

# Holographic QCD and gravitational form factors

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# References

This talk is based on:

- **2106.00752** (NPB)
- **2204.08857** (PRD)
- **2103.03186** (PRD)
- **1910.04707** (PRD)

in collaboration with **Ismail Zahed**.

- The AdS/CFT correspondence can be used to compute correlation functions of local operators:

$$Z_{\text{gauge}}(J\mathcal{O}, N_c, \lambda) \equiv Z_{\text{gravity}}(\phi_0, g_5, \alpha'/R^2), \quad \text{where } J \equiv \phi_0.$$

- Correlation functions are evaluated via Witten diagrams in AdS.
- For non-conformal theories with a mass gap (dual to a deformed AdS background), scattering amplitudes can likewise be computed using these Witten diagrams in AdS

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \quad \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1),$$

with  $0 \leq z \leq \infty$ , connects the UV boundary ( $z \rightarrow 0$ ) to the IR ( $z \rightarrow \infty$ ), and mass gap/confinement induced by a background dilaton field  $\phi(z) = \kappa^2 z^2$ .

# Two-point function

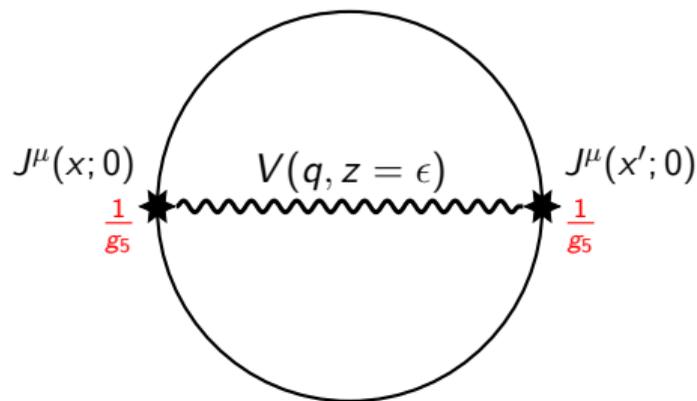


Figure: Witten diagram for the two-point function of the current operator with  $g_5^2 \sim \frac{1}{N_c}$ .

- The bulk-to-boundary propagator for the virtual photon is

$$\mathcal{V}(Q, z) = g_5 \sum_n \frac{F_n \phi_n(z)}{Q^2 + m_n^2} = \Gamma\left(1 + \frac{Q^2}{4\kappa_V^2}\right) \kappa_V^2 z^2 \mathcal{U}\left(1 + \frac{Q^2}{4\kappa_V^2}; 2; \kappa_V^2 z^2\right),$$

where  $\Gamma$  is the Gamma function and  $\mathcal{U}$  is the Tricomi confluent hypergeometric function.

# Spin-1 (Electromagnetic) Form Factors of Proton

- The Dirac and Pauli form factors of the proton are defined by

$$\langle p_2 | J_{em}^\mu(0) | p_1 \rangle = \bar{u}(p_2) \left( F_1(q^2) \gamma^\mu + F_2(q^2) \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} \right) u(p_1).$$

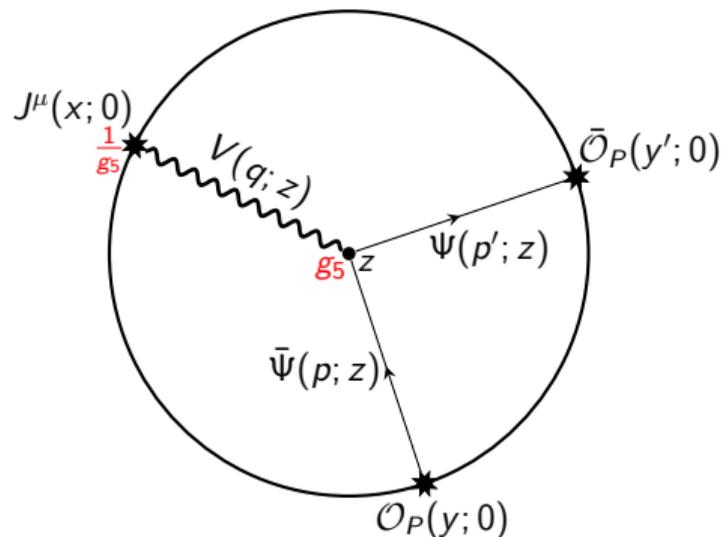


Figure: Witten diagram for EM form factor due to the exchange of vector mesons with  $g_5^2 \sim \frac{1}{N_c}$ .

# Spin-1 (Electromagnetic) Form Factors of Proton

- The scattering amplitude in AdS is

$$S_{Dirac}^{EM}[i, f] = (2\pi)^4 \delta^4(p' - p - q) \frac{1}{g_5} \times g_5 \bar{u}_{s_f}(p') \epsilon_\mu(q) \gamma^\mu u_{s_i}(p) \\ \times \frac{1}{2} \int \frac{dz}{z^{2M}} e^{-\phi} \mathcal{V}(Q, z) (\psi_L^2(z) + \psi_R^2(z)).$$

# Spin-1 (Electromagnetic) Form Factors of Proton

- The electric and magnetic Sachs form factors are given by

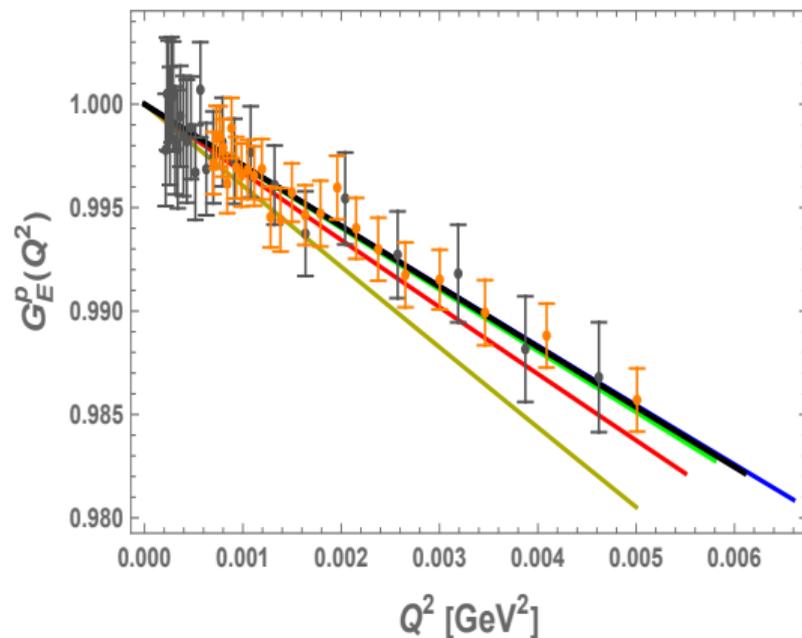
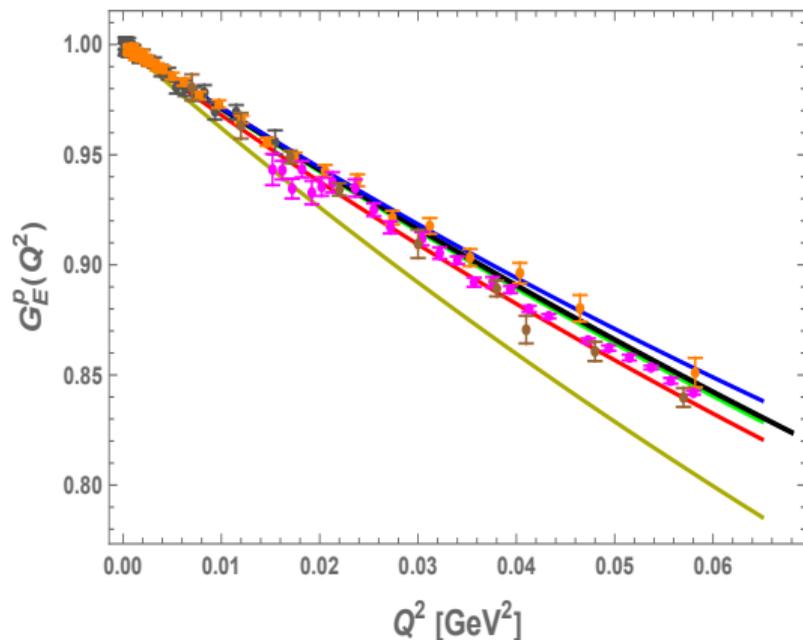
$$G_E^{p,n}(Q) = F_1^{p,n}(Q) - \frac{Q^2}{4m_N^2} F_2^{p,n}(Q), \quad G_M^{p,n}(Q) = F_1^{p,n}(Q) + F_2^{p,n}(Q).$$

- In AdS/QCD, they can be explicitly expressed via Beta functions [Abidin & Carlson (2009), Vega (2010)], for example

$$F_1^P(Q, \kappa, \tau) = \left(\frac{a}{2} + \tau\right) B(\tau, a+1) + \eta_P (\tau - 1) \frac{a(a(\tau - 1) - 1)}{(a + \tau)(a + \tau + 1)} B(\tau - 1, a + 1)$$

with  $a = \frac{Q^2}{4\kappa^2}$  where  $\kappa$  sets the mass scale, and Beta functions  $B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$ .

# Comparison to Data



Sample comparisons with world data (brown), Arrington (red), Mainz A1 (magenta), and PRad (orange/gray with blue fit). See [Bernauer:2013], [Xiong:2019]. Our prediction (black).

# Charge Radius of the Proton

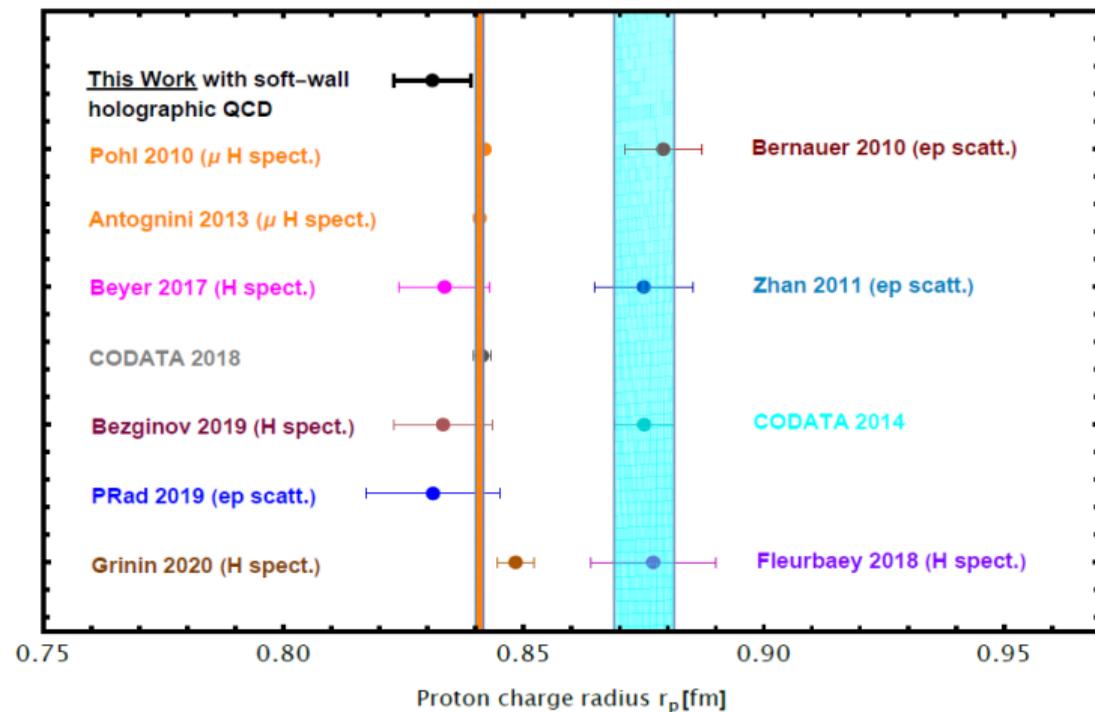


Figure: Various experimental determinations of the proton charge radius.

# Spin-2/0 (Gravitational) Form Factors of Proton

- The gravitational form factors (GFFs) of the proton are defined via the energy-momentum tensor (EMT):

$$\langle p_2 | T^{\mu\nu}(0) | p_1 \rangle = \bar{u}(p_2) \left( A(k) \gamma^{(\mu} p^{\nu)} + B(k) \frac{i p^{(\mu} \sigma^{\nu)\alpha} k_\alpha}{2m_N} + C(k) \frac{k^\mu k^\nu - \eta^{\mu\nu} k^2}{m_N} \right) u(p_1),$$

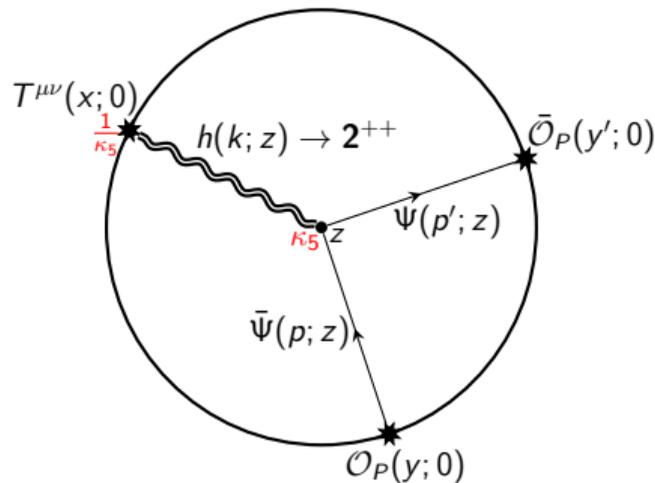
with  $k = p_2 - p_1$ . Often one writes  $D(k) \equiv 4 C(k)$ .

- In holographic QCD, one can compute these via Witten diagrams coupling AdS metric fluctuations  $h_{\mu\nu}$  to the bulk Dirac fermion representing the proton.
- The bulk metric fluctuations decompose into spin-2 (transvers-traceless part  $h$ ) and spin-0 (traceful part  $f$ ) [Kanitscheider (2008)]:

$$h_{\mu\nu}(k, z) \supset \left[ \epsilon_{\mu\nu}^{TT} h(k, z) \right] + \left[ \frac{1}{3} \eta_{\mu\nu} f(k, z) \right].$$

- For non-degenerate  $2^{++}$  and  $0^{++}$  glueball spectra, the holographic coupling includes both transverse-traceless (spin-2) and scalar (spin-0) fluctuations, respectively.

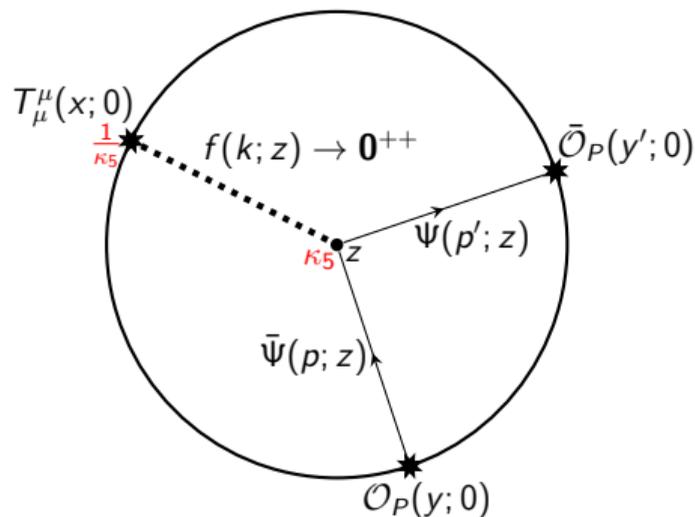
# Spin-2/0 (Gravitational) Form Factors of Proton



**Figure:** Witten diagram for spin-2 gravitational form factor due to the exchange  $2^{++}$  glueballs with  $\kappa_5^2 \sim \frac{1}{N_c^2}$ .

$$A(K, \kappa_T) = \frac{1}{2} \int dz \sqrt{g} e^{-\phi} z (\psi_R^2(z) + \psi_L^2(z)) \sum_{n=0}^{\infty} \frac{\sqrt{2} \kappa F_n \psi_n(z)}{K^2 + m_n^2}.$$

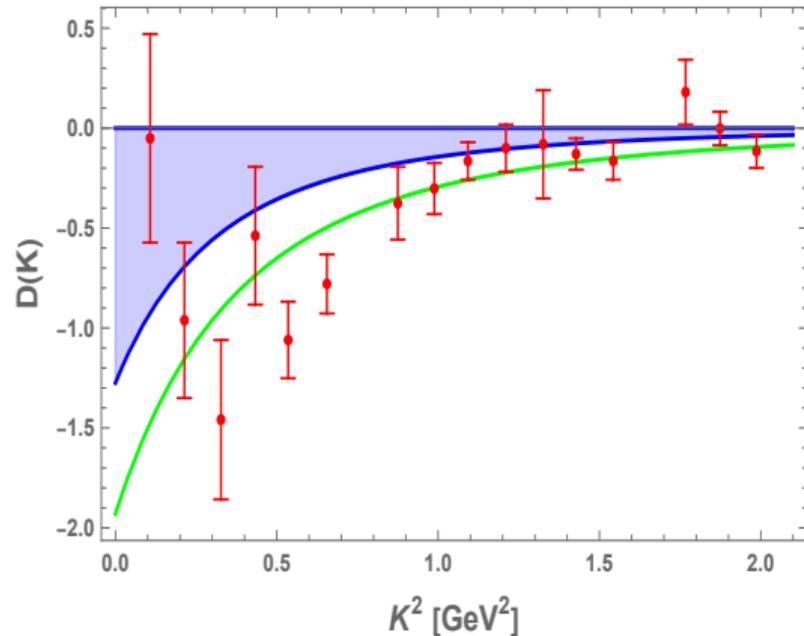
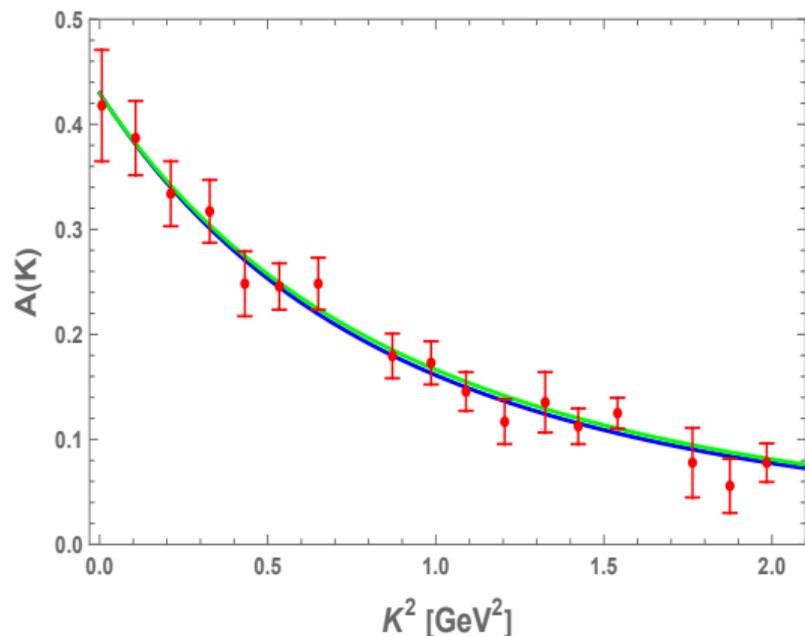
# Holographic Gravitational Form Factors



**Figure:** Witten diagram for scalar gravitational form factor due to the exchange  $0^{++}$  glueballs.

$$D(K, \kappa_T, \kappa_S) = -\frac{4 m_N^2}{3 K^2} \left[ A(K, \kappa_T) - A_S(K, \kappa_S) \right],$$

# Comparison with Lattice Data



Recent lattice QCD results [Pefkou:2021] (red points) compared to holographic fits (blue curves) with  $\kappa_T = 0.388$  GeV,  $\kappa_S = 0.217$  GeV. The green line is a tripole fit to the same lattice data.

# Scalar and Mass Radii of Proton

- Define the scalar radius  $r_{GS}$  from the trace part of the EMT [Ji:2021]:

$$\langle r_{GS}^2 \rangle = -\frac{6}{A_S(0)} \left. \frac{dA_S(K)}{dK^2} \right|_{K=0}, \quad A_S(K) = A(K) - \frac{K^2}{4m_N^2} B(K) + \frac{3K^2}{4m_N^2} D(K).$$

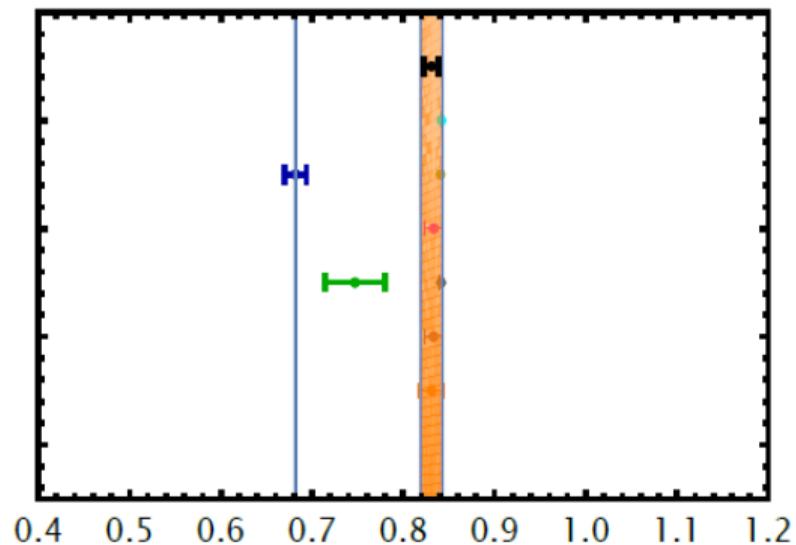
- Similarly, the mass radius  $r_{GM}$  of the proton is:

$$\langle r_{GM}^2 \rangle = -\frac{6}{A_M(0)} \left. \frac{dA_M(K)}{dK^2} \right|_{K=0},$$

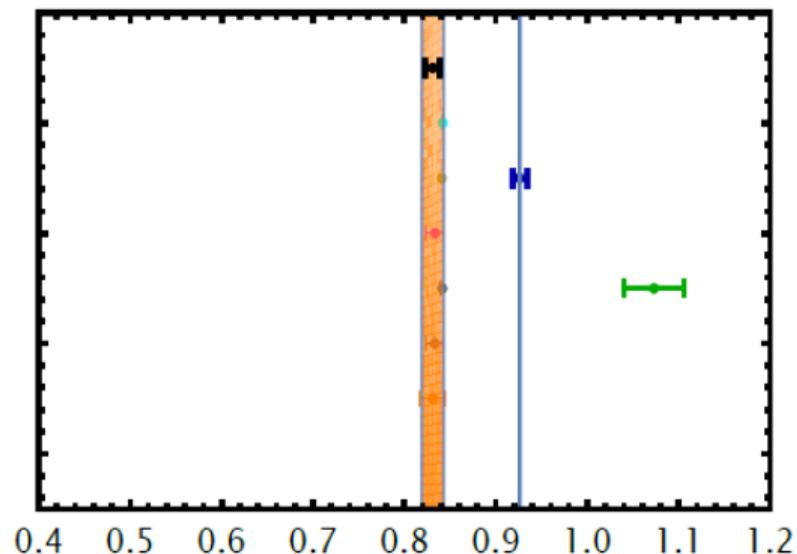
where

$$A_M(K) = A(K) - \frac{K^2}{4m_N^2} B(K) + \frac{K^2}{4m_N^2} D(K).$$

# Numerical Results for Radii



Proton gluonic mass radius  $r_{GM}$  [fm] with  $D(0) < 0$



Proton gluonic scalar radius  $r_{GS}$  [fm] with  $D(0) < 0$

Our holographic predictions for  $(r_{GM}, r_{GS})$  shown in blue. The green point is the lattice fit from [Pefkou:2021]. Also shown the charge radius in orange for comparison.

# Photoproduction of Heavy Mesons Near Threshold

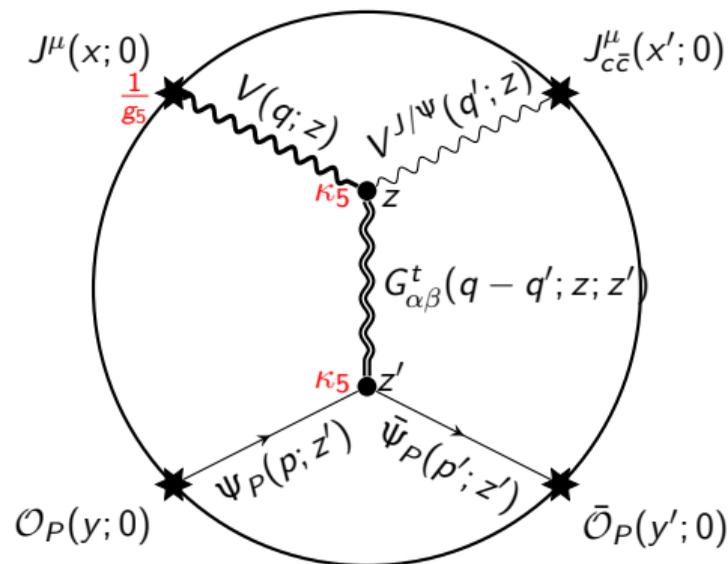


Figure: Witten diagrams for the holographic photo/electroproduction of  $J/\psi$  with  $g_5^2 \sim \frac{1}{N_c}$  and  $\kappa_5^2 \sim \frac{1}{N_c^2}$ .

# Photoproduction of heavy mesons near threshold

- the differential cross section for photoproduction of heavy vector mesons ( $J/\psi$  or  $\Upsilon$ ), near threshold, is given by

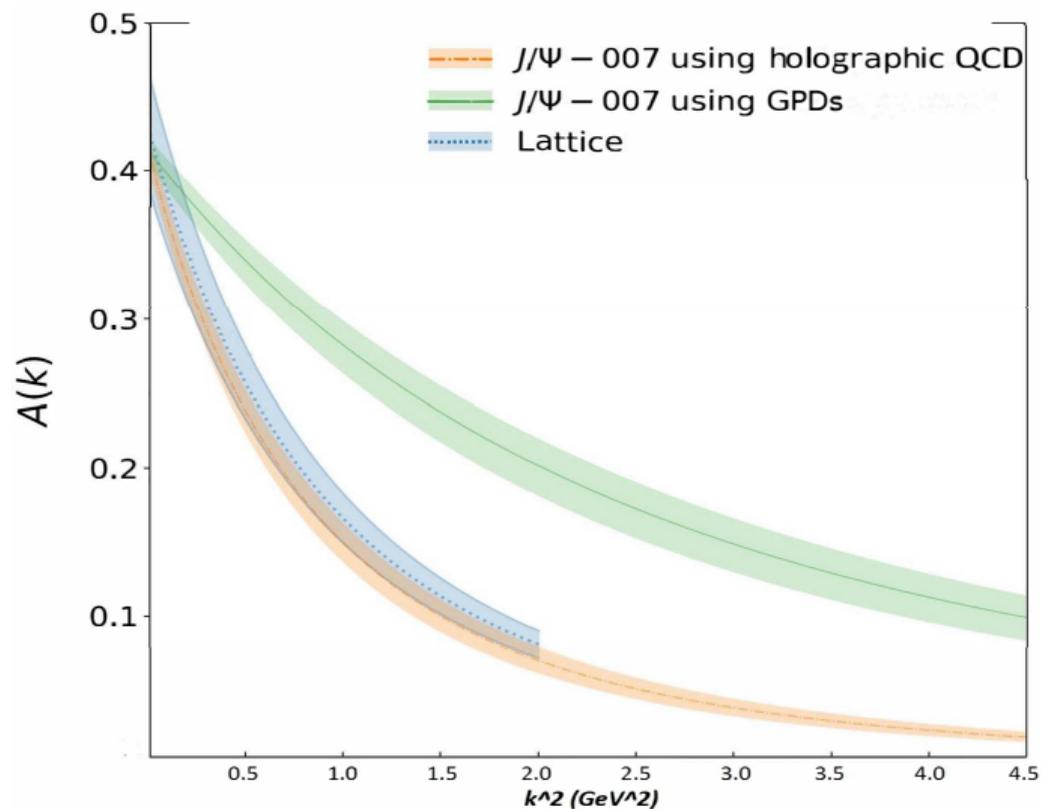
$$\frac{d\sigma}{dt} = \mathcal{N}^2 \times [A(t) + \eta^2 D(t)]^2 \\ \times \frac{1}{A^2(0)} \times \frac{1}{32\pi(s - m_N^2)^2} \times F(s, t, M_V, m_N) \times \left(1 - \frac{t}{4m_N^2}\right)$$

with the normalization factor  $\mathcal{N}$  defined as

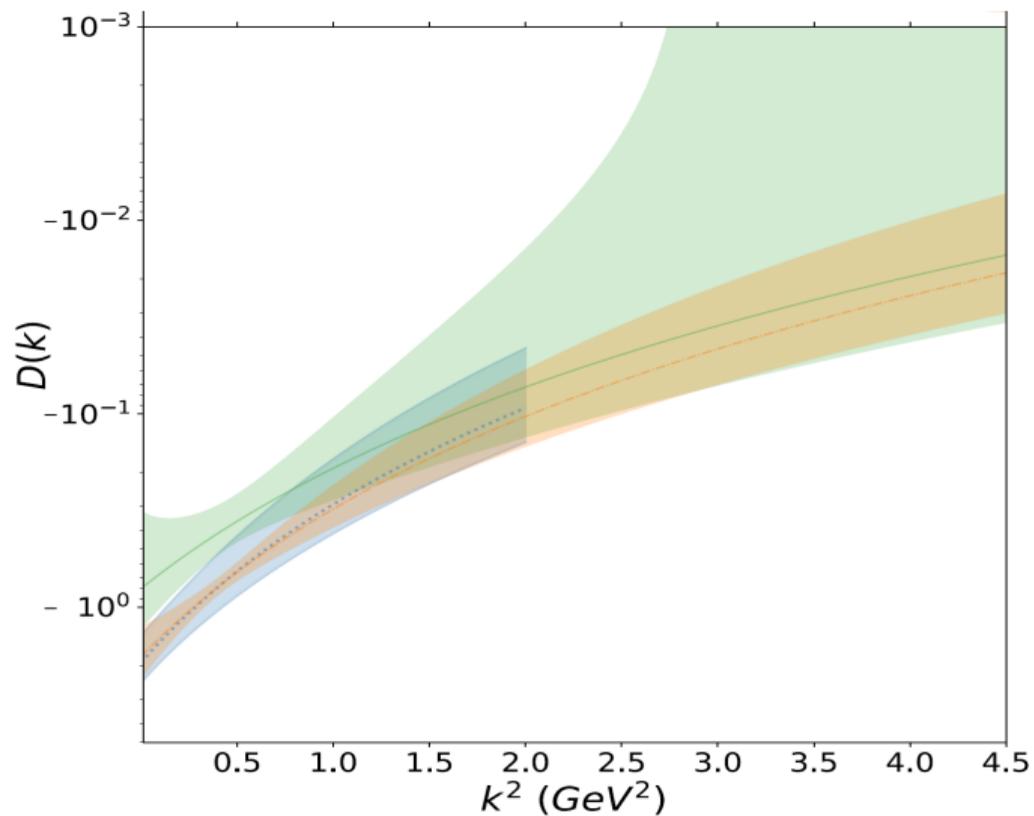
$$\mathcal{N}^2 \equiv e^2 \times \left(\frac{f_V}{M_V}\right)^2 \times \mathbb{V}_{hAA}^2 \times (2\kappa_5^2)^2 \times A^2(0) = 7.768 \text{ GeV}^{-4}$$

- note that  $F(s, t) \sim s^4 \sim 1/\eta^4$  with the amplitude  $\mathcal{A} \sim s^2 \times A(t) + s^0 \times D(t)$  as expected from  $2^{++}$  and  $0^{++}$  glueball t-channel exchanges

# Extraction of $A(t)$ and $D(t)$



# Spin-0 Contribution



# Summary

- The  $J/\psi$ -007 Collaboration at JLab [Duran et al. (2022)], using our holographic amplitude, has extracted the values of the proton's **mass radius**  $\approx 0.755 \pm 0.035$  fm and **scalar radius**  $\approx 1.069 \pm 0.056$  fm.
- These are in excellent agreement with lattice QCD results [Pefkou et al. (2021)] of  $\approx 0.7464 \pm 0.025$  fm (**mass radius**) and  $\approx 1.073 \pm 0.066$  fm (**scalar radius**).
- The gluonic mass distribution, therefore, is determined to lie predominantly within the proton's charge distribution. Meanwhile, the scalar (trace) distribution appears to extend further, suggesting a long-range confining component.

# Thank You!