



Introduction to $\mu \rightarrow e$ Conversion

Yoshitaka KUNO
RCNP, Osaka University

April 14th, 2025
Workshop on “Lepton Flavour Change in Nuclei”
ECT*, Trento, Italy

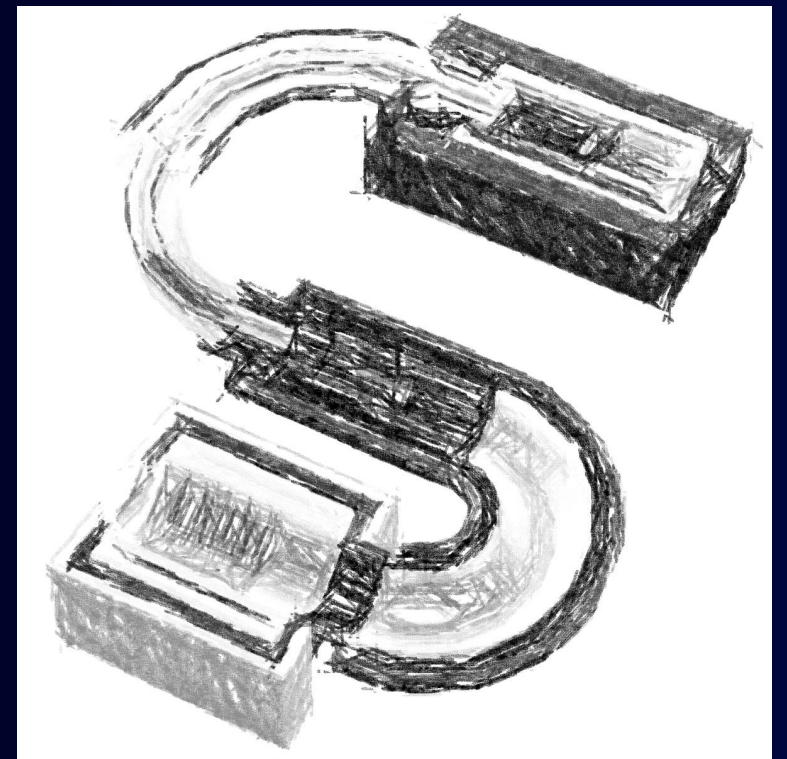
Thanks for the organizers, my last visit to ECT was 1998.*

Outline

muon to electron conversion

$$\mu^{-} \rightarrow e^{-}$$

- 1: CLFV Physics
- 2: Muon CLFV
- 3: What is $\mu^{-} \rightarrow e^{-}$ Conversion ?
- 4: $\mu^{-} \rightarrow e^{-}$ Conversion Experiment
- 5: $\mu^{-} \rightarrow e^{-}$ Conversion Phenomenology
- 6: $\mu^{-} \rightarrow e^{+}$ Conversion
- 7: Future : PRISM/PRIME
- 8: Summary

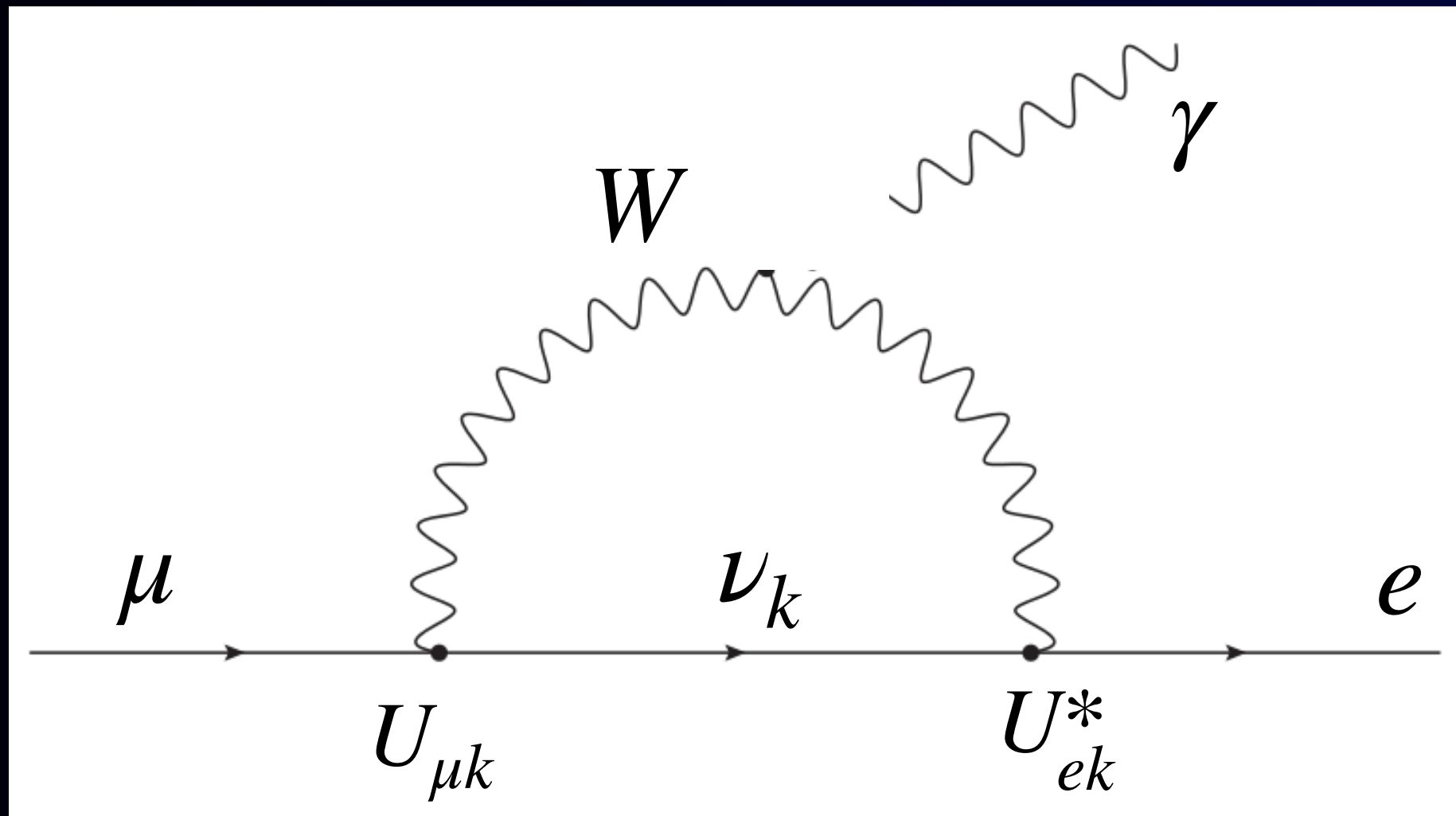


CLFV Physics

Charged Lepton Flavour Violation (CLFV)



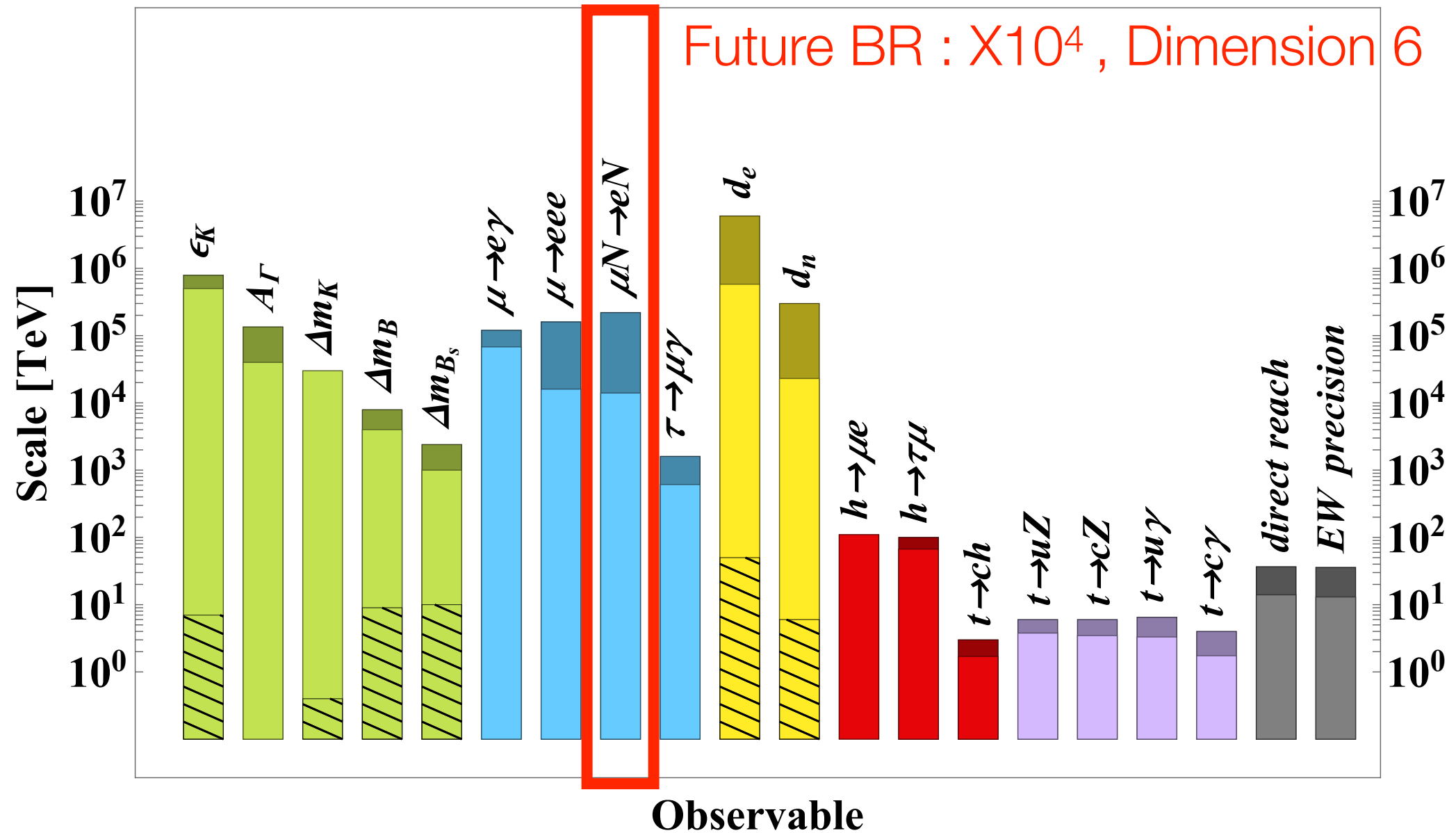
CLFV in the Standard Model



$$\text{BR}(\mu \rightarrow e \gamma) = \frac{3\alpha}{32\pi} \left| \sum_{k=1,2,3} U_{\mu k} U_{ek}^* \frac{m_{\nu_k}^2}{M_W^2} \right|^2$$

$$\text{BR} \sim \mathcal{O}(10^{-54})$$

BSM Physics Scale Reach



from European particle Physics Strategy Update (2019)

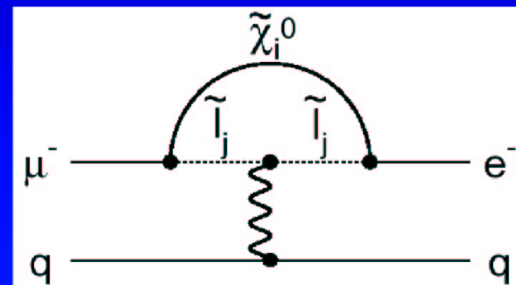
light colour: present
dark colour: future prospect

Various CLFV Theoretical Models

Sensitivity to Different Muon Conversion Mechanisms

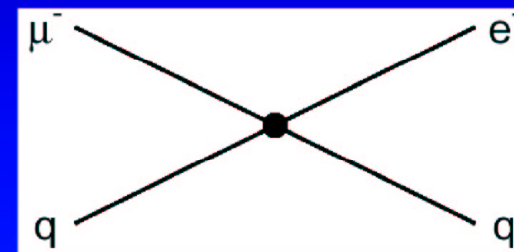


Supersymmetry
Predictions at 10^{-15}



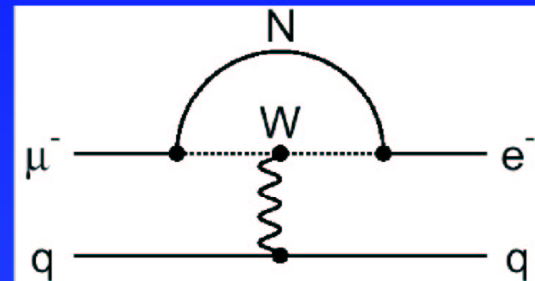
Compositeness

$$\Lambda_c = 3000 \text{ TeV}$$



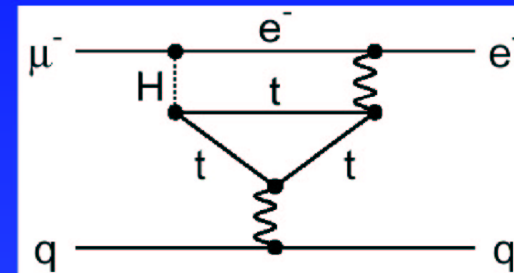
Heavy Neutrinos

$$|U_{\mu N}^* U_{eN}|^2 = 8 \times 10^{-13}$$



Second Higgs doublet

$$g_{H\mu e} = 10^{-4} \times g_{H\mu\mu}$$

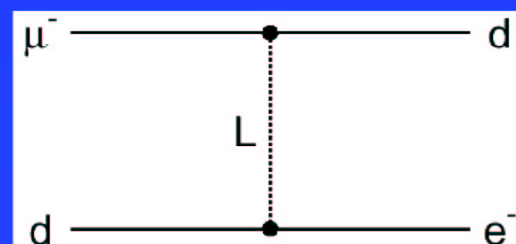


Leptoquarks

$$M_L =$$

$$3000 (\lambda_{\mu d} \lambda_{ed})^{1/2} \text{ TeV}/c^2$$

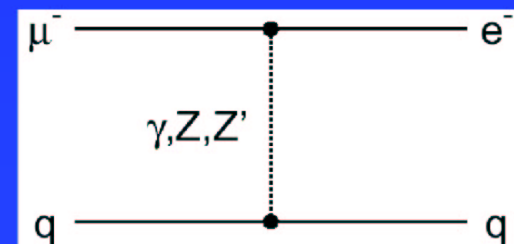
After W. Marciano



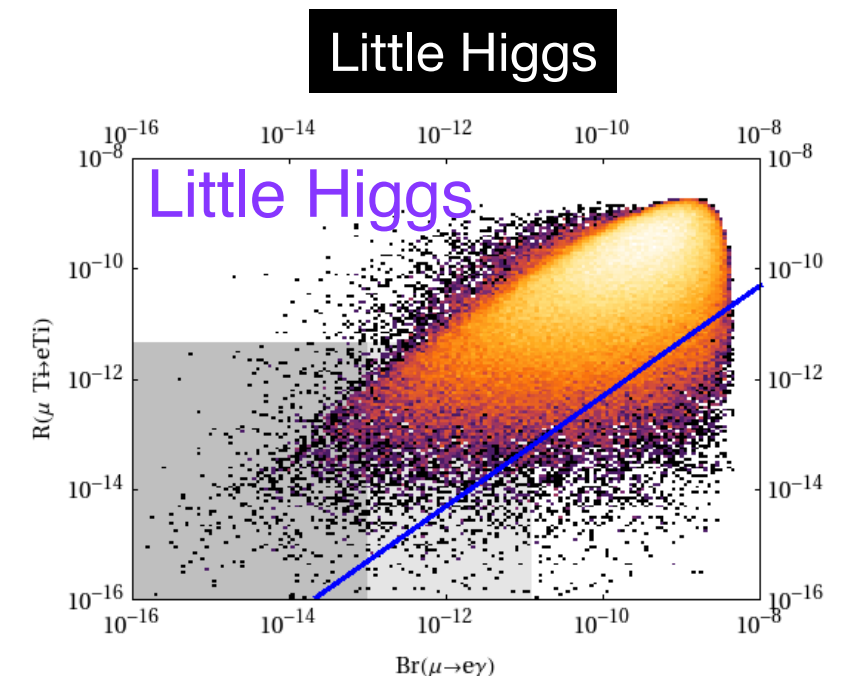
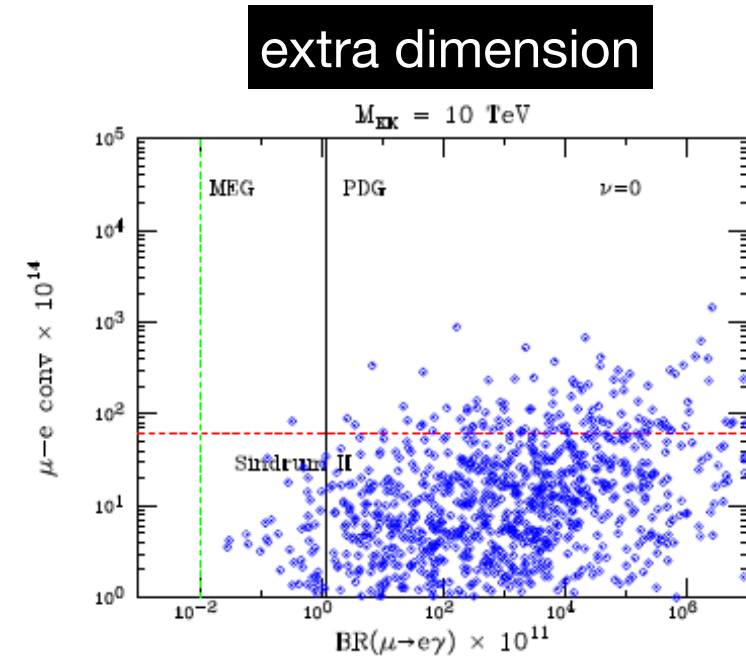
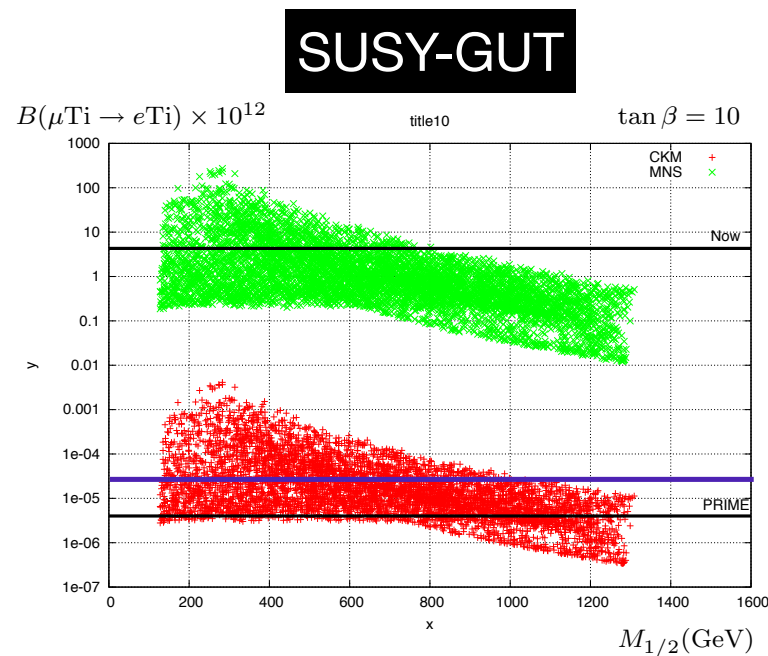
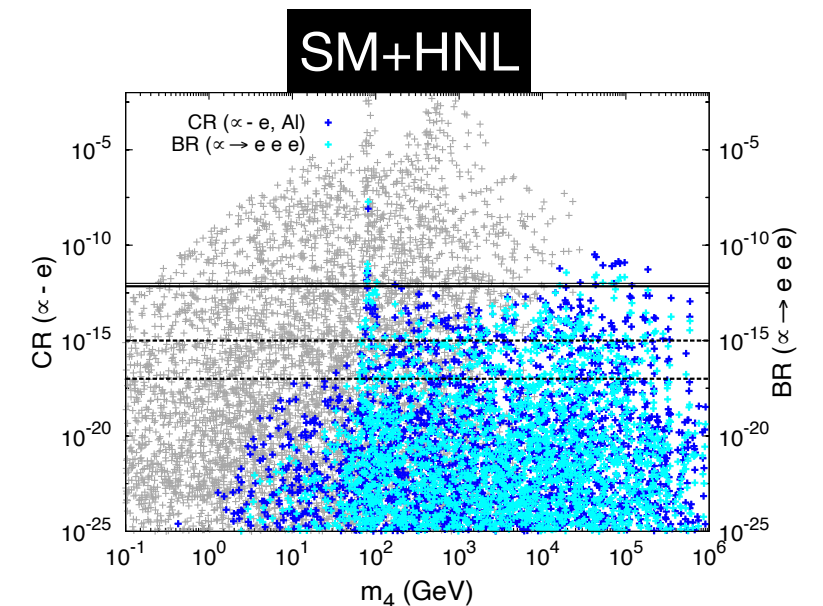
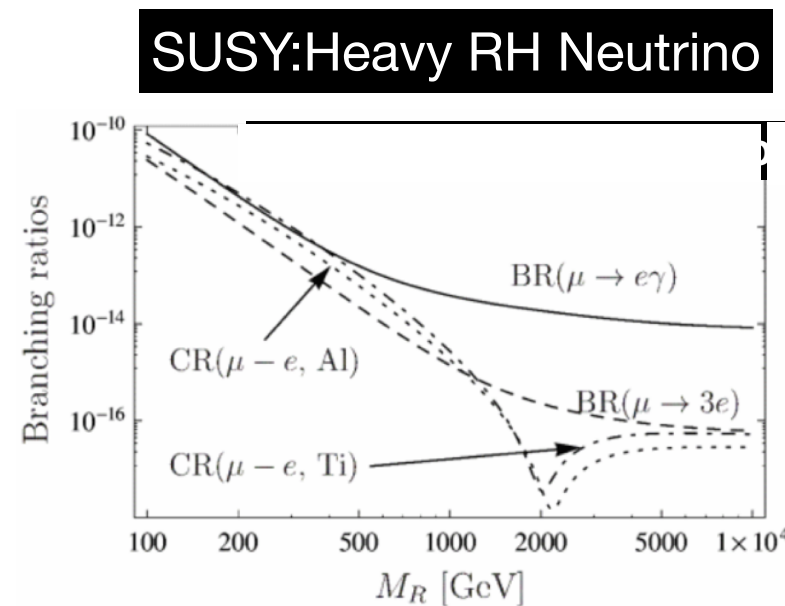
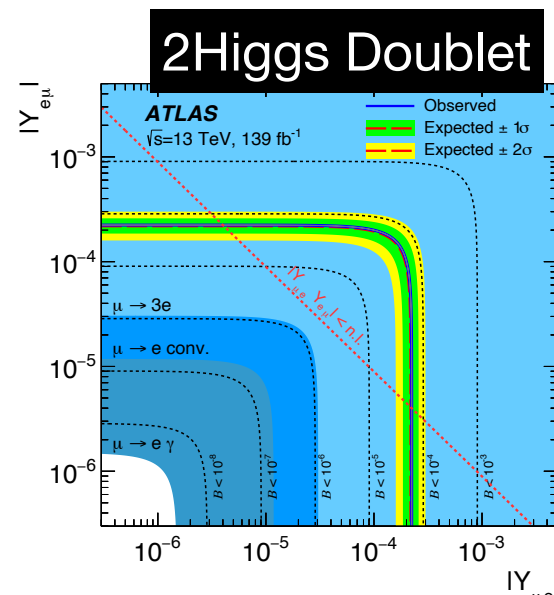
Heavy Z' ,
Anomalous Z
coupling

$$M_{Z'} = 3000 \text{ TeV}/c^2$$

$$B(Z \rightarrow \mu e) < 10^{-17}$$



Model Dependent CLFV Predictions



Many BSM models predict sizable CLFV rates.

Muon CLFV



Present Upper Limits for CLFV Searches

process	present limit	future
$\tau \rightarrow \mu\eta$	$< 6.5 \times 10^{-8}$	$10^{-9} - 10^{-10}$
$\tau \rightarrow \mu\gamma$	$< 6.8 \times 10^{-8}$	
$\tau \rightarrow \mu\mu\mu$	$< 3.2 \times 10^{-8}$	
$\tau \rightarrow eee$	$< 3.6 \times 10^{-8}$	
$K_L \rightarrow e\mu$	$< 4.7 \times 10^{-12}$	
$K^+ \rightarrow \pi^+ e^- \mu^+$	$< 1.3 \times 10^{-11}$	
$B^0 \rightarrow e\mu$	$< 7.8 \times 10^{-8}$	
$B^+ \rightarrow K^+ e\mu$	$< 9.1 \times 10^{-8}$	
$Z^0 \rightarrow e\mu$	$< 7.5 \times 10^{-7}$	
$Z^0 \rightarrow e\tau$	$< 1.2 \times 10^{-5}$	
$Z^0 \rightarrow \mu\tau$	$< 9.8 \times 10^{-6}$	
$H^0 \rightarrow e\mu$	$< 3.5 \times 10^{-4}$	
$H^0 \rightarrow e\tau$	$< 3.7 \times 10^{-3}$	
$H^0 \rightarrow \mu\tau$	$< 2.5 \times 10^{-3}$	
$\mu^+ \rightarrow e^+ \gamma$	$< 4.2 \times 10^{-13}$	10^{-14} (MEG II)
$\mu^+ \rightarrow e^+ e^+ e^-$	$< 1.0 \times 10^{-12}$	10^{-16} (Mu3e)
$\mu^- \text{Au} \rightarrow e^- \text{Au}$	$< 7.0 \times 10^{-13}$	10^{-17} (COMET, Mu2e)
$\mu^- \text{Ti} \rightarrow e^+ \text{Ca}$	$< 3.6 \times 10^{-11}$	10^{-17} (COMET, Mu2e)
$\mu^+ e^- \rightarrow \mu^- e^+$	$< 8.3 \times 10^{-11}$	

$\times 10^{-4}$

Muon CLFV History



Pontecorvo in 1948

Muon Michel decay
(1948)

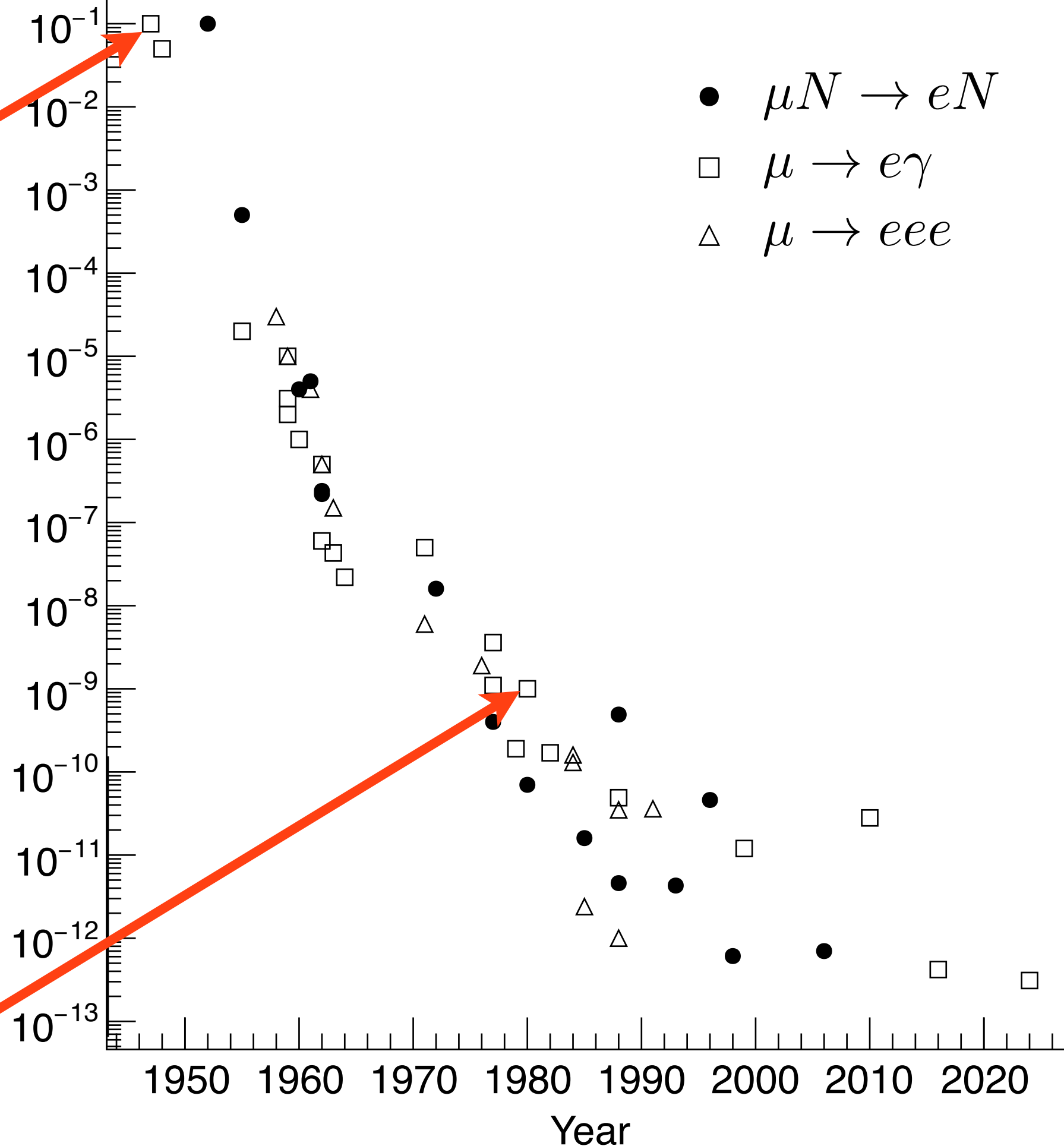
Accelerator muons
(1950's)

Feinberg's $\mu \rightarrow e\gamma$
crisis (1955)

Meson Factory Era
(1970)

False rumor of
 $\mu \rightarrow e\gamma$ at SIN (1976)

Upper Limits of Branching Ratios



$\mu^+ \rightarrow e^+ \gamma$ Rumors (1976)

7.1 History - how it all began

In the fall of 1976 rumors spread about an experiment performed at SIN for the search of the decay $\mu \rightarrow e \gamma$. A debate was going on, whether or not the decay had been observed. The rumors traveled from SIN via email to R. Eichler at Stanford and from him to a graduate student in the lecture-class of James Bjorken. The next week, J. Bjorken in turn gave the students an exercise to compute the decay rate and also confronted his colleague Steven Weinberg with the rumor. It took a few weeks after Weinberg's talk at the APS meeting to reach the New York Times. There it read on February 8th 1977: *Experimenters in Switzerland have reportedly observed an "impossible" transmutation of atomic particles. This has thrown the world community of theoretical physicists into a frenzy of speculations, calculations and publications (S. Weinberg).* This inspired R. Hofstadter of Stanford to initiate an experiment at LAMPF for $\mu^+ \rightarrow e^+ \gamma$ to try to resolve the dispute around the SIN experiment.

Permanent link:

by R. Eichler

<https://doi.org/10.3929/ethz-b-000526612>

$\mu^+ \rightarrow e^+ \gamma$: History

Year	90% CL on $\mathcal{B}(\mu \rightarrow e\gamma)$	Collaboration/Lab	Reference
1947	1.0×10^{-1}	Chalk River	Hincks and Pontecorvo [1948]
1948	.04	Washington University	Sard and Althaus [1948]
1955	2.0×10^{-5}	Nevis	Steinberger and Lokanathan [1955]
1959	7.5×10^{-6}	Liverpool	O'Keefe <i>et al.</i> [1959]
1959	2.0×10^{-6}	Nevis	Berley <i>et al.</i> [1959]
1959	1.0×10^{-5}	Rochester	Davis <i>et al.</i> [1959]
1959	1.2×10^{-6}	CERN	Ashkin <i>et al.</i> [1959]
1960	1.2×10^{-6}	LBL	Frankel <i>et al.</i> [1960]
1961	2.5×10^{-5}	Carnegie	Crittenden <i>et al.</i> [1961]
1962	1.9×10^{-7}	LBL	Frankel <i>et al.</i> [1962]
1962	6.0×10^{-8}	Nevis	Bartlett <i>et al.</i> [1962]
1963	4.3×10^{-8}	LBL	Frankel <i>et al.</i> [1963]
1964	2.2×10^{-8}	Chicago	Parker <i>et al.</i> [1964]
1971	2.9×10^{-8}	Dubna	Korenchenko <i>et al.</i> [1971]
1977	3.6×10^{-9}	TRIUMF	Depommier <i>et al.</i> [1977]
1977	1.1×10^{-9}	SIN	Povel <i>et al.</i> [1977]
1979	1.9×10^{-10}	LAMPF	Bowman <i>et al.</i> [1979]
1982	1.7×10^{-10}	LAMPF	Kinnison <i>et al.</i> [1982]
1986	4.9×10^{-11}	LAMPF/Crystal Box	Bolton <i>et al.</i> [1986, 1988]
1999	1.2×10^{-11}	LAMPF/MEGA	Brooks <i>et al.</i> [1999]
2010	2.8×10^{-11}	PSI/MEG	Adam <i>et al.</i> [2010]
2011	2.4×10^{-12}	PSI/MEG	Adam <i>et al.</i> [2011]

What is
 $\mu^- \rightarrow e^-$ Conversion



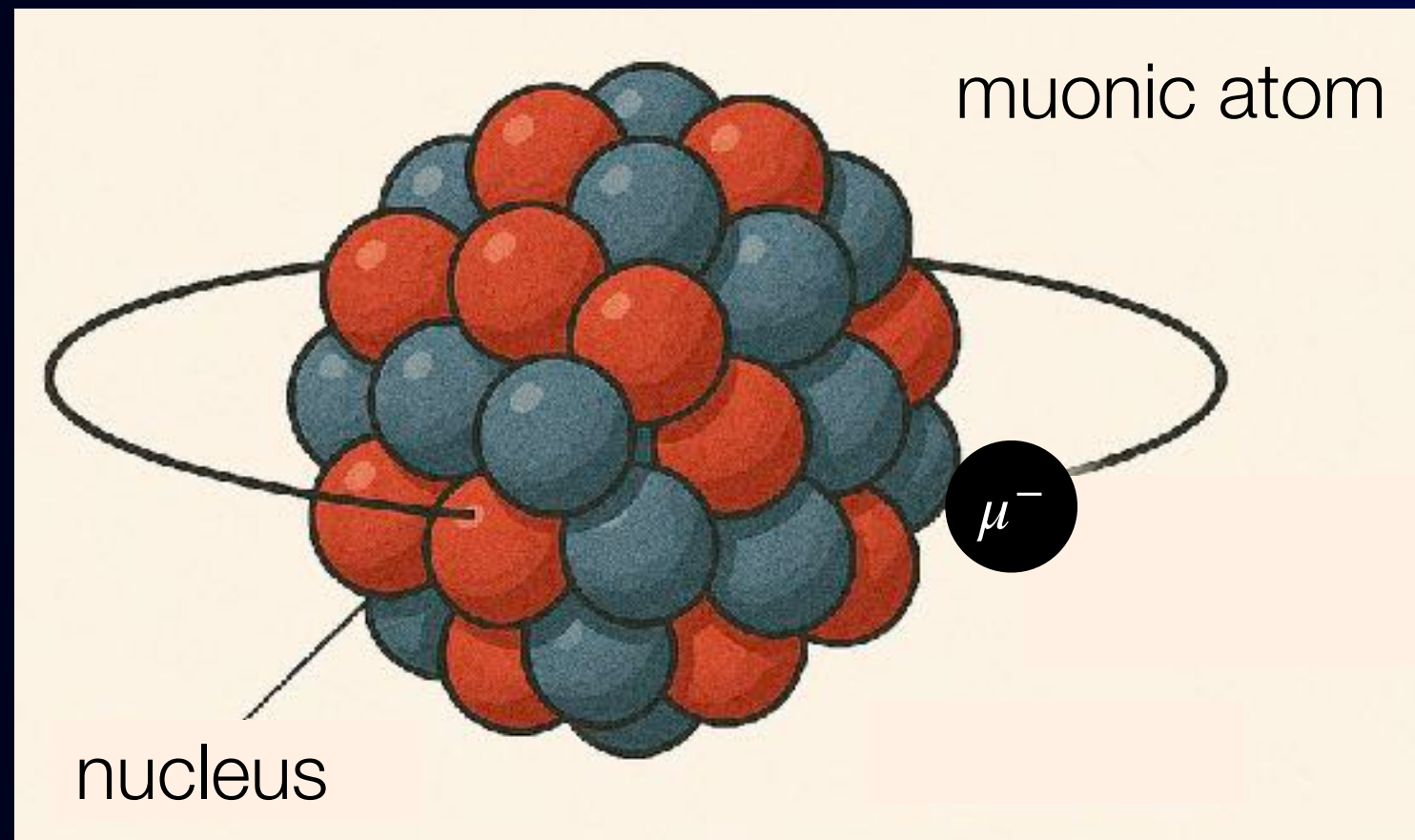
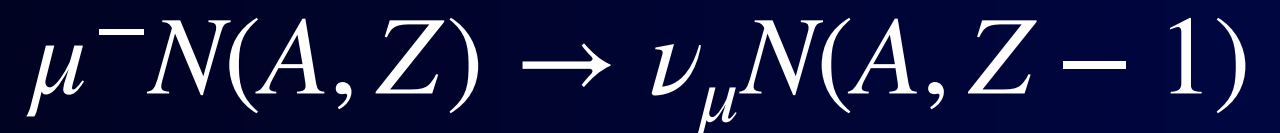
Muonic Atom

- μ^- is stopped in material and it forms a muonic atom.
- μ^- cascades down from excited states to the ground (1s) state by emitting Auger electrons and muonic X rays.
- In the ground state, μ^- either decays in orbit or is captured by a nucleus.

Bound Muon Decay in Orbit (DIO)



Nuclear Muon Capture (NMC)



Lifetime of Muonic Atoms

$$\frac{1}{\tau_{\mu atom}} = \Lambda_{cap} + Q\Lambda_{free}$$

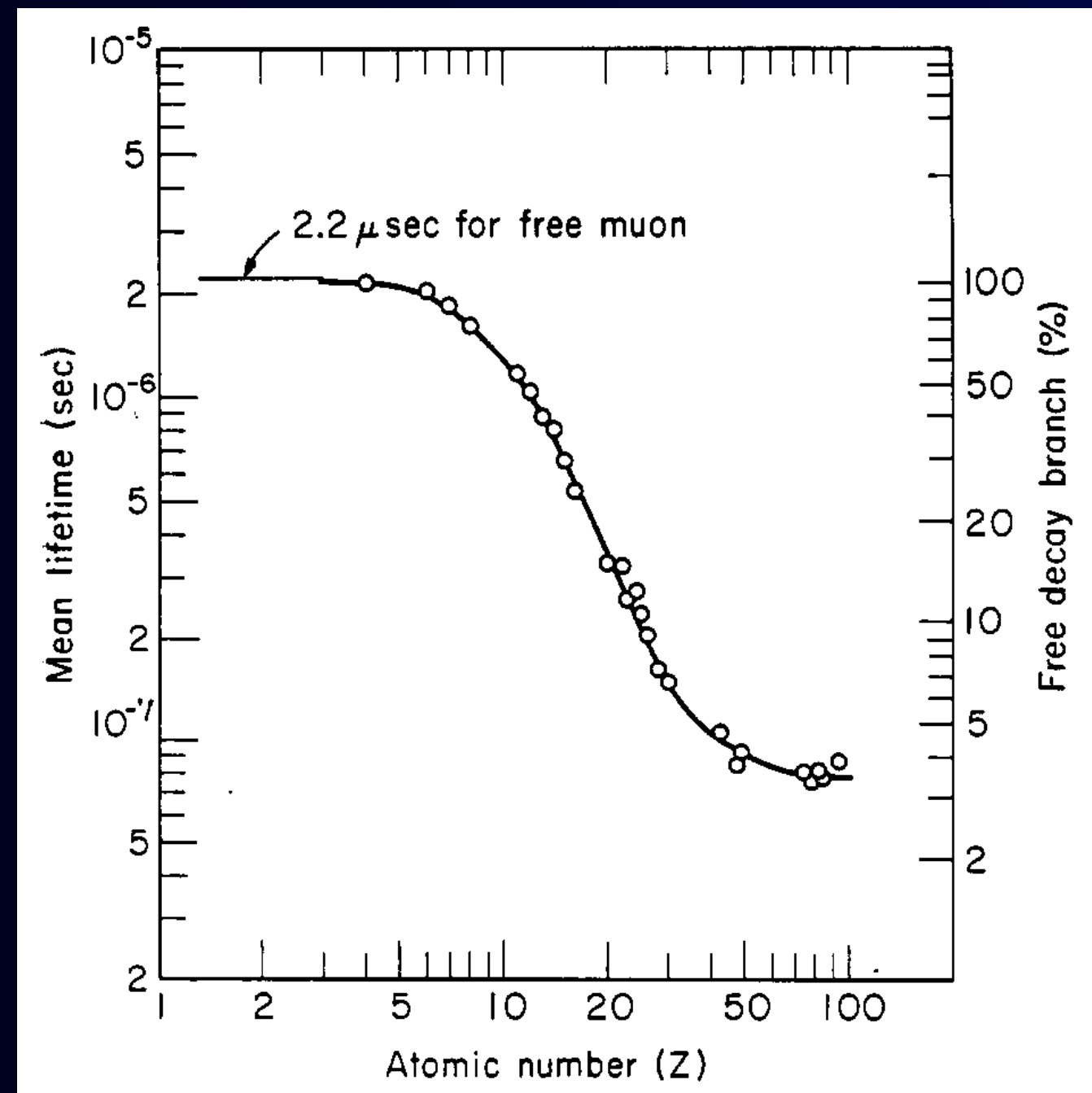
$$\Lambda_{CAP} \propto Z_{eff}^4$$

Q : Huff factor

$Q = 0.975$ for Iron ($Z=26$)

$Q = 0.844$ for lead ($Z=82$)

nucleus	z	lifetime (ns)
C	6	2027
Al	13	864
Ti	22	330
Cu	29	164
Pb	82	74

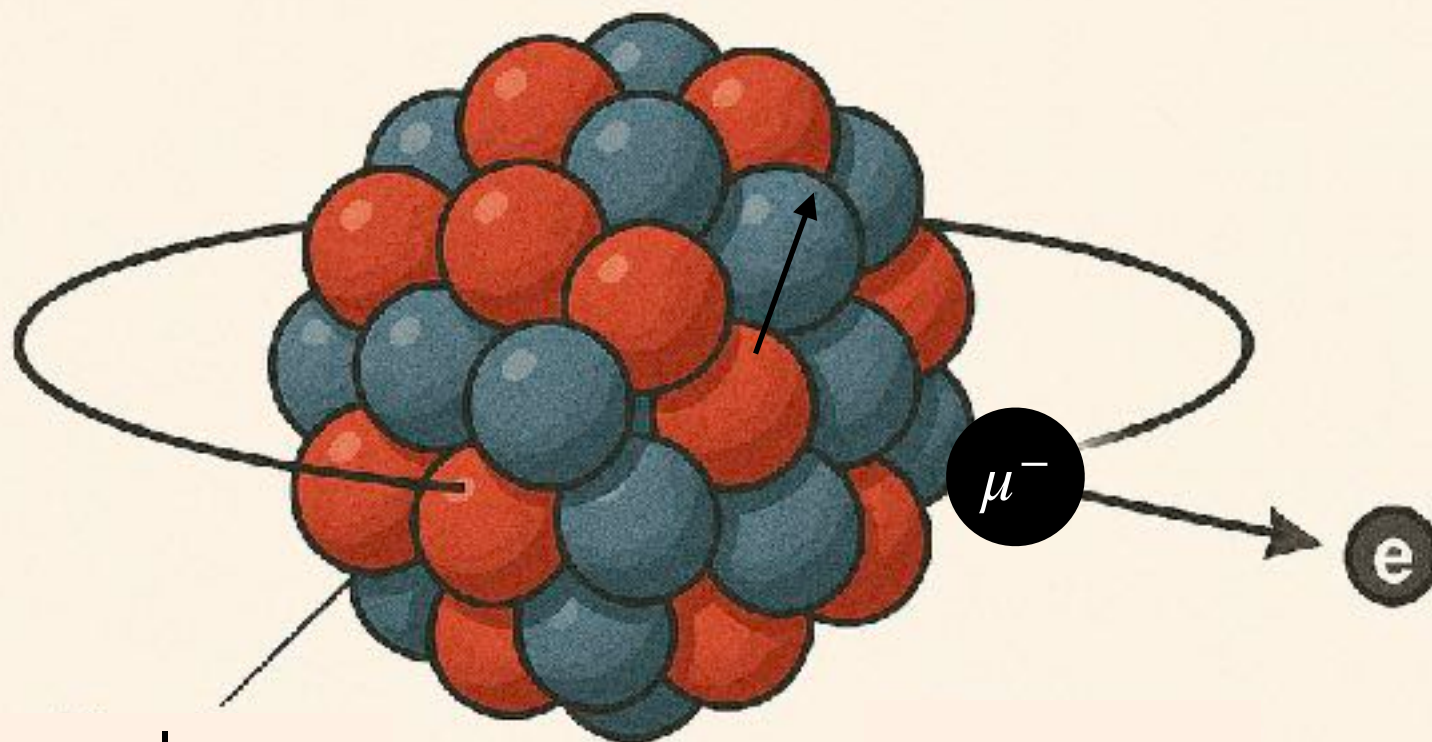


$\mu^- \rightarrow e^-$ Conversion : Signal

Neutrinoless Muon to Electron Conversion

$$\mu^- N(A, Z) \rightarrow e^- N(A, Z)$$

$$\Delta L_\mu = -\Delta L_e = -1$$



Event Signature for the transition to the ground state :

a single mono-energetic electron

$$E_{\mu e} = \frac{(m_N + m_\mu - B_\mu)^2 - m_N^2 + m_e^2}{2(m_N + m_\mu - B_\mu)} \\ \sim m_\mu - B_\mu - E_{recoil}$$

nucleus	z	$E_{\mu e}$ (MeV)
Al	13	104.97
S	16	104.70
Ti	22	104.30
Cu	82	103.50
Au	79	95.56
Pb	82	94.02

Coherency

Coherency

When the initial and final states are the same (ground state), all the nucleons can contribute to the process, and the rate of conversion is enhanced by A^2 (or Z^2).

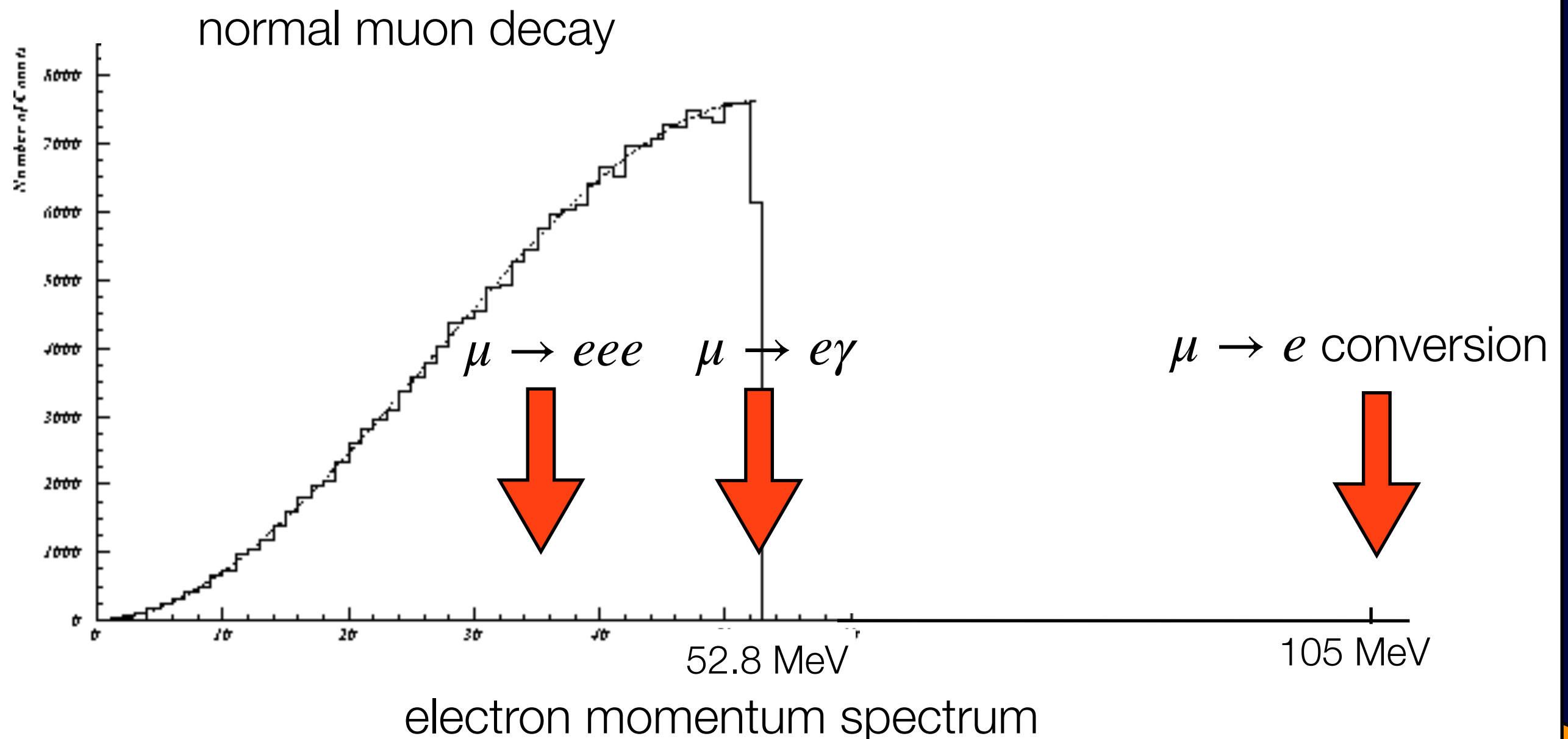
Transitions to excited states are suppressed.

Definition of Conversion Rate

$$\text{CR}(\mu^- \text{N} \rightarrow e^- \text{N}) \equiv \frac{\Gamma(\mu^- \text{N} \rightarrow e^- \text{N})}{\Gamma(\mu^- \text{N} \rightarrow \nu_\mu X)}$$

Experimental Advantages of $\mu^- \rightarrow e^-$ Conversion (1)

(1) Separation of Signals and Normal Muon Decay



Experimental Advantages of $\mu^- \rightarrow e^-$ Conversion (2)

(2) Ability to Use High Intensity Beams

	Measurements	Major background	intensity (R_μ)
$\mu \rightarrow e\gamma$	coincidence of decay particles	accidentals ($\propto R_\mu^2$)	$<10^8/\text{sec}$
$\mu \rightarrow eee$	coincidence of decay particles	accidentals ($\propto R_\mu^2$)	$<10^{(8-9)}/\text{sec} ?$
μ -e conversion	no coincidence	beam related	$10^{11}/\text{sec}$

accidental background = more than one muon decays at the same time to mimic the event signature

Disadvantages of $\mu^- \rightarrow e^-$ Conversion

(R_μ)

The cost of muon beam lines is high!

Backgrounds for $\mu^- \rightarrow e^-$ Conversion

Muon-induced backgrounds

Bound muon decay in orbit (DIO)

$$(\mu^- + N \rightarrow e^- \nu_\mu \bar{\nu}_e + N)$$

Radiative nuclear muon capture (RMC)

$$(\mu^- N \rightarrow \nu_\mu N' \gamma, \quad \gamma \rightarrow e^+ e^-)$$

Particles from muon nuclear capture

Beam-related backgrounds

Radiative nuclear pion capture (RPC)

$$(\pi^- N \rightarrow N' \gamma, \quad \gamma \rightarrow e^+ e^-)$$

Beam electrons

Muon decay in flights

Neutron induced background

Antiproton induced background

Other backgrounds

Cosmic-ray induced background

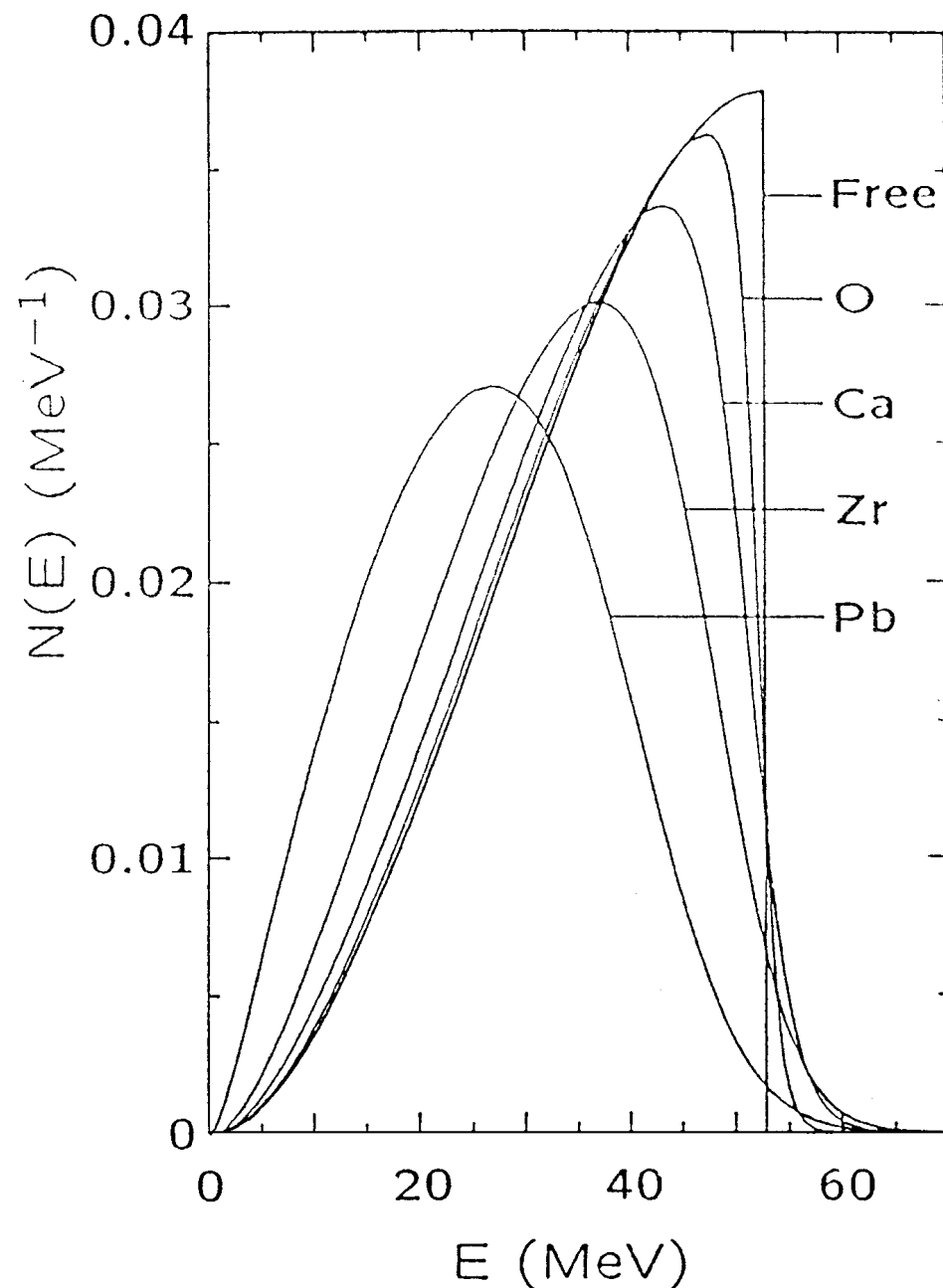
False tracking

others

Bound Muon Decay in Orbit (DIO)

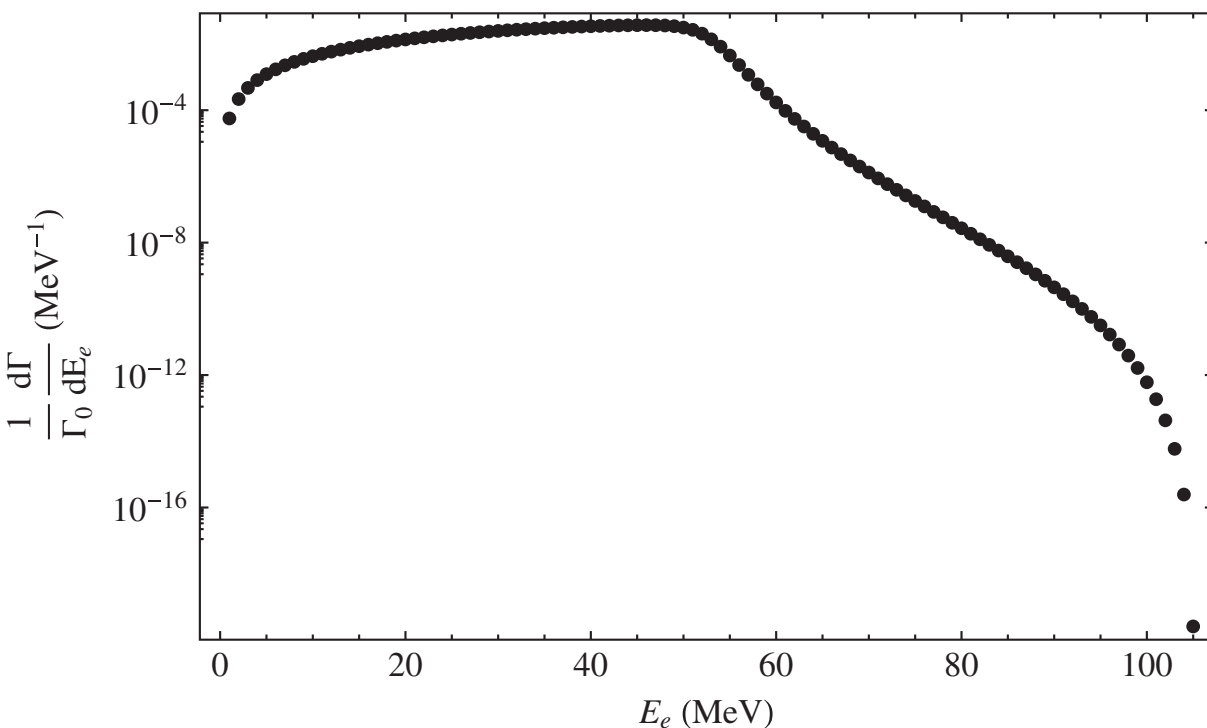


DIO spectra

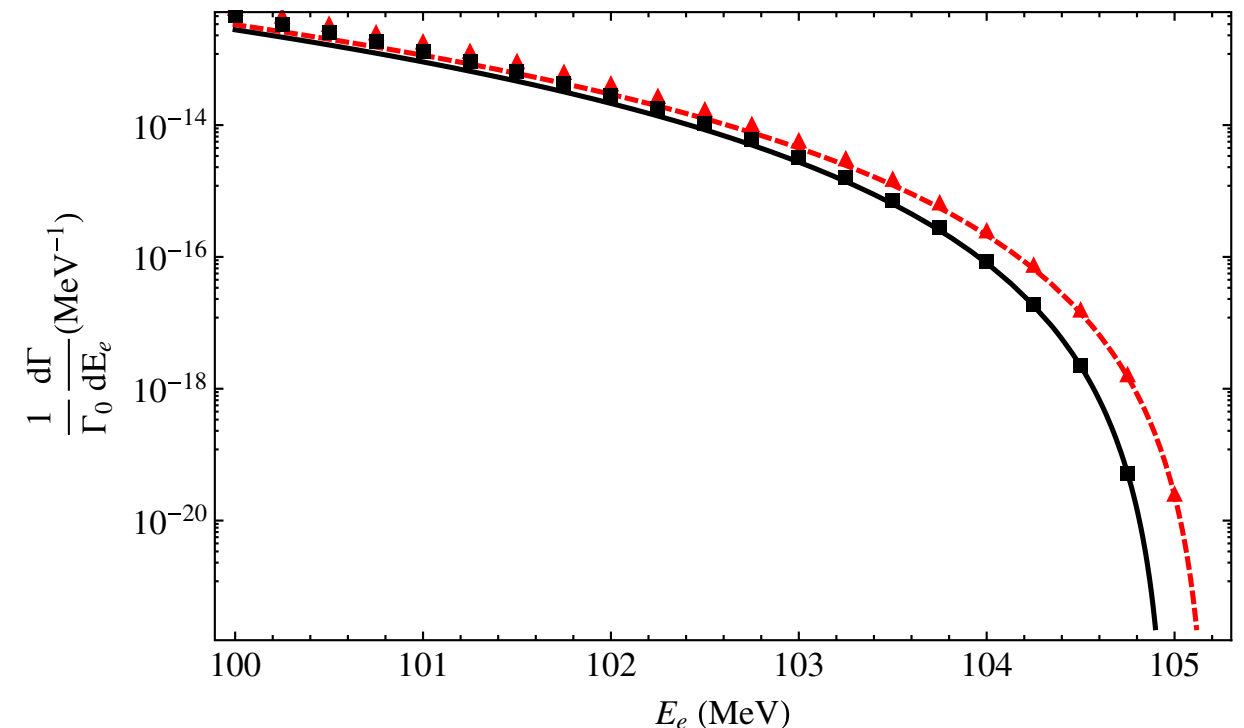


Bound Muon Decay in Orbit (DIO)

DIO spectra for Al



DIO endpoint spectrum for Al

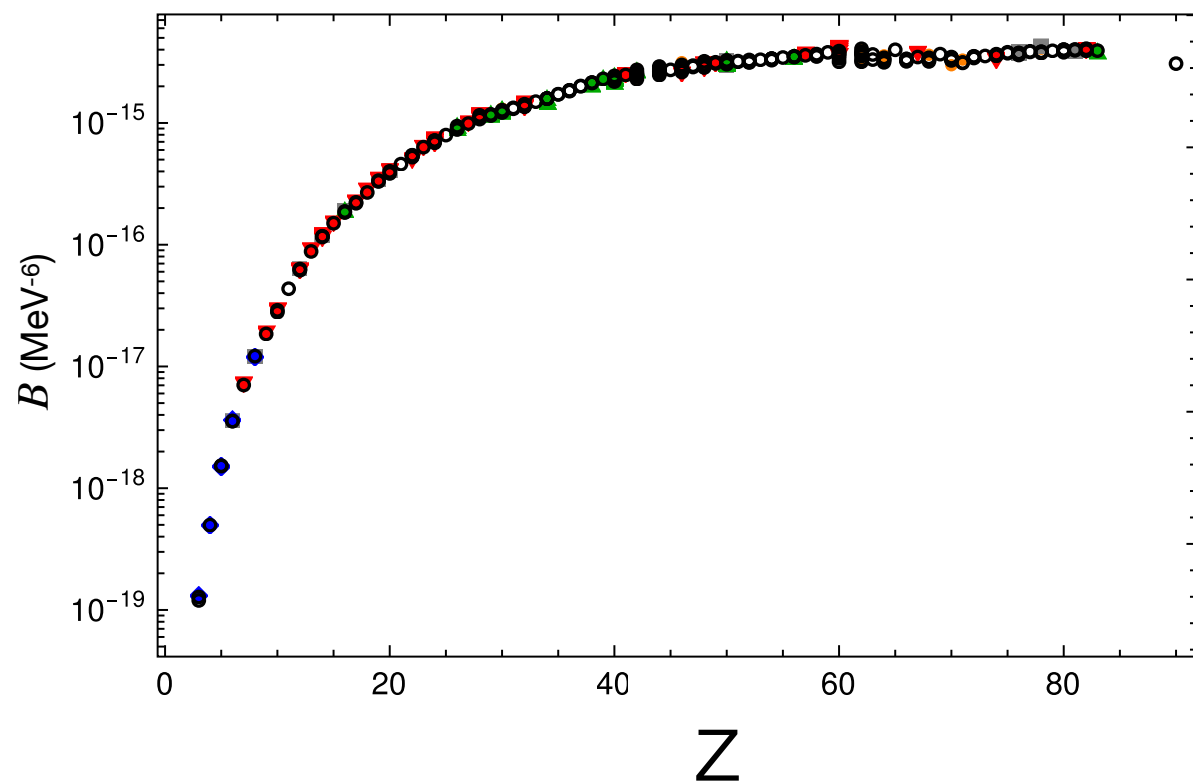


$$\frac{1}{\Gamma} \frac{d\Gamma}{dE_e} = B(E_{\mu e} - E_e)^5$$

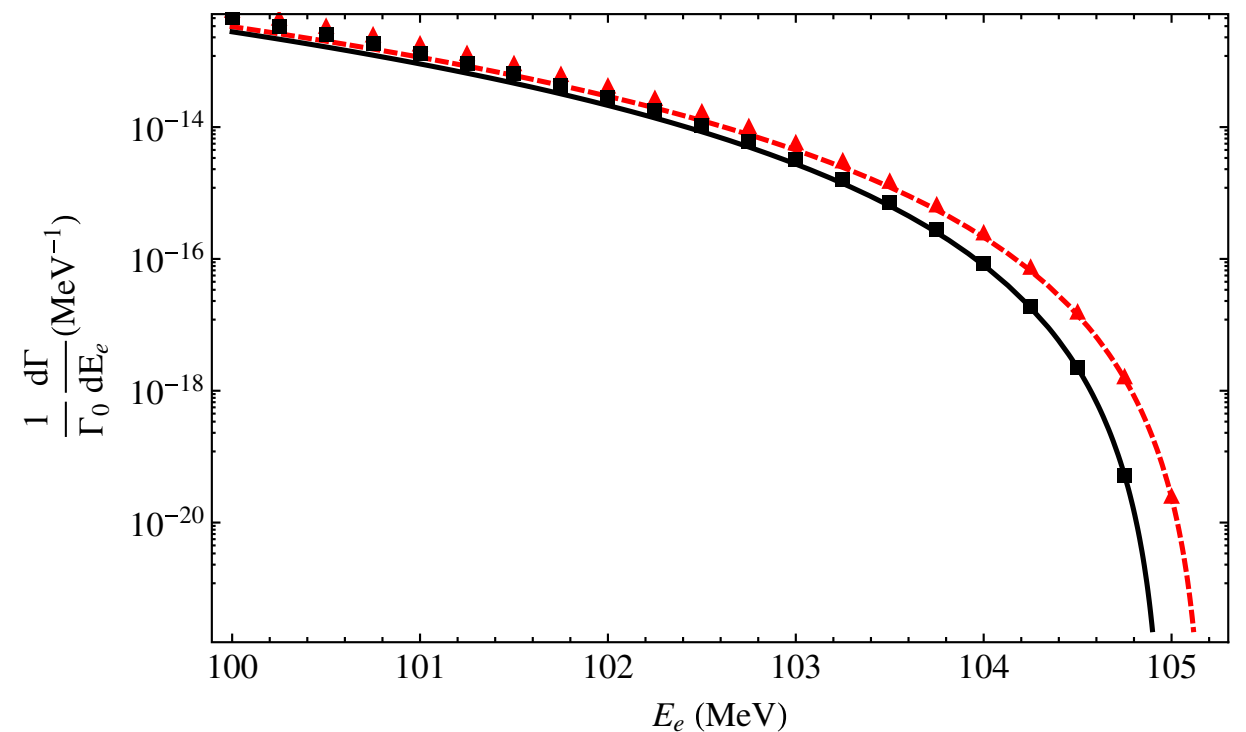
$$E_{\mu e} = m_{\mu} - B_{\mu} - \frac{E_e^2}{2M}$$

Bound Muon Decay in Orbit (DIO)

B coefficient



DIO endpoint spectrum for Al

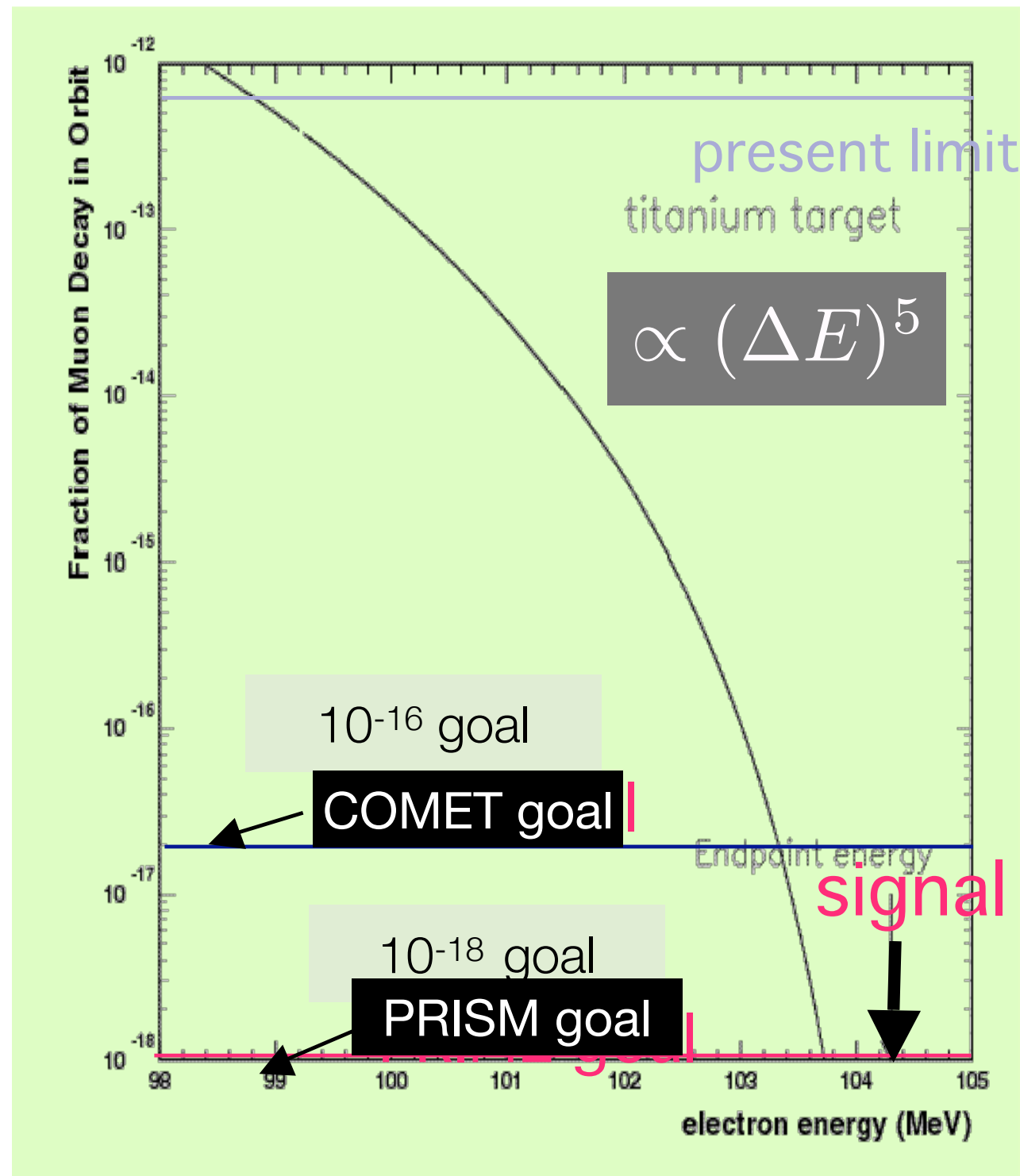


$$\frac{1}{\Gamma} \frac{d\Gamma}{dE_e} = B(E_{\mu e} - E_e)^5$$

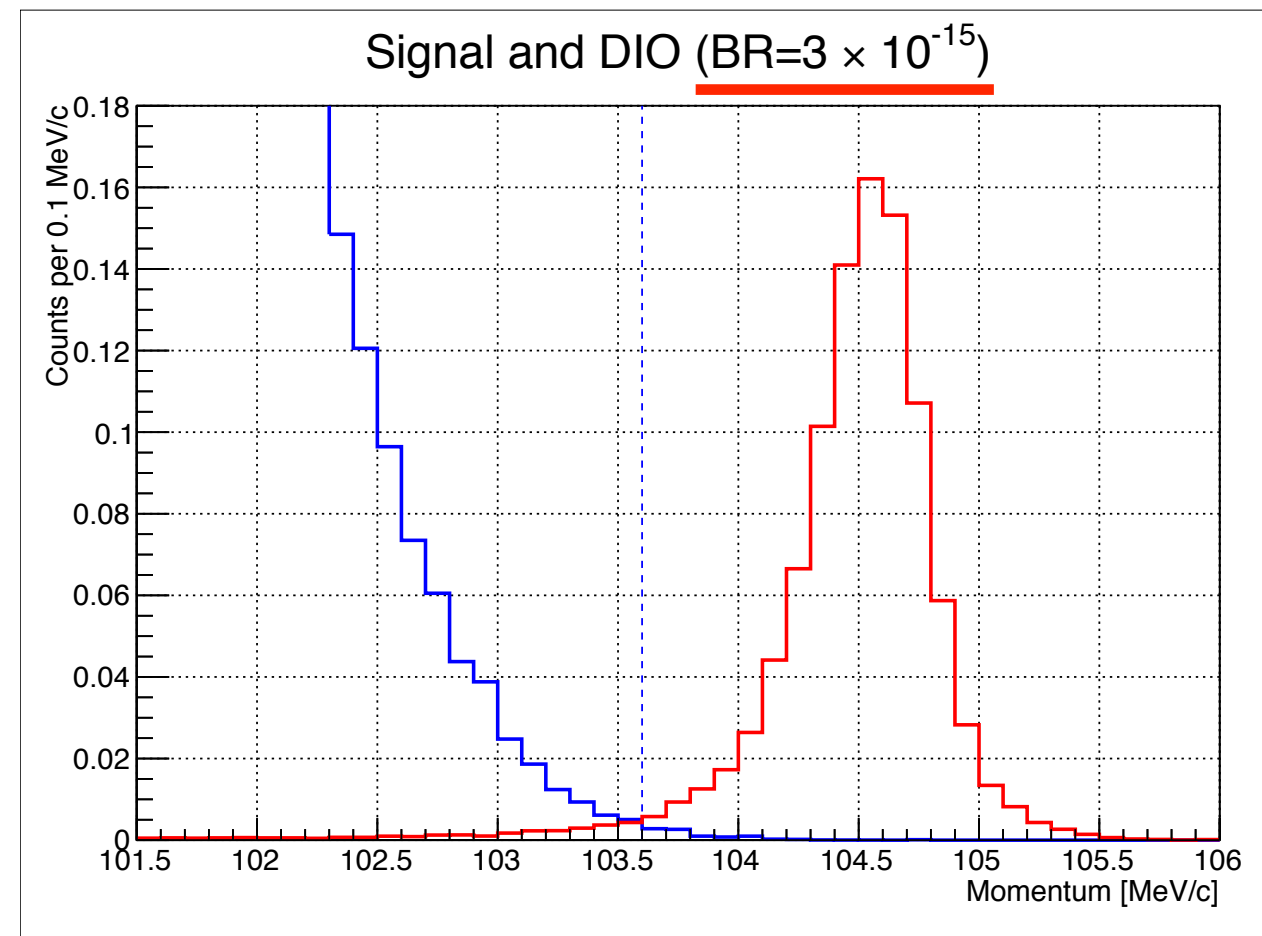
$$E_{\mu e} = m_{\mu} - B_{\mu} - \frac{E_e^2}{2M}$$

Nucleus	$B(\text{MeV}^{-6})$
Al($Z = 13$)	8.98×10^{-17}
Ti($Z = 22$)	4.94×10^{-16}
Cu($Z = 29$)	1.14×10^{-15}
Se($Z = 34$)	1.62×10^{-15}
Sb($Z = 51$)	3.57×10^{-15}
Au($Z = 79$)	4.79×10^{-15}

Bound Muon Decay in Orbit (DIO)



from COMET Phase-I TDR



Good momentum resolution is needed.

Radiative Muon Capture (RMC)



Using an appropriate target

Maximum photon energy (endpoint) :

$$E_{\gamma}^{\max} = m_{\mu} - B_{\mu} - E_{\text{rec}} + M(A, Z) - M(A, Z - 1)$$

Mass Requirement of Muon target $N(A, Z)$:

$$E_{\mu e} > E_{\gamma}^{\max} \Rightarrow M(A, Z) < M(A, Z - 1)$$

	$E_{\mu e}$	$E_{\gamma}^{\max}$	$k_{\max}$
Al	104.9	101.8	90
Au	95.56	93.81	88
Ti	104.3	102.5	93

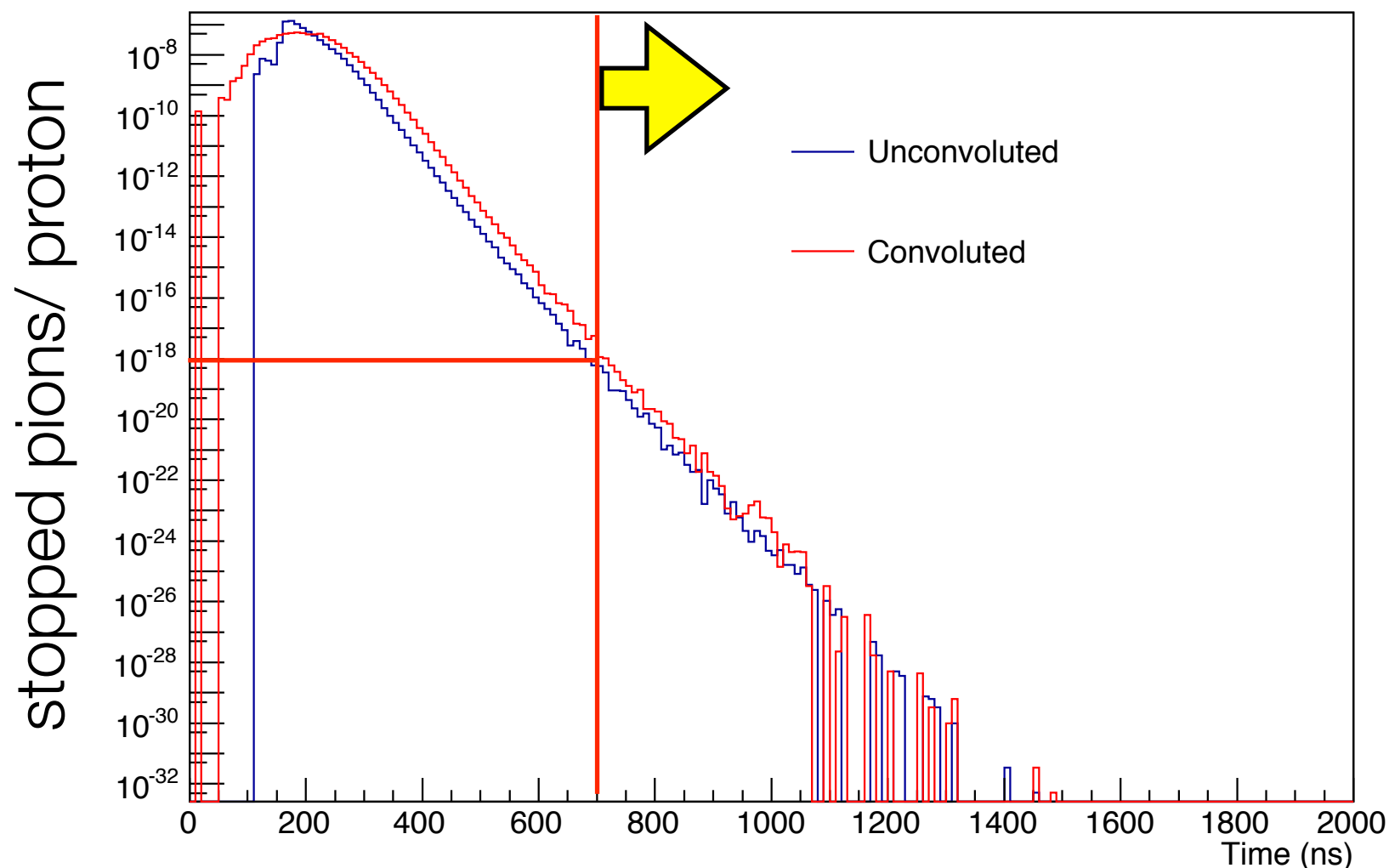
$k_{\max}$ is the empirical endpoint of the spectrum determined by the data that is fitted to the Primakoff (closure approximation) model?

Radiative Pion Capture (RPC)

$$\pi^- + N \rightarrow N' + \gamma; \quad \gamma \rightarrow e^- e^+ \quad (m_\pi > m_\mu)$$

Using a pulsed beam, a delayed time window is employed for measurement.

Total proton on target (POT): 10^{21}
 $10^{21} \times 10^{-2} \times 10^{-3} \times 10^{-18} = 0.01$



The measurement window starts at 700 ns after the beam arrival makes total 10^{-18} suppression

High Z targets cannot be used due to their short lifetimes.

$\mu^- \rightarrow e^-$ Conversion
Experiments



$\mu^- \rightarrow e^-$ Conversion : History

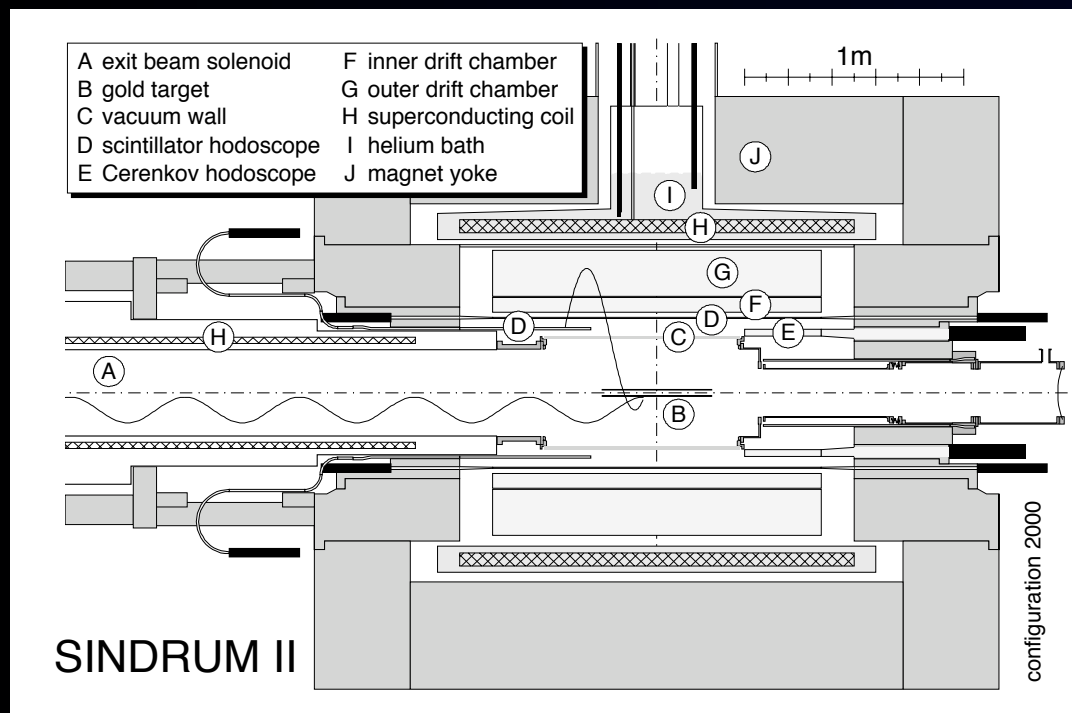
Year	90% Limit	Lab/Collaboration	Reference	Material
1952	1.0×10^{-1}	Cosmic Ray	Lagarrigue and Peyrou [1952]	Sn, Sb
1955	5.0×10^{-4}	Nevis	Steinberger and Wolfe [1955]	Cu
1961	4.0×10^{-6}	LBL	Sard <i>et al.</i> [1961]	Cu
1961	5.9×10^{-6}	CERN	Conversi <i>et al.</i> [1961]	Cu
1962	2.2×10^{-7}	CERN	Conforto <i>et al.</i> [1962]	Cu
1964	2.2×10^{-7}	Liverpool	Bartley <i>et al.</i> [1964]	Cu
1972	1.6×10^{-8}	SREL	Bryman <i>et al.</i> [1972]	Cu
1977	4.0×10^{-10}	SIN	Badertscher <i>et al.</i> [1977]	S
1982	7.0×10^{-11}	SIN	Badertscher <i>et al.</i> [1982]	S
1988	4.6×10^{-12}	TRIUMF	Ahmad <i>et al.</i> [1988]	Ti
1993	4.3×10^{-12}	SINDRUM II	Dohmen <i>et al.</i> [1993]	Ti
1996	4.6×10^{-11}	SINDRUM II	Honecker <i>et al.</i> [1996]	Pb
2006	7.0×10^{-13}	SINDRUM II	Bertl <i>et al.</i> [2006]	Au

Summary of Current Limits on $\mu^- \rightarrow e^-$ Conversion

	Z	spin	CR upper limit
sulfur	16	0	7×10^{-11}
titanium	22	0, 5/2, 7/2	4.3×10^{-12}
copper	29	3/2	1.6×10^{-8}
gold	79	5/2	7×10^{-13}
lead	82	0, 1/2	4.6×10^{-11}

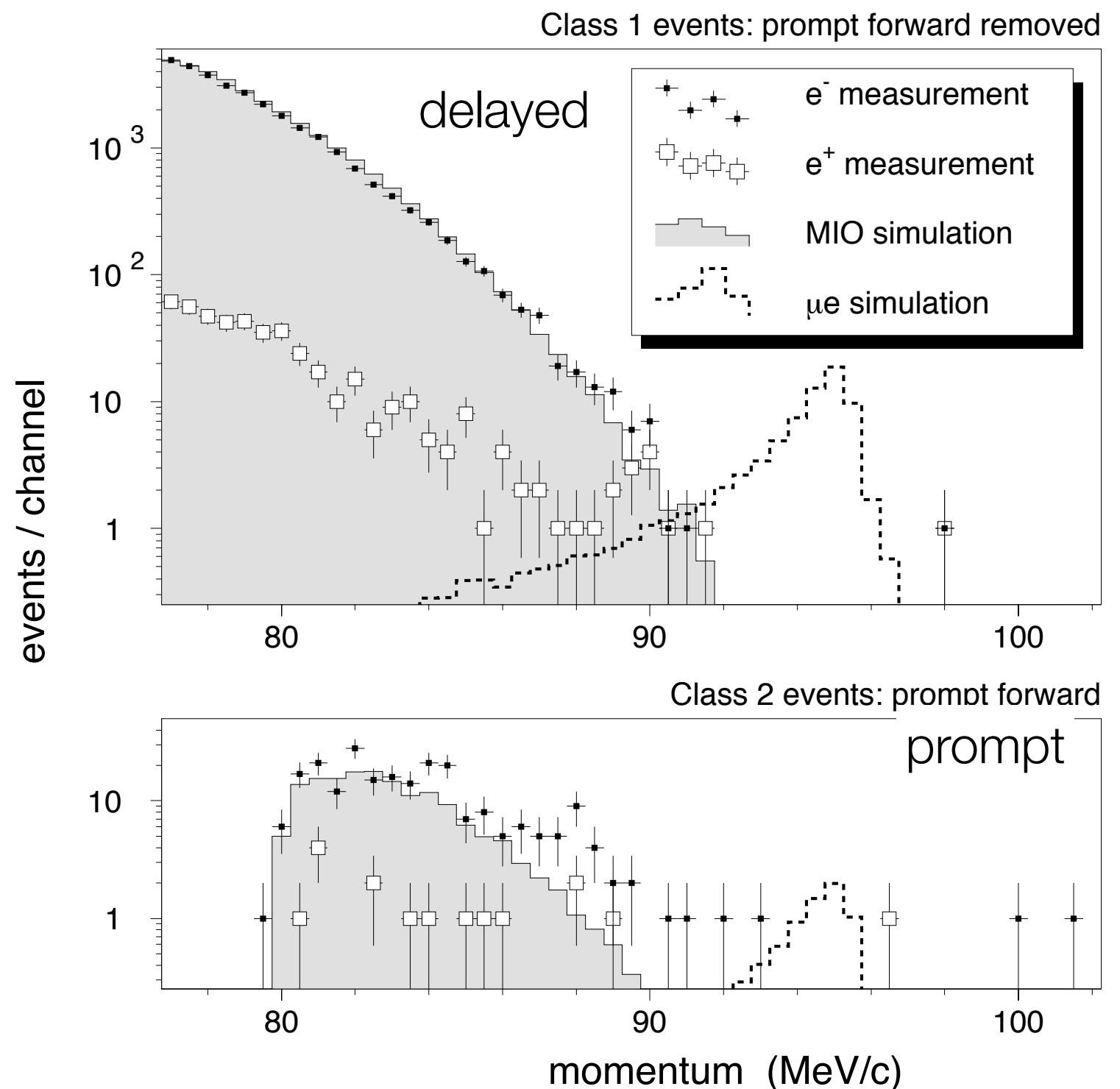
Current Limits on $\mu^- \rightarrow e^-$ Conversion (2004)

SINDRUM-II (PSI)



$$B(\mu^- + Au \rightarrow e^- + Au) < 7 \times 10^{-13}$$

- one e^- and one e^+ background at 98 MeV/c ?
- e^+ peak at 90 MeV/c ?



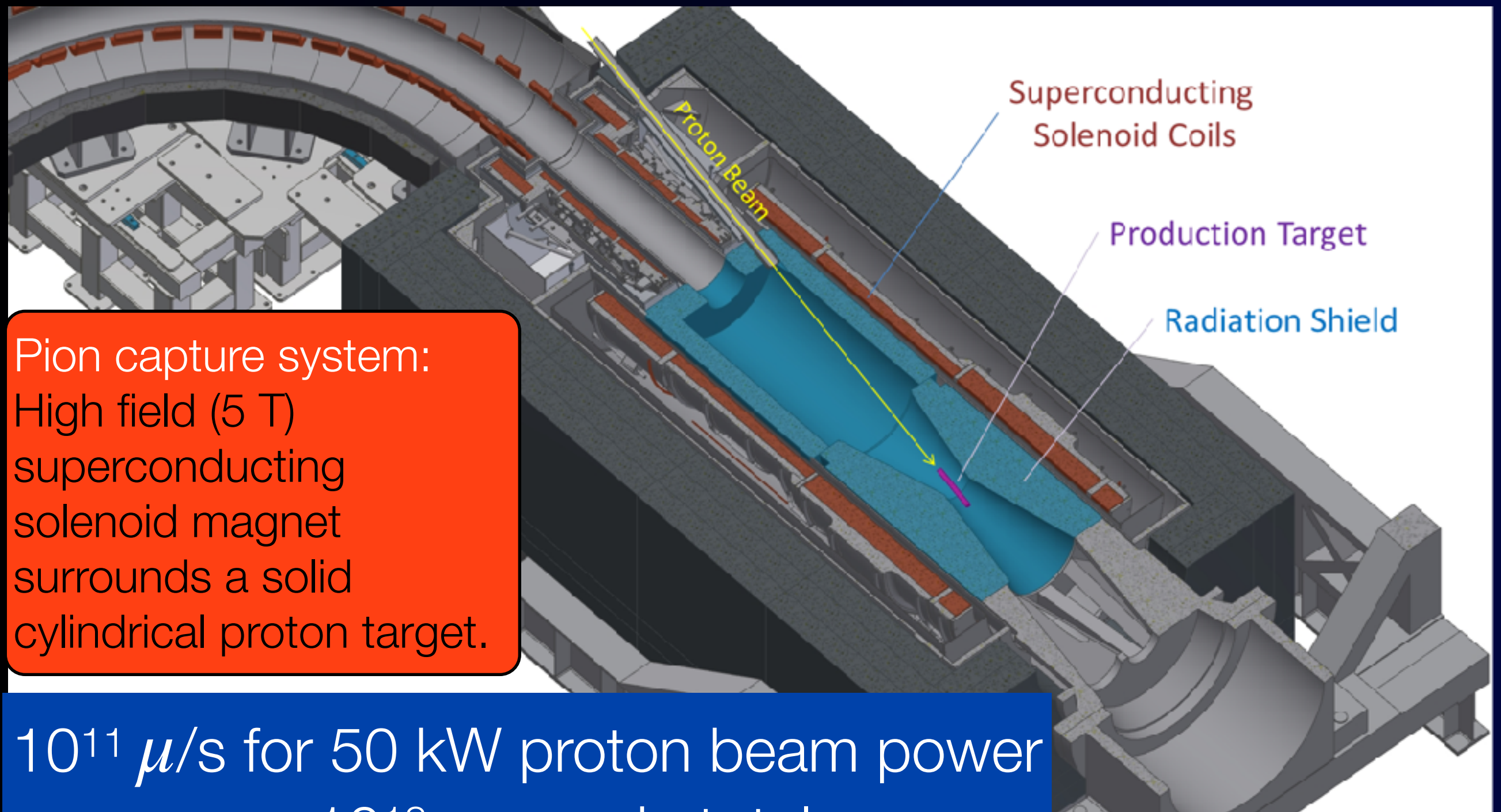
Improvement of
 $\mu^- \rightarrow e^-$ Conversion
Sensitivity



$$B(\mu N \rightarrow e N) \leq 10^{-16}$$

with a factor of 10,000 improvement

Improvements for Signal Sensitivity

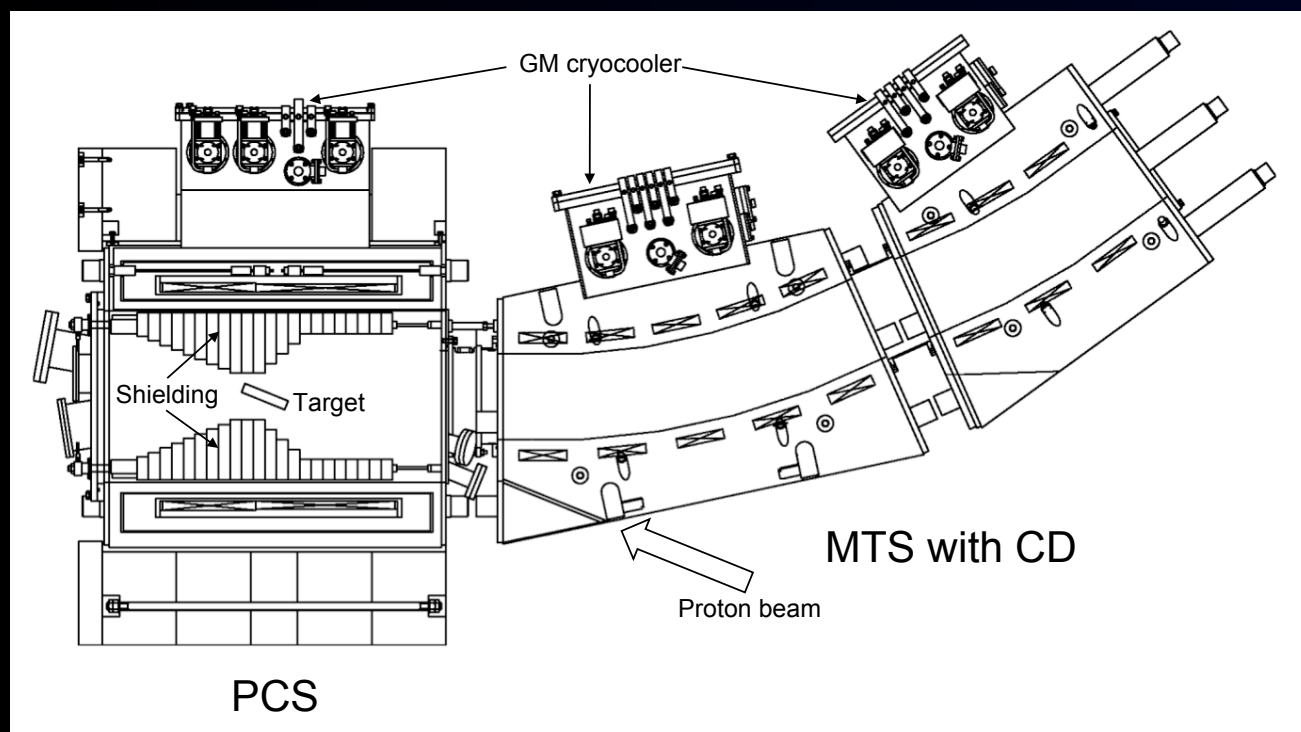


Pion capture system:
High field (5 T)
superconducting
solenoid magnet
surrounds a solid
cylindrical proton target.

$10^{11} \mu/\text{s}$ for 50 kW proton beam power
or 10^{18} muons in total

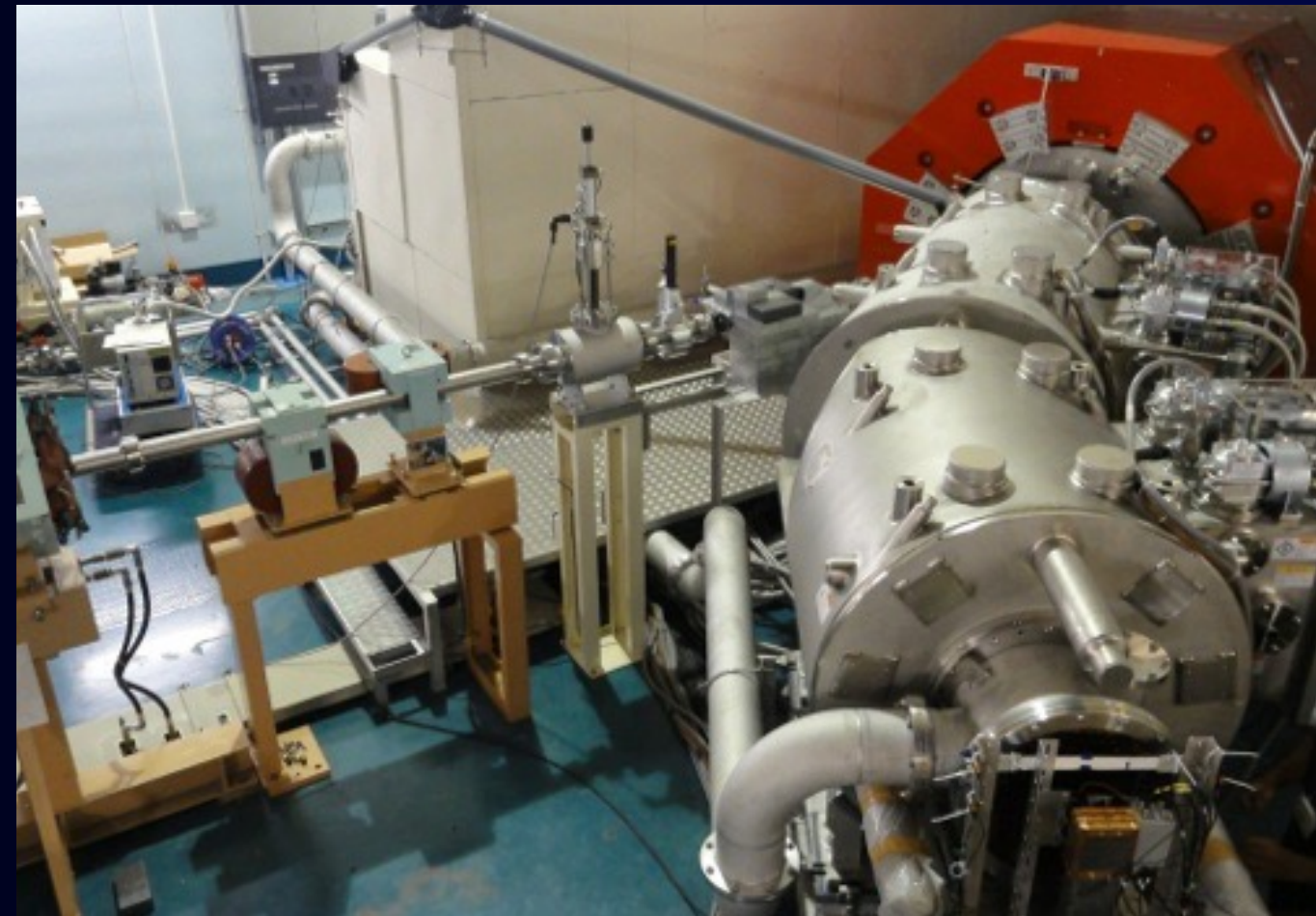
MuSIC at RCNP, Osaka University

MuSIC (Muon Science Intense Channel) since 2011



PCS : 3.5T solenoid field
and graphite target

MuSIC: 10^5 muons/sec/W
PSI: 3×10^2 muons/sec/W



(RCNP proton cyclotron 400 W)
(PSI proton cyclotron 1.2 MW)

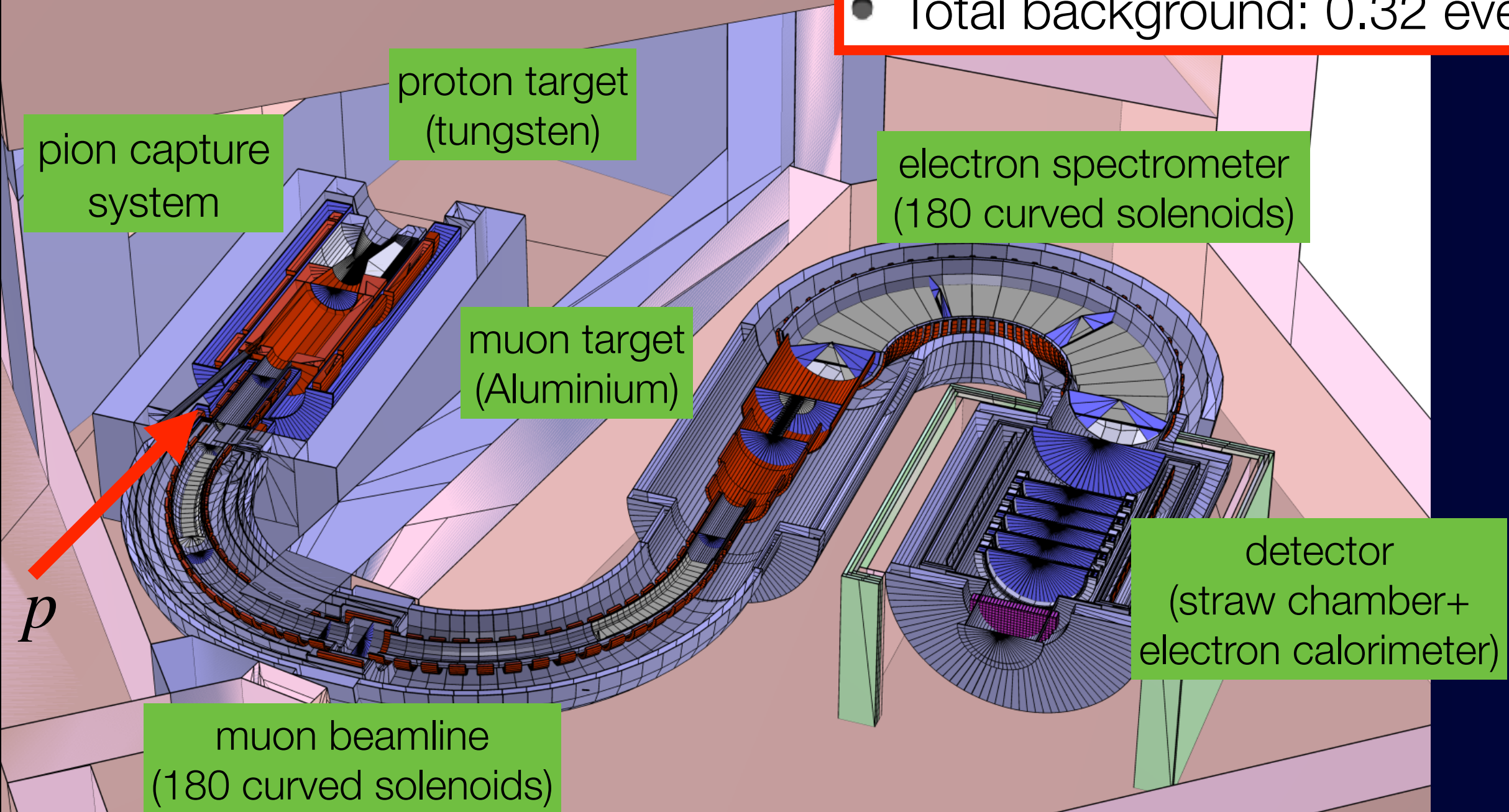
New Experiments of
 $\mu^- \rightarrow e^-$ Conversion



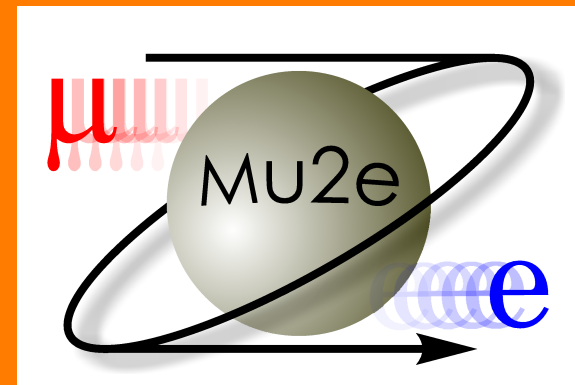
COMET at J-PARC

- Proton beam, 8 GeV, 56kW
- 2×10^{11} stopped muons/s
- Running time: 2/3 years (2×10^7 s)
- Single event sensitivity : 1.4×10^{-17}
- 90% CL limit : $< 3.2 \times 10^{-17}$
- Total background: 0.32 events

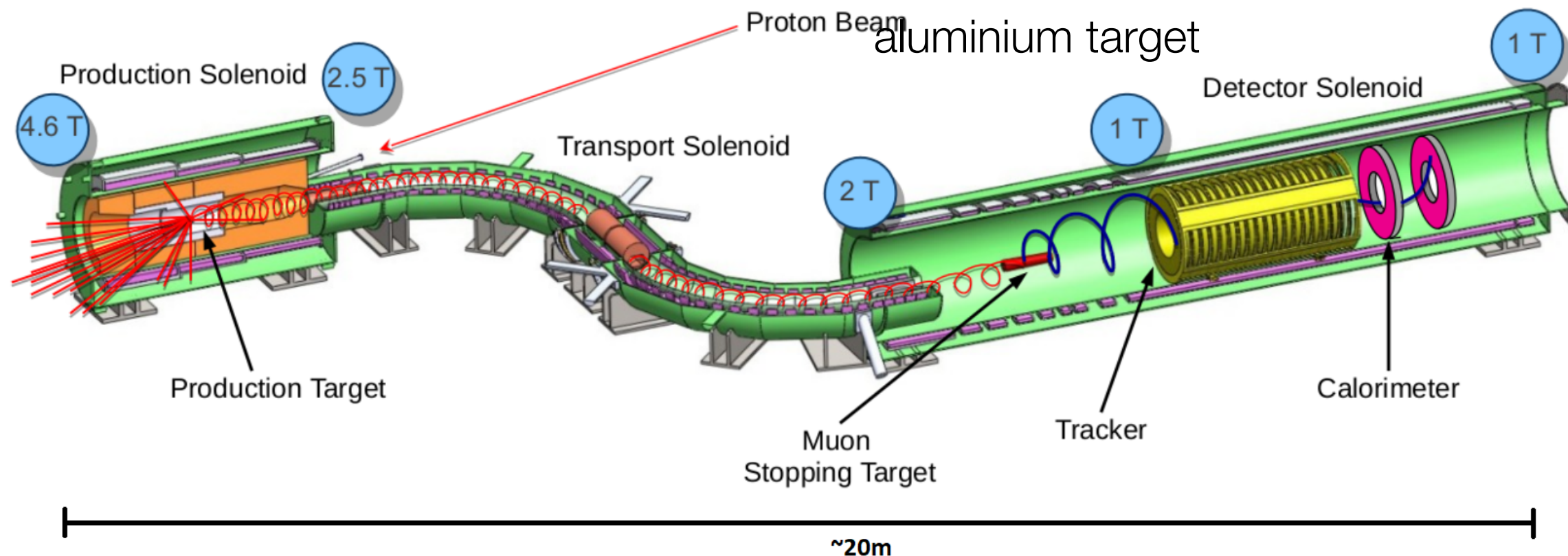
Phase-II



Mu2e at Fermilab



6×10^{10} muons/s from 8 kW, 8 GeV proton beam



Sensitivity : $< 6 \times 10^{-17}$ (90% CL)

a factor of 10,000 improvement

Run time: 3 years (2×10^7 sec/year)
commissioning in late 2020s

Mu2e-II

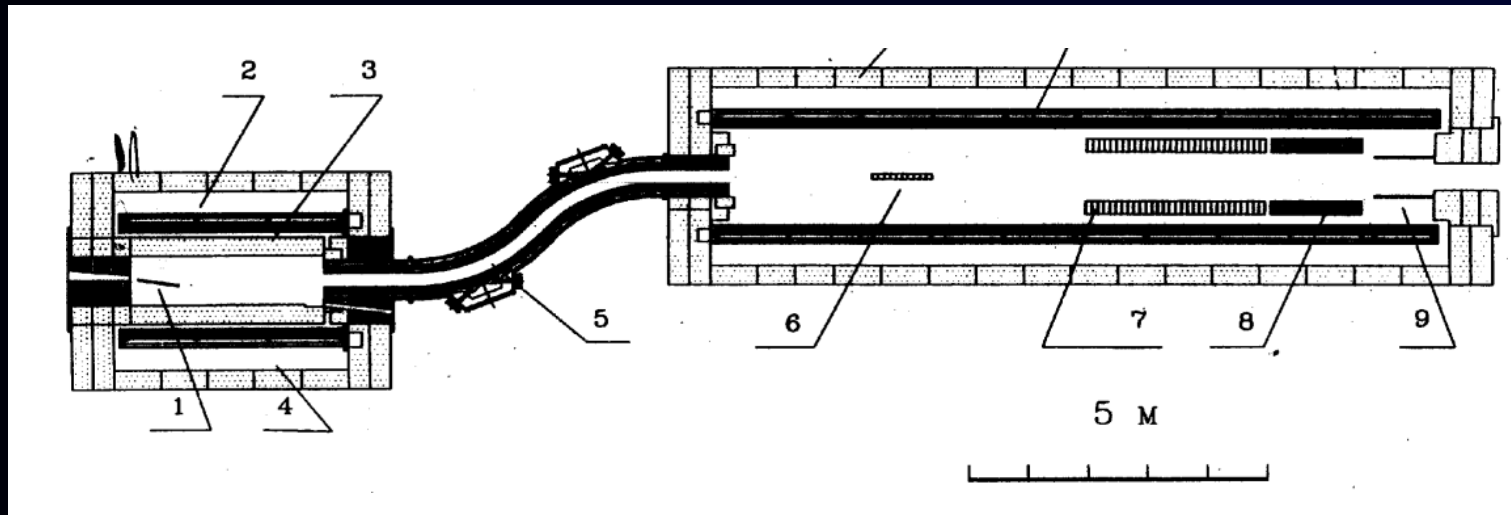
- 800 MeV, 100 kW from PIP-II

- aim at $< O(10^{-17})$ with 3 years

a factor of 10 better from Mu2e

From MELC to MECO (1992 - 2005)

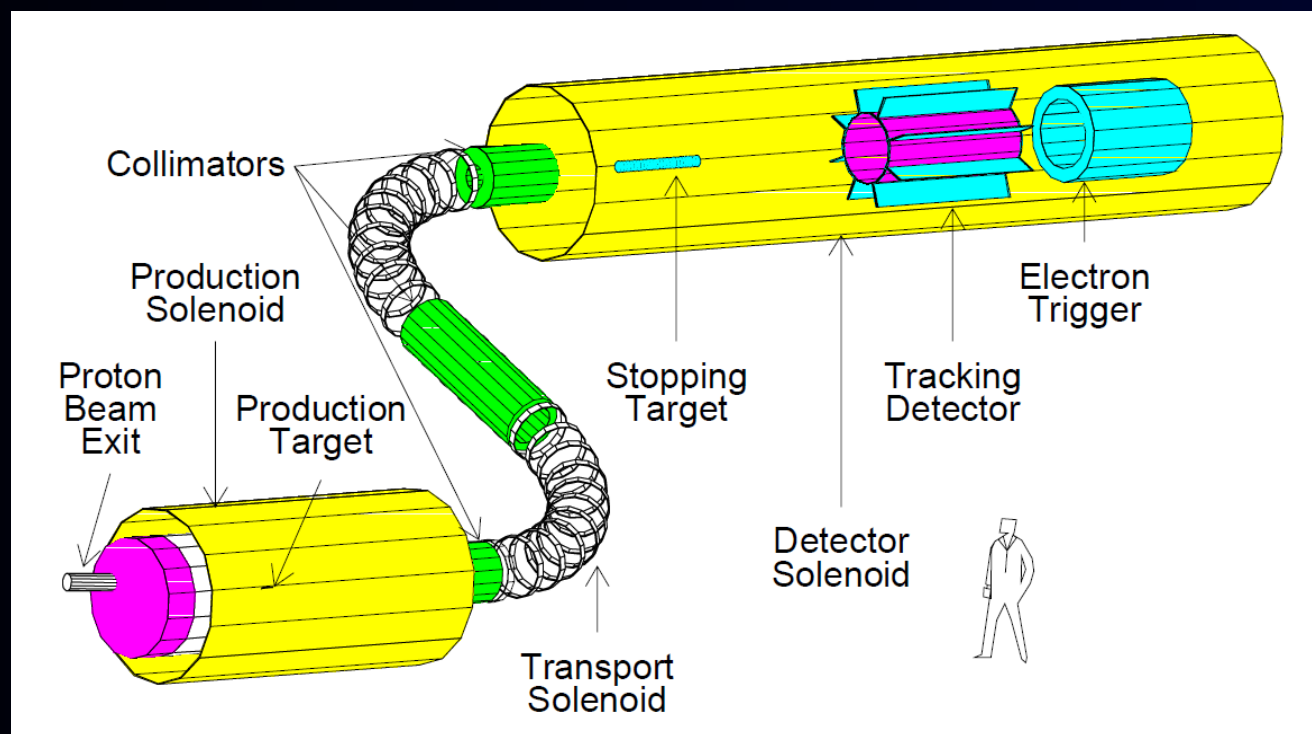
MELC



Proposal (1992)
at Moscow
Meson Factory

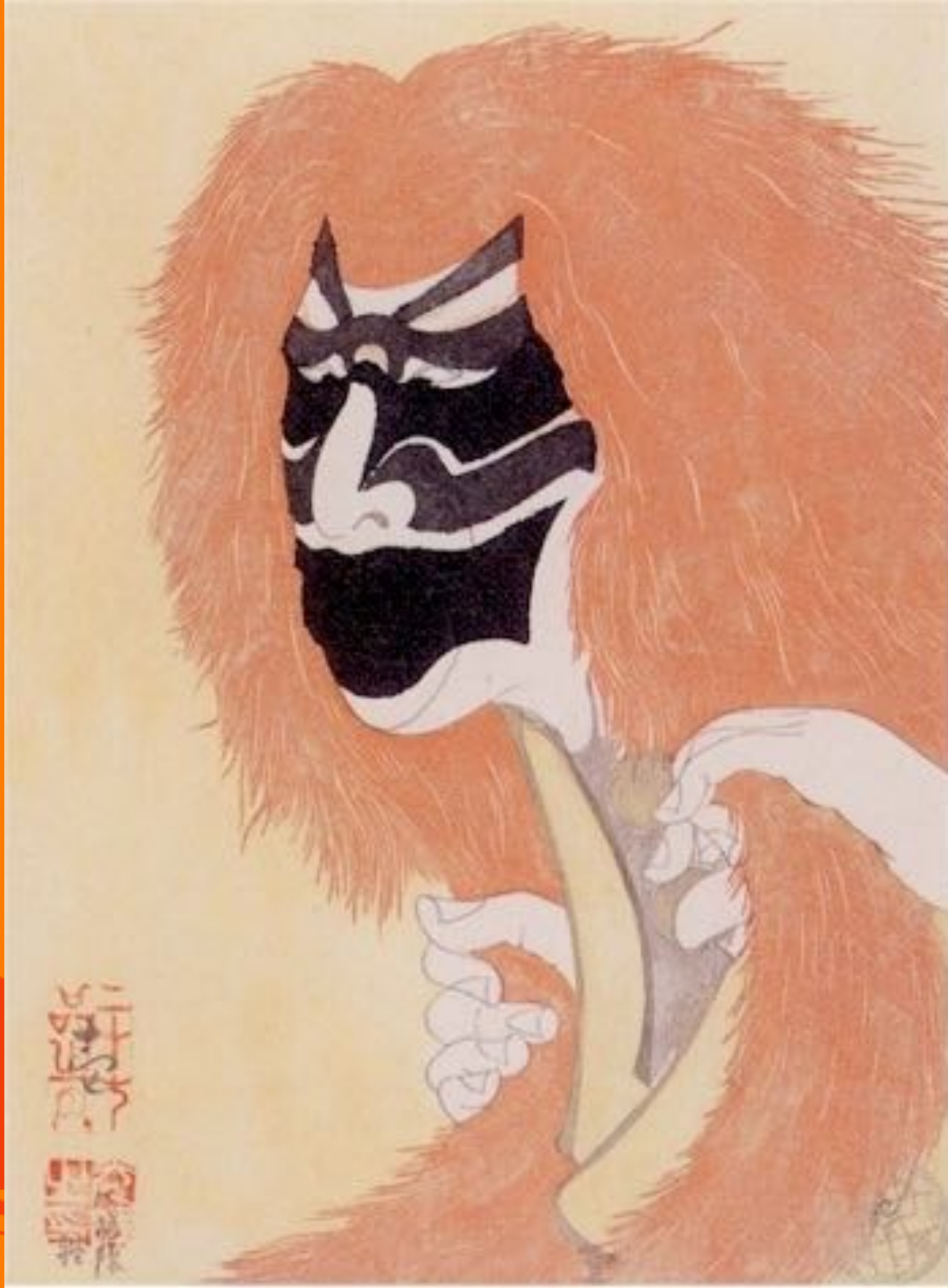
R. M. Dzhilkibaev and V. M. Lobashev, Sov. J. Nucl. Phys. 49, 384 (1989)

MECO



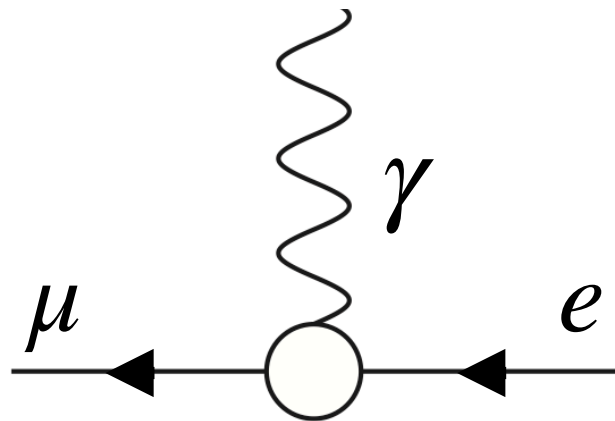
BNL E940 (1997)
one of the RSVP (rare
symmetry violating
processes) with KOPIO
terminated in 2005

$\mu^- \rightarrow e^-$ Conversion
Phenomenology

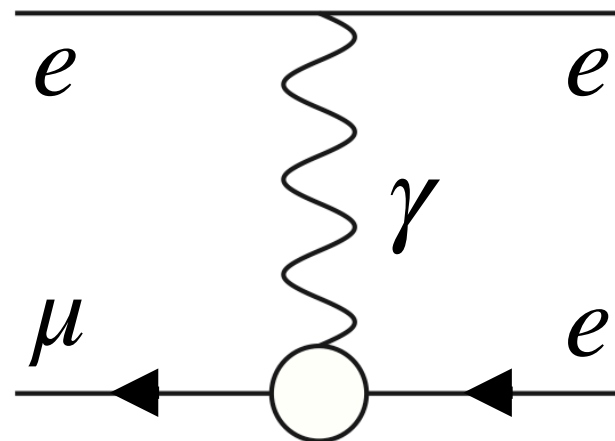


“Golden” $\mu \rightarrow e$ CLFV Processes in EFT

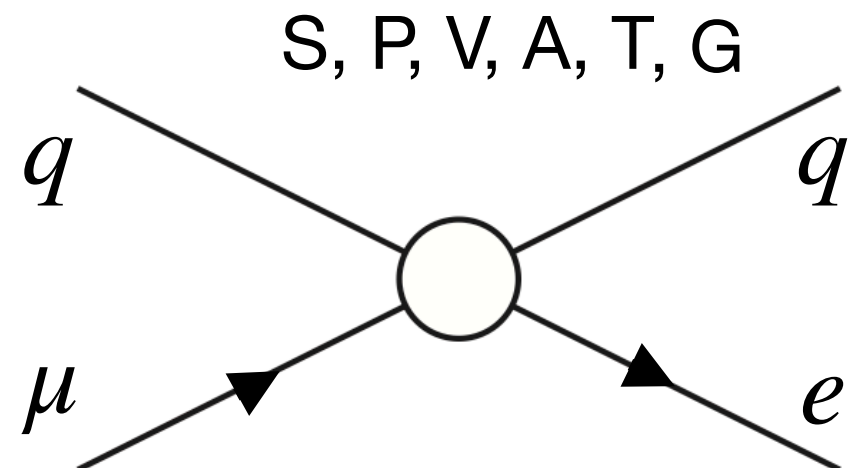
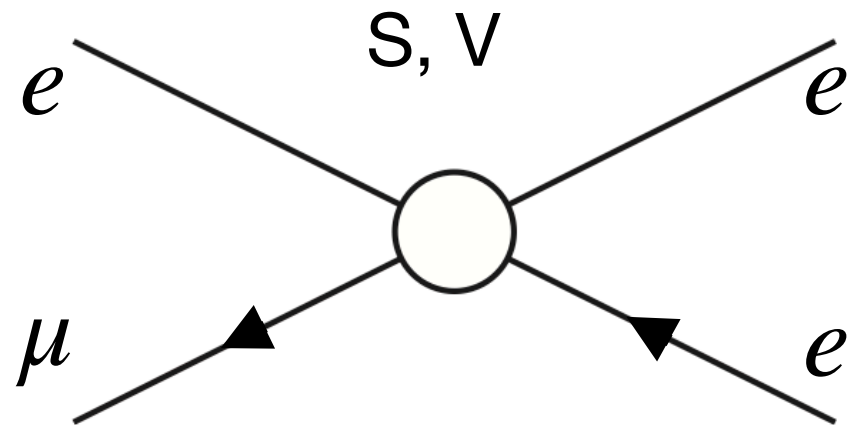
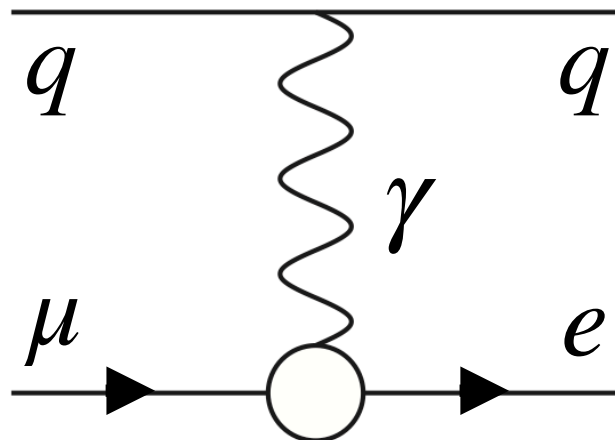
$$\mu^+ \rightarrow e^+ \gamma$$



$$\mu^+ \rightarrow e^+ e^+ e^-$$

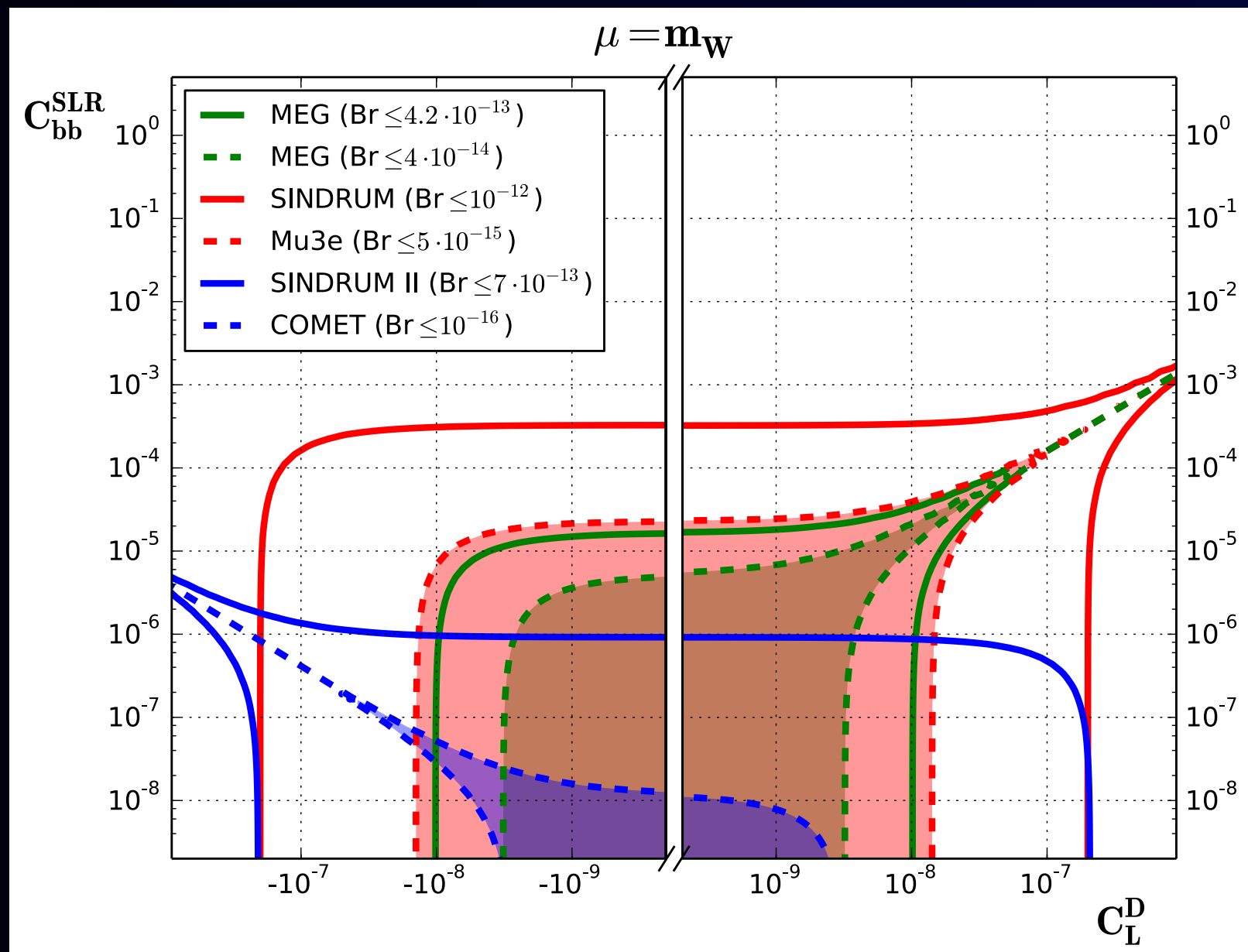


$$\mu^- N \rightarrow e^- N$$

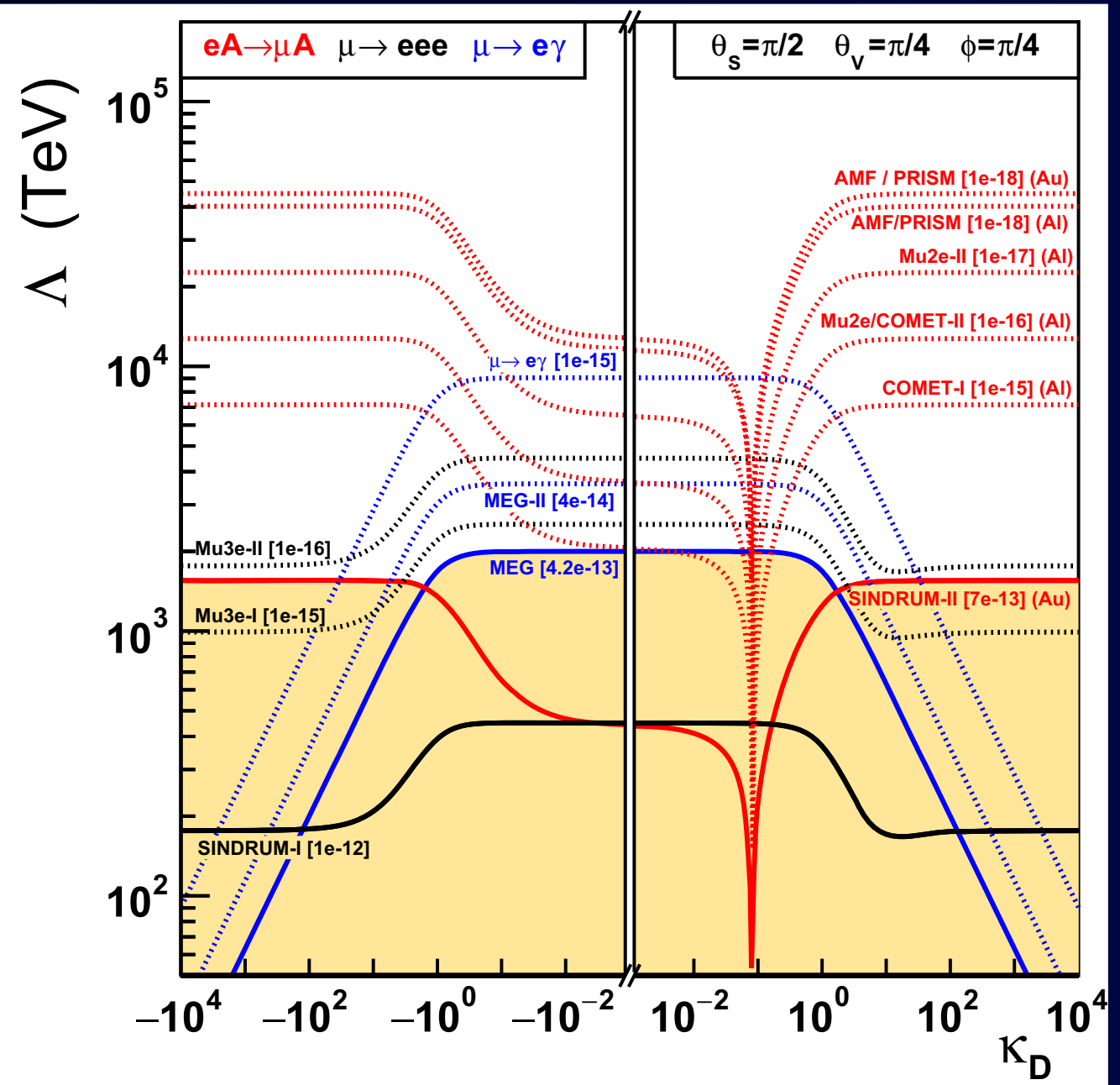
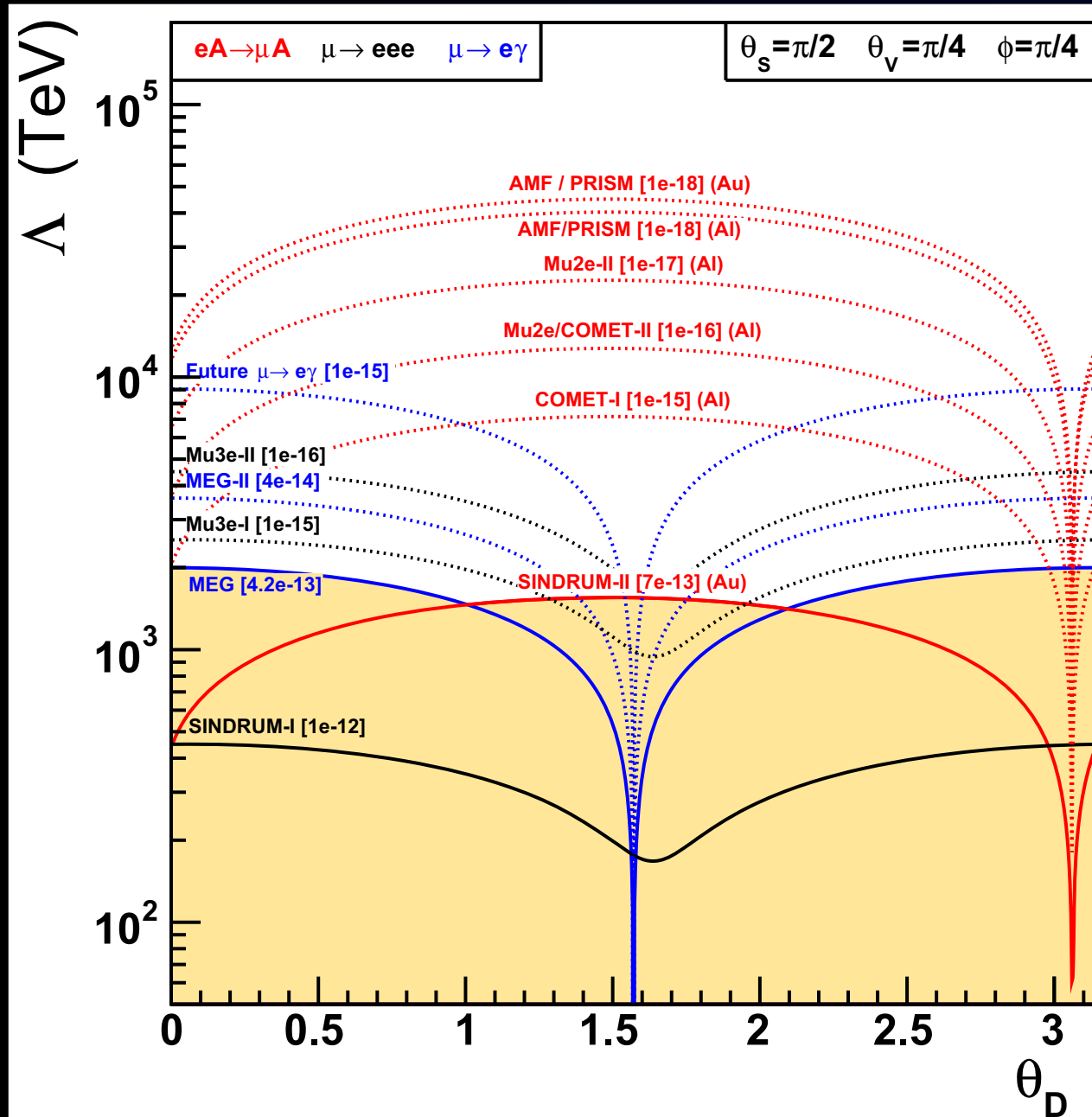


CLFV in EFT : RGE

RGE mixes different operators.



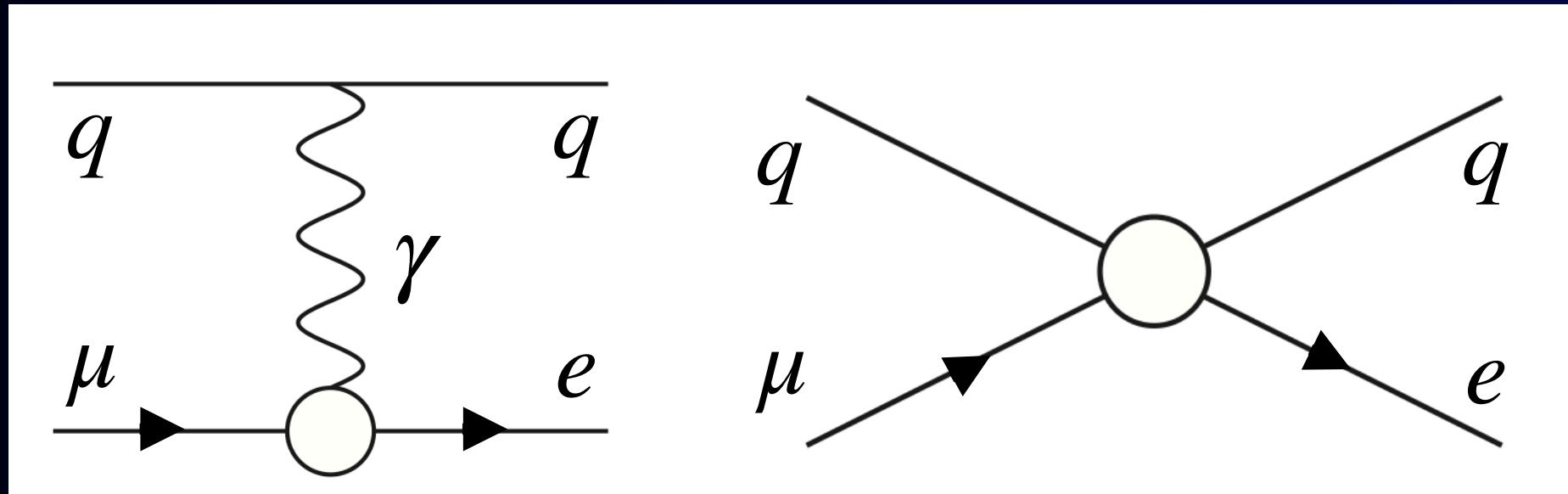
CLFV in EFT : Dipole vs. Contact Interactions



$\kappa_D = \cotan(\theta_D - \pi/2)$ $|\kappa_D| \ll 1$: dipole dominant, $|\kappa_D| \gg 1$: four-fermion dominant

EFT for Coherent $\mu^- \rightarrow e^-$ Conversion

up to dimension 6

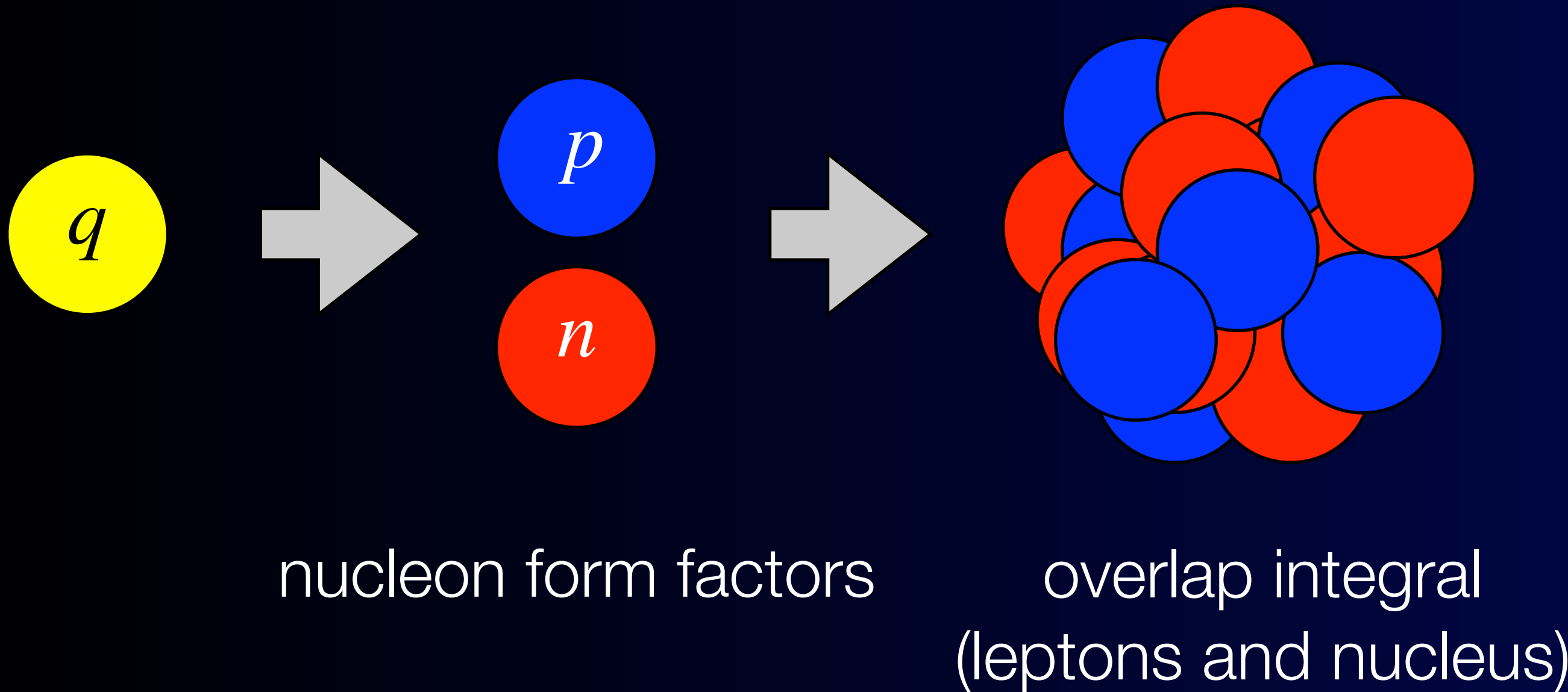


Spin-independent
Coherent

Dipole (L/R)

Scalar (L/R)
Vector (L/R)

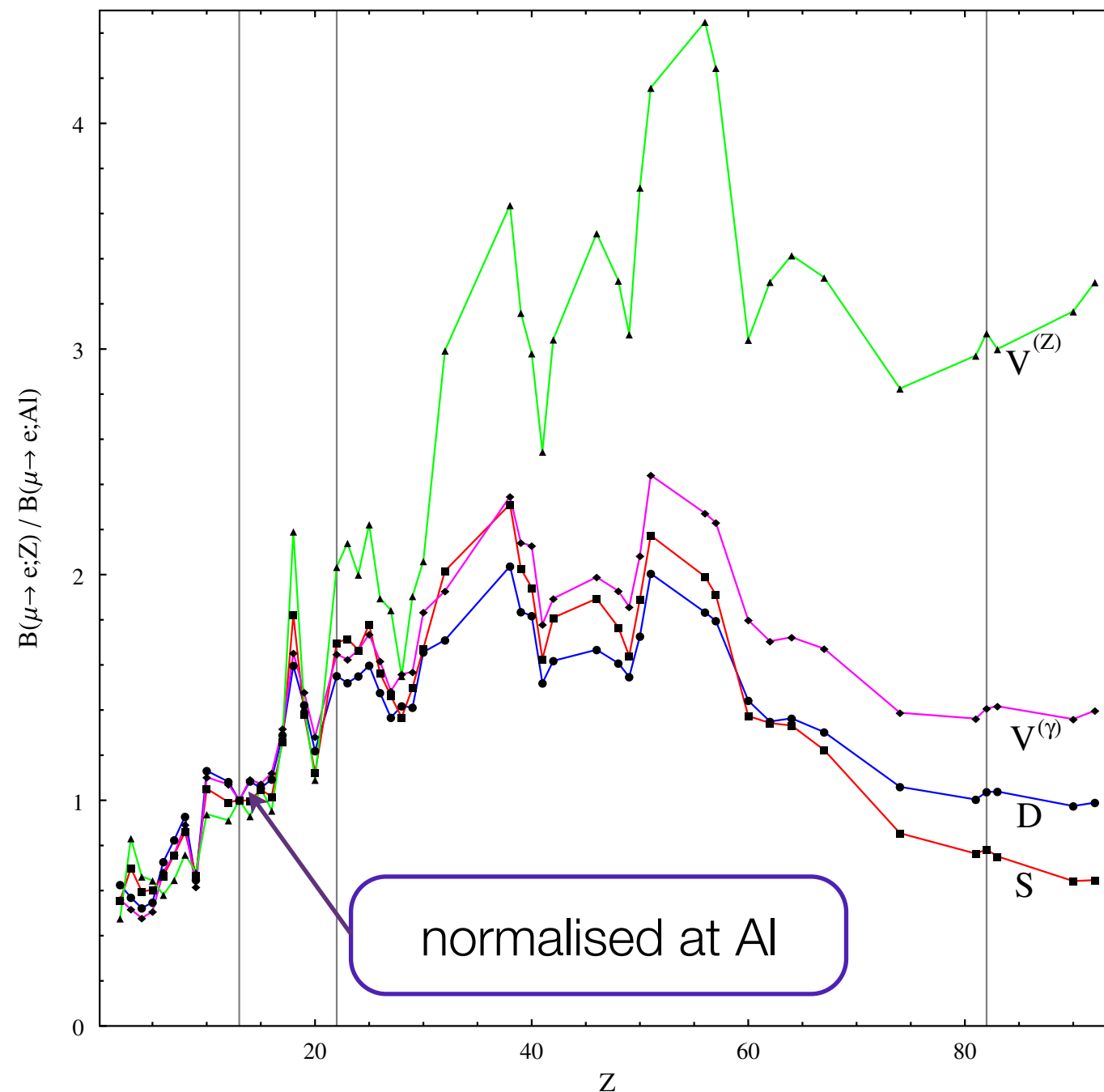
Rate of $\mu^- \rightarrow e^-$ Conversion



“One of the major topics in this workshop”

Model Discrimination with Different Muon Targets

5% for light targets,
20% for heavy targets



(single and two operators models)

vector int. (with Z boson)

Left-right symmetric models

vector int. (with photon)

dipole int. SUSY-GUT, SUSY-Seesaw

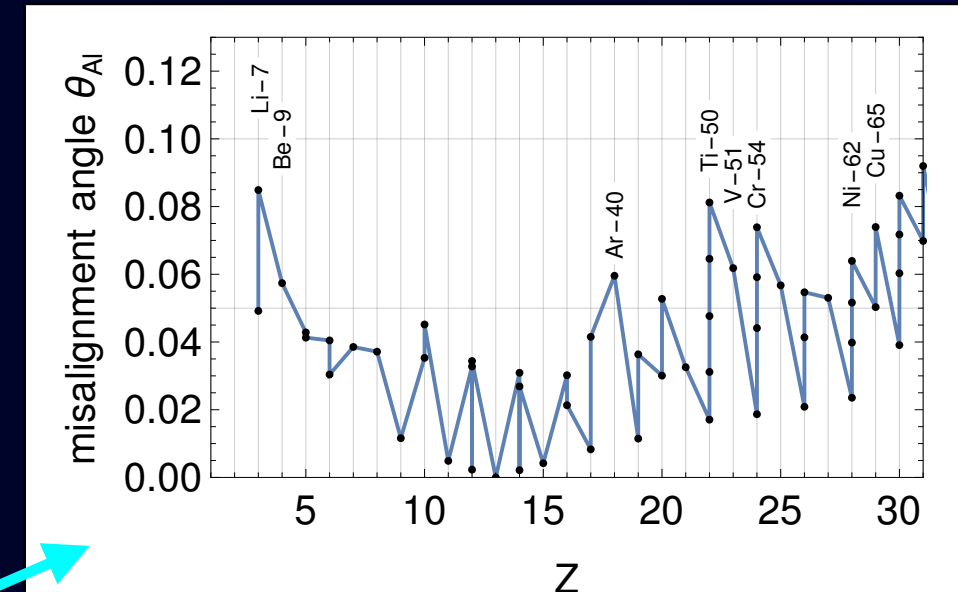
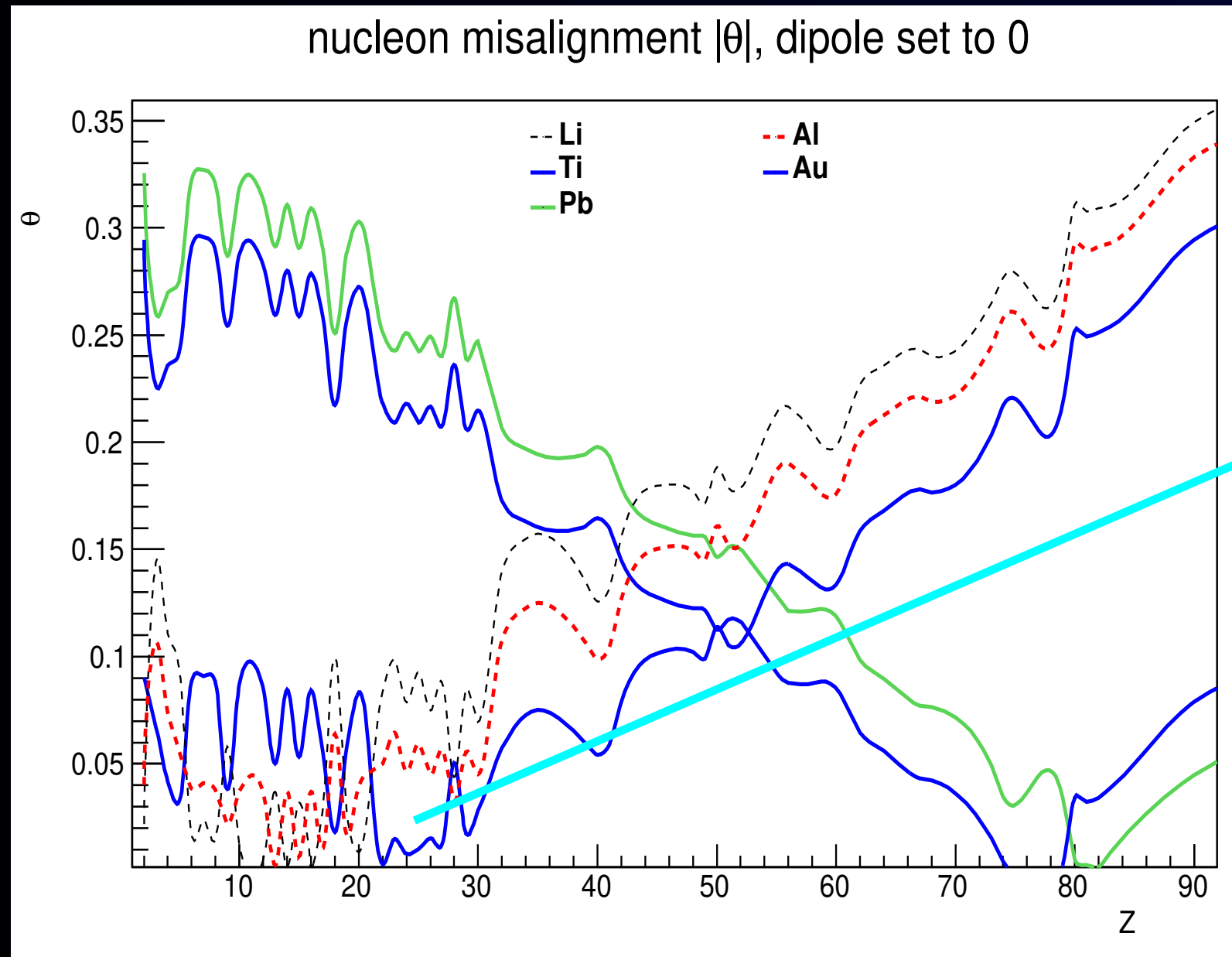
scalar int. R-violating SUSY,
SUSY-Seesaw

R. Kitano, M. Koike and Y. Okada, Phys.Rev. D66 (2002) 096002; D76 (2007) 059902

V. Cirigliano, R. Kitano, Y. Okada, and P. Tuzon, Phys. Rev. D80 (2009) 013002

Model Discrimination with Different Muon Targets (2)

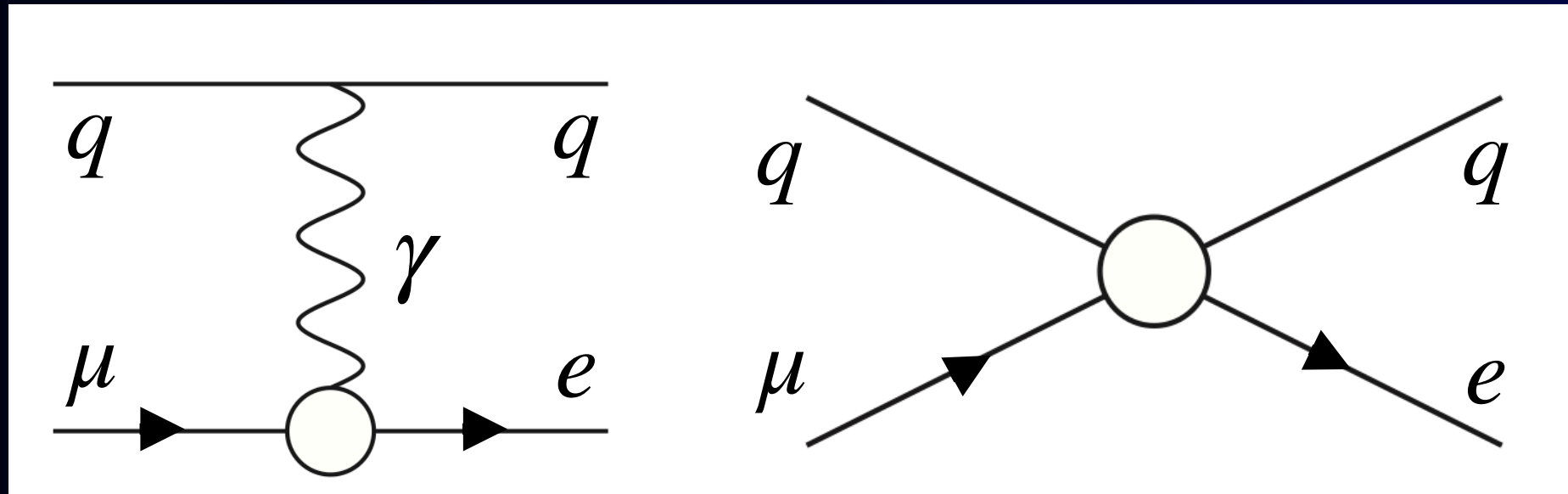
Scalar (n,p)+Vector (n,p)(+Dipole)



choose a target to maximize the constraints on the operators, represented by a large misalignment angle.

Spin Dependent (SD) $\mu^- \rightarrow e^-$ Conversion

up to dimension 6



Spin-independent

Coherent

Dipole (L/R)

Scalar (L/R)

Vector (L/R)

Spin-dependent

Incoherent

Pseudoscalar (L/R)

Axial vector (L/R)

Tensor (L/R)

Inelastic $\mu^- \rightarrow e^-$ Conversion

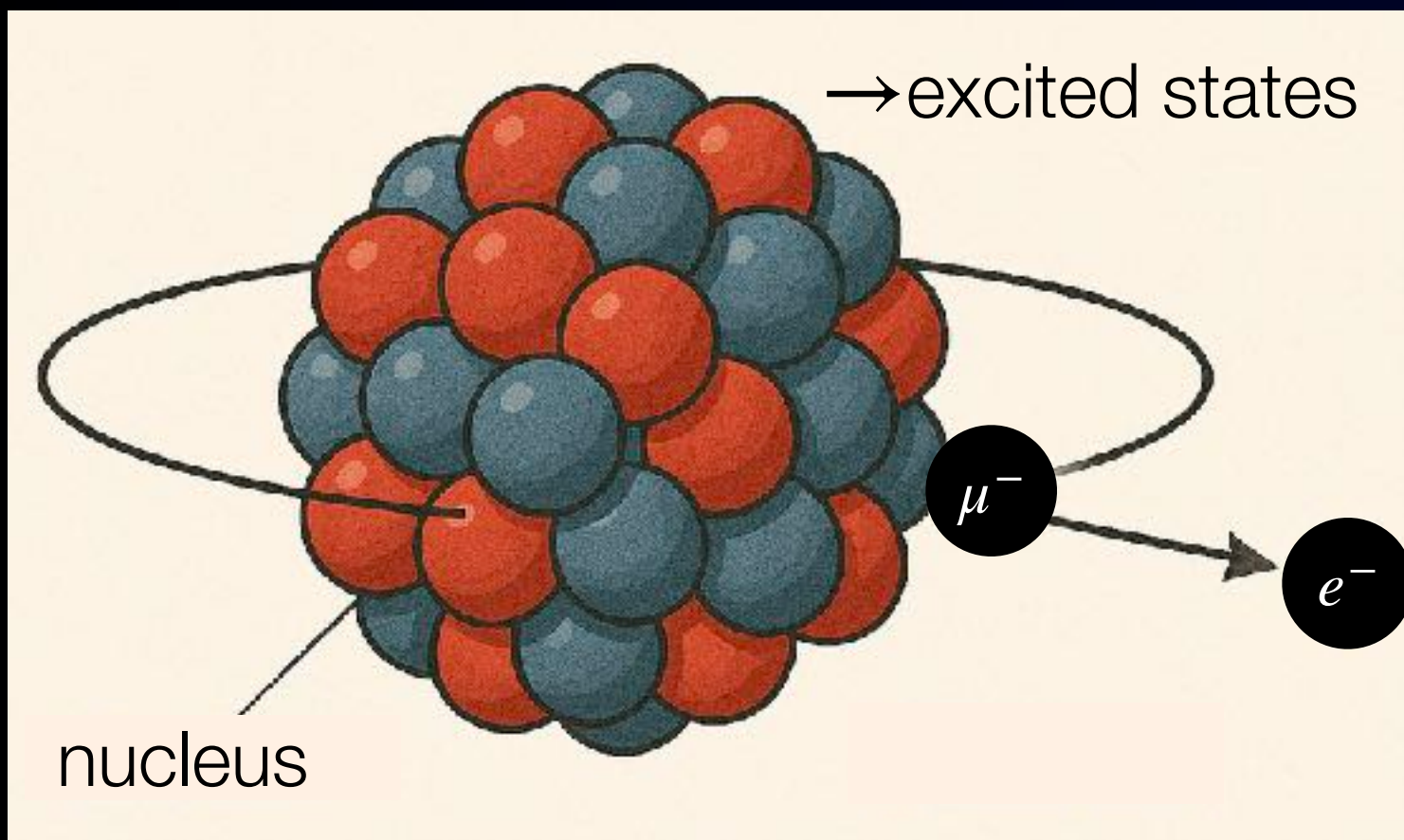
“Inelastic” $\mu^- \rightarrow e^-$ Conversion

$$\mu^- N(A, Z) \rightarrow e^- N(A, Z)^*$$

- Conversion to excited states.
- incoherent processes

Event Signature for the transition to the excited state :

a single mono-energetic electron



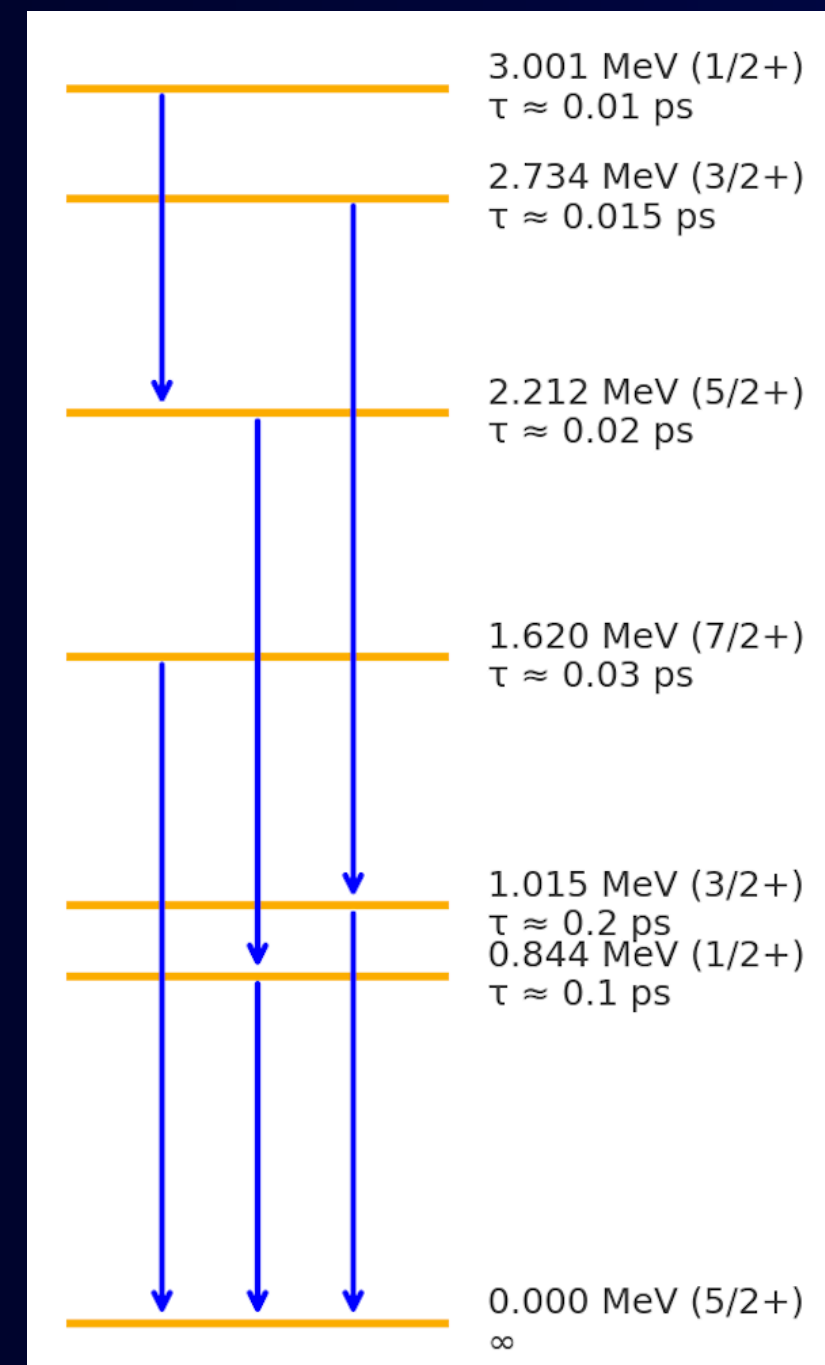
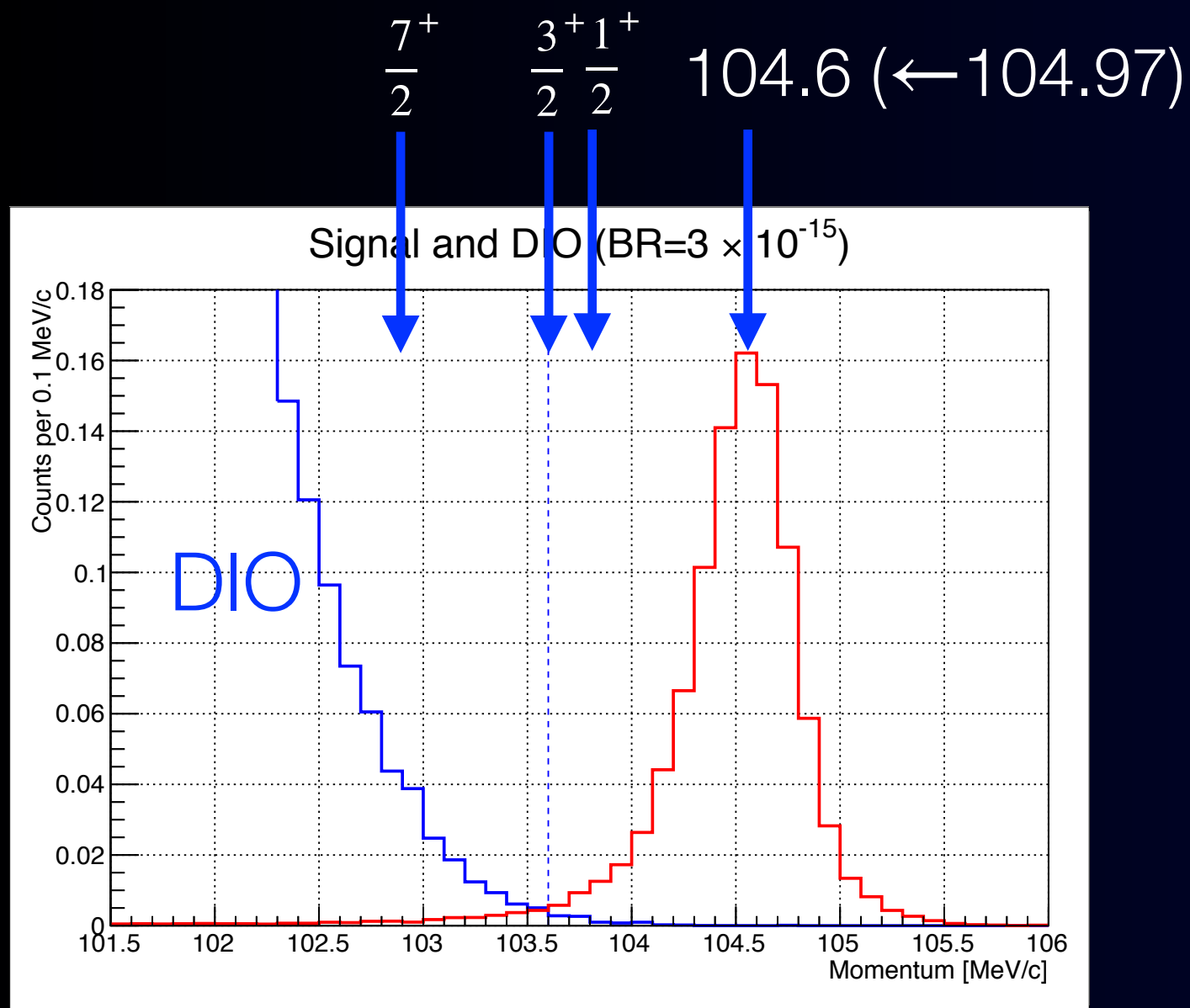
$$E_{\mu e}^* = E_{\mu e}(\text{gs}) - E_{\text{ex}}$$

when the energy levels of excited states are well separated.

Inelastic $\mu^- \rightarrow e^-$ Conversion

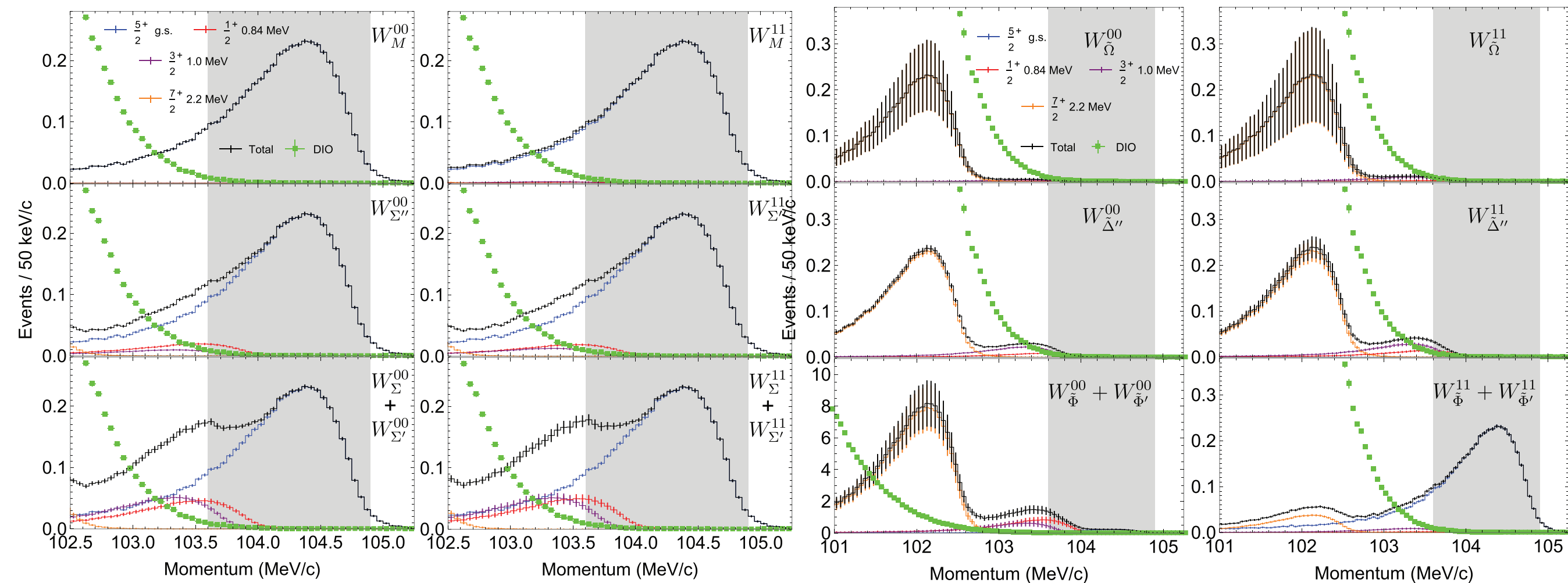
$\mu^- \rightarrow e^-$ conversion to the excited states

aluminum



from COMET Phase-I TDR: (notes, DIO at 10^{-15})

Inelastic $\mu^- \rightarrow e^-$ Conversion (Aluminium)



Mu2e spectrum response

default signal region (103.6-105 MeV/c)
DIO (green)

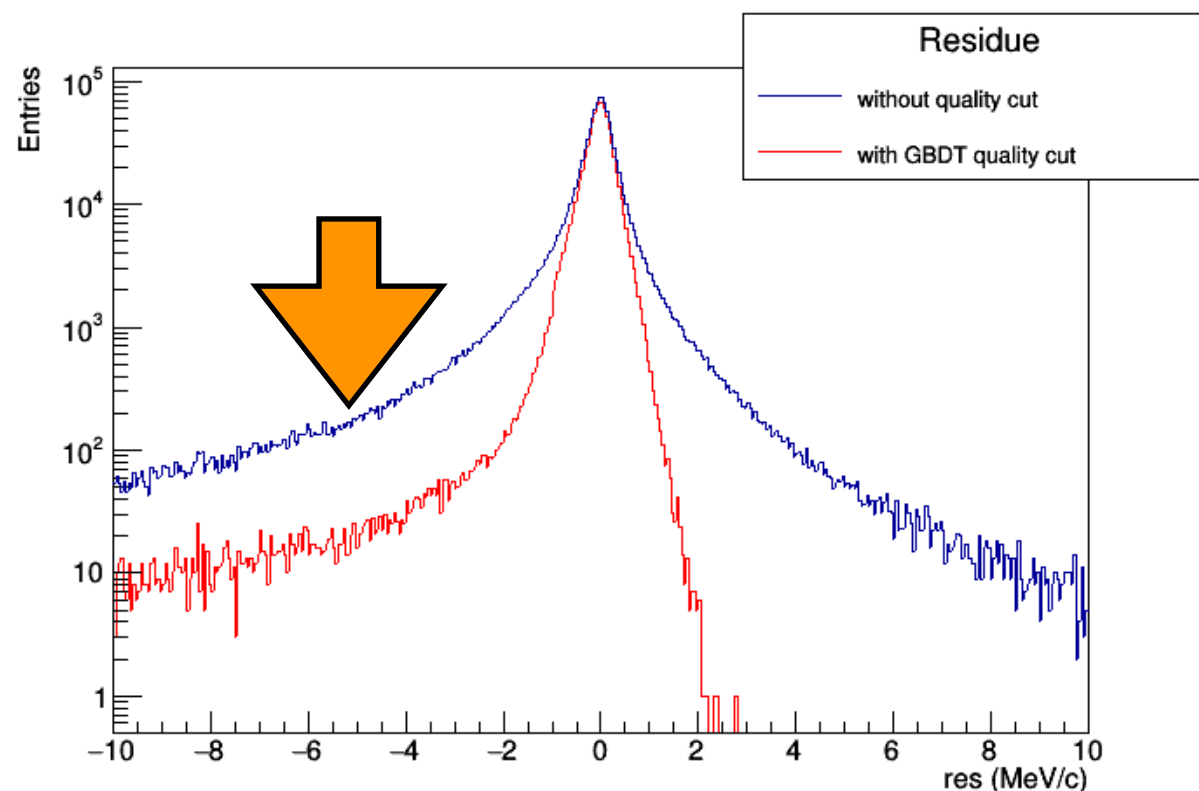
The peak of elastic conversion can be either coherent or incoherent ?

Experimental Observation of Inelastic $\mu^- \rightarrow e^-$ Conversion

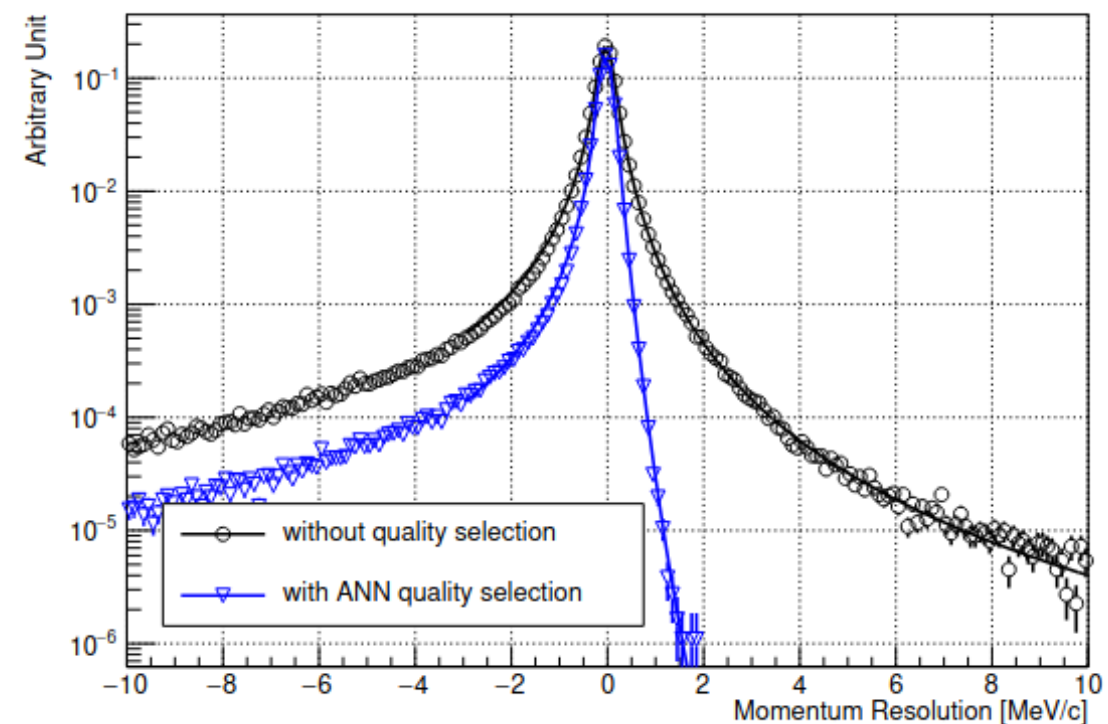
Experimental considerations for potential observation?

- (1) Suppression of low energy tail in the spectrum by GBDT.
- (2) Suppression of DIO electrons

Use the light target with low Z for the small “B” coefficient (p.24).

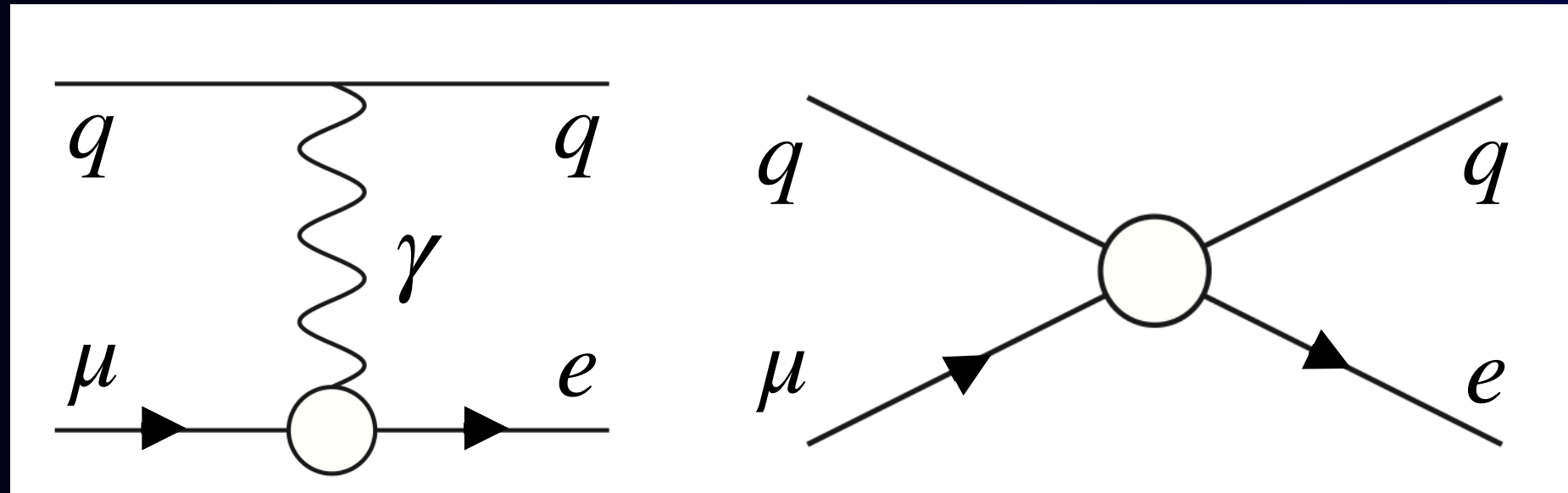


COMET.



Mu2e

Diphoton for $\mu^- \rightarrow e^-$ Conversion



Spin-independent
Coherent

Dipole (L/R)

Scalar (L/R)
Vector (L/R)

Spin-dependent
Incoherent

Pseudoscalar (L/R)
Axial vector (L/R)
Tensor (L/R)

up to dimension 7

Diphoton (L/R)

Polarized $\mu^- \rightarrow e^-$ Conversion

$\mu^- \rightarrow e^-$ conversion with **spin polarized muons** could be used to determine the chirality of the outgoing electron.

Creating a highly spin-polarized muonic atom for CLFV measurements could be extremely challenging!

- High-intensity muon sources, such as COMET/Mu2e
- Atomic capture
- Hyperfine interaction, when a nucleus has a spin

Re-polarization of muonic atom

$\mu^- \rightarrow e^+$ Conversion



$\mu^- \rightarrow e^+$ Conversion in a Muonic Atom

$$\mu^- + N(A, Z) \rightarrow e^+ + N(A, Z - 2)$$

- Lepton number violation (LNV) and charged lepton flavour violation (CLFV)
- Long range interaction
 - Exchange of light Majorana neutrino is small ($< 10^{-40}$)
 - $\langle m_{\mu e} \rangle = | \sum U_{\mu i} U_{ei} m_{\nu_i} |$
- Short range interaction
 - TeV LNV physics

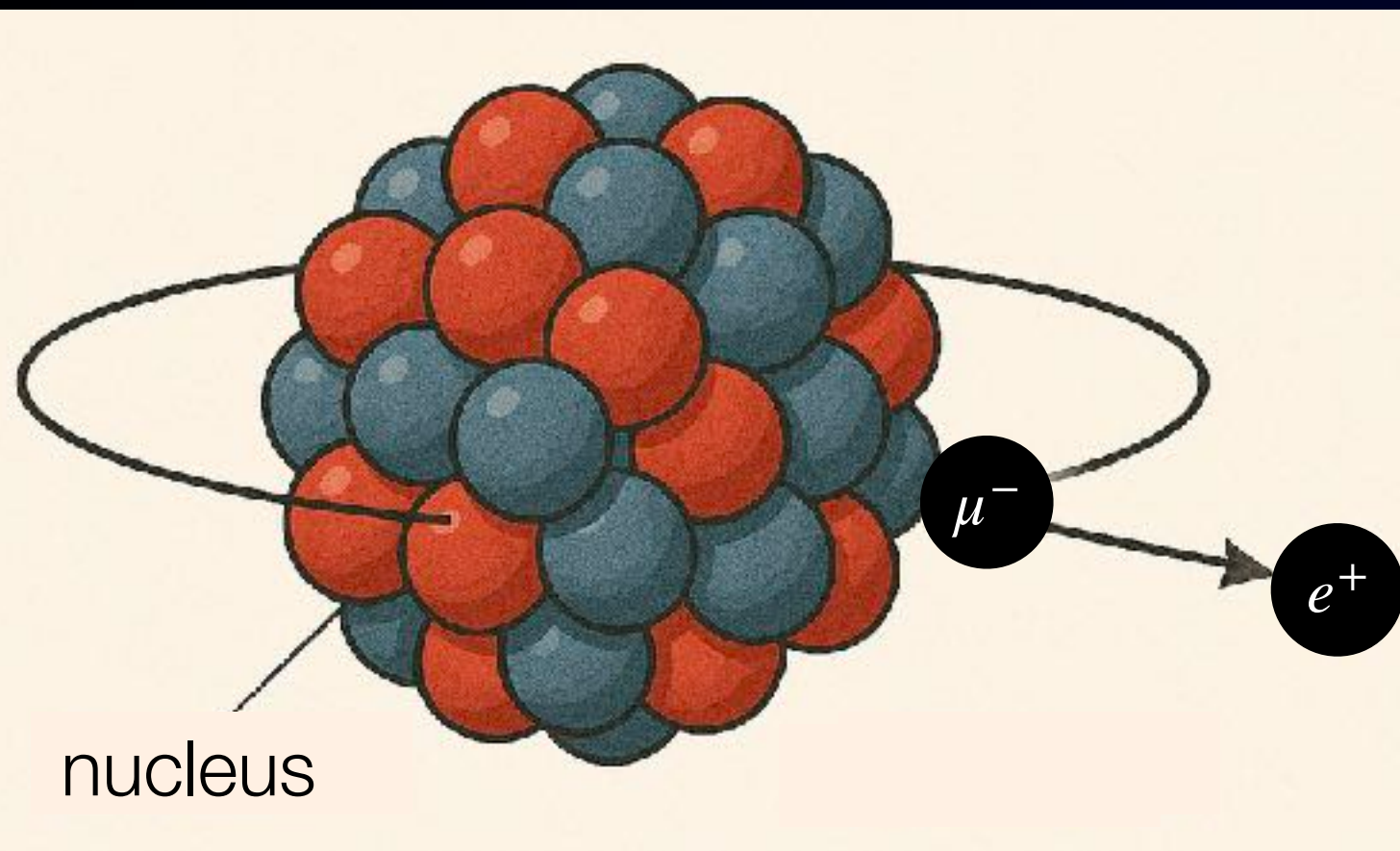
$\mu^- \rightarrow e^+$ Conversion in a Muonic Atom

$$\mu^- + N(A, Z) \rightarrow e^+ + N(A, Z - 2)$$

The final state is either ground state or excited states.

Event Signature for the transition to the ground state :

$$E_{\mu e^+} = m_\mu - B_\mu - E_{rec} + M(A, Z) - M(A, Z - 2)$$



Incoherent

Background for $\mu^- \rightarrow e^+$ Conversion : RMC

$$\mu^- + N(A, Z) \rightarrow N(A, Z - 1) + \nu + \gamma; \quad \gamma \rightarrow e^+ e^-$$

Using an appropriate target

Maximum photon energy (endpoint) :

$$E_{\gamma}^{\max} = m_{\mu} - B_{\mu} - E_{\text{rec}} + M(A, Z) - M(A, Z - 1)$$

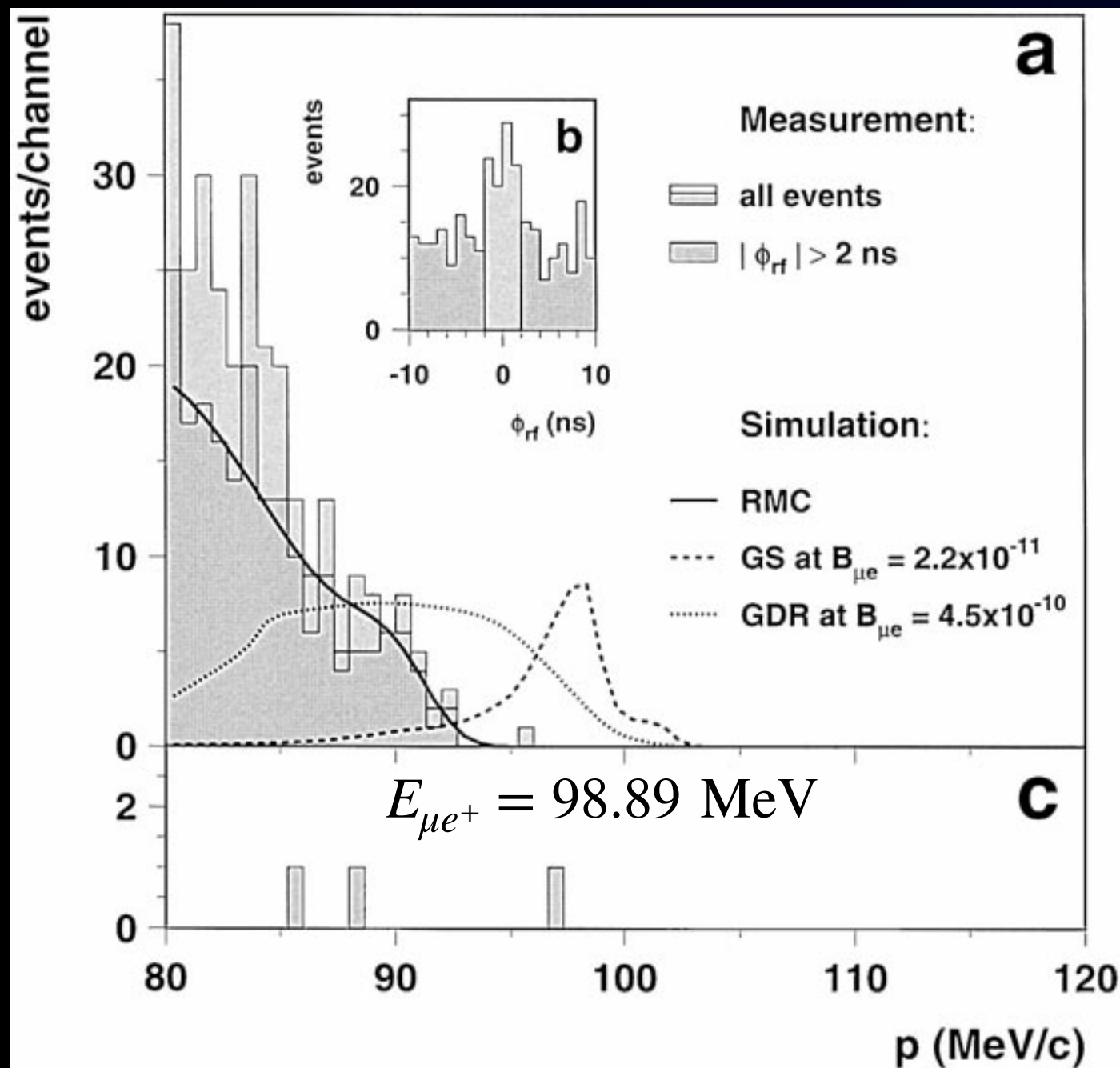
Mass Requirement of Muon target $N(A, Z)$:

$$E_{\mu e^+} > E_{\gamma}^{\max} \quad \Rightarrow \quad M(A, Z - 2) < M(A, Z - 1)$$

$$E_{\mu e} > E_{\gamma}^{\max} \quad \Rightarrow \quad M(A, Z) < M(A, Z - 1)$$

ex. ^{32}S , ^{40}Ca , ^{48}Ti , ^{50}Cr , ^{54}Fe , ^{58}Ni , ^{64}Zn , ^{70}Ge (for light and medium nuclei)

$\mu^- \rightarrow e^+$ Conversion : Current Limits (1998)



Current limits

$$\mu^- + \text{Ti} \rightarrow e^+ + \text{Ca}(\text{gs}) \leq 1.7 \times 10^{-12}$$

$$\mu^- + \text{Ti} \rightarrow e^+ + \text{Ca}(\text{ex}) \leq 3.6 \times 10^{-11}$$

- (a) positron spectrum
- (b) prompt and delayed timing
- (c) cosmic-ray background

RMC endpoint is 93 MeV/c, instead of maximum off 99 MeV/c (maybe $^{48}\text{Sc}(0^+; 6.7\text{MeV})$ excited ?)

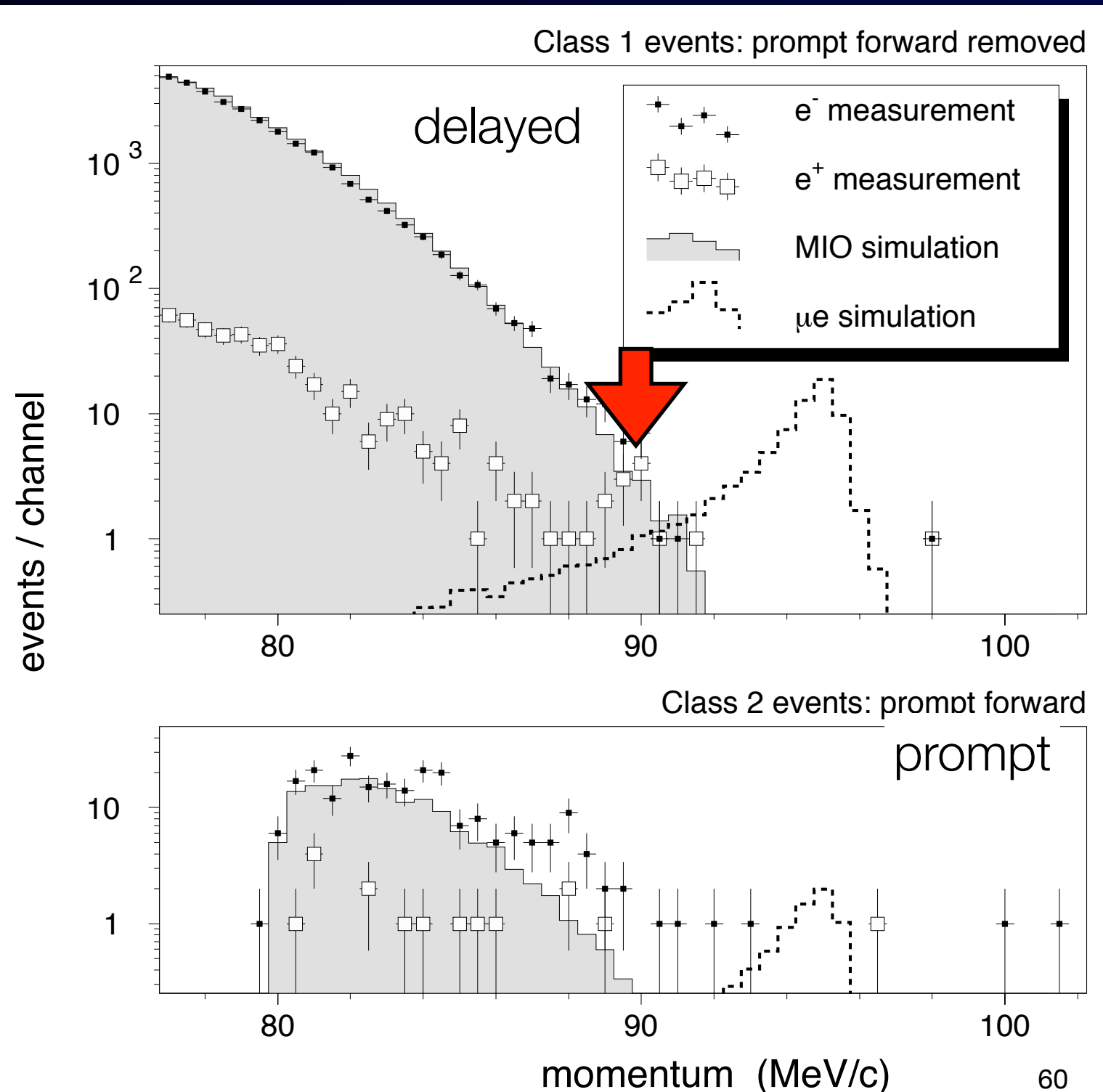
The sensitivity of COMET/Mu2e should be comparable to $\mu^- \rightarrow e^-$ Conversion.

$\mu^- \rightarrow e^+$ Conversion in Gold ?????

e^+ peak at 90 MeV/c ?

- The signal energy of $\mu^- + \text{Au} \rightarrow e^+ + \text{Ir}$ is 91.96 MeV.
- The energy shift (from the material in front) can be 0.6 MeV.
- The expected peak energy is 91.36 MeV.
- It is about 1 MeV higher than the observed peak.
- k_{max} for RMC in the Primakoff model is 88 MeV.

arXiv:2009.00214v1



$\mu^- \rightarrow e^+$ Conversion : History

Process	90%-C.L. upper limit	Place	Year	Reference	
$\mu^- + \text{Cu} \rightarrow e^+ + \text{Co}$	2.6×10^{-8}	SREL	1972	Bryman <i>et al.</i> (1972)	①
$\mu^- + \text{S} \rightarrow e^+ + \text{Si}$	9×10^{-10}	SIN	1982	Badertscher <i>et al.</i> (1982)	②
$\mu^- + \text{Ti} \rightarrow e^+ + \text{Ca}(\text{gs})$	9×10^{-12}	TRIUMF	1988	Ahmad <i>et al.</i> (1988)	③
$\mu^- + \text{Ti} \rightarrow e^+ + \text{Ca}(\text{ex})$	1.7×10^{-10}	TRIUMF	1988	Ahmad <i>et al.</i> (1988)	
$\mu^- + \text{Ti} \rightarrow e^+ + \text{Ca}(\text{gs})$	4.3×10^{-12}	PSI	1993	Dohmen <i>et al.</i> (1993)	④
$\mu^- + \text{Ti} \rightarrow e^+ + \text{Ca}(\text{ex})$	8.9×10^{-11}	PSI	1993	Dohmen <i>et al.</i> (1993)	
$\mu^- + \text{Ti} \rightarrow e^+ + \text{Ca}(\text{gs})$	1.7×10^{-12}	PSI	1998	Kaulard <i>et al.</i> (1998)	⑤
$\mu^- + \text{Ti} \rightarrow e^+ + \text{Ca}(\text{ex})$	3.6×10^{-11}	PSI	1998	Kaulard <i>et al.</i> (1998)	

$\mu^- \rightarrow e^-$ conversion

1972	1.6×10^{-8}	SREL	Bryman <i>et al.</i> [1972]	Cu	①
1977	4.0×10^{-10}	SIN	Badertscher <i>et al.</i> [1977]	S	
1982	7.0×10^{-11}	SIN	Badertscher <i>et al.</i> [1982]	S	②
1988	4.6×10^{-12}	TRIUMF	Ahmad <i>et al.</i> [1988]	Ti	③
1993	4.3×10^{-12}	SINDRUM II	Dohmen <i>et al.</i> [1993]	Ti	④
1996	4.6×10^{-11}	SINDRUM II	Honecker <i>et al.</i> [1996]	Pb	
2006	7.0×10^{-13}	SINDRUM II	Bertl <i>et al.</i> [2006]	Au	

$\mu^- \rightarrow e^+$ experiments were made at the same time as $\mu^- \rightarrow e^-$ experiments.

Where is ⑤ for $\mu^- \rightarrow e^-$?

SINDRUM II

$\mu^- \rightarrow e^-$ Conversion on Ti (1998)

Wintz, P., 1998, in Proceedings of the First International Symposium on Lepton and Baryon Number Violation, edited by H.V. Klapdor-Kleingrothaus and I.V. Krivosheina (Institute of Physics, Bristol/Philadelphia), p. 534.

$$\text{CR}(\mu^- + \text{Ti} \rightarrow e^- + \text{Ti}) < 6.1 \times 10^{-13}$$

Lepton-Baryon'98,
Trento, ECT*, Italy, April 20-25, 1998

Future : PRISM/PRIME



$$B(\mu N \rightarrow e N) \leq 10^{-18}$$

with a factor of 1,000,000 improvement

Muon Storage Ring : Merit (1)

1 Allowing the use of high-Z target material

Flight length of 17 m for
COMET/Mu2

Pion survival rate $\sim 5 \times 10^{-4}$

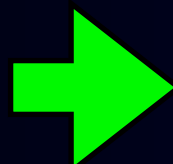


Flight length of 200 m

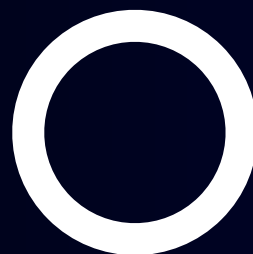
Pion survival rate $< 10^{-39}$

The measurement can start from about the
beam arrival time, since no pions remain,

linac



ring



Muon Storage Ring

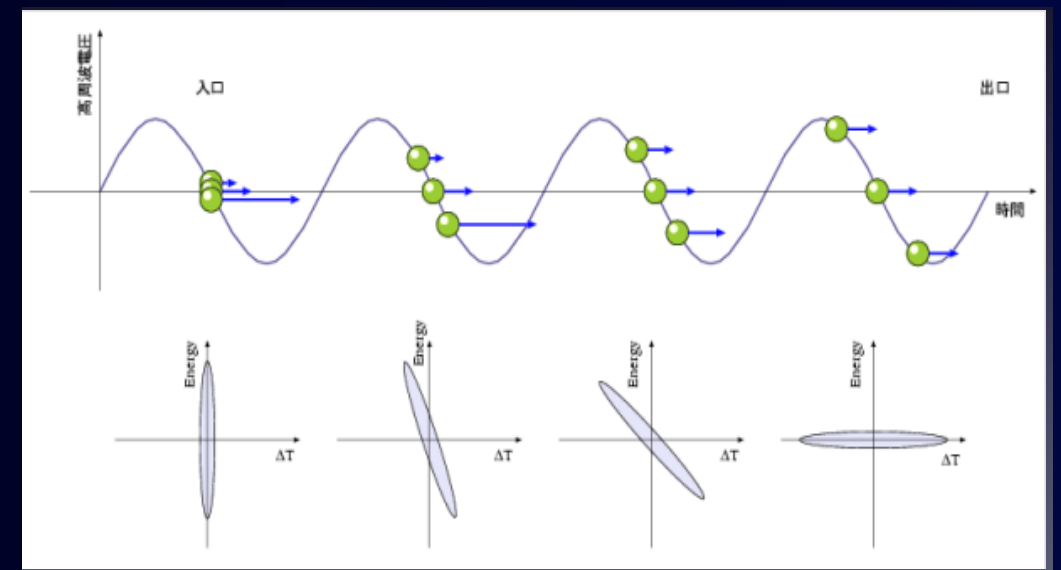
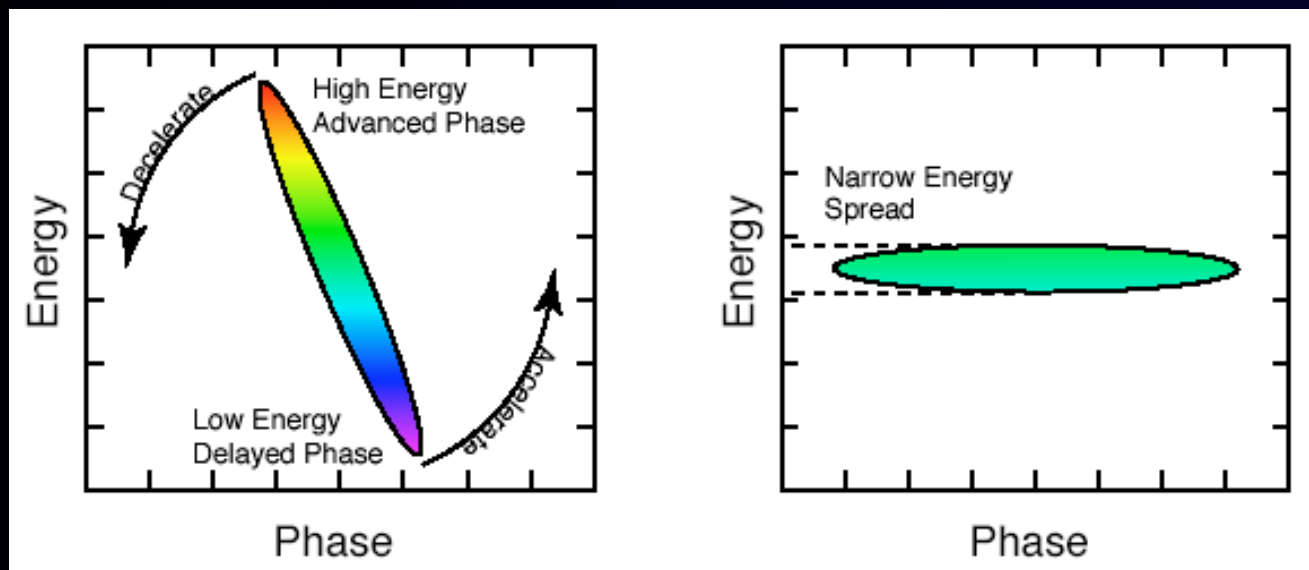
diameter 13 m, 5 turns

Muon Storage Ring : Merit (2)

2 Allowing the use of thinner targets

Phase rotation (synchrotron oscillation)

By accelerating slow muons and decelerating fast muons, a beam energy spread can be narrowed.



smaller number of RF's

Muon Storage Ring

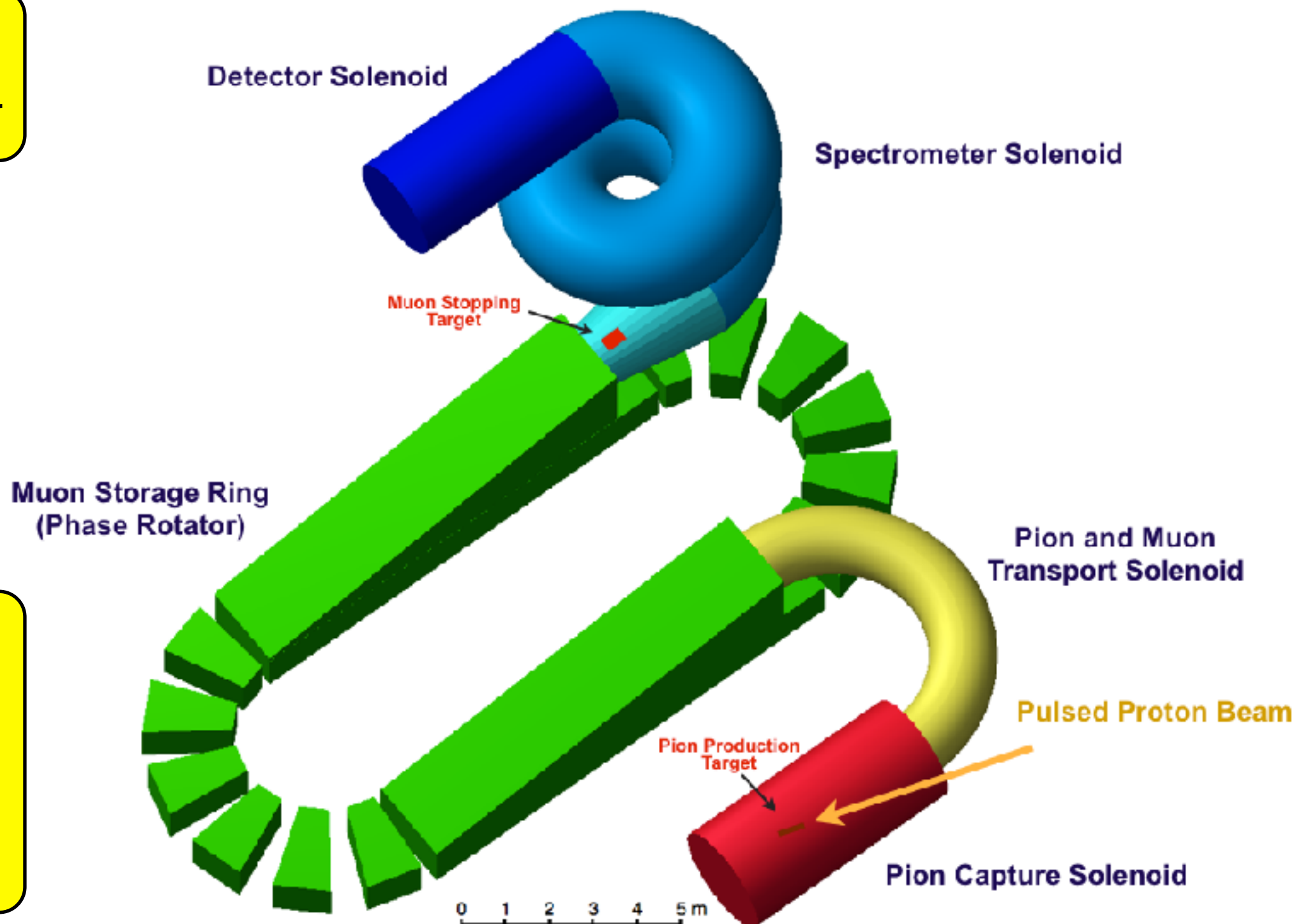
PRISM=Phase Rotated Intense Slow Muon source



PRISM/PRIME (2003)

PRIME
detector

PRISM
muon
storage
ring

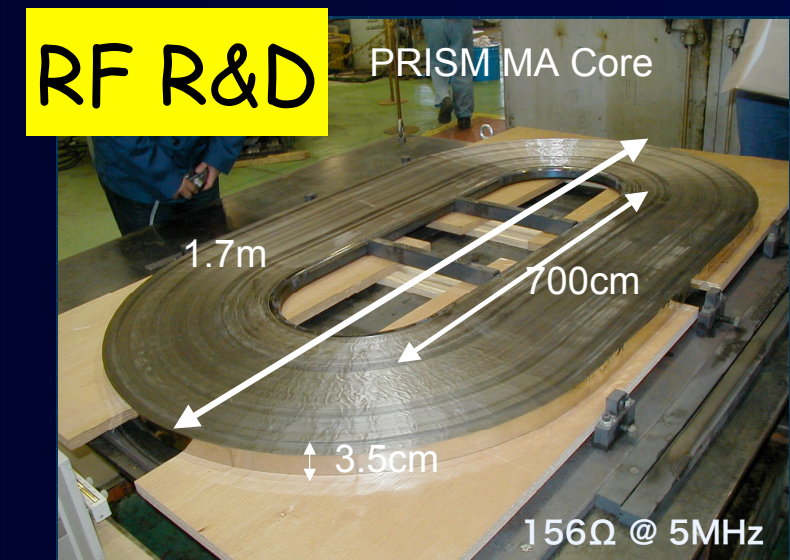


PRISM FFA R&D at Osaka University (2003 - 2007)



Fixed Field Alternating Gradient Synchrotron (FFA)

- FFA is suitable for acceptance low-energy muons
 - large beam acceptance
 - fast beam acceleration
 - synchrotron oscillation



FFA R&D at Osaka

- Scaling FFA with DFD triplet magnets

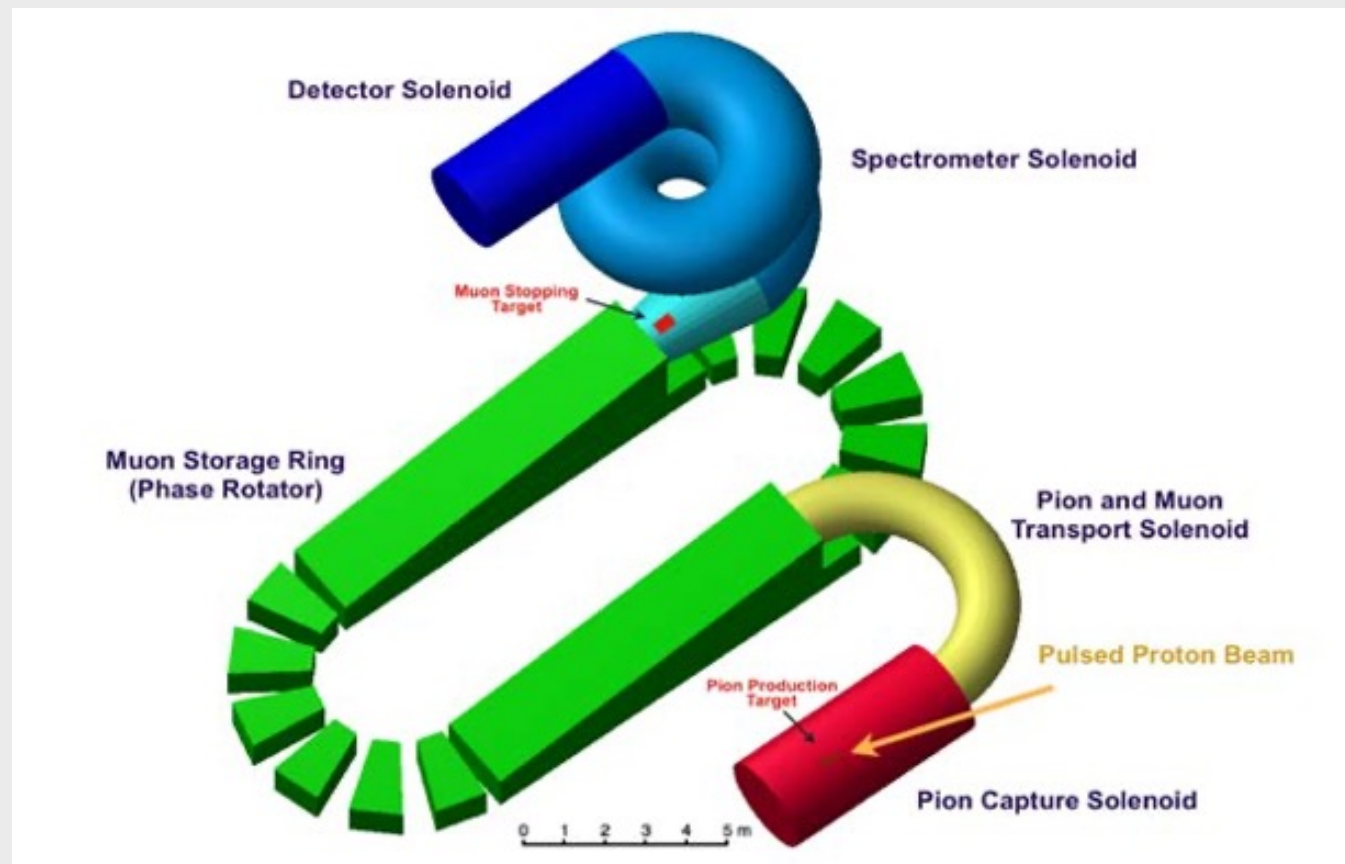
$$B(r) \propto \left(\frac{r}{r_0}\right)^5$$



Advanced Muon Facility at Fermilab in the US (2022)

One Concept for $\mu^- N \rightarrow e^- N$

- Spiral Detector Solenoid greatly reduces rate seen by detector, opens up new detector designs (from PRISM)



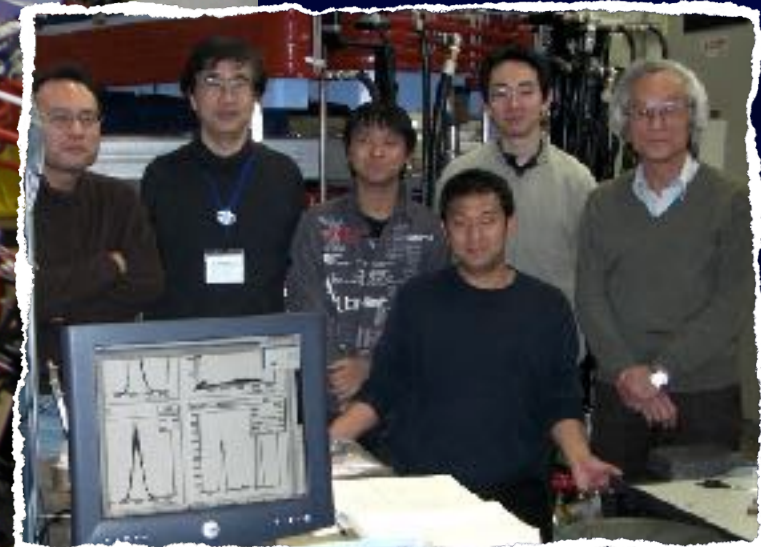
ENIGMA
experiment

Future advanced muon facility at Fermilab discussed in Snowmass 2022 workshop by the FNAL people.

PRISM FFA Phase Rotation at Osaka University (2003 - 2007)



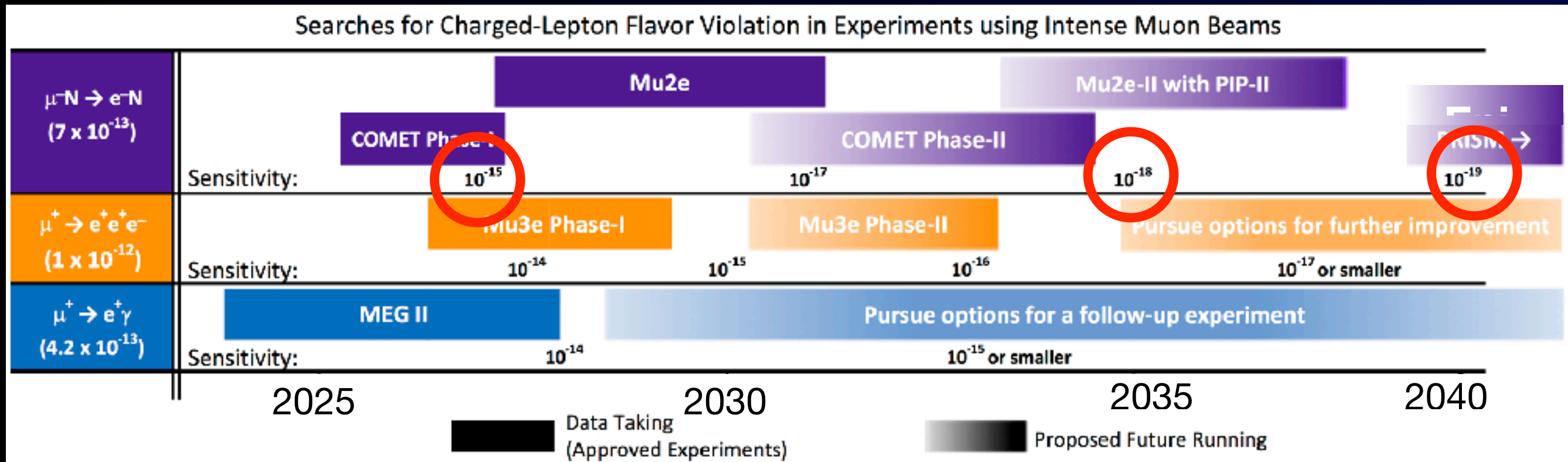
PRISM-FFAG (6 sectors) in RCNP, Osaka



T. Nakanishi, Ms.thesis (2008)

Demonstration of phase rotation has been made.

Global Timeline of Muon CLFV



$\times 100$

$\times 10,000$

$\times 1,000,000$

$\times 100,000$

modified from the muon CLFV white paper for the 2020 update of European Strategy of Particle Physics

Summary



Summary

The history, experiments, and phenomenology of $\mu \rightarrow e$ conversion are presented, along with the planned experiments and long-term improvements.

Although the $\mu \rightarrow e$ conversion is an **indirect** search for BSM, it would be crucial to develop the necessary tools to understand the structure of BSM if it were discovered.



Backup

