Transition GPDs and perspectives for mechanical properties of hadrons

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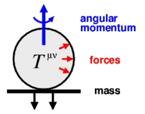
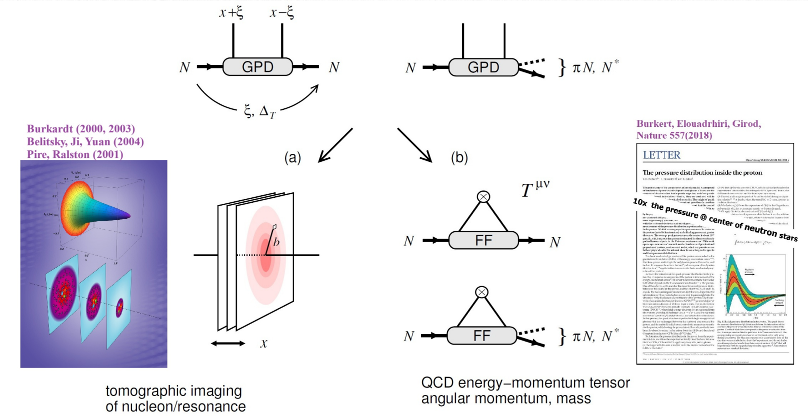
Mechanical properties of hadrons: Structure, dynamics, visualization,

ECT* Trento, 31 March - 4 April 2025

In collaboration with J.-Y. Kim, S. Son, P. Sznajder, M. Vanderhaeghen, C. Weiss, H.-Y. Won



GPDs and Transition GPDs



arXiv:2405.15386 [hep-ph] 24 May 2024

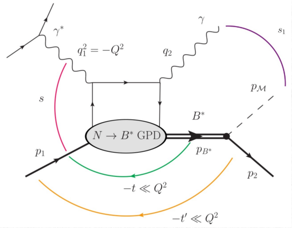
Exploring Baryon Resonances with Transition Generalized Parton Distributions: Status and Perspectives

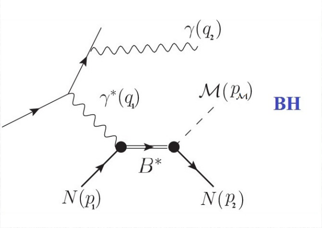
S. Diehl[1,2,a,b], K. Joo[2,a], K. Semenov-Tian-Shansky[3,a], C. Weiss[4,a], V. Braun[5], W.C. Chang[6], P. Chatagnon[4], M. Constantinou[7], Y. Guo[8], P. T. P. Hutauruk[9], H.-S. Jo[3], A. Kim[2], J.-Y. Kim[4], P. Kroll[10], S. Kumano[11], C.-H. Lee[12], S. Liuti[13], R. McNulty[14], H.-D. Son[15], P. Sznajder[16], A. Usman[17], C. Van Hulse[18], M. Vanderhaeghen[19], M. Winn[20]

to appear soon at EPJ A

What is non-diagonal DVCS?

$$\gamma^*(q_1) + N(p_1) \to \gamma(q_2) + \left[\mathcal{M}(p_\mathcal{M})N\left(p'\right)\right];\ \mathcal{M} = \pi, \eta, \rho, \omega \ \ldots$$





- Factorized description in terms of $N \to B^*$ GPDs in the generalized Bjorken kinematics:

$$-q_1^2;\ s = (p_1 + q_1)^2;\ s_1 = (p_\mathcal{M} + q_2)^2 - \text{ large}; \qquad x_B = \frac{-q_1^2}{2p_1 \cdot q_1} - \text{ fixed};$$

$$-t = -(p_{B^*} - p_1)^2;\ \ -t' = -(p_2 - p_1)^2;\ \ W^2_{\mathcal{M}N} = (p_1 + p_\mathcal{M})^2 \quad \text{of hadronic scale.}$$

- Meson-nucleon system resonates at $W^2_{\mathcal{M}N} = M^2_{B^*}$.
- Status of factorization: same as for the DVCS&DVMP: X. Ji et al.'98, J. Collins et al.'97,99.
- Rates are the same order as in usual DVCS.

Some motivation

E.m./weak probe : QCD structure :

$$\gamma \Leftrightarrow \langle B^* | \bar{q} \hat{Q}_{\rm e.m.} \gamma_\mu q | N \rangle$$

$$W^\pm,\ Z^0 \Leftrightarrow \langle B^* | \bar{q} \hat{Q}_w \gamma_\mu (1-\gamma_5) q | N \rangle$$

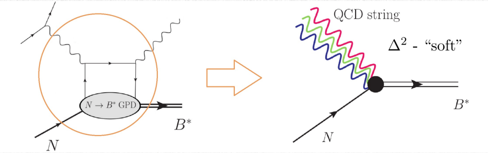
- Only $C=-1$ probe;
- Local in space-time;
- No direct access to gluons;

Hadronic probe : QCD structure :

$$\pi,\ K \Leftrightarrow \langle B^* | ??? | N \rangle$$

- QCD structure unknown;

- DVCS creates a low-energy QCD string = a tower of local probes of arbitrary spin-J.



$$\bar{\Psi}(n) P \exp\left(i \int_{-n}^{n} dz^\mu A_\mu(z) \right) \Psi(-n) = \bar{\Psi} \text{●—●} \Psi = \sum_{J=0}^{\infty} \left[\text{●—●} \right]_J Y_{JM}$$

E.g. "G probe" : QCD structure : $G \Leftrightarrow \langle B^* | \underbrace{\bar{\Psi}\gamma_\mu(\partial_\nu - A_\nu)\Psi + \frac{1}{4} F^a_{\mu\alpha} F^a_{\nu\alpha}}_{\text{QCD EMT}} | N \rangle$

Physical contents I: a unique option for baryon spectroscopy

2008 White paper

Baryon spectroscopy in non-diagonal DVCS

M. Amarian[1], M.V. Polyakov[2], K.M. Semenov-Tian-Shansky[2], I. Strakovsky[3]

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1. Excitation of resonances by non-local QCD quark light-cone operators:

$$\left\langle N^* \left| \bar{\Psi}_\alpha(0) \mathrm{P} e^{ig \int_0^z dx_\mu \lambda^c A_c^\mu} \Psi_\beta(z) \right| N \right\rangle$$

★ excitation by probes of arbitrary spin (not just $J = 1$);

2. Possible generalization to the gluon light-cone operators. ★ explicit access to the gluonic DOFs:

$$\langle N^* | G^a_{\alpha\beta}(0)\ [0, z]^{ab}\ G^b_{\mu\nu}(z) | N \rangle$$

3. Direct access to Im (beam spin asymmetry) and Re (charge asymmetry) of the amplitude $A^{\mathrm{DVCS}}_{N \to B^*}$. Without complicated PWA!

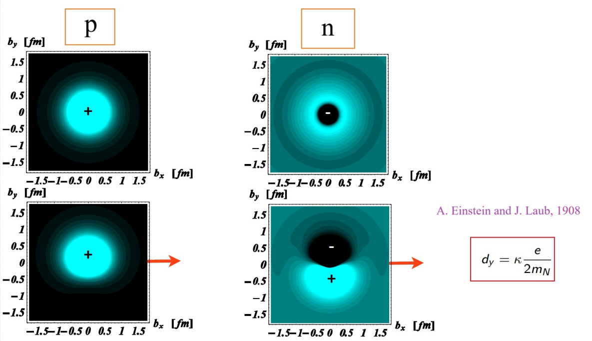
- Hunt for exotics: possible access to non-usual spin-flavor configurations: *e.g.* SU(6) $[20, 1^+]$: $N = 2$ orbital excitation of the SU(6) 20-plet.
- Symmetry argument by R. Feynman'1972: *"Two quark at least must have their motion changed to get to the* $[20, 1^+]$ *from the fundamental* $[56, 0^+]$*."*

Hadronic imaging: nucleon FF case

- FFs as quark transverse charge densities in the IMF, J. Miller'07, C. Carlson, M. Vanderhaeghen'07;

$$\rho_T^N(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{b}} \frac{1}{2P^+} \left\langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2} \right| J^+(0) \left| P^+, -\frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2} \right\rangle$$

$$= \int_0^\infty \frac{dQ}{2\pi} Q J_0(bQ) F_1\left(Q^2\right) + \sin\left(\phi_b - \phi_S\right) \int_0^\infty \frac{dQ}{2\pi} \frac{Q^2}{2M} J_1(bQ) F_2\left(Q^2\right).$$



Hadronic imaging: FF case, $N \to \Delta(1232)$ transition charge densities

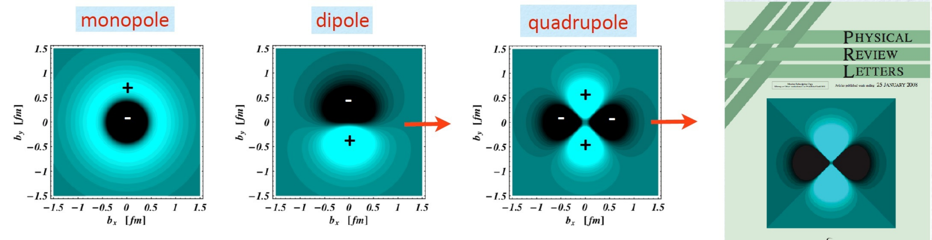
- $N \to \Delta$ transition: 3 FFs, H. Jones, M. Scadron'73:
  G_M-Magnetic, G_E-Electric, G_C-Coulomb quadruple;

$$\rho_T^{N\Delta}(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}} \frac{1}{2P^+} \left\langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp^\Delta = +\frac{1}{2} \middle| J^+(0) \middle| P^+, -\frac{\vec{q}_\perp}{2}, s_\perp^N = +\frac{1}{2} \right\rangle$$

$$= \int_0^\infty \frac{dQ}{2\pi} \frac{Q}{2} \left\{ J_0(bQ) G^+_{+\frac{1}{2}+\frac{1}{2}} \longrightarrow \text{monopole} \right.$$

$$- \sin(\phi_b - \phi_S) J_1(bQ) \left[\sqrt{3} G^+_{+\frac{3}{2}+\frac{1}{2}} + G^+_{+\frac{1}{2}-\frac{1}{2}} \right] \longrightarrow \text{dipole}$$

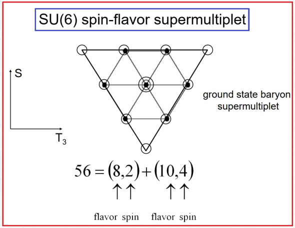
$$\left. - \cos 2(\phi_b - \phi_S) J_2(bQ) \sqrt{3} G^+_{+\frac{3}{2}-\frac{1}{2}} \right\} \longrightarrow \text{quadrupole}$$



C. Carlson, M.Vanderhaeghen'2007

How to interpret hadron imaging with transition FFs (and tGPDs)

- A correspondence between $N \to \Delta$ transitional FFs and quantities, admitting a probabilistic interpretation, is based on the $\mathrm{SU}(2N_f)$ spin-flavor symmetry (symmetry of QCD at large N_c).



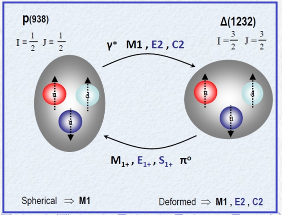
- $SU(6)$ symmetry predictions for $G_{M,E,C}(t)$:

$$G_M(0) = \frac{2\sqrt{2}}{3}\mu_p = 2.63\,\mu_N \qquad \text{Exp: } (3.46 \pm 0.03)\mu_N$$

Clebsh-Gordan coefficient

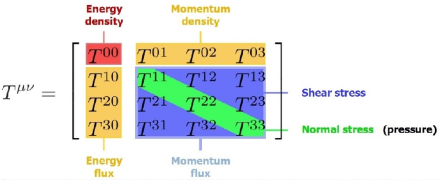
$G_{E,C} \sim \mathrm{SU}(6)$ breaking effects;

- Non-zero $G_{E,C}$ signal deviation from $\mathrm{SU}(6)$ caused by
  - quark mass differences;
  - relativistic effects;
  - long-range QCD effects (pion cloud);
- One may compute systematically the $1/N_c$ corrections;
- Try lattice QCD and/or functional approach;



QCD EMT $N \to B^*$ transition FFs

Gravitational FFs of the proton, see e.g. **V.D. Burkert et al. 2303.08347** **Burkert, Elouadrhiri, Girod, Nature 557(2018)**



M. Polyakov' 03:

$$T^{ij}(\vec{r}) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right) s(r) + \delta^{ij}\, p(r)$$

$$s(r) = -\frac{1}{4M_N}\, r\, \frac{\mathrm{d}}{\mathrm{d}r}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\widetilde{D}(r)$$

$$p(r) = \frac{1}{6M_N}\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}r^2\frac{\mathrm{d}}{\mathrm{d}r}\widetilde{D}(r)$$

$$\widetilde{D}(r) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{r}} D(-\vec{\Delta}^2).$$

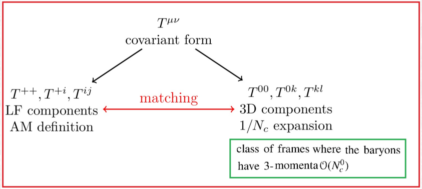
- Study of QCD EMT $N \to B^*$ matrix elements complements studies of e.m. transition FFs;
- Possible access to transition spin contents (for $N \to N^*$, Δ);
- AM as a transverse density at fixed light-front time C. Lorce, L. Mantovani, B. Pasquini'18;
- Transition angular momentum $N \to \Delta$: J.-Y. Kim et al.'23;

For large-N_c:

$$J^V_{p\to p} = \frac{1}{\sqrt{2}} J^V_{p\to\Delta^+} = 5 J^V_{\Delta^+\to\Delta^+} = O(N_c);$$

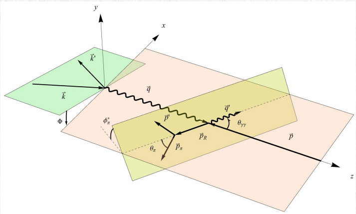
Large J^{u-d} flavor asymmetry!

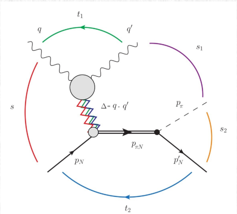
$$\int_{-1}^{1} dx\, x H_M(x,\xi,0) = 2 J^V_{p\to\Delta^+};$$



Kinematics and decay angular distribution

$$e(k) + N(p_N) \to e'(k') + \gamma^*(q) + N(p_N) \to e'(k') + \gamma(q') + \pi(p_\pi) + N'(p'_N)$$

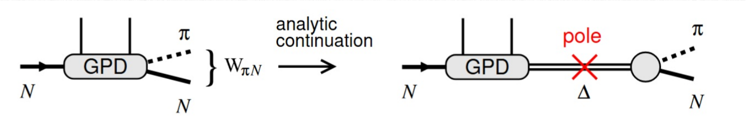




- $\gamma^* N \to B^* \gamma$: $\gamma^* N$ CMS;
- $B^* \to \pi N'$: $\pi N'$ CMS $\equiv$ $(\pi N')$ at rest;
  - $W_{\pi N}{}^2$: πN invariant mass;
  - θ^*_π, φ^*_π: decay angles in πN rest frame; $\Leftrightarrow$ s_1, t_2

$$\frac{d^7\sigma}{\underbrace{dQ^2\, dx_B}_{\text{lepton side}}\ \underbrace{dt\, d\Phi}_{\gamma^* N \to \gamma B^*}\ \underbrace{dW^2_{\pi N} d\Omega^*_\pi}_{B^* \to \pi N}}$$

Non-diagonal reactions: integrated cross section



- Narrow resonance limit: $\Gamma_R \ll M_R$;
- The Breit-Wigner spectral function with the energy-dependent decay width;

$$\int dW_{\pi N}^2 \int d\Omega_\pi^* \frac{d^7\sigma}{dQ^2 dx_B dt d\Phi dW_{\pi N}^2 d\Omega_\pi^*}$$

$$\approx \frac{1}{(2\pi)^4} \frac{x_B\, y^2}{32\, Q^4 \sqrt{1 + \frac{4M_N^2 x_B^2}{Q^2}}} \overline{\sum_i} \sum_f \left| \mathcal{M}(e^- N \to e^- \gamma R(M_R, \lambda_R)) \right|^2 .$$

- Fully consistent with the result for a stable particle.

Compton FFs for $N \to \pi N$ DVCS

- LO, leading twist-2: same hard part

$$C^{\pm}(x,\xi) = \frac{1}{x-\xi+i\varepsilon} \pm \frac{1}{x+\xi+i\varepsilon};$$

Diagonal DVCS

$$\{\mathcal{H}, \tilde{\mathcal{H}}\}(\xi, t);$$

Non-diagonal DVCS

$$\{\mathcal{H}, \tilde{\mathcal{H}}\}(\xi, t;\ W^2_{\pi N},\ \theta^*_\pi,\ \varphi^*_\pi);$$

A dream of an ideal experiment:

- Good coverage in θ^*_π, φ^*_π;
- Scan over $W^2_{\pi N}$.

A test ground: $N \to \Delta(1232)$ DVCS

$$\gamma^*(q) + N^p(p_N) \to \gamma(q') + \Delta^+(p_\Delta) \to \gamma(q') + \pi^0(p_\pi) + N^p(p'_N)$$

K. Goeke, M.Polyakov and M. Vanderhaeghen'01:

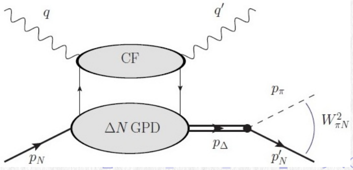
- 4 vector $N \to \Delta$ GPDs $H_{E,M,C,X}$ (and also 4 axial-vector $C_{1,2,3,4}$):

$$\frac{1}{2\pi}\int dy^- e^{ixP^+y^-} \langle \Delta(p_\Delta) |\bar{\Psi}(-y/2)\gamma\cdot n\tau_3\Psi(y/2)| N(p_N)\rangle \Big|_{y^+=\vec{y}_\perp=0}$$

$$= \sqrt{\frac{2}{3}}\bar{\mathcal{U}}^\beta(p_\Delta)\Big\{H_M(x,\xi,t)\left(-\mathcal{K}^M_{\beta\mu}\right)n^\mu + H_E(x,\xi,t)\left(-\mathcal{K}^E_{\beta\mu}\right)n^\mu$$

$$+H_C(x,\xi,t)\left(-\mathcal{K}^C_{\beta\mu}\right)n^\mu + \underbrace{H_X(x,\xi,t)\left(-\mathcal{K}^X_{\beta\mu}\right)n^\mu}_{\text{omitted structure, see J.-Y. Kim, H.-Y. Won et al.'25}} \Big\}u(p_N),$$

- 1 + 2 relevant in the large N_c limit;
- Early analysis: P. Guichon, L. Mossé and M. Vanderhaeghen'03;
- K.S. and M. Vanderhaeghen, PRD 108 (2023): study for CLAS@12 conditions;



Sum rules and large-N_c relations

- Unpolarized GPDs are related to e.m. form factors Jones and Scadron'73, see a review G. Ramalho, M.T. Peña'24:

$$\int_{-1}^{1} dx H_{M,E,C} = 2G_{M,E,C}(t); \quad \int_{-1}^{1} dx H_X = 0;$$

- Polarized transition GPDs are related to axial form factors Adler'75; (can be accessed in neutrino-production reactions):

$$\int_{-1}^{1} dx C_{1,2,3} = 2C^A_{5,6,3}(t).$$

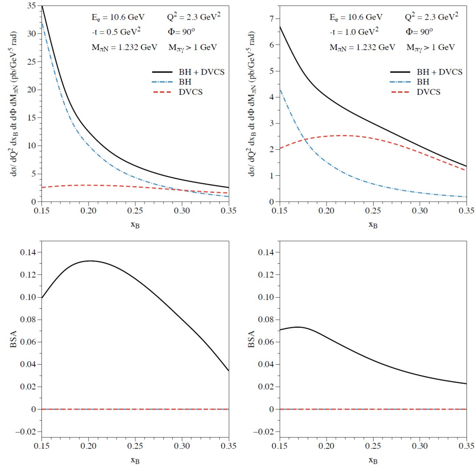
- Large-N_c relations for octet-to-decuplet transition GPDs, Goeke et al.'01:

$$H_M = \frac{2}{\sqrt{3}}\left[E^u - E^d\right]; \quad C_1 = \sqrt{3}\left[\tilde{H}^u - \tilde{H}^d\right]; \quad C_2 = \frac{\sqrt{3}}{4}\left[\tilde{E}^u - \tilde{E}^d\right];$$

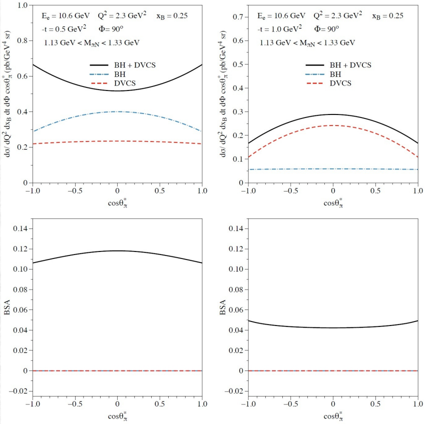
- Pion pole contribution into C_2:

$$\lim_{t \to m_\pi^2} C_2(x,\xi,t) = \sqrt{3}\,\frac{g_A m_N^2}{m_\pi^2 - t}\,\theta[\xi - |x|]\,\frac{1}{\xi}\,\Phi_\pi\left(\frac{x}{\xi}\right);$$

Cross sections and BSA for JLab@12 GeV



- Δ in helicity ±1/2 state: $\frac{1}{4}\left(1+3\cos^2\theta^*_\pi\right)$
- Δ in helicity ±3/2 state: $\frac{3}{4}\sin^2\theta^*_\pi$



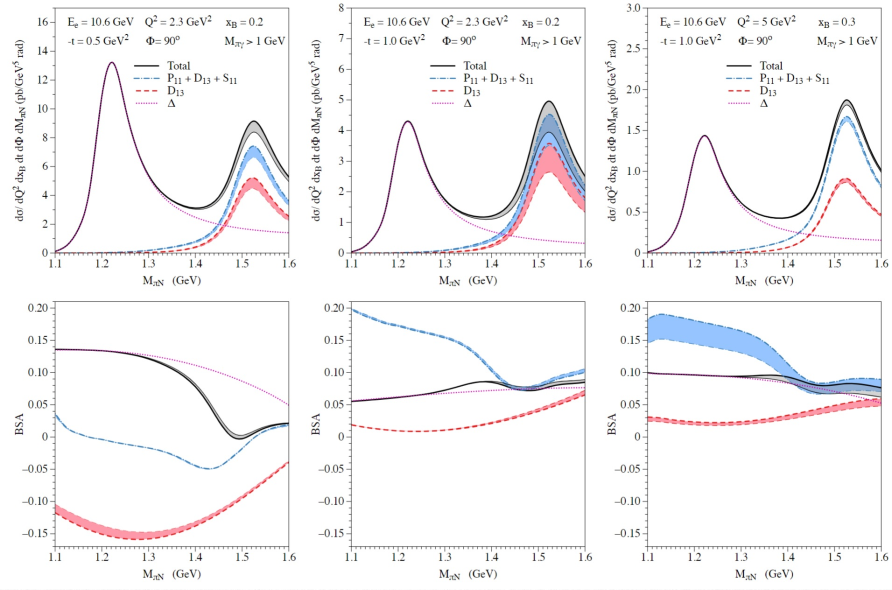
Going to the 2nd resonance region

- Formalism extended to $N \to N^*$ DVCS for $N^* = \mathrm{P}_{11}(1440)$, $\mathrm{D}_{13}(1520)$, $\mathrm{S}_{11}(1535)$:
  - for spin-$\frac{1}{2}$ resonances at twist-2: 2 unpolarized GPDs (vector operator), 2 polarized GPDs (axial-vector operator);
  - for spin-$\frac{3}{2}$ resonances at twist-2: 4 unpolarized GPDs (vector operator), 4 polarized GPDs (axial-vector operator);

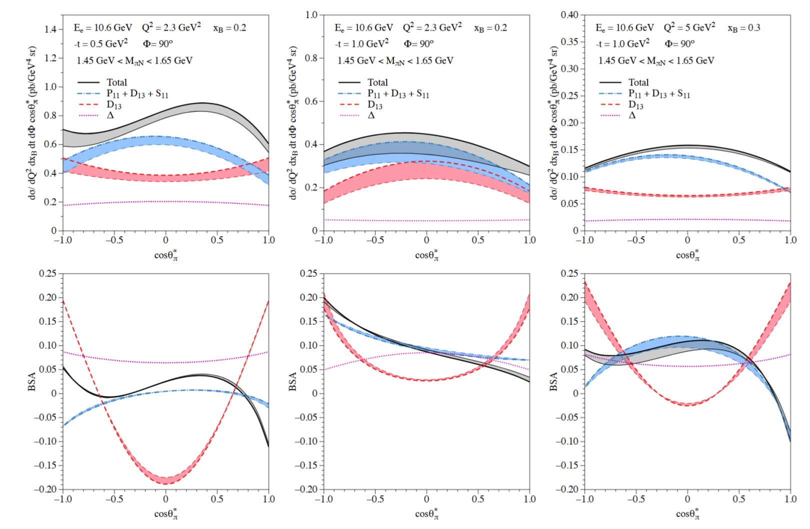
$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \sum_q e_q^2 \langle R(p_R, s_R)| \bar{q}\left(-\frac{\lambda n}{2}\right) \gamma \cdot n\, \gamma_5\, q\left(\frac{\lambda n}{2}\right) |N(p, s_N)\rangle$$
$$= \bar{R}_\beta\,(p_R, s_R) \left\{ \tilde{H}_1^{pD_{13}}(x, \xi, \Delta^2)\, n^\beta + \tilde{H}_2^{pD_{13}}(x, \xi, \Delta^2) \left(\frac{\Delta \cdot n}{M_N^2}\right) \Delta^\beta \right.$$
$$\left. + \tilde{H}_3^{pD_{13}}(x, \xi, \Delta^2) \frac{1}{M_N} \left(n^\beta \gamma \cdot \Delta - \Delta^\beta \gamma \cdot n\right) + \tilde{H}_4^{pD_{13}}(x, \xi, \Delta^2) \frac{2}{M_N^2} \left(n^\beta\, \bar{P} \cdot \Delta - \Delta^\beta\right) \right\} \gamma_5\, N\,(p, s_N)$$

- t-dependence of GPDs (first moments):
  - \- unpolarized GPDs: first moments constrained by data on e.m. transition FFs (CLAS@6 GeV)
  - \- polarized GPDs: 2 dominant axial FFs constrained using PCAC + pion pole dominance:
    - normalization at $t = 0$ given by $(f_{\pi NN^*}/m_\pi) 2 f_\pi$ (Goldberger-Treiman relation);
    - t-dependence: dipole ($M_A = 1$ GeV) and pion-pole $\sim 1/(t - m_\pi^2)$;
    - isoscalar axial FF neglected;
- x & ξ dependence of GPDs: RDDA $b = 1$ and $b = \infty$ with $q(x) \sim x^{-0.5}(1 - x)^3$

Cross section and BSA

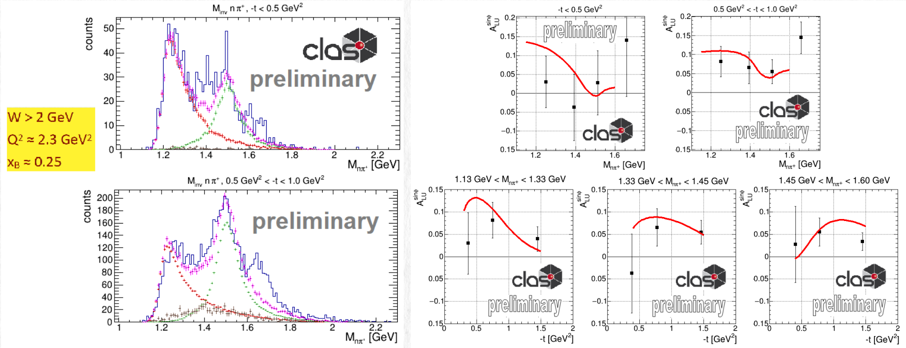


Pion angular distribution



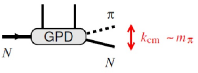
- Forward-backward asymmetry: due to interference between even/odd partial waves.

$e^- p \to e^- \gamma \pi^+ n$ with CLAS@12 GeV

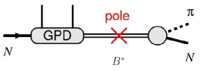


- Preliminary data confirm that with increasing of $-t$ the 2nd resonance region becomes more pronounced. S. Diehl et al. arXiv:2405.15386 [hep-ph];
- BSA compared to predictions of K.S. and M.Vanderhaeghen, Phys.Rev.D 108(2023);
- Possible JLab@20 upgrade: statistics may increase by a factor 100 - 1000;
- Access to the complete angular distribution of the cross section $\Rightarrow$ PW analysis?

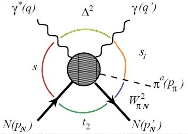
A unified description: $N \to \pi N$ transition GPDs



Chiral regime



Resonance regime



- Unpolarized $N \to \pi N$ GPDs M. Polyakov and S. Stratmann, arXiv:hep-ph/0609045 :

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x \bar{P}\cdot n} \left\langle N\left(p'_N\right) \pi^a(k_\pi) | \bar{\Psi}(-\frac{\lambda n}{2}) \not{n} \Psi(\frac{\lambda n}{2}) | N(p_N) \right\rangle = \frac{i g_A}{m_N f_\pi} \sum_{i=1}^{4} \bar{u}\left(p'_N\right) \Gamma_i \tau^a H_i^{(0)}(x, \xi, t_1, \alpha, W^2_{\pi N}, t_2) u(p_N)$$

$$\Gamma_1 = \gamma_5; \quad \Gamma_2 = \frac{m_N \not{n}}{n \cdot \bar{P}} \gamma_5; \quad \Gamma_3 = \frac{\not{k}_\pi}{m_N} \gamma_5; \quad \Gamma_4 = \frac{\not{k}_\pi \not{n}}{m_N} \gamma_5; \quad \boxed{\alpha = \frac{n \cdot k_\pi}{n \cdot (p'_N + k_\pi)}}$$

- Polynomiality of the Mellin moments; access to the EMT FFs;
- In chiral regime $N \to \pi N$ GPDs from χPT: soft pion theorems; First principle calculations!
- $\Delta(1232)$ can be included in χPT;

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Chiral theory of nucleons and pions in the presence of an external gravitational field

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Gravitational $p \to \Delta^+$ transition form factors in chiral perturbation theory

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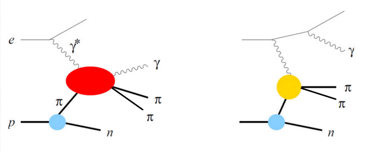
- In resonance regime: constraints from unitarity (Watson'54 final state interaction theorem);

A test ground: $\pi \to \pi\pi$ ND DVCS

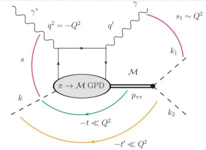
S. Son and K.S., JHEP 01 (2025);

- The Sullivan-type process:

$$e(l) + p(p) \to e(l') + \gamma(q') + \pi^+(k) + n(p') \to e(l') + \gamma(q') + \pi^+(k_1) + \pi^0(k_2) + n(p')$$



- No complications due to spin-$\frac{1}{2}$;
- Access to the meson spectrum: $\rho(770)$, $f_2(1270)$ *etc*;
- An option for the EIC?



$$Y_\ell^m(\theta_\pi^*, \varphi_\pi^*) = (-1)^m \sqrt{\frac{(2\ell+1)}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}} P_\ell^m(\cos\theta - \pi^*) e^{im\varphi_\pi^*}$$

the real-valued spherical harmonics read :

$$Y_\ell^m = \begin{cases} \frac{1}{\sqrt{2}}\left(\mathrm{Y}_{\ell,|m|} - i\,\mathrm{Y}_{\ell,-|m|}\right) & \text{if } m < 0; \\ \mathrm{Y}_{\ell,0} & \text{if } m = 0; \\ \frac{(-1)^m}{\sqrt{2}}\left(\mathrm{Y}_{\ell,|m|} + i\,\mathrm{Y}_{\ell,-|m|}\right) & \text{if } m > 0; \end{cases}$$

- **N.B.** $\gamma^* N \to \rho N' \to \pi\gamma N'$ a background for $N \to \Delta$ DVCS.

A test ground: $\pi \to \pi\pi$ ND DVCS

$$\frac{1}{2}\int \frac{d\lambda}{2\pi} e^{i\lambda x \bar{P}\cdot n} \langle \pi(k_1)\pi(k_2)|\bar{\psi}\left(-\frac{\lambda n}{2}\right)\begin{bmatrix} \not{n} \\ \not{n}\gamma_5 \end{bmatrix}\psi\left(\frac{\lambda n}{2}\right)|\pi(p_\pi)\rangle = \begin{bmatrix} \frac{1}{2\bar{P}\cdot n} i\varepsilon(n,\bar{P},\Delta,k_1)\frac{1}{f_\pi^3} H_{\pi\to\pi\pi}(x,\xi,\Delta^2,W_{\pi\pi}^2,\theta_\pi^*,\varphi_\pi^*); \\ \frac{1}{2\bar{P}\cdot n}(\bar{P}\cdot n)\frac{1}{f_\pi}\tilde{H}_{\pi\to\pi\pi}(x,\xi,\Delta^2,W_{\pi\pi}^2,\theta_\pi^*,\varphi_\pi^*); \end{bmatrix}$$

- PW expansion in angles θ_π^* and φ_π^* for vector GPD:

$$H_{\pi\to\pi\pi}(x,\xi,t,W_{\pi\pi}^2,\theta_\pi^*,\varphi_\pi^*) = \frac{1}{\sqrt{1-\cos^2\theta_\pi^*}\sin\varphi_\pi^*}\sum_{\ell=1}^{\infty}\sum_{m=-\ell}^{-1} H_{\pi\to\pi\pi}^{\ell m}(x,\xi,t,W_{\pi\pi}^2)\, Y_{\ell m}(\theta_\pi^*,\varphi_\pi^*);$$

N.B. Spherical harmonics in are odd under $\varphi_\pi^* \to -\varphi_\pi^*$.

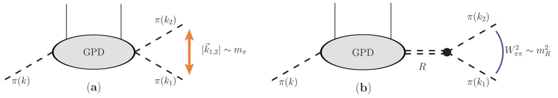
- PW expansion in angles θ_π^* and φ_π^* for axial-vector GPD:

$$\tilde{H}_{\pi\to\pi\pi}(x,\xi,t,W_{\pi\pi}^2,\theta_\pi^*,\varphi_\pi^*) = \sum_{\ell=0}^{\infty}\sum_{m=0}^{\ell} \tilde{H}_{\pi\to\pi\pi}^{\ell m}(x,\xi,t,W_{\pi\pi}^2)\, Y_{\ell m}(\theta_\pi^*,\varphi_\pi^*);$$

N.B. Spherical harmonics in are even under $\varphi_\pi^* \to -\varphi_\pi^*$.

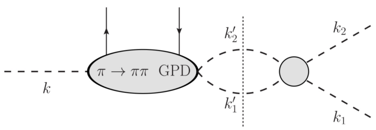
How to go beyond the threshold?

- Two regimes



(a) $\pi \to \pi\pi$ transition GPD in the chiral regime; **(b)** $\pi \to \pi\pi$ transition GPD in the resonance region;

- The Watson'54 final state interaction theorem for $\pi \to \pi\pi$ transition GPD:



- The Omnes solution (for $N = 0$) $\delta^I_\ell\left(W^2_{\pi\pi}\right)$ are the $\pi\pi$ scattering phases:

$$\tilde{H}^I_{\ell;m}(x,\xi,W^2) = \tilde{H}^I_{\ell;m}(x,\xi,W^2=0)\exp\left[\frac{1}{\pi}\int_{4m_\pi^2}^{\infty} d\omega \frac{\delta^I_\ell(\omega)}{\omega - m^2_{\pi\pi} - i\epsilon}\right]$$

- $\pi \to \pi\pi$ and $N \to \pi N$ transition GPDs are complex functions above threshold!

What we want?

$$\int_{-1}^{1} dx\, x\, H(x,\xi,t) = M_2(t) + \xi^2 \frac{4}{5} d_1(t);$$

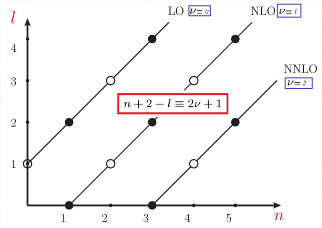
What we get from experiment:

$$D(t) = \sum_{\substack{i \\ \text{odd}}} d_i(t);$$

- Mellin moments expanded in PWs of the t-channel reaction $\gamma^*\gamma \to h\bar{h}$: M. Polyakov, A. Shuvaev'02, K. Kumericki, D. Müller, and K. Passek-Kumericki'08, D. Müller, M. Polyakov and K.S.'15;

Conformal PW expansion:

$$H_+(x,\xi,t) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \sum_{\substack{l=0 \\ \text{even}}}^{n+1} B_{nl}(t)\, \theta\left(1-\frac{x^2}{\xi^2}\right)\left(1-\frac{x^2}{\xi^2}\right) C_n^{\frac{3}{2}}\left(\frac{x}{\xi}\right) P_l\left(\frac{1}{\xi}\right)$$

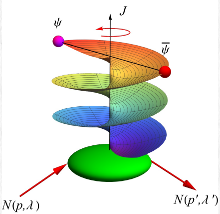


- Polynomiality implemented via the Wigner-Ekkart theorem ($l \leq n+1$).
- Discrete symmetries (C, T) through the selection rules for I^{PC} (X. Ji, R. Lebed'01).
- Generalized FFs $B_{nl}(t)$ are renormalized multiplicatively.

Can we handle with the QCD string?

- How to decompose QCD string into probes of different spin-J?

$$\bar{\Psi}(z)\gamma_\mu[z;\,0]\Psi(0)\big|_{z\to 0} = z^\nu \underbrace{\bar{\Psi}\gamma_\mu\nabla_\nu\Psi}_{\text{Spin}-2} + z^\nu z^\rho \underbrace{\bar{\Psi}\gamma_\mu\nabla_\nu\nabla_\rho\Psi}_{\text{Spin}-3} + \ldots$$



- Expansion of the Compton FFs in the SO(3) PWs of the cross channel ($\gamma^*\gamma \to h\bar{h}$):

$$F_J(t) \equiv \frac{2J+1}{2}\int_{-1}^{1} d(\cos\theta_t)\, P_J(\cos\theta_t)\mathcal{H}^{(+)}(\cos\theta_t, t); \quad \cos\theta_t = -\frac{1}{\xi\beta} + \mathcal{O}\left(1/Q^2\right).$$

- Froissart-Gribov projection (Gribov'61, Froissart'61) of the Compton FFs allows the calculation of $F_J(t)$ from the absorptive part of the amplitude (GPDs on the $x=\xi$ line);

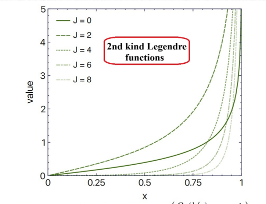
$$\mathrm{Re}\,\mathcal{H}_+(\xi,t) \overset{\text{LO}}{=} \mathcal{P}\int_0^1 dx\, \frac{2xH_+(x,x,t)}{\xi^2 - x^2} + 4D(t).$$

- Froissart-Grbov projection in the context of DVCS: K. Kumericki, D. Müller, and K. Passek-Kumericki, Eur. Phys. J. C **58**, (2008); M. Polyakov, Phys. Lett. B **659**, (2008); K.S. and P. Sznajder, PRD **109**, (2024);

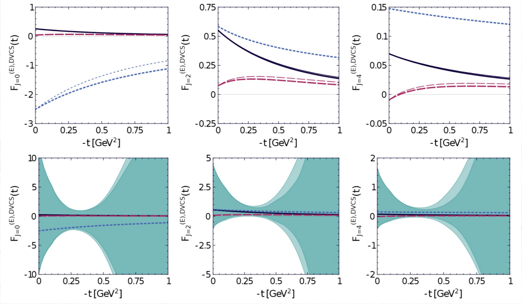
Froissart-Gribov: usual DVCS

$$F_J(t) = 2(2J+1)\int_0^1 dx \left[\frac{\mathcal{Q}_J(1/x)}{x^2} - \frac{1}{x}\delta_{J,0}\right] H_+(x,x,t) + 4D(t)\delta_{J,0}\,.$$

- Possible benefits of $F_J(t)$ as observables?
- Shortcomings:
  - Analytic continuation in t;
  - Target mass corrections ($\beta \neq 1$) mix Js;



First weight functions $2(2J+1)\left(\frac{\mathcal{Q}_J(1/x)}{x^2} - \delta_{J0}\frac{1}{x}\right)$



- GK (solid black), MMS (dashed red) and KM (dotted blue) GPD models;
- V.s. the global analysis of DVCS data, H. Moutarde, P. Sznajder, and J. Wagner, Eur. Phys. J. C 79, (2019);

Sum rules for the Mellin moments of GPDs

- Polynomiality property of GPDs ($h_{N,k}(t)$ are in one-to-one correspondence with $B_{n,l}(t)$)

$$\int_0^1 dx x^N H_+(x,\xi,t) = \sum_{\substack{k=0 \\ \text{even}}}^{N+1} h_{N,k}(t)\xi^k, \quad \text{for odd} \quad N;$$

- The sum rule for $J=0,\,2$ FG projection truncated at $\nu=1$ ("next-to-minimalist" contribution):

$$F_{J=0}(t) = 4\left(B_{1,0}(t)+\ldots\right) = \frac{5}{3}h_{1,0}(t) + 5h_{1,2}(t) + \left\{\begin{matrix}\text{contribution of conformal PWs} \\ \text{with } \nu \geq 2\end{matrix}\right\};$$

$$F_{J=2}(t) = 4\left(B_{1,2}(t)+B_{3,2}(t)+\ldots\right)$$
$$= -\frac{7}{6}h_{1,0}(t) + 9h_{3,0}(t) + \frac{21}{2}h_{3,2}(t) + \left\{\begin{matrix}\text{contribution of conformal PWs} \\ \text{with } \nu \geq 2\end{matrix}\right\},$$

- Coefficients $h_{N,k}$ can be studied with methods of lattice QCD. A new way to connect lattice results to the data!

On mixing due to target mass $\neq 0$ I

- The modification of the double PW expansion (dual parametrization) explicit inclusion of threshold $\beta \equiv \sqrt{1-\frac{4m^2}{t}}$ corrections in the summation of t-channel spin-l exchanges:

$$H_+(x,\xi,t) = 2\sum_{\substack{n=1\\ \text{odd}}}^{\infty}\sum_{\substack{l=0\\ \text{even}}}^{n+1} \beta^l \bar{B}_{n,l}(t)\,\theta\left(1-\frac{x^2}{\xi^2}\right)\left(1-\frac{x^2}{\xi^2}\right) C_n^{3/2}\left(\frac{x}{\xi}\right) P_l\left(\frac{1}{\xi\beta}\right).$$

- The appropriate system of orthogonal polynomials for the CFF is still $P_l(\beta\cos\theta_t)$, (see the integration limits in the dispersive integral for the LO CFF: [0; 1] and not [0; $1/\beta$]).
- Reexpansion of $P_l(\cos\theta_t)$ back to $P_l(\beta\cos\theta_t) \equiv P_l\left(\frac{1}{\xi}\right)$.
- Some guiding principles:
  - Two expansions give the same coefficients $h_{N,k}(t)$ at power ξ^k of N-th Mellin moments of a GPD.
  - The coefficients $h_{N,k}(t)$ must be regular in the $t \to 0$ limit $\Rightarrow$ assume specific singularities for the generalized FFs $\bar{B}_{n,l}(t)$ at $t=0$.

On mixing due to target mass $\neq 0$ II

- Consider $h_{N,k=0}(t)$: no modification for $\nu = 0$ contribution ($Q_0(y,t)$):

$$B_{n,n+1}(t) = \bar{B}_{n,n+1}(t);$$

- Consider $h_{N,k=2}(t)$: for $\nu = 1$ ($Q_2(y,t)$): admixture of higher $J = l + 2$ spin:

$$\underbrace{B_{n,n-1}(t)}_{\text{Occurs in } J=n-1\text{-th FG projection}} = \underbrace{\bar{B}_{n,n-1}(t)}_{\text{spin } n-1} - (1-\beta^2)\left(\frac{1}{2} - n\right) \underbrace{\bar{B}_{n,n+1}(t)}_{\text{spin } n+1}.$$

- In general, the mixing for $B_{l+2\nu-1,l}(t)$ involves higher spin contributions up to $l + 2\nu$.

- FG projections $F_J(t)$ get admixture from higher spins:

$$F_J(t) = 4\int_0^1 dy x^{J-1} N(y,t) = 4\Big[\underbrace{B_{J-1\,J}(t)}_{\text{pure spin-}J} + \underbrace{B_{J+1\,J}(t)}_{\text{spin } J,\, J+2} + \underbrace{B_{J+3\,J}(t)}_{\text{spin } J,\, J+2,\, J+4} + \dots\Big]$$

- Mixing can be tamed once we may truncate summation in ν at some $\nu_{\max}$!
- This assumption can be tested through saturation of sum rules.

Can we treat the analytic properties in t better?

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Dispersive evaluation of the D-term form factor in deeply virtual Compton scattering

B. Pasquini [a,b,*], M.V. Polyakov [c], M. Vanderhaeghen [d,e]

- Unsubtracted DR in t for the D-term FF at $\frac{s-u}{4m_N}=0$:

$$4D^q(t)=\frac{1}{\pi}\int_{4m_\pi^2}^{+\infty}\mathrm{d}t'\,\frac{\mathrm{Im}_t\,F^q\,(0,t')}{t'-t}+\frac{1}{\pi}\int_{-\infty}^{-a}\mathrm{d}t'\,\frac{\mathrm{Im}_t\,F^q\,(0,t')}{t'-t};$$

- Phenomenological input for the absorptive part: $\pi\pi$ intermediate state

$$2\,\mathrm{Im}\,T^{\gamma^*\gamma\to N\bar{N}}=\frac{1}{(4\pi)^2}\frac{p_\pi}{\sqrt{t}}\int d\Omega_\pi\left[T^{\gamma^*\gamma\to\pi\pi}\right]\cdot\left[T^{\pi\pi\to N\bar{N}}\right]^*;$$

- Can one extend this kind of technique for the FG projection FFs?

Summary and Outlook

1. New tool for baryon spectroscopy: arbitrary spin-J probe and PW analysis of excited states;
2. Access to $N \to N^*$ EMT matrix elements: mechanical properties of resonances;
3. $N \to \pi N$, $N \to \eta N$ GPDs provide a lab for χPT on the light cone: soft pion theorems and chiral expansion. Can we build a new bridge between the chiral regime and resonance physics?
4. GPD formalism worked out for $N \to \Delta(1232)$, $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$. Can be studied at JLab@12 GeV and an option for JLab@22 GeV;
5. Need to study for the kinematical conditions of EIC and EicC;
6. Froissart-Gribov projection as a means to focus on specific J in the cross channel;
7. "Electric" and "magnetic" spin-J radiuses of a nucleon: nucleon seen by a spin-J probe?

$$\frac{1}{F_J^{(E,M)}(0)} \frac{dF_J^{(E,M)}(t)}{dt}\Big|_{t=0} = -\frac{\left(r_J^{(E,M)}\right)^2}{6};$$

Thank you for your attention!