# NUCLEAR EFFECTIVE FIELD THEORIES and FUNDAMENTAL SYMMETRIES

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# Outline

- Symmetries
- Effective field theory
- Chiral EFT
- Neutrinoless double-beta decay
- Conclusion

# Symmetries

Standard Model (SM) "explains" everything, except:

- neutrino masses
  - sterile neutrinos?
  - Majorana neutrinos ←→ lepton-number (L) violation
- galaxy rotations, lensing
  - dark matter?
  - modification of gravity?
- matter-antimatter imbalance
  - time-reversal (T) violation
  - baryon-number (B) violation

⇒ Physics beyond the SM (BSM)

neutrinoless double-beta decay:
$nn \to ppee$
in nucleus

nucleon and nuclear electric dipole moments

nucleon decay and neutron-antineutron oscillation:
$N \to l\,X,\ n \leftrightarrow \bar{n}$
free and in nucleus

$Q$

$M_{\mathrm{BSM}} \sim ?$

unknown physics

$M_{\mathrm{EW}} \sim v, m_Z, m_W$

$\sim 100\ \mathrm{GeV}$

$M_{\mathrm{nuc}} \sim f_\pi, 1/r_{NN}, m_\pi, \ldots$

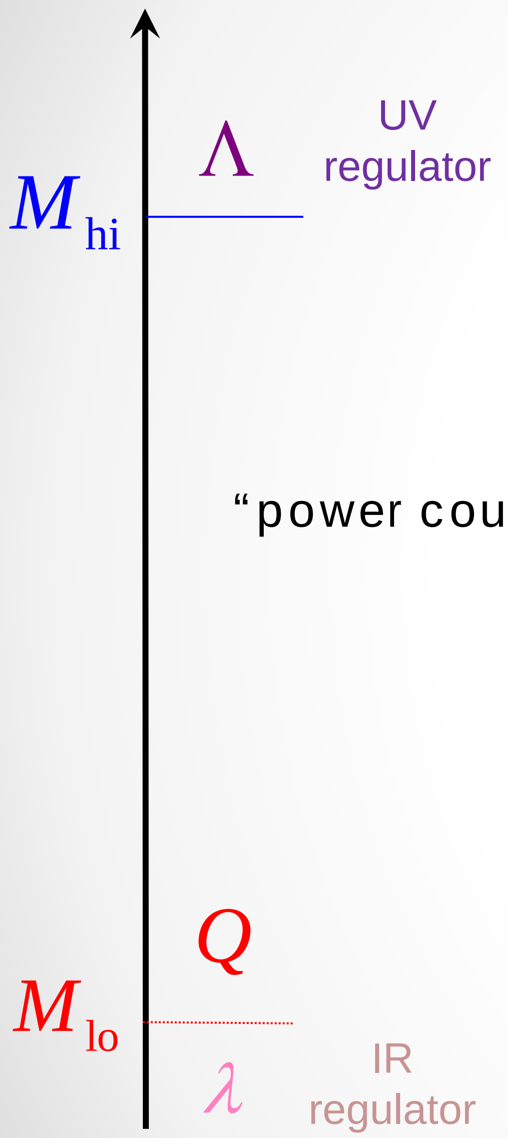
$\sim 100\ \mathrm{MeV}$

relevant for
precision experiments
with hadrons and nuclei

...

# Effective Field Theory©

Euler, Heisenberg '36
Weinberg '67 … '79
…




most general Lagrangian

$$\mathcal{L}_{\rm EFT} = \sum_{\nu=0}^{\infty} \underbrace{\gamma_i^{(\nu)}\left(\frac{M_{\rm lo}}{\Lambda}, \frac{\lambda}{M_{\rm lo}}\right) \frac{M_{\rm lo}^{s}}{M_{\rm hi}^{\nu}}}_{\text{“low-energy constants”/ “Wilson coefficients”}} \underbrace{O^{(\nu)}\left(\partial^d \psi^N\right)}_{\text{operators}}$$

"power counting" $\quad \nu = \nu(s, d, N, \ldots)$

most general S matrix

non-analytic functions, from (finite or infinite number of) loops

$$S^{(\bar{\nu})}(Q \sim M_{\rm lo} \ll M_{\rm hi}) - 1 \propto \sum_{\nu=0}^{\bar{\nu}} \left[\frac{Q}{M_{\rm hi}}\right]^{\nu} \overbrace{F^{(\nu)}\left(\frac{Q}{M_{\rm lo}}, \frac{Q}{\Lambda}, \frac{\lambda}{Q}; \gamma_i^{(\le\nu)}\left(\frac{M_{\rm lo}}{\Lambda}, \frac{\lambda}{M_{\rm lo}}\right)\right)}$$

$$\times\left\{1 + \mathcal{O}\left(\frac{Q^{\bar{\nu}+1}}{M_{\rm hi}^{\bar{\nu}+1}}, \underbrace{\frac{Q^{\bar{\nu}+1}}{M_{\rm hi}^{\bar{\nu}}\Lambda}, \frac{\lambda Q^{\bar{\nu}}}{M_{\rm hi}^{\bar{\nu}+1}}}_{\text{RENORMALIZATION}}\right)\right\}$$

$\mathrm{N}^{\bar{\nu}}\mathrm{LO}$

CONTROLLED UNCERTAINTY

MODEL INDEPENDENCE

For predictive power we need to know, *a priori*, how to truncate the infinite series

"power counting"

renormalization + naturalness

't Hooft '79
Veltman '80
...

dependence on regulator cutoff reflects

sensitivity to physics at the breakdown scale

LEC ~ part that cancels cutoff dependence in observables

+ finite part with $\Lambda \to M_{\mathrm{hi}}$

unless protected by symmetry

*Entities must not be multiplied beyond necessity*

Wikipedia

William of Ockham

# The Way of EFT

$Q$

$M_{\not{L}} \sim ?$

unknown physics

$M_{\mathrm{EW}} \sim v, m_Z, m_W$

$\sim 100\ \mathrm{GeV}$

Standard Model + higher-dim

“SMEFT”

$M_{\mathrm{QCD}} \sim m_N, m_\rho, 4\pi f_\pi, \ldots$

$\sim 1\ \mathrm{GeV}$

QCD + QED + higher-dim

“WEFT”

$M_{\mathrm{nuc}} \sim f_\pi, 1/r_{NN}, m_\pi, \ldots$

$\sim 100\ \mathrm{MeV}$

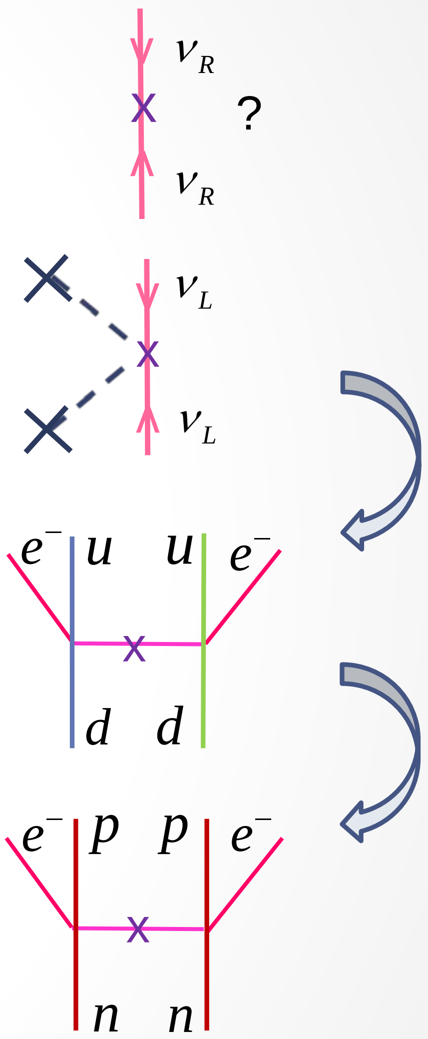
Chiral EFT

($\chi$PT)

...

...

## Example: $L$



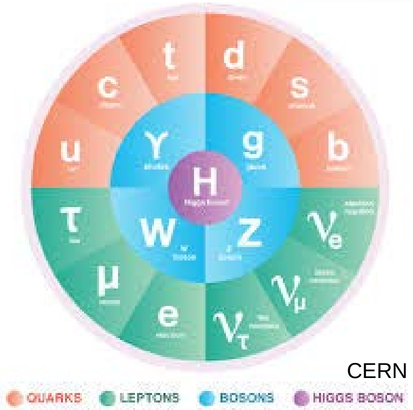

run RG

match with lattice, ...

...

SMEFT

$Q \sim M_{\mathrm{EW}}$




d.o.f.s

$\mathrm{SO}(3,1)$ global,

symmetries

$\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge

approximate
*B*, *L*, *T*

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\dim=5} + \mathcal{L}_{\dim=6} + \ldots + \mathcal{L}_{\dim=9} + \ldots$$

Weinberg '67
Salam '68
...

Weinberg '79

Rao, Shrock '82
...
Buchoff, Wagman '16

$\Delta L = \pm 2$ | $\cancel{P}, \cancel{T}$ | $\Delta B = \Delta L = \pm 1$ | $\Delta B = \pm 2$

Weinberg '79'89
Wilczek, Zee '79
Abbott, Wise '80
Claudson, Wise, Hall '82
...
Buchmüller, Wyler '86
de Rújula *et al.* '91
...
Ng, Tulin '11

neutrinoless double-beta decay

nuclear electric dipole moments

nuclear decay into mesons (+ lepton)

# QC(+E)D (LITE)

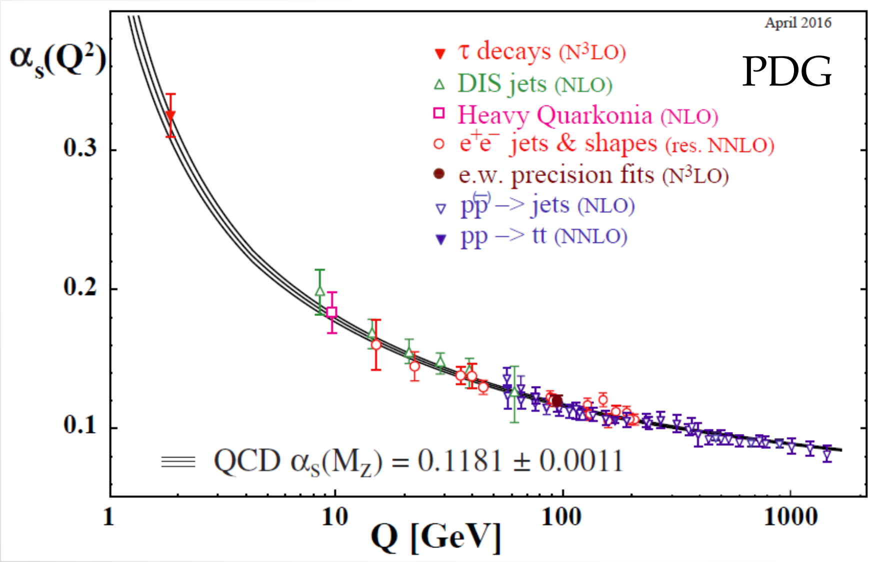
d.o.f.s  quarks: $q=\begin{pmatrix}u\\d\end{pmatrix}$  gluons: $G_\mu^a$  (+ photon)

symmetries  SO(3,1) global, $\mathrm{SU}(3)_\mathrm{c}$ $(+\mathrm{U}(1)_\mathrm{em})$ gauge

$Q \ll M_{\mathrm{EW}}$

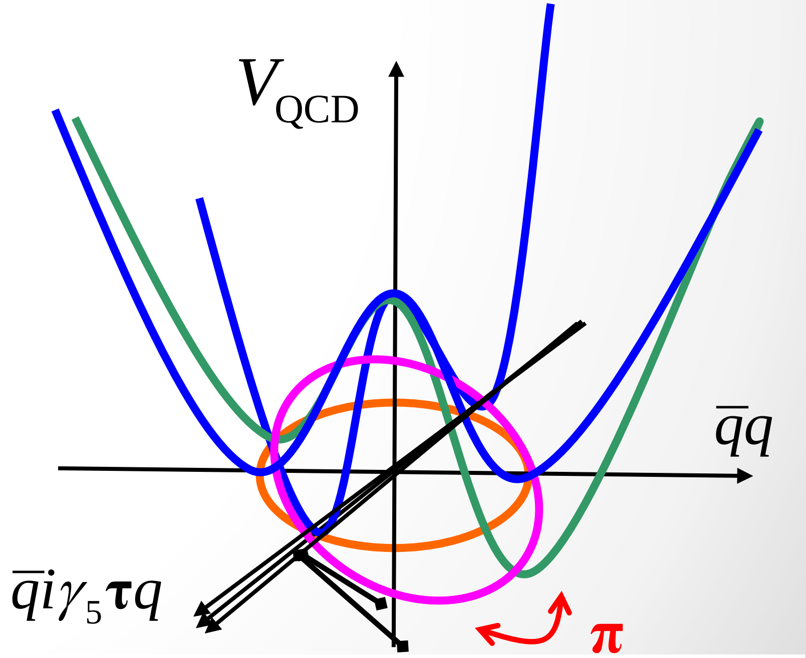
$$\mathcal{L}_{\mathrm{QCD}} = \underbrace{\bar{q}\left(i\partial\!\!\!/ + g_s G\!\!\!/\right)q - \frac{1}{2}\mathrm{Tr}\,G^{\mu\nu}G_{\mu\nu}}_{M_{\mathrm{QCD}} \sim m_N, m_\rho, 4\pi f_\pi, \ldots \sim 1\ \mathrm{GeV}} + \underbrace{\bar{m}\,\bar{q}\left(1-\varepsilon\tau_3\right)q}_{m_\pi \sim \sqrt{\bar{m} M_{\mathrm{QCD}}} \simeq 140\ \mathrm{MeV}} + \ldots$$

Basic
mass scales



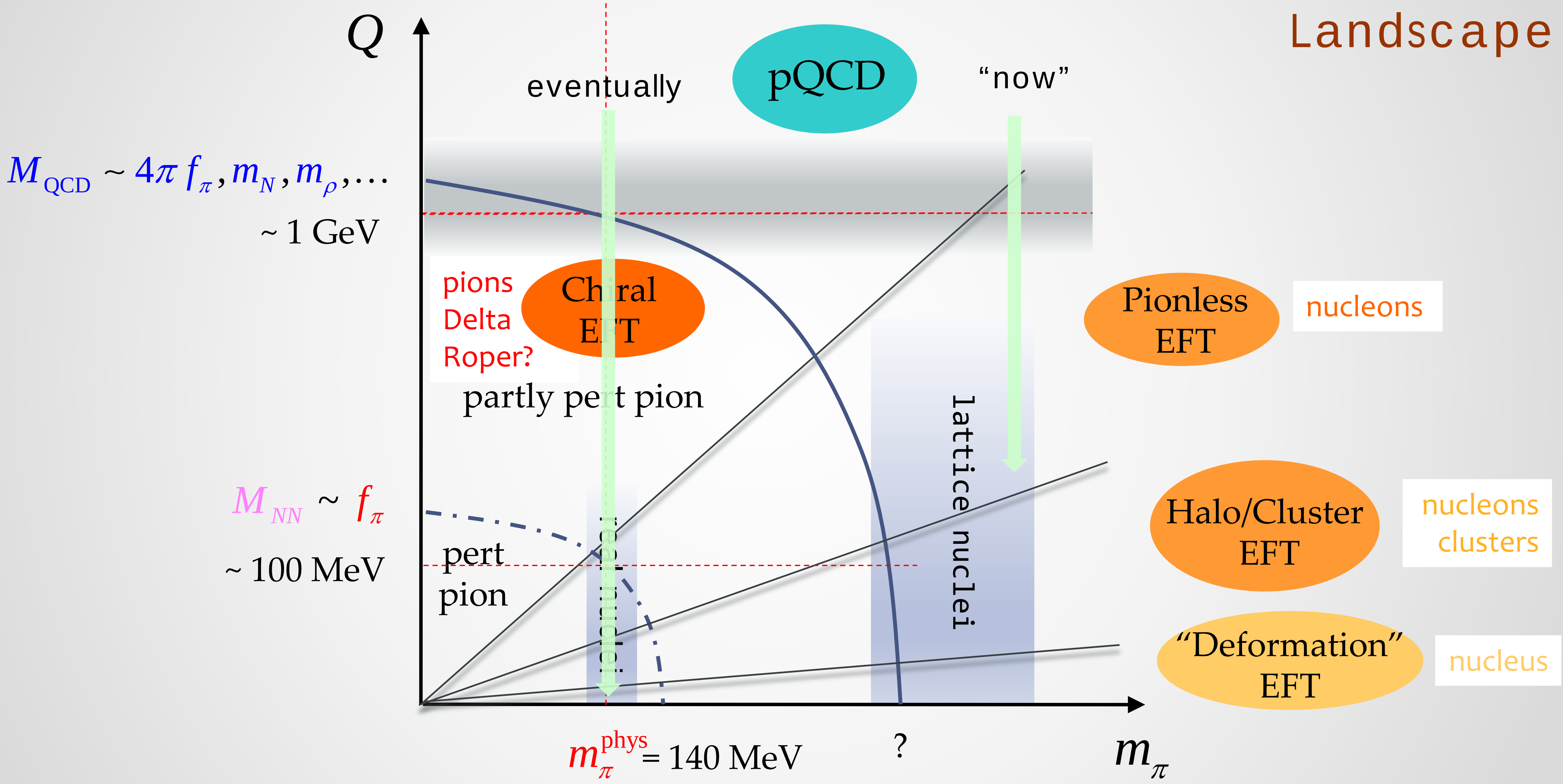

Gross, Wilczek ’73
Politzer ’73
…

Nambu ’64
…



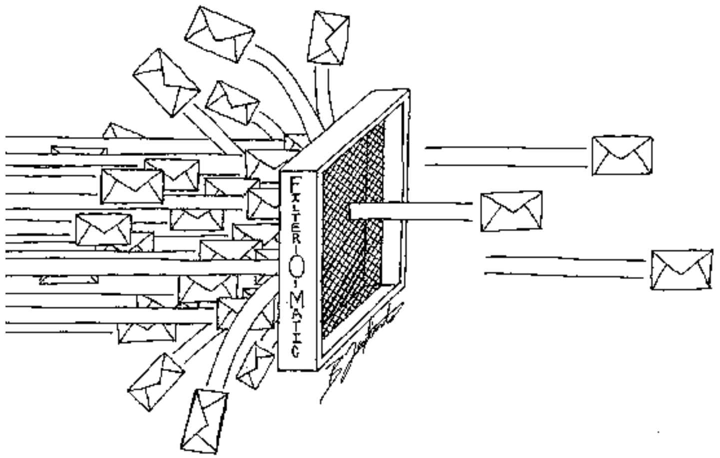

$$f_\pi \sim M_{\mathrm{QCD}}/4\pi + \mathcal{O}(\bar{m}) \simeq 100\ \mathrm{MeV}$$

# Nuclear EFT Landscape



# An advantage of Chiral EFT

Possibility to disentangle symmetry-violating sources:
each breaks chiral symmetry in a particular way
and produces *different* hadronic interactions




(For Pionless EFT, only isospin is left…)

# Through the nuclear chiral filter, an example

De Vries, Timmermans, Mereghetti + vK '11
Mereghetti *et al.*'11
De Vries *et al.*'11
...

Chiral EFT

$\not{P}, \not{T}$ sources

| $Q \sim f_\pi$ | | $\theta$ term | qEDM | qCEDM | gCEDM, PSC | LRC |
|---|---|---|---|---|---|---|
| $^1$H | $d_p/d_n$ | $\mathcal{O}(1)$ | $\mathcal{O}(1)$ | $\mathcal{O}(1)$ | $\mathcal{O}(1)$ | $\mathcal{O}(1)$ |
| $^2$H | $d_d/d_n$ | $\mathcal{O}(1)$ | $\mathcal{O}(1)$ | $\mathcal{O}\left(\frac{M_{\text{QCD}}^2}{Q^2}\right)$ | $\mathcal{O}(1)$ | $\mathcal{O}\left(\frac{M_{\text{QCD}}^2}{Q^2}\right)$ |
| $^3$He | $d_h/d_n$ | $\mathcal{O}\left(\frac{M_{\text{QCD}}^2}{Q^2}\right)$ | $\mathcal{O}(1)$ | $\mathcal{O}\left(\frac{M_{\text{QCD}}^2}{Q^2}\right)$ | $\mathcal{O}(1)$ | $\mathcal{O}\left(\frac{M_{\text{QCD}}^2}{Q^2}\right)$ |
| $^3$H | $d_t/d_h$ | $\mathcal{O}(1)$ | $\mathcal{O}(1)$ | $\mathcal{O}(1)$ | $\mathcal{O}(1)$ | $\mathcal{O}(1)$ |

electric dipole moments

\+ specific relations

*e.g.*

$$\begin{cases} d_h + d_t \simeq 0.84(d_n + d_p) & \text{qEDM and } \theta \text{ term} \\ d_h - d_t \simeq 0.94(d_n - d_p) & \text{qEDM} \\ d_h + d_t \simeq 3d_d & \text{qCEDM and LRC} \end{cases}$$

Farley *et al.* '04
...

storage-ring measurements could teach us about sources!

Weinberg ’90’91’92
Rho ’91
Ordóñez, vK ’92
vK ’94
Ordóñez, Ray, vK ’94,’96
…

# Chiral EFT

$Q \sim m_\pi \ll M_{\mathrm{QCD}}$ Chiral EFT

- nucleons and *pions* *(and Deltas, Ropers?)*
- SM symmetries (including approximate chiral symmetry)

$$\mathcal{L}^{(\Delta B=0)}_{\pi\mathrm{EFT}} = \frac{1}{2}\left[(\partial_\mu \boldsymbol{\pi})^2 - m_\pi^2 \boldsymbol{\pi}^2\right] + \ldots$$

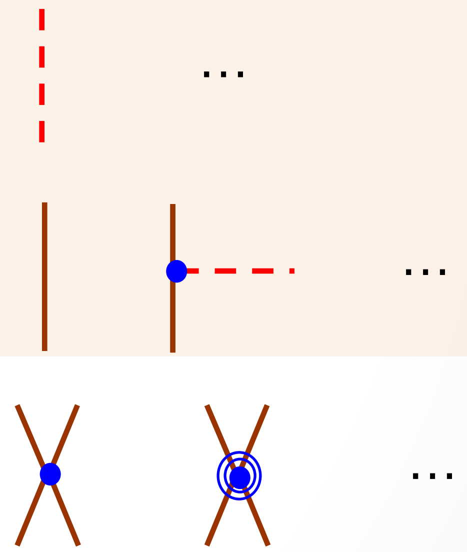
$$+ N^+ \left( i\partial_0 + \frac{\nabla^2}{2m_N} + \frac{g_A}{2 f_\pi} \boldsymbol{\tau}\vec{\sigma}\cdot\vec{\nabla}\boldsymbol{\pi} + \ldots \right) N$$

$$- \sum_{I=0,1} \left(N^T P_2^{(I)} N\right)^+ N^T P_2^{(I)} \left(C_{0I} + m_\pi^2 D_{2I} + C_{2I}\nabla^2 + \ldots\right) N$$

$$+ \ldots$$

more derivatives,
more fields,
isospin violation

projector on isospin *I*

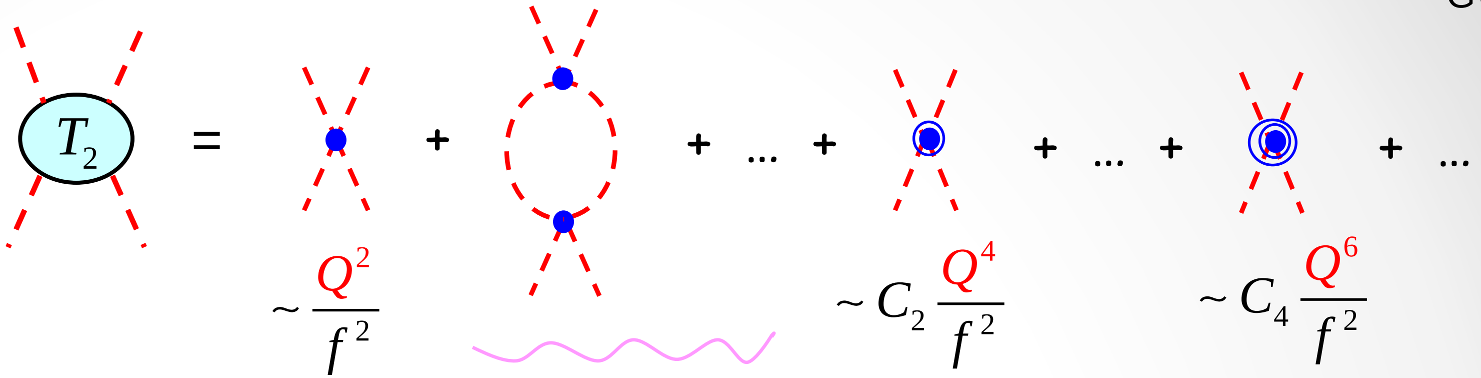



perturbative

naïve dimensional analysis (NDA)

Manohar, Georgi ‘87

Quintessential example:
pion-pion scattering




$$Q = \mathcal{O}(p) = \mathcal{O}(m_\pi)$$

$$\sim \frac{1}{(4\pi)^2 f^4}\left[\cancel{\gamma_4 \Lambda^4} + \gamma_2 \Lambda^2 Q^2 + \gamma_0 Q^4 \left(\ln(\mu/\Lambda) + \ln(Q/\mu)\right) + \gamma_{-2}\frac{Q^6}{\Lambda^2} + \ldots\right]$$

chiral symmetry $\qquad \gamma_i = \mathcal{O}(1)$

$$\sim \frac{Q^2}{f^2(\Lambda)}\left[1 + \gamma_2\left(\frac{\Lambda}{4\pi f(\Lambda)}\right)^2 + \ldots\right] \qquad \sim \frac{Q^4}{f^2}\left(C_2(\Lambda) + \gamma_0 \frac{\ln(\mu/\Lambda)}{(4\pi f(\Lambda))^2} + \ldots\right) \qquad \sim \frac{Q^6}{f^2}\left(C_4(\Lambda) + \frac{\gamma_{-2}}{(4\pi f(\Lambda)\Lambda)^2} + \ldots\right)$$

renormalization

$$\frac{1}{f^2(\Lambda)} \sim \frac{1}{f_\pi^2}\left[1 - \gamma_2\left(\frac{\Lambda}{4\pi f_\pi}\right)^2 + \ldots\right] \qquad C_2(\Lambda) \sim -\gamma_0 \frac{\ln(\mu/\Lambda)}{(4\pi f_\pi)^2} + C_{2\pi} + \ldots \qquad C_4(\Lambda) \sim -\frac{\gamma_{-2}}{(4\pi f_\pi \Lambda)^2} + C_{4\pi} + \ldots$$

naturalness $\quad \boxed{f_\pi = \mathcal{O}(M_{\rm hi}/4\pi)} \quad \boxed{C_{2n\pi} = \mathcal{O}\left(M_{\rm hi}^{-2n}\right)} \quad \Rightarrow$ expansion in $Q^2/M_{\rm hi}^2$

perturbation theory self-consistent

arbitrary diagram

naturalness

$(4\pi)^{-2}$ per loop

number of fields in operator

Weinberg '89

$$\gamma_i = \mathcal{O}\left(\frac{(4\pi)^{N-2}}{M_{\text{hi}}^{D-4}}\gamma_i^{\text{red}}\right)$$

dimension of operator

reduced coupling

$$\gamma_i^{\text{red}} = \mathcal{O}\left(\left(g^{\text{red}}\right)^{\#}\right)$$

reduced underlying theory parameter

number of insertions

NDA: naïve dimensional analysis

examples:

$$f_\pi^{-2} = \mathcal{O}\left(\frac{(4\pi)^2}{M_{\text{hi}}^2}\left(f_\pi^{-2}\right)^{\text{red}}\right) \Rightarrow \boxed{f_\pi = \mathcal{O}\left(M_{\text{hi}}/4\pi\right)}$$

$$\left(f_\pi^{-2}\right)^{\text{red}} = \mathcal{O}\left(\left(\frac{g_s}{4\pi}\right)^n\right)$$

$$\frac{C_{2n\pi}}{f_\pi^2} = \mathcal{O}\left(\frac{(4\pi)^2}{M_{\text{hi}}^{2n+2}}\left(\frac{C_{2n\pi}}{f_\pi^2}\right)^{\text{red}}\right) \Rightarrow \boxed{C_{2n\pi} = \mathcal{O}\left(M_{\text{hi}}^{-2n}\right)}$$

$$\left(\frac{C_{2n\pi}}{f_\pi^2}\right)^{\text{red}} = \mathcal{O}\left(\left(\frac{g_s}{4\pi}\right)^n\right)$$

$$g_s = \mathcal{O}(4\pi)$$

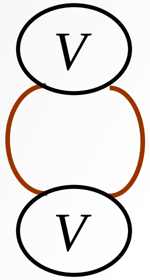
consistency

**renormalization + naturalness: NDA when perturbative**

# Are nuclear amplitudes perturbative?

Weinberg's IR enhancement

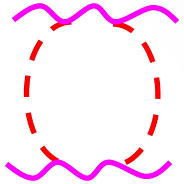
(after renormalization)




$$\sim \frac{m_N Q}{4\pi}$$

4π enhancement compared to NDA

vs.



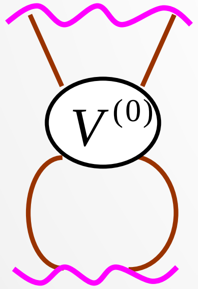
$$\sim \frac{Q^2}{(4\pi)^2}$$

Chiral Perturbation Theory

$$V(Q, M_{lo}, M_{hi}, \lambda, \Lambda) \propto \sum_{\mu=\mu_{\min}}^{\infty} \underbrace{\left[\frac{Q}{M_{hi}}\right]^{\mu} \tilde{F}^{(\mu)}\left(\frac{Q}{M_{lo}}, \frac{Q}{\Lambda}, \frac{\lambda}{Q}; \gamma_i^{(\leq\mu)}\left(\frac{M_{lo}}{\Lambda}, \frac{\lambda}{M_{lo}}\right)\right)}_{V^{(\mu)}(Q, M_{lo}, M_{hi}, \lambda, \Lambda)}$$

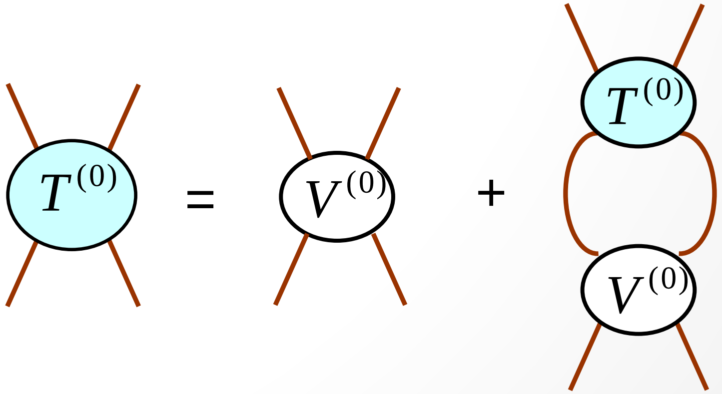
(after renormalization)

$$V^{(0)} \sim \frac{4\pi}{m_N M_{\mathrm{lo}}}$$




$$\sim \frac{Q}{M_{\mathrm{lo}}}$$

⇒ expansion in $Q/M_{\mathrm{lo}}$ ⇒




b.s. at

$$B \sim \frac{M_{\mathrm{lo}}^2}{m_N}$$

but still keep perturbative expansion in $Q/M_{hi}$

CANNOT JUST COUNT POWERS OF $Q$

$$V^{(\mu)}(Q \sim M_{\rm lo}) \sim \frac{4\pi}{m_N M_{\rm lo}} \left(\frac{Q}{M_{\rm hi}}\right)^{\mu}$$

various orders in the potential $\xrightarrow{\text{DWBA}}$ same order in amplitude

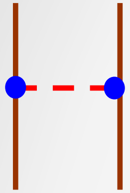
e.g. $V^{(1)2}, V^{(2)}$ in $T^{(2)}$

$$\left.\begin{array}{c}\text{potential must depend on regulator} \\ \text{each order in potential} \\ \text{must contain enough LECs}\end{array}\right\} \Rightarrow \text{amplitude renormalization}$$

↳ not trivial when resuming higher orders

e.g. $V^{(1)\infty}$

# nonperturbative pions



$$\sim \frac{1}{f_\pi^2} \sim \frac{4\pi}{m_N M_{NN}}$$ LO

$$\Rightarrow \quad B \sim \frac{M_{NN}^2}{m_N} \approx 10\ \text{MeV}$$

$$M_{NN} \equiv \frac{4}{g_A^2}\frac{4\pi f_\pi}{m_N} f_\pi \simeq 300\ \text{MeV}$$

Kaplan, Savage, Wise '98

$$\left.\begin{array}{r} \boxed{m_N = \mathcal{O}(4\pi f_\pi)} \\ \boxed{g_A = \mathcal{O}(1)} \end{array}\right\} \quad \boxed{M_{NN} = \mathcal{O}(f_\pi)}$$

suggest

$$\boxed{\begin{array}{c} M_{\text{lo}} \sim M_{NN}, m_\pi \\ M_{\text{hi}} \sim M_{QCD} \equiv 4\pi f_\pi, m_N, \ldots \end{array}}$$

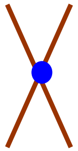
Weinberg's prescription

Weinberg '90'91
Ordóñez, vK '92
…

1. NDA for short-range interactions

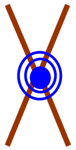
2. solve dynamical equation *exactly*
   with truncated potential

not consistent with RG!



~~dim. anal.~~ $\cancel{\sim \frac{1}{M_{QCD}^2}}$

NDA $\sim \frac{1}{f_\pi^2} \sim \frac{4\pi}{m_N M_{\text{lo}}}$ LO



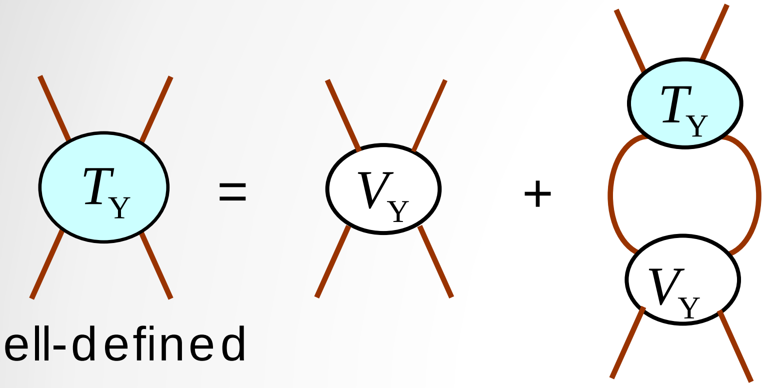
NDA $\sim \frac{4\pi}{m_N M_{\text{lo}}} \frac{Q^2}{M_{\text{hi}}^2}$ $\text{N}^2\text{LO}$

Kaplan, Savage, Wise ’96
Beane, Bedaque, Savage, vK ‘01
Long, Yang ‘13

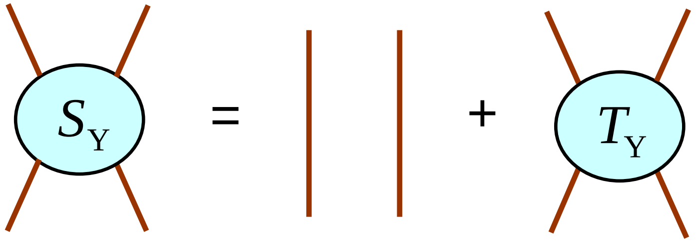
$^1S_0$

$$V_{21}^{(0)}(\vec{q}) = C_{01} + \frac{4\pi}{m_N M_{NN}}\left[1 - \frac{m_\pi^2}{\vec{q}^{\,2} + m_\pi^2}\right] \equiv \frac{4\pi}{m_N M_{NN}}\tilde{C}_{01} + V_Y(\vec{q})$$

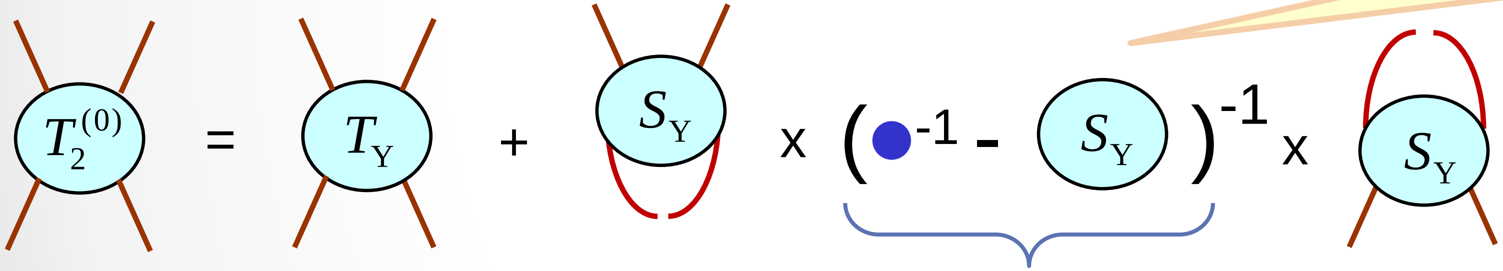
singular regular

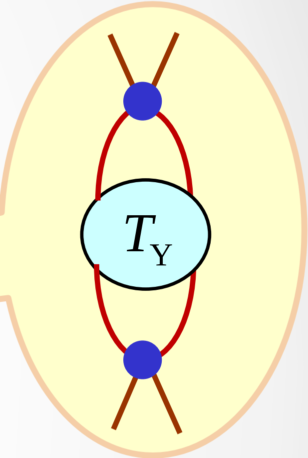



well-defined




well-defined




$$\propto \tilde{C}_{01}^{-1}(\Lambda) + \frac{\Lambda}{M_{NN}} + \#\frac{m_\pi^2}{M_{NN}^2}\ln\left(\frac{\Lambda}{M_{NN}}\right) + \#\frac{k^2}{M_{NN}\Lambda} + \dots$$

$$\propto \frac{1}{M_{NN}^2}\int d^3l_1 \int d^3l_2 \, \frac{1}{l_1^2}\frac{1}{l_2^2}\frac{m_\pi^2}{(l_1 - l_2)^2}$$

$$\propto \frac{m_\pi^2}{M_{NN}^2}\ln\Lambda$$

LO

NOT NDA!

$$D_{21} = \mathcal{O}\left(\frac{4\pi}{m_N M_{NN}^3}\right)$$

$$C_{21} = \mathcal{O}\left(\frac{4\pi}{m_N M_{NN}^2 M_{hi}}\right)$$

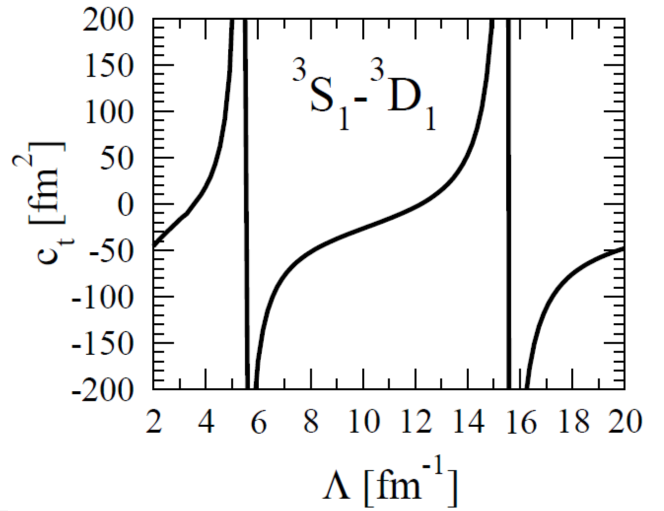
NLO

NOT NDA!

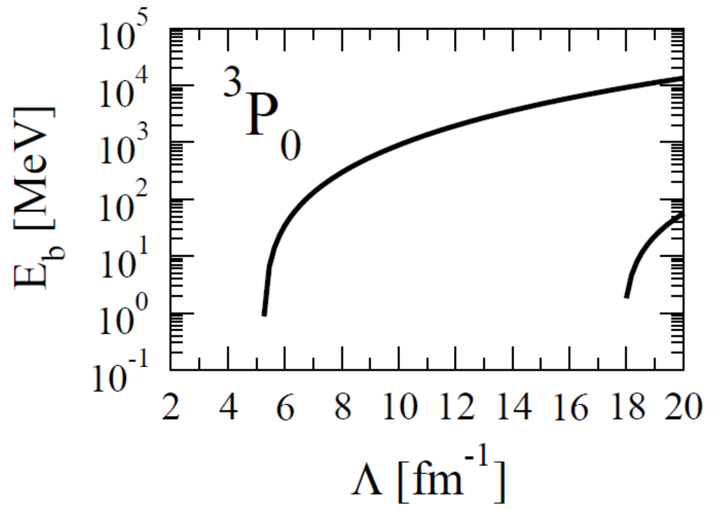
# Attractive-tensor channels

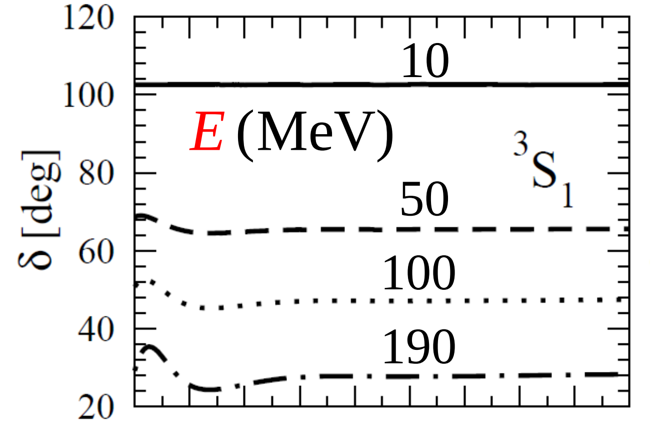
$$V_{20}^{(0)}(\vec{q}) = C_{00} - \frac{4\pi}{m_N M_{NN}} \frac{S_{12}(\vec{q})}{\vec{q}^2 + m_\pi^2} \qquad S_{12}(\vec{q}) = \vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q} - \vec{q}^2 \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

singular




Frederico, Timóteo, Tomio ’99
Beane, Bedaque, Savage, vK ’02
Nogga, Timmermans, vK ’05
…

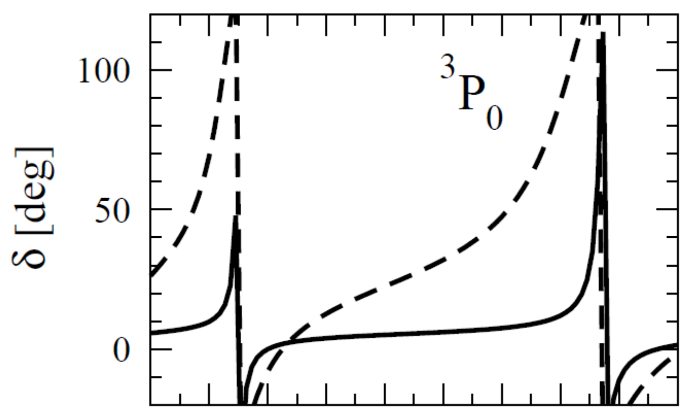



Nogga, Timmermans, vK,
*Phys. Rev. C* 72 (2005) 054006

Problems!

Nogga, Timmermans, vK ’05
Pavón Valderrama, Ruiz-Arriola ’06
…

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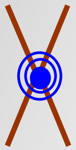
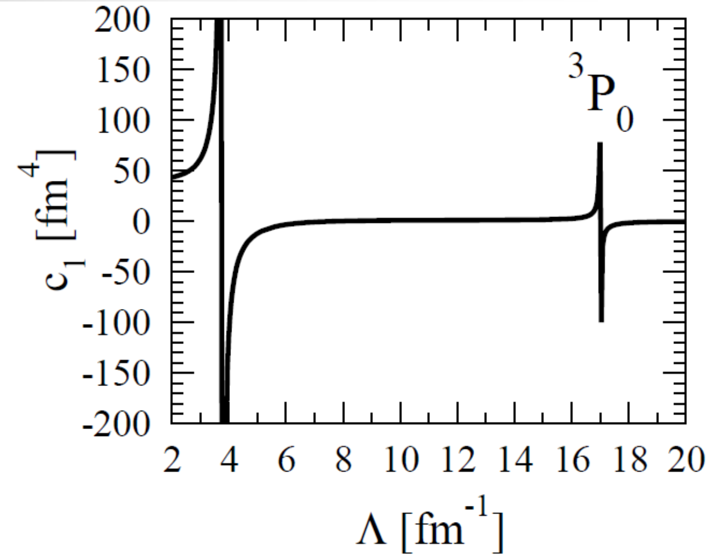
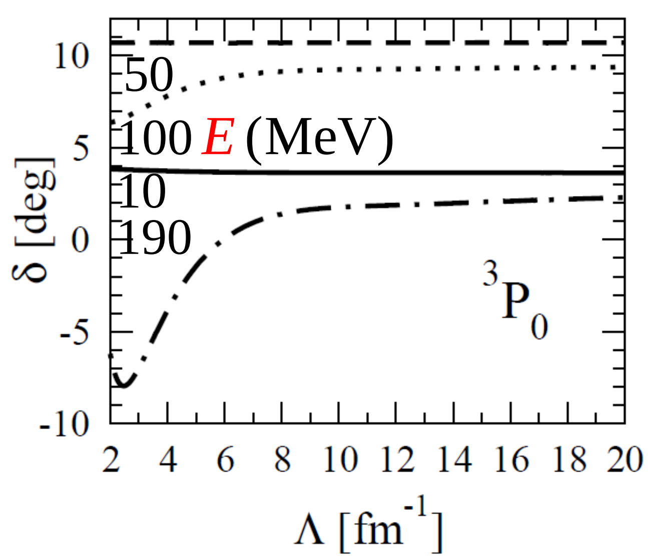


$$\overset{\text{NDA}}{\sim} \cancel{\frac{4\pi}{m_N M_{lo}}} \frac{Q^2}{M_{hi}^2} \qquad \sim \frac{Q^2}{f_\pi^4} \sim \frac{4\pi}{m_N M_{NN}} \frac{Q^2}{M_{lo}^2}$$

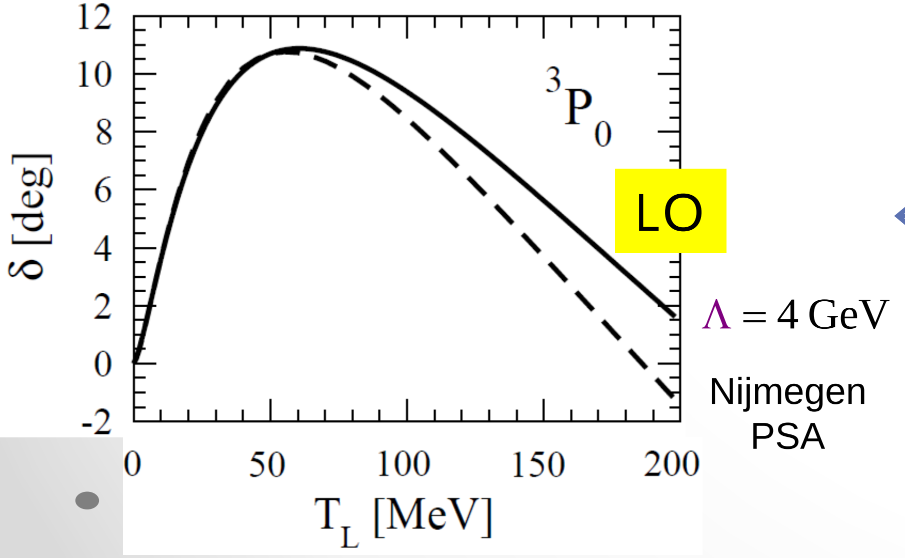
LO

NOT NDA!



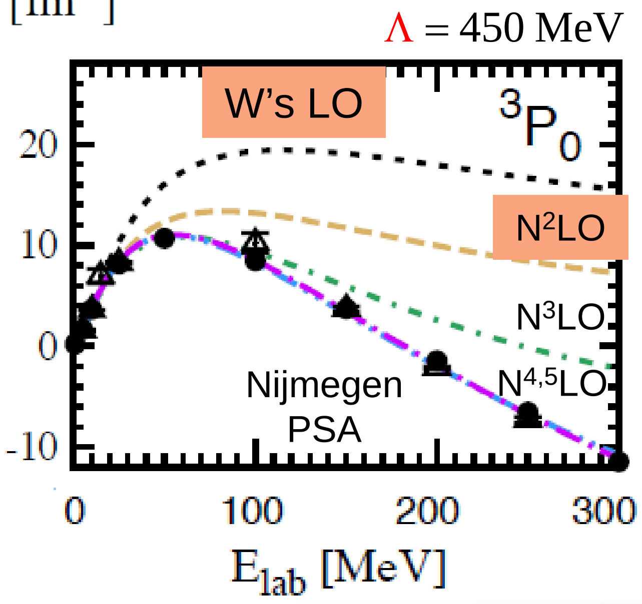




same for every attractive eigenchannel with fully iterated one-pion exchange

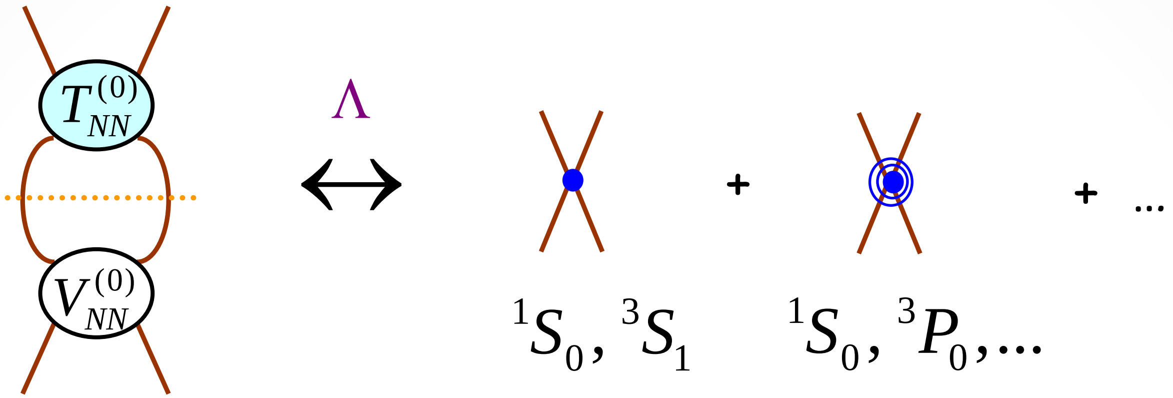



Nogga, Timmermans, vK, *Phys. Rev. C* 72 (2005) 054006

cf.




Reinert, Krebs, Epelbaum, *Eur. Phys. J. A* 54 (2018) 86




+ NATURALNESS

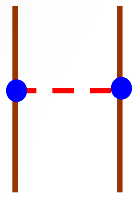
= NDA

≠ NDA

renormalization + naturalness: deviation from NDA already at LO

Predictive power?

## ➢ *(partly)* perturbative pions



$$\sim \frac{4\pi}{m_N M_{\rm lo}} \left( \frac{M_{\rm lo}}{M_{\rm hi}} \right)^{n_l} \qquad \begin{cases} n_l = 0, & l < l_c \\ n_l \geq 1, & l \geq l_c \end{cases}$$

LO

N$^{n_l}$LO

$$l_c = 0$$

effectively $M_{\rm hi} \sim M_{NN}$

does not seem to converge
beyond Pionless EFT
**in low attractive-tensor waves**

Kaplan, Savage, Wise '98
...

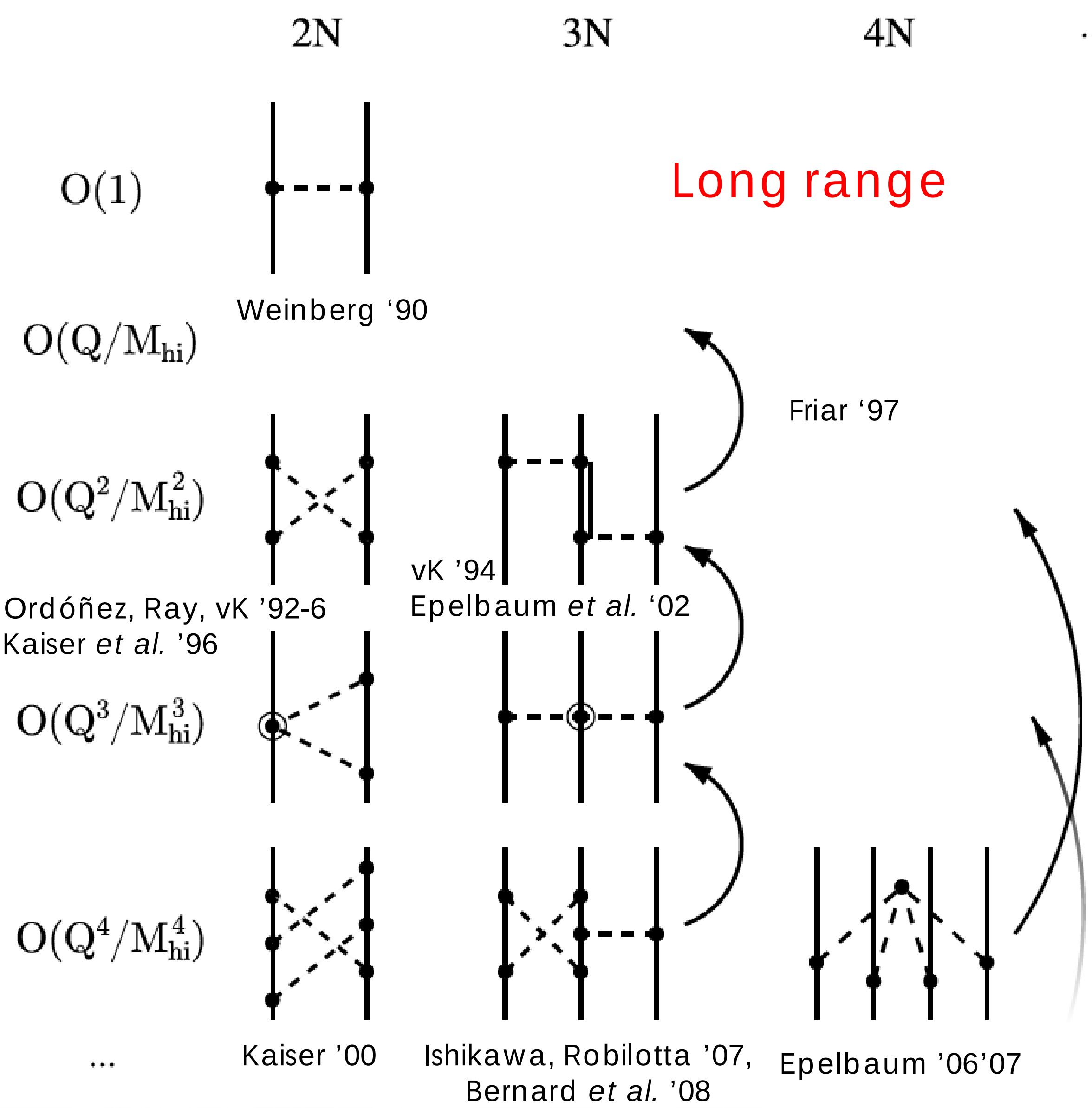
Fleming, Mehen,
Stewart '01
...

$$l_c > 0$$

accounts for
angular-momentum barrier

Nogga, Timmermans, vK '05
Birse '06
Pavón Valderrama '11'11
Long ,Yang '11'12'12
...

# (isospin-symmetric) potential




Hammer, König, vK, *Rev. Mod. Phys.* 92 (2020) 025004

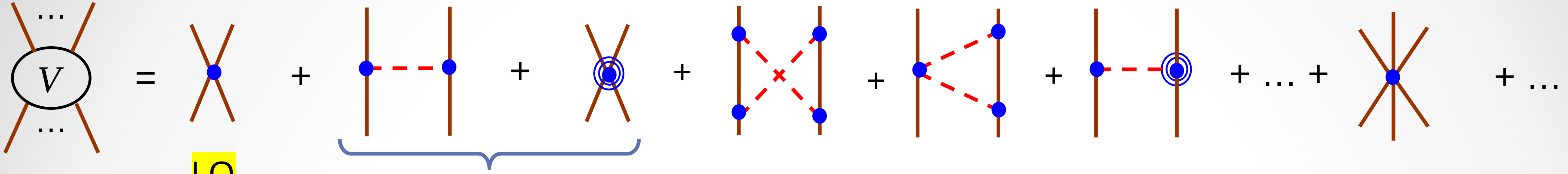
## Short range

vK, *Front. in Phys.* 8 (2020) 79

| | $^1S_0$ | $^3S_1$ | $\epsilon_1$ | $^3P_0$, $^3P_2$ | $^1P_1$, $^3P_1$ | $\epsilon_2$ | $^3D_2$, $^3D_3$ |
|---|---|---|---|---|---|---|---|
| LO | 1 | 1 | | $p'p$ | | | $p'^2p^2$ |
| NLO | $p'^2+p^2$ | | | | | | |
| N$^2$LO | $p'^4+p^4$ | $p'^2+p^2$ | $p^2$ | $p'p\,(p'^2+p^2)$ | $p'p$ | $p'p\,p^2$ | $p'^2p^2\,(p'^2+p^2)$ |
| N$^3$LO | $p'^6+p^6$ | | | | | | |

Table 3: Schematic momentum dependence of the lowest-order contact interactions in the $2N$ system up to $D$ waves, according to Refs. [62, 20, 31, 104, 105, 68].

Bedaque, Beane, Savage, vK '02
Nogga, Timmermans, vK '05
Long, vK '08
Pavón Valderrama '11
Long, Yang, '11'12
...

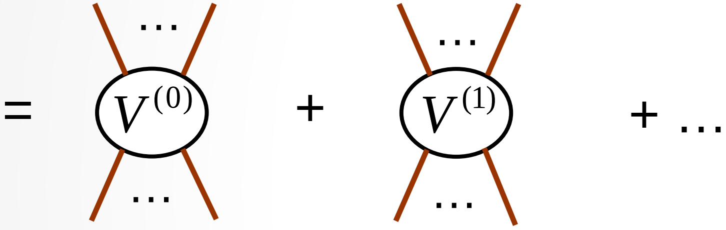


LO

LO for $Q \gtrsim M_{NN}$, NLO for $Q \lesssim M_{NN}$

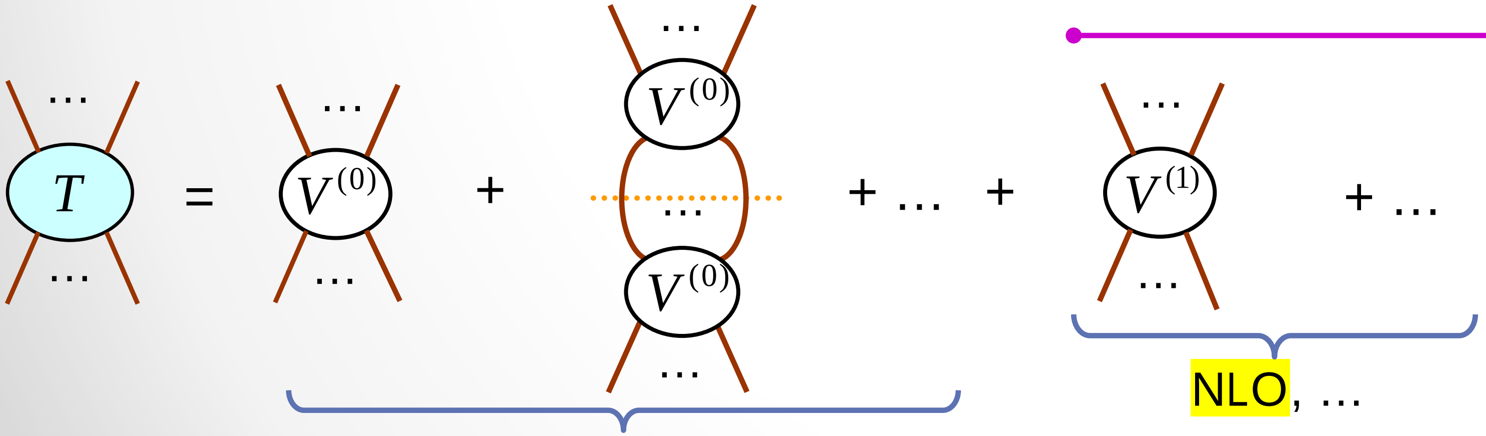
$$M_{NN} \equiv \frac{4}{g_A^2}\frac{4\pi f_\pi}{m_N} f_\pi \simeq 300\ \text{MeV}$$

Kaplan, Savage, Wise '98



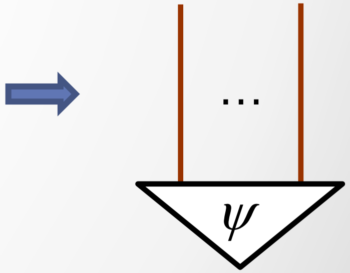
LO NLO

distorted-wave perturbation theory



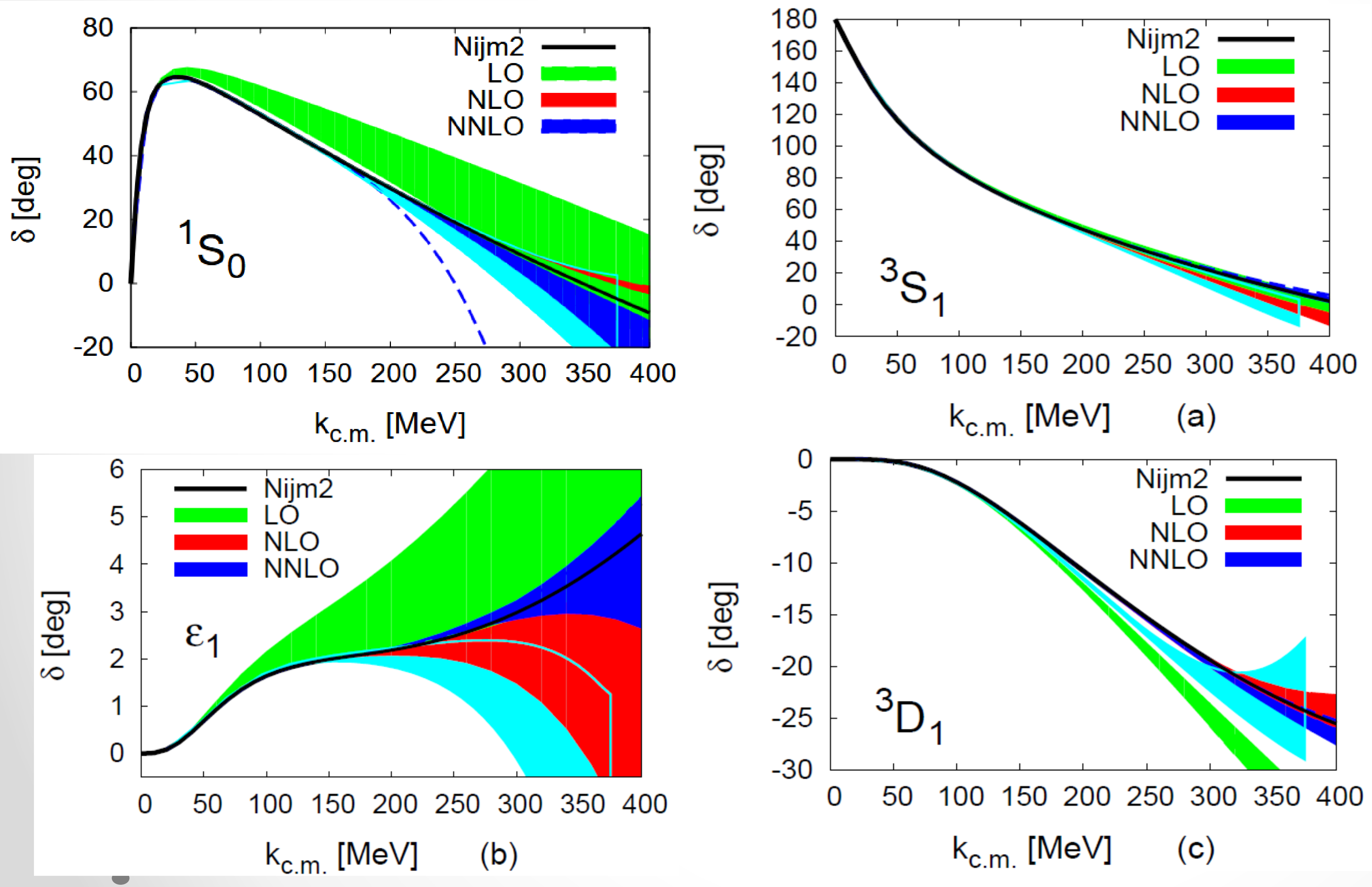
NLO, …

LO



Pavón Valderrama '11'12
Long, Yang '12'13
...

# $A = 2$



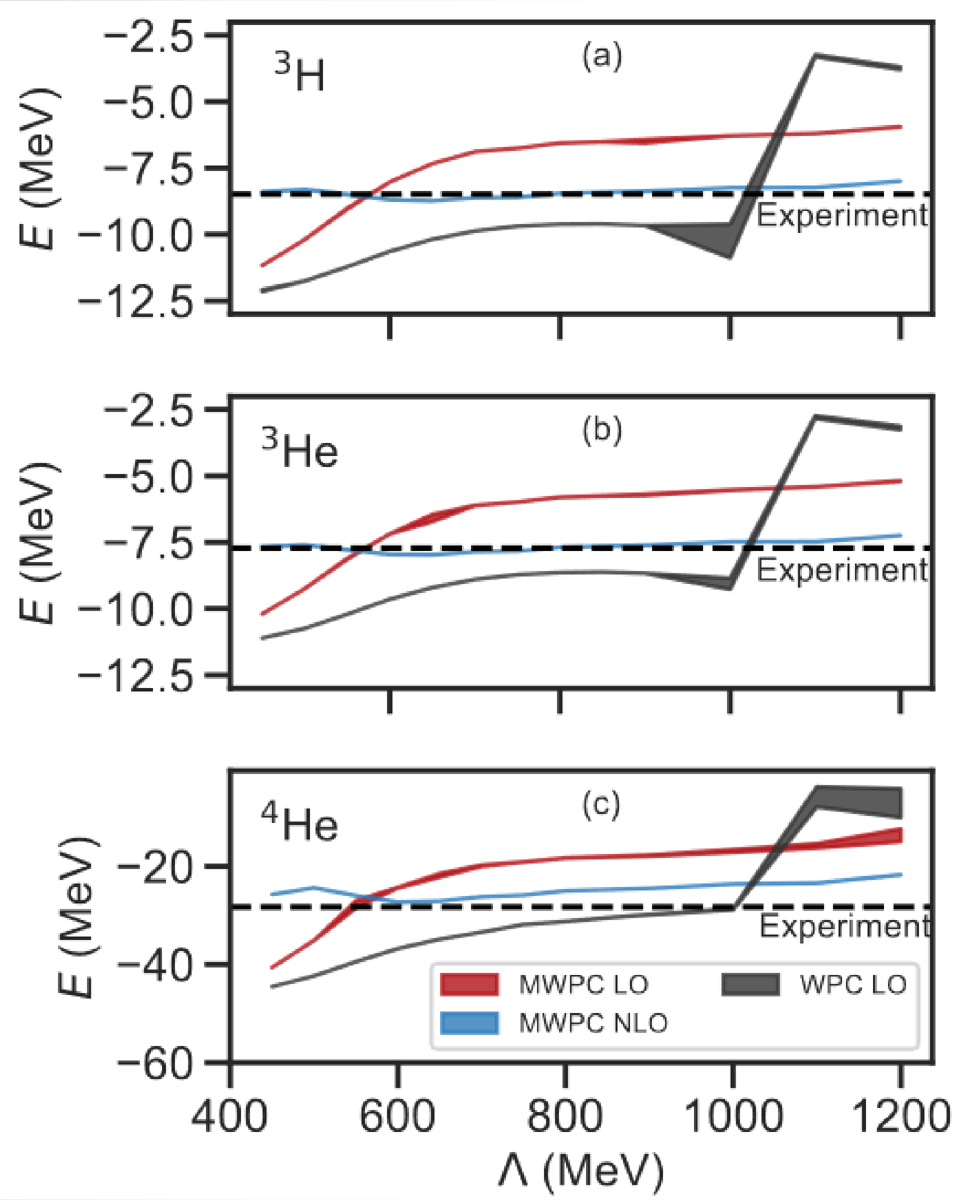

Pavón Valderrama, *Phys. Rev. C* 83 (2011) 024003

cyan:
NNLO in Weinberg's scheme

bands:
coordinate-space cutoff variation 0.6 – 0.9 fm

Nogga, Timmermans, vK '05
Song, Lazauskas, vK '17
Yang, Ekström, Forssén, Hagen '21
Yang, Ekström, Forssén, Hagen, Rupak, vK '23
...

$A \geq 3$



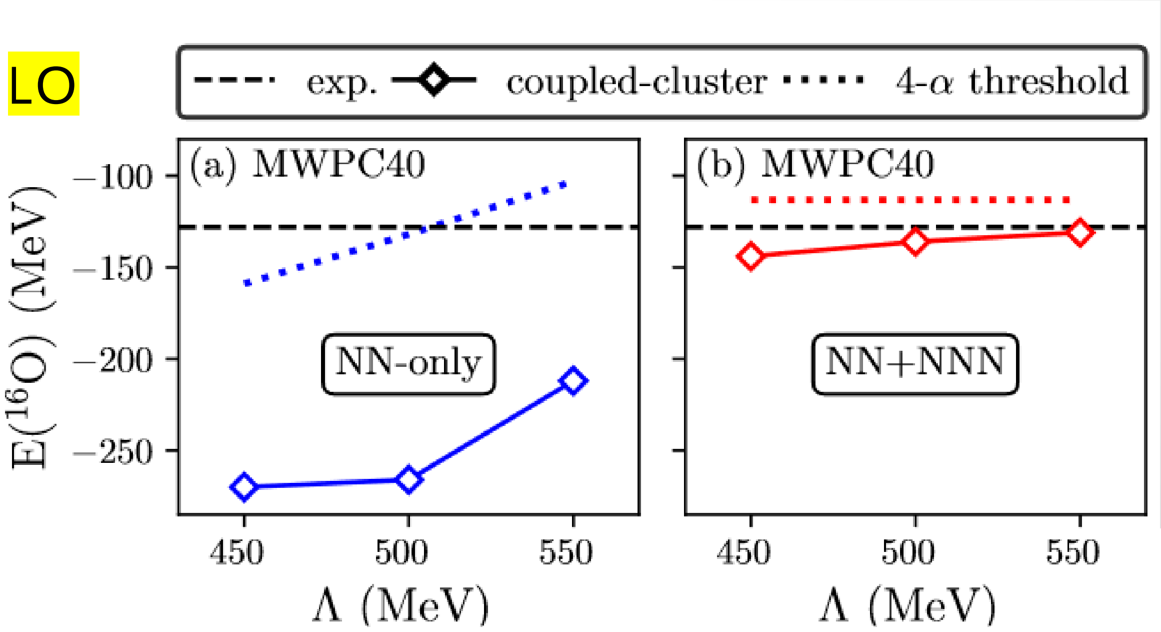

NDA "LO"
LO
NLO
(Deltaless)

Yang, Ekström, Forssén, Hagen,
*Phys. Rev. C* 103 (2021) 054304

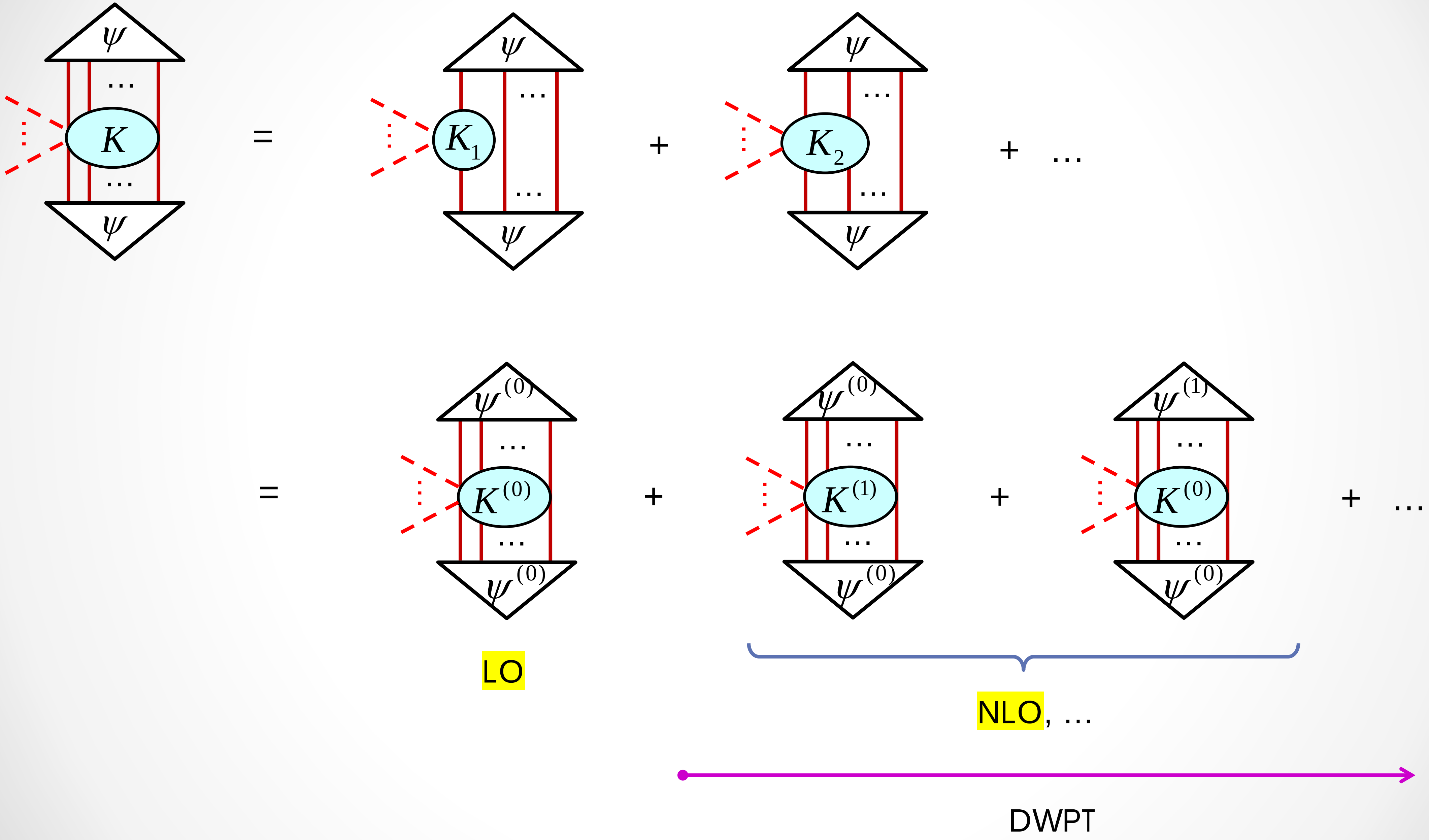
works well for light nuclei

combinatorial enhancement of few-body forces for larger nuclei?

LO




Yang, Ekström, Forssén, Hagen, Rupak, vK,
Eur. Phys. J. A 59 (2023) 233



LO
NLO, …
DWPT

# Nuclear effective field theory: status and perspectives

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*University of Arizona,*
*Tucson, AZ 85721,*
*USA*

(Dated: July 1, 2019)

The nuclear physics landscape has been redesigned as a sequence of effective field theories (EFTs) connected to the Standard Model through symmetries and lattice simulations of Quantum Chromodynamics (QCD). EFTs in this sequence are expansions around different low-energy limits of QCD, each with its own characteristics, scales, and ranges of applicability regarding energy and number of nucleons. We review each of the three main nuclear EFTs—Chiral, Pionless, Halo/Cluster—highlighting their similarities, differences, and connections. In doing so, we survey the structural properties and reactions of nuclei that have been derived from the *ab initio* solution of the few- and many-body problem built upon EFT input.

arXiv:1906.12122v1 [nucl-th] 28 Jun 2019

# Most applications of chiral potentials and kernels to date violate RG invariance

- DWBA requires new codes
- Renormalization not fully appreciated?

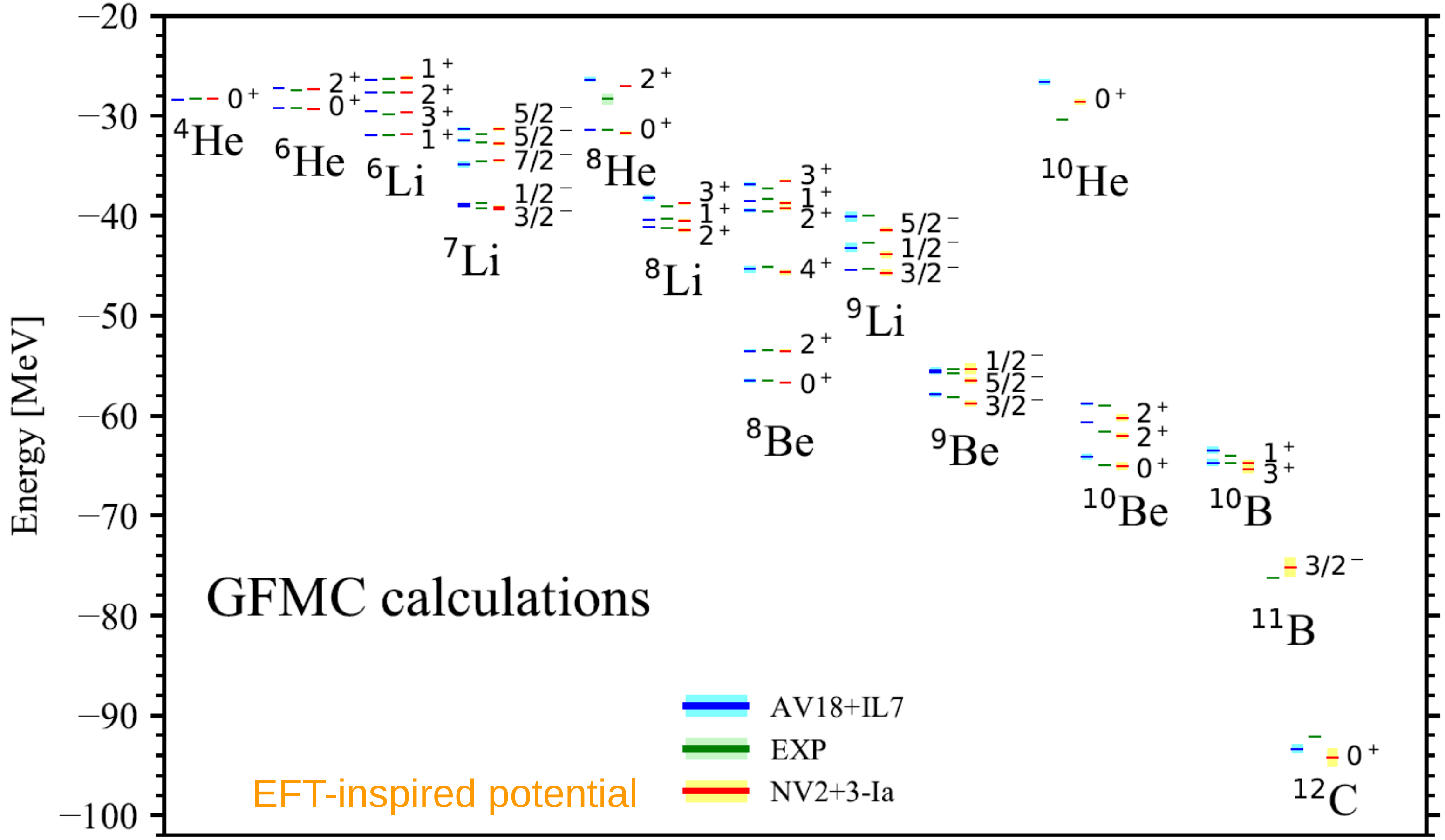
Wikipedia


Thomas Murner, 1512

# Still, a very good model

## Examples

Piarulli *et al.*, *Phys. Rev. Lett.* 120 (2018) 052503



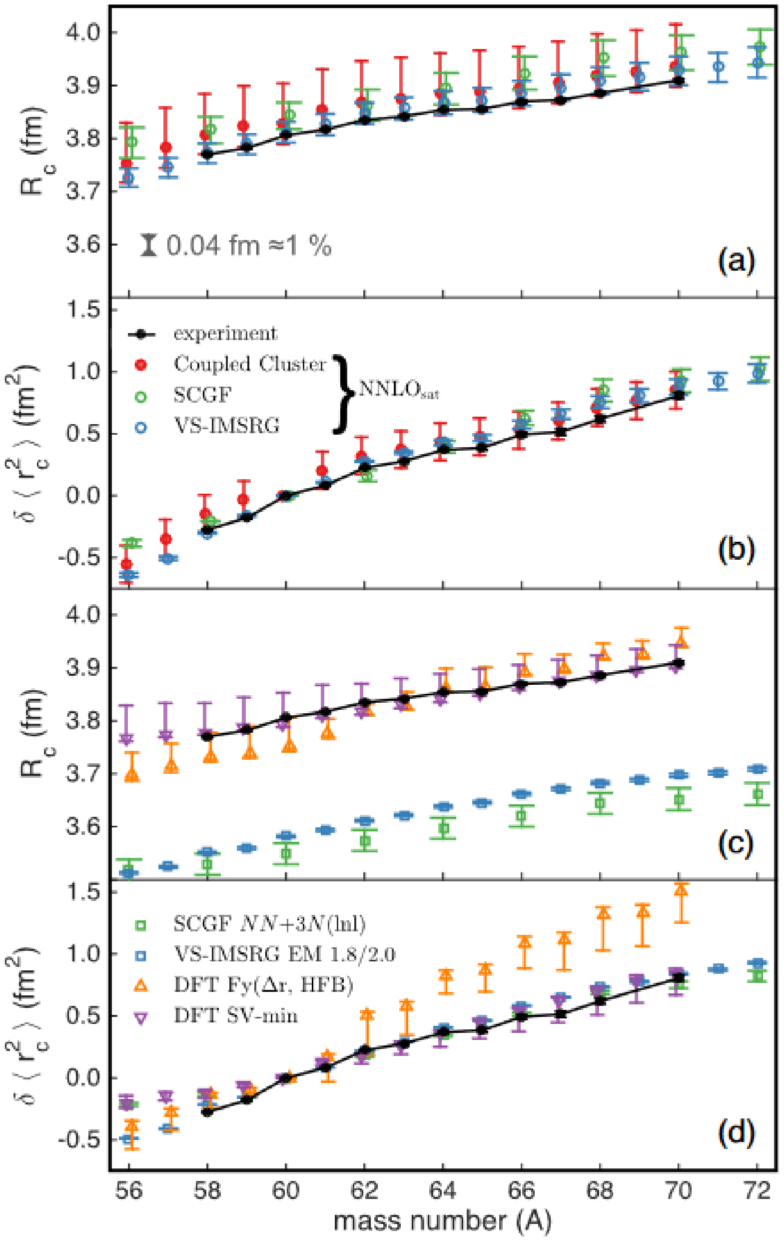




FIG. 2. Nuclear charge radii $R_c$ (a,c) and differentials $\delta\langle r_c^2\rangle^{60,A}$ (b,d) of Ni isotopes with respect to $^{60}$Ni as reference. Experimental data are compared to theoretical results. See text for details.

Malbrunot-Ettanauer *et al.*, *Phys. Rev. Lett.* 128 (2022) 022502

# So, why not pick one cutoff and stick to NDA, as Weinberg suggested?

relation to QCD?

Lynn *et al.*,
*Phys. Rev. Lett.* 116 (2016) 062501

A
R
T
I
F
A
C
T
S

*e.g.*

$$V_{E\tau} = \frac{c_E}{\Lambda_\chi F_\pi^4} \sum_{i<j<k} \sum_{\text{cyc}} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k \delta_{R_{3N}}(\mathbf{r}_{kj}) \delta_{R_{3N}}(\mathbf{r}_{ij}),$$

$$V_{E\mathbb{1}} = \frac{c_E}{\Lambda_\chi F_\pi^4} \sum_{i<j<k} \sum_{\text{cyc}} \delta_{R_{3N}}(\mathbf{r}_{kj}) \delta_{R_{3N}}(\mathbf{r}_{ij}),$$

$$V_{E\mathcal{P}} = \frac{c_E}{\Lambda_\chi F_\pi^4} \sum_{i<j<k} \sum_{\text{cyc}} \mathcal{P}\, \delta_{R_{3N}}(\mathbf{r}_{kj}) \delta_{R_{3N}}(\mathbf{r}_{ij}).$$

→ 0 in
neutron systems

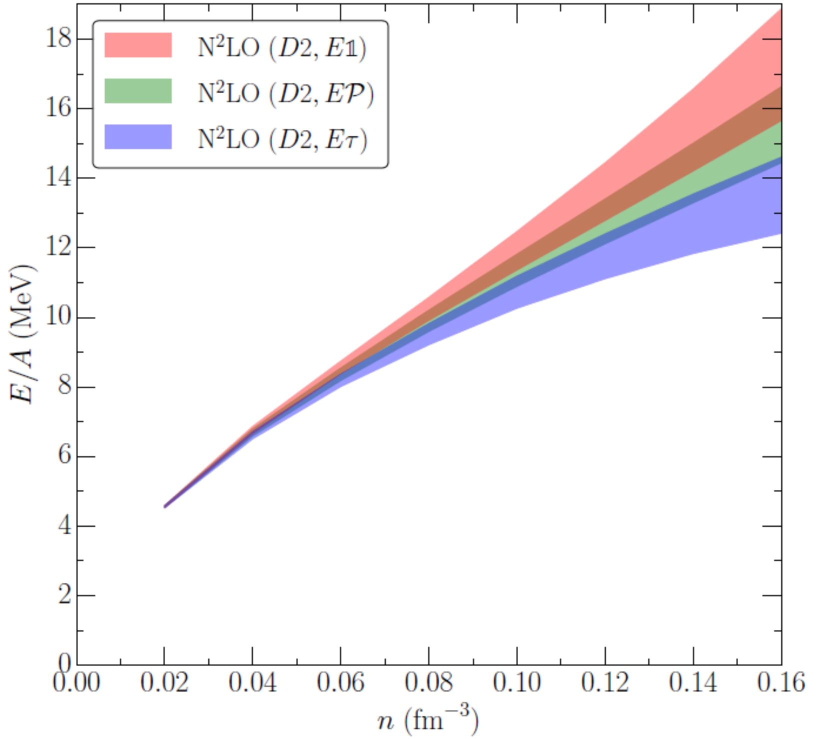



FIG. 3. The energy per particle in neutron matter as a function of density for the NN and full 3N interactions at N$^2$LO with $R_0 = 1.0$ fm. We use $V_{D2}$ and different 3N contact structures: The blue band corresponds to $V_{E\tau}$, the red band to $V_{E\mathbb{1}}$, and the green band to $V_{E\mathcal{P}}$. The green band coincides with the NN + $2\pi$-exchange-only result because both $V_D$ and $V_E$ vanish in this case. The bands are calculated as described in the text.

need a theory!

# Conclusion

EFTs connect symmetry violation
from beyond the Standard Model to nuclear physics
in a controlled and systematic way

Renormalization and naturalness do not imply
naïve dimensional analysis in nonperturbative context

Nuclear observables can be properly renormalized
but most-used approach is model-dependent

# EFT for neutrinoless double-beta decay

Unless $\Delta L = -\Delta B$ , expected BSM dominance:

$Q \sim M_{\mathrm{EW}}$

Weinberg '79
Weldon + Zee '80
...

Majorana '37
...

$$c_5^{\mathrm{red}} = \mathcal{O}\left(\frac{e^2}{(4\pi)^2}\right)$$

$$\boxed{c_5 = \mathcal{O}\left((4\pi)^2 c_5^{\mathrm{red}}\right) = \mathcal{O}\left(e^2\right)}$$

$$\mathcal{L}_{\mathrm{dim}=5} = \frac{c_5}{M_{\cancel{L}}}\left[\left(\ell^T C\tilde{\varphi}\right)\left(\tilde{\varphi}^T \ell\right) + \mathrm{H.c.}\right] = \frac{c_5 v^2}{2M_{\cancel{L}}}\left[\overline{\nu_L}\left(\nu_L\right)^c + \mathrm{H.c.}\right] + \ldots$$

$|\Delta L| = 2$

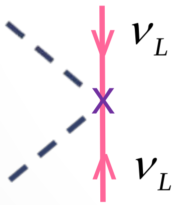
$C = i\gamma_2\gamma_0$

$(\nu_L)^c \equiv C\gamma^{0T}(\nu_L)^\dagger$


*Chi l'ha visto ?*

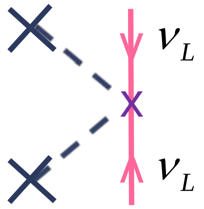
Ettore Majorana, ordinario di fisica teorica all' Università di Napoli, è misteriosamente scomparso dagli ultimi di marzo. Di anni 31, alto metri 1,70, snello, con capelli neri, occhi scuri, una lunga cicatrice sul dorso di una mano. Chi ne sapesse qualcosa è pregato di scrivere al R. P. E. Marianecci, Viale Regina Margherita 66 - Roma.

Wikipedia/Public domain



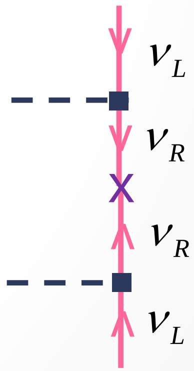

$\langle\varphi\rangle \sim v/\sqrt{2}$

$v \simeq 246\ \mathrm{GeV}$




Majorana mass

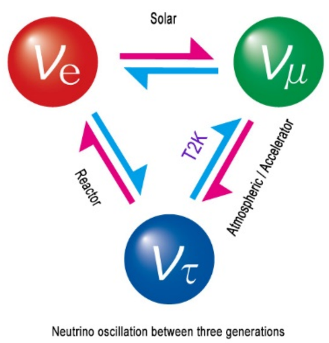
?




$M_{\cancel{L}} \sim M_R \gg M_{\mathrm{EW}}$

$$P_{i\to j} \approx \sin^2(2\theta_{ij})\sin^2\left(\frac{L}{4E}\left(m_i^2 - m_j^2\right)\right)$$

(two flavors)

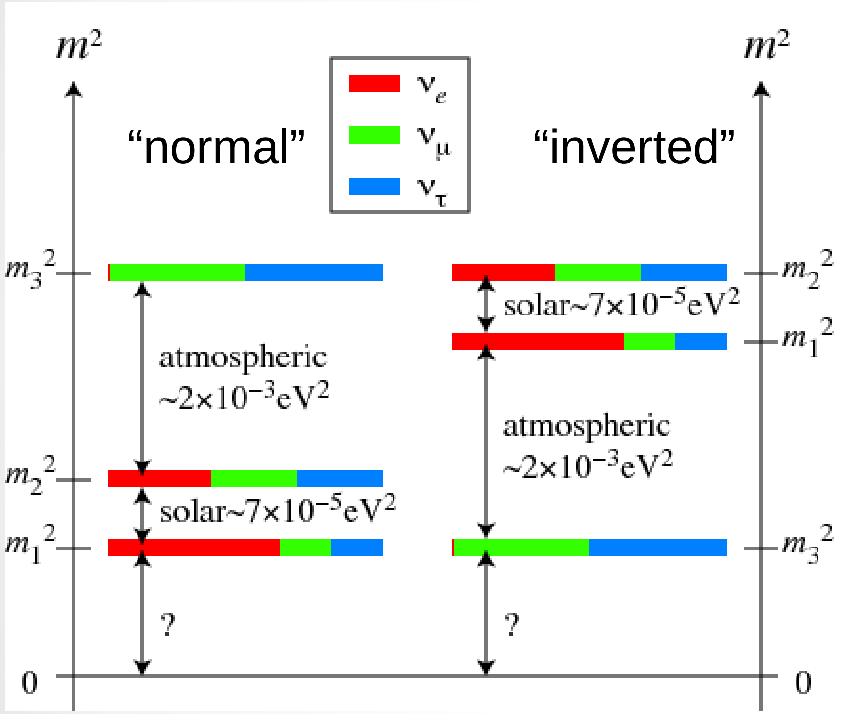


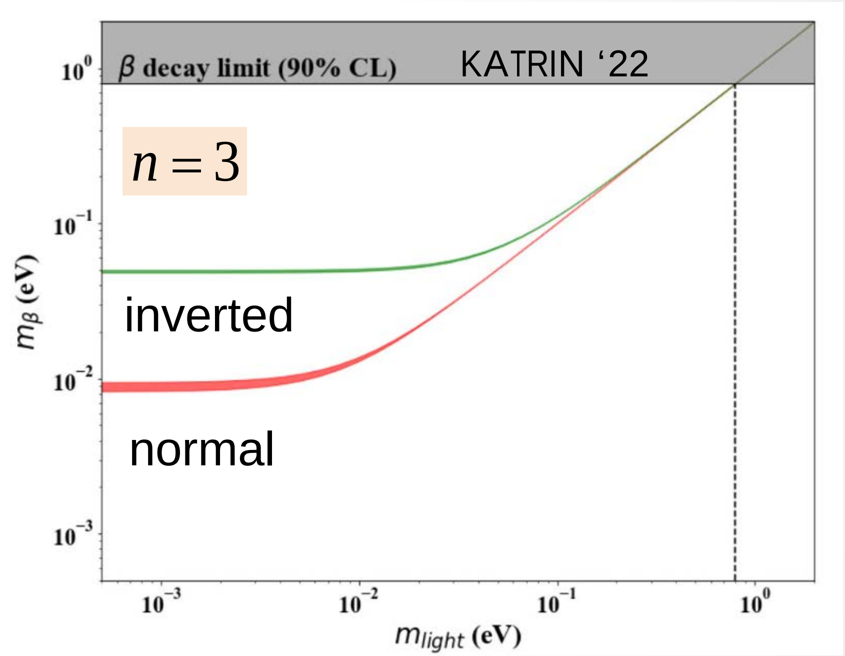

$$m_\beta^2 \equiv \sum_{i=1}^{n} \left|U_{ei}\right|^2 m_{\nu i}^2$$

PMNS matrix elements

neutrino masses

King, *PoS* PLANCK2015 (2015) 068




Gómez-Cadenas *et al.* *Riv. Nuovo Cim.* 46 (2023) 619

neutrino mass discovered!

coincidence?

$$M_{\not L} \sim \frac{c_5}{m_\nu/0.1\ \text{eV}} \cdot 10^{15}\ \text{GeV}$$

comparable to GUT scale!

coincidence?

# $0\nu 2\beta$ decay

lots of nucleons for lots of time ⟹ most sensitive probe of $B-L$ violation

single-beta decay

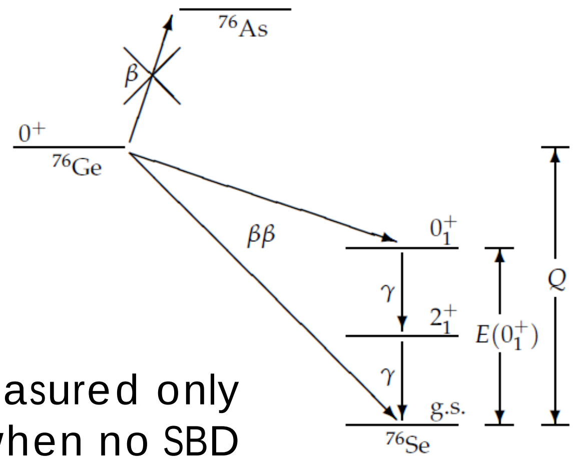
$$^{A}Z \to {}^{A}(Z+1) + e^{-} + \bar{\nu}_e \qquad \left(T_{1/2}^{(\beta)}\right)^{-1} \propto \left(\underbrace{G_F f_\pi^2}_{\sim 10^{-7}}\right)^2$$

two-neutrino double-beta decay

$$^{A}Z \to {}^{A}(Z+2) + 2e^{-} + 2\bar{\nu}_e \qquad \left(T_{1/2}^{(2\nu 2\beta)}\right)^{-1} \propto \left(G_F f_\pi^2\right)^4$$

rare, e.g. $T_{1/2}^{(2\nu 2\beta)}\left({}^{76}\mathrm{Ge} \to {}^{76}\mathrm{Se}\right) = (1.926 \pm 0.094)\cdot 10^{21}\,\mathrm{y}$

GERDA Collab. '15




measured only when no SBD

neutrinoless double-beta decay

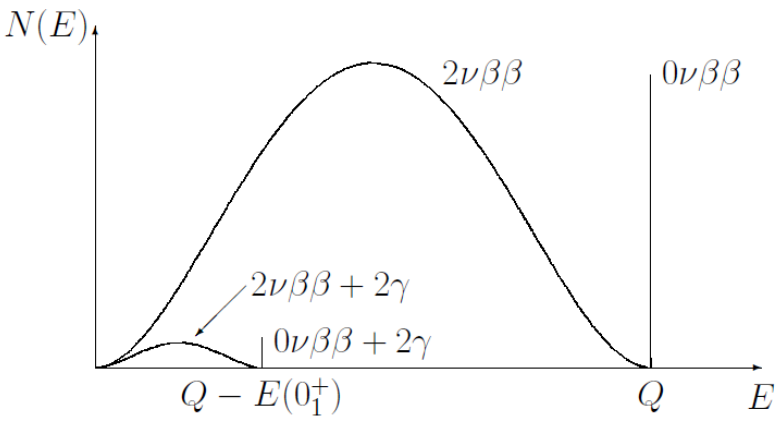
$$^{A}Z \to {}^{A}(Z+2) + 2e^{-} \qquad \Delta L = 2$$

rarer still, e.g.

$$T_{1/2}^{(0\nu 2\beta)}\left({}^{76}\mathrm{Ge} \to {}^{76}\mathrm{Se}\right) > 1.8\cdot 10^{26}\,\mathrm{y}\ \ (90\%\ \mathrm{c.l.})$$

GERDA Collab. '20

Racach '37
Furry '39
...




Duerr *et al.*, *Phys. Rev. D* 84 (2011) 093004 e.g. Haxton, Stephenson ‘84

$$\left(T_{1/2}^{(0\nu 2\beta)}\right)^{-1} \propto \left|M^{(0\nu)}\right|^2 \left|m_{\beta\beta}\right|^2$$

nuclear matrix element

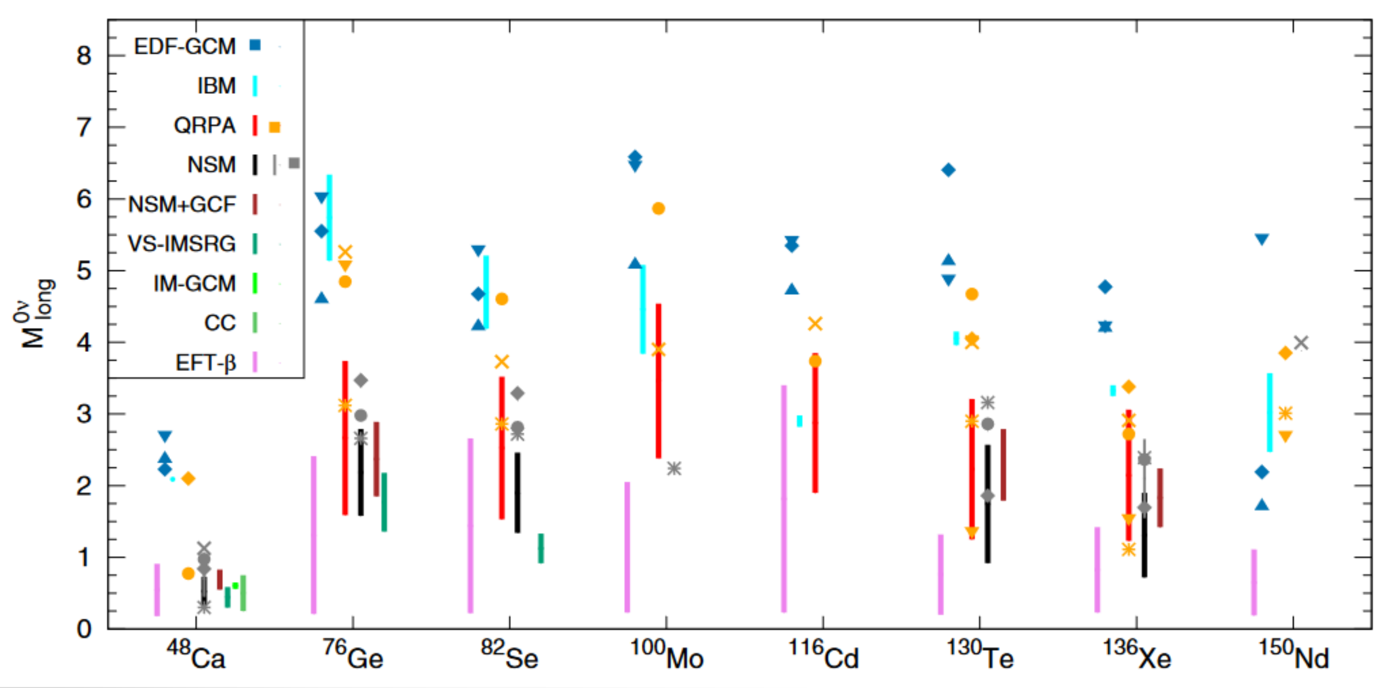
effective Majorana mass $m_{\beta\beta} \equiv \sum_{i=1}^{3} U_{ei}^2 m_{\nu i}$

$$\begin{cases} c_{ij} \equiv \cos\theta_{ij} \\ s_{ij} \equiv \sin\theta_{ij} \end{cases}$$

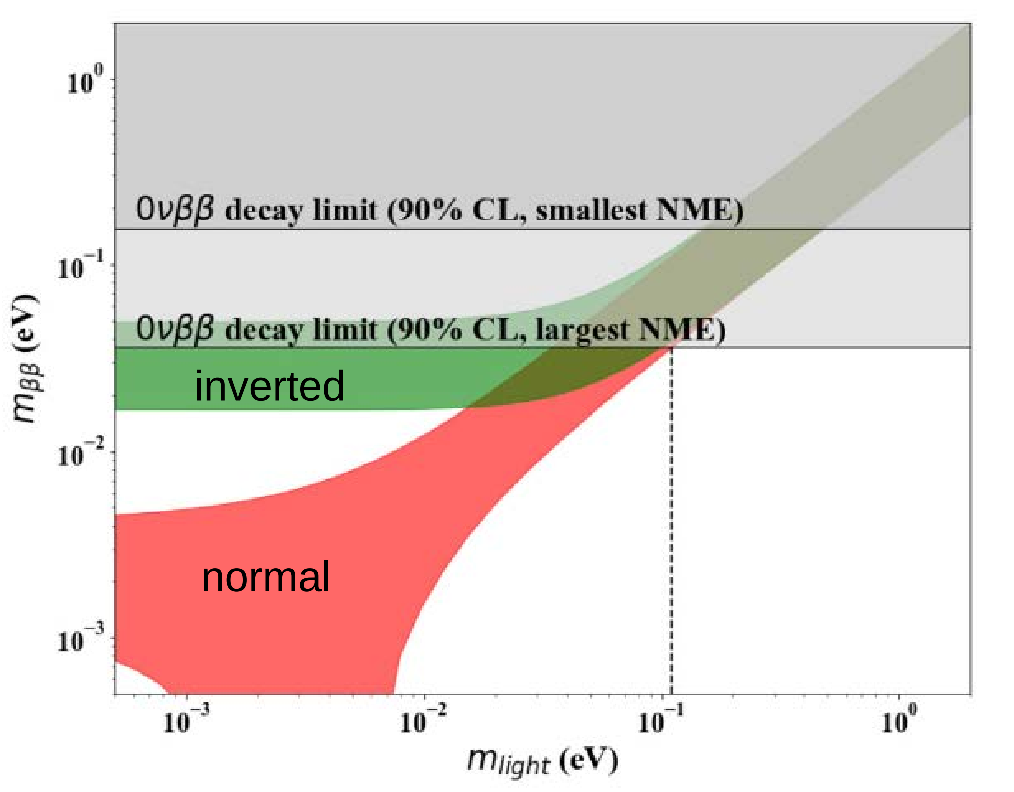
Majorana phases

Dirac phase

$$m_{\beta\beta} = m_{\nu 1}\, c_{12}^2 c_{13}^2 + m_{\nu 2}\, s_{12}^2 c_{13}^2\, e^{i\alpha_{21}} + m_{\nu 3}\, s_{13}^2\, e^{i(\alpha_{31}-2\delta)}$$




Gómez-Cadenas *et al. Riv. Nuovo Cim.* 46 (2023) 619



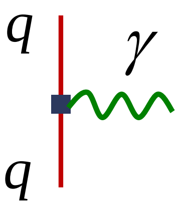
# Renormalization of 0$\nu$2ß decay

Cirigliano, Dekens, De Vries, Graesser, Mereghetti, Pastore + vK ’18
Cirigliano, Dekens, De Vries, Graesser, Mereghetti, Pastore, Piarulli, vK + Wiringa ‘19

$Q \ll M_{\mathrm{EW}}$

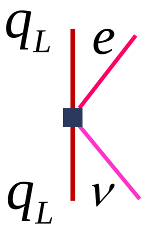
$$\mathcal{L}_{\mathcal{SM}} = \mathcal{L}_{\mathrm{QCD}} + \bar{q}_R \gamma^\mu e A_\mu Q_R^{\mathrm{EM}} q_R + \bar{q}_L \gamma^\mu \left[ e A_\mu Q_L^{\mathrm{EM}} - 2\sqrt{2} G_F \left( V_{ud} \bar{e}_L \gamma_\mu \nu_{eL} Q_L^{\mathrm{W}} + \mathrm{H.c.} \right) \right] q_L + \ldots$$

$$Q_L^{\mathrm{EM}} = Q_R^{\mathrm{EM}} = \frac{\tau_3}{2}$$




$$Q_L^{\mathrm{W}} = \tau^+$$

$$Q_R^{\mathrm{W}} = 0$$

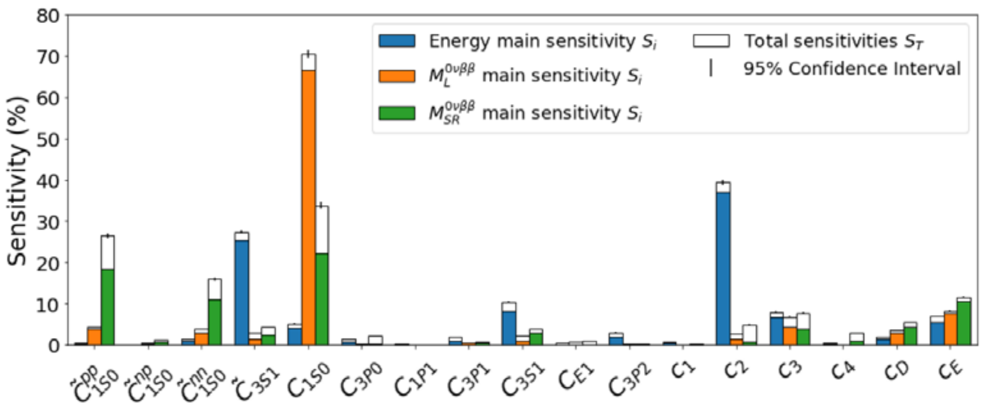



EM isoscalar interactions
neutral weak interactions
...

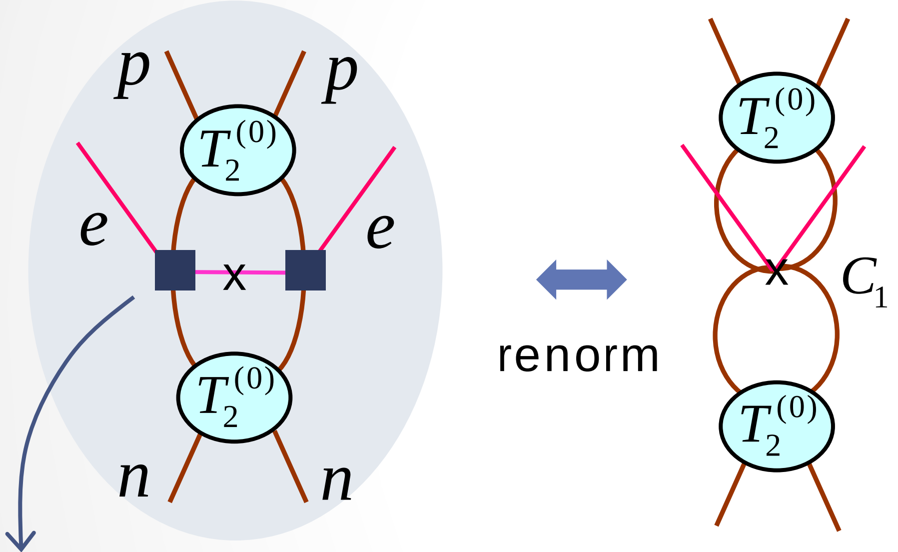
$$\mathcal{L}_{\mathrm{dim}=5} = -\frac{m_{\beta\beta}}{2} \nu_{eL}^T C \nu_{eL} + \ldots \qquad m_{\beta\beta} \equiv \sum_{i=1}^{3} U_{ei}^2 m_{\nu i} \propto \frac{v^2}{M_{\not{L}}}$$

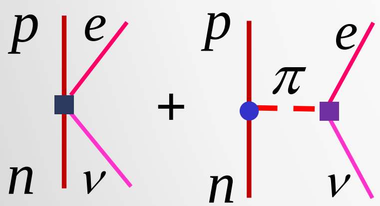
$^1S_0$

cf. chiral pot model




Belley *et al.*, arXiv:2408.02169




$$\propto \int d^3 l_1 \int d^3 l_2 \, \frac{m_N}{l_1^2} \frac{m_N}{l_2^2} \frac{1}{(l_1 - l_2)^2} \propto m_N^2 \ln \Lambda$$

bad news:
unknown parameter

good news:
no new unknown parameter
at NLO

# Alternative: numerical solution of Schrödinger equation in coordinate space

$^1S_0$

$$\tilde{C}\,\delta^{(3)}(\vec{r}) \to \tilde{C}(R)\,\delta_R^{(3)}(\vec{r}) \quad \Rightarrow \quad \tilde{C}(R) = \#R + \#\frac{m_\pi^2}{M_{NN}} R^2 \ln\left(\frac{R}{R_*}\right) + \ldots$$

Beane, Bedaque, Savage + vK '02

here $\delta_R^{(3)}(\vec{r}) = \dfrac{e^{-r^2/R^2}}{\pi^{3/2} R^3}$

$$M_{NN} \equiv \frac{4\pi f_\pi}{g_A^2 m_N} f_\pi \sim f_\pi$$

determined from scattering length

Kaplan, Savage, Wise '98

$$C_1\,\delta^{(3)}(\vec{r}) \to C_1(R)\,\delta_R^{(3)}(\vec{r})$$

$$A^{(L)}_{\Delta L=2} = -\int d^3r\,\psi^-_{\vec{p}'}(\vec{r})\,V_\nu^{(0)}(\vec{r})\,\psi^+_{\vec{p}}(\vec{r})$$

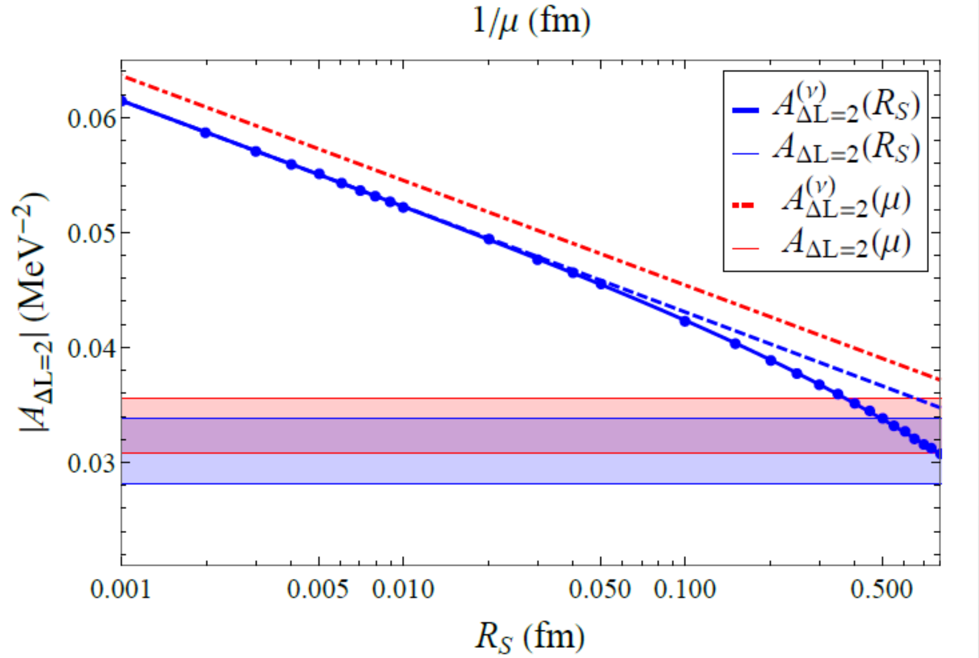
$$A^{(S)}_{\Delta L=2} = -\int d^3r\,\psi^-_{\vec{p}'}(\vec{r})\,V_{\mathrm{ct}}^{(0)}(\vec{r})\,\psi^+_{\vec{p}}(\vec{r})$$

$$A_{\Delta L=2} = A^{(L)}_{\Delta L=2} + A^{(S)}_{\Delta L=2}$$

$$\Downarrow$$

$$\frac{M_{NN}^2 C_1(\mu)}{\tilde{C}^2(\mu)} \simeq -\left(1+2g_A^2\right)\ln\left(\frac{R}{R_\nu}\right) + \#R + \ldots$$

determined how?

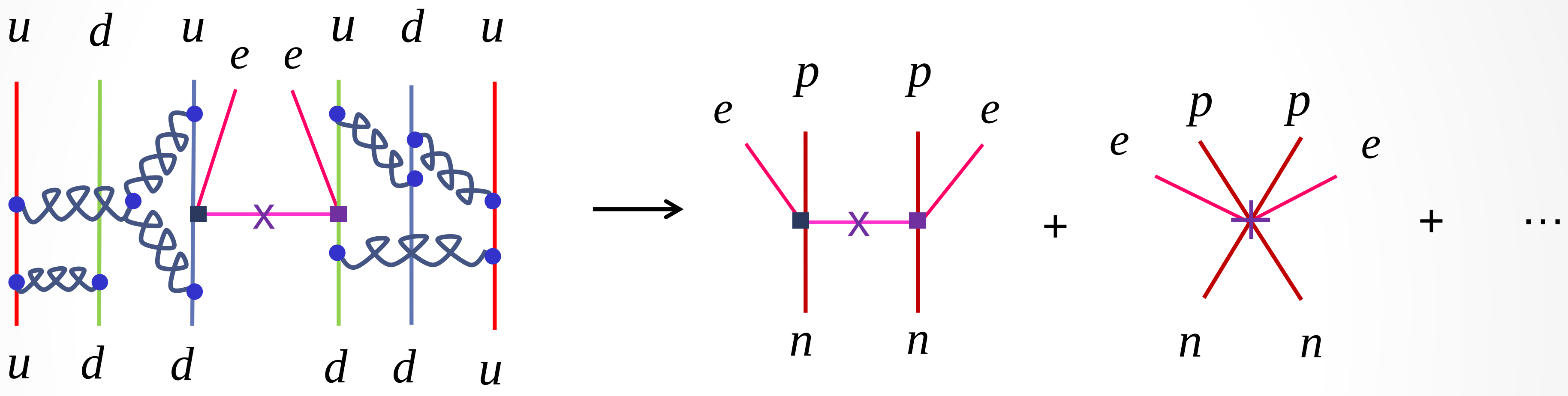



Cirigliano *et al.*, *Phys. Rev. Lett.* 120 (2018) 202001

$$|\vec{p}| = 1\ \mathrm{MeV} \quad |\vec{p}'| = 38\ \mathrm{MeV} \quad \vec{p}_{e1} = \vec{p}_{e2} = 0$$

# Short-range interaction

$Q \sim M_{\text{QCD}}$ $\qquad$ $Q \ll M_{\text{QCD}}$




correlations at distances $\lesssim 1/M_{QCD}$
not accounted for internucleon potential

⇨ ≠ correlations missed in single-particle basis

cf. Miller, Spencer '76

needed for model-independent definition
of light-neutrino exchange

⇨ ≠ a form-factor refinement

( ~10% in *ab initio* calculations)

…
Vergados '81
…

Pastore *et al.* '18

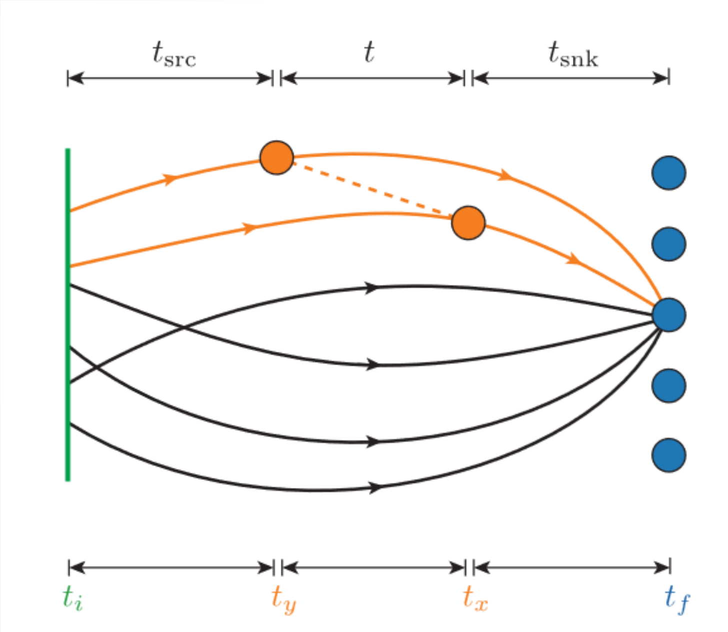
$nn \to ppee$

calculable with lattice QCD

match to EFT at two-body level,
use *ab initio* methods for many bodies

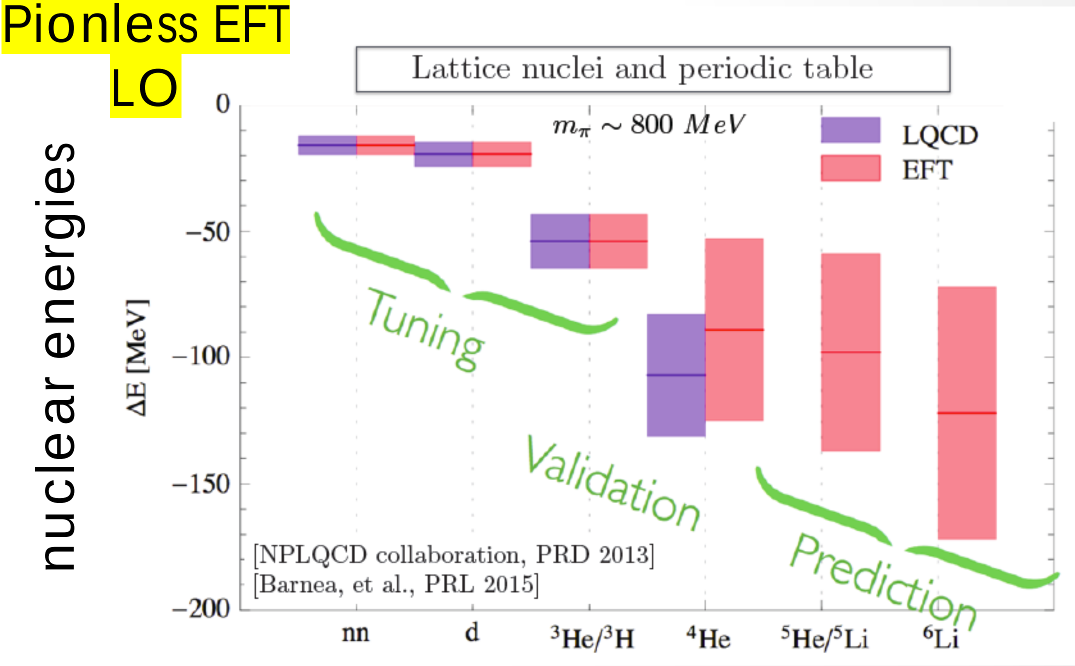
Davoudi, Kadan ’21
Davoudi *et al.* (NPLQCD Collab) ‘24

cf. Barnea, Contessi, Gazit, Pederiva + vK ’15
...




Davoudi *et al., Phys. Rev. D* 109 (2024) 114514

Pionless EFT
LO




Z. Davoudi

but a long way away...

# Another way?

$$C_1 \propto \left\langle pp \left| \frac{1}{\vec{q}^{\,2}} \right| nn \right\rangle$$ same as electromagnetism for $I=2$

chiral symmetry

$u = \exp\left(i\boldsymbol{\tau}\bullet\boldsymbol{\pi}/2f_\pi\right)$

$$\begin{cases} O_1 = N^\dagger u^\dagger Q_L u N \, N^\dagger u^\dagger Q_L u N - \frac{1}{6}\mathrm{Tr}\left(u^{\dagger 2} Q_L u^2 Q_L\right) N^\dagger \boldsymbol{\tau} N \bullet N^\dagger \boldsymbol{\tau} N + \left(L \leftrightarrow R\right) \\ O_2 = 2\left[ N^\dagger u^\dagger Q_L u N \, N^\dagger u Q_R u^\dagger N - \frac{1}{6}\mathrm{Tr}\left(u^{\dagger 2} Q_L u^2 Q_R\right) N^\dagger \boldsymbol{\tau} N \bullet N^\dagger \boldsymbol{\tau} N \right] \end{cases}$$

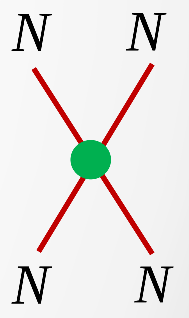
$\Rightarrow$

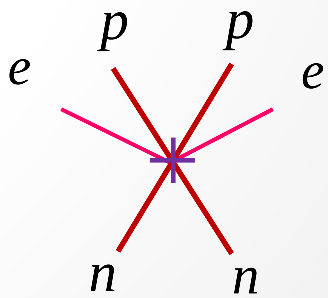
$$\mathcal{L}_{\chi\mathrm{EFT}} = \ldots + \frac{e^2}{8}\left(C_1 + C_2\right)\left[ N^\dagger \tau_3 N \, N^\dagger \tau_3 N - \frac{1}{3} N^\dagger \boldsymbol{\tau} N \bullet N^\dagger \boldsymbol{\tau} N \right]$$

$$+ 2G_F^2 \, m_{\beta\beta} \, C_1 \left[ V_{ud}^2 \, \bar{e}_L C \bar{e}_L^T \; N^\dagger \tau^+ N \, N^\dagger \tau^+ N + \mathrm{H.c.} \right]$$

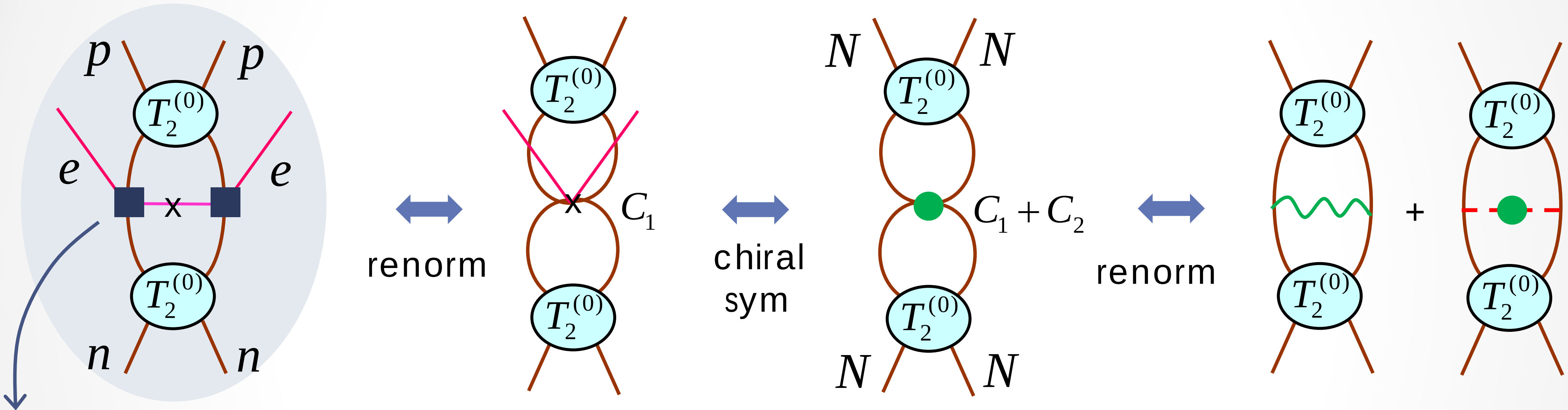
$$+ \ldots$$

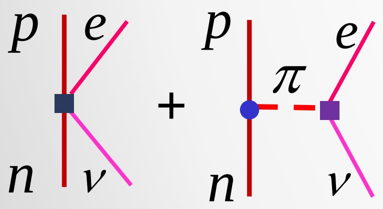
multi-pion interactions



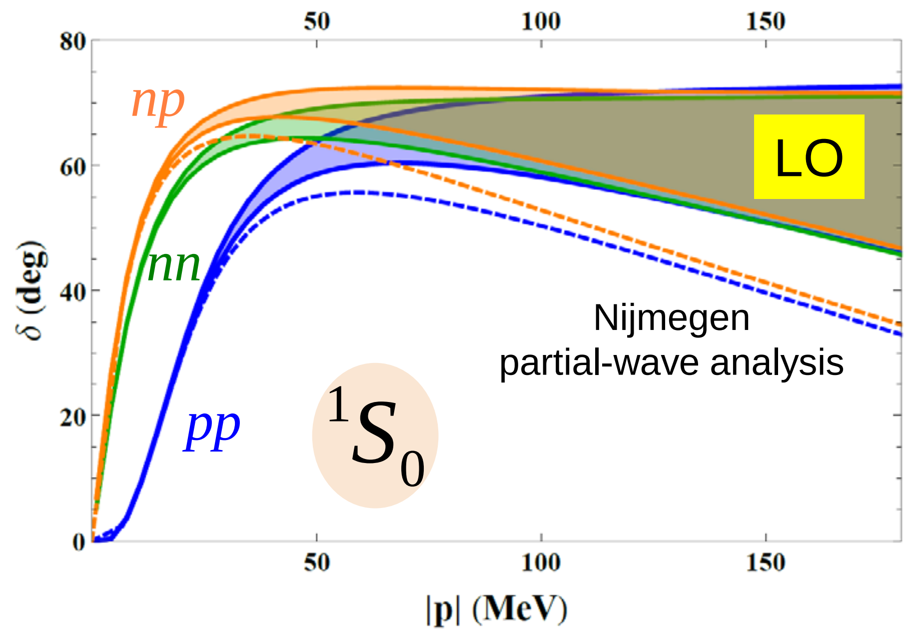



$^1S_0$




$$\propto \int d^3 l_1 \int d^3 l_2 \, \frac{m_N}{l_1^2} \frac{m_N}{l_2^2} \frac{1}{(l_1 - l_2)^2} \propto m_N^2 \ln \Lambda$$

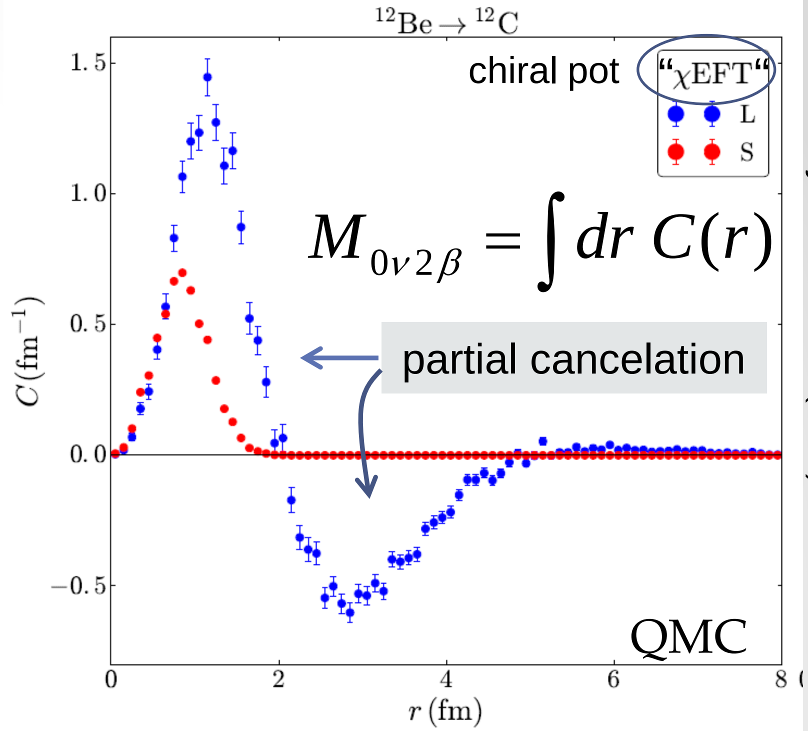



Cirigliano *et al.*, *Phys. Rev. C* 100 (2019) 055504

$$C_1 + C_2$$

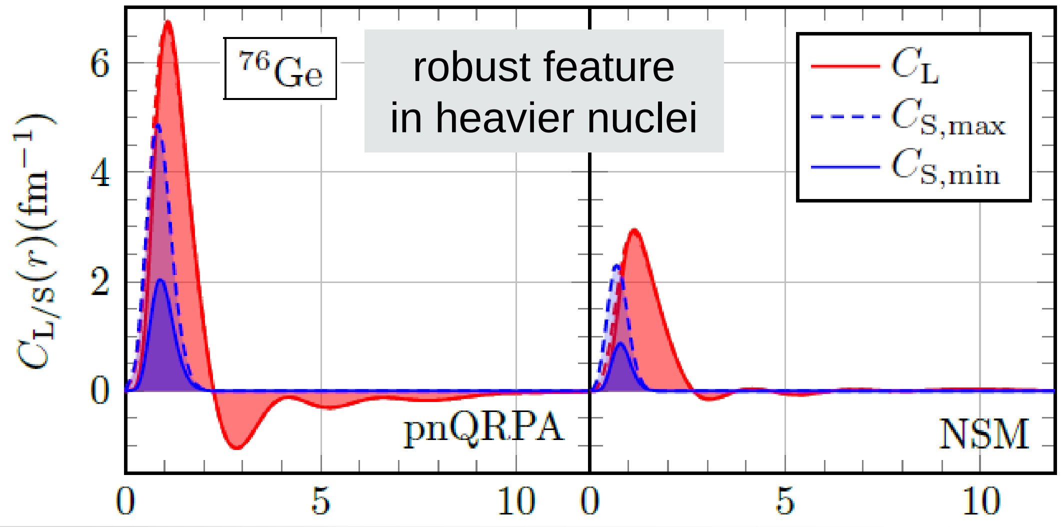
assuming $C_1 \sim C_2$

$$\frac{M_S}{M_L} \simeq 0.75$$



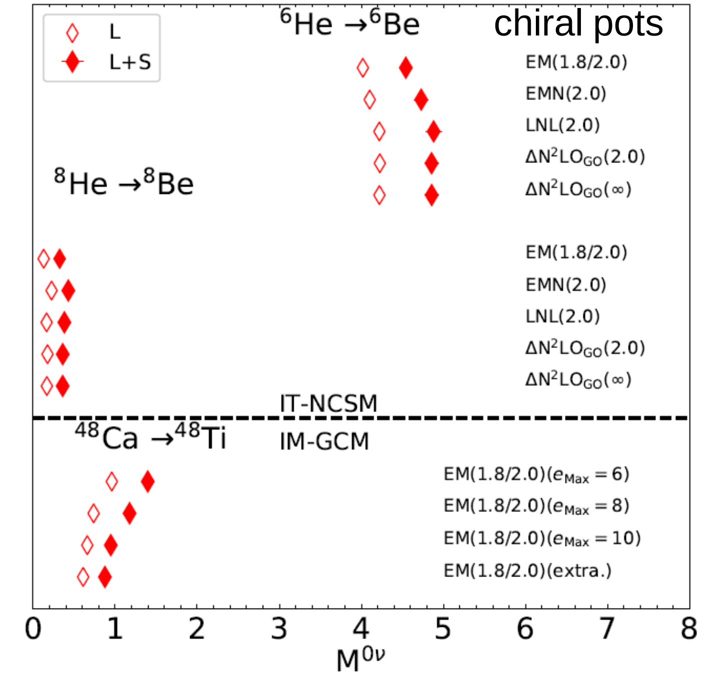

Cirigliano *et al.*, *Phys. Rev. C* 100 (2019) 055504

Jokiniemi, Soriano, Menéndez, *Phys. Lett. B* 823 (2021) 136720




$$\frac{M_S}{M_L} \simeq 0.15 - 0.80$$

more
*ab initio*
calculations
needed



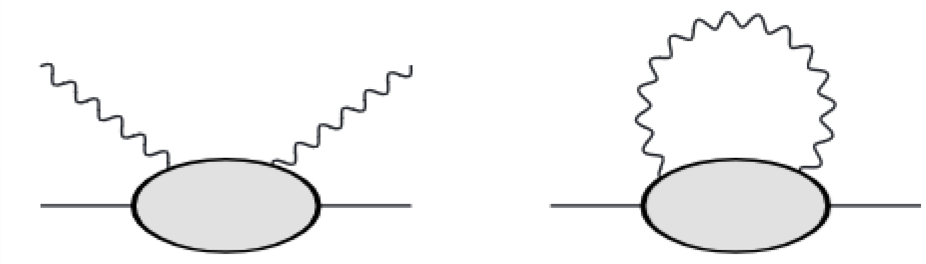

Wirth, Yao, Hergert, *Phys. Rev. Lett.* 127 (2021) 242502

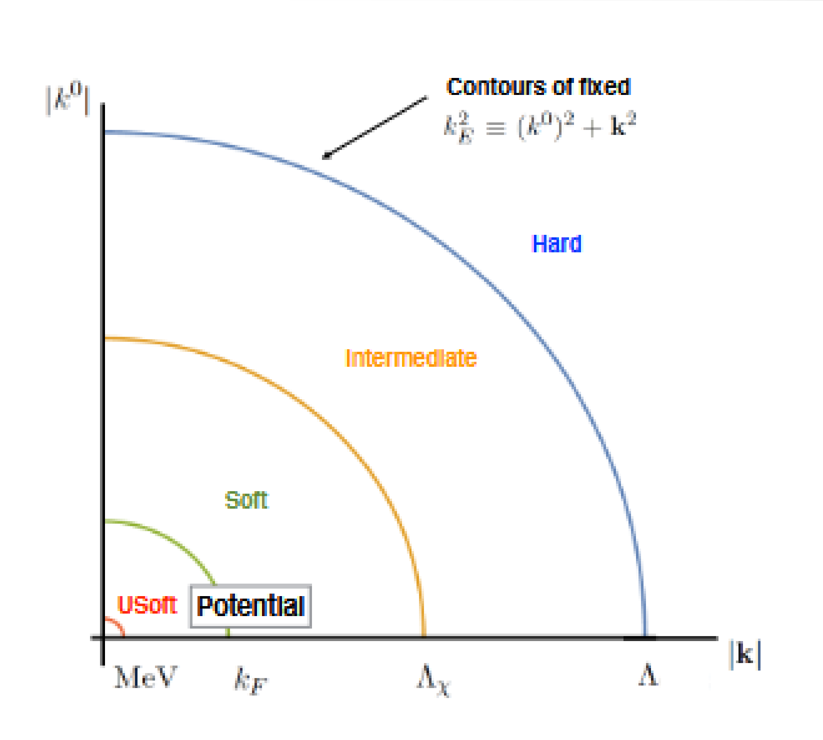
Cirigliano, Dekens, De Vries,
Hoferichter, Mereghetti, ’21’21

# Dispersion estimate

analogous to Cottingham sum rule
for electromagnetic contribution to hadron masses

$$\mathcal{A}_\nu \propto \int \frac{d^4k}{(2\pi)^4} \frac{g_{\mu\nu}}{k^2 + i\epsilon} \int d^4x\, e^{ik\cdot x} \langle pp | T\{ j^\mu_{\mathrm{w}}(x) j^\nu_{\mathrm{w}}(0) \} | nn \rangle$$






EFT model OPE

Cirigliano *et al.*, *JHEP* 05 (2021) 289

➡ consistent with *NN* estimate of $C_1 + C_2$ with $C_1 \sim C_2$

$$C_1 \propto \left\langle pp \middle| \frac{1}{\vec{q}^{\,2}} \middle| nn \right\rangle$$ same as electromagnetism for $I=2$

chiral symmetry

$u = \exp\left(i\boldsymbol{\tau}\bullet\boldsymbol{\pi}/2 f_\pi\right)$

$$\begin{cases} O_1 = N^\dagger u^\dagger Q_L u N\, N^\dagger u^\dagger Q_L u N - \frac{1}{6}\mathrm{Tr}\left(u^{\dagger 2} Q_L u^2 Q_L\right) N^\dagger \boldsymbol{\tau} N \bullet N^\dagger \boldsymbol{\tau} N + (L \leftrightarrow R) \\ O_2 = 2\left[ N^\dagger u^\dagger Q_L u N\, N^\dagger u Q_R u^\dagger N - \frac{1}{6}\mathrm{Tr}\left(u^{\dagger 2} Q_L u^2 Q_R\right) N^\dagger \boldsymbol{\tau} N \bullet N^\dagger \boldsymbol{\tau} N \right] \end{cases}$$

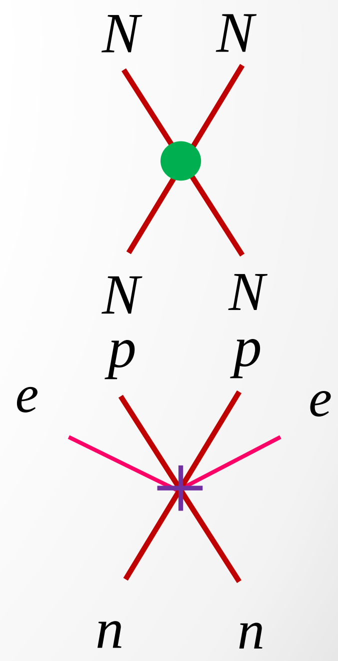
$$\Rightarrow \quad \mathcal{L}_{\chi\mathrm{EFT}} = \ldots + \frac{e^2}{8}(C_1 + C_2)\left[ N^\dagger \tau_3 N\, N^\dagger \tau_3 N - \frac{1}{3} N^\dagger \boldsymbol{\tau} N \bullet N^\dagger \boldsymbol{\tau} N \right]$$

$$+ 2 G_F^2\, m_{\beta\beta}\, C_1 \left[ V_{ud}^2\, \overline{e}_L C \overline{e}_L^T\; N^\dagger \tau^+ N\, N^\dagger \tau^+ N + \mathrm{H.c.} \right]$$

$$+ \ldots$$

multi-pion interactions

multi-pion E&M interactions can in principle separate $C_{1,2}$

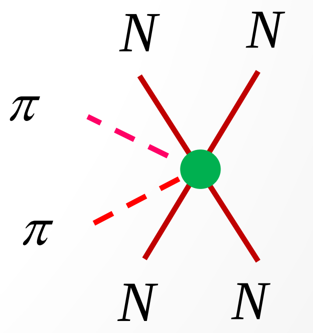


isospin violation

$$\mathcal{L}_{\chi\mathrm{EFT}} = \ldots + \frac{e^2}{8}\left[ N^\dagger \tau_a N\, N^\dagger \tau_b N - \frac{1}{3}\delta_{ab} N^\dagger \boldsymbol{\tau} N \bullet N^\dagger \boldsymbol{\tau} N \right]$$

$$\times\left\{ \left(C_1 + C_2\right)\left[ \delta_{a3}\delta_{b3} - \frac{1}{f_\pi^2}\left( \pi^2 \delta^{ab} - \frac{1}{2}\pi_3\left(\pi_a\delta_{b3} + \pi_b\delta_{a3}\right)\right)\right] - \frac{C_1 - C_2}{f_\pi^2}\left( \pi^a\pi^b + \pi^2\delta^{ab} + \pi_3\left(\pi_a\delta_{b3} + \pi_b\delta_{a3}\right)\right)\right\}$$

$$+\ldots$$



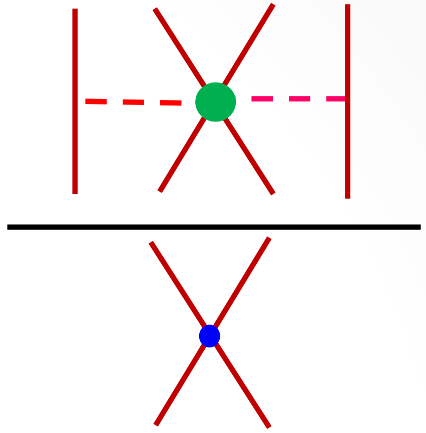

$$\propto \frac{e^2}{f_\pi^2} C_{1,2}$$

Some possibilities:

1. charge-independence breaking
   in nuclear potential at higher orders

Lieffers, Mereghetti,
in preparation

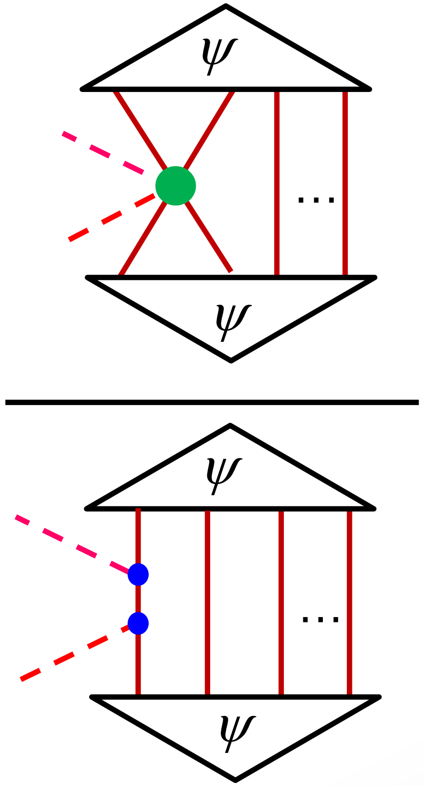
e.g.,



$\sim \dfrac{\alpha}{4\pi}$

2. charge-independence breaking
   in pion-nucleus scattering

Wu, Fleming, Mereghetti, vK,
in preparation




$\sim \alpha$

# Conclusion

EFTs connect symmetry violation
from beyond the Standard Model to nuclear physics
in a controlled and systematic way

Renormalization and naturalness do not imply
naïve dimensional analysis in nonperturbative context

Nuclear observables can be properly renormalized
but most-used approach is model-dependent

Renormalization of neutrinoless double-beta decay requires
short-range physics missed by nuclear models