Dispersion relations beyond the lowest order approximation: D-term and hadronic properties

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Outline

- Overview: GPDs, EMT, the *D*-term and our goals.
- Processes of interest.
- CFFs at NLO and kinematic twist-4.
- Analyticity of the scattering amplitude.
- Dispersion relation beyond Born and Bjorken approx.
- Subtraction constant and double distributions.
- *D*-term extraction at LO and NLO.
- Take aways.

Overview

Generalized Parton Distributions

GPD

Generalized Parton Distribution ≈ "3D version of a PDF (Parton Distribution Function)." With x the average fraction of the hadron's longitudinal momentum carried by a quark:

$$H_f(x,\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix\bar{p}^+z^-} \langle p'|\bar{\mathfrak{q}}_f(-z/2)\gamma^+\mathcal{W}[-z/2,z/2]\mathfrak{q}_f(z/2)|p\rangle\Big|_{z_\perp=z^+=0}$$

$$t=\Delta^2=(p'-p)^2, \quad \xi=-\frac{\Delta n}{2\bar{p}n}, \quad \rho=\xi\frac{qq'}{\Delta q'}, \quad \bar{p}=\frac{p+p'}{2}$$

Importance

- Connected to **QCD energy-momentum tensor**. GPDs are a way to study "mechanical" properties and to address the hadron's spin puzzle (*X. Ji's sum rule*[*]).
- **Tomography:**[$] distribution of quarks in terms of the longitudinal momentum and in the transverse plane.

$$f(x,\vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{4\pi^2}\, e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp} H_f(x,0,\vec{\Delta}_\perp^2)$$

[*] PRD 55 (1997) 7114-7125; [$] Burkardt, Int. J. Mod. Phys. A 21 (2006) 926-929.

QCD energy-momentum tensor (EMT), $\Theta^{\mu\nu}$

$\Theta^{\mu\nu}$ parameterization $\to$ gravitational form factors (GFFs):

$$\langle p', s'|\Theta_a^{\mu\nu}(0)|p, s\rangle =$$

$$= \bar{u}(p', s')\Bigg\{ \frac{\bar{p}^\mu \bar{p}^\nu}{M} A_a(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} C_a(t) + M\eta^{\mu\nu}\bar{C}_a(t) + \frac{\bar{p}^{\{\mu} i\sigma^{\nu\}\rho}\Delta_\rho}{4M}\left[A_a(t) + B_a(t)\right] + \frac{\bar{p}^{[\mu} i\sigma^{\nu]\rho}\Delta_\rho}{4M} D_a(t) \Bigg\} u(p, s)\,.$$

a = quarks and gluons.

GPDs and GFFs:

$$\int dx\, x\, H_f(x, \xi, t) = A_f(t) + 4\xi^2 C_f(t)\,, \quad \int dx\, H_g(x, \xi, t) = A_g(t) + 4\xi^2 C_g(t)\,,$$

$$\int dx\, x\, E_f(x, \xi, t) = B_f(t) - 4\xi^2 C_f(t)\,, \quad \int dx\, E_g(x, \xi, t) = B_g(t) - 4\xi^2 C_g(t)\,.$$

Ji's sum rules: $J_{f,\,g} = \dfrac{A_{f,\,g}(0) + B_{f,\,g}(0)}{2} = \dfrac{1}{2}\displaystyle\int dx\, x^{\mathscr{P}_{f,\,g}} \left[H_{f,\,g}(x, \xi, 0) + E_{f,\,g}(x, \xi, 0)\right]\,,$

$\mathscr{P}_{f,\,g} = 1$, quarks; 0, gluons.

QCD energy-momentum tensor (EMT), $\Theta^{\mu\nu}$

With the double distribution parameterization of GPDs

$$H_a(x,\xi,t) = \iint_{\mathbb{D}} d\beta d\alpha\ \delta(x-\beta-\alpha\xi)\left[\beta^{\mathscr{P}_a} F_a(\beta,\alpha,t) + \xi^{1+\mathscr{P}_a} D_a(\alpha,t)\delta(\beta)\right] ,$$

and the 1st moments of GPDs seen before:

$$C_a(t) = \frac{1}{4}\int_{-1}^{1} d\alpha\ \alpha^{1-\mathscr{P}_a} D_a(\alpha,t) .$$

C_a is related to the pressure inside the hadron:[‡, ‡‡]

$$p_a(r) = M \int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3}\, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \left\{-\bar{C}_a(t) + \frac{2}{3}\frac{t}{M^2}\, C_a(t)\right\} .$$

$\boldsymbol{D_a}$-term $\mapsto$ GFF[§] $\boldsymbol{C_a}$ $\mapsto$ **pressure** p_a .

Our goal: Extraction of the *D*-term.

[‡]Polyakov, Schweitzer, Int. J. Mod. Phys. A33(26), 1830025 (2018).

[‡‡]Dutrieux, Lorcé, Moutarde, Sznajder, Trawiński, Wagner, EPJC 81 (2021) 4, 300.

$\sum_a \bar{C}_a = 0$. $\bar{C}_a$ is related to higher-twist distributions: Leader, Lorcé, Phys. Rept. 541(3), 163 (2014); Leader, PLB 720, 120 (2013) [Erratum: PLB 726, 927–927 (2013)]; Tanaka, PRD 98(3), 034009 (2018).

[§]**See today's talks by C. Weiss, F. Yuan, Y. Hatta. Wed's talk on the interpretation of GFFs by X. Ji.**

D-term extraction

D_a-term $\mapsto$ GFF C_a $\mapsto$ **pressure** p_a .

2 ways for extracting the D-term:

1. Scattering amplitude: $\mathcal{H}(\xi, t) = \int \frac{dx}{\xi}\, C(x/\xi) H(x, \xi, t) \Rightarrow$ inversion problem $(x, \xi, t) \to (\xi, t)$:

   $H(x, \xi, t) \mapsto$ D_a-term $\mapsto$ GFF C_a $\mapsto$ **pressure** p_a .

2. Scattering amplitude + dispersion relation $\Rightarrow$ inversion problem $(\alpha, t) \to t$ (simpler):

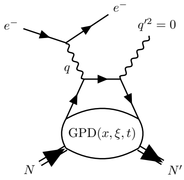
$$\mathcal{H}(\xi, t) = \int \frac{dx}{\xi}\, C(x/\xi) H(x, \xi, t)\,, \quad \& \quad \int d\alpha\, D(\alpha, t) \underset{\mathrm{LT}}{\sim} h_0^{++}(t) \sim \mathrm{Re}\,(\mathcal{H}) + \int \frac{\mathrm{Im}\,(\mathcal{H})}{\xi - \xi'}\,.$$

Our goal: Extraction of the D-term with the least impact from deconvolution $\Leftrightarrow$ 2nd via above.

Golden channels for GPD analysis

Deeply virtual Compton scattering (DVCS)

- In the 1990s, Müller et al.,† Ji* and Radyushkin# introduced GPDs. First process to study is DVCS:



- At LO ($O(\alpha_s^0)$) and LT ($\Lambda/Q^2 \to 0$, $\Lambda \in \{|t|, M^2\}$):

$$\mathcal{H}_{\rm DVCS} = -{\rm PV}\left(\int_{-1}^{1} dx \frac{1}{x-\xi} H^{(+)}(x,\xi,t)\right) + \int_{-1}^{1} dx \; i\pi\delta(x-\xi) H^{(+)}(x,\xi,t)\,,$$

$$H^{(+)}(x,\xi,t) = H(x,\xi,t) - H(-x,\xi,t)\,.$$

- $\xi = \frac{-n\Delta}{2\bar{p}n}\,,\quad \bar{p} = \frac{p+p'}{2}\,,\quad \Delta = p' - p\,,\quad t = \Delta^2\,.$

† Fortsch. Phys. 42 (1994) 101-141. * PRD 55 (1997) 7114-7125. # PLB 449 (1999) 81-88.

Higher-twist corrections to DVCS off proton: Braun et al., PRD 89 (2014) 7, 074022; & arXiv:2501.08185 (2025).

See Thursday's talk by M. Čuić on extraction of CFFs from DVCS data.

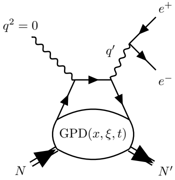
Timelike Compton scattering (TCS)

- Complementary to DVCS.

LT: Berger, Diehl and Pire, EPJC 23, 675–689 (2002).

Higher twists: VMF, Pire, Sznajder & Wagner, arXiv:2503.02461 (2025)
$\hookrightarrow$ **TCS as $q^2 \to 0$ limit of DDVCS off spin-0 target.**

1st measurement of TCS off proton: P. Chatagnon et al., PRL **127**, 262501 (2021).

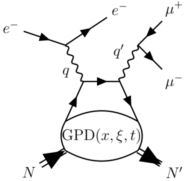


$$\mathcal{H}_{\rm TCS} = -{\rm PV}\left(\int_{-1}^{1} dx \frac{1}{x+\xi} H^{(+)}(x,\xi,t)\right) + \int_{-1}^{1} dx \;\; i\pi\delta(x+\xi) H^{(+)}(x,\xi,t)\,.$$

- Like DVCS but $\xi \to -\xi$.

Double deeply virtual Compton scattering (*D*DVCS)

- **DDVCS vs DVCS/TCS:** extra virtuality $\Rightarrow$ *generalized* Björken variable $\rho \Rightarrow$ GPDs for $x = \rho \neq \xi$.



- At LO LT:

$$\mathcal{H}_{\rm DDVCS} = -{\rm PV}\left(\int_{-1}^{1} dx \frac{1}{x-\rho} H^{(+)}(x,\xi,t)\right) + \int_{-1}^{1} dx \;\; i\pi\delta(x-\rho) H^{(+)}(x,\xi,t)\,.$$

$$\xi = -\frac{\Delta n}{2\bar{p}n}, \quad \rho = \xi \frac{Q^2 - Q'^2}{Q^2 + Q'^2}\,, \qquad \rho = \xi \text{ (DVCS)}\,, \quad \rho = -\xi \text{ (TCS, LT approach)}\,.$$

LO + LT: Belitsky & Müller, PRL 90, 022001 and PRD 68, 116005 (2003); Guidal & Vanderhaeghen, PRL 90, 012001 (2003); Deja, VMF, Pire, Sznajder & Wagner, PRD 107, 094035 (2023); Alvarado, Hoballah & Voutier, arXiv:2502.02346 (2025).

Higher twists: VMF, Pire, Sznajder & Wagner, arXiv:2503.02461 (2025).

Kinematic higher-twist corrections

- LT: Q^2, $Q'^2 \to \infty$, Bjorken limit.
- Kinematic power corrections = kinematic higher-twist corrections:

$$\text{kin. power corrections} = O\left(\left(\frac{|t|}{Q^2}\right)^P, \left(\frac{M^2}{Q^2}\right)^P\right),$$

$$P = \frac{\tau_{\text{kin}} - 2}{2}.$$

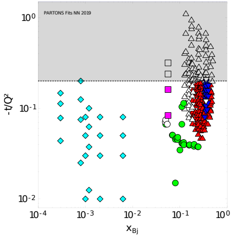
- Similarly for Q'^2.
- LT = kinematic leading twist = kinematic twist-2, $\tau_{\text{kin}} = 2$.
- We employ the conformal techniques by Braun, Ji & Manashov in JHEP 03 (2021) 051, and JHEP 01 (2023) 078.

Why to go beyond leading twist?

1. Nucleon tomography is a Fourier transform in $\Delta_\perp = (p' - p)_\perp$ that requires data on a sizable range of t:

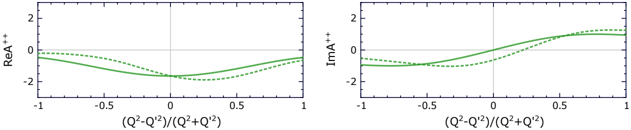
$$f(x, \vec{b}_\perp) = \int d^2\vec{\Delta}_\perp \; e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp} H_f(x, 0, t = -\vec{\Delta}_\perp^2)/(4\pi^2)\,.$$

2. Increase the range of useful experimental data:



Data: Hall A (▼, ▽), CLAS (▲, △), HERMES (●, ○), COMPASS (■, □) and HERA H1 and ZEUS (◆, ◇). The gray bands (open markers) indicate phase-space areas (experimental points) being excluded in the analysis of Moutarde, Sznajder, Wagner, EPJC 79, 614 (2019).

3. Universality tests $\rightarrow \mathcal{A}^{++}_{\mathrm{DVCS}} \overset{\mathrm{LO,\,HT}}{\neq} \left(\mathcal{A}^{++}_{\mathrm{TCS}}\right)^*$:



$\xi = 0.2$, $\mathbb{Q}^2 = 1.9\ \mathrm{GeV}^2$, $t = -0.6\ \mathrm{GeV}^2$.
π-GPD model: J. M. Morgado-Chávez et al., PRD 105, 094012 (2022). **Solid line: LT.** **Dashed line: LT + tw-4.**

Plot from arXiv:2503.02461.

DVCS' $\mathcal{H}^{++}$ = LT + tw-4 + O(tw-6), at LO

CFF for in- and out-photons of equal transverse helicity (+):*,**

$$\mathcal{A}^{++} = \mathcal{H}^{++} = \int_{-1}^{1} dx\, \frac{1}{\xi}\Bigg\{ -\left(1 - \frac{1}{2}\frac{t}{\mathbb{Q}^2}\right)\frac{\xi}{x-\xi+i0} H^{(+)}$$

$$- 2\frac{t}{\mathbb{Q}^2}\left[\frac{\xi}{x+\xi}\ln\left(\frac{x-\xi+i0}{-2\xi+i0}\right) + \frac{\xi L_{\rm DVCS}}{4}\right] H^{(+)}$$

$$+ \frac{t}{\mathbb{Q}^2}\xi\partial_\xi\left[\left(\frac{\xi}{x+\xi}\ln\left(\frac{x-\xi+i0}{-2\xi+i0}\right) + \frac{\xi L_{\rm DVCS}}{2}\right) H^{(+)}\right]$$

$$- \frac{\xi^2\bar{p}_\perp^2}{\mathbb{Q}^2} 2\xi^2\partial_\xi^2\left[\left(\frac{\xi}{x+\xi}\ln\left(\frac{x-\xi+i0}{-2\xi+i0}\right) + \frac{\xi L_{\rm DVCS}}{2}\right) H^{(+)}\right]\Bigg\}$$

$$+ O(\text{tw-6})\,,$$

$$\xi L_{\rm DVCS} = \frac{4\xi}{x-\xi}\left[\mathrm{Li}_2\left(\frac{x+\xi}{2\xi-i0}\right) - \mathrm{Li}_2(1)\right]\,, \quad \text{scale of DVCS} \to \mathbb{Q}^2 = Q^2 + t\,.$$

*Directly in **DVCS:** Braun, Manashov & Pirnay, PRD 86 (2012) 014003; and Braun, Ji & Manashov, JHEP 01 (2023) 078.

**Beginning of this March, $\mathcal{H}^{++}$ as the $q'^2 \to 0$ limit of **DDVCS** in **arXiv:2503.02461** (VMF, Pire, Sznajder & Wagner).

LT NLO coefficient functions

$$T_{\rm LT}^{++}(x/\xi) = \left[C_0(x/\xi) + C_1(x/\xi) + \ln\left(\frac{Q^2}{\mu_{\rm F}^2}\right) C_{\rm coll}(x/\xi)\right] - (x/\xi \to -x/\xi)\,,$$

$$C_0(x/\xi) = -\frac{\xi}{x+\xi-i0}\,,$$

$$C_1(x/\xi) = \frac{\alpha_S C_F}{4\pi}\frac{\xi}{x+\xi-i0}\left[9 - 3\frac{x+\xi}{x-\xi}\ln\left(\frac{x+\xi}{2\xi}-i0\right) - \ln^2\left(\frac{x+\xi}{2\xi}-i0\right)\right],$$

$$C_{\rm coll}(x/\xi) = \frac{\alpha_S C_F}{4\pi}\frac{\xi}{x+\xi-i0}\left[-3 - 2\ln\left(\frac{x+\xi}{2\xi}-i0\right)\right],$$

$+$ gluon terms.

NLO and higher-twist coefficient functions:

$C_i \to \mathrm{Im}(C_i) \propto \theta(x-\xi)$, $\delta(x-\xi)$, accessing therefore:

$$\begin{cases} \text{DGLAP:} & |x| > |\xi| \;\checkmark \\ \text{ERBL:} & |x| < |\xi| \;\times \end{cases}$$

Belitsky, Müller, PLB 417 (1998) 129; Ji, Osborne, PRD 58 (1998) 094018; Ji, Osborne, PRD 57 (1998) 1337; Mankiewicz, Piller, Stein, Vänttinen, Weigl, PLB 425 (1998) 186; Pire, Szymanowski & Wagner, PRD 83 (2011) 034009.

Dispersion relation

Analyticity of the scattering amplitude

Following procedure in Dutrieux, Meisgny, Mezrag & Moutarde, EPJC 85 (2025) 1, 105:

Fixed t + momentum conservation: amplitude $= \mathcal{F}_s(s)$.

Causality $\Rightarrow$ extension to the upper-half of the complex plane: $\mathcal{F}_s(s) \to \mathcal{F}_s(s + i0)$.

$$s + u = -\mathbb{Q}^2 \left[1 - \frac{2M^2}{\mathbb{Q}^2} \right] .$$

1. $s + u < 0$ if $M^2/\mathbb{Q}^2 < 1/2 \Rightarrow$
   2. $\exists$ region for both $s, u < 0 \Rightarrow$
      3. no particle production & $\mathrm{Im}(\mathcal{F}) = 0$ **(optical theorem)** $\Rightarrow$
         4. analytic continuation to the lower-half of the complex plane:

            Schwartz's reflexion principle.

Extension to the lower-half part of the complex plane

Principle (Schwartz's reflexion)

Let $f(z) : A \subset \mathbb{C} \to \mathbb{C}$ be a function analytic in A which intersects the real axis in an interval $I = A \cap \mathbb{R} \neq \emptyset$ and such that $f(z)$ is real in I. Then,

$$f^*(z) = f(z^*) \quad \forall z \in A\,.$$

For our amplitude:

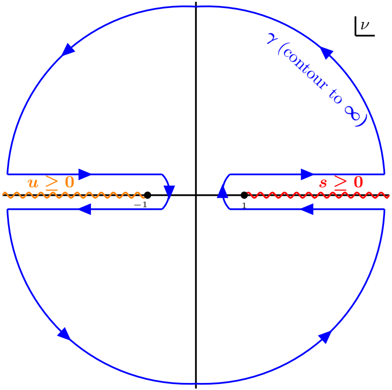
$$\mathcal{F}_s(s - i0) = \mathcal{F}_s^*(s + i0)\,.$$

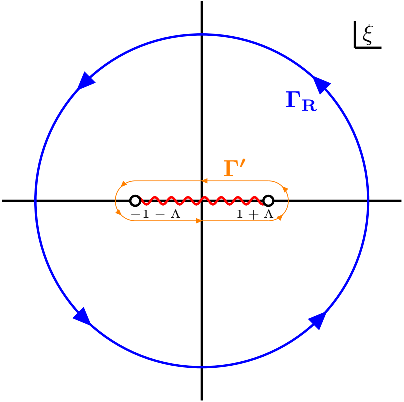
Change of variables:

$$\begin{cases} s \to \nu = \dfrac{s-u}{-(s+u)} = \dfrac{1}{\xi} \cdot \dfrac{1}{1 - 2M^2/\mathbb{Q}^2}\,, \\ \\ \mathcal{F}_s(s) \to \mathcal{F}(\nu)\,. \end{cases}$$

Domain of (granted) analyticity in ν and ξ

$$\nu \simeq 1/\xi + \Lambda \,, \qquad \Lambda = 2M^2/\mathbb{Q}^2$$





With the path γ depicted in ν's complex plane, one reproduces the dispersion relation introduced in:

Diehl & Ivanov, EPJC 52 (2007) 919-932.

Towards the dispersion relation (DR)

Incoming-photon helicity: A
Outgoing-photon helicity: B

Analyticity $\Rightarrow \mathcal{H}^{AB}(\xi) = \sum_{j=0}^{\infty} h_j^{AB} \frac{1}{\xi^j}$,

$\boldsymbol{h_j^{AB}} \in \mathbb{R}$ (Schwartz) **.**

Integration over $\boldsymbol{\Gamma_{\mathrm{R}}}$ and $\boldsymbol{\Gamma'}$:

1) $$\oint_{\Gamma_{\mathrm{R}}} d\xi' \, \frac{\mathcal{H}^{AB}(\xi')}{\xi' - \xi} \left(\frac{\xi'}{\xi}\right)^n = 2\pi i \sum_{j=0}^{n} h_j^{AB} \frac{1}{\xi^j}$$

2) $$\oint_{\Gamma'} d\xi' \, \frac{\mathcal{H}^{AB}(\xi')}{\xi' - \xi} \left(\frac{\xi'}{\xi}\right)^n =$$

$$= \mathrm{PV} \int_{-(1+\Lambda)}^{1+\Lambda} d\xi' \, \frac{\mathcal{H}^{AB}(\xi' - i0) - \mathcal{H}^{AB}(\xi' + i0)}{\xi' - \xi} \left(\frac{\xi'}{\xi}\right)^n$$

$$+ i\pi \left[\mathcal{H}^{AB}(\xi - i0) + \mathcal{H}^{AB}(\xi + i0)\right]$$

Schwartz + homotopy of Γ_{R} and Γ' = **dispersion relation (DR):**

$$\sum_{j=0}^{n} h_j^{AB} \frac{1}{\xi^j} = \frac{1}{\pi} \mathrm{PV} \int_{-(1+\boldsymbol{\Lambda})}^{1+\boldsymbol{\Lambda}} d\xi' \, \frac{\mathrm{Im}(\mathcal{H}^{AB}(\xi'))}{\xi' - \xi} \left(\frac{\xi'}{\xi}\right)^n + \mathrm{Re}(\mathcal{H}^{AB}(\xi))$$

DGLAP and $\mathrm{Im}(\mathcal{H}^{AB}(\xi'))$

$|\xi| > 1 \Rightarrow$ GPDs on $x \in (-\xi, \xi) \Rightarrow \underbrace{\mathrm{Im}(\mathcal{H}^{++}(\xi))}_{\text{DGLAP region } |x|>|\xi|} = 0$ for $|\xi| > 1$:

$$\sum_{j=0}^{n} h_j^{AB} \frac{1}{\xi^j} = \frac{1}{\pi} \mathrm{PV} \int_{-1}^{1} d\xi' \, \frac{\mathrm{Im}(\mathcal{H}^{AB}(\xi'))}{\xi' - \xi} \left(\frac{\xi'}{\xi}\right)^n + \mathrm{Re}(\mathcal{H}^{AB}(\xi))$$

Same result as in Dutrieux, Meisgny, Mezrag & Moutarde, EPJC 85 (2025) 85:105 → all orders in pQCD but LT: **we proved its validity including kinematic twist corrections.**

Subtraction constant and DDs

$$\sum_{j=0}^{n} h_j^{AB} \frac{1}{\xi^j} \Rightarrow \boldsymbol{h_0^{++}} \textbf{ = subtraction constant of the DR}\,.$$

With quark GPD

$$H(x,\xi,t) = \iint_{\mathbb{D}} d\beta d\alpha\ \delta(x-\beta-\alpha\xi)\left[F(\beta,\alpha,t)+\xi D(\alpha,t)\delta(\beta)\right]\,,$$

we get:

Subtraction constant beyond Born and Bjorken approximations (PRELIMINARY)

$$h_0^{++} = \int_{-1}^{1} d\alpha\ \left[\mathrm{Re}\left(T_0^{++}(\alpha)\right) + \frac{t}{\mathbb{Q}^2}\mathrm{Re}\left(T_1^{++}(\alpha)\right)\right] \boldsymbol{D(\alpha)}$$
$$- 4\frac{M^2 - t/4}{\mathbb{Q}^2} \iint_{\mathbb{D}} d\beta d\alpha\ \boldsymbol{F(\beta,\alpha)}\beta\,\mathrm{Re}\left(T_1^{++\,(1)}(\alpha)\right)\,,$$

where $f^{(n)}(\alpha) = \partial_\alpha^n f(\alpha)$ and

$$T_0^{++} \sim \text{LT} + \text{tw-4},\ \text{LT} = \text{LO} + \text{NLO},$$

$\boldsymbol{T_1^{++}} \sim$ **LO + tw-4** $\Rightarrow$ $\boldsymbol{F(\beta,\alpha)}$ **contribution.**

$\hookrightarrow$ **novel result not reported before**

$\boldsymbol{F(\beta,\alpha)}$ **affects determination of the** $\boldsymbol{D}$**-term from the subtraction constant and data in CFFs combined with dispersion relation.**

$F(\beta,\alpha)$-term by means of CFFs

Recap, DR's subtraction constant:

$$h_0^{++} = \left(\int D\right) - 4\frac{M^2 - t/4}{\mathbb{Q}^2}\iint_{\mathbb{D}} d\beta d\alpha\ \boldsymbol{F(\beta,\alpha)}\beta\,\mathrm{Re}\left(T_1^{++\,(1)}(\alpha)\right).$$

From the DR, we extract the coefficient h_2^{++} which leads us to:

$$\frac{2}{\pi}\mathrm{PV}\int_0^1 d\xi'\ \xi'\mathrm{Im}\left(\mathcal{H}^{++}_{\mathrm{tw}-4}(\xi',t) - \mathcal{H}^{++}_{\mathrm{tw}-4}(\xi',0)\right) =$$

$$= \iint_{\mathbb{D}} d\beta d\alpha\ \beta\left[\frac{t}{\mathbb{Q}^2}F(\beta,\alpha,t)\left\{-3\,T_1^{++\,(1)}(\alpha) + \mathbf{R}^{(1)}(\alpha) - \beta^2\,T_1^{++\,(3)}(\alpha)\right\}\right.$$

$$\left. - \beta^2\frac{4M^2}{\mathbb{Q}^2}\,T_1^{++\,(3)}(\alpha)\left(F(\beta,\alpha,t) - F(\beta,\alpha,0)\right)\right]$$

$$\underbrace{\simeq}_{(?)}(-3+\kappa)\frac{t}{\mathbb{Q}^2}\iint_{\mathbb{D}} d\beta d\alpha\ F(\beta,\alpha)\beta\ T_1^{++\,(1)}(\alpha)\,.$$

PRELIMINARY, we are working on this

Parenthesis stands for the order in ∂_α . $\mathbf{R}^{(1)}(\alpha) \mapsto \kappa\,T_1^{++\,(1)}(\alpha) + \epsilon_\kappa(\alpha)\,,\ |\kappa| = O(1).$

$F(\beta,\alpha)$-term in h_0^{++} might be obtained from data on CFFs $\longrightarrow$ avoid modelling $F(\beta,\alpha)$.

What do we have so far?

1. New dispersion relation:

$$h_0^{++} = \mathrm{Re}\left(\mathcal{H}^{++}(\xi)\right) - \frac{2}{\pi}\mathrm{PV}\int_0^1 d\xi' \, \frac{\xi' \, \mathrm{Im}\left(\mathcal{H}^{++}(\xi')\right)}{\xi^2 - \xi'^2}$$

$$= \left(\int D\right) - 4\frac{M^2 - t/4}{\mathbb{Q}^2}\iint_{\mathbb{D}} d\beta d\alpha \; \boldsymbol{F(\beta, \alpha)} \beta \, \mathrm{Re}\left(T_1^{++\,(1)}(\alpha)\right) .$$

2. Simplification for model independence (**preliminary**):

$$h_0^{++} \simeq \left(\int D\right) + \left(\int \xi' \mathrm{Im}\left(\mathcal{H}^{++}_{\mathrm{tw}-4}(\xi', t) - \mathcal{H}^{++}_{\mathrm{tw}-4}(\xi', 0)\right)\right) \Rightarrow$$

$$\Rightarrow \left(\int D\right) \simeq \underbrace{\mathrm{Re}\left(\mathcal{H}^{++}(\xi)\right) - \frac{2}{\pi}\mathrm{PV}\int_0^1 d\xi' \, \frac{\xi' \, \mathrm{Im}\left(\mathcal{H}^{++}(\xi')\right)}{\xi^2 - \xi'^2}}_{h_0^{++}}$$

$$- \left(\int \xi' \mathrm{Im}\left(\mathcal{H}^{++}_{\mathrm{tw}-4}(\xi', t) - \mathcal{H}^{++}_{\mathrm{tw}-4}(\xi', 0)\right)\right) .$$

3. **Re** $\left(\mathcal{H}^{++}\right)$ **and Im** $\left(\mathcal{H}^{++}\right)$ $\mapsto$ ***D*-term** $\mapsto$ GFF C_a $\mapsto$ **pressure** in the hadron.
4. Need enough sensitivity as to isolate the kinematic twist-4 component of $\mathcal{H}^{++}$.

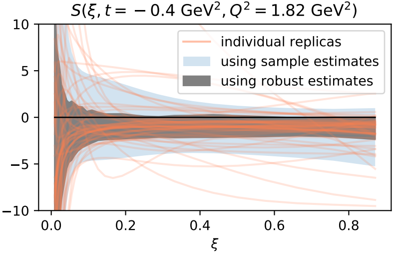
D-term extraction

Subtraction constant from CFFs

DR choosing $n = 0$ to extract h_0^{++} :

$$h_0^{++} = \mathrm{Re}\left(\mathcal{H}^{++}(\xi)\right) - \frac{2}{\pi}\mathrm{PV}\int_0^1 d\xi' \; \frac{\xi' \,\mathrm{Im}\left(\mathcal{H}^{++}(\xi')\right)}{\xi^2 - \xi'^2}$$

With NLO coefficient functions $\rightarrow$ 100 sets of CFFs[&] $\rightarrow$ neural network extraction of 100 samples/**replicas** of $h_0^{++} = S$ at NLO and LT:



Example of 50 replicas for $\boldsymbol{S = h_0^{++}}$ from Dutrieux, Meisgny, Mezrag & Moutarde, EPJC 85 (2025) 85:105.

[&] Dutrieux, Lorcé, Moutarde, Sznajder, Trawiński & Wagner, EPJC 81(4), 300 (2021).

D-term and subtraction constant

Parameterization of the D-term:

$$D_f(\alpha, t, \mu^2) = (1-\alpha^2) \sum_{\text{odd } n} d_{n,\,f}(t,\mu^2) \underbrace{C_n^{(3/2)}(\alpha)}_{\text{Gegenbauer poly.}} \,,$$

$$D_g(\alpha, t, \mu^2) = \frac{3}{2}(1-\alpha^2)^2 \sum_{\text{odd } n} d_{n,g}(t,\mu^2) C_{n-1}^{(5/2)}(\alpha) \,,$$

$$d_{n,\,a}(t,\mu^2) = d_{n,\,a}(0,\mu^2) \cdot (1 - t[\mathrm{GeV}^2]/0.8^2)^{-3} \,, \quad a \in \{f, g\} \,.$$

Extracting[+] $d_{n,\,a}(0,\mu^2)$ from h_0^{++} $\Rightarrow$ C_a GFF $\Rightarrow$ hadron's pressure.

At LO and LT (no $F(\beta,\alpha)$ at LT):

$$h_0^{++} = \frac{4}{(1-t[\mathrm{GeV}^2]/0.8^2)^3} \sum_f e_f^2 \sum_{\text{odd } n} d_{n,\,f}(0,\mu^2) \,.$$

Match the formula above (and NLO equivalent) to the replicas for h_0^{++}.

[+] **See Thursday's talk by M. Higuera-Angulo on extraction of GFFs from DVCS data.**

Shadow D-term

Definition (Shadow D-term)

Function that introduces no contribution to the DR's subtraction constant at a given energy scale μ_0^2 .

- **At LO and LT (no $F(\beta, \alpha)$):**

$$h_0^{++} = \frac{4}{(1 - t[\mathrm{GeV}^2]/0.8^2)^3} \sum_f e_f^2 \sum_{\mathrm{odd}\ n} d_{n,f}(0, \mu^2) .$$

Truncating for $n \in \{1, 3\}$, a **LO shadow D-term at a scale μ_0^2** is given by the condition:

$$d_{1,f}(0, \mu_0^2) = -d_{3,f}(0, \mu_0^2) = \lambda . \qquad (1)$$

- **Evolution $\mu_0^2 \to \mu^2$ at LT:**

$h_{0,S}^{++} \to$ contribution to h_0^{++} from the shadow D-term:

$$d_{n,f}(0, \mu^2) = \left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{\frac{2\gamma_n}{\beta_0}} d_{n,f}(0, \mu_0^2) \Rightarrow h_{0,S}^{++} \overset{\text{lin.}}{\simeq} \lambda \left[1 - \frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right] .$$

- **Uncertainty estimator:**

$$\text{Experimental uncertainty: } \Delta h_0^{++} \Rightarrow \sigma_{S,\,d1f} \approx \sigma_{S,\,d3f} \approx \frac{\Delta h_0^{++}}{1 - \frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}} . \qquad (2)$$

LO h_0^{++} fit: $n = 1$ vs $n = 3$, radiative gluons

$$d_{1,\,uds} := d_{1,\,u} = d_{1,\,d} = d_{1,\,s}\,.$$

LO h_0^{++}, $n = 1$, radiative gluons
$d_{1,\,uds}(0, \mu_0^2)$ free (only)

$$d_{1,\,uds}(0, 2\ \mathrm{GeV}^2) = -0.6 \pm 1.1$$
$$d_{1,\,g}(0, 2\ \mathrm{GeV}^2) = -0.8 \pm 1.5$$
$$d_{1,\,c}(0, 2\ \mathrm{GeV}^2) = -0.003 \pm 0.005$$

LO h_0^{++}, $n = 3$, radiative gluons
$d_{1,\,uds}(0, \mu_0^2)$, $d_{3,\,uds}(0, \mu_0^2)$ free

$$d_{1,\,uds}(0, 2\ \mathrm{GeV}^2) = -2.1 \pm 26.6$$
$$d_{3,\,uds}(0, 2\ \mathrm{GeV}^2) = 1.5 \pm 26.5$$
$$d_{1,\,g}(0, 2\ \mathrm{GeV}^2) = -2.9 \pm 37$$
$$d_{3,\,g}(0, 2\ \mathrm{GeV}^2) = 0.2 \pm 4.1$$

$n = 1$ vs $n = 3$:

$$d_{1,\,uds} \approx -d_{3,\,uds} \overset{(1)}{\Rightarrow} \text{shadow } D\text{-term} \xRightarrow{\substack{(2),\\ \mu_0^2=1.4,\\ \mu^2=2.5}} \sigma_{d1f} \approx \sigma_{d3f} \approx 25.5\,.$$

Conclusion: most of the uncertainty is contamination by LO shadow D-term.

Fits hereafter from Dutrieux, Meisgny, Mezrag & Moutarde, EPJC 85 (2025) 85:105.

LO h_0^{++} fit: $d_{1,a}$, unconstrained gluons

LO h_0^{++}, $n=1$, radiative gluons
$d_{1,uds}(0,\mu_0^2)$ free (only)

$d_{1,uds}(0, 2\ \mathrm{GeV}^2) =$	-0.6 ± 1.1
$d_{1,g}(0, 2\ \mathrm{GeV}^2) =$	-0.8 ± 1.5
$d_{1,c}(0, 2\ \mathrm{GeV}^2) =$	-0.003 ± 0.005

LO h_0^{++}, $n=1$, free gluons
$d_{1,uds}(0,\mu_0^2)$, $d_{1,g}(0,\mu_0^2)$ free

$d_{1,uds}(0, 2\ \mathrm{GeV}^2) =$	-0.6 ± 1.1
$d_{1,g}(0, 2\ \mathrm{GeV}^2) =$	-11 ± 132
$d_{1,c}(0, 2\ \mathrm{GeV}^2) =$	-0.04 ± 0.47

Radiative (generation at $\mu_g^2 = 0.09$) vs free gluons:

$$d_{1,uds}(\mu^2) = \Gamma_1^{qq}(\mu^2,\mu_0^2)\left[1 + \frac{\Gamma_1^{qg}(\mu^2,\mu_0^2)\Gamma_1^{gq}(\mu_0^2,\mu_g^2)}{\Gamma_1^{qq}(\mu^2,\mu_0^2)\Gamma_1^{qq}(\mu_0^2,\mu_g^2)}\right] \times d_{1,uds}(\mu_0^2)\,.$$

$$\mu_0^2 = 1\,,\ \mu^2 = 2.5 \Rightarrow \frac{\Gamma_1^{qg}(\mu^2,\mu_0^2)}{\Gamma_1^{qq}(\mu^2,\mu_0^2)} \approx \frac{1}{60}\,.$$

Gluon distribution needs to be 60 times larger than the quark distribution to contribute similarly.

Conclusion: At LO, DVCS is NOT sensitive to $d_{1,g}$. Evolution does not allow for assessment on $d_{1,g}$.

LO vs NLO h_0^{++} fits: $n = 1$, radiative gluons

LO h_0^{++}, $n=1$, radiative gluons $d_{1,\,uds}(0,\mu_0^2)$ free (only)		NLO h_0^{++}, $n=1$, radiative gluons $d_{1,\,uds}(0,\mu_0^2)$ free (only)	
$d_{1,\,uds}(0, 2\ \mathrm{GeV}^2) =$	-0.6 ± 1.1	$d_{1,\,uds}(0, 2\ \mathrm{GeV}^2) =$	-0.7 ± 1.3
$d_{1,\,g}(0, 2\ \mathrm{GeV}^2) =$	-0.8 ± 1.5	$d_{1,\,g}(0, 2\ \mathrm{GeV}^2) =$	-0.9 ± 1.8
$d_{1,\,c}(0, 2\ \mathrm{GeV}^2) =$	-0.003 ± 0.005	$d_{1,\,c}(0, 2\ \mathrm{GeV}^2) =$	-0.003 ± 0.006

LO vs NLO:

$$\mu_g^2 = 1\ (\text{gluon radiation threshold}) \Rightarrow d_{n,\,g}(0,2) = \frac{\Gamma_n^{gq}(2,1)}{\Gamma_n^{qq}(2,1)} d_{n,\,uds}(0,2)\,.$$

$$h_0^{++} \overset{\mathrm{NLO}}{\approx} -0.3 d_{1,\,g} + 2.65 d_1\big|_{\mathrm{quarks}}$$

Conclusion: gluons account for a 10% effect in h_0^{++}, hence the LO-NLO similarity.

NLO h_0^{++} fit: $n = 1$ vs $n = 3$, radiative gluons

NLO h_0^{++}, $n = 1$, radiative gluons
$d_{1,\,uds}(0, \mu_0^2)$ free (only)

$$d_{1,\,uds}(0, 2\ \mathrm{GeV}^2) = -0.7 \pm 1.3$$
$$d_{1,\,g}(0, 2\ \mathrm{GeV}^2) = -0.9 \pm 1.8$$
$$d_{1,\,c}(0, 2\ \mathrm{GeV}^2) = -0.003 \pm 0.006$$

NLO h_0^{++}, $n = 3$, radiative gluons
$d_{1,\,uds}(0, \mu_0^2)$, $d_{3,\,uds}(0, \mu_0^2)$ free

$$d_{1,\,uds}(0, 2\ \mathrm{GeV}^2) = -1.7 \pm 21$$
$$d_{3,\,uds}(0, 2\ \mathrm{GeV}^2) = 0.7 \pm 15$$
$$d_{1,\,g}(0, 2\ \mathrm{GeV}^2) = -2 \pm 30$$
$$d_{3,\,g}(0, 2\ \mathrm{GeV}^2) = 0.1 \pm 2.3$$

***n* = 1 vs *n* = 3:**

$$\mu_g^2 = 0.09 \text{ (gluon radiation threshold)} \Rightarrow d_{n,\,g}(0, 2) = \frac{\Gamma_n^{gq}(2, 0.09)}{\Gamma_n^{qq}(2, 0.09)} d_{n,\,uds}(0, 2)\,.$$

$$h_0^{++} \overset{\mathrm{NLO}}{=} (2.65-0.36) d_{1,\,uds}(0, 2) + (3.36+0.05) d_{3,\,uds}(0, 2) \Rightarrow d_{1,\,uds}^{\mathrm{shadow}}(0, 2) = -1.5 d_{3,\,uds}^{\mathrm{shadow}}(0, 2)\,.$$

$$\sigma_{d1f} \approx 1.5 \sigma_{d3f} \approx 1.5 \times 15 \approx 22.5\,.$$

Conclusion: uncertainties are due to NLO shadow *D*-term.

NLO h_0^{++} fit: $d_{1,a}$, unconstrained gluons

LO h_0^{++}, $n = 1$, radiative gluons
$d_{1,uds}(0, \mu_0^2)$ free (only)

$$d_{1,uds}(0, 2\ \mathrm{GeV}^2) = -0.6 \pm 1.1$$
$$d_{1,g}(0, 2\ \mathrm{GeV}^2) = -0.8 \pm 1.5$$
$$d_{1,c}(0, 2\ \mathrm{GeV}^2) = -0.003 \pm 0.005$$

NLO h_0^{++}, $n = 1$, free gluons
$d_{1,uds}(0, \mu_0^2)$, $d_{1,g}(0, \mu_0^2)$ free

$$d_{1,uds}(0, 2\ \mathrm{GeV}^2) = -1.1 \pm 7.7$$
$$d_{1,g}(0, 2\ \mathrm{GeV}^2) = -6 \pm 78$$
$$d_{1,c}(0, 2\ \mathrm{GeV}^2) = -0.02 \pm 0.27$$

Radiative vs free gluons:

Larger quark uncertainty for free gluons:

$$h^{++}_{0,\mathbf{g}} \propto \alpha_s(\mu^2)\Gamma^{\mathbf{gg}}(\mu^2, \mu_0^2) d_{1,\mathbf{g}}(0, \mu_0^2) \approx \alpha_s(\mu^2)\Gamma^{\mathbf{qq}}(\mu^2, \mu_0^2) d_{1,\mathbf{g}}(0, \mu_0^2)\,.$$

Conclusion: gluon and quark distributions are strongly correlated.

Take aways

- Analytic properties of the scattering amplitude $\leftrightarrow$ dispersion relation at all orders.
- Shadow contributions to the D-term can help understanding uncertainties in its extraction $\leftrightarrow$ inverse/deconvolution problem.
- Kin. twist-4 effects: **the subtraction constant seems to depend not only on the D-term, but also on $F(\beta, \alpha)$. This is a new result, not published before.**
- Our scientific program:
  1. assessment of F-term in h_0^{++} as a moment of Im$\mathcal{H}$.
  2. assessment of F's impact in the extraction of d_is.
  3. do twist corrections reduce the uncertainty in the d_is?

Thank you!

Complementary slides

Conformal group (CG)

Set of transformations $z \to z'$ such that the metric is re-scaled as:

$$g_{\mu\nu}(z) \to g'_{\mu\nu}(z') = \Omega^2(z) g_{\mu\nu}(z) \,.$$

CG = Poincaré + dilations + $\underbrace{\text{special conformal transformations}}_{\text{inversion + translation + inversion}}$.

Main tool: shadow-operator formalism

- Use of conformal symmetry to constrain the coefficients of the expansion around the light-cone:

$$\mathscr{P}_{\ell_n,n} = \int d^D z\ \widetilde{\mathcal{O}}_{\alpha_1\cdots\alpha_n}(z)|0\rangle\langle 0|\mathcal{O}^{\alpha_1\cdots\alpha_n}(z)\,,$$

$\widetilde{\mathcal{O}}$ is the *shadow operator* of $\mathcal{O}$ with scaling dimension $\widetilde{\ell}_n = D - \ell_n$ for a D-dimensional spacetime ,

$$\mathcal{O}_1(z_1)\mathcal{O}_2(z_2) = \sum_{\ell_n,n}\int d^D z\ \left[\langle \mathcal{O}_1(z_1)\mathcal{O}_2(z_2)\widetilde{\mathcal{O}}_{\beta_1\cdots\beta_n}(z)\rangle e^{izr}\right]\ \mathcal{O}^{\beta_1\cdots\beta_n}(y)\Big|_{y=0}\,,$$

$$r^\mu = -i\partial_y^\mu\,,$$

$\langle \mathcal{O}_1(z_1)\mathcal{O}_2(z_2)\widetilde{\mathcal{O}}_{\beta_1\cdots\beta_n}(z)\rangle \to$ **up to a constant,** it is given by the constraints to impose conformal covariance by means of:

$$\Lambda^\mu(z_1,z_2,z_3) = \frac{1}{2}\partial_3^\mu \ln\frac{(z_2-z_3)^2}{(z_1-z_3)^2}\,,\quad \partial_3^\mu = \frac{\partial}{\partial z_{3,\,\mu}}\,,$$

$$Z(z_1,z_2,z_3) = \left(\frac{(z_1-z_2)^2}{(z_1-z_3)^2(z_2-z_3)^2}\right)^{T/2}\,,\quad T\in\mathbb{R}\,,$$

$$\mathcal{I}^\mu_\nu(z) = \delta^\mu_\nu - 2\frac{z^\mu z_\nu}{z^2}\,.$$

The conformal basis

- At LO, the expansion of $j^\mu j^\nu$ is given in a basis of 3 operators: $\mathcal{O}_N^{(k)}$ for $k \in \{0,1,2\}$ with

$$\mathcal{O}_N^{(k)}(y) = \partial_y^{\alpha_1} \cdots \partial_y^{\alpha_k} \mathcal{O}_{\alpha_1 \cdots \alpha_k \alpha_{k+1} \cdots \alpha_N}(y) z_{12}^{\alpha_{k+1}} \cdots z_{12}^{\alpha_N}\,.$$

- Not all traces have been removed, so write this operators by means of their geometric LT components. In general, for a scalar operator ($\partial^\mu = \partial/\partial y_\mu$):

$$\mathcal{O}(y) = [\mathcal{O}(y)]_{\mathrm{LT}} - \sum_{k=1}^{\infty} \int_0^1 dt\, \left(\frac{-y^2}{4}\right)^k \frac{(\partial^2)^k}{k!(k-1)!} \frac{(1-t)^{k-1}}{t^k} \mathcal{O}(ty)$$

$$= [\mathcal{O}(y)]_{\mathrm{LT}} + \frac{y^2}{4} \int_0^1 \frac{dt}{t} \left[\partial^2 \mathcal{O}(ty)\right]_{\mathrm{LT}} + \frac{y^4}{32} \int_0^1 dt\, \frac{1-t}{t^3} \left[\partial^4 \mathcal{O}(ty)\right]_{\mathrm{LT}} + O(y^6)\,.$$

Connection to light-ray operators

- Light-ray operator definition:

$$\mathscr{O}(\lambda_1,\lambda_2) = \sum_f \left(\frac{e_f}{e}\right)^2 \frac{1}{2}[\bar{q}_f(\lambda_1 z)\not{z} q_f(\lambda_2 z) - (\lambda_1 \leftrightarrow \lambda_2)]_{\mathrm{LT}}\,, \quad \mathbf{z^2 \neq 0\,.}$$

- Braun et al. proved them to be related to the conformal basis by:

$$\mathscr{O}(\lambda_1,\lambda_2) = \sum_{\substack{N>0,\\ \text{even}}} \rho_N \lambda_{12}^{N-1} \int_0^1 du\,(u\bar{u})^N \left[\mathcal{O}_N^{(0)}(\lambda_{21}^u z)\right]_{\mathrm{LT}}\,.$$

- $\mathcal{O}_N^{(k)} \to \mathscr{O}$ by the above integral and similar relations for $k \neq 0$.

Braun-Ji-Manashov conformal OPE

$$T^{\mu\nu} = i \int d^4z\, e^{iq'z} \langle p' | \mathbb{T}\{ j^\nu(z) j^\mu(0) \} | p \rangle =$$

$$\frac{1}{i\pi^2} i \int d^4z\, e^{iq'z} \Bigg\{ \frac{1}{(-z^2+i0)^2} \left[g^{\nu\mu}\mathscr{O}(1,0) - z^\nu \partial^\mu \int_0^1 du\; \mathscr{O}(\bar{u},0) - z^\mu(\partial^\nu - i\Delta^\nu) \int_0^1 dv\; \mathscr{O}(1,v) \right]$$

$$- \frac{1}{-z^2+i0} \left[\frac{i}{2}(\Delta^\mu\partial^\nu - (\nu \leftrightarrow \mu)) \int_0^1 du \int_0^{\bar{u}} dv\; \mathscr{O}(\bar{u},v) - \frac{t}{4} z^\nu \partial^\mu \int_0^1 du\; u \int_0^{\bar{u}} dv\; \mathscr{O}(\bar{u},v) \right]$$

$$+ \cdots$$

Operators $\mathscr{O}$ above are understood as matrix elements, that is:

$$\langle p' | \mathscr{O}(\lambda_1,\lambda_2) | p \rangle = \frac{2i}{\lambda_{12}} \iint_{\mathbb{D}} d\beta d\alpha\; \underbrace{\left[e^{-i\ell_{\lambda_1,\lambda_2} z} + O(z^2) \right]}_{\text{LT}} \Phi^{(+)}(\beta,\alpha,t)\,,$$

where

$$\ell_{\lambda_1,\lambda_2} = -\lambda_1\Delta - \lambda_{12}\left[\beta\bar{p} - \tfrac{1}{2}(\alpha+1)\Delta\right]$$

and

$$\Phi^{(+)}(\beta,\alpha,t) = \partial_\beta F + \partial_\alpha G \quad \leftrightarrow \quad H(x,\xi,t) = \iint_{\mathbb{D}} d\beta d\alpha\; \delta(x-\beta-\alpha\xi)[F+\xi G]\,.$$

More details on: Braun, Ji & Manashov, JHEP 03 (2021) 051; JHEP 01 (2023) 078.

Compton tensor for spin-0 target & helicity amplitudes

- Spin-0 target $\Rightarrow$ vector component of $T^{\mu\nu}$ is enough.
- Parameterization of $T^{\mu\nu} \to$ **helicity amplitudes, $\mathcal{A}^{AB}$.**
- Spin-0 $\Rightarrow$ total of **5 independent** $\mathcal{A}^{AB}$s thanks to parity conservation.

$$
\begin{aligned}
T^{\mu\nu} = {} & \mathcal{A}^{00} \frac{-i}{QQ'R^2} \left[(qq')(Q'^2 q^\mu q^\nu - Q^2 q'^\mu q'^\nu) + Q^2 Q'^2 q^\mu q'^\nu - (qq')^2 q'^\mu q^\nu \right] \\
& + \mathcal{A}^{+0} \frac{i\sqrt{2}}{R|\bar{p}_\perp|} \left[Q' q^\mu - \frac{qq'}{Q'} q'^\mu \right] \bar{p}_\perp^\nu - \mathcal{A}^{0+} \frac{\sqrt{2}}{R|\bar{p}_\perp|} \bar{p}_\perp^\mu \left[\frac{qq'}{Q} q^\nu + Q q'^\nu \right] \\
& + \mathcal{A}^{+-} \frac{1}{|\bar{p}_\perp|^2} \left[\bar{p}_\perp^\mu \bar{p}_\perp^\nu - \widetilde{\bar{p}}_\perp^\mu \widetilde{\bar{p}}_\perp^\nu \right] - \mathcal{A}^{++} g_\perp^{\mu\nu} \,,
\end{aligned}
$$

- Read out projectors $\to \mathcal{A}^{AB} = \mathbf{\Pi}^{(AB)}_{\mu\nu} T^{\mu\nu}$.
- Scale of DDVCS: $\mathbb{Q}^2 = Q^2 + Q'^2 + t$.

Double DVCS' $\mathcal{A}^{++} = \text{LT} + \text{tw-4} + \text{O(tw-6)}$, at LO

$$
\begin{aligned}
\mathcal{A}^{++} = \int_{-1}^{1} dx \Bigg\{ & -\left(1 - \frac{t}{2\mathbb{Q}^2} + \frac{t(\xi-\rho)}{\mathbb{Q}^2}\partial_\xi\right)\frac{H^{(+)}}{x-\rho+i0} \\
& + \frac{t}{\xi\mathbb{Q}^2}\Bigg[\mathbb{P}_{(\mathrm{i})} + \mathbb{P}_{(\mathrm{ii})} - \frac{\xi L}{2} + \frac{\widetilde{\mathbb{P}}_{(\mathrm{iii})} - \widetilde{\mathbb{P}}_{(\mathrm{i})}}{2} \\
& \qquad - \frac{\xi}{x+\xi}\left(\ln\left(\frac{x-\rho+i0}{\xi-\rho+i0}\right) - \frac{\xi+\rho}{2\xi}\ln\left(\frac{-\xi-\rho+i0}{\xi-\rho+i0}\right) - \widetilde{\mathbb{P}}_{(\mathrm{i})}\right)\Bigg] H^{(+)} \\
& - \frac{t}{\mathbb{Q}^2}\partial_\xi\Bigg[\Bigg(\mathbb{P}_{(\mathrm{i})} + \mathbb{P}_{(\mathrm{ii})} - \frac{\xi L}{2} \\
& \qquad - \frac{\xi}{x+\xi}\left(\ln\left(\frac{x-\rho+i0}{\xi-\rho+i0}\right) - \frac{\xi+\rho}{2\xi}\ln\left(\frac{-\xi-\rho+i0}{\xi-\rho+i0}\right) - \widetilde{\mathbb{P}}_{(\mathrm{i})}\right)\Bigg) H^{(+)}\Bigg] \\
& + \frac{\xi^2\bar{p}_\perp^2}{\mathbb{Q}^2} 2\xi\partial_\xi^2\left[\left(\mathbb{P}_{(\mathrm{i})} + \mathbb{P}_{(\mathrm{ii})} - \frac{\xi L}{2} + \frac{\widetilde{\mathbb{P}}_{(\mathrm{iii})} + \widetilde{\mathbb{P}}_{(\mathrm{i})}}{2}\right) H^{(+)}\right]\Bigg\} \\
& + O(\mathrm{tw\text{-}6}) \,,
\end{aligned}
$$

- $\xi^2\bar{p}_\perp^2 = \xi^2 M^2 - t\left(\xi^2 - 1\right)/4\,.$
- **All amplitudes in VMF, Pire, Sznajder & Wagner, arXiv:2503.02461 (2025).**

- Coefficient functions of $\mathcal{A}^{++}$:

$$\mathbb{P}_{(\mathrm{i})}(x/\xi,\rho/\xi) = \frac{\xi-\rho}{x-\xi}\mathrm{Li}_2\left(-\frac{x-\xi}{\xi-\rho+i0}\right),$$

$$\widetilde{\mathbb{P}}_{(\mathrm{i})}(x/\xi,\rho/\xi) = -\frac{\xi-\rho}{x-\xi}\ln\left(\frac{x-\rho+i0}{\xi-\rho+i0}\right),$$

$$\mathbb{P}_{(\mathrm{ii})}(x/\xi,\rho/\xi) = \frac{\xi-\rho}{x+\xi}\left[\mathrm{Li}_2\left(-\frac{x-\xi}{\xi-\rho+i0}\right)-(x\to-\xi)\right],$$

$$\widetilde{\mathbb{P}}_{(\mathrm{iii})}(x/\xi,\rho/\xi) = -\frac{\xi+\rho}{x+\xi}\ln\left(\frac{x-\rho+i0}{-\xi-\rho+i0}\right),$$

$$L = \int_0^1 dw\,\frac{-4}{x-\xi-w(x+\xi)}\int_0^1 du\,\ln\left(1+\frac{\bar{u}[x-\xi-w(x+\xi)]}{\xi-\rho+i0}\right)C_{\bar{u},\bar{u}w},$$

$$C_{\bar{u},v} = \ln\left(\frac{\bar{u}-v}{1-v}\right)+\frac{1}{1-v}.$$

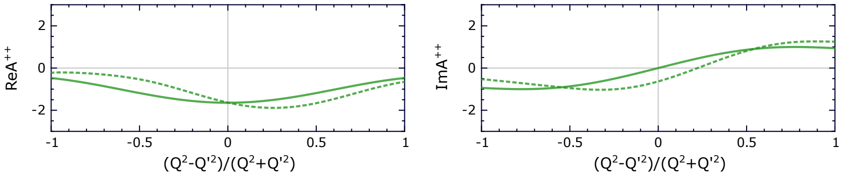
- From the DDVCS result:

$$\begin{cases}\rho\to\xi\Rightarrow \text{DVCS}^{\#},\\ \rho\to-\xi(1-2t/Q'^2)\Rightarrow \text{TCS to twist-4 accuracy.}\end{cases}$$

#DVCS for spin-0 target was already computed in: Braun, Ji & Manashov, JHEP 01 (2023) 078.

Phenomenology for pion target

π-GPD model: J. M. Morgado-Chávez et al., PRD 105, 094012 (2022).



$\xi = 0.2$, $\mathbb{Q}^2 = 1.9\ \mathrm{GeV}^2$, $t = -0.6\ \mathrm{GeV}^2$.
Solid line: LT. Dashed line: LT + tw-4.

- The higher-twist corrections **break** the simple LO LT relation: $\mathcal{A}^{++}_{\mathrm{DVCS}} \overset{\mathrm{LO,\ LT}}{=} \left(\mathcal{A}^{++}_{\mathrm{TCS}}\right)^*$.