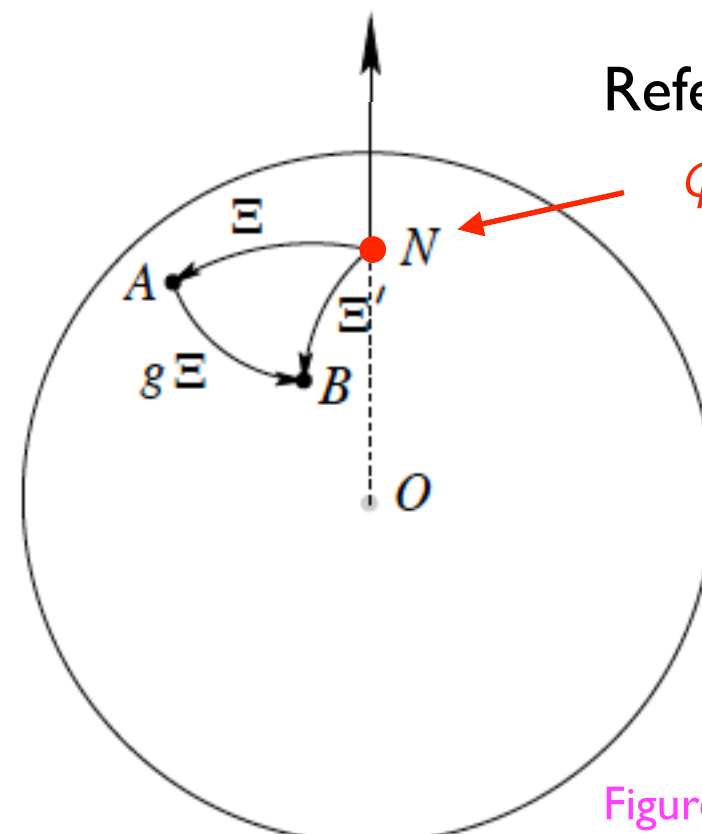


Figure from A. Manohar hep-ph/9606222

Fields and their transformations (2)

Callan, Coleman, Wess, Zumino '69

- SSB pattern G→H: GB fields ~ coordinates of the vacuum manifold G/H

- GBs & massive fields (ψ) transformation

$$\Xi(\pi) = e^{i\pi^a A^a}$$

$$\boxed{\Xi(\pi') = g\ \Xi(\pi)\ h^{-1}(g,\pi)}$$

$h \in H$
$g \in G$

$\Psi \qquad h_\Psi(h)$

$$\boxed{\Psi \xrightarrow{g} \Psi' = h_\Psi(h(g,\pi))\ \Psi}$$

Representation of unbroken subgroup H under which ψ transforms

Non-linear representation of the group G, linear when restricted to H

Back to $SU(n)_L \times SU(n)_R$ $n = 2,3$

Transformation

$$g = \begin{pmatrix} L & 0 \\ 0 & R \end{pmatrix}$$

Generators

$$V^a = T^a_L + T^a_R$$ Unbroken

$$A^a = T^a_L - T^a_R$$ Broken

$$V^a = \begin{pmatrix} T^a & 0 \\ 0 & T^a \end{pmatrix}$$

$$A^a = \begin{pmatrix} T^a & 0 \\ 0 & -T^a \end{pmatrix}$$

SU(n) generators

$$\Xi(\pi) = e^{i\pi^a A^a} = \begin{pmatrix} u(\pi) & 0 \\ 0 & u^\dagger(\pi) \end{pmatrix}$$

$$\boxed{u(\pi) = e^{i\pi^a T^a}}$$

$$\boxed{\begin{pmatrix} u(\pi) & 0 \\ 0 & u^\dagger(\pi) \end{pmatrix} \longrightarrow \begin{pmatrix} L & 0 \\ 0 & R \end{pmatrix} \begin{pmatrix} u(\pi) & 0 \\ 0 & u^\dagger(\pi) \end{pmatrix} \begin{pmatrix} h^{-1} & 0 \\ 0 & h^{-1} \end{pmatrix}}$$

h^{-1}, element of $SU(n)_V$

$$u \to L\,u\,h^{-1} = h\,u\,R^\dagger$$

$$u^2 \equiv U \to L\,U\,R^\dagger$$