Near-threshold vector meson production and gravitational form factors

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BNL/RIKEN BNL

Mechanical properties of hadrons, ECT*, March 31-April 4, 2025

Proton electromagnetic form factors

EM form factors from elastic scattering

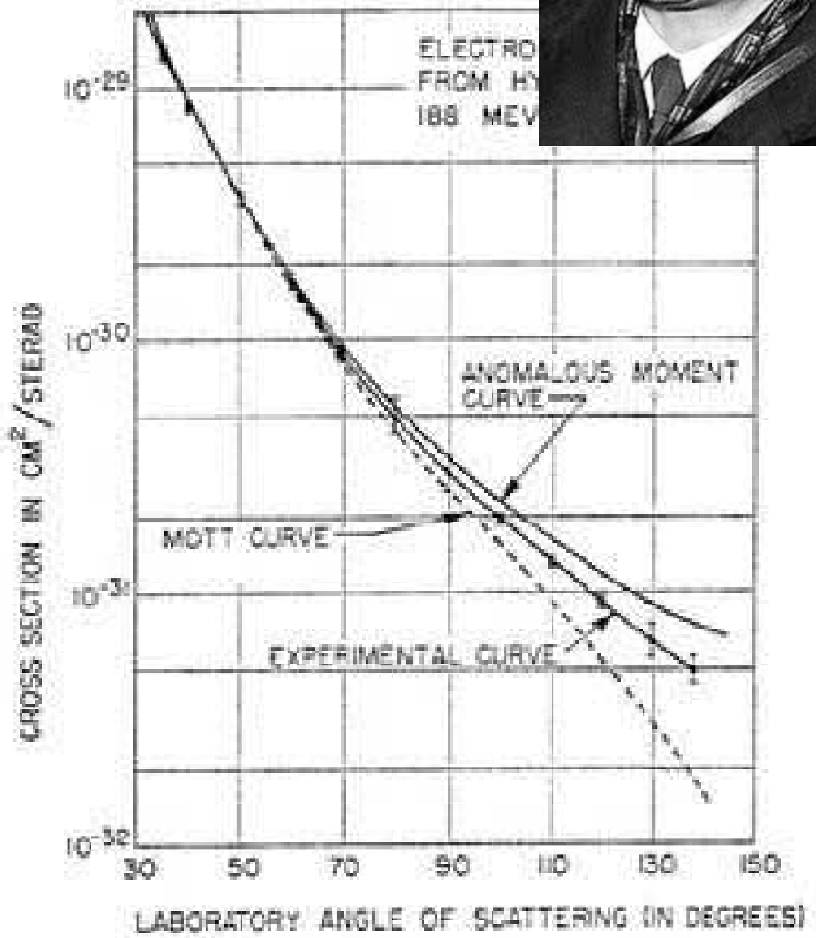
$$\langle p'|J^\mu(0)|p\rangle = \bar{u}(p')\left[\gamma^\mu F_1 + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}F_2\right]u(p)$$

Electric form factor

$$G_E(t) = F_1(t) + \frac{t}{4M^2}F_2(t)$$

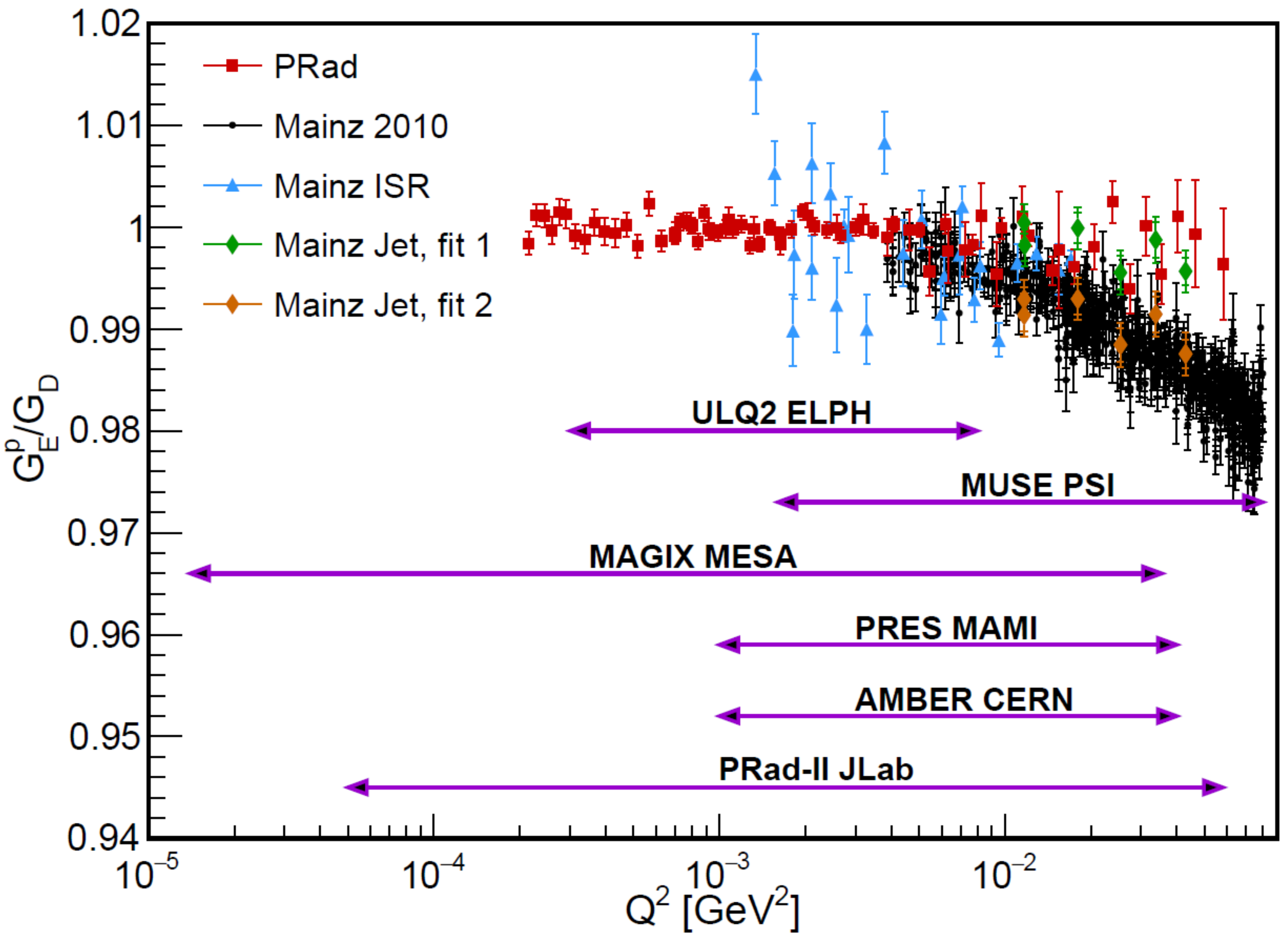
Proton charge radius

$$\langle r^2\rangle = 6\frac{dG_E(t)}{dt}\bigg|_{t=0}$$

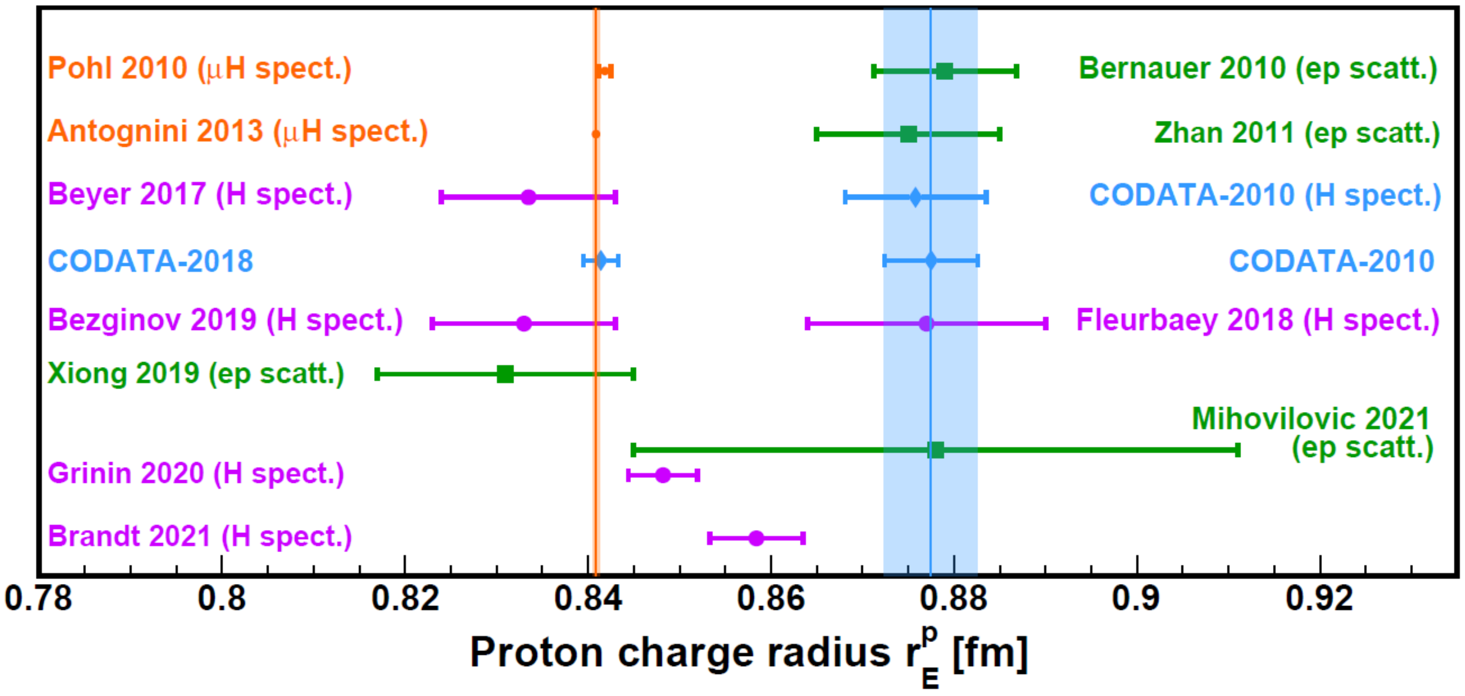


Elastic scattering 70 years later

Xiong, Peng, 2302.13818



Proton radius puzzle?



Both CODATA and PDG now recommend the smaller value ~0.84fm.

Several future experiment planned, aim for less than 1% precision

PRad (2019) $r_p = 0.831 \pm 0.007_{\text{stat}} \pm 0.012_{\text{syst}}$

Radius zoo

2312.12984

Charge radius
$$\langle r^2\rangle_c = \frac{\int d\boldsymbol{x} x^2 \rho_c(\boldsymbol{x})}{\int d\boldsymbol{x} \rho_c(\boldsymbol{x})} = \frac{6}{G_E(0)} \frac{dG_E(t)}{dt}\bigg|_{t=0}$$

Magnetic radius
$$\langle r^2\rangle_M = \frac{6}{G_M(0)} \frac{dG_M(t)}{dt}\bigg|_{t=0}$$

Baryon number radius
$$\langle r^2\rangle_B = \frac{\int d\boldsymbol{x} x^2 \rho_B(\boldsymbol{x})}{\int d\boldsymbol{x} \rho_B(\boldsymbol{x})}$$

Mass radius
$$\langle r^2\rangle_m = \frac{\int d\boldsymbol{x}\, x^2 T^{00}(\boldsymbol{x})}{\int d\boldsymbol{x} T^{00}(\boldsymbol{x})} = 6\frac{dA(t)}{dt}\bigg|_{t=0} - \frac{3D(0)}{2M^2}$$

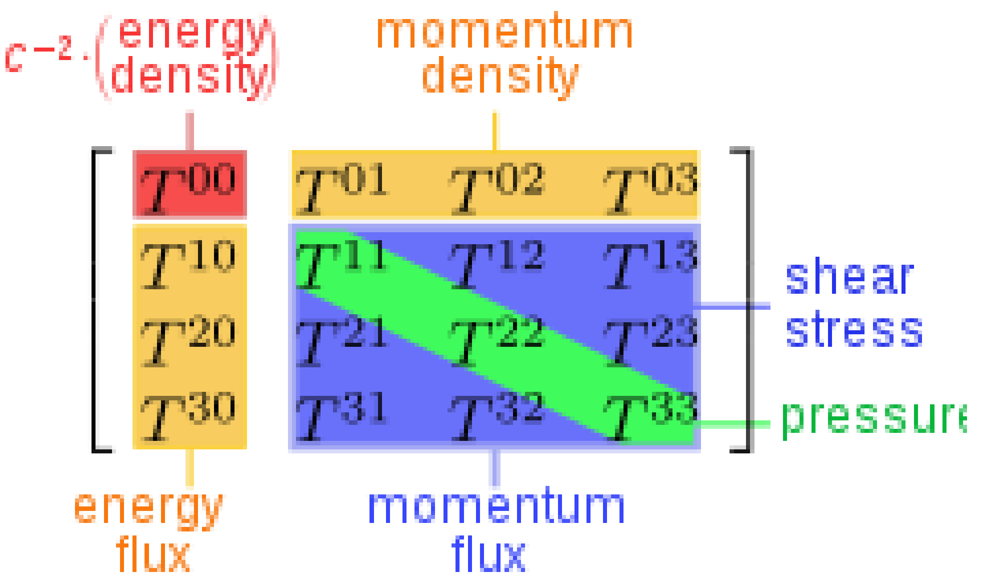
Scalar radius
$$\langle r^2\rangle_s = \frac{\int d\boldsymbol{x}\, x^2 T^{\mu}_{\mu}(\boldsymbol{x})}{\int d\boldsymbol{x} T^{\mu}_{\mu}(\boldsymbol{x})} = 6\frac{dA(t)}{dt}\bigg|_{t=0} - \frac{9D(0)}{2M^2}$$

Tensor radius
$$\langle r^2\rangle_t \equiv \frac{\int d\boldsymbol{x}\, x^2 \left(T^{00}(\boldsymbol{x}) + \frac{1}{2}T_{ii}(\boldsymbol{x})\right)}{\int d\boldsymbol{x}\, \left(T^{00} + \frac{1}{2}T_{ii}\right)} = 6\frac{dA(t)}{dt}\bigg|_{t=0}$$

Mechanical radius
$$\langle r^2\rangle_{mech} = \frac{\int d\boldsymbol{x} x^2 \frac{x_i x_j}{x^2} T_{ij}(\boldsymbol{x})}{\int d\boldsymbol{x} \frac{x_i x_j}{x^2} T_{ij}(\boldsymbol{x})} = \frac{6D(0)}{\int_{-\infty}^{0} dt D(t)}$$

…

Gravitational form factors



QCD energy momentum tensor

$$T^{\mu\nu} = \sum_f \bar{\psi}_f \gamma^{(\mu} iD^{\nu)} \psi_f - F^{\mu\rho} F^{\nu}{}_{\rho} + \frac{g^{\mu\nu}}{4} F^{\alpha\beta} F_{\alpha\beta}$$

Associated form factors

$$\langle P' | T^{\mu\nu} | P \rangle = \bar{u}(P') \left[A(t) \gamma^{(\mu} \bar{P}^{\nu)} + B(t) \frac{\bar{P}^{(\mu} i\sigma^{\nu)\alpha} \Delta_\alpha}{2M} + D(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4M} \right] u(P)$$

GFFs for quarks and gluons

Separately defined for quarks and gluons

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \bar{u}(P')\Big[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D_{q,g}\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M} + \bar{C}_{q,g}M\eta^{\mu\nu}\Big]u(P)$$

hidden form factor

$$\bar{C}_q + \bar{C}_g = 0$$

$$\langle P|(T_{q,g})^\mu_\mu|P\rangle = 2M^2(A_{q,g} + 4\bar{C}_{q,g})$$

Connection to the trace anomaly and gluon condensate → Origin of hadron masses

$\langle P'|F^2|P\rangle$ not an independent form factor

Relation between $\bar{C}_{q,g}(0)$ and $\langle P|F^{\mu\nu}F_{\mu\nu}|P\rangle$ in MSbar

1 loop, 2 loop: YH, Rajan, Tanaka (2018)
3 loop: Tanaka (2019)
4 loop: Ahmed, Chen, Czakon (2022)

$$\left\langle \mathrm{Tr}\left([\Theta_g]_R^{\overline{\mathrm{MS}}}\right)\right\rangle_{\mathrm{P}} = \langle [O_F]_R\rangle_{\mathrm{P}} \left(-0.437676\,\alpha_s - 0.261512\,\alpha_s^2 - 0.183827\,\alpha_s^3 - 0.256096\,\alpha_s^4\right)$$
$$+ \langle [O_m]_R\rangle_{\mathrm{P}} \left(0.495149\,\alpha_s + 0.776587\,\alpha_s^2 + 0.865492\,\alpha_s^3 + 0.974674\,\alpha_s^4\right) ,$$

$$\left\langle \mathrm{Tr}\left([\Theta_q]_R^{\overline{\mathrm{MS}}}\right)\right\rangle_{\mathrm{P}} = \langle [O_F]_R\rangle_{\mathrm{P}} \left(0.079578\,\alpha_s + 0.058870\,\alpha_s^2 + 0.021604\,\alpha_s^3 + 0.013675\,\alpha_s^4\right)$$
$$+ \langle [O_m]_R\rangle_{\mathrm{P}} \left(1 + 0.141471\,\alpha_s - 0.008235\,\alpha_s^2 - 0.064351\,\alpha_s^3 - 0.065869\,\alpha_s^4\right)$$

Similar relation between non-forward $\bar{C}_{q,g}(t)$ and $\langle P'|F^2|P\rangle$ YH, Rajan, Tanaka (2019)

Nucleon D-term in the Sakai-Sugimoto model

Fujita, YH, Sugimoto, Ueda (2022)

Baryons = instantons in D8 branes in type-IIA superstring

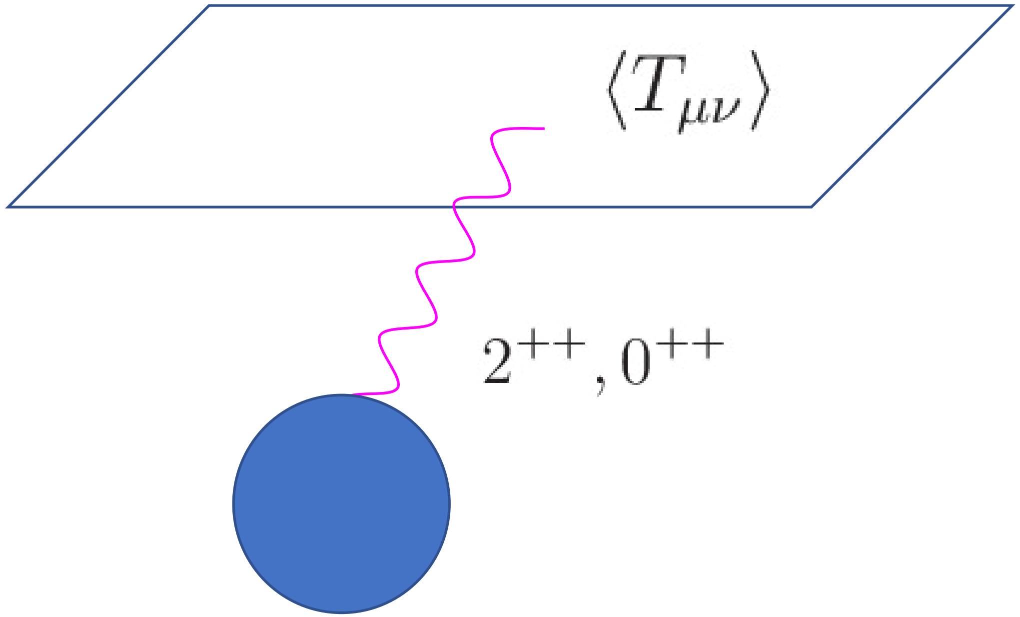
QFT energy momentum tensor from holographic renormalization

Graviton in 7D AdS = QCD glueballs

Glueball dominance in large-Nc QCD

$$D(|\vec{k}|) \sim \sum_{n=1}^{\infty} \frac{c_n^{\mathrm{T}}(|\vec{k}|)}{\vec{k}^2 + (m_n^{\mathrm{T}})^2} + \sum_{n=1}^{\infty} \frac{c_n^{\mathrm{S}}(|\vec{k}|)}{\vec{k}^2 + (m_n^{\mathrm{S}})^2}$$

see also, Mamo, Zahed (2021)



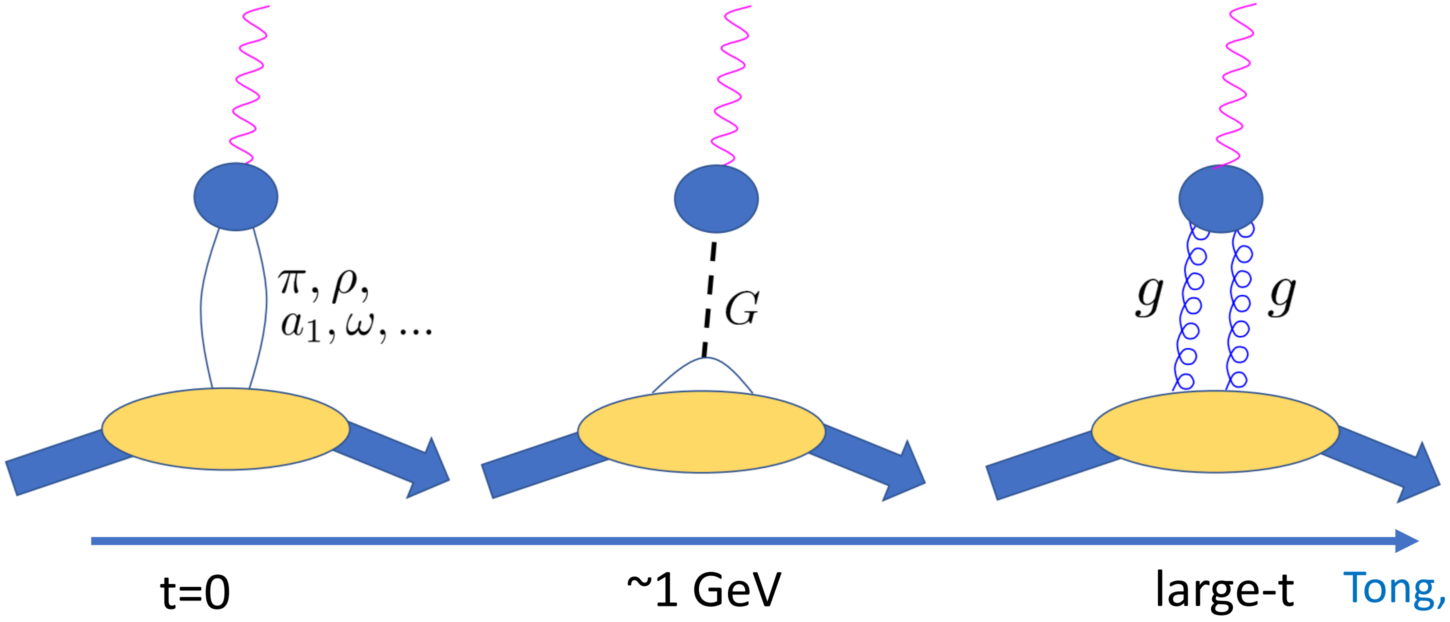
At $t = |\vec{k}|^2 = 0$, the infinite sum can be performed in a closed form

Numerical result (revised in Sugimoto, Tsukamoto, 2503.19492)

$$D(0) = -3.42 + 1.36 = -2.06$$

Negative (attractive) contribution from isovector mesons $\pi, \rho, a_1, \cdots$

Positive (replusive) contribution from isoscalar mesons ω

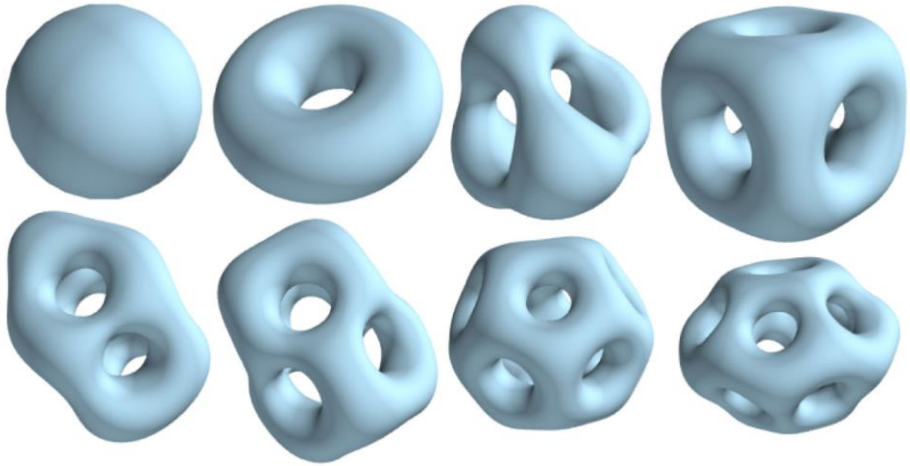


Tong, Ma, Yuan (2021)

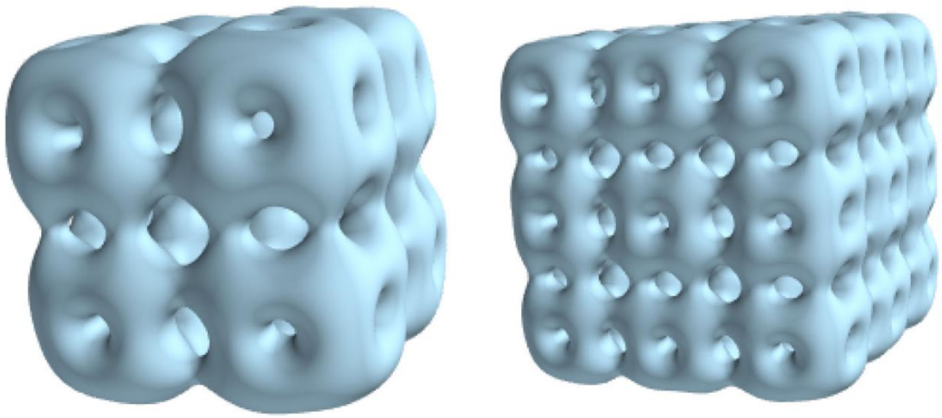
D-term of atomic nuclei in the Skyrme model

Martin-Caro, Huidobro, YH 2304.05994; 2312.12984

$B = 1 \sim 8$



$B = 32$ $\quad$ $B = 108$



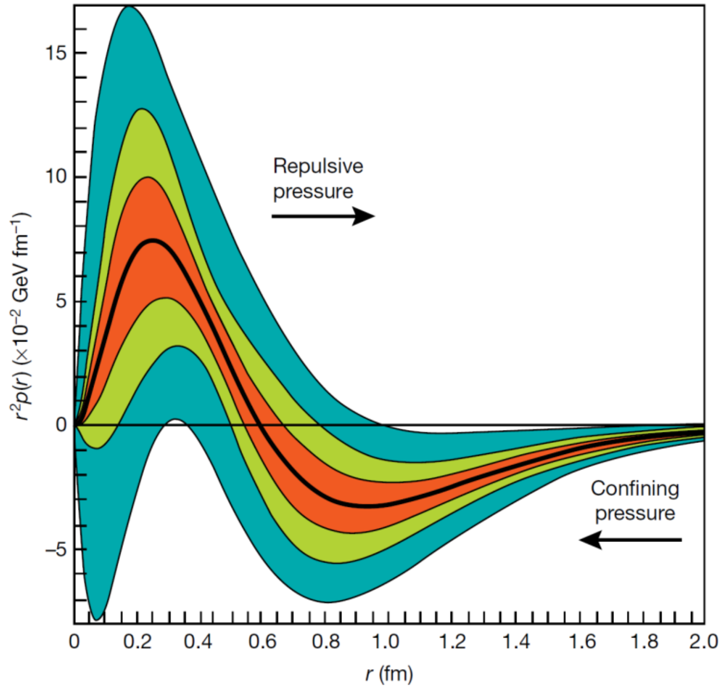
B	1	2	3	4	5	6	7	$8a$	$8b$	32	108
$D(0)$	−3.701	−13.126	−26.757	−43.304	−62.72	−85.95	−106.596	−128.368	−140.816	-1.874×10^3	-2.152×10^4

The value D(0) grows quickly with increasing B

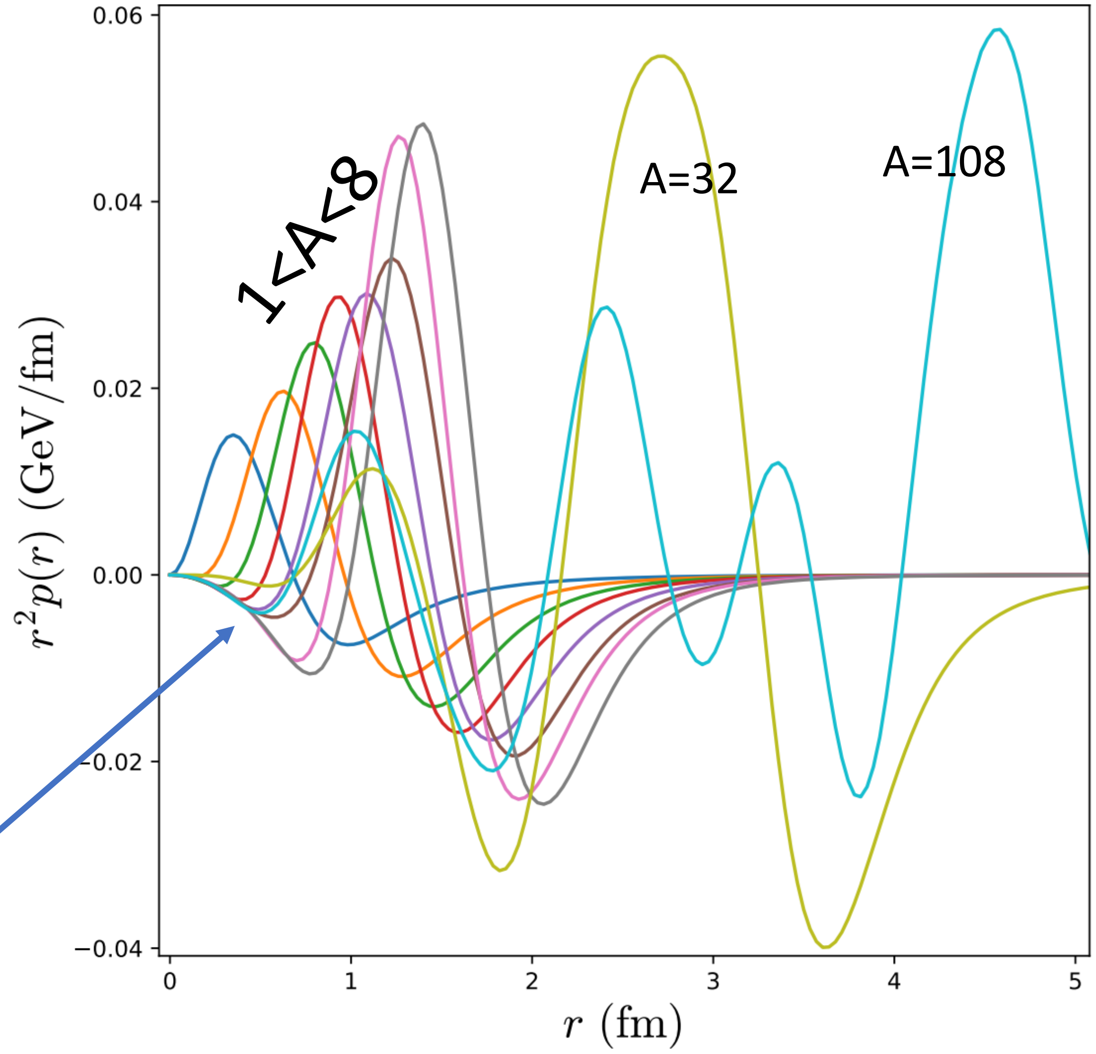
cf. Polyakov (2003); Liuti, Taneja (2005); Guzey, Siddikov (2005)

`Pressure' inside nucleon and nuclei

Burkert, Elouadrhiri, Girod (2018)



Martin-Caro, Huidobro, YH, 2312.12984

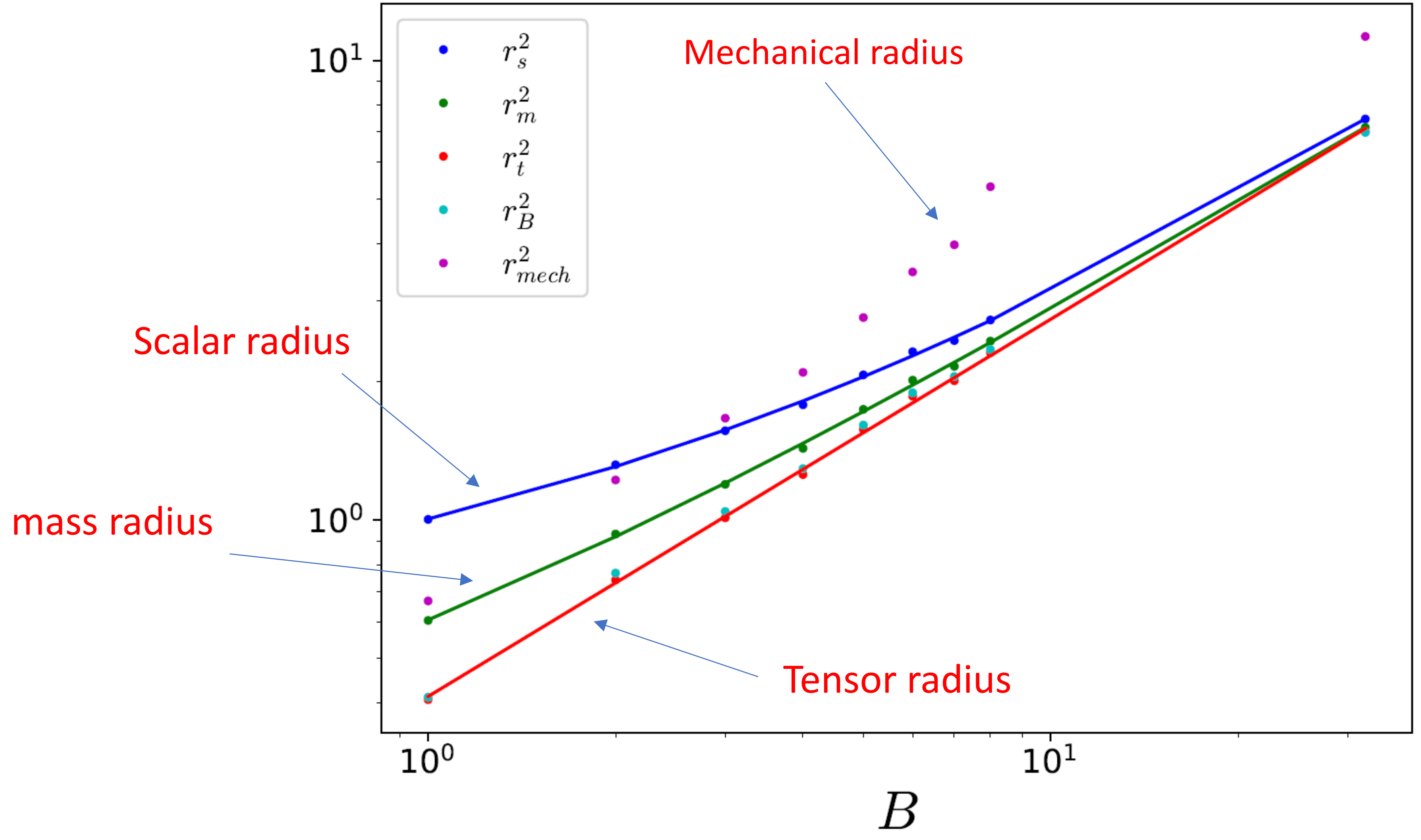


Negative pressure near the core for nuclei A>1

see also, Freese, Cosyn (2022), He, Zahed (2023)

Nuclear radii

Martin-Caro, Huidobro, YH, 2312.12984



$$\langle r^2 \rangle_s = \langle r^2 \rangle_m - \frac{3D(0)}{M^2}$$

$$\frac{D(0)}{M^2} \propto B^{\beta-2}$$

Experimental study of GFFs?

- Introduced theoretically in the 60s.
- Received far less attention than EM form factors, not because they are less interesting/important.
- The obvious reason: We cannot measure them directly!

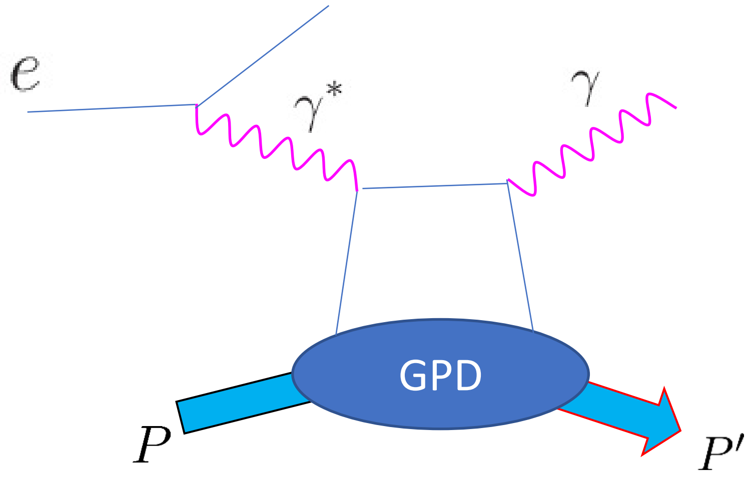
One-graviton exchange cross section $$\frac{d\sigma}{dt} \sim G_N^2 \frac{s^2}{t^2}$$

$$G_N \sim 1/M_P^2 \qquad M_P \sim 10^{19} \text{ GeV}$$

- There are, however, indirect ways to measure them.

Quark D-term from Deeply Virtual Compton Scattering

$$D = D_u + D_d + D_s + D_g + \cdots$$



$D_{u,d}$ related to the subtraction constant in the dispersion relation for the Compton form factor Teryaev (2005)

$$\mathrm{Re}\mathcal{H}_q(\xi,t) = \frac{1}{\pi}\int_{-1}^{1} dx \mathrm{P}\frac{\mathrm{Im}\mathcal{H}_q(x,t)}{\xi - x} + 2\int_{-1}^{1} dz \frac{D_q(z,t)}{1-z}$$

$$\int_{-1}^{1} dz z D_q(z,t) = D_q(t)$$

1 graviton ≈ 2 photons

1+1=2

After all, 1 graviton $\neq$ 2 photons

$$\int_{-1}^{1} dz \frac{D_q(z,t)}{1-z}$$

what is measurable

$$\int_{-1}^{1} dz z D_q(z,t)$$

what we want

2-photon state couples to operators with arbitrary spin.
How can one isolate the spin-2 component?

1+1= anything

$$\frac{1}{1-z} = 1 + z + z^2 + z^3 + \cdots$$

spin-2 (EMT) → z

spin-4 → z^3

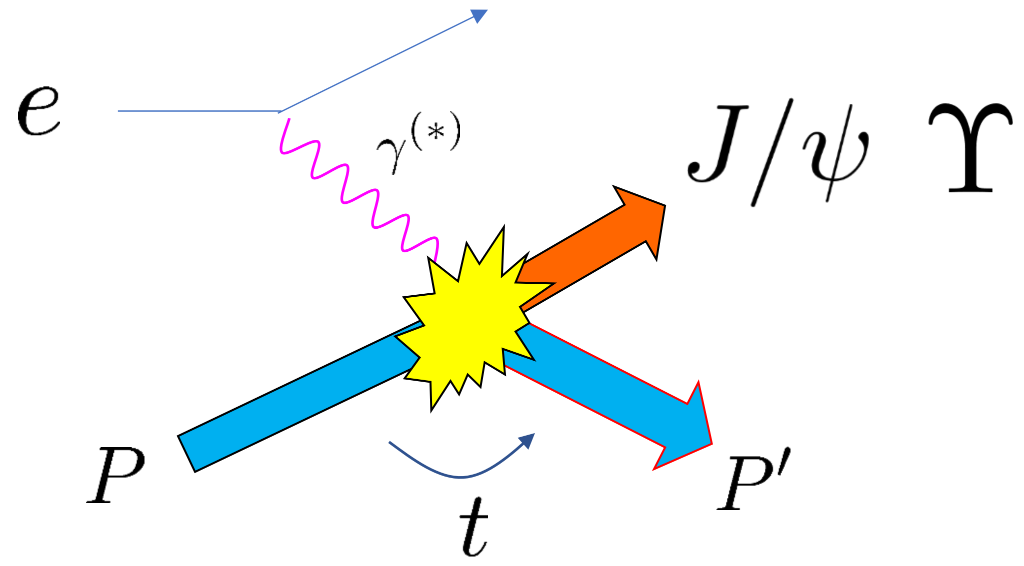
$$d_1^{uds}(t=0, 2\ \mathrm{GeV}^2) = -1.7 \pm 21$$
$$d_3^{uds}(t=0, 2\ \mathrm{GeV}^2) = 0.7 \pm 15$$
$$d_1^{g}(t=0, 2\ \mathrm{GeV}^2) = -2 \pm 30$$
$$d_3^{g}(t=0, 2\ \mathrm{GeV}^2) = 0.1 \pm 2.3$$

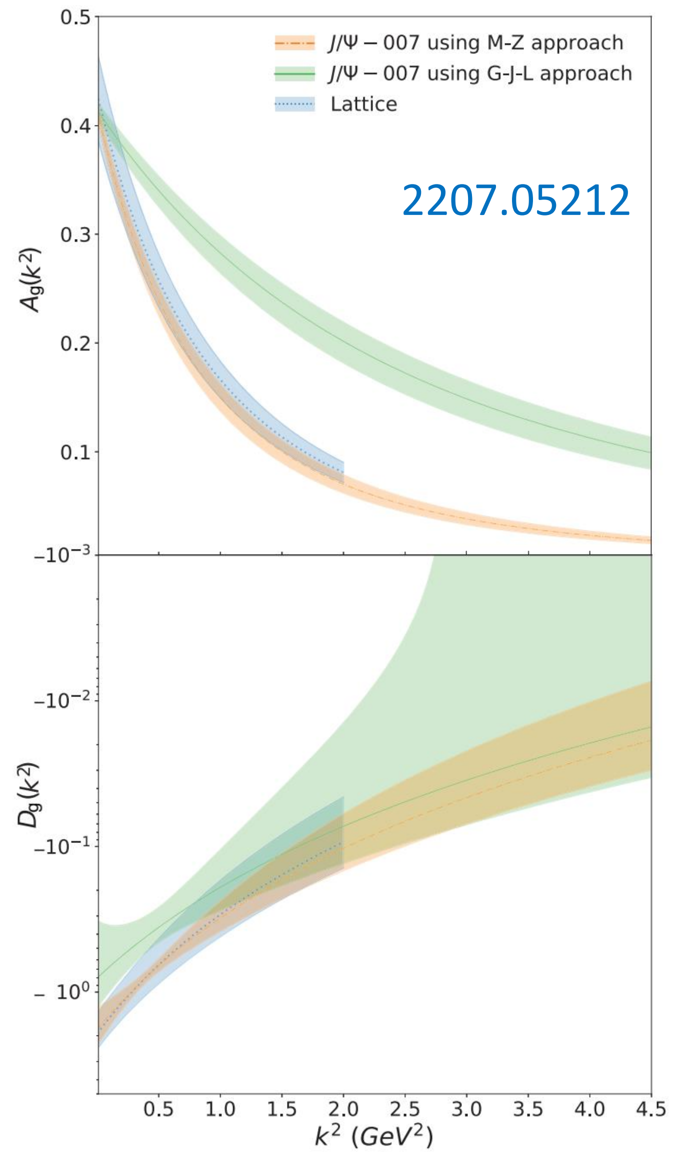
(NLO n=3 radiativ

current precision: 1000%

Dutrieux, Meisgny, Mezrag, Moutarde (2024)

Quarkonium photo-production near threshold





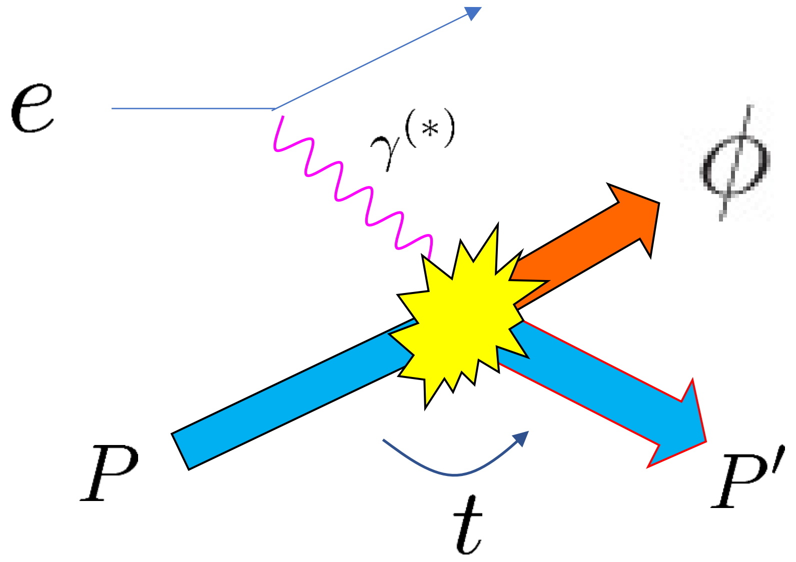
Ongoing experiments at JLab, future measurement at EIC?

Originally proposed by Kharzeev, Satz, Syamtomov, Zinovev (1997) to probe the gluon condensate.

One can also study gluon GFFs in this process YH, Yang (2018)

ϕ -meson electro-production near threshold

YH, Strikman (2021)
YH, Klest, Passek-K, Schoenleber (2025)



Mostly sensitive to gluon GFFs,
but also strangeness GFFs

GPD factorization only for the longitudinally polarized photon

L/T separation crucial → SoLID and EIC?

Again, 1 graviton $\neq$ 2 gluons

what is measurable

$$\int_{-1}^{1} \frac{dx}{x} \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H_g(x,\xi,t)$$

what we want

$$\int_{-1}^{1} dx H_g(x,\xi,t) = A_g(t) + \xi^2 D_g(t)$$

Essentially the same problem as in the extraction of quark D-term from DVCS

HOWEVER, two important differences

Leading contribution from gluon GPD
There is a tunable skewness parameter ξ which becomes large near the threshold.

Threshold approximation

YH, Strikman 2102.12631 (Mellin moment)
Guo, Ji, Liu 2103.11506 (Mellin moment)
Guo, Ji, Yuan 2308.13006 (conformal moment)

what is measurable — what we want

$$\int_{-1}^{1} dx \frac{1}{\xi - x - i\epsilon} \left\{ \begin{array}{l} \frac{1}{2} H^{q(+)}(x,\xi,t,\mu^2) \\ \frac{1}{x} H^{g}(x,\xi,t,\mu^2), \end{array} \right. \approx \frac{2}{\xi^2}\frac{5}{4}\left(A^a(t,\mu^2) + \xi^2 D^a(t,\mu^2)\right)$$

Keep only the first term in the conformal partial wave expansion

Very good approximation when $\xi = \mathcal{O}(1)$ and for gluon and strangeness GPDs
(but not for light-quark GPDs)

Recently extended to NLO Guo, Yuan, Zhao, 2501.10532 → talk by Feng
YH, Klest, Passek-K, Schoenleber, 2501.12343
YH, Schoenleber 2502.12061

Example: NLO ϕ -electroproduction

YH, Klest, Passek-K, Schoenleber (2025)

Compare the full NLO amplitude (Muller et al. (2013)) with the truncated version, also at NLO

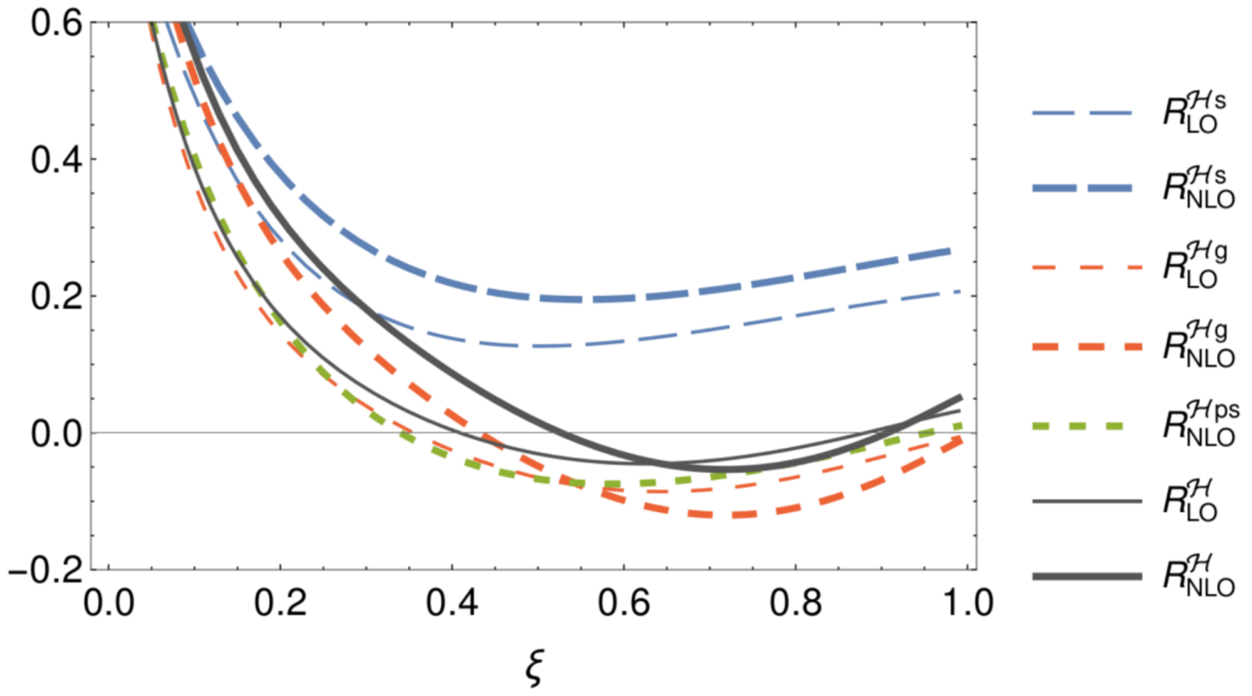
$$\mathcal{H}(\xi,t,Q^2) \approx \frac{2\kappa}{\xi^2}\frac{15}{2}\left[\left\{\alpha_s(\mu)+\frac{\alpha_s^2(\mu)}{2\pi}\left(25.7309-2n_f+\left(-\frac{131}{18}+\frac{n_f}{3}\right)\ln\frac{Q^2}{\mu^2}\right)\right\}(A_s(t,\mu)+\xi^2 D_s(t,\mu))\right.$$
$$\left.+\frac{\alpha_s^2}{2\pi}\left(-2.3889+\frac{2}{3}\ln\frac{Q^2}{\mu^2}\right)\sum_q (A_q+\xi^2 D_q)+\frac{3}{8}\left\{\alpha_s+\frac{\alpha_s^2}{2\pi}\left(13.8682-\frac{83}{18}\ln\frac{Q^2}{\mu^2}\right)\right\}(A_g+\xi^2 D_g)\right]$$

Goloskokov-Kroll (GK) model for nucleon GPD

Truncation error

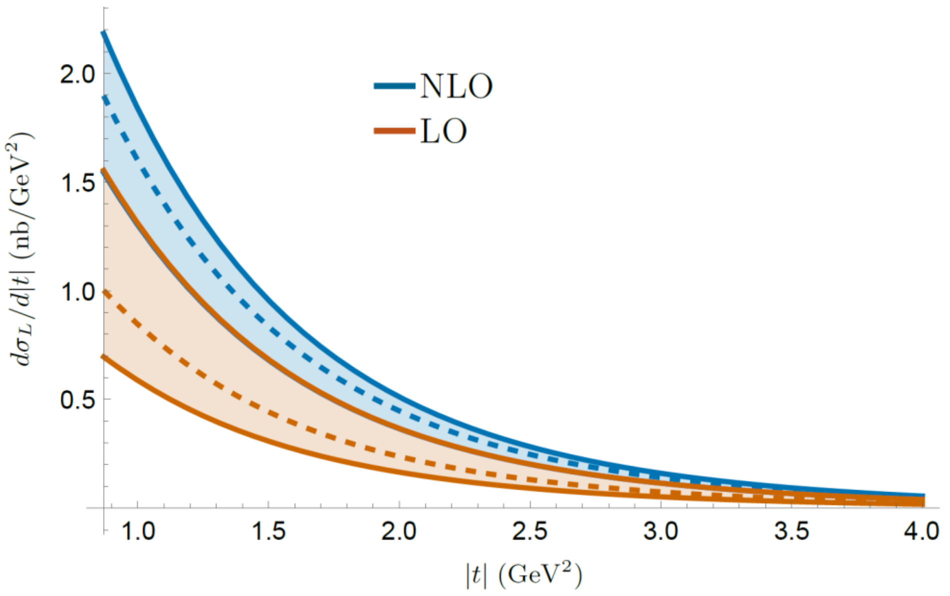
$$R = 1 - \frac{|\mathcal{H}_{\text{full}}|}{\mathcal{H}_{\text{trunc}}}$$

less than 10% for $\xi \gtrsim 0.4$



ϕ -electroproduction at NLO

YH, Klest, Passek-K, Schoenleber (2025)

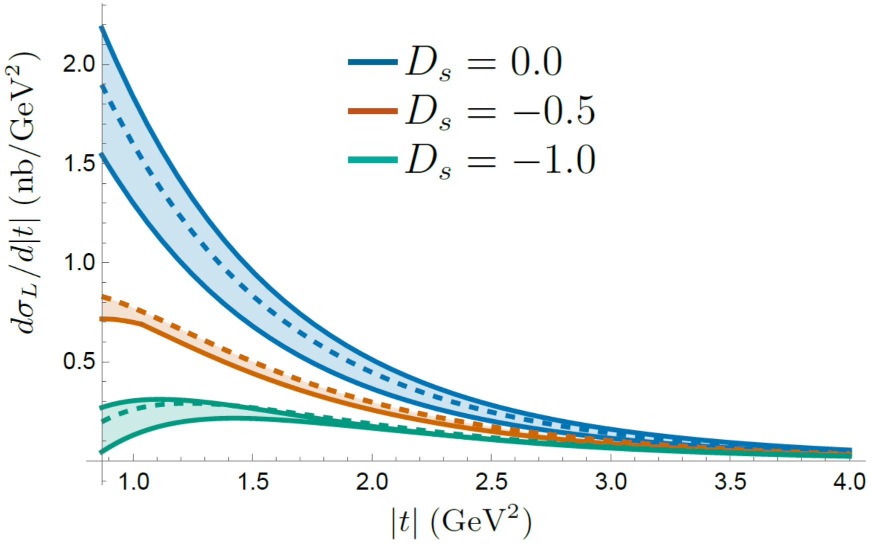


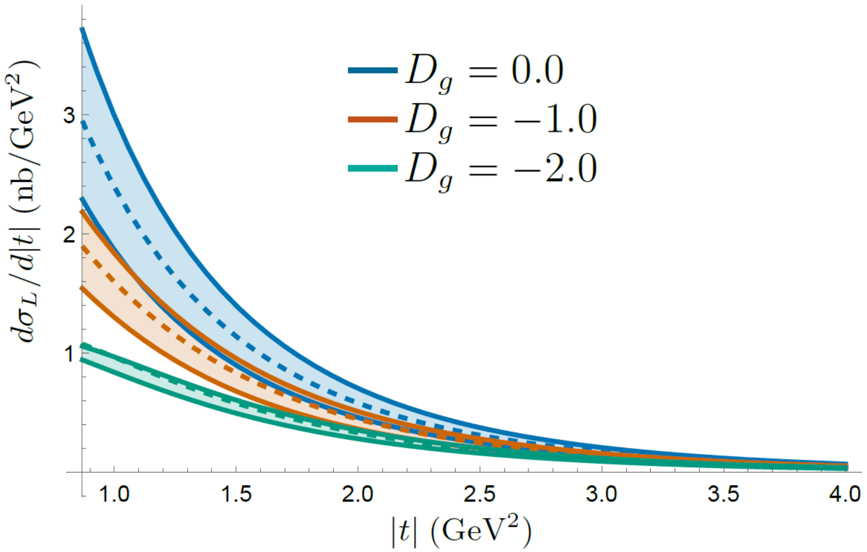
Dominated by gluons.

Cancellation between LO strangeness and NLO valence

Strangeness is important if $D_s = O(1)$

Combined fit to J/psi production data desirable

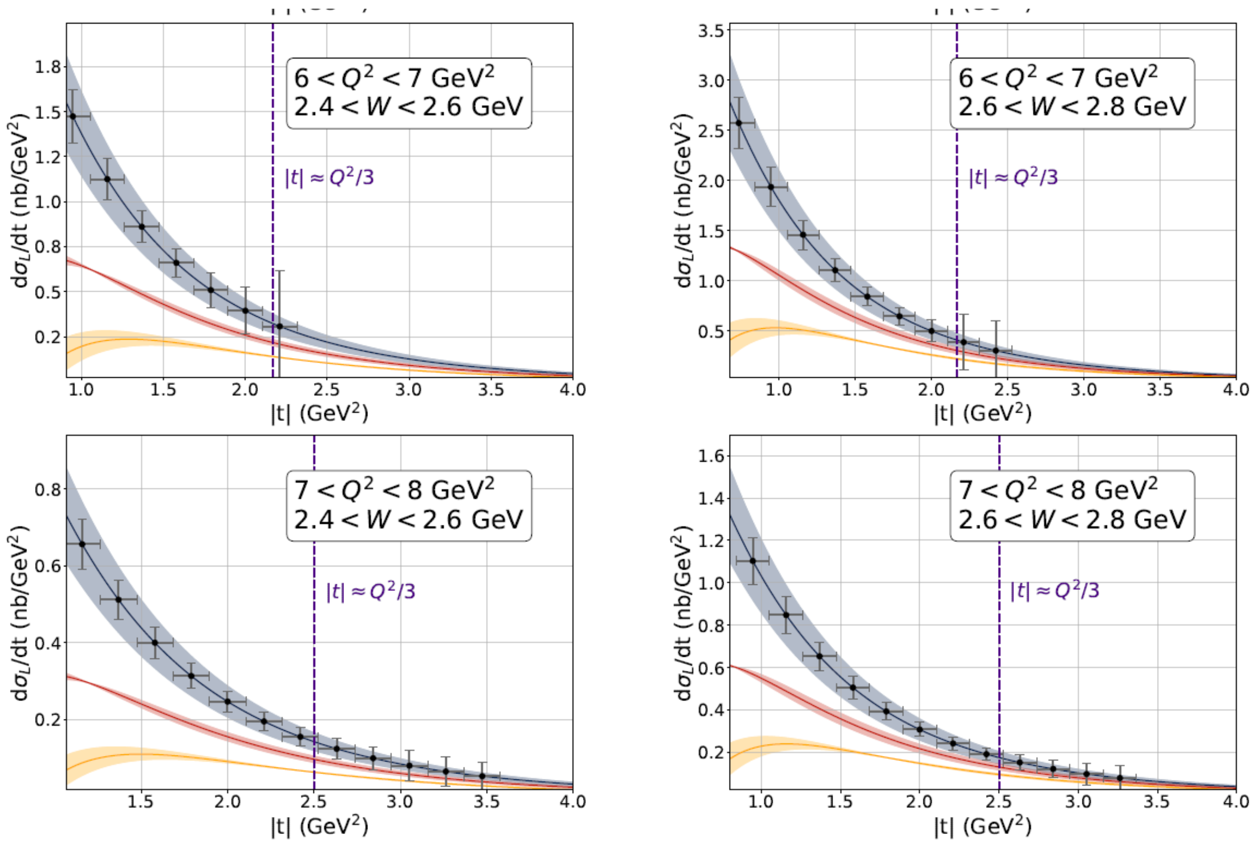


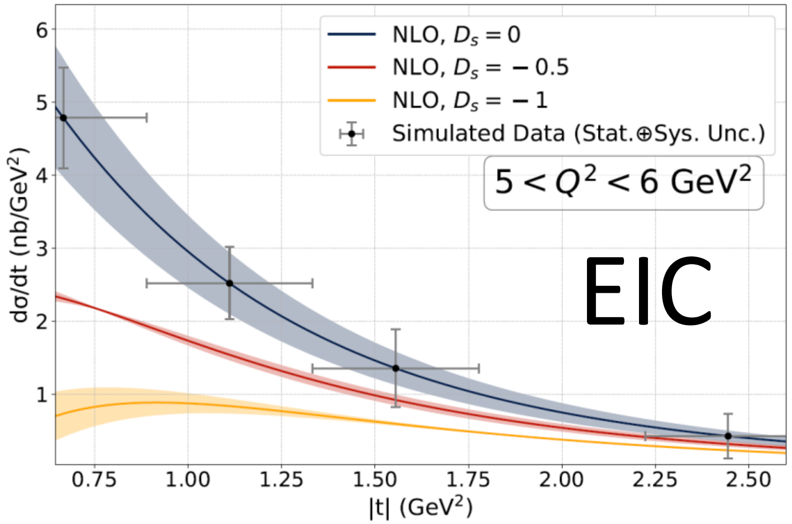


ϕ -electroproduction: Monte Carlo simulation

YH, Klest, Passek-K, Schoenleber (2025)

SoLID





Looks like a feasible measurement!

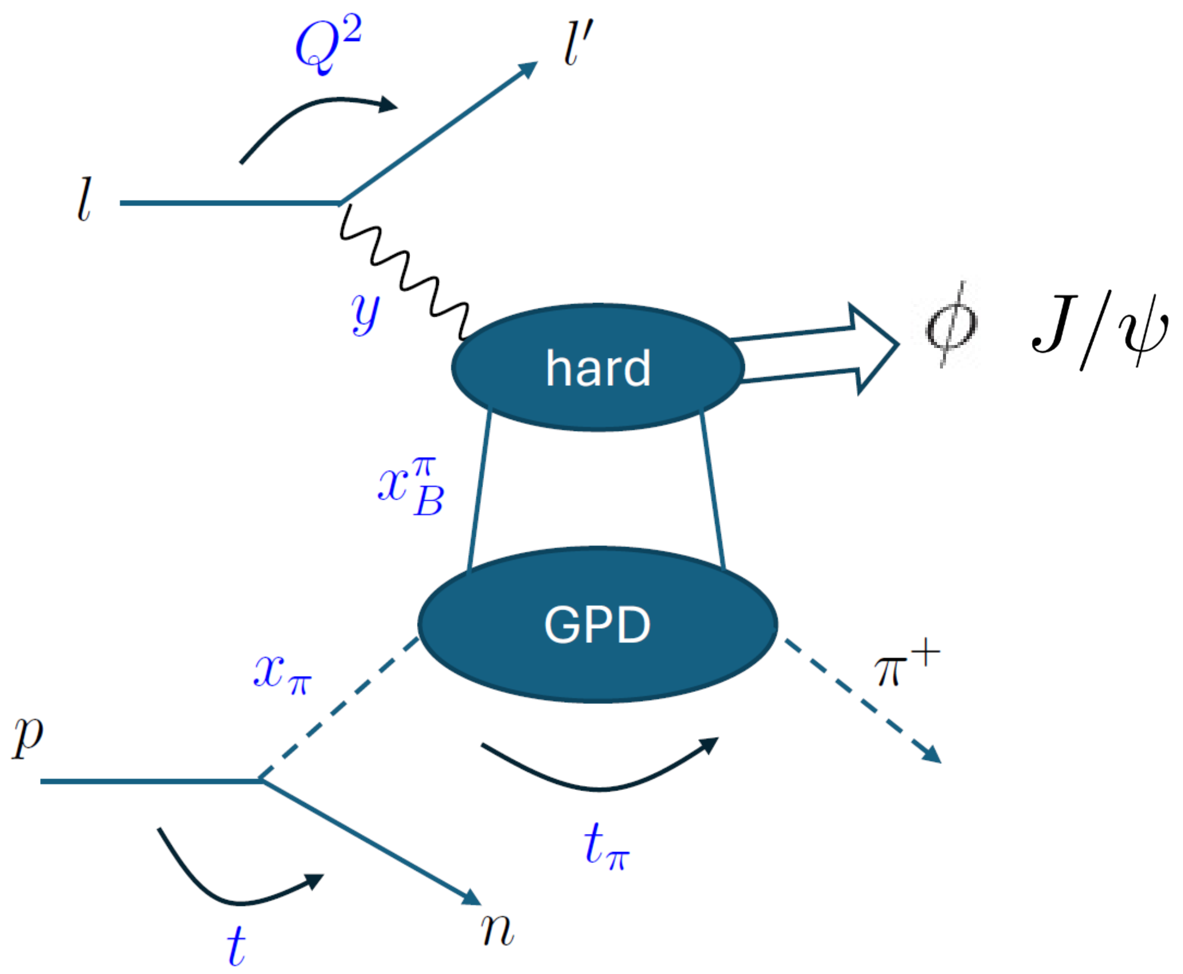
Pion GFFs from Sullivan process

Originally proposed in 1972 to access the pion EM form factors

Pion GPDs from DVCS

Amrath, Diehl, Lansberg (2008)
Chavez, et al. (2022)

Pion GFFs from
J/ψ photoproduction
ϕ electroproduction
near threshold YH, Schoenleber (2025)



Sullivan process near threshold

Measure the cross section $\frac{d\sigma}{dx_B dx_\pi}$

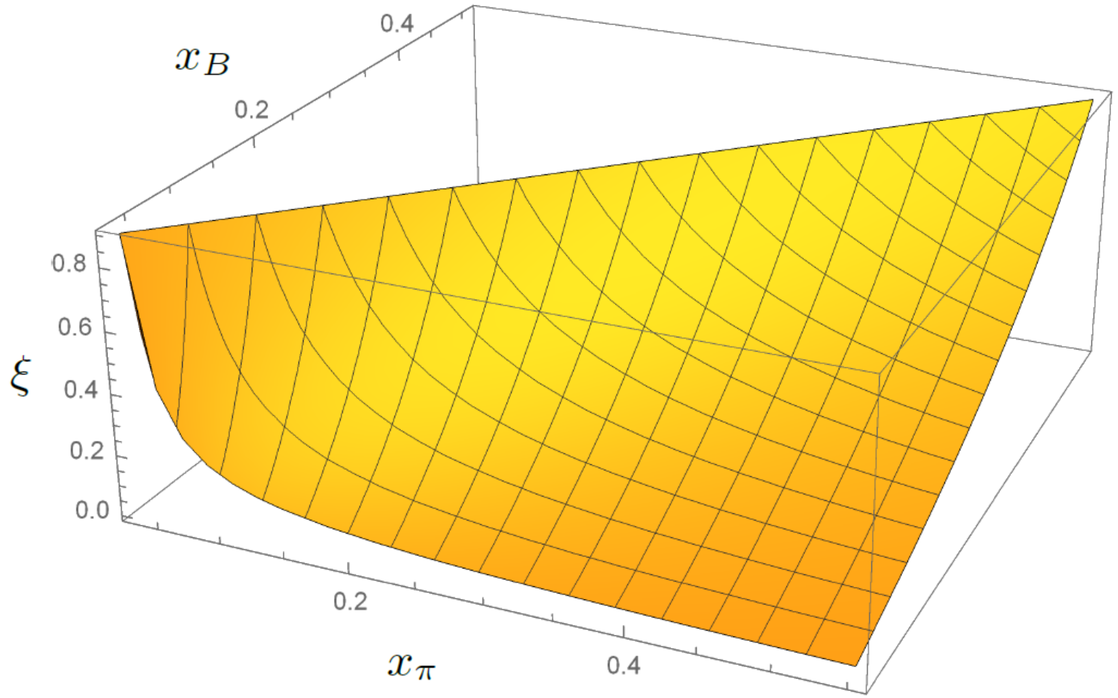
$$x_\pi = \frac{p_\pi \cdot l}{p \cdot l} \qquad x_B = \frac{Q^2}{2p \cdot q}$$

Threshold region along the diagonal line

$$x_B \approx x_\pi$$

Thanks to the light pion mass, relatively easier to achieve large skewness while keeping t small

$$t_{min} = -\frac{4\xi^2 m_\pi^2}{1-\xi^2}$$



Effect of off-shell pion? Broniowski, Shastry, Ruiz-Arriola (2022)

Threshold approximation

Input: Pion GPD at $\mu^2 = 10\,\mathrm{GeV}^2$

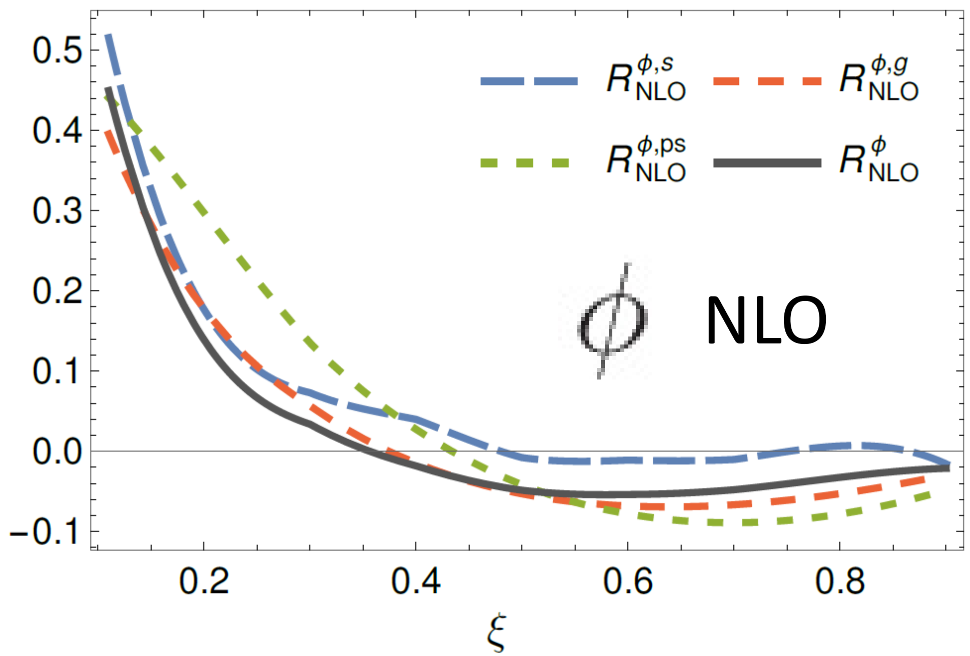
Chavez et al. 2110.06052

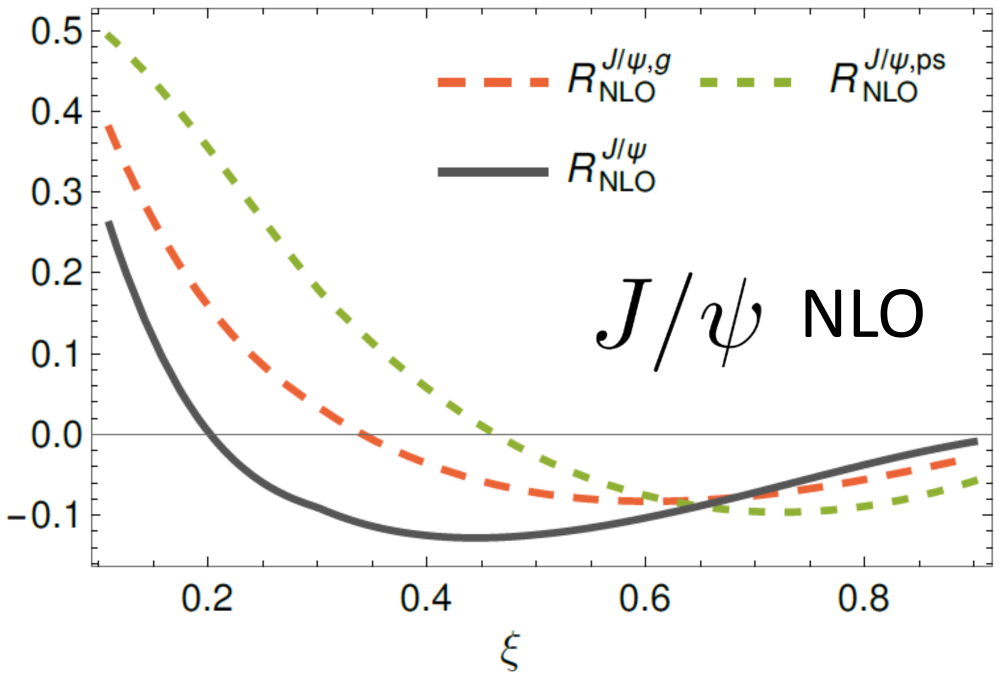
Soft pion theorem

$$D_a(0) = -A_a(0)$$

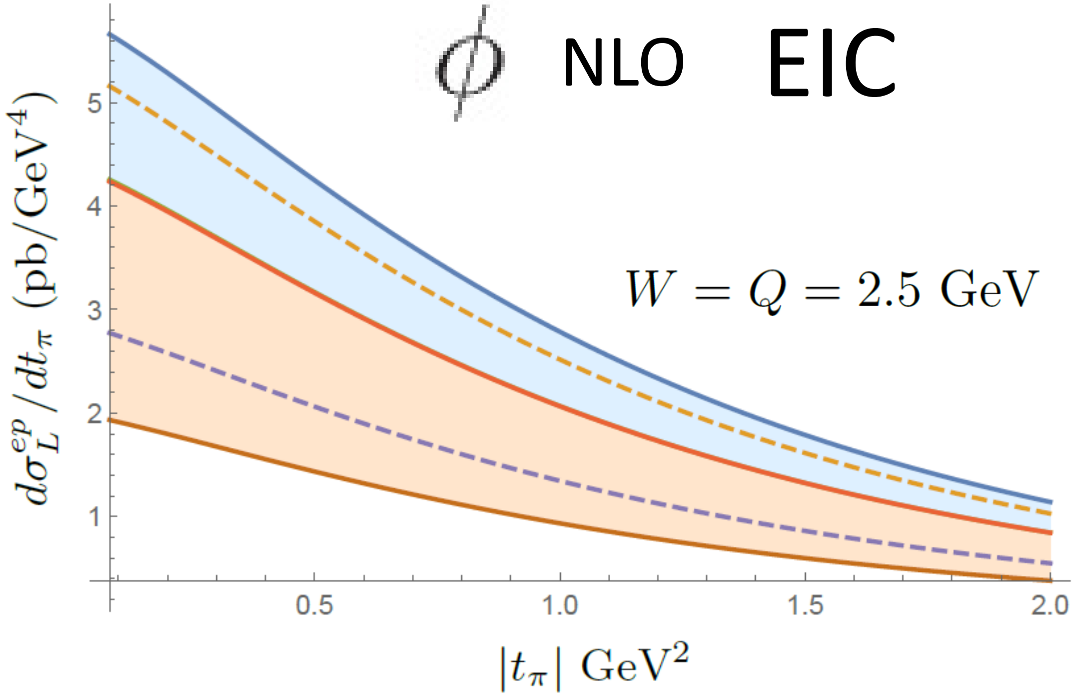
Truncation error

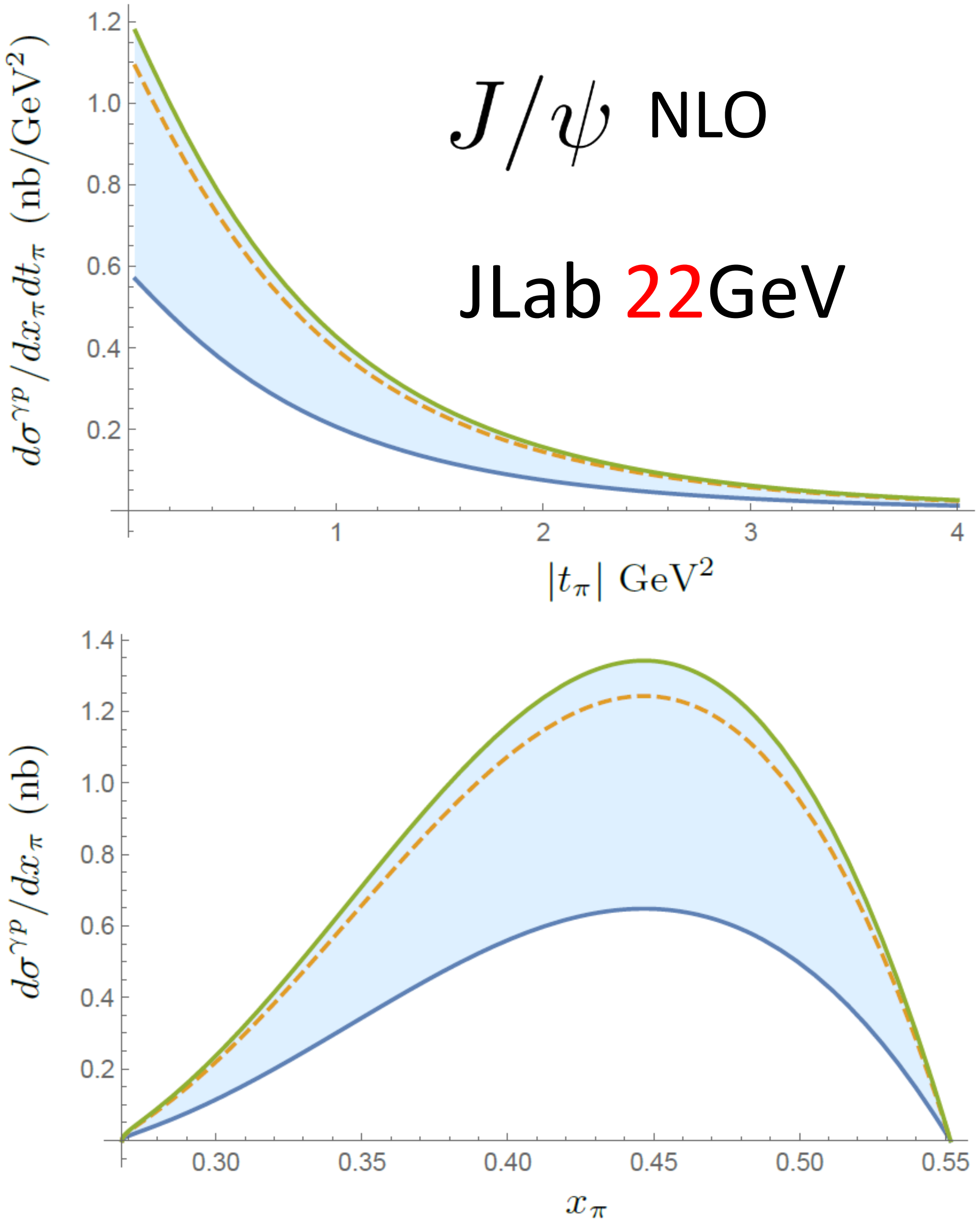
$$R = 1 - \frac{|\mathcal{H}_{\rm full}|}{\mathcal{H}_{\rm trunc}} \qquad 5 \sim 10\%$$





Prediction for JLab and EIC





Cross section well in the measurable range

Direct measurement of GFFs?

- Graviton exchange suppressed by the Planck energy $M_P \sim 10^{19}\ \mathrm{GeV}$
- But in some BSM scenarios, the effective Planck energy could be in the TeV region.
  e.g. extra dimension models.
- These models typically predict massive gravitons.
- Long history of tests of Newton's inverse-square law

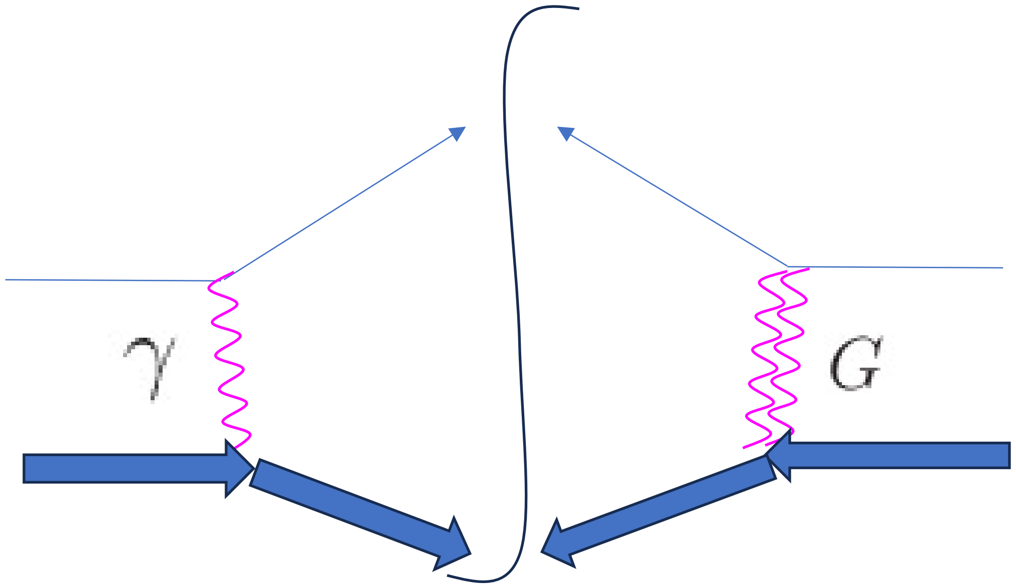
$$V(r) = -G\frac{m_1\ m_2}{r}\left[1 + \alpha\ e^{-r/\lambda}\right]$$

TeV-scale elastic ep, eA scattering

YH, 2311.14470

$$\delta\mathcal{L} = \kappa h_{\mu\nu} T^{\mu\nu}$$

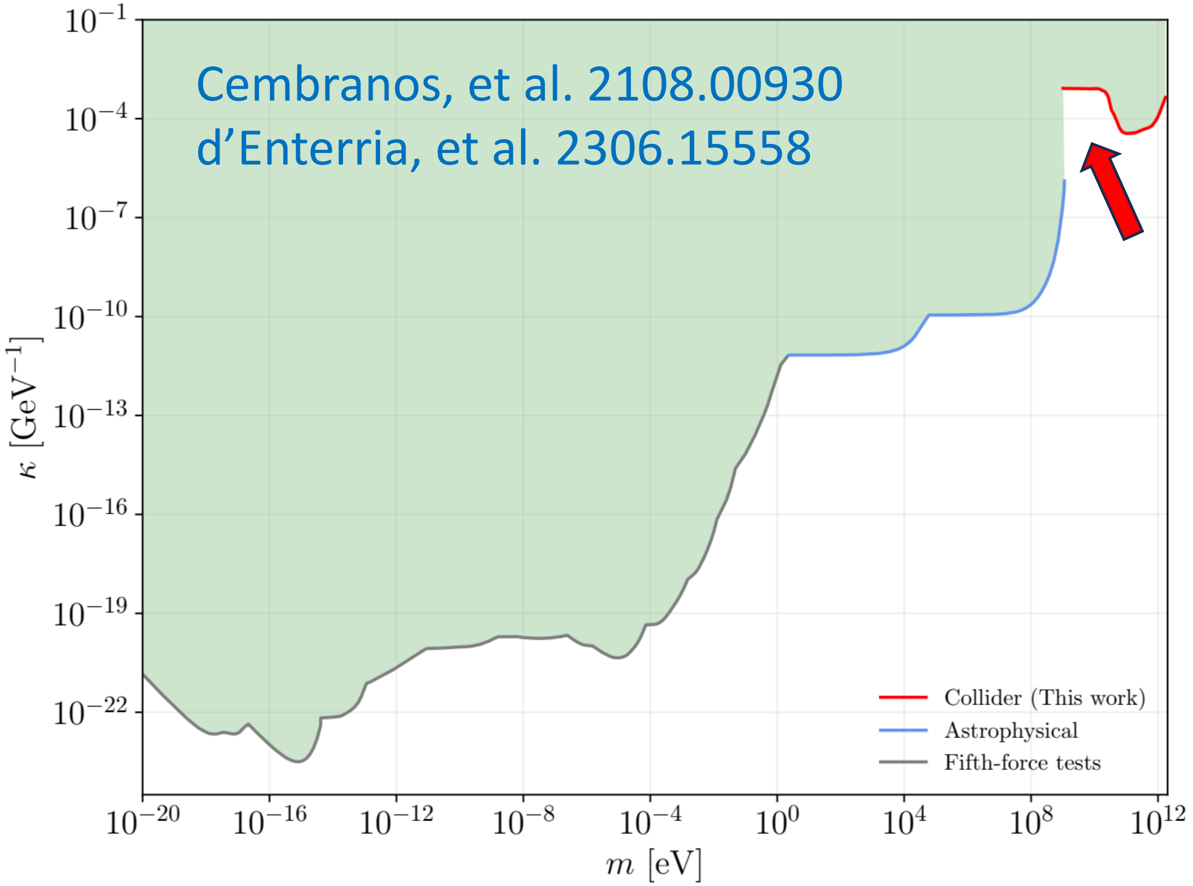
assume $\kappa \sim 1\,\mathrm{TeV}^{-1}$

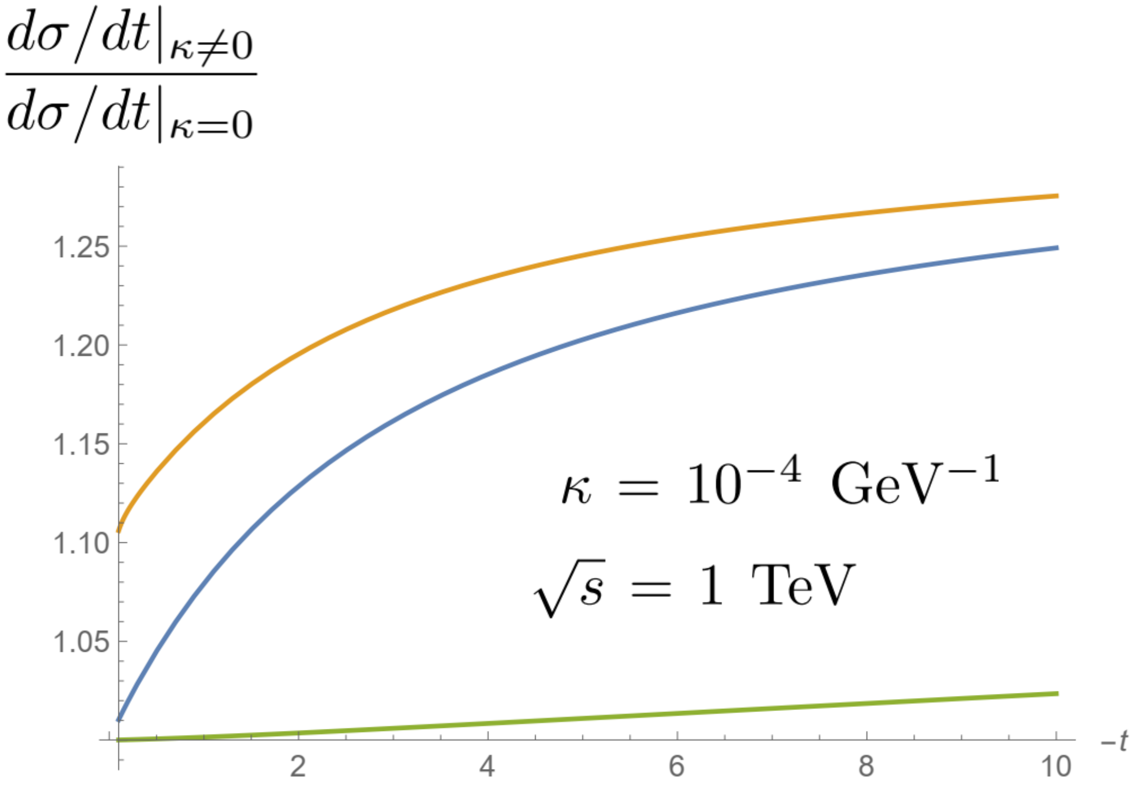


Rosenbluth

$$\frac{d\sigma}{dt} = \frac{4\pi\alpha_{em}^2}{t^2}\left\{\left(1+\frac{t-2M^2}{s}+\frac{M^4}{s^2}\right)\left(F_1^2(t)-\frac{tF_2^2(t)}{4M^2}\right)+\frac{t^2}{2s^2}(F_1(t)+F_2(t))^2\right\}$$
$$+\frac{\alpha_{em}\kappa^2 s}{t(t-m^2)}\left\{\left(1+\frac{3(t-2M^2)}{2s}\right)\left(A(t)F_1(t)-\frac{tB(t)F_2(t)}{4M^2}\right)+\mathcal{O}(s^{-2})\right\}+\mathcal{O}(\kappa^4)$$

Evading the LHC constraints





Where to look for?

MuIC : a future Muon-ion collider at BNL Acosta, Li 2107.02073

Conclusions

- EM form factors: very active field even after 70 years, aiming for 1% precision

- GFFs: just the beginning!

- Heavy meson threshold production.
  Large skewness enhances the sensitivity to GFFs.