Mechanical properties of N and π: some general conclusions, meson dominance

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Mechanical properties of hadrons, 31 March - 4 April 2025, ECT*

More details and references in
Phys. Lett. B 859 (2024) 139138 = 2405.07815 [hep-ph]
2412.00848 [hep-ph], 2503.09297 [hep-ph]

Recap of ERA's talk

Spin decomposition, pQCD, 2π threshold

N

$-tD(t) = \frac{1}{3}\left[4m_N^2\left(\frac{\Theta(t)}{m_N} - A(t)\right) - tB(t)\right]$ Raman: $-\frac{1}{4}g^{\mu\nu} \to -\frac{1}{3}[g^{\mu\nu} - q^\mu q^\nu/q^2]$

LO pQCD $-t \to \infty$: $-tD(t) \sim -\frac{\alpha(t)^2}{(-t)^2}$ Feng Yuan's talk

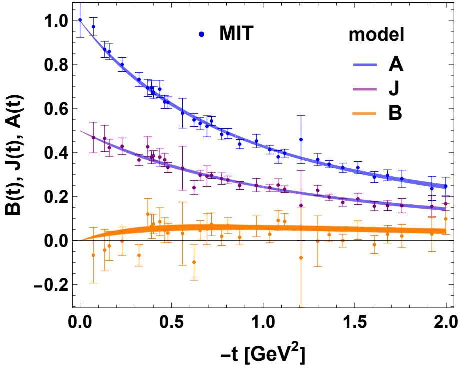
2π threshold: $\mathrm{Im}\, D(t) \sim -\sqrt{t - 4m_\pi^2}$ – S-wave dominates [Feng-Kun Guo's talk]

π

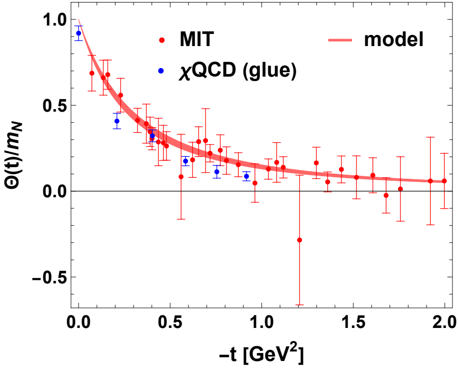
$-tD_\pi(t) = \frac{1}{3}\left[2\Theta_\pi(t) - \left(4m_\pi^2 - t\right) A_\pi(t)\right]$

LO pQCD $-t \to \infty$: $-tD(t) \sim -16\pi f_\pi^2 \alpha(t)$ – very weak

2π threshold: $\mathrm{Im}\, D_\pi(t) \sim -\sqrt{t - 4m_\pi^2}$

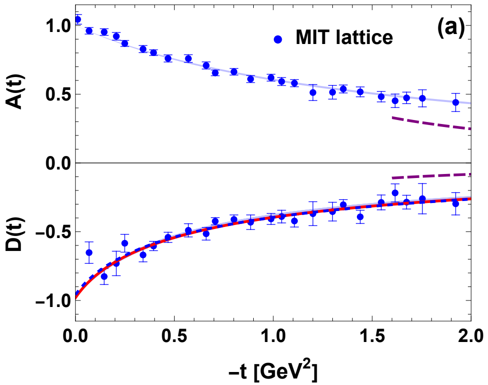


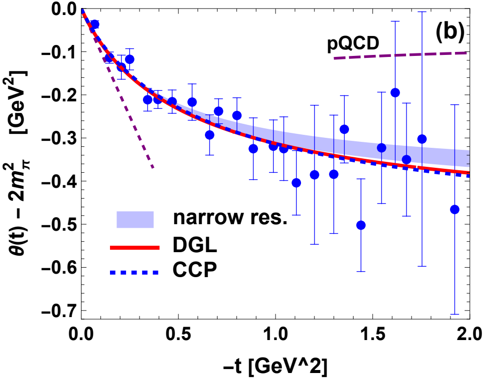
Distribution of mass



$(B = 2J - A)$ $f_2(1275)$ + 3 more f_2 states, $m_\sigma = 0.65(3)$ GeV, $f_0(975)$

Fit to the MIT lattice data – π

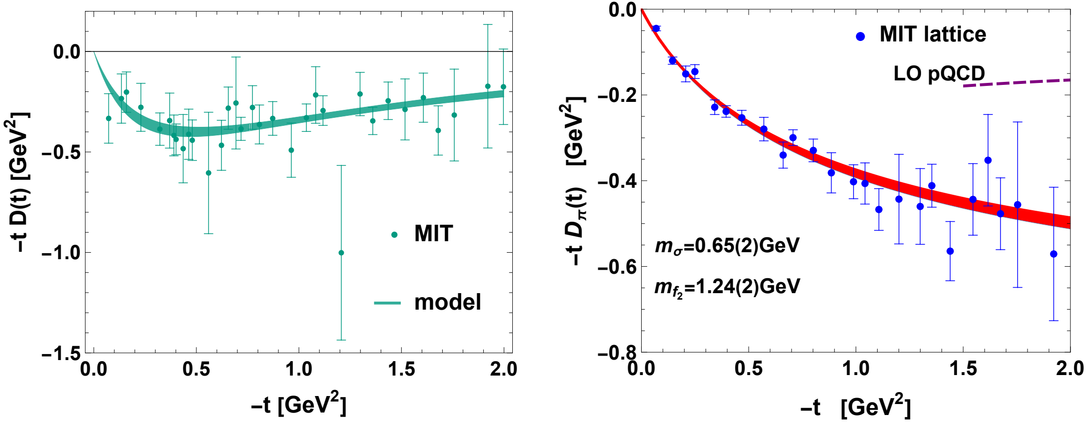




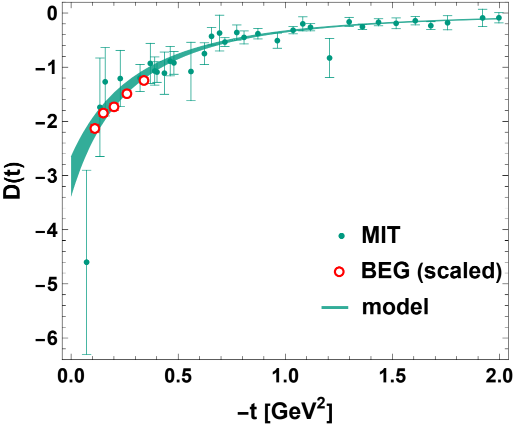
$$m_{f_2} = 1.275 \text{ MeV}, \; m_\sigma = 0.64(2) \text{ GeV}$$

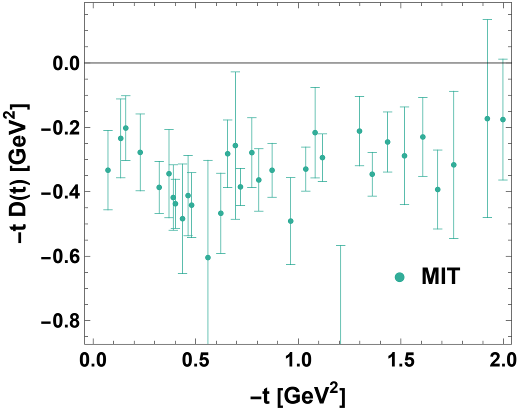
MIT data for $-tD(t)$

Recall p and s are $\sim$ Fourier-Bessel transforms of $q_\perp^2 D(-q_\perp^2)$



MIT data for N





Transverse densities

Transverse density form factor F:

$$F(b) = \int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\mathbf{b}\cdot\mathbf{q}_\perp} F(-q_\perp^2) = \frac{1}{2\pi} \int q_\perp \, dq_\perp J_0(b q_\perp) F(-q_\perp^2), \qquad F = A, B, \Theta, D$$

With the dispersion relation

$$F(-q_\perp^2) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\mathrm{Im} F(s)}{s + q_\perp^2},$$

yields [Miller, Strikman, Weiss 2011]

$$F(b) = \frac{1}{2\pi^2} \int_{4m_\pi^2}^{\infty} ds K_0(b\sqrt{s}) \, \mathrm{Im}\, F(s)$$

For the case of J we take the front form def. $s^z J(b) \equiv \langle J^z_{\mathrm{Bel}}(b) \rangle$ [Lorcé, Mantovani, Pasquini 2017]:

$$J(b) = \frac{b}{4\pi^2} \int_{4m_\pi^2}^{\infty} ds \sqrt{s} K_1(b\sqrt{s}) \, \mathrm{Im}\, J(s)$$

Nucleon

Transverse pressure and stress

[Panteleeva, Polyakov 2021]

$$T^{ij}(b) = \delta^{ij} p(b) + \left(\frac{b^i b^j}{b^2} - \frac{1}{2}\delta^{ij}\right) s(b) = \frac{1}{4m_N}\int \frac{d^2 q_\perp}{(2\pi)^2} e^{-i\mathbf{b}\cdot\mathbf{q}_\perp} (q_\perp^i q_\perp^j - \delta_{ij} q_\perp^2) D(-q_\perp^2)$$

$$p(b) = -\frac{1}{16\pi m_N}\int_0^\infty q_\perp\, dq_\perp\, J_0(q_\perp b) q_\perp^2 D(-q_\perp^2), \quad s(b) = -\frac{1}{8\pi m_N}\int_0^\infty q_\perp\, dq_\perp\, J_2(q_\perp b) q_\perp^2 D(-q_\perp^2)$$

$J_0(z) = 1 - z^2/4 + \dots \rightarrow$ near the origin $p(b) > 0$ and concave if $D(-q_\perp^2) < 0$.
Similarly, $J_2(z) = z^2/8 + \dots \rightarrow$ vanishing and convex $s(b)$ at $b = 0$

The spectral representation:

$$p(b) = \frac{1}{16\pi^2 m_N}\int_{4m_\pi^2}^\infty ds K_0(b\sqrt{s}) s \,\mathrm{Im}\, D(s), \quad s(b) = -\frac{1}{8\pi^2 m_N}\int_{4m_\pi^2}^\infty ds K_2(b\sqrt{s}) s \,\mathrm{Im}\, D(s)$$

Meson/glueball dominance model (see ERA)

$$A(t) = \frac{1 - c_A t + c_2 t^2}{(1 - t/m_{f_2}^2)(1 - t/m_{f_2'}^2)(1 - t/m_{f_2''}^2)(1 - t/m_{f_2'''}^2)}$$

$$J(t) = \frac{1 - c_J t + c_2 t^2}{2(1 - t/m_{f_2}^2)(1 - t/m_{f_2'}^2)(1 - t/m_{f_2''}^2)(1 - t/m_{f_2'''}^2)}$$

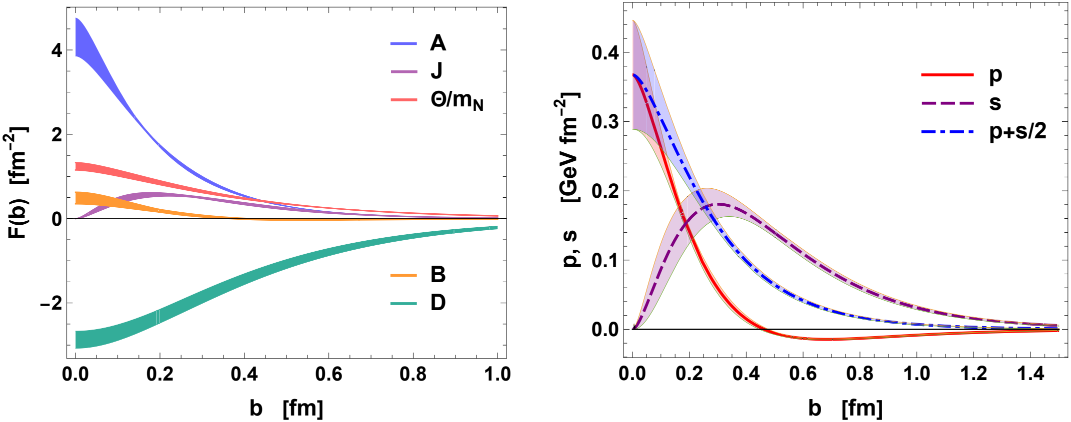
$$\to \qquad B(t) = \frac{(c_J - c_A) t}{(1 - t/m_{f_2}^2)(1 - t/m_{f_2'}^2)(1 - t/m_{f_2''}^2)(1 - t/m_{f_2'''}^2)}$$

In the scalar channel

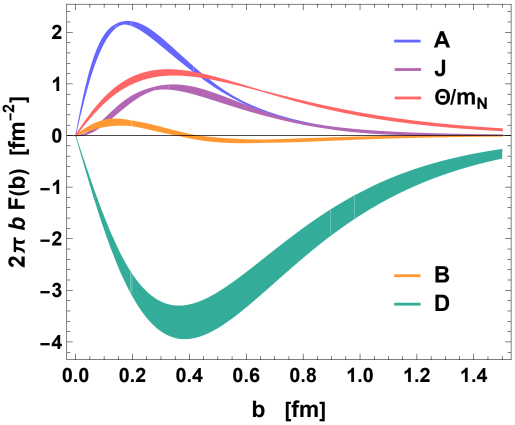
$$\Theta(t) = \frac{m_N}{(1 - t/m_\sigma^2)(1 - t/m_{f_0}^2)},$$

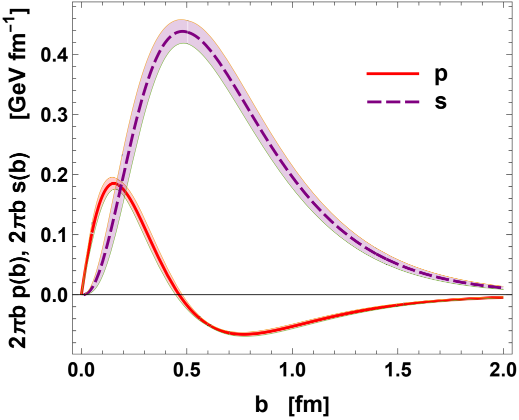
$$\begin{aligned}
&\mathrm{I:}\ m_{f_0} = 0.98, m_{f_2} = 1.275, m_{f_2'} = 1.517, m_{f_2''} = 1.936, m_{f_2'''} = 2.011[\mathrm{GeV}] \\
&\ c_A = 0.47(4)\mathrm{GeV}^{-2}, c_J = 0.69(5)\mathrm{GeV}^{-2}, c_2 = 0.10(4)\mathrm{GeV}^{-4}, m_\sigma = 0.64(4)\mathrm{GeV} \\
&\mathrm{II:}\ m_{f_0} = 0.98, m_{f_2} = 1.275, m_{f_2'} = 1.430, m_{f_2''} = 1.517, m_{f_2'''} = 1.565[\mathrm{GeV}] \\
&\ c_A = 0.83(6)\mathrm{GeV}^{-2}, c_J = 1.12(7)\mathrm{GeV}^{-2}, c_2 = 0.25(5)\mathrm{GeV}^{-4}, m_\sigma = 0.64(4)\mathrm{GeV}
\end{aligned}$$

Transverse densities and mechanical in meson dominance



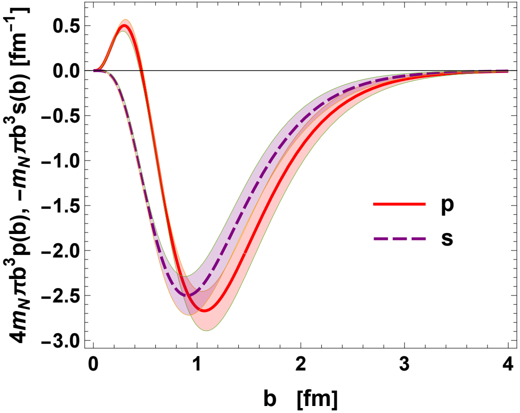
$2\pi b \times$ densities/mechanical





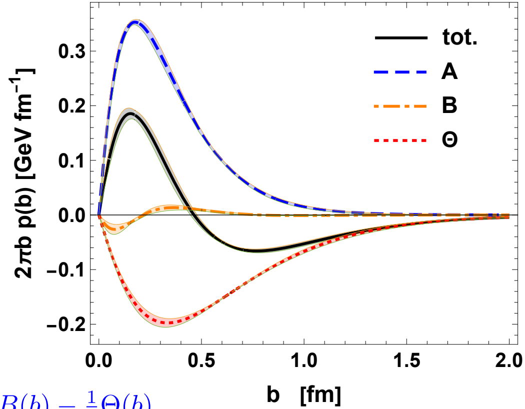
$$\int_0^\infty 2\pi b\, db\, p(b) = 0$$

$2\pi b^3 \times$ mechanical



quantities integrate to $D(0) = -3.0(4)$

Anatomy of pressure



$$p(b) = \frac{m_N}{6} A(b) + \frac{1}{24 m_N} \nabla_b^2 B(b) - \frac{1}{6}\Theta(b)$$

2^{++} repulsion in the core, 0^{++} repulsion in the tail [cf. Ji, Yang 2025, Fujii, Kawaguchi, Tanaka 2025]

In meson dominance it simply reflects the hierarchy of masses

Transverse radii

$$\langle b^2\rangle_F = \frac{\int_0^\infty 2\pi b\, b^2 F(b)}{\int_0^\infty 2\pi b\, F(b)} = \frac{4}{F(0)} \left.\frac{dF(t)}{dt}\right|_{t=0}$$

In our model

$$\langle b^2\rangle_A = 4\left(-c_A + \frac{1}{m_{f_2}^2} + \frac{1}{m_{f_2'}^2} + \frac{1}{m_{f_2''}^2} + \frac{1}{m_{f_2'''}^2}\right) = [0.34(1)\ \mathrm{fm}]^2$$

c_A approximately cancels the contribution $1/m_{f_2''}^2 + 1/m_{f_2'''}^2$

$$\langle b^2\rangle_\Theta = 4\left(\frac{1}{m_\sigma^2} + \frac{1}{m_{f_0}^2}\right) = [0.60(3)\ \mathrm{fm}]^2$$

$$\langle b^2\rangle_{\mathrm{mech}} = \frac{\int_0^\infty 2\pi b\, b^2 [p(b) + \frac{1}{2}s(b)]}{\int_0^\infty 2\pi b [p(b) + \frac{1}{2}s(b)]} = \frac{4D(0)}{\int_0^\infty d(-t) D(t)} = [0.48(3)\ \mathrm{fm}]^2$$

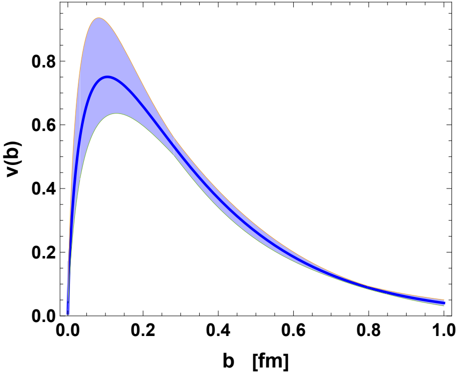
Hierarchy reflects the meson mass pattern

$$\langle b^2\rangle_A < \langle b^2\rangle_{\mathrm{mech}} < \langle b^2\rangle_\Theta$$

Rotational velocity profile

Taking the classical analogy $\mathbf{J} = \mathbf{r} \times \mathbf{p} = m\,\mathbf{r} \times \mathbf{v}$,

$$v(b) = \frac{J(b)}{b\,\Theta(b)}$$



$v(b)$ grows linearly at low b – no vortex singularity

Sum rules for mechanical properties near the origin (general)

$$F(b) = \frac{1}{2\pi^2} \int_{4m_\pi^2}^{\infty} ds K_0(b\sqrt{s}) \, \mathrm{Im}\, F(s), \qquad F = A, B, \Theta, D$$

At low b: $K_0(b\sqrt{s}) \sim -\frac{1}{2}\log(b^2 s) + \mathrm{const}$, but the superconvergence sum rule (see ERA, also [WB, ERA 2024, ERA, Sanchez Puertas, Weiss 2025]) $\int_{4m_\pi^2}^{\infty} ds\, s\, \mathrm{Im}\, F(s) = 0$ cancels the $\log b$ and constant terms $\rightarrow$

$b = 0$ sum rules

$$F(b=0) = -\frac{1}{4\pi^2} \int_{4m_\pi^2}^{\infty} ds \log s \, \mathrm{Im}\, F(s)$$

$$J(b) = -\frac{b^2}{16\pi^2} \int_{4m_\pi^2}^{\infty} ds\, s \log s \, \mathrm{Im}\, J(s) + \mathcal{O}(b^4 \log b)$$

$$p(b=0) = -\frac{1}{32\pi^2 m_N} \int_{4m_\pi^2}^{\infty} ds\, s \log s \, \mathrm{Im}\, D(s)$$

$$\left.\frac{ds(b^2)}{db^2}\right|_{b=0} = \frac{1}{64\pi^2 m_N} \int_{4m_\pi^2}^{\infty} ds\, s^2 \log s \, \mathrm{Im}\, D(s)$$

...

No singularities (for N), all parts of the spectrum contribute $\rightarrow$ uncertainty

Large-b asymptotics

Follows from the behavior at the 2π threshold [Cao, Guo, Li, Yao 2024] (ERA)

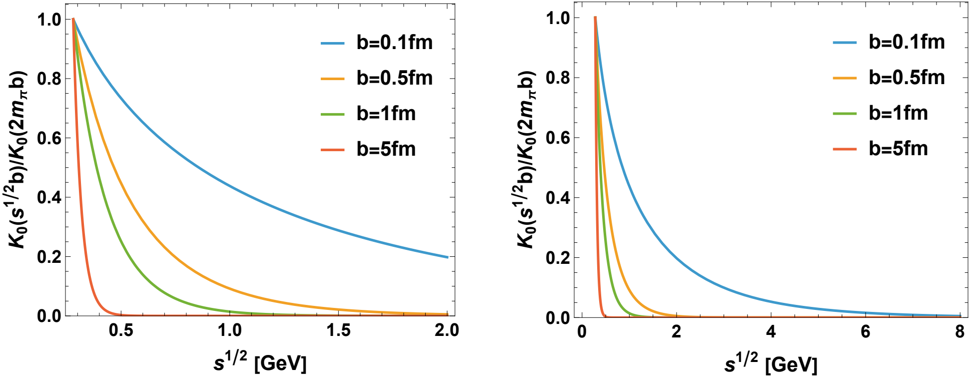
$$F(b) = \frac{1}{2\pi^2}\int_{4m_\pi^2}^{\infty} ds K_0(b\sqrt{s})\,\mathrm{Im}\,F(s) \to$$

$$\int_{4m_\pi^2}^{\infty} ds K_0(b\sqrt{s})(s-4m_\pi^2)^{J+1/2} = 2^{2J+3}\Gamma\left(J+\frac{3}{2}\right)\left(\frac{b}{m_\pi}\right)^{-J-\frac{3}{2}} K_{J+\frac{3}{2}}(2bm_\pi) \sim \frac{1}{b^{J+2}}e^{-2m_\pi b}$$

valid for $b \gg 1/m_\pi$

$$\begin{aligned} A(b), B(b) &\sim +e^{-2m_\pi b}/b^4 \\ J(b) &\sim +e^{-2m_\pi b}/b^3 \\ \Theta(b), -p(b), s(b) &\sim +e^{-2m_\pi b}/b^2 \end{aligned}$$

How close to the origin can we go?



e.g., for $b = 1$ fm, we need the spectral density up to $s^{1/2} \simeq 1.1$ GeV
(cf. a detailed analysis for the EM case in [ERA, Sanchez-Puertas, Weiss 2025])

$2D \leftrightarrow 3D$

For N – Abel transform [Panteleeva, Polyakov 2021, Freese, Miller 2021], for π 3D makes no sense

Radii hierarchy

$$\langle r^2\rangle_A^{1/2} < \langle r^2\rangle_J^{1/2} < \langle r^2\rangle_E^{1/2} < \langle r^2\rangle_{\rm mech}^{1/2} < \langle r^2\rangle_\Theta^{1/2}$$
$$0.51(1) < 0.57(3) < 0.67(2) < 0.72(5) < 0.90(4)[\text{fm}]$$

$b = 0$ sum rules

$$F(r=0) = -\frac{1}{4\pi^2}\int_{4m_\pi^2}^{\infty} ds\sqrt{s}\,\text{Im}\,F(s)$$

$$J^{\rm mon}(r) = -\frac{2}{3}J^{\rm quad}(r) = \frac{r^2}{36\pi^2}\int_{4m_\pi^2}^{\infty} ds\, s^{3/2}\text{Im}\,J(s) + \mathcal{O}(r^3) \text{ [Lorce et al. 2017]}$$

$$p(r=0) = -\frac{1}{24\pi^2 m_N}\int_{4m_\pi^2}^{\infty} ds\, s^{3/2}\,\text{Im}\,D(s)$$

$$\left.\frac{ds(r)}{dr^2}\right|_{r=0} = \frac{1}{240\pi^2 m_N}\int_{4m_\pi^2}^{\infty} ds\, s^{5/2}\,\text{Im}\,D(s)$$

$\ldots$

Large-r asymptotics

$$F(r) = \frac{1}{4\pi^2}\int_{4m_\pi^2}^{\infty} ds \frac{e^{-b\sqrt{s}}}{r}\,\mathrm{Im}\,F(s) \to$$

$$\int_{4m_\pi^2}^{\infty} ds \frac{e^{-b\sqrt{s}}}{r}(s-4m_\pi^2)^{J+1/2} = \frac{1}{\sqrt{\pi}} 2^{2J+4}\Gamma\left(J+\frac{3}{2}\right)\left(\frac{m}{r}\right)^{J+2} K_{J+2}(2mr) \sim \frac{1}{r^{J+5/2}} e^{-2m_\pi r}$$

valid for $r \gg 1/m_\pi$

$$\begin{aligned} A(r), B(r) &\sim +e^{-2m_\pi r}/r^{9/2} \\ J(r) &\sim +e^{-2m_\pi r}/r^{7/2} \\ \Theta(r), -p(r), s(r) &\sim +e^{-2m_\pi r}/r^{5/2} \end{aligned}$$

Different from $1/r^6$ in the Chiral Soliton [Bochum 2007] or Skyrmion [Cebulla et al. 2007] at $m_\pi = 0$, or [Alharazin, Djukanovic, Gegelia, Polyakov 2020]

Pion

Limits of transverse densities

Details to appear in Memorial Vol. of APPB for D. Diakonov, V. Petrov, and M. Polyakov
2412.00848 [hep-ph]

pQCD: singularities at $b = 0$

$$A(-Q^2) \simeq \frac{48\pi f_\pi^2 \alpha(-Q^2)}{Q^2} = \frac{192\pi^2 f_\pi^2}{\beta_0 Q^2 \ln(Q^2/\Lambda^2)} \to A(b) \simeq \frac{49\pi f_\pi^2}{\beta_0} \ln\ln\left(\frac{1}{b^2\Lambda^2}\right) \quad \text{[cf. Miller 2009]}$$

$$\Theta(-Q^2) \simeq -4\beta_0 \alpha(-Q^2)^2 f_\pi^2 = -\frac{64\pi^2 f^2}{\beta_0 \ln^2(Q^2/\Lambda^2)} \to \Theta(b) \simeq -\frac{128\pi f_\pi^2}{\beta_0} \frac{1}{b^2 \ln^3\left(\frac{1}{b^2\Lambda^2}\right)}$$

2π: $b \to \infty$

$$A(b) = m_\pi^2 |A(s=4m_\pi^2)| \frac{5e^{-2bm_\pi}}{8\pi(bm_\pi)^4} \left(\begin{cases} 0.00175(3) \\ 0.00178(3) \end{cases} + \mathcal{O}[(bm_\pi)^{-2}] \right)$$

$$\Theta(b) = m_\pi^2 |\Theta(s=4m_\pi^2)| \frac{e^{-2bm_\pi}}{2\pi(bm_\pi)^2} \left(\begin{cases} 0.220(5) \\ 0.220(8) \end{cases} + \mathcal{O}[(bm_\pi)^{-2}] \right)$$

$\Theta(b)$ must change sign, $A(b)$ positive definite

Spectral modeling

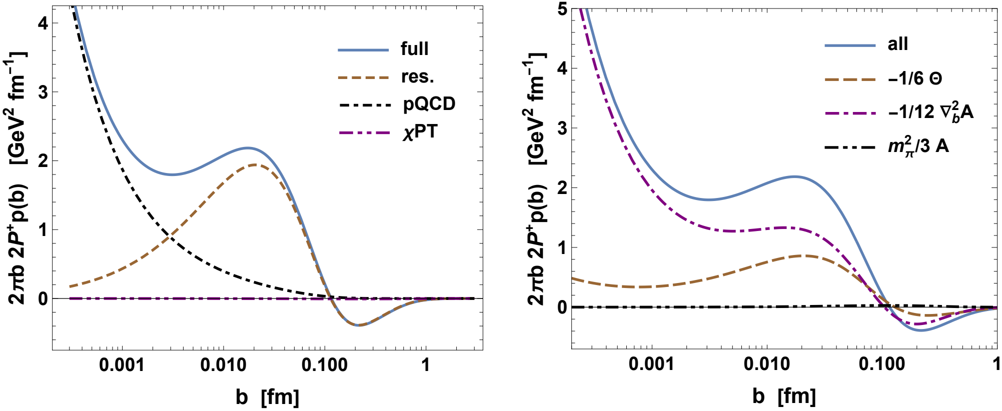
Spectral modeling of $\mathrm{Im}\, A(s)$ and $\mathrm{Im}\, \Theta(s)$:

$$\mathrm{Im}\, f(s) = \mathrm{Im}\, f_\chi(s)\theta(\Lambda_\chi^2 - s) + \mathrm{Im}\, f_R(s) + \mathrm{Im}\, f_p(s)\theta(s - \Lambda_p^2)$$

Two resonances per channel: $\mathrm{Im}\, f_R(s) = c_1\delta(s - M_1^2) + c_2\delta(s - M_2^2)$

Negative strength of the second resonance needed to satisfy the superconvergence sum rules $\int ds\, \mathrm{Im}\, A(s) = 0$ and $\int ds\, \mathrm{Im}\, \Theta(s) = 0$

Pressure



$(M_2 = 5\ \text{GeV})$

Summary tables

Pion

quantity	low limit		intermediate range	high limit	
$\mathrm{Im}\, A(s)$	$+$	2π	changes sign	$-$	pQCD
$\mathrm{Im}\, D(s)$	$-$		changes sign	$+$	
$\mathrm{Im}\, \Theta(s)$	$+$		changes sign	$-$	
$A(-Q^2)$	1	sym.		$+$	pQCD
$D(-Q^2)$	$-1 + \mathcal{O}(m_\pi^2)$			$-$	
$\Theta(-Q^2)$	$2m_\pi^2$		changes sign	$-$	
$A(b)$	$+\infty$	pQCD	positive definite	$+$	2π
$\Theta(b)$	$-\infty$		changes sign	$+$	
$p(b)$	$+\infty$		changes sign	$-$	

Nucleon

quantity	low limit		intermediate range	high limit	
$\mathrm{Im}\,A(s)$	$+$	2π	changes sign	$+$	pQCD
$\mathrm{Im}\,J(s)$	$+$		changes sign	$+$	
$\mathrm{Im}\,B(s)$	$+$		changes sign	$+$	
$\mathrm{Im}\,D(s)$	$-$		changes sign	$+$	
$\mathrm{Im}\,\Theta(s)$	$+$		changes sign	$-$	
$A(-Q^2)$	1	sym.		$+$	pQCD
$J(-Q^2)$	$\frac{1}{2}$			$+$	
$B(-Q^2)$	0			$-$	
$D(-Q^2)$				$-$	
$\Theta(-Q^2)$	m_N		changes sign	$-$	
$A(b)$	$+$		positive definite	$+$	2π
$\Theta(b)$				$+$	
$p(b)$			changes sign	$-$	