GPDs from Lattice QCD
towards mechanical properties

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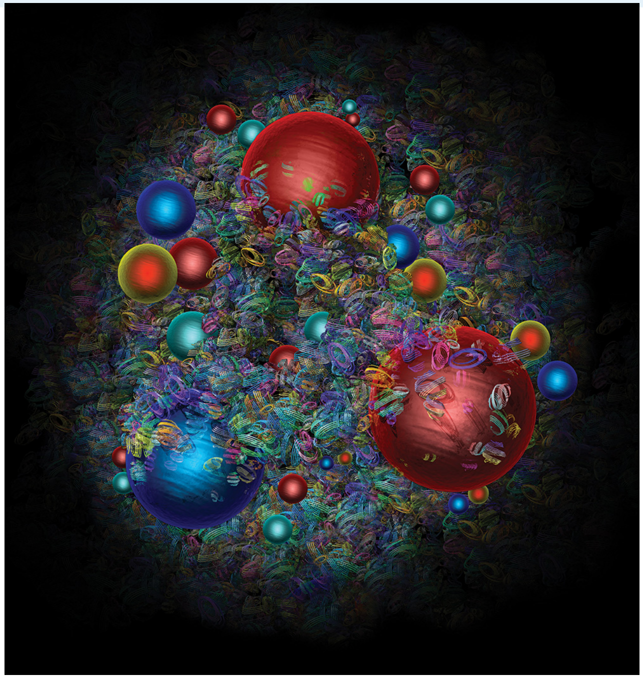
NATIONAL SCIENCE CENTRE POLAND

Supported by the National Science Center of Poland
SONATA BIS grant No. 2016/22/E/ST2/00013 (2017-2022)
OPUS grant No. 2021/43/B/ST2/00497 (2022-2026)

Many thanks to my Collaborators for work presented here:

C. Alexandrou, M.C. Banuls, S. Bhattacharya, M. Constantinou
J. Dodson, X. Gao, K. Hadjiyiannakou, K. Jansen, D. Lin, A. Metz
J. Miller, S. Mukherjee, N. Nurminen, P. Petreczky, A. Scapellato
M. Schneider, F. Steffens, P. Sznajder, J. Wagner, Y. Zhao

One of the central aims of hadron physics:
to understand better nucleon's 3D structure.



- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise? NAS report 2018
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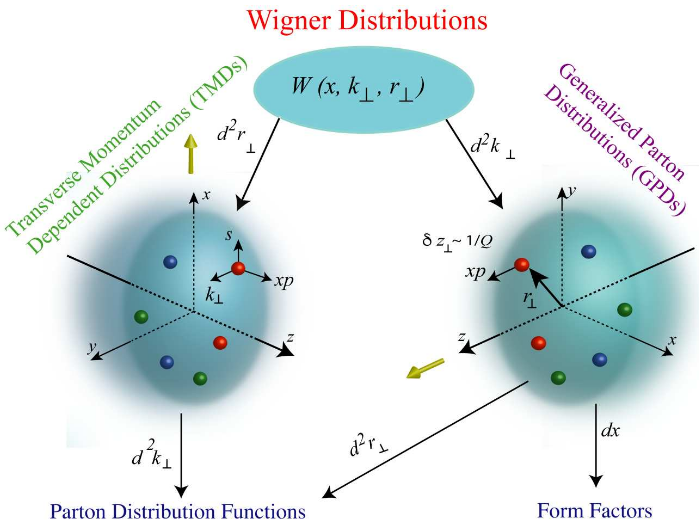
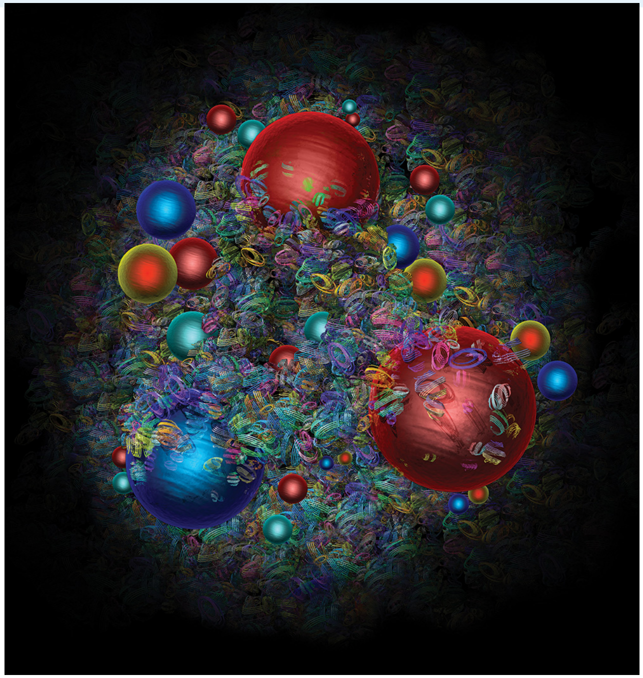


Nucleon structure and GPDs



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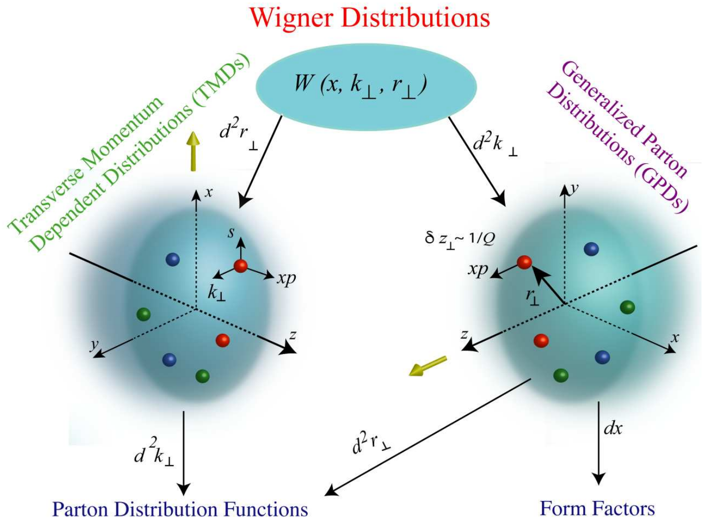
- much more difficult to extract than PDFs,
- but they provide a wealth of information:
  - ⋆ spatial distribution of partons in the transverse plane,
  - ⋆ **mechanical properties of hadrons**,
  - ⋆ hadron's spin decomposition,
- reduce to PDFs in the forward limit, e.g. $H(x,0,0)=q(x)$,
- their moments are form factors, e.g. $\int dx\, H(x,\xi,t)=F_1(t)$.



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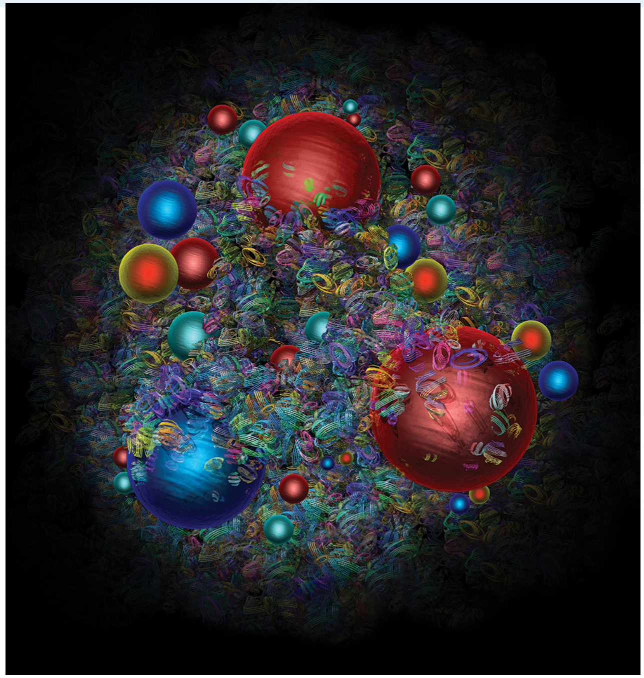

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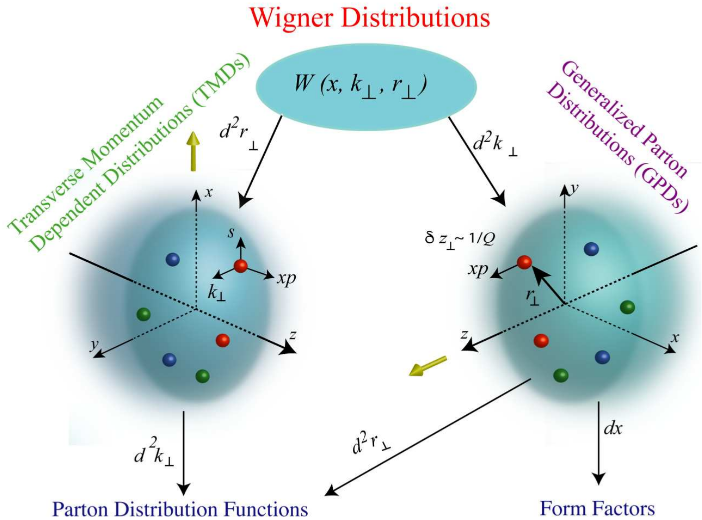

This talk: mostly **zero-skewness GPDs**.

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- Which lattice observables one can use?
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- Examples:
  - ⋆ **hadronic tensor** – K.-F. Liu, S.-J. Dong, 1993
  - ⋆ **auxiliary scalar quark** – U. Aglietti et al., 1998
  - ⋆ **auxiliary heavy quark** – W. Detmold, C.-J. D. Lin, 2005
  - ⋆ **auxiliary light quark** – V. Braun, D. Müller, 2007
  - ⋆ **quasi-distributions** – X. Ji, 2013
  - ⋆ **"good lattice cross sections"** – Y.-Q. Ma, J.-W. Qiu, 2014,2017
  - ⋆ **pseudo-distributions** – A. Radyushkin, 2017
  - ⋆ **"OPE without OPE"** – QCDSF, 2017

But maybe we can aim for direct access?

Direct access to partonic distributions **is possible** in lattice field theory (maybe one day in lattice QCD!) using:

- tensor network methods with
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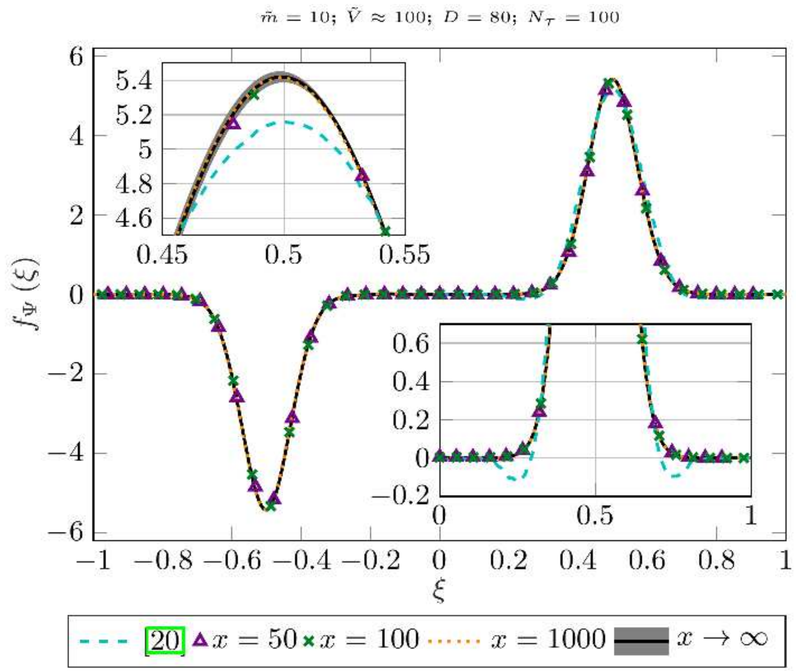
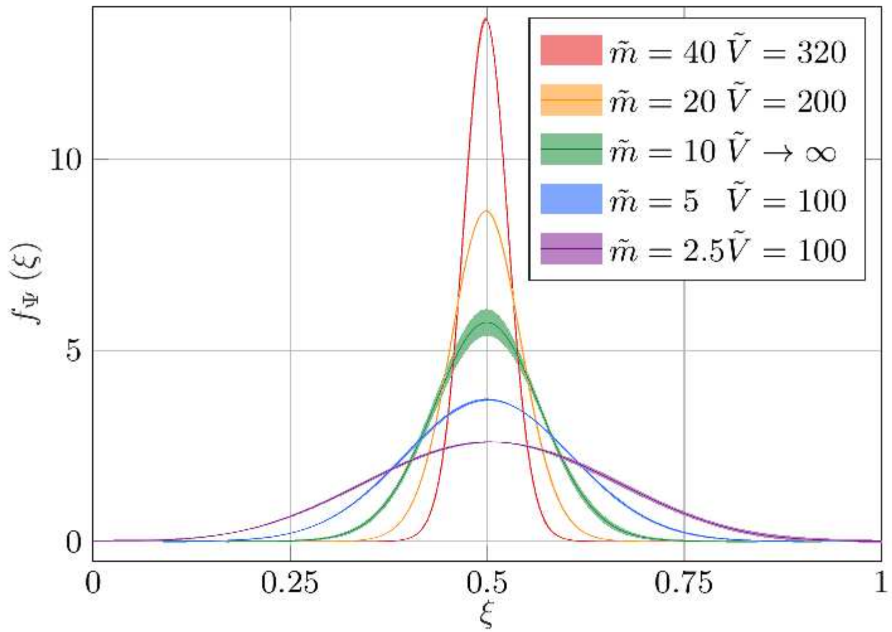
PDF for the **massive Schwinger model**:

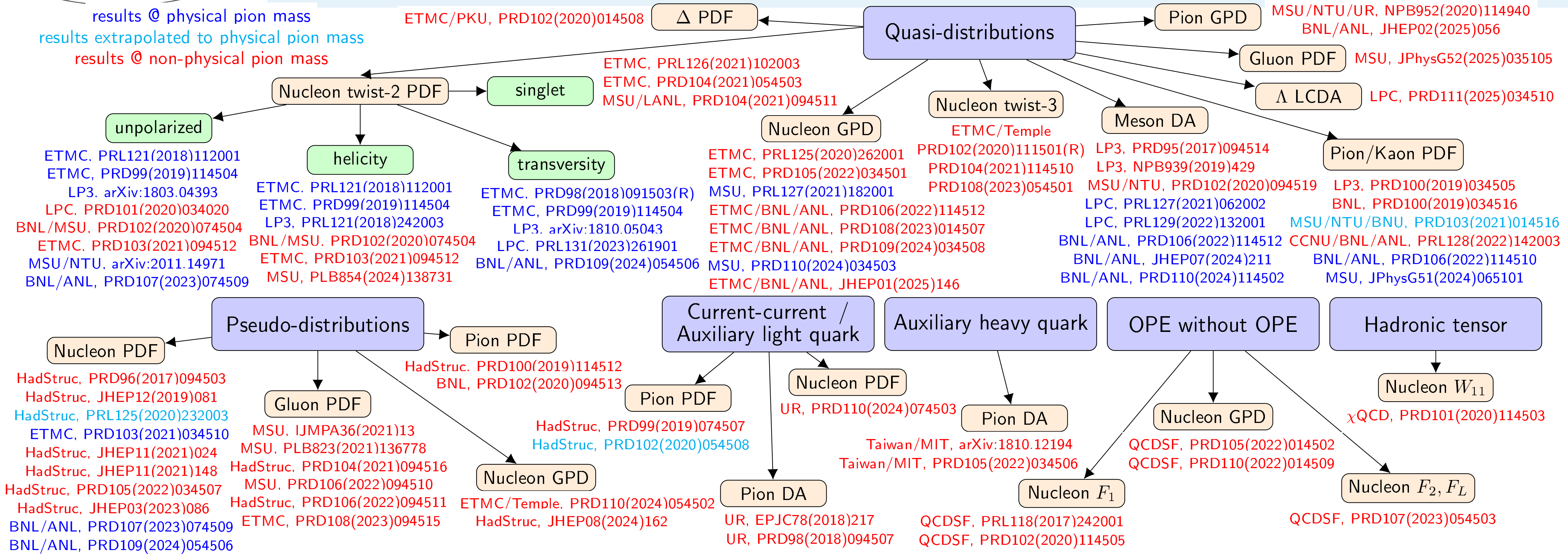
M.C. Bañuls, KC, D. Lin, M. Schneider, coming soon (LATTICE2024 proceedings arXiv:2409.16996)





See also (quantum computing perspective): S. Grieninger, K. Ikeda, I. Zahed, PRD110(2024)076008

Lattice PDFs/GPDs: dynamical progress



REVIEWS

K. Cichy, *Progress in x-dependent partonic distributions from lattice QCD*, plenary talk LATTICE 2021, 2110.07440

K. Cichy, *Overview of lattice calculations of the x-dependence of PDFs, GPDs and TMDs*, plenary talk of Virtual Tribute to Quark Confinement 2021, 2111.04552

K. Cichy, M. Constantinou, *A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results*, invited review for a special issue of Adv. High Energy Phys. 2019 (2019) 3036904, 1811.07248

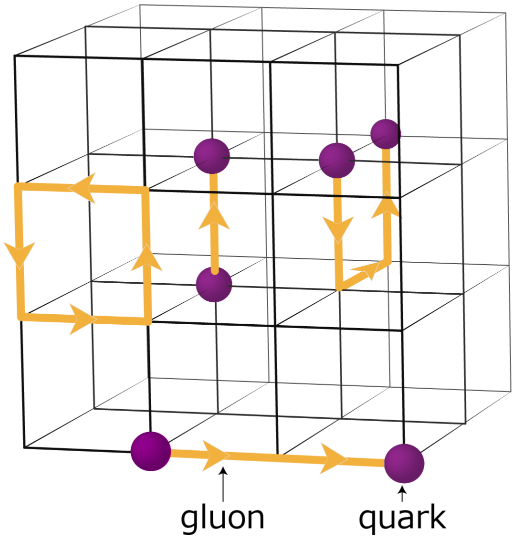
M. Constantinou, *The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD* (would-be) plenary talk of LATTICE 2020, EPJA 57 (2021) 77, 2010.02445

X. Ji et al., *Large-Momentum Effective Theory*, Rev. Mod. Phys. 93 (2021) 035005

M. Constantinou et al., *Parton distributions and LQCD calculations: toward 3D structure*, PPNP 121 (2021) 103908

Lattice QCD – brief reminder

- QCD put on a **Euclidean** lattice: quarks → sites, gluons → links)
- various discretizations can be used for quarks and gluons
- typical lattice parameters:
  - ⋆ $L/a = 32, 48, 64, 80, 96, 128$
  - ⋆ $a \in [0.04, 0.15]$ fm
  - ⋆ $L \in [2, 10]$ fm
  - ⋆ $m_\pi L \geq 3 - 4$
  - ⋆ $\Rightarrow$ ∞-dim path integral → $10^8 - 10^9$-dim integral



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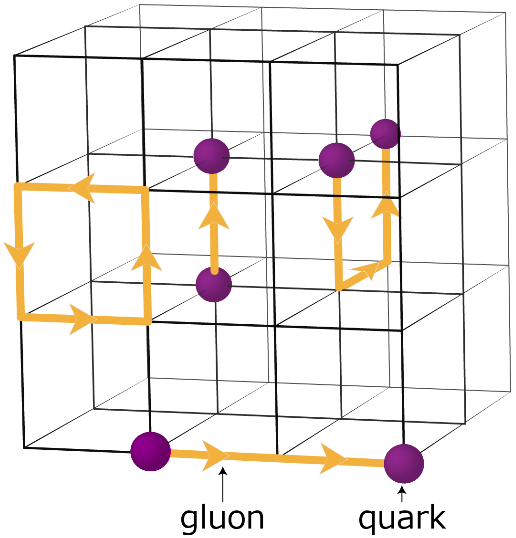
- Monte Carlo simulations to evaluate the discretized path integral
- feasible, but still requires huge computational resources of $\mathcal{O}(1 - 1000)$ million core-hours, depending on the question asked



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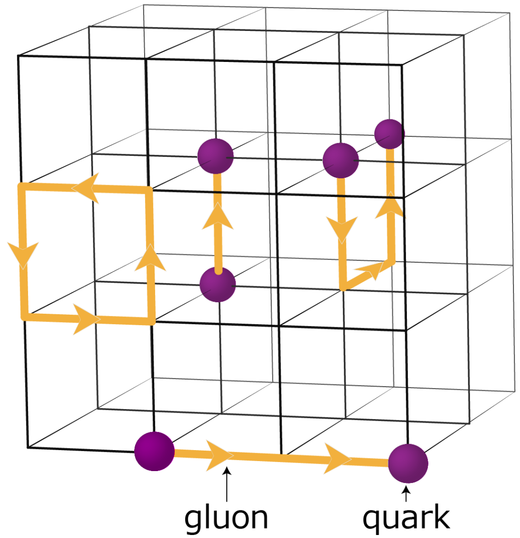

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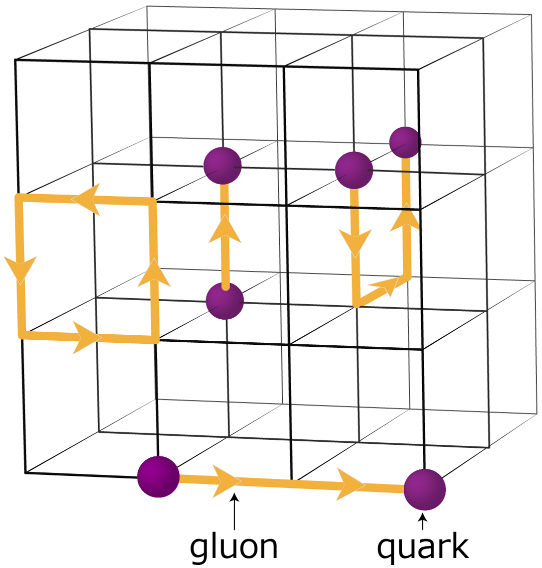
- Lattice QCD offers a way for a careful *ab initio* study of non-perturbative aspects of QCD.
- Its huge strength: possibility to control all systematic effects: *cut-off effects, finite volume effects, renormalization quark mass effects, isospin breaking, excited states, . . .*
- For many aspects, already precision results with percent/per mille total uncertainty.
- However, many aspects (the difficult ones like hadron structure!) with still only exploratory studies.

Lattice QCD – what one should keep in mind

- Main difficulty for quasi/pseudo PDFs/GPDs: **needs large hadron boost!**
- Problem: *signal-to-noise ratio decays* **exponentially** *with increasing boost.*



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  - ⋆ $P_3 = 0.83$ GeV with 1000 measurements
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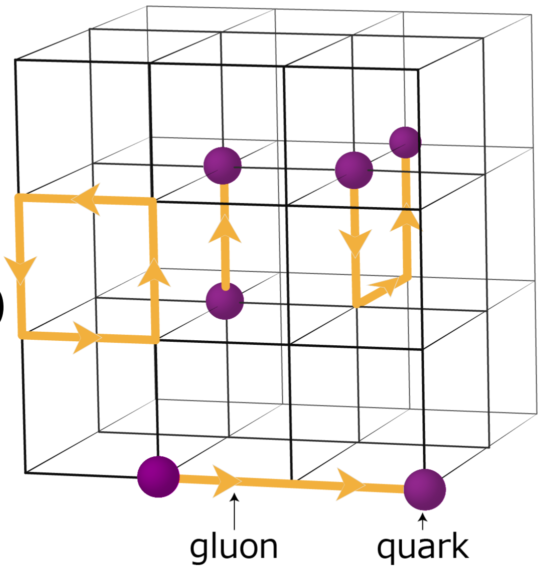
  - ⋆ $P_3 = 1.67$ GeV with 100000 measurements
  - ⋆ $P_3 = 2.1$ GeV – **cost becoming fairly prohibitive...**

  And this is twice too large pion mass.
- At the physical point, many excited states necessitate increased source-sink separation, which further **exponentially** worsens the signal.



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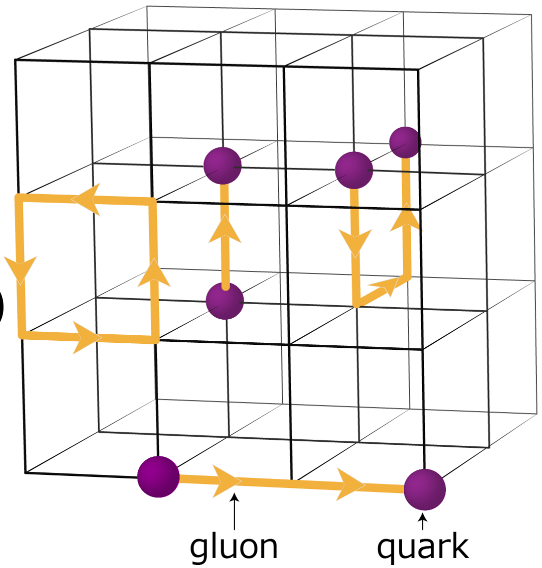

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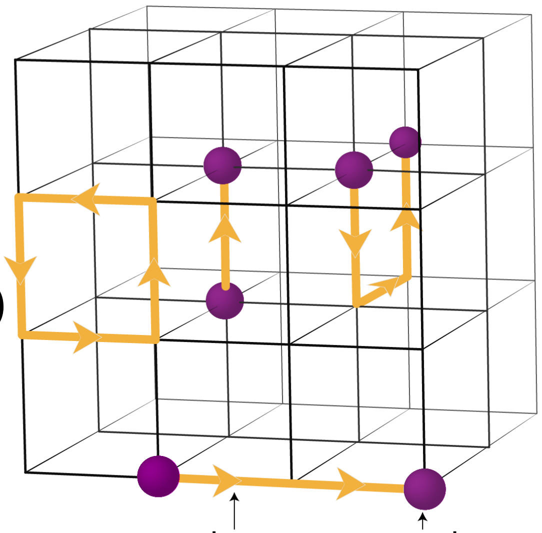

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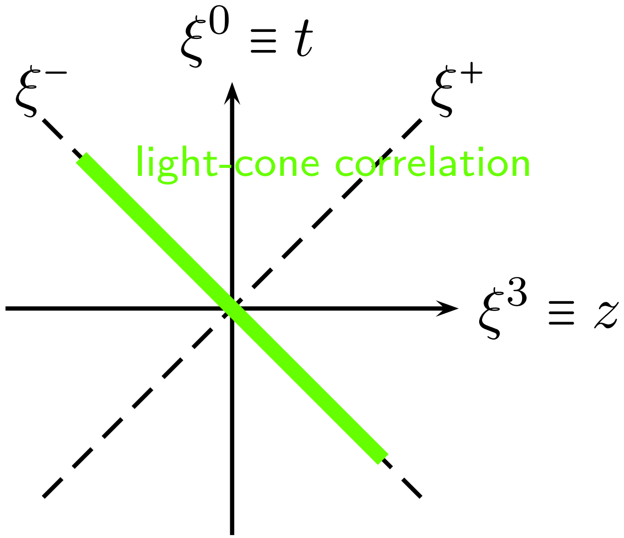
- Overall, **expect complementary role of lattice**.
- Robust quantitative statements: *low moments, form factors*.
- x-dependence: breakthrough in recent years, but a long way to go to solid quantitative statements.

Quasi-distributions

X. Ji, *Parton Physics on a Euclidean Lattice*, Phys. Rev. Lett. **110** (2013) 262002



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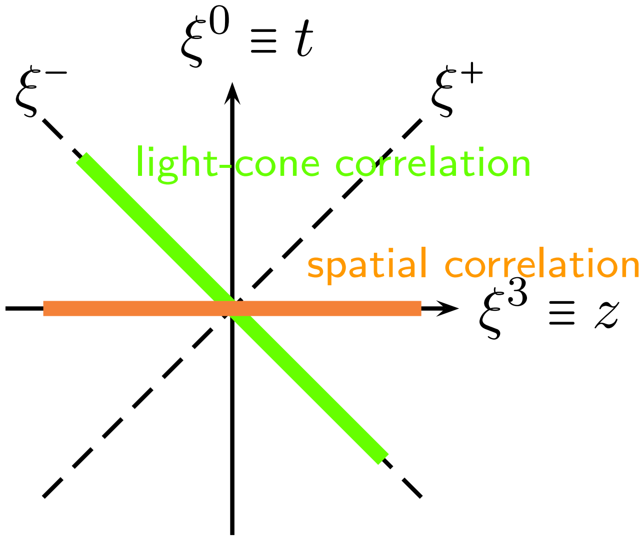

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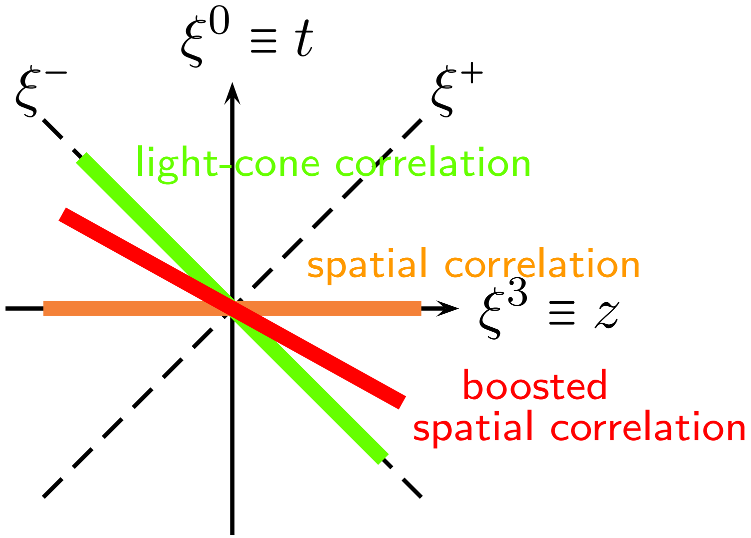

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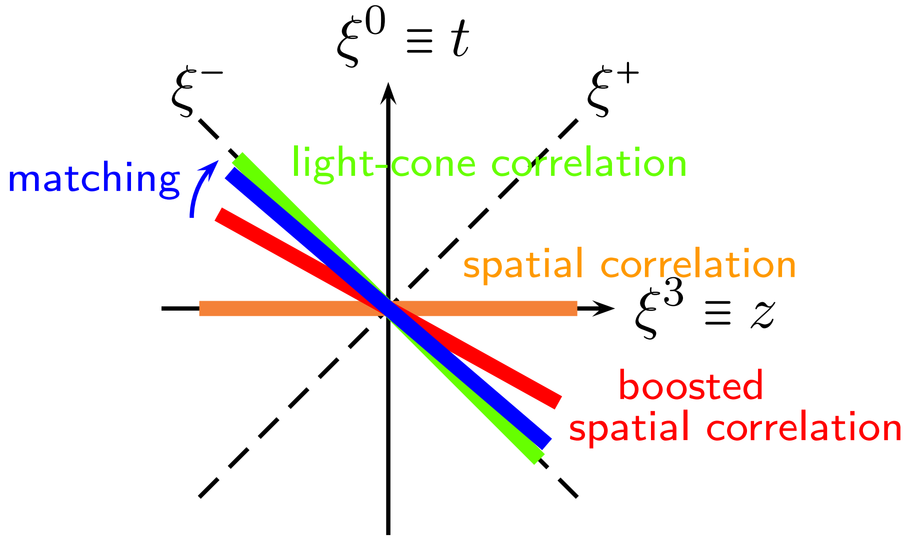

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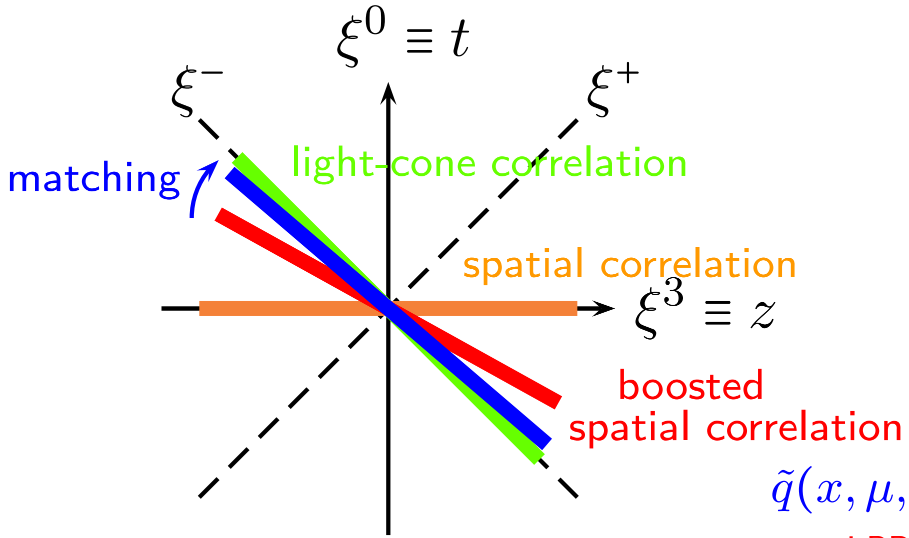

Euclidean matrix element:

$\langle P_f|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|P_i\rangle$

Its Fourier transform (quasi-distribution)
can be matched onto the light-cone distribution:
(Large Momentum Effective Theory (LaMET))

$$\tilde{q}(x,\mu,P_3) = \int_{-1}^{1}\frac{dy}{|y|}\,C\left(\frac{x}{y},\frac{\mu}{P_3}\right)q(y,\mu) + \mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, M_N^2/P_3^2\right)$$

quasi-PDF — pert.kernel — PDF — higher-twist effects

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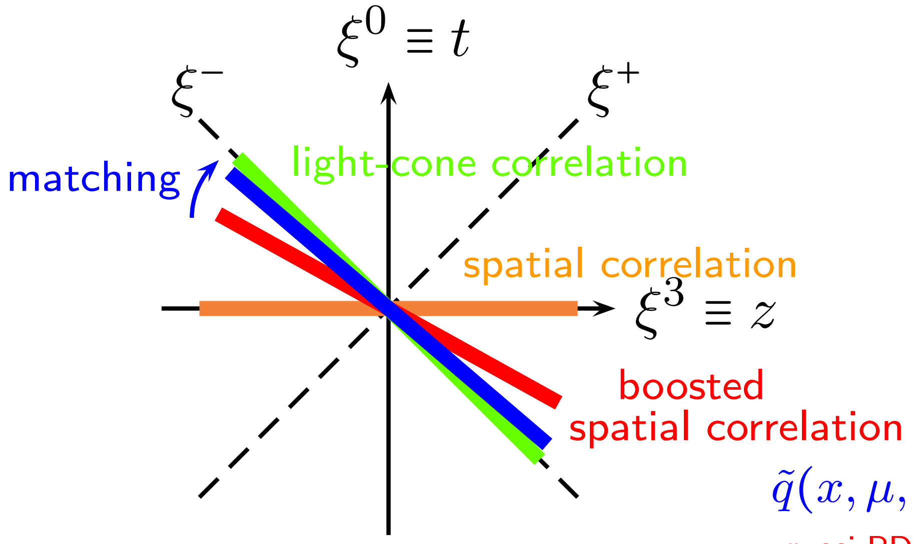

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Dirac structures Γ for different GPDs:

VECTOR: γ_0,γ_3: H,E (unpolarized twist-2),
γ_1,γ_2: G_1,G_2,G_3,G_4 (vector twist-3).

AXIAL VECTOR: $\gamma_5\gamma_0,\gamma_5\gamma_3$: $\tilde{H}$,$\tilde{E}$ (helicity twist-2),
$\gamma_5\gamma_1,\gamma_5\gamma_2$: $\tilde{G}_1$,$\tilde{G}_2$,$\tilde{G}_3$,$\tilde{G}_4$ (axial vector twist-3).

TENSOR: $\gamma_1\gamma_3,\gamma_2\gamma_3$: H_T,E_T,$\tilde{H}_T$,$\tilde{E}_T$ (transversity twist-2),
$\gamma_1\gamma_2$: H_2',E_2' (tensor twist-3).

Need different projectors
to disentangle 2/4 GPDs

UNPOL: $\mathcal{P} = \frac{1+\gamma_0}{4}$

POL-k: $\mathcal{P} = \frac{1+\gamma_0}{4}i\gamma_5\gamma_k$

Quasi-GPDs lattice procedure

spatial correlation in a boosted nucleon
$\langle N(\vec{P}')|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N(\vec{P})\rangle$
$\vec{P}' = \vec{P} + \vec{\Delta}$, $\vec{\Delta}$ – momentum transfer
lattice computation of bare ME

different insertions and projectors
several $\vec{\Delta}$ vectors
symmetric: each $\vec{\Delta}$ separate calc.
asymmetric: many $\vec{\Delta}$ at once!

extraction of amplitudes and/or GPDs
frame-dependent formulas

amplitudes frame-invariant
possible different definitions of GPDs

LaMET (quasi)

SDF (pseudo)

renormalization
e.g. RI, MMS, hybrid

renormalization
e.g. (double) ratio

logarithmic and power divergences
in bare MEs/GPDs

reconstruction of x-dependence (F.T. in z)
e.g. Backus-Gilbert

matching to light cone
in ν-space

reconstruction:
non-trivial ("inverse problem")

matching to light cone
in x-space

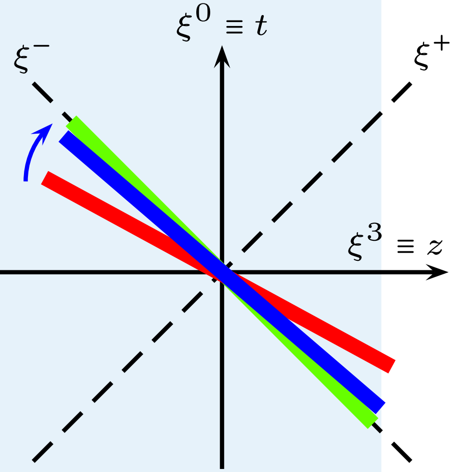
reconstruction of x-dependence (F.T. in ν)
e.g. fitting ansatz

matching:
needs large boosts
valid up to HTEs

x-dependent **light-cone GPD**

moments of **light-cone GPD**

final goal!



Setup

Lattice setup:

- fermions: $N_f = 2$ twisted mass fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 1.778$
- gauge field configurations generated by ETMC
- lattice spacing $a \approx 0.093$ fm,
- $32^3 \times 64 \Rightarrow L \approx 3$ fm,
- $m_\pi \approx 260$ MeV.

Kinematics:

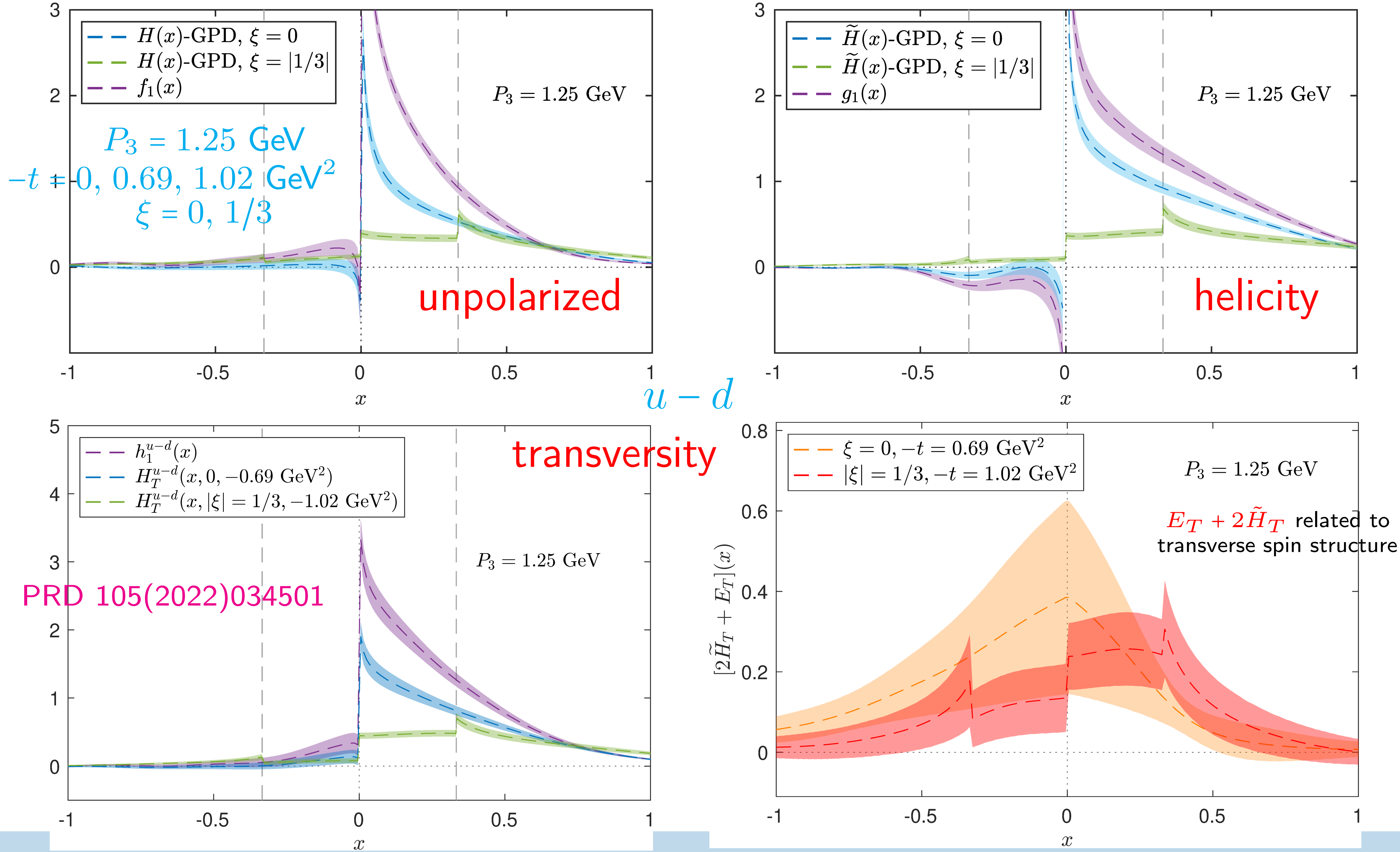
- three nucleon boosts: $P_3 = 0.83, 1.25, 1.67$ GeV,
- momentum transfers: $-t \leq 2.76$ GeV^2, most data: $-t = 0.64,\ 0.69$ GeV^2,
- skewness: $\xi = 0, 1/3$.

up to $\mathcal{O}$(250K) measurements (≈ 500 confs, 32 src positions, 16 permut. of $\vec{\Delta}$).

Twist-2 unpolarized+helicity GPDs C. Alexandrou et al. (ETMC), PRL 125(2020)262001
Twist-2 transversity GPDs C. Alexandrou et al. (ETMC), PRD 105(2022)034501
Twist-2 unpolarized GPDs S. Bhattacharya et al. (ETMC/BNL/ANL) PRD 106(2022)114512
Twist-2 unpolarized GPDs (OPE) S. Bhattacharya et al. (ETMC/BNL/ANL) PRD 108(2023)014507
Twist-3 axial GPDs S. Bhattacharya et al. (ETMC/Temple), PRD 108(2023)054501
Twist-2 helicity GPDs S. Bhattacharya et al. (ETMC/BNL/ANL) PRD 109(2024)034508
Twist-2 unpolarized GPDs (pseudo-GPDs) S. Bhattacharya et al. (ETMC/Temple) PRD110(2024)054502
Twist-2 helicity GPDs (OPE) S. Bhattacharya et al. (ETMC/ANL/BNL/LANL) JHEP01(2025)146
Twist-2 transversity GPDs S. Bhattacharya et al. (ETMC/BNL/ANL) coming soon

First extractions of x-dependent GPDs

C. Alexandrou, KC, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, PRL 125(2020)262001



Standard symmetric (Breit) frame:

source momentum: $P_i = (E, \vec{P} - \vec{\Delta}/2)$,
sink momentum: $P_f = (E, \vec{P} + \vec{\Delta}/2)$.

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Lattice perspective:
construction of the 3-point correlation functions required for the MEs needs the calculation of the all-to-all propagator
preferred way: "sequential propagator" – implies separate inversions (most costly part!) for each P_f.
Hence, **separate calculation for each momentum transfer $\vec{\Delta}$!**

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Lattice perspective:

Several momentum transfer vectors $\vec{\Delta}$ can be obtained within a single calculation!

Lorentz-covariant parametrization

Main theoretical tool: Lorentz-covariant parametrization of matrix elements:

unpolarized: S. Bhattacharya et al., PRD106(2022)114512

$$F^{\mu}(z,P,\Delta)=\bar{u}(p',\lambda')\Big[\frac{P^{\mu}}{m}A_1+mz^{\mu}A_2+\frac{\Delta^{\mu}}{m}A_3+im\sigma^{\mu z}A_4+\frac{i\sigma^{\mu\Delta}}{m}A_5+\frac{P^{\mu}i\sigma^{z\Delta}}{m}A_6+\frac{z^{\mu}i\sigma^{z\Delta}}{m}A_7+\frac{\Delta^{\mu}i\sigma^{z\Delta}}{m}A_8\Big]u(p,\lambda),$$

helicity: S. Bhattacharya et al., PRD109(2024)034508

$$F^{[\gamma^{\mu}\gamma_5]}=\bar{u}(p',\lambda')\Big[\frac{i\epsilon^{\mu Pz\Delta}}{m}\widetilde{A_1}+\gamma^{\mu}\gamma_5\widetilde{A_2}+\gamma_5\Big(\frac{P^{\mu}}{m}\widetilde{A_3}+mz^{\mu}\widetilde{A_4}+\frac{\Delta^{\mu}}{m}\widetilde{A_5}\Big)+m\not{z}\gamma_5\Big(\frac{P^{\mu}}{m}\widetilde{A_6}+mz^{\mu}\widetilde{A_7}+\frac{\Delta^{\mu}}{m}\widetilde{A_8}\Big)\Big]u(p,\lambda)$$

transversity: S. Bhattacharya et al., coming soon

$$F^{[i\sigma^{\mu\nu}\gamma_5]}=\bar{u}(p',\lambda')\Big[\sum_{i=1}^{12}\Gamma_i^{\mu\nu}A_{Ti}\Big]u(p,\lambda)$$

- most general parametrization in terms of 8 or 12 linearly-independent Lorentz structures,
- 8/12 Lorentz-invariant amplitudes $A_i/\widetilde{A_i}/A_{Ti}(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$.

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Example: (γ_0 insertion, unpolarized projector)

symmetric frame:

$$\Pi_0^s(\Gamma_0)=C\left(\frac{E\left(E(E+m)-P_3^2\right)}{2m^3}A_1+\frac{(E+m)\left(-E^2+m^2+P_3^2\right)}{m^3}A_5+\frac{EP_3\left(-E^2+m^2+P_3^2\right)z}{m^3}A_6\right),$$

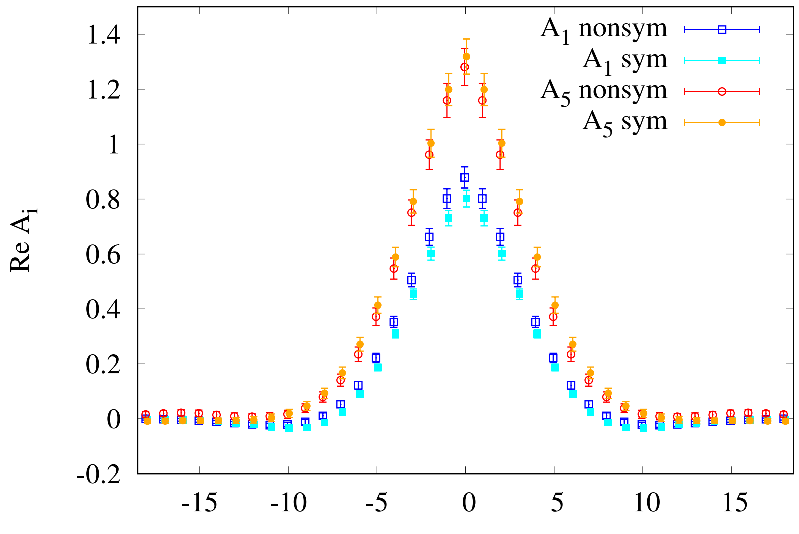
asymmetric frame:

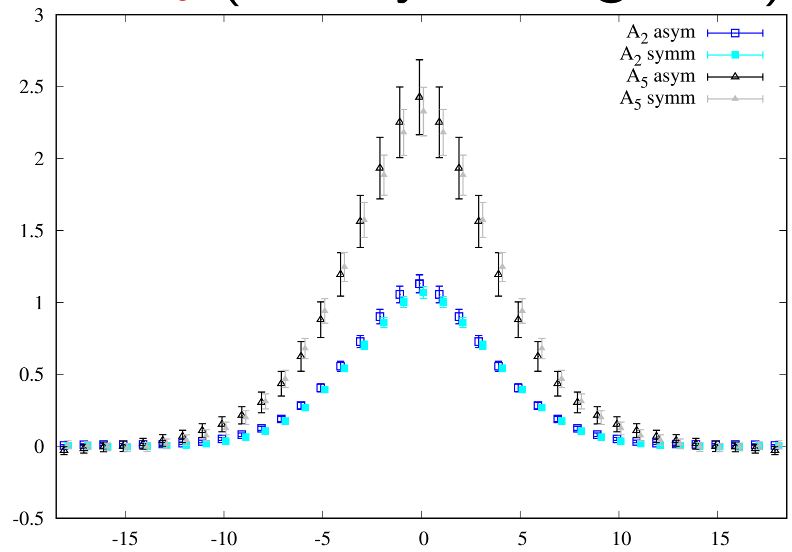
$$\Pi_0^a(\Gamma_0) = C\Bigg(-\frac{(E_f+E_i)(E_f-E_i-2m)(E_f+m)}{8m^3}A_1-\frac{(E_f-E_i-2m)(E_f+m)(E_f-E_i)}{4m^3}A_3+\frac{(E_i-E_f)P_3z}{4m}A_4$$
$$+\frac{(E_f+E_i)(E_f+m)(E_f-E_i)}{4m^3}A_5+\frac{E_f(E_f+E_i)P_3(E_f-E_i)z}{4m^3}A_6+\frac{E_fP_3(E_f-E_i)^2z}{2m^3}A_8\Bigg).$$

- matrix elements $\Pi_{\mu}(\Gamma_{\nu})$ or $\Pi_{\mu5}(\Gamma_{\nu})$ are **frame-dependent**
- but the amplitudes A_i, $\widetilde{A_i}$ or A_{Ti} are **frame-invariant**.

A_1, A_5 (unpolarized leading ones) — $\widetilde{A_2}$, $\widetilde{A_5}$ (helicity leading ones)

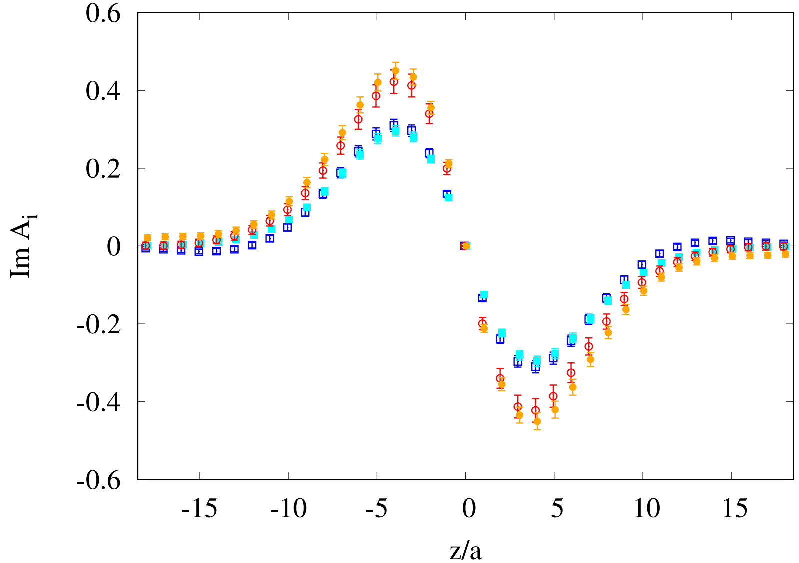
Re

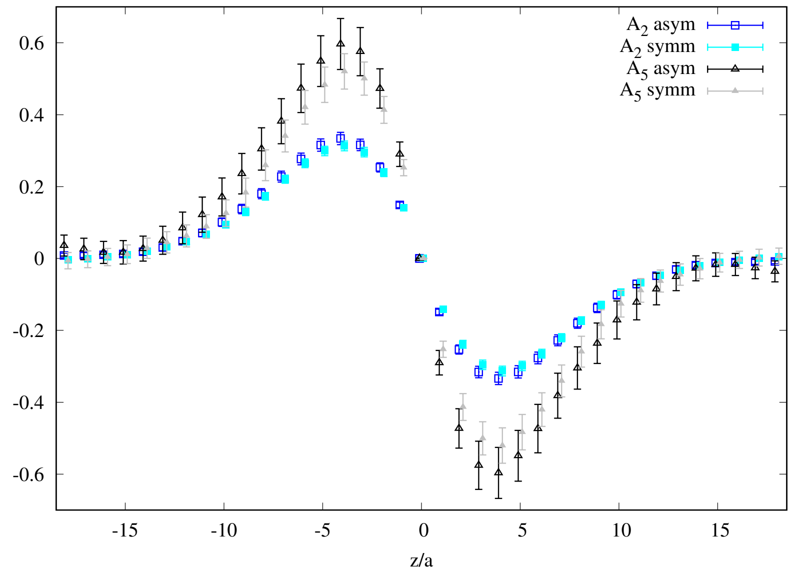




PRD106(2022)114512 S. Bhattacharya et al. PRD109(2024)034508

Im





GPDs – possible definitions

Defining H and E GPDs in the standard way, expressions are frame-dependent:

SYMMETRIC frame:

$$F_{H(0)} = A_1 + \frac{z(\Delta_1^2 + \Delta_2^2)}{2P_3} A_6\,, \qquad F_{E(0)} = -A_1 + 2A_5 + \frac{z\left(4E^2 - \Delta_1^2 - \Delta_2^2\right)}{2P_3} A_6\,.$$

ASYMMETRIC frame:

$$F_{H(0)} = A_1 + \frac{\Delta_0}{P_0} A_3 + \frac{m^2 z \Delta_0}{2P_0 P_3} A_4 + \frac{z(\Delta_0^2 + \Delta_\perp^2)}{2P_3} A_6 + \frac{z(\Delta_0^3 + \Delta_0 \Delta_\perp^2)}{2P_0 P_3} A_8\,,$$

$$F_{E(0)} = -A_1 - \frac{\Delta_0}{P_0} A_3 - \frac{m^2 z(\Delta_0 + 2P_0)}{2P_0 P_3} A_4 + 2A_5 - \frac{z\left(\Delta_0^2 + 2P_0\Delta_0 + 4P_0^2 + \Delta_\perp^2\right)}{2P_3} A_6 - \frac{z\Delta_0\left(\Delta_0^2 + 2\Delta_0 P_0 + 4P_0^2 + \Delta_\perp^2\right)}{2P_0 P_3} A_8\,.$$

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One can also modify the definition to make it Lorentz-invariant and arrive at:

ANY frame:

$$F_H = A_1\,, \qquad F_E = -A_1 + 2A_5 + 2zP_3A_6\,.$$

With respect to the standard definition, removed/reduced contribution from A_3, A_4, A_6, A_8.

In terms of matrix elements: standard definition – only $\Pi_0(\Gamma_0)$, $\Pi_0(\Gamma_{1/2})$,

LI definition – additionally: $\Pi_{1/2}(\Gamma_3)$ (both frames), $\Pi_{1/2}(\Gamma_3)$, $\Pi_{1/2}(\Gamma_0)$, $\Pi_1(\Gamma_2)$, $\Pi_2(\Gamma_1)$ (asym.).

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Two definitions of $\widetilde{H}$:

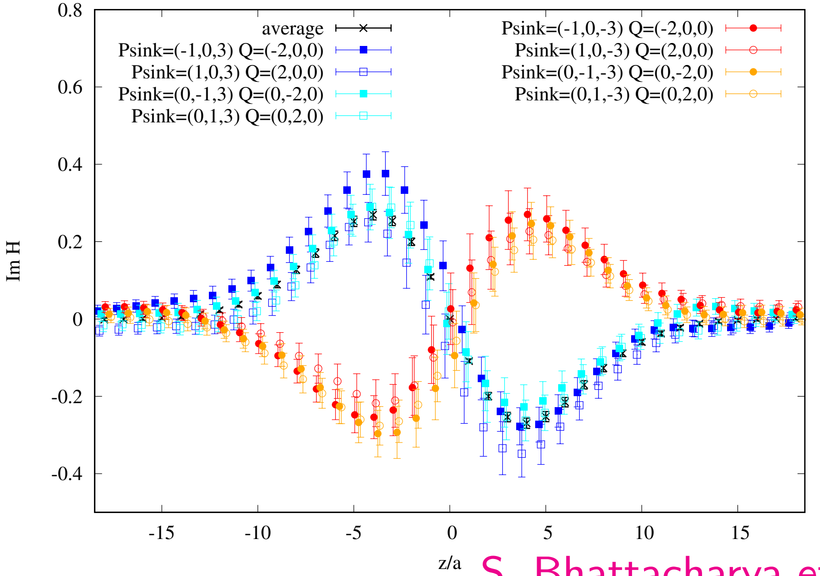
standard ($\gamma_5\gamma_3$ operator): $F_{\tilde{H}} = \widetilde{A_2} + zP_3\widetilde{A_6} - m^2 z^2 \widetilde{A_7}$,

another ($\gamma_5\gamma_i$ operators, $i = 0, 1, 2$): $F_{\tilde{H}} = \widetilde{A_2} + zP_3\widetilde{A_6}$.

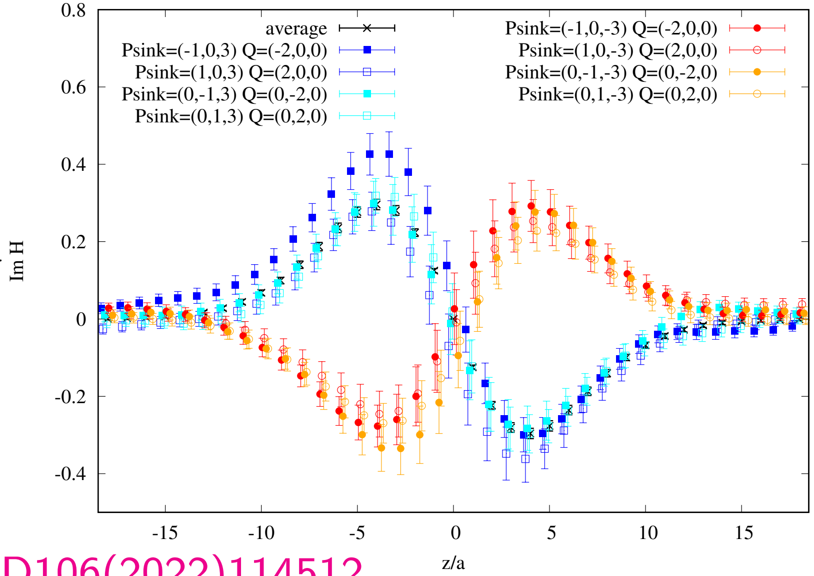
$\widetilde{E}$ impossible to extract at zero skewness: $F_{\tilde{E}} = 2\,\frac{P \cdot z}{\Delta \cdot z}\widetilde{A_3} + 2\widetilde{A_5}$.

Signal comparison for different definitions

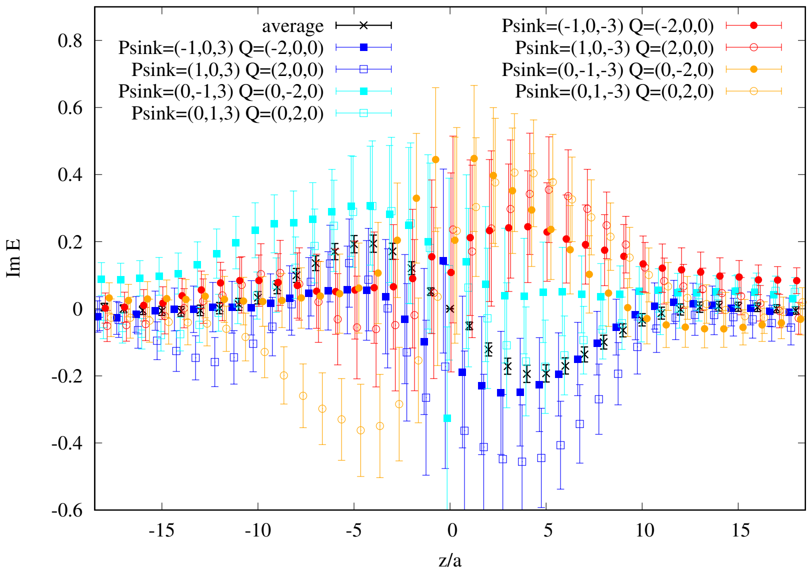
standard | Lorentz-invariant



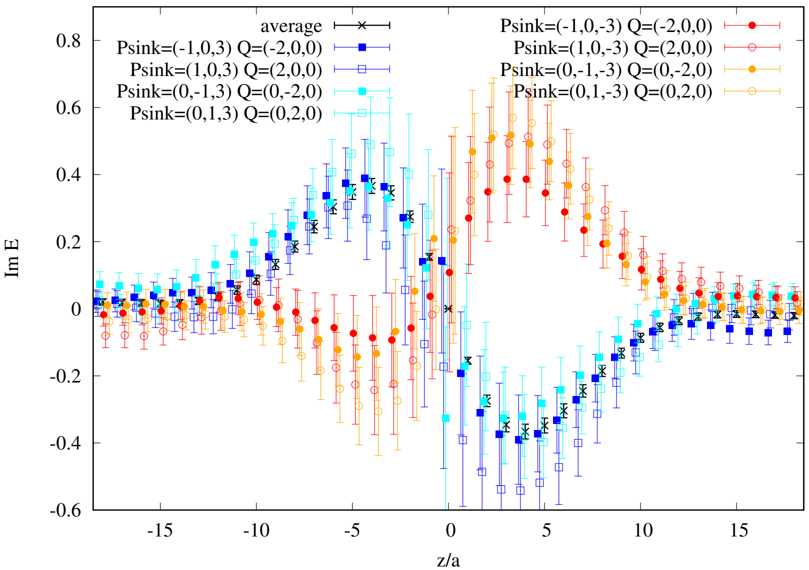
Im H



S. Bhattacharya et al., PRD106(2022)114512



Im E



Convergence of different definitions of $\tilde{H}/H/E$

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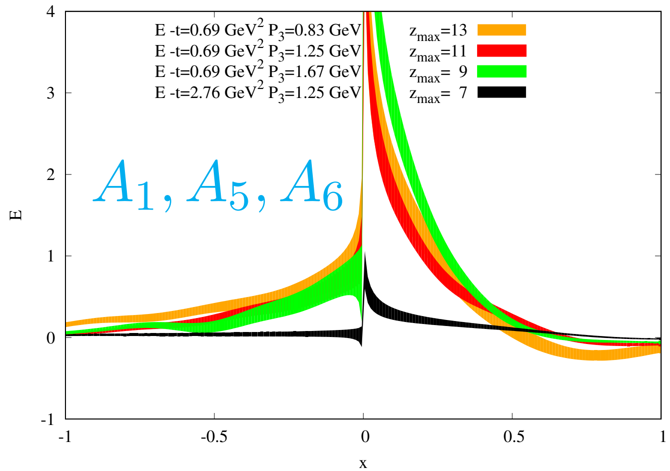
UNPOLARIZED



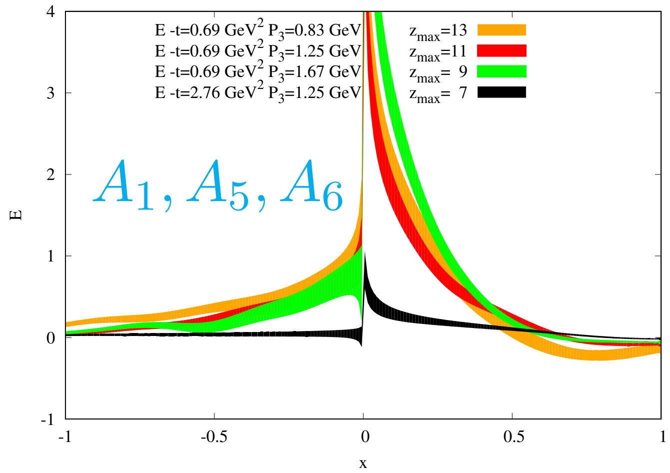
γ_0 operator (non-LI)

E-GPD

Convergence of different definitions of $\tilde{H}/H/E$

STANDARD

UNPOLARIZED



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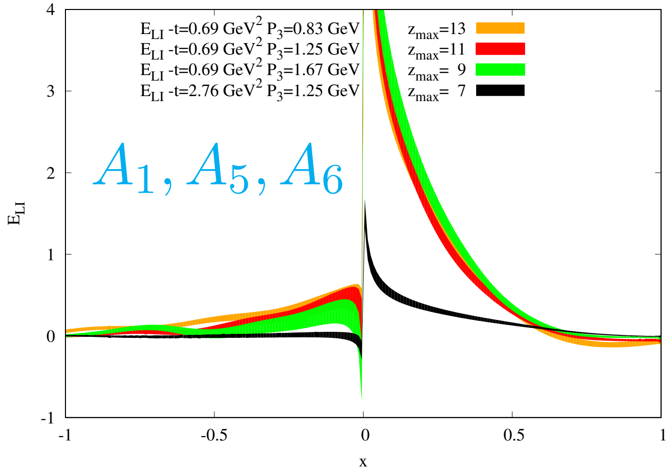
E-GPD

LORENTZ–INV.

γ_0, γ_T operators (LI)

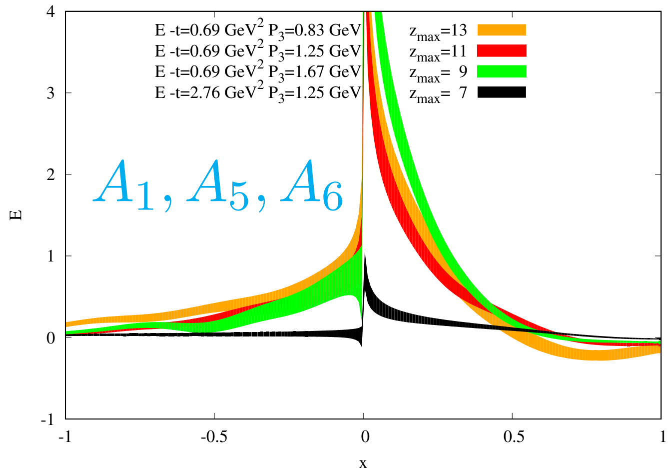


Convergence of different definitions of $\tilde{H}/H/E$

STANDARD

LORENTZ–INV.

UNPOLARIZED



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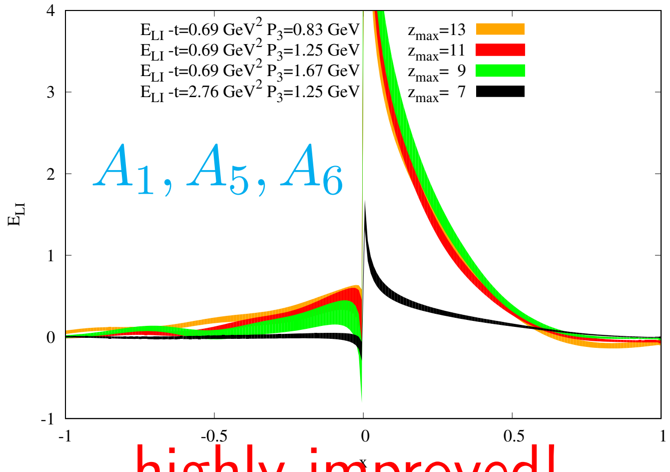
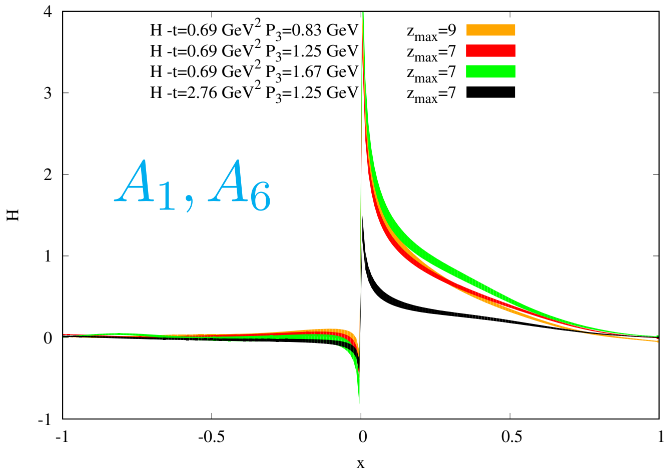
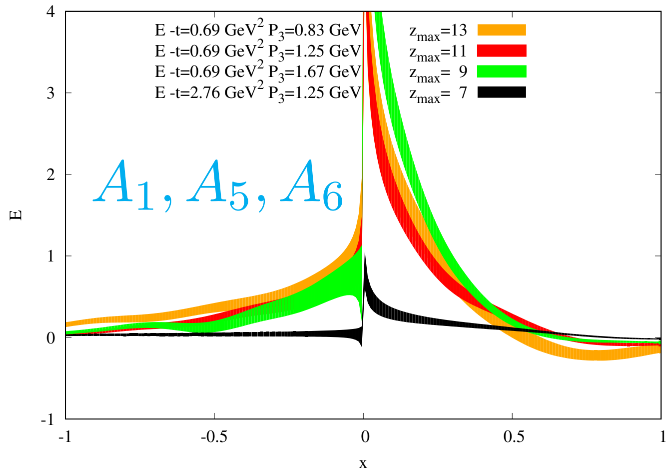


highly-improved!

STANDARD

UNPOLARIZED





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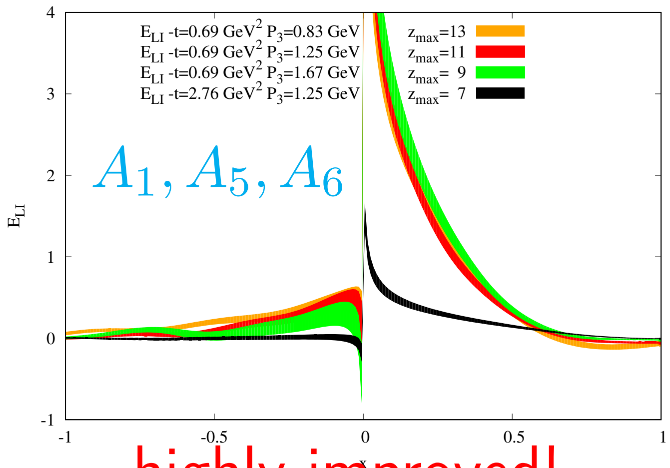

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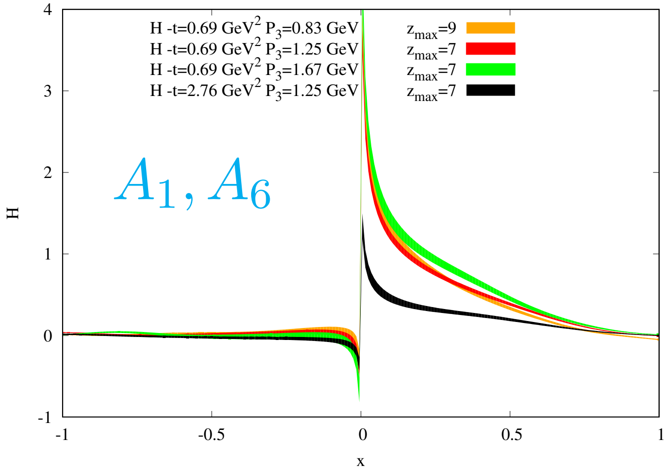
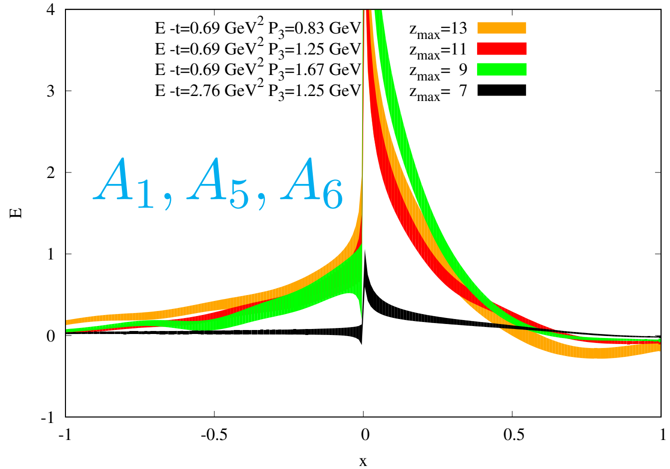
UNPOLARIZED

STANDARD





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H-GPD E-GPD

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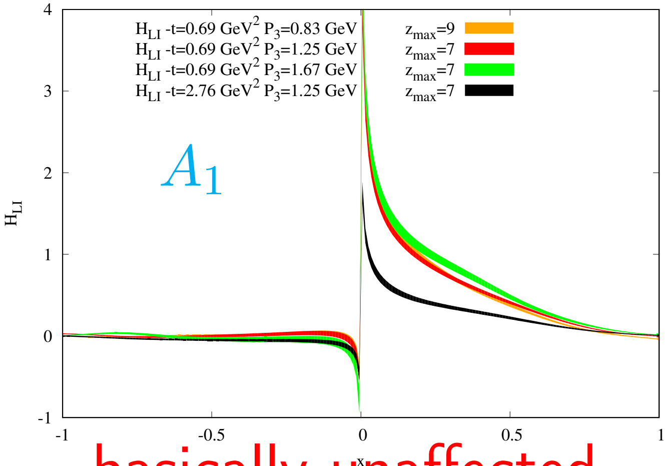
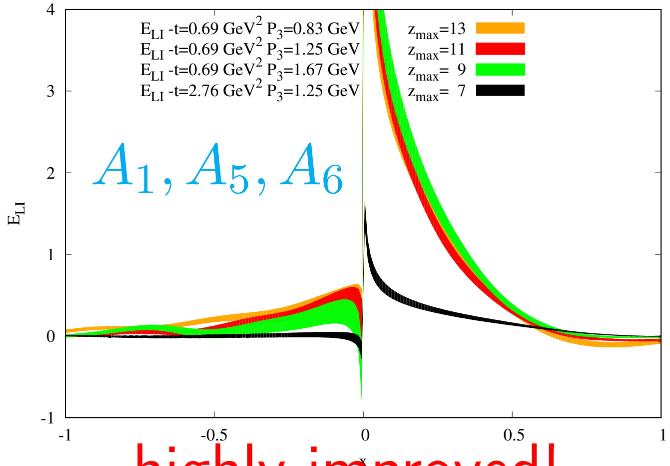
LORENTZ-INV.





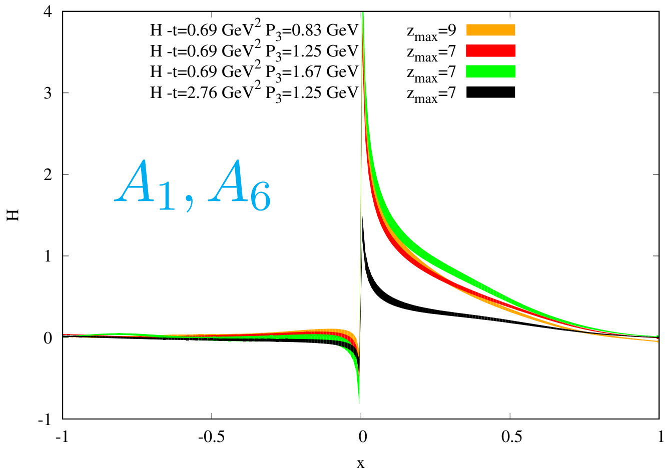
basically unaffected highly-improved!

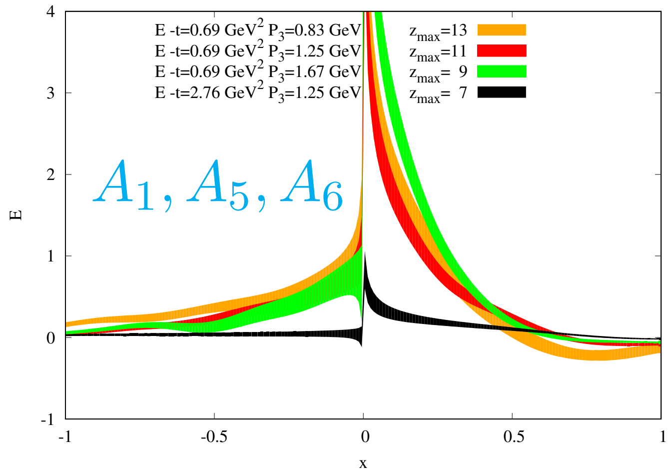
Convergence of different definitions of $\tilde{H}/H/E$

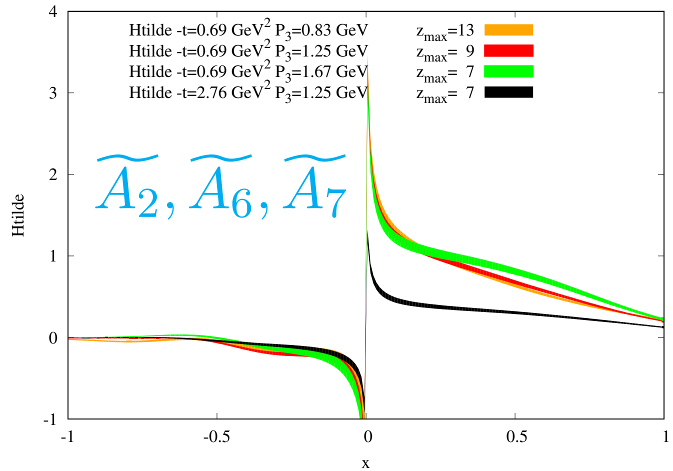
STANDARD

UNPOLARIZED

HELICITY







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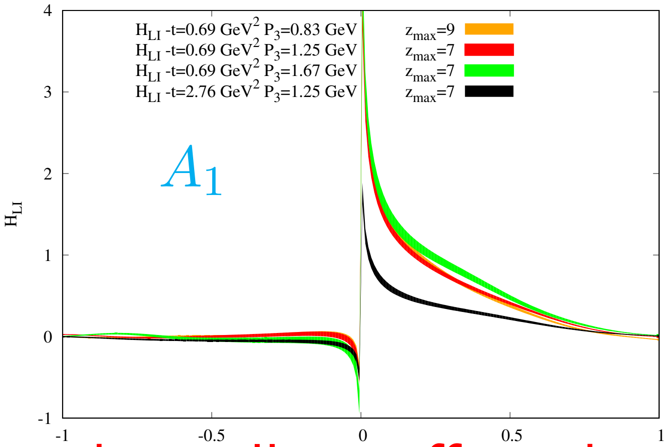
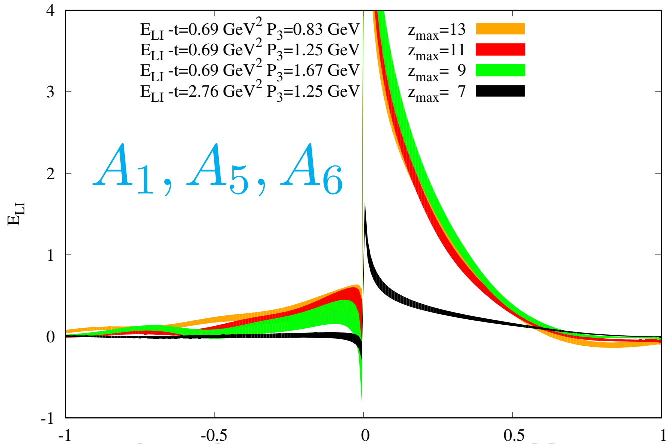




basically unaffected

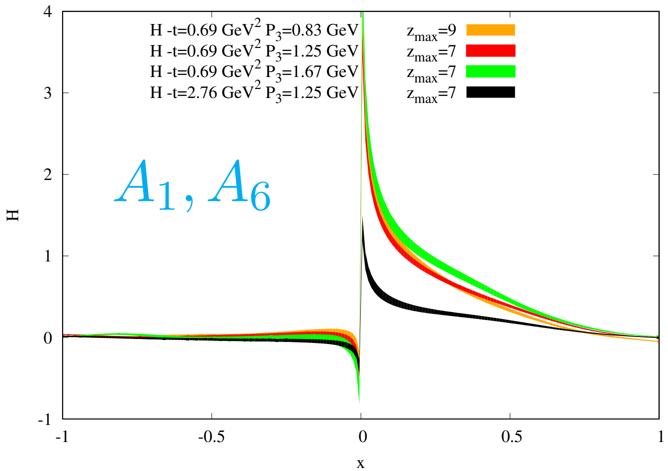
highly-improved!

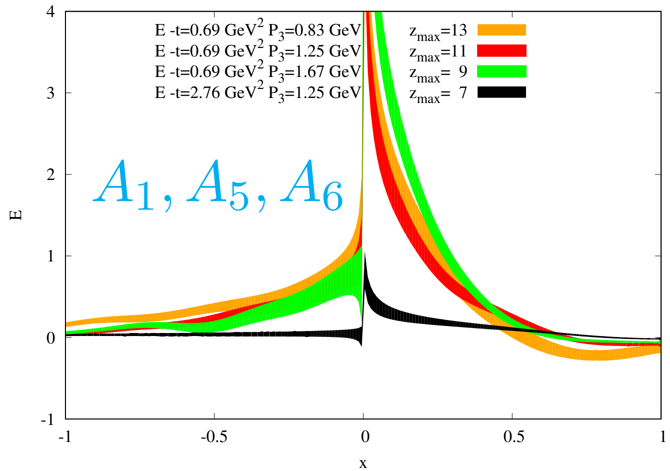
Convergence of different definitions of $\tilde{H}/H/E$

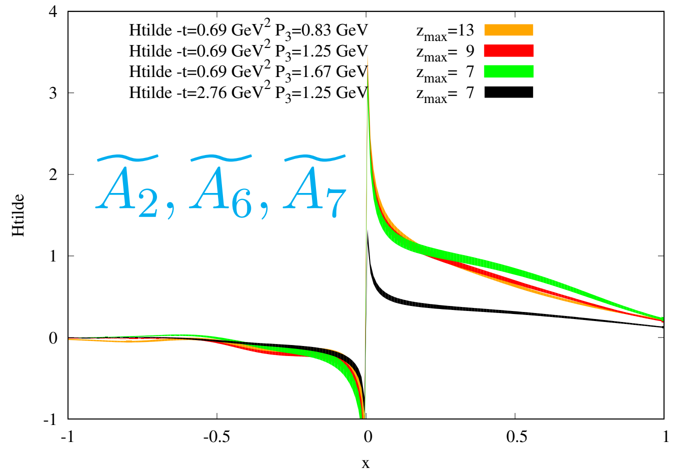
STANDARD

UNPOLARIZED

HELICITY







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LORENTZ-INV.

H-GPD

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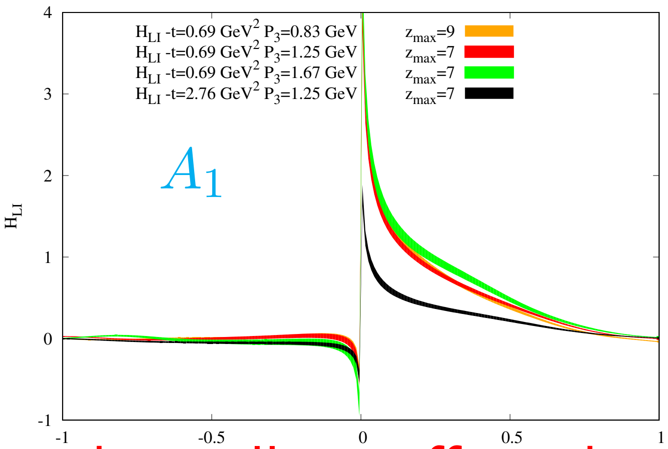
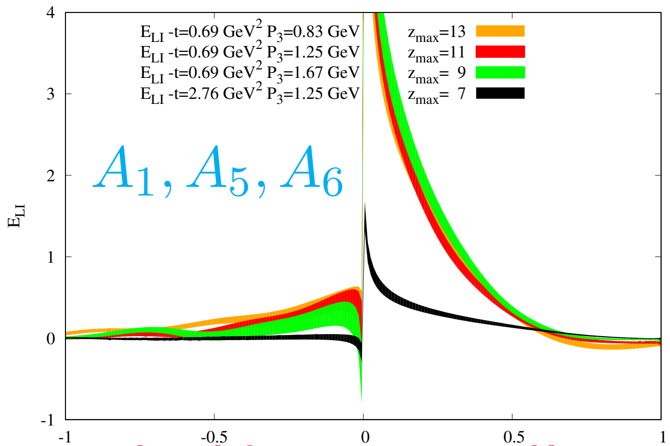
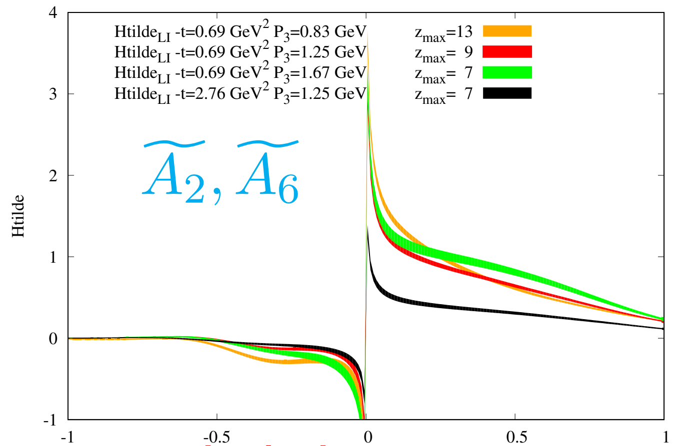
γ_0, γ_T operators (LI)

$\gamma_5\gamma_0,\ \gamma_5\gamma_T$ operators (LI)





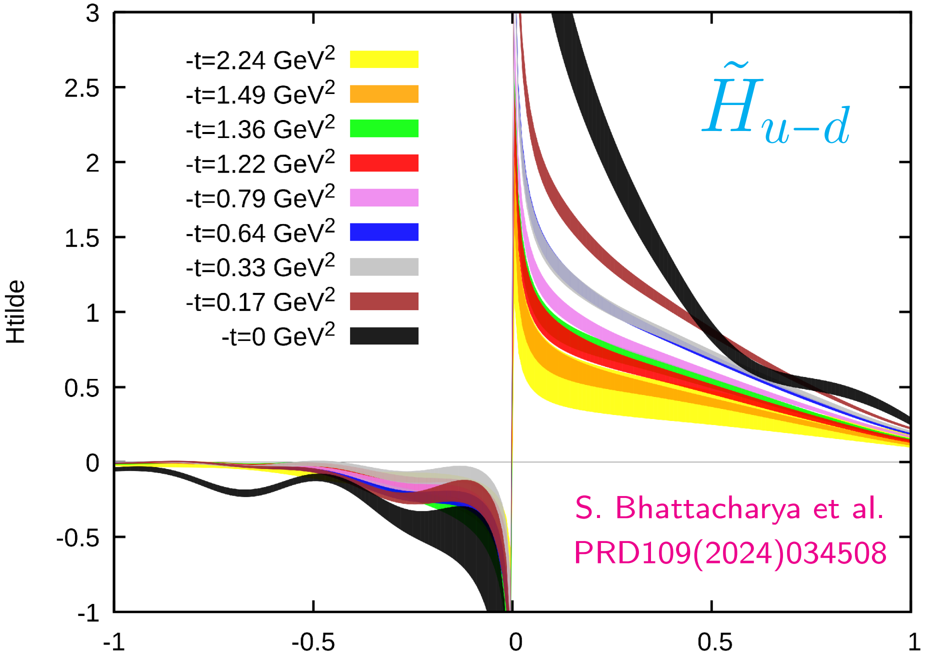


basically unaffected

highly-improved!

slightly worse

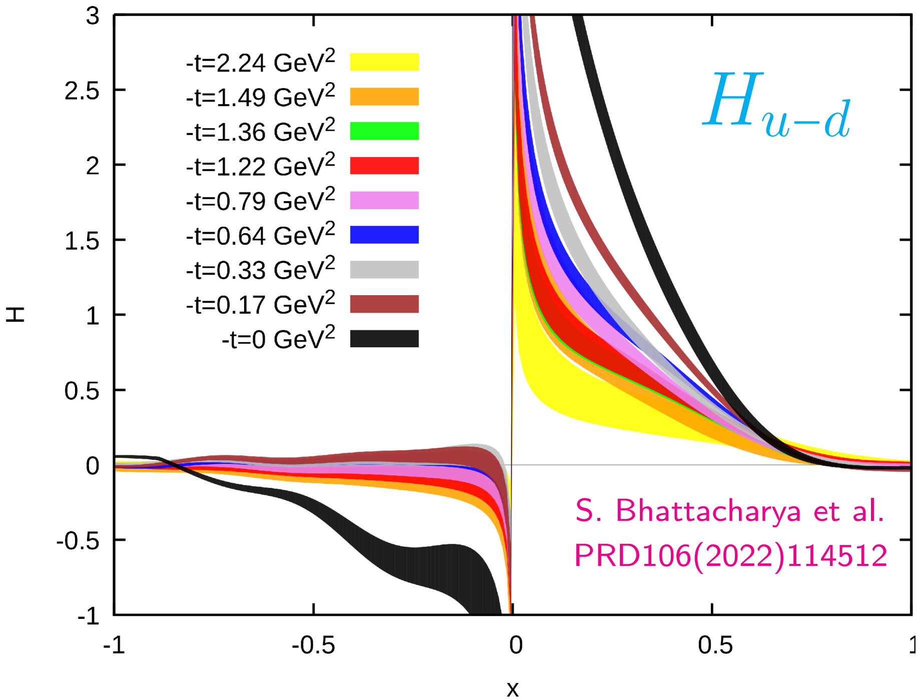
t-dependence of $\tilde{H}/H/E$ GPDs (quasi)

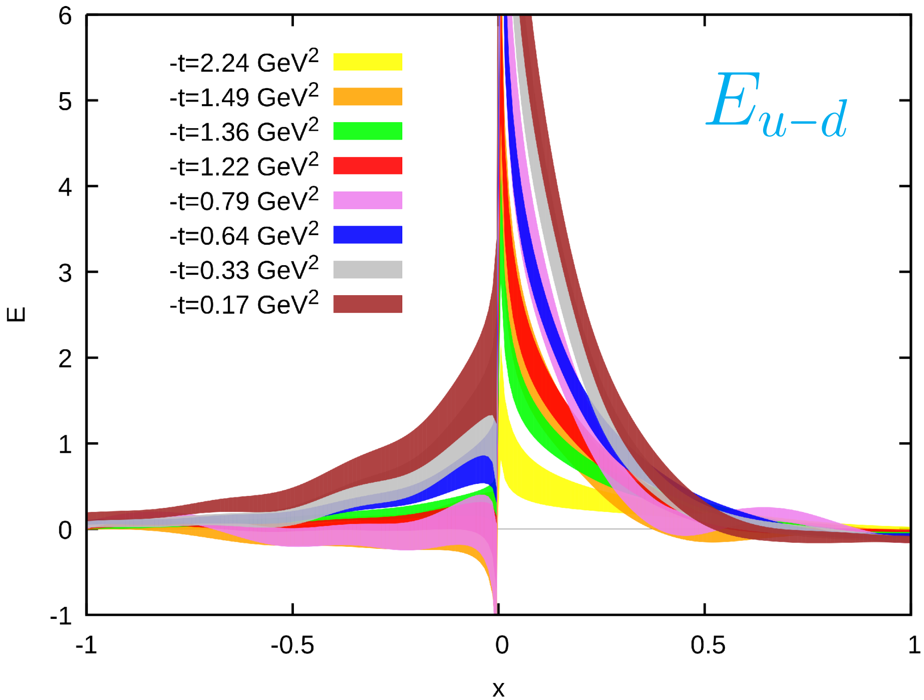


$\Delta = (1,0,0) \Rightarrow -t = 0.17$ GeV^2
$\Delta = (1,1,0) \Rightarrow -t = 0.33$ GeV^2
$\Delta = (2,0,0) \Rightarrow -t = 0.64$ GeV^2
$\Delta = (2,1,0) \Rightarrow -t = 0.79$ GeV^2
$\Delta = (2,2,0) \Rightarrow -t = 1.22$ GeV^2
$\Delta = (3,0,0) \Rightarrow -t = 1.36$ GeV^2
$\Delta = (3,1,0) \Rightarrow -t = 1.49$ GeV^2
$\Delta = (4,0,0) \Rightarrow -t = 2.24$ GeV^2

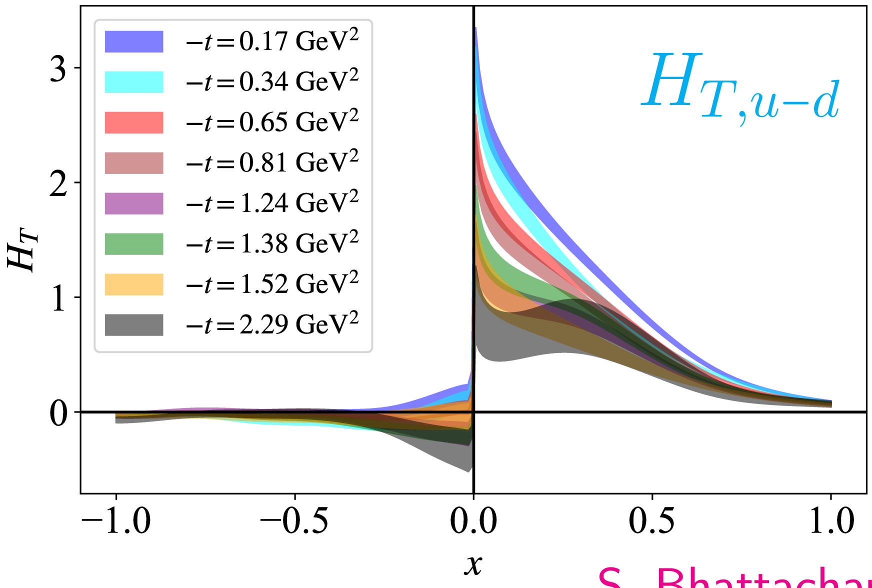
Impact parameter distribution:

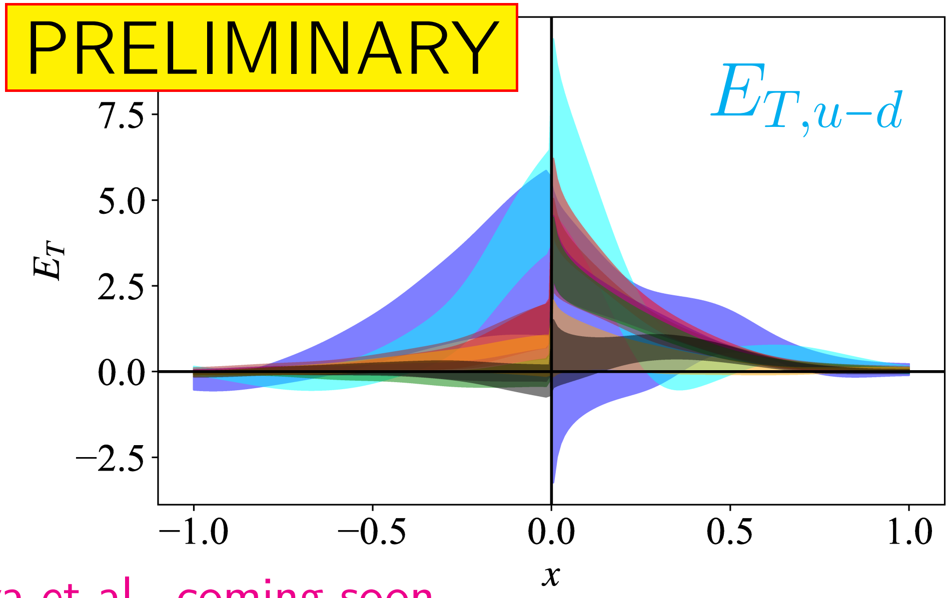
$$GPD(x, b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-ib_\perp \cdot \Delta_\perp} GPD(x,t)$$



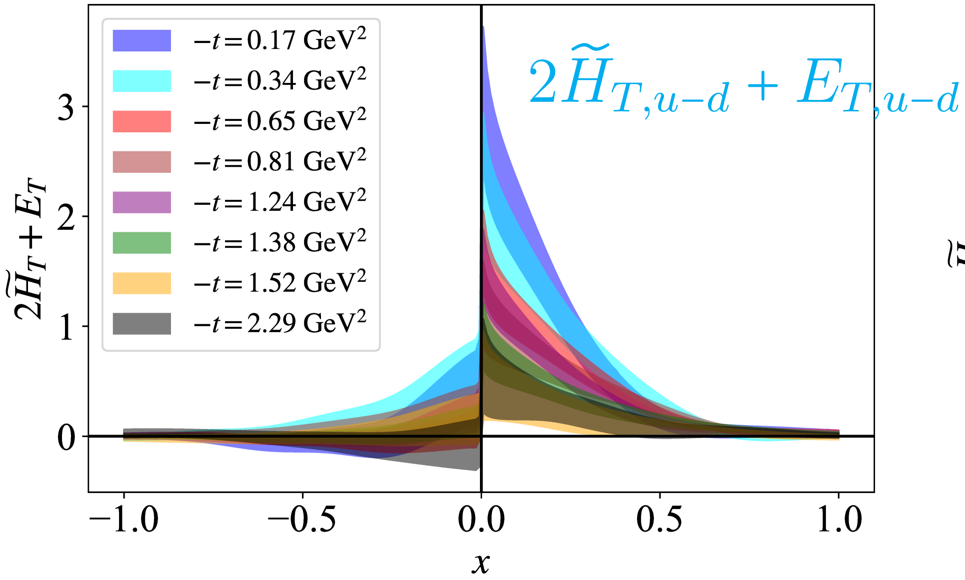


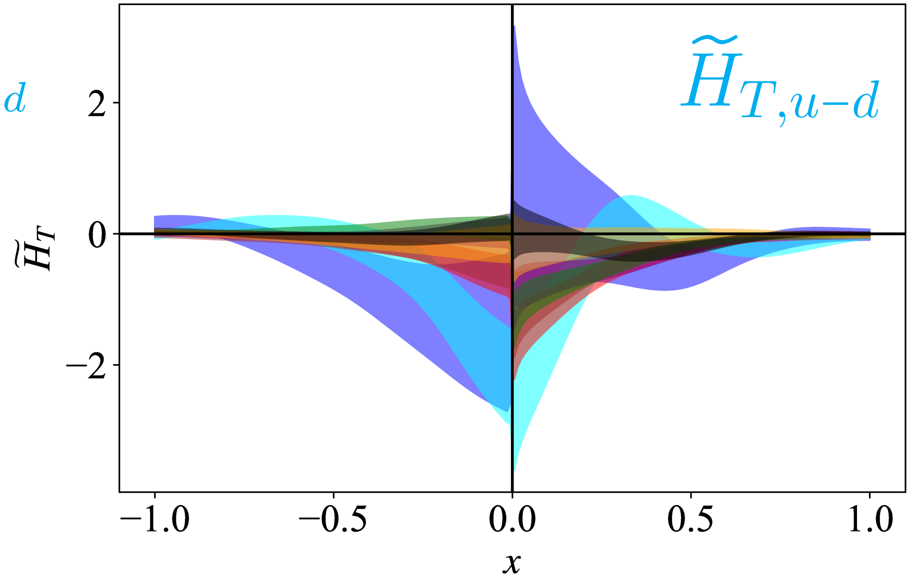
t-dependence of $H_T/E_T/\widetilde{H}_T/2\widetilde{H}_T + E_T$ GPDs (quasi)





S. Bhattacharya et al., coming soon





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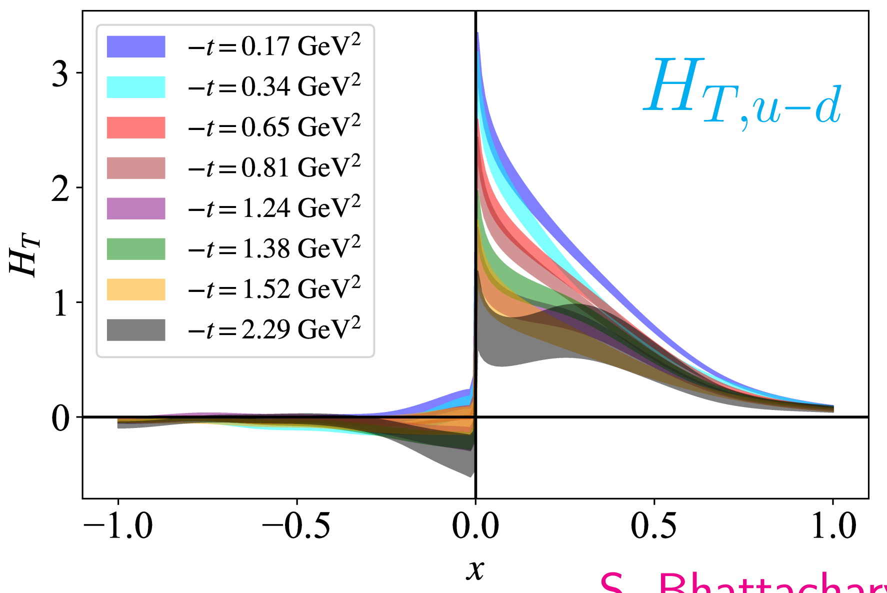

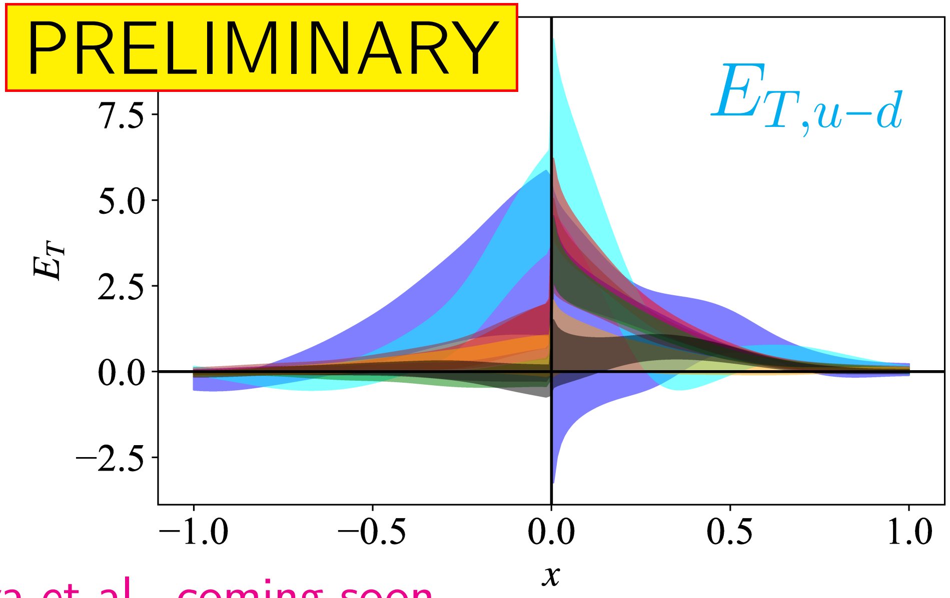

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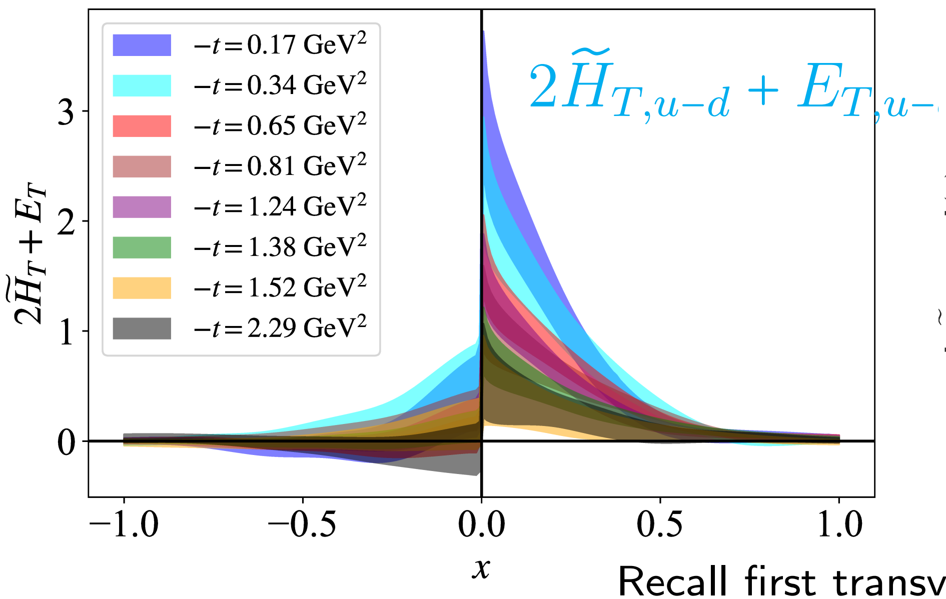
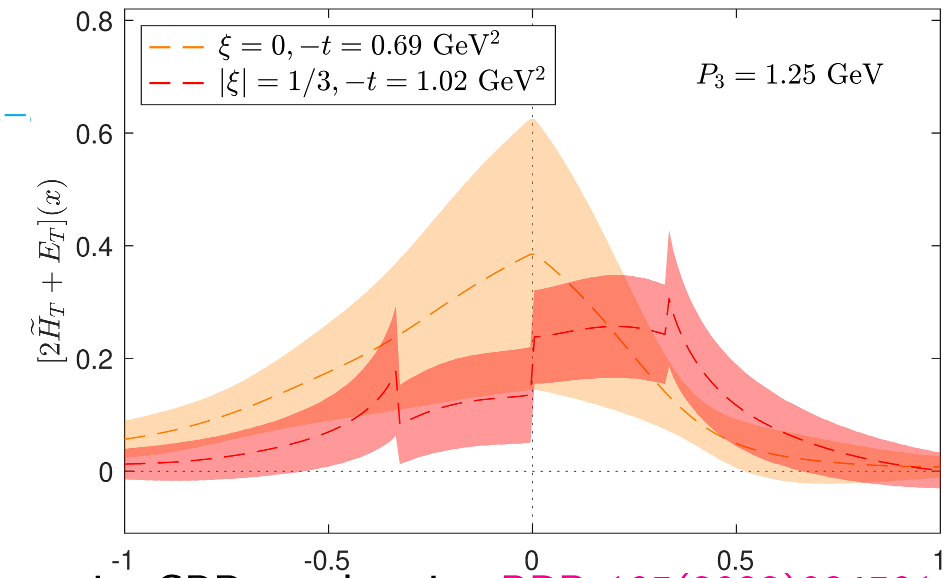
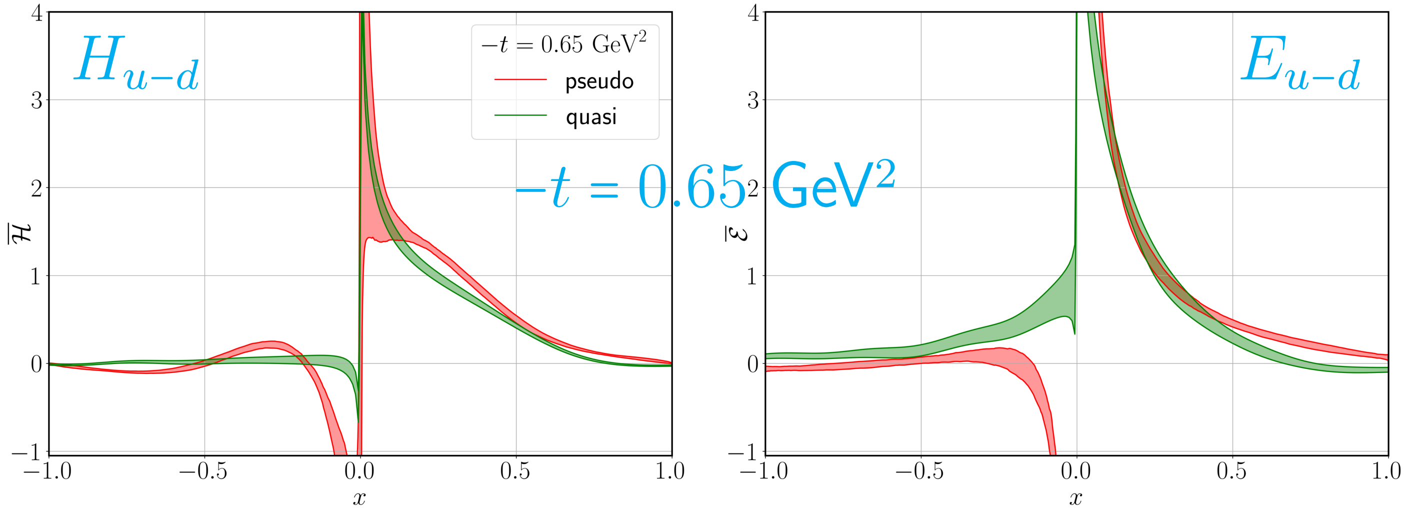




Recall first transversity GPDs exploration PRD 105(2022)034501

GPDs from quasi and pseudo

The same lattice data can also be analyzed within the approach of pseudo-GPDs



S. Bhattacharya, KC, M. Constantinou, A. Metz, N. Nurminen, F. Steffens, PRD110(2024)054502

Note, however, the different status of x-dependence reconstruction:

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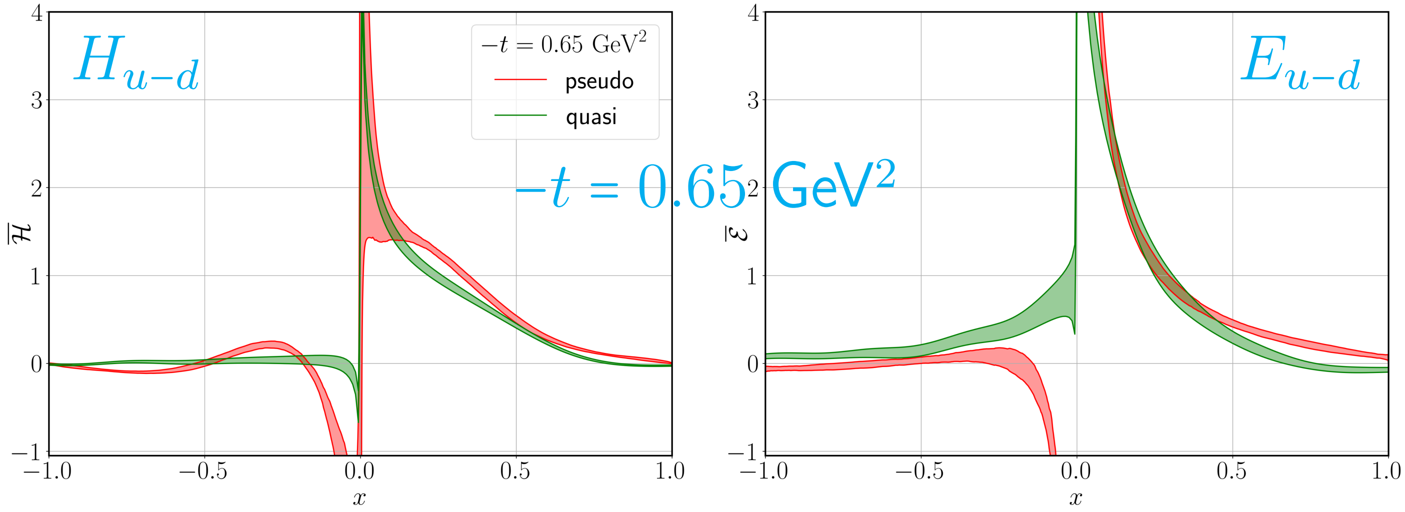

S. Bhattacharya, KC, M. Constantinou, A. Metz, N. Nurminen, F. Steffens, PRD110(2024)054502

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  reason: power corrections of $\mathcal{O}(\Lambda^2_{\rm QCD}/x^2 P_3^2)$, $\mathcal{O}(\Lambda^2_{\rm QCD}/(1-x)^2 P_3^2)$.

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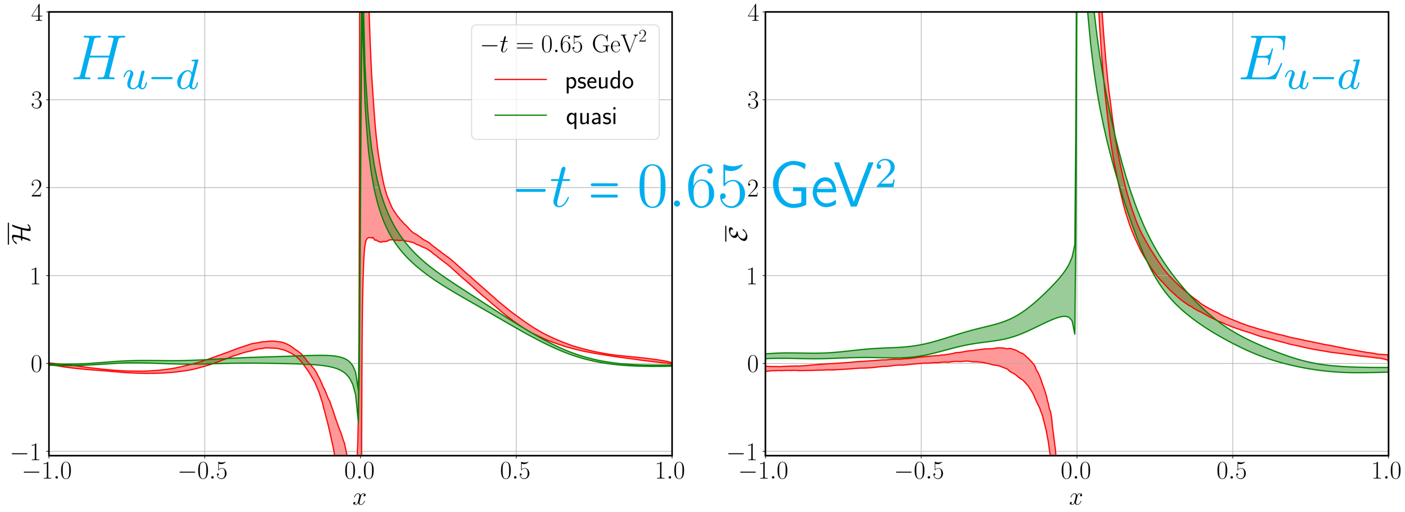

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  model-independent – **fully-reliable** GPDs moments ($\nu_{\rm max} \Rightarrow \langle x^{n_{\rm max}}\rangle$)

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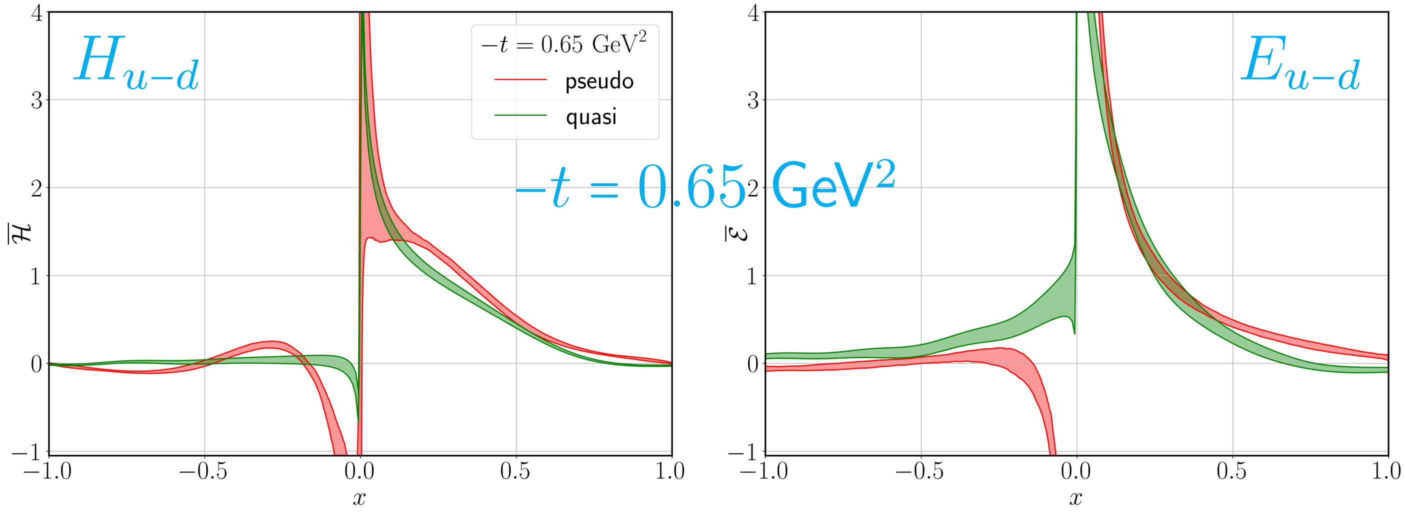

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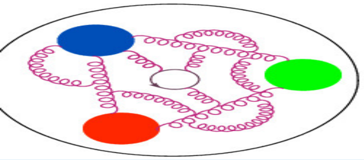
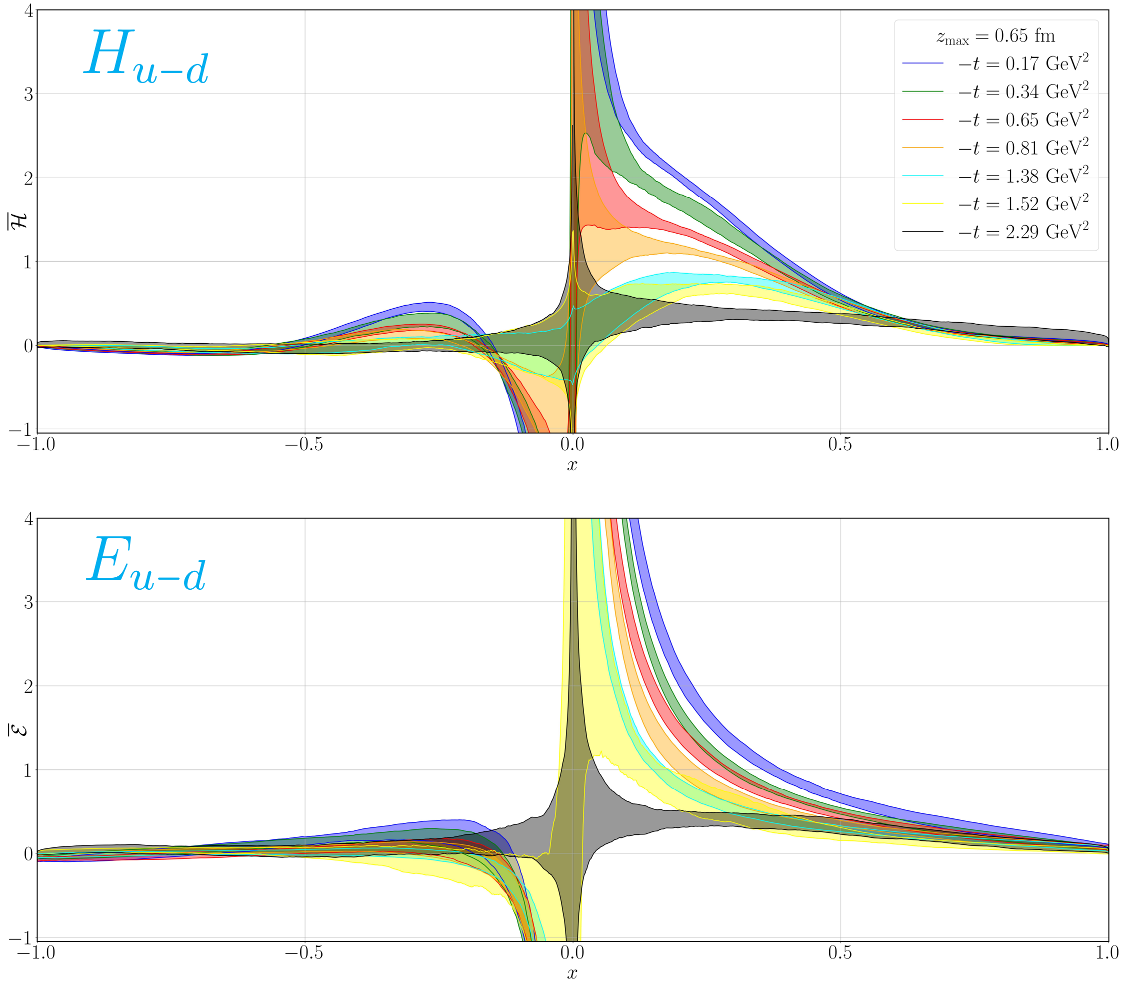
$\Rightarrow$ **COMPLEMENTARITY** – e.g. extract $x \in [0.2, 0.8]$ from quasi
+ add constraints from lowest 6 moments from pseudo

See also: X. Ji, LaMET vs. SDE: Contrast and Complementarity, arXiv:2209.09332



t-dependence of H/E GPDs (pseudo)

S. Bhattacharya, KC, M. Constantinou, A. Metz, N. Nurminen, F. Steffens, PRD110(2024)054502



Qualitatively similar picture to the one from quasi-GPDs.
Quantitative conclusions after careful estimation of systematics!

Short-distance factorization (SDF) can also be used to extract moments of GPDs.

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For ratio-renormalized H/E: $\mathcal{F}^{\overline{\mathrm{MS}}}(z,P,\Delta) = \sum_{n=0} \frac{(-izP)^n}{n!} C_n^{\overline{\mathrm{MS}}}(\mu^2 z^2)\langle x^n \rangle + \mathcal{O}(\Lambda_{\mathrm{QCD}}^2 z^2)$,

$C_n^{\overline{\mathrm{MS}}}(\mu^2 z^2)$ – Wilson coefficients (NNLO for $u-d$, NLO for $u+d$)

GPDs moments from OPE of non-local operators

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Moments of impact parameter parton distributions in the transverse plane:

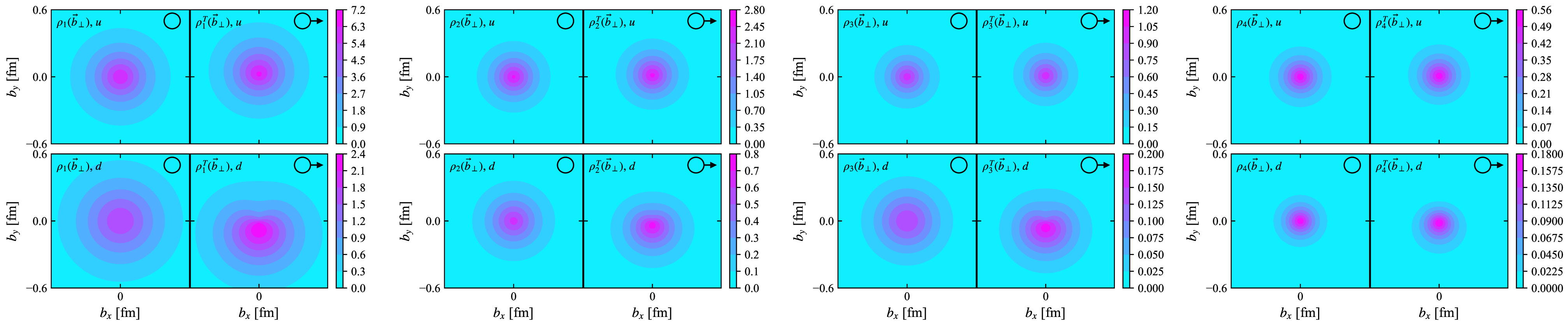
$$\rho_{n+1}(\vec{b}_\perp)=\int\frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}A_{n+1,0}(-\vec{\Delta}_\perp^2)e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp},$$

$$\rho^T_{n+1}(\vec{b}_\perp)=\int\frac{d^2\vec{\Delta}_\perp}{(2\pi)^2}\left[A_{n+1,0}(-\vec{\Delta}_\perp^2)+i\frac{\Delta_y}{2M}B_{n+1,0}(-\vec{\Delta}_\perp^2)\right]e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp}.$$

1st moment | 2nd moment | 3rd moment | 4th moment

u

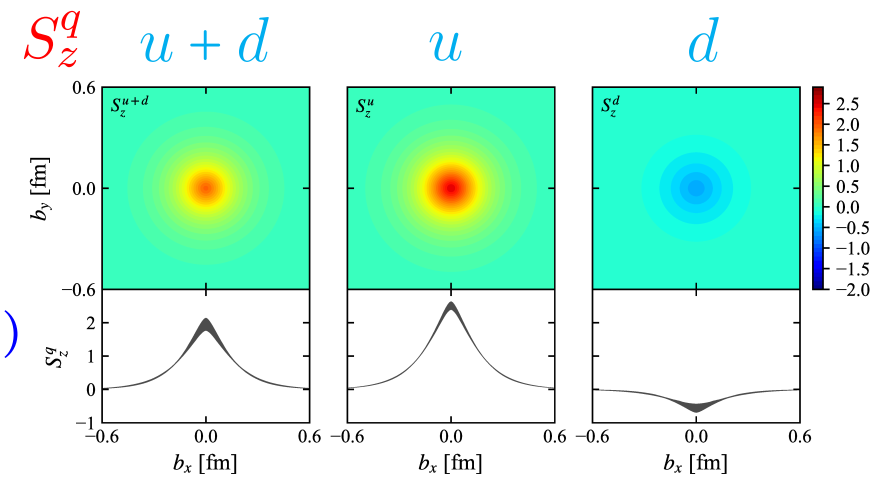
d

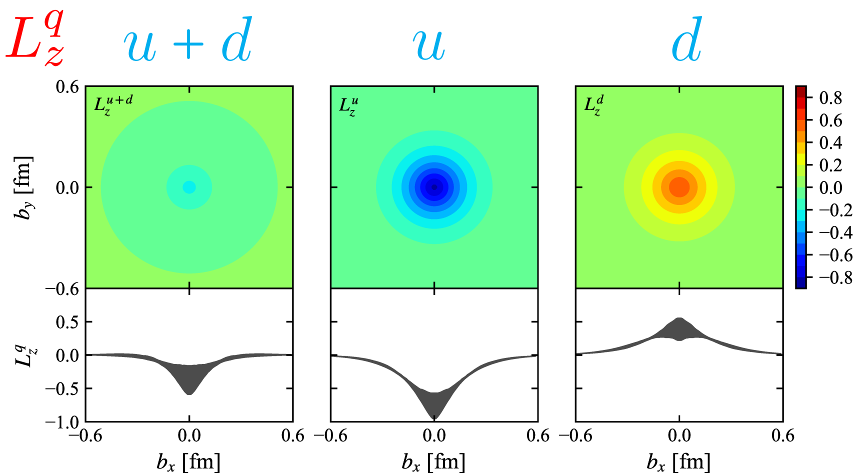


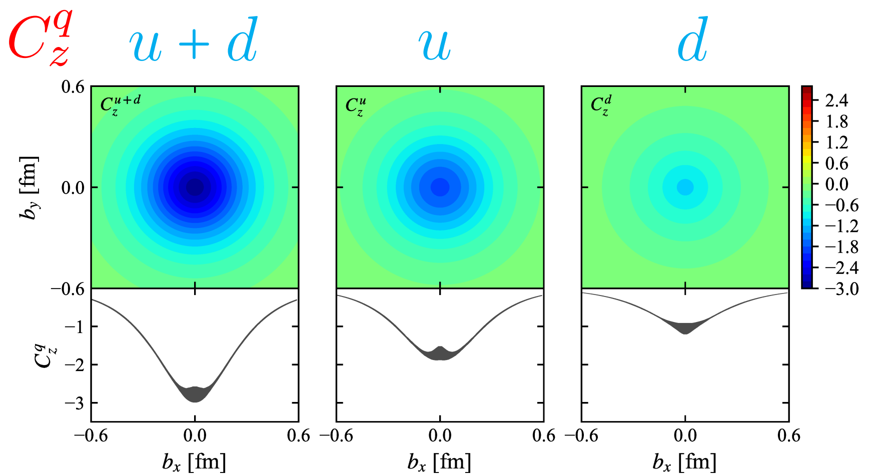
S. Bhattacharya, KC, M. Constantinou, X. Gao, A. Metz, J. Miller, S. Mukherjee, P. Petreczky, F. Steffens, Y. Zhao, PRD108(2023)014507

Similar extraction of moments $\tilde{A}^{u\pm d}_{n,0}$ using SDF can be done in the axial vector sector.
In particular, one can extract moments related to:

- quark helicity: $S_z^q = \frac{1}{2}\int_{-1}^{1} dx \widetilde{H}^q(x,0,0) = \frac{1}{2}\tilde{A}^q_{1,0}(0)$
- quark OAM: $L_z^q = J_z^q - S_z^q = \frac{1}{2}(A^q_{2,0}(0) + B^q_{2,0}(0)) - \frac{1}{2}\tilde{A}^q_{1,0}(0)$
- spin-orbit correlation: $C_z^q = \frac{1}{2}(\tilde{A}^q_{2,0}(0) - A^q_{1,0}(0))$
  (ignoring term suppressed by $m_q/2m_N$ with $E_T + 2\tilde{H}_T$)







S. Bhattacharya, KC, M. Constantinou, X. Gao, A. Metz, J. Miller, S. Mukherjee, P. Petreczky, F. Steffens, Y. Zhao, JHEP01(2025)146

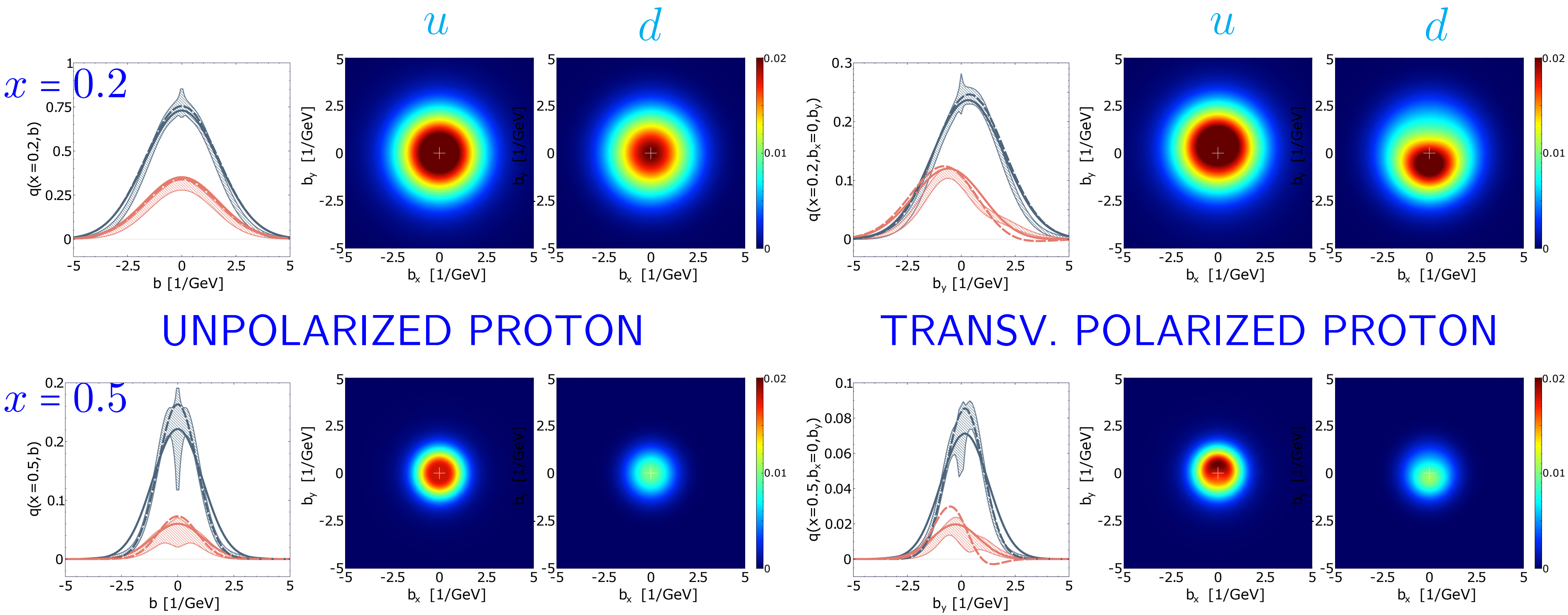
Another opportunity for **COMPLEMENTARITY**:
combine lattice data with phenomenological/experimental data

Combining lattice with pheno/exp data

Another opportunity for **COMPLEMENTARITY**:

combine lattice data with phenomenological/experimental data:

- lattice: double ratios removing explicit info on PDFs and EFFs (thus, lots of systematics mildened),
- pheno/exp: proton's and neutron's magnetic and electric EFFs and their ratios.



KC, M. Constantinou, P. Sznajder, J. Wagner, Phys. Rev. D110 (2024) 114025

Twist-3 GPDs

PDFs/GPDs can be classified according to their twist, which describes the order in $1/Q$ at which they appear in the factorization of structure functions.

Twist-2 – probability densities for finding partons carrying fraction x of the hadron momentum.

Twist-3 – no density interpretation, contain important information about qgq correlations, appear in QCD factorization theorems for a variety hard scattering processes, interesting connections with TMDs, important for JLab12 and EIC, but difficult to measure.

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Exploratory studies: S. Bhattacharya, KC, M. Constantinou A. Metz, A. Scapellato, F. Steffens

- matching for twist-3 PDFs: g_T, h_L, e
  S. Bhattacharya et al., PRD102(2020)034005, PRD102(2020)114025
  BC-type sum rules S. Bhattacharya, A. Metz, PRD105(2022)054027
  Note: neglected qgq correlations
  see also: V. Braun, Y. Ji, A. Vladimirov, JHEP 05(2021)086, 11(2021)087
- lattice extraction of $g_T^{u-d}(x)$ and $h_L^{u-d}(x)$
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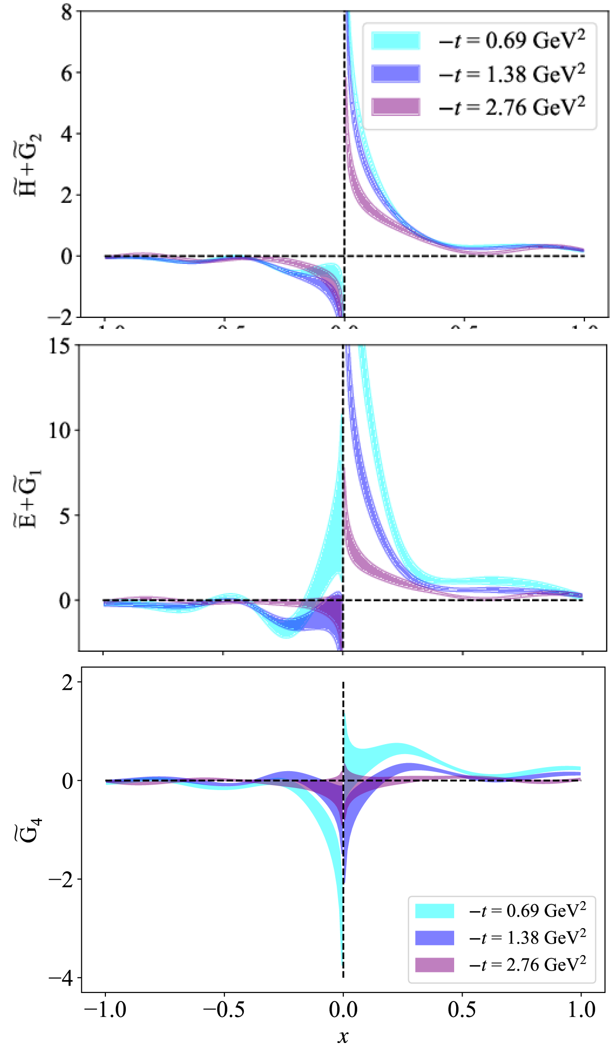

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  S. Bhattacharya et al., PRD102(2020)111501(R), PRD104(2021)114510
- first exploration of twist-3 axial GPDs: $\widetilde{G}_1, \widetilde{G}_2, \widetilde{G}_3, \widetilde{G}_4$
  S. Bhattacharya et al., PRD108(2023)054501

$$\mathcal{F}^{[\gamma_j \gamma_5]} = -i\frac{\Delta_j \gamma_5}{2m} F_{\widetilde{E}+\widetilde{G}_1} + \gamma_j \gamma_5 \; F_{\widetilde{H}+\widetilde{G}_2} + \frac{\Delta_j \gamma_3 \gamma_5}{P_3} F_{\widetilde{G}_3} - \frac{\mathrm{sign}[P_3]\, \epsilon_\perp^{j\,\rho} \Delta_\rho \gamma_3}{P_3} F_{\widetilde{G}_4}$$

Conclusions and prospects

- Main message: **probing nucleon's 3D structure with LQCD becomes feasible!**
- Recent breakthrough for GPDs: **computationally more efficient calculations in non-symmetric frames**.
- Also, **new definitions of GPDs with different convergence properties** – e.g. faster convergence in some instances.
- A lot of follow-up work in progress: **non-zero skewness**, other twist-3 GPDs, meson GPDs, extensions of kinematics.
- Obviously, GPDs much more challenging than PDFs.
- Several challenges have to be overcome – control of lattice and other systematics.
- Quantification of systematics very laborious, but crucial.
- **Consistent progress will ensure complementary role to phenomenology and experiment!**

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Thank you for your attention!

Backup slides

Lorentz-covariant parametrization

Main theoretical tool: S. Bhattacharya et al., PRD106(2022)114512

Lorentz-covariant parametrization of matrix elements (e.g. vector case):

$$F^{\mu}(z,P,\Delta)=\bar{u}(p',\lambda')\Big[\frac{P^{\mu}}{m}A_1+mz^{\mu}A_2+\frac{\Delta^{\mu}}{m}A_3+im\sigma^{\mu z}A_4+\frac{i\sigma^{\mu\Delta}}{m}A_5+\frac{P^{\mu}i\sigma^{z\Delta}}{m}A_6+\frac{z^{\mu}i\sigma^{z\Delta}}{m}A_7+\frac{\Delta^{\mu}i\sigma^{z\Delta}}{m}A_8\Big]u(p,\lambda),$$

(inspired by: S. Meissner, A. Metz, M. Schlegel, JHEP08(2009)056).

- most general parametrization in terms of 8 linearly-independent Lorentz structures,
- 8 Lorentz-invariant amplitudes $A_i(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$.

Example: (γ_0 insertion, unpolarized projector)

symmetric frame:

$$\Pi_0^s(\Gamma_0)=C\left(\frac{E\left(E(E+m)-P_3^2\right)}{2m^3}A_1+\frac{(E+m)\left(-E^2+m^2+P_3^2\right)}{m^3}A_5+\frac{EP_3\left(-E^2+m^2+P_3^2\right)z}{m^3}A_6\right),$$

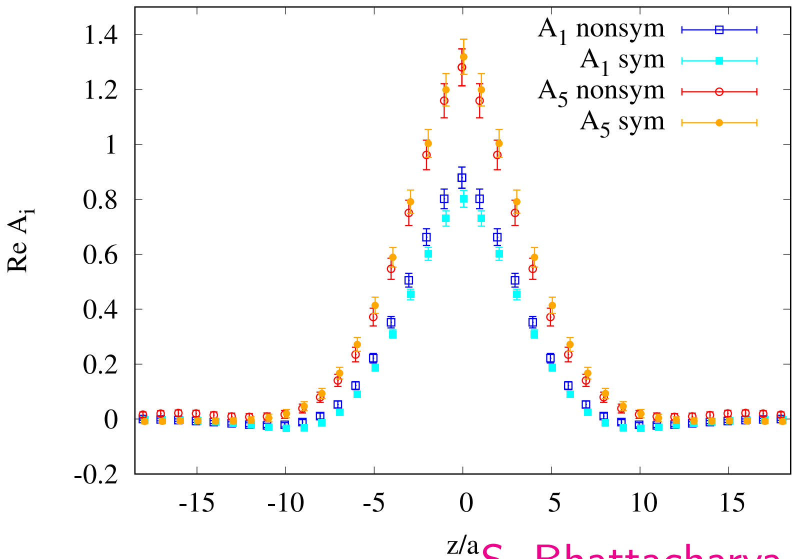
asymmetric frame:

$$\Pi_0^a(\Gamma_0) = C\Bigg(-\frac{(E_f+E_i)(E_f-E_i-2m)(E_f+m)}{8m^3}A_1-\frac{(E_f-E_i-2m)(E_f+m)(E_f-E_i)}{4m^3}A_3+\frac{(E_i-E_f)P_3z}{4m}A_4$$
$$+\frac{(E_f+E_i)(E_f+m)(E_f-E_i)}{4m^3}A_5+\frac{E_f(E_f+E_i)P_3(E_f-E_i)z}{4m^3}A_6+\frac{E_fP_3(E_f-E_i)^2z}{2m^3}A_8\Bigg).$$

- matrix elements $\Pi_\mu(\Gamma_\nu)$ are **frame-dependent**,
- but the amplitudes A_i are **frame-invariant**.

Proof of concept (comparison between frames)

A_1, A_5 (leading ones) A_2, A_3, A_4, A_6, A_7, A_8 (suppressed ones)



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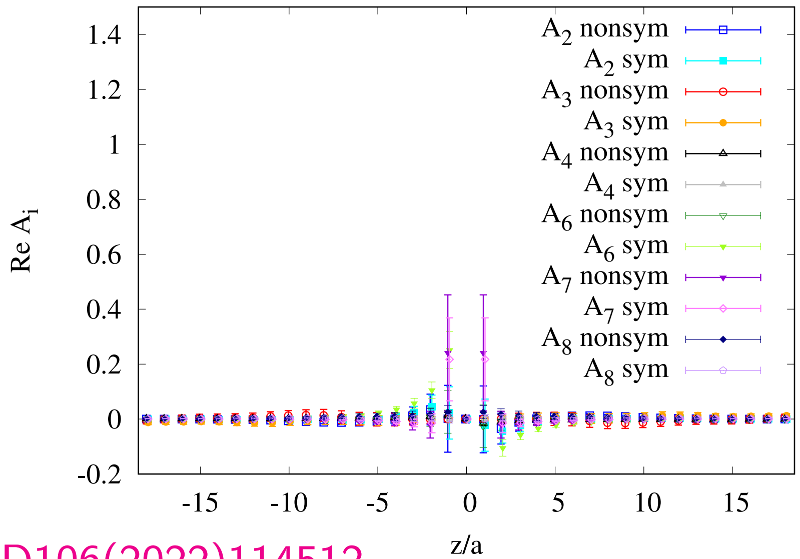
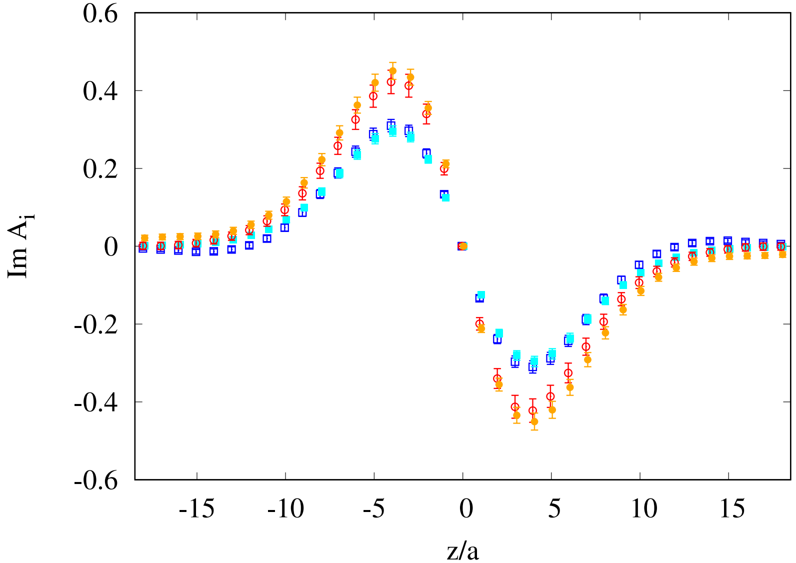


S. Bhattacharya et al., PRD106(2022)114512



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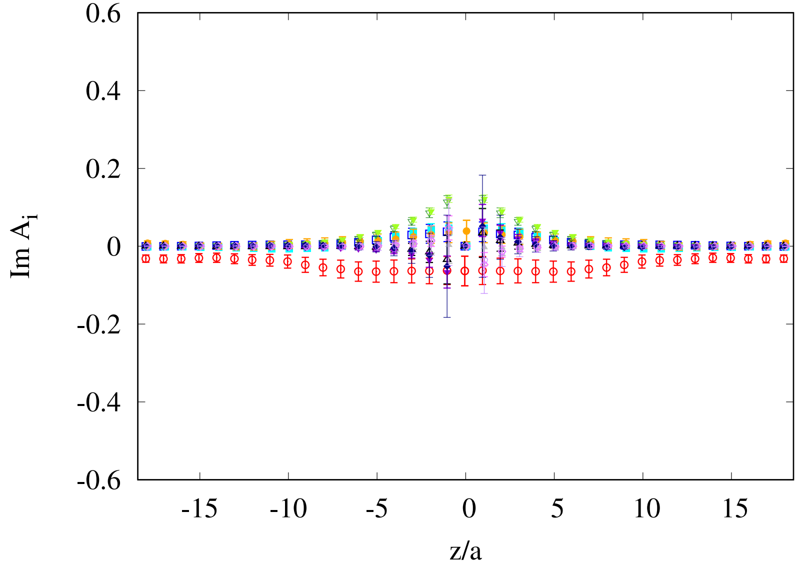


H and E GPDs – possible definitions

Defining H and E GPDs in the standard way, expressions are frame-dependent:

SYMMETRIC frame:

$$F_{H(0)} = A_1 + \frac{z(\Delta_1^2 + \Delta_2^2)}{2P_3} A_6 \,,$$

$$F_{E(0)} = -A_1 + 2A_5 + \frac{z\left(4E^2 - \Delta_1^2 - \Delta_2^2\right)}{2P_3} A_6 \,.$$

ASYMMETRIC frame:

$$F_{H(0)} = A_1 + \frac{\Delta_0}{P_0} A_3 + \frac{m^2 z \Delta_0}{2P_0 P_3} A_4 + \frac{z(\Delta_0^2 + \Delta_\perp^2)}{2P_3} A_6 + \frac{z(\Delta_0^3 + \Delta_0 \Delta_\perp^2)}{2P_0 P_3} A_8 \,,$$

$$F_{E(0)} = -A_1 - \frac{\Delta_0}{P_0} A_3 - \frac{m^2 z(\Delta_0 + 2P_0)}{2P_0 P_3} A_4 + 2A_5 - \frac{z\left(\Delta_0^2 + 2P_0\Delta_0 + 4P_0^2 + \Delta_\perp^2\right)}{2P_3} A_6 - \frac{z\Delta_0\left(\Delta_0^2 + 2\Delta_0 P_0 + 4P_0^2 + \Delta_\perp^2\right)}{2P_0 P_3} A_8 \,.$$

One can also modify the definition to make it Lorentz-invariant and arrive at:

ANY frame:

$$F_H = A_1 \,,$$

$$F_E = -A_1 + 2A_5 + 2zP_3 A_6 \,.$$

With respect to the standard definition, removed/reduced contribution from A_3, A_4, A_6, A_8.

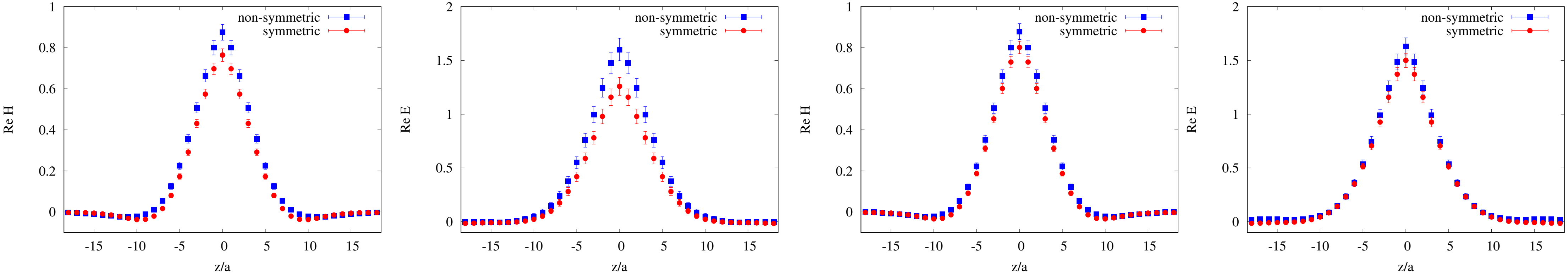
In terms of matrix elements: standard definition – only $\Pi_0(\Gamma_0)$, $\Pi_0(\Gamma_{1/2})$,

LI definition – additionally: $\Pi_{1/2}(\Gamma_3)$ (both frames), $\Pi_{1/2}(\Gamma_3)$, $\Pi_{1/2}(\Gamma_0)$, $\Pi_1(\Gamma_2)$, $\Pi_2(\Gamma_1)$ (asym.).

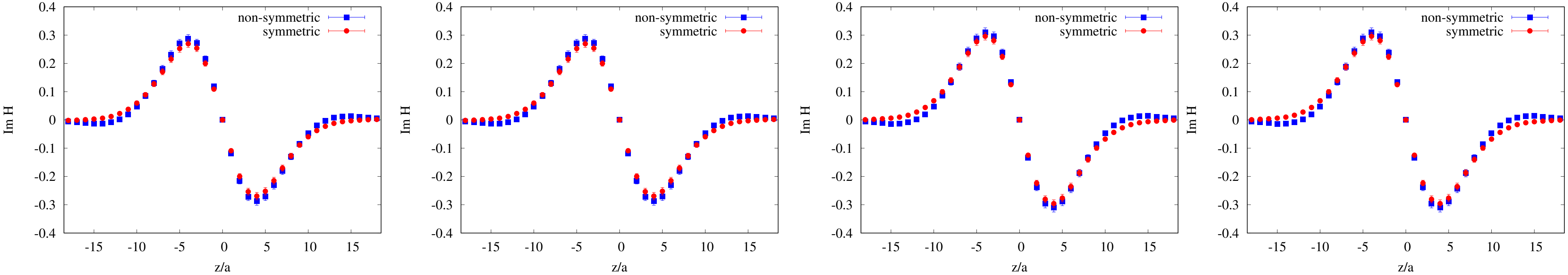
H and E GPDs – comparison of definitions

STANDARD DEFINITION | LORENTZ-INVARIANT DEFINITION

H-GPD | E-GPD | H-GPD | E-GPD

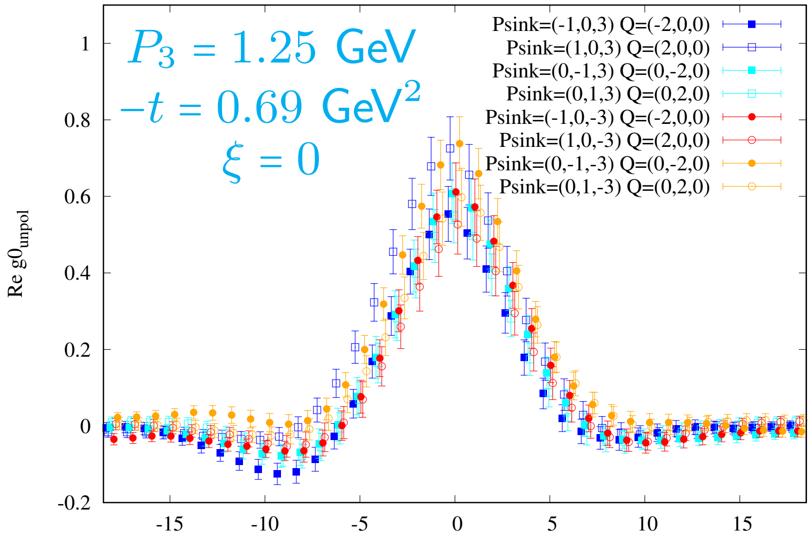


S. Bhattacharya et al., PRD106(2022)114512

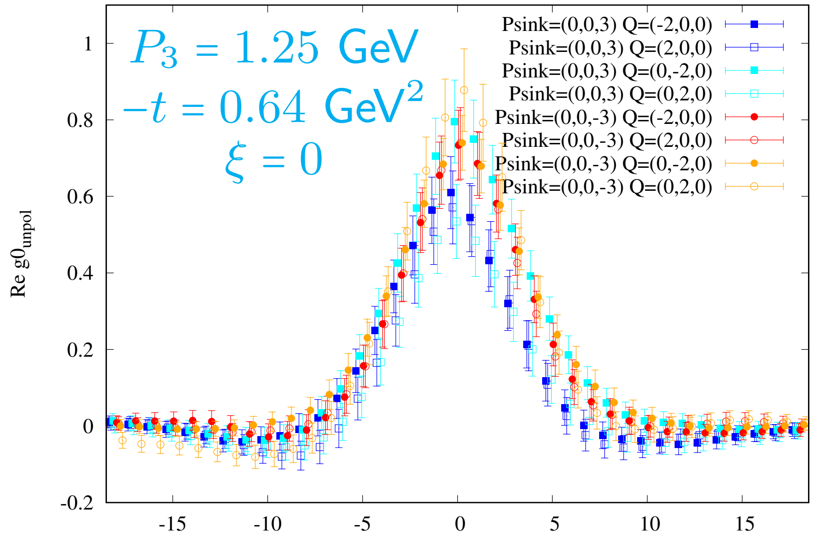


Bare matrix elements of $\Pi_0(\Gamma_0)$

symmetric frame



non-symmetric frame



Re

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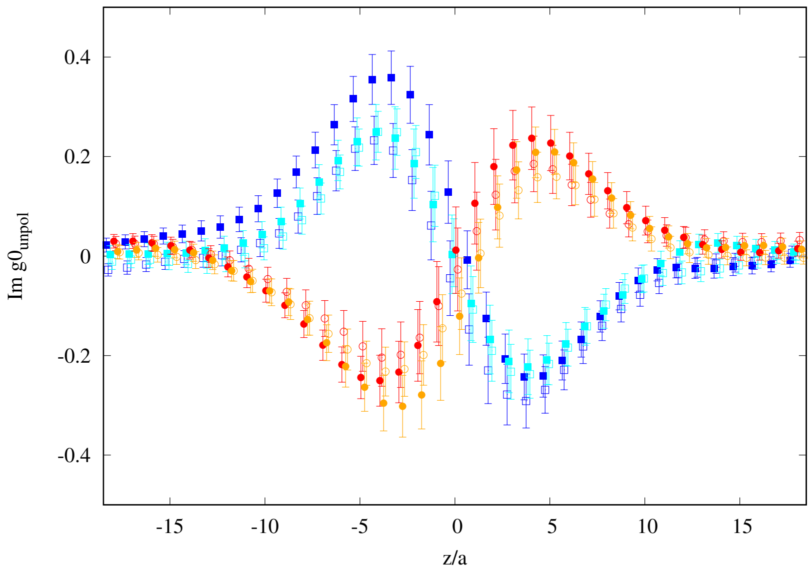
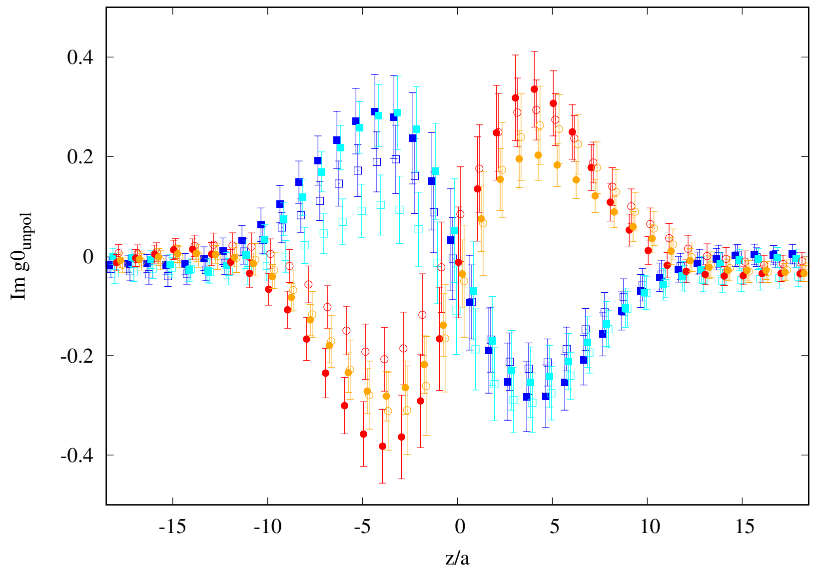


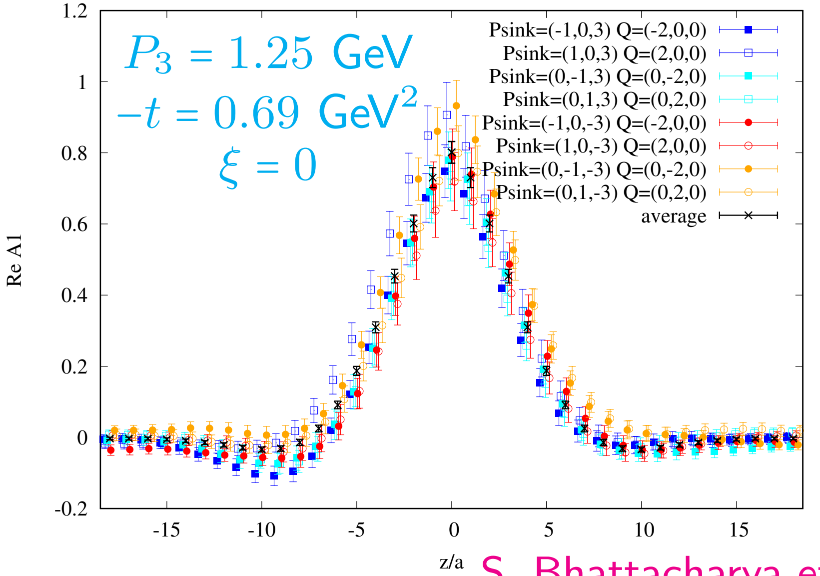


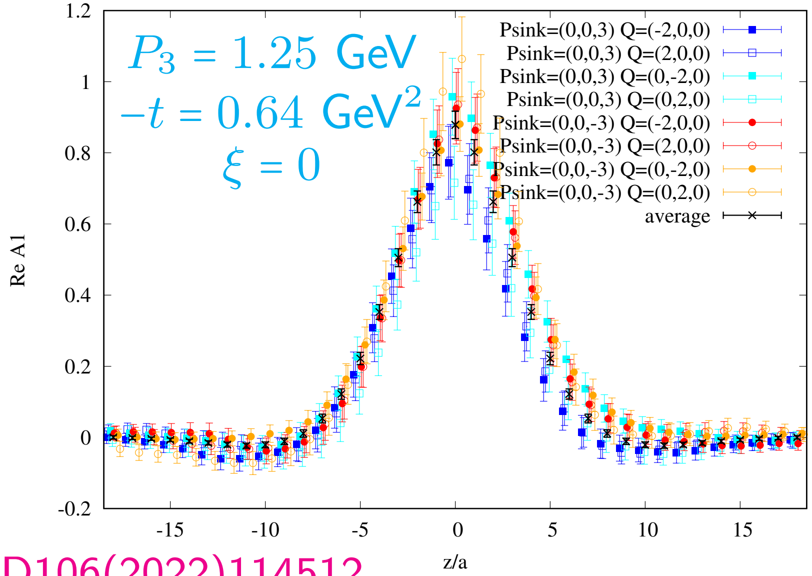
Im

Example amplitude A_1

symmetric frame | non-symmetric frame

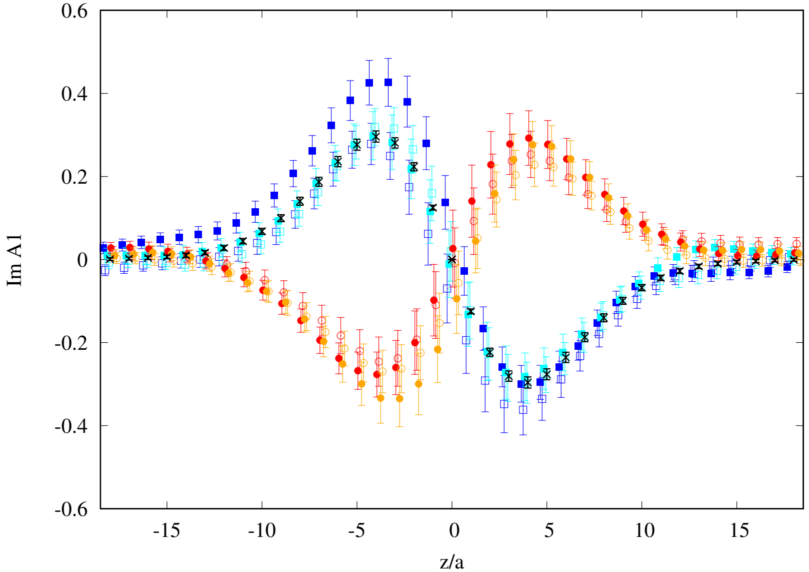
Re

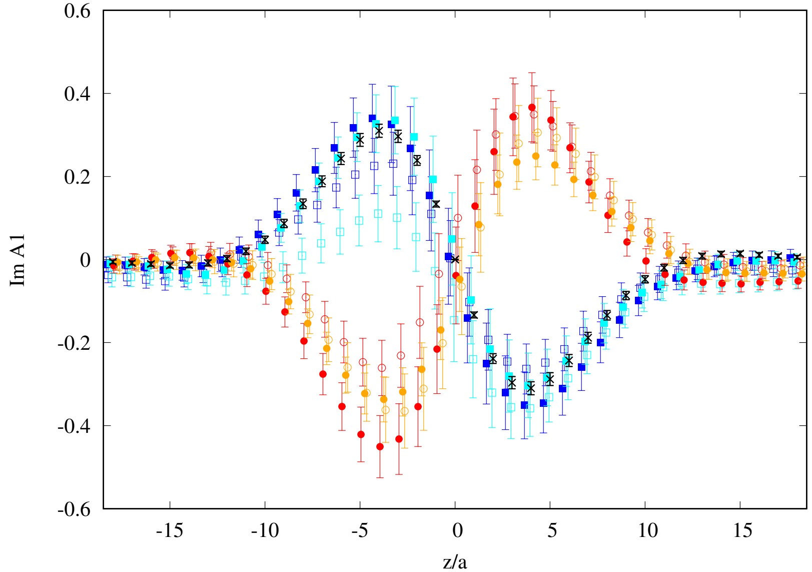




S. Bhattacharya et al., PRD106(2022)114512

Im

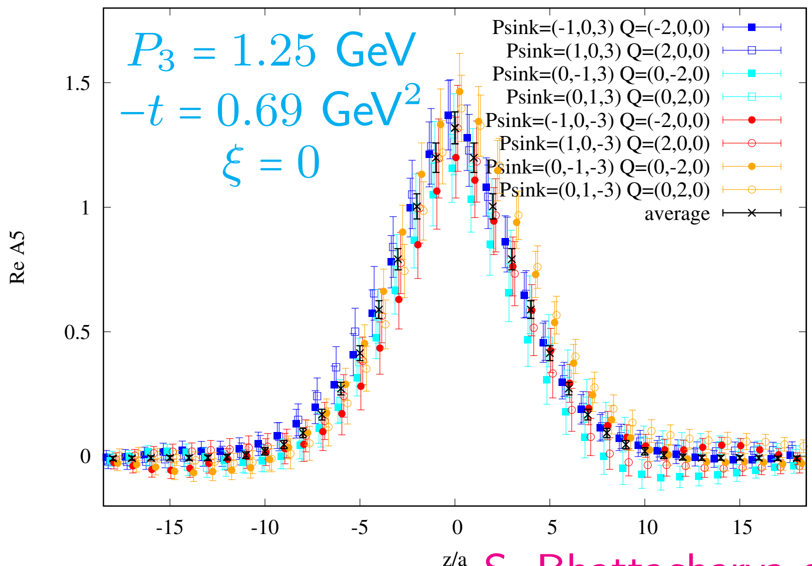


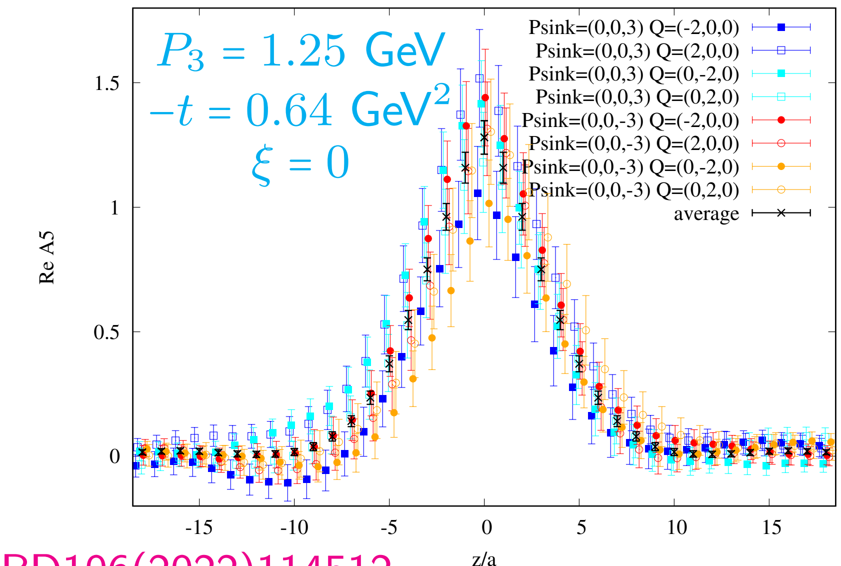


Example amplitude A_5

symmetric frame | non-symmetric frame

Re





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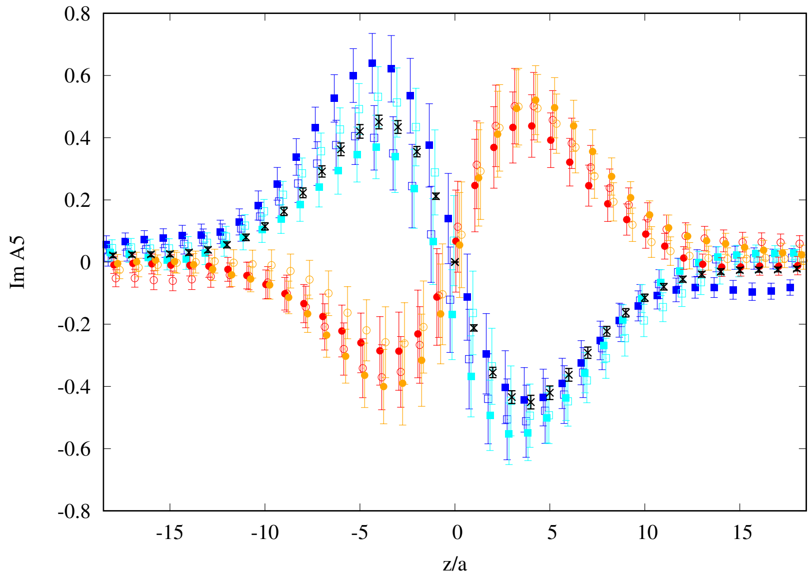
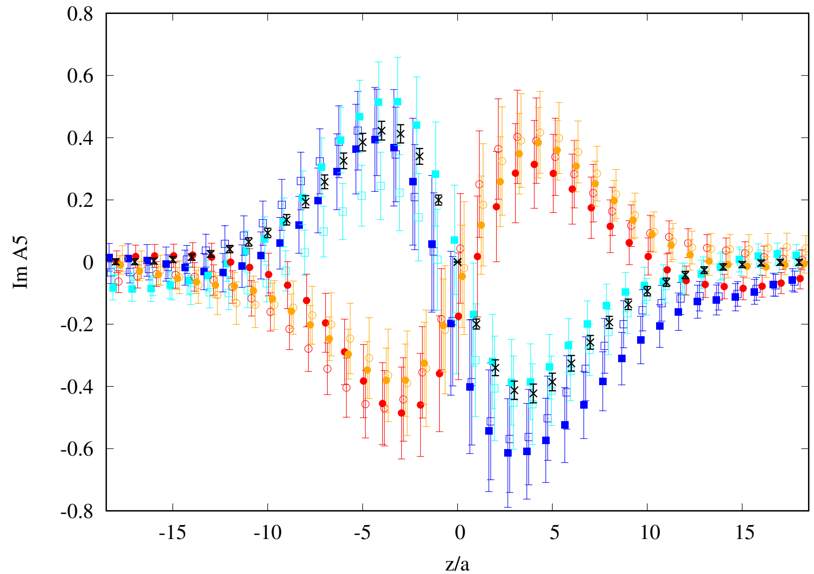
Im

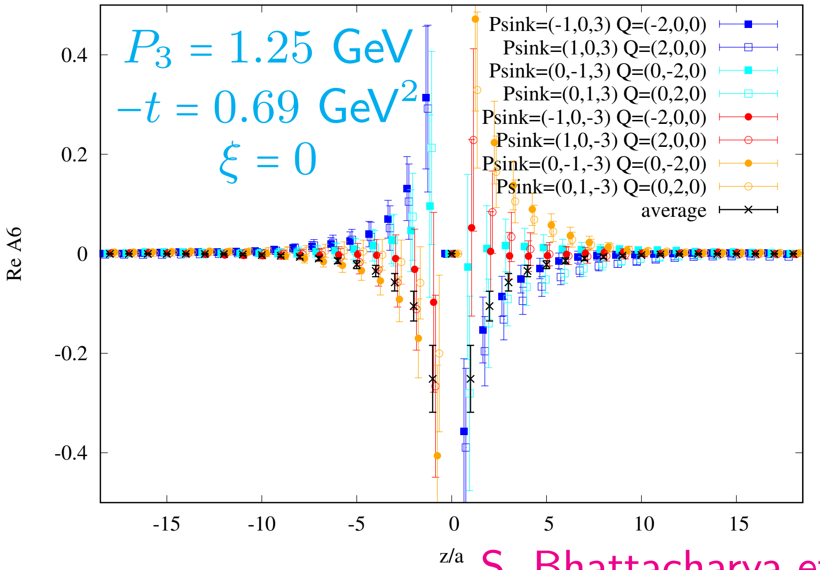


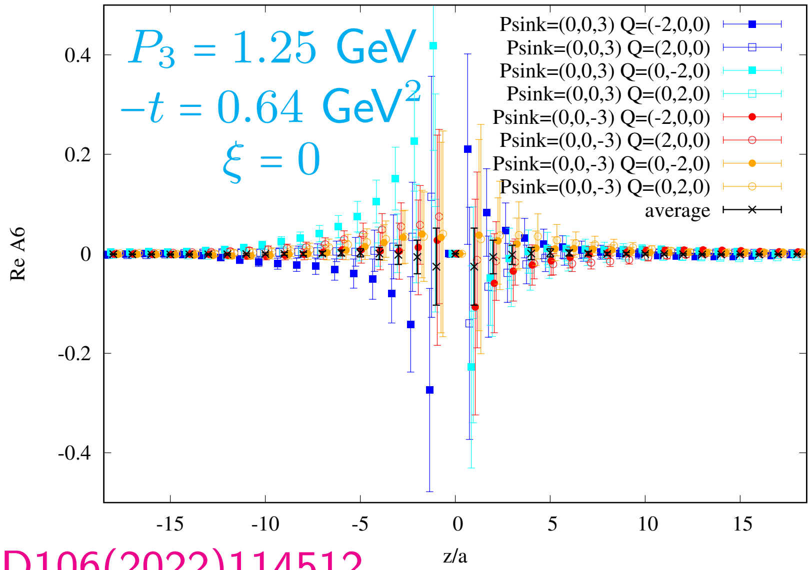


Example amplitude A_6

symmetric frame

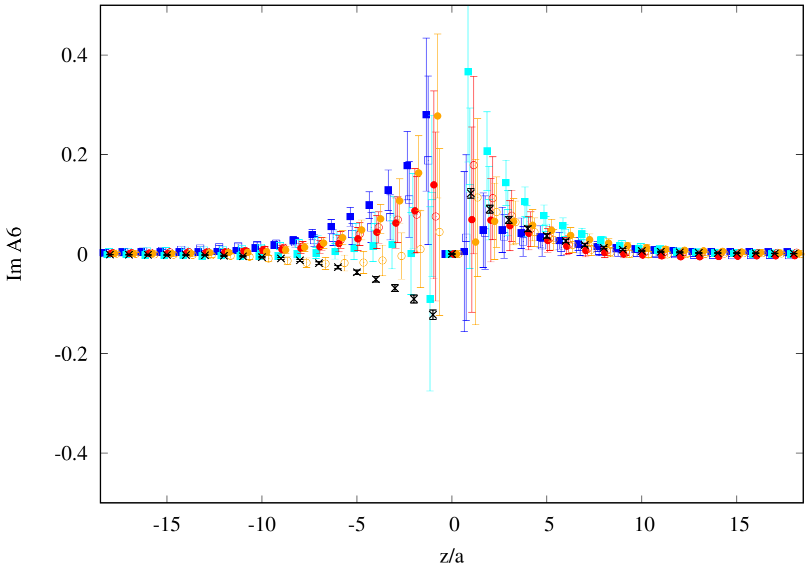
non-symmetric frame

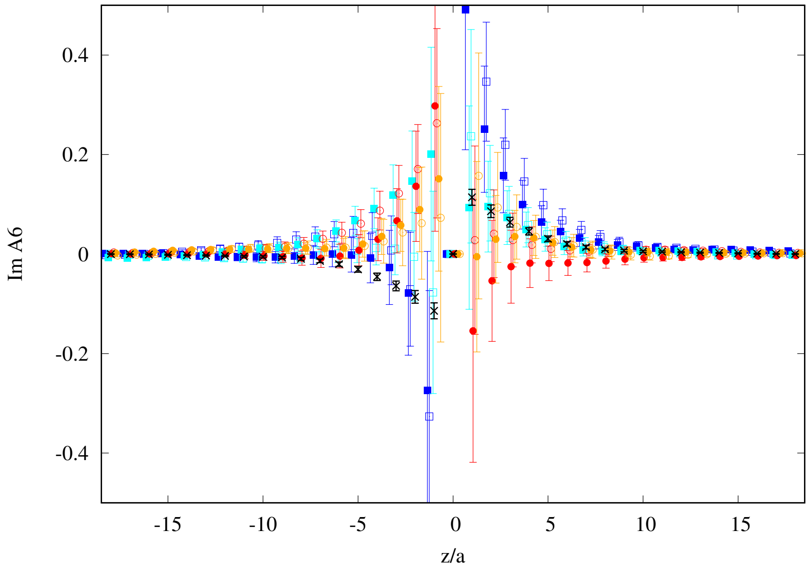




Re

S. Bhattacharya et al., PRD106(2022)114512

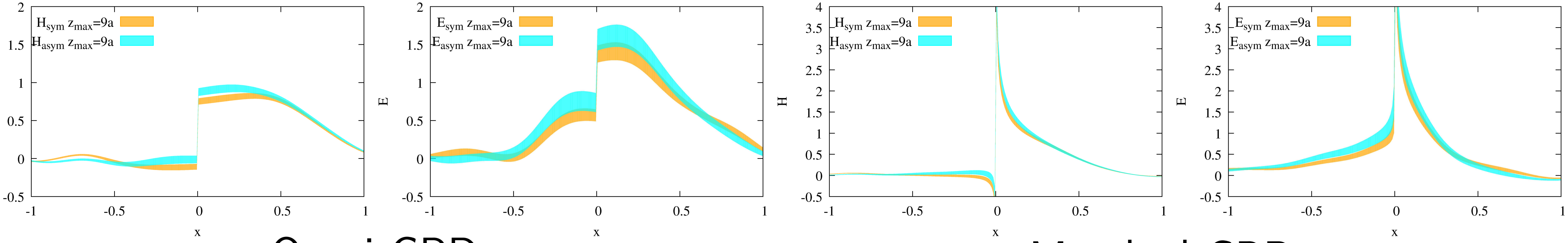




Im

Quasi- and matched H and E GPDs

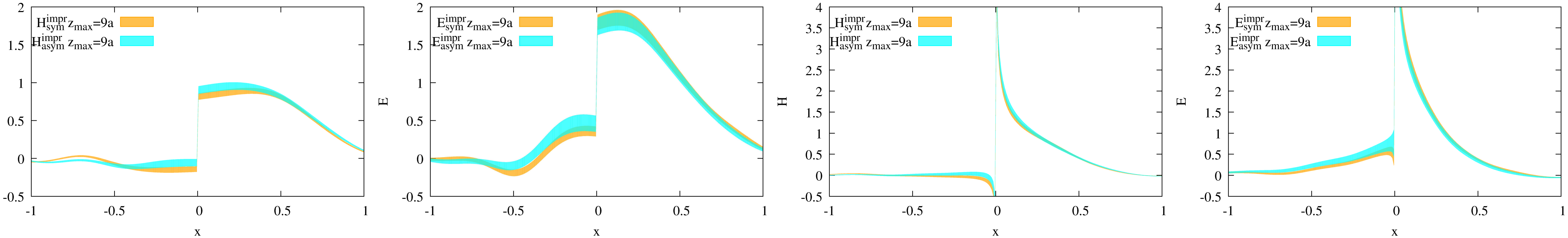
STANDARD DEFINITION



Quasi-GPDs S. Bhattacharya et al., PRD106(2022)114512 Matched GPDs

H-GPD E-GPD H-GPD E-GPD

LORENTZ-INVARIANT DEFINITION



Twist-3 PDFs

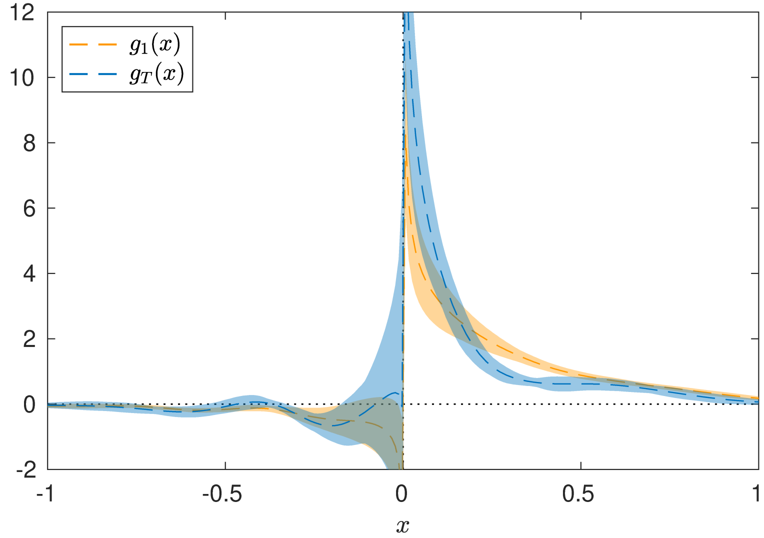
PDFs/GPDs can be classified according to their twist, which describes the order in $1/Q$ at which they appear in the factorization of structure functions.

LT: **twist-2** – probability densities for finding partons carrying fraction x of the hadron momentum.

Twist-3:

QUASI	TMF	$m_\pi = 260$ MeV	$a = 0.093$ fm

- no density interpretation,
- contain important information about qgq correlations,
- appear in QCD factorization theorems for a variety of hard scattering processes,
- have interesting connections with TMDs,
- important for JLab's 12 GeV program + for EIC,
- however, measurements very difficult.



Exploratory studies:

- matching for twist-3 PDFs: g_T, h_L, e
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 034005
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 114025

  BC-type sum rules S. Bhattacharya, A. Metz, Phys. Rev. D105 (2022) 05402

  Note: neglected qgq correlations
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- lattice extraction of $g_T^{u-d}(x)$ and $h_L^{u-d}(x)$
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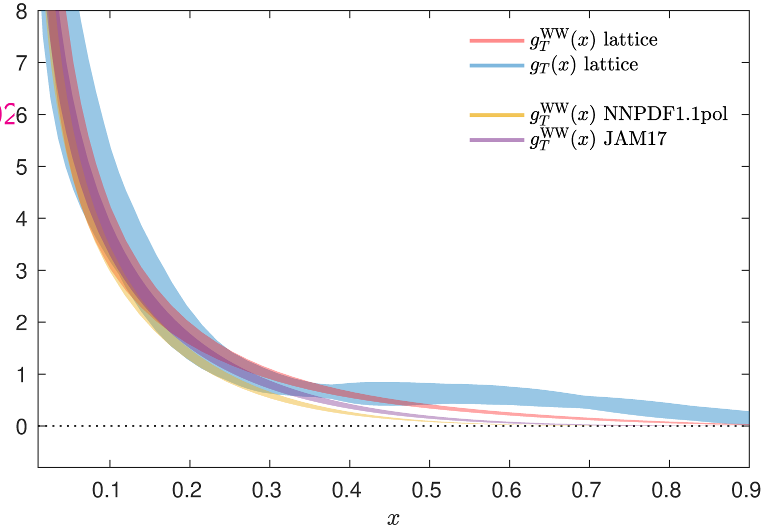
  S. Bhattacharya et al., Phys. Rev. D102 (2020) 111501(R)
  S. Bhattacharya et al., Phys. Rev. D104 (2021) 114510



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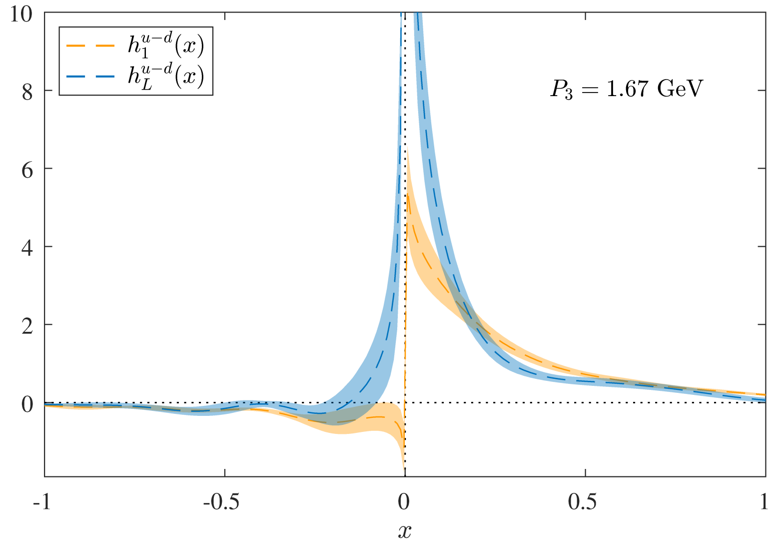

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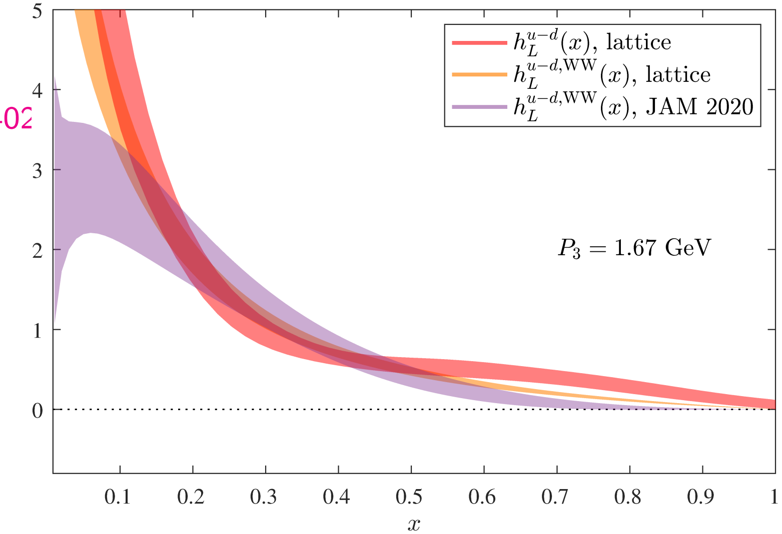


Very recently, we combined our explorations of GPDs and of twist-3 distributions
S. Bhattacharya et al., PRD108(2023)054501

Twist-3 axial GPDs: $\widetilde{G}_1$, $\widetilde{G}_2$, $\widetilde{G}_3$, $\widetilde{G}_4$

$$\mathcal{F}^{[\gamma_j \gamma_5]} = -i \frac{\Delta_j \gamma_5}{2m} F_{\widetilde{E}+\widetilde{G}_1} + \gamma_j \gamma_5 \; F_{\widetilde{H}+\widetilde{G}_2} + \frac{\Delta_j \gamma_3 \gamma_5}{P_3} F_{\widetilde{G}_3} - \frac{\mathrm{sign}[P_3]\, \varepsilon_\perp^{j\,\rho} \Delta_\rho \gamma_3}{P_3} F_{\widetilde{G}_4}$$

Contributions from different insertions and projectors ($\vec{\Delta} = (\Delta_1, 0, 0)$):

$\Pi(\gamma^2\gamma^5, \Gamma_0)$: $\widetilde{H} + \widetilde{G}_2$ and $\widetilde{G}_4$,
$\Pi(\gamma^2\gamma^5, \Gamma_2)$: $\widetilde{H} + \widetilde{G}_2$ and $\widetilde{G}_4$,
$\Pi(\gamma^1\gamma^5, \Gamma_1)$: $\widetilde{H} + \widetilde{G}_2$ and $\widetilde{E} + \widetilde{G}_1$,
$\Pi(\gamma^1\gamma^5, \Gamma_3)$: $\widetilde{G}_3$.

Twist-3 GPDs in coordinate space

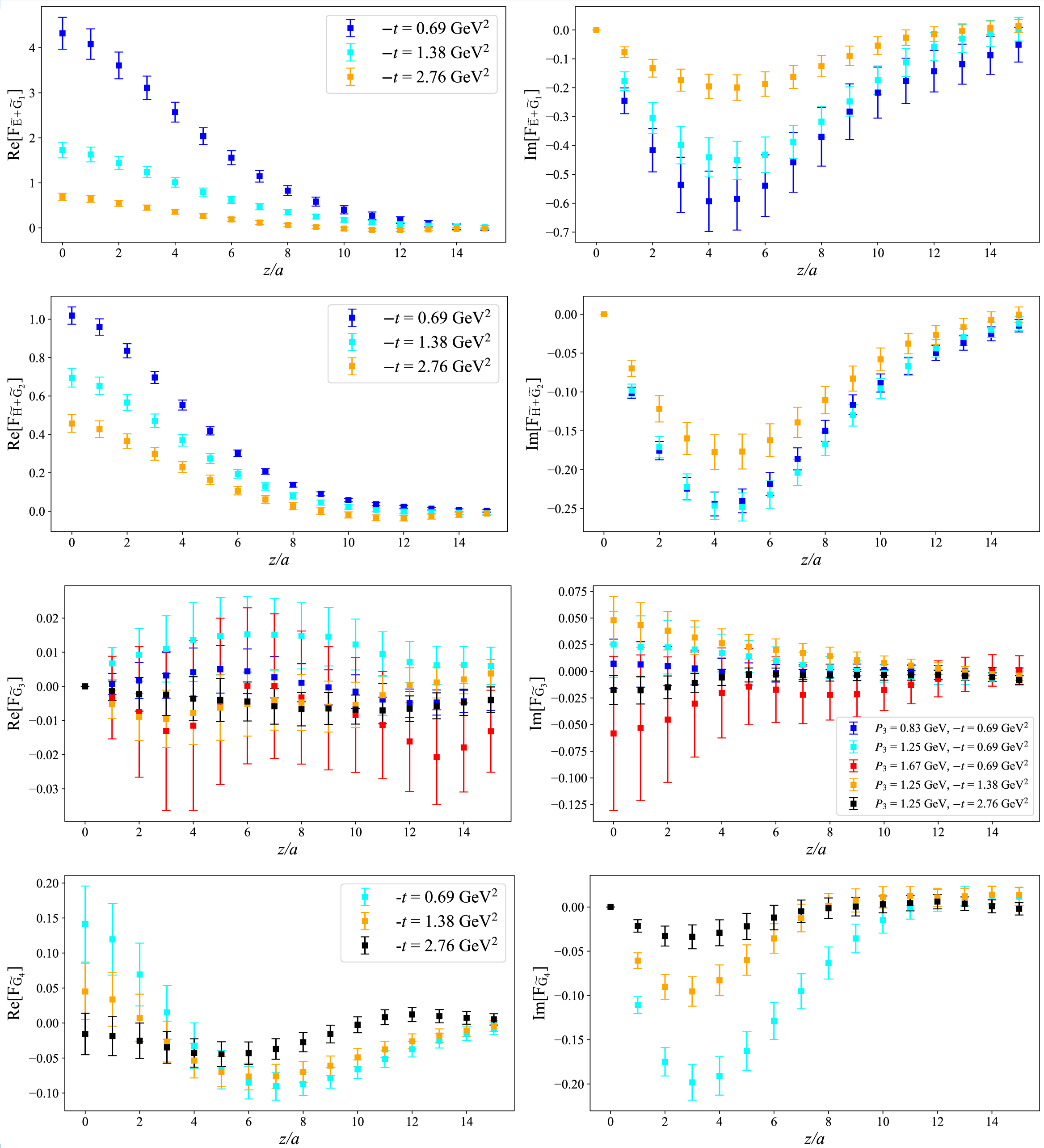
S. Bhattacharya et al. PRD108(2023)054501

$\widetilde{E} + \widetilde{G}_1$

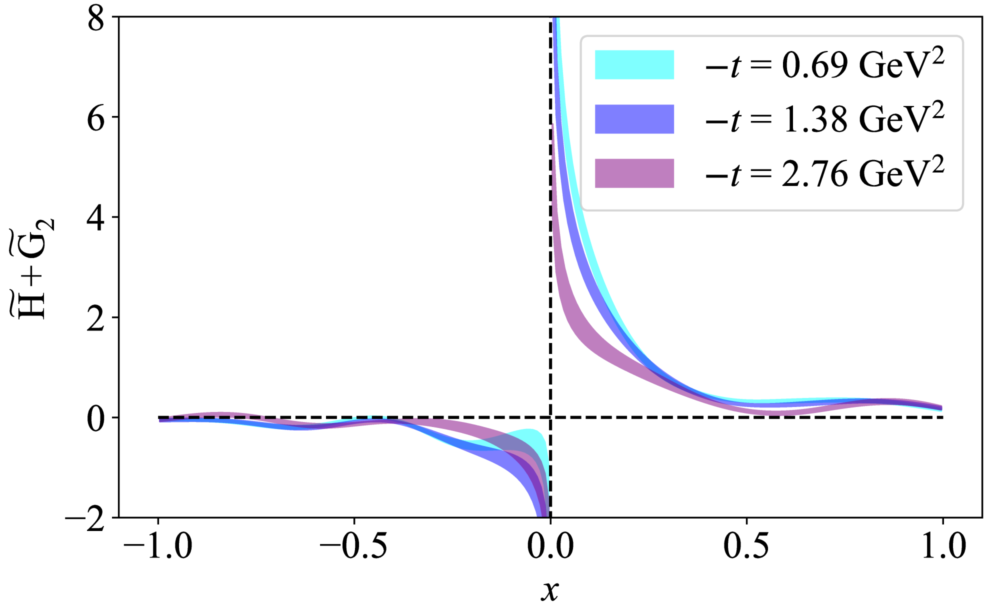
$\widetilde{H} + \widetilde{G}_2$

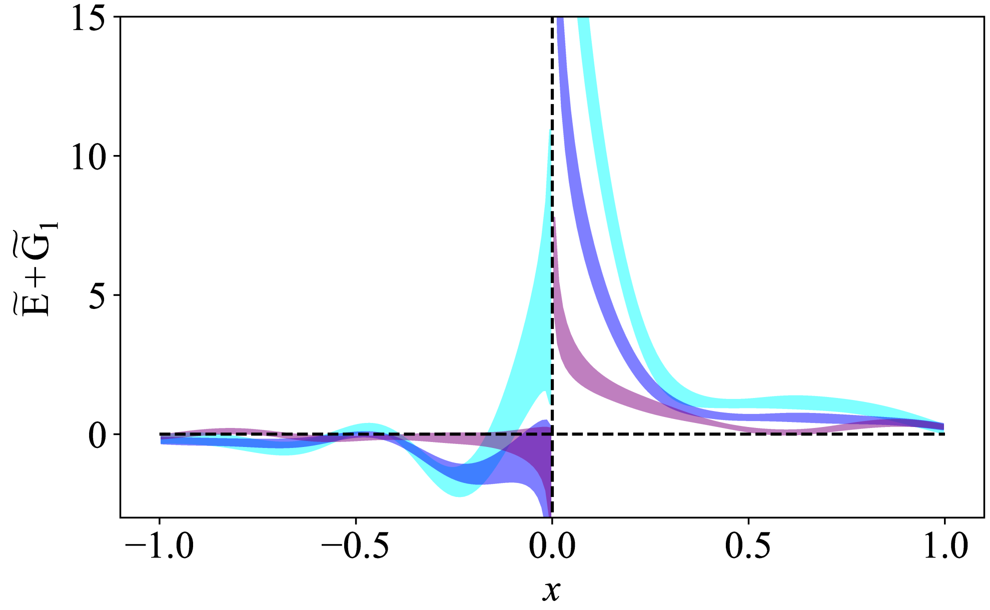
$\widetilde{G}_3$

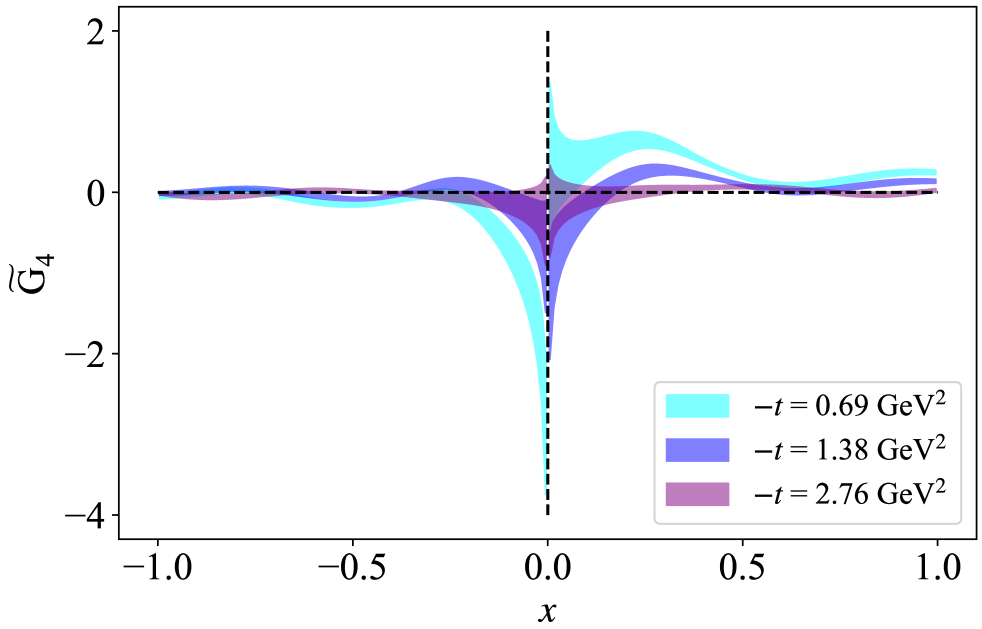
$\widetilde{G}_4$



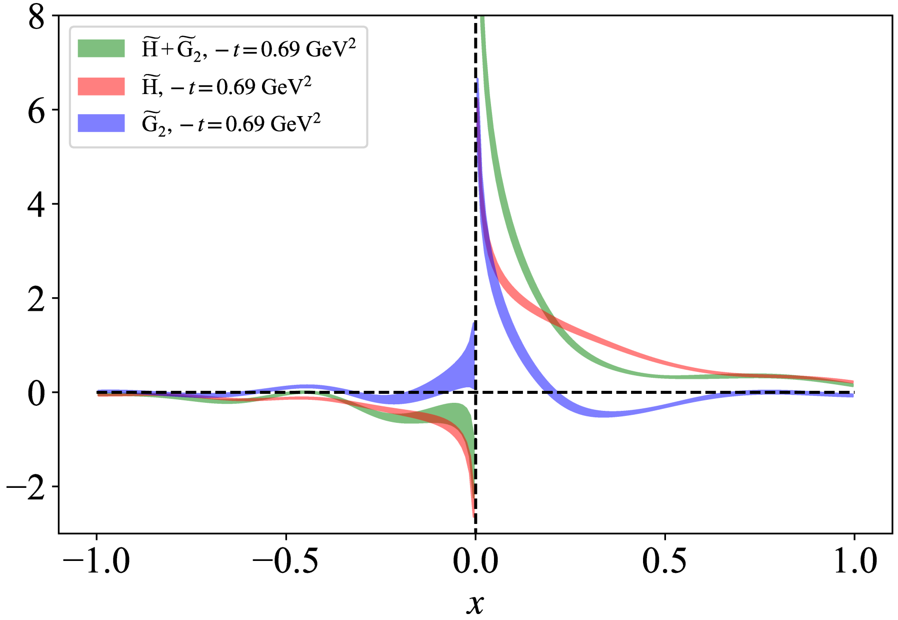
Twist-3 GPDs in x-space



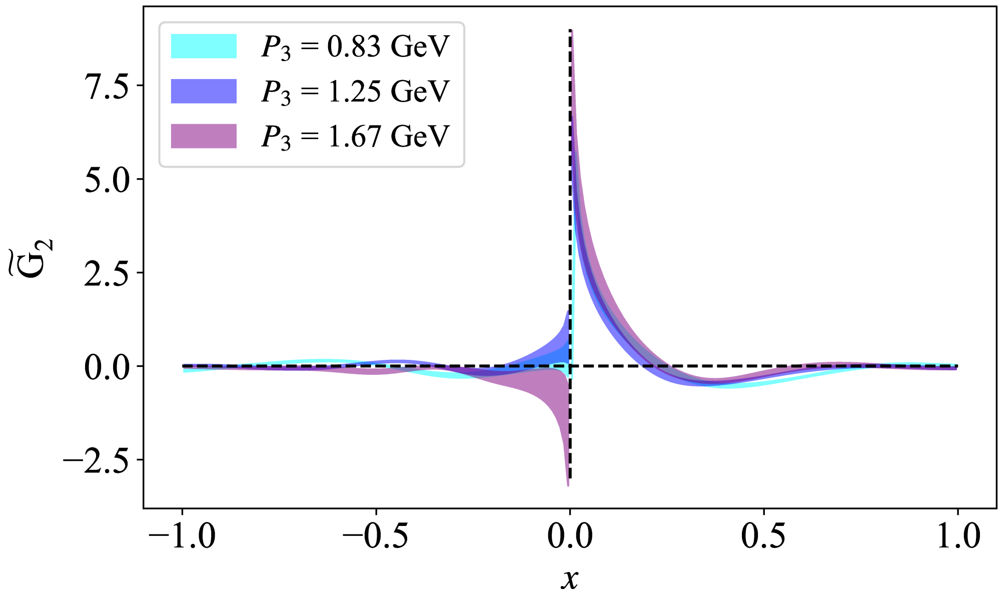


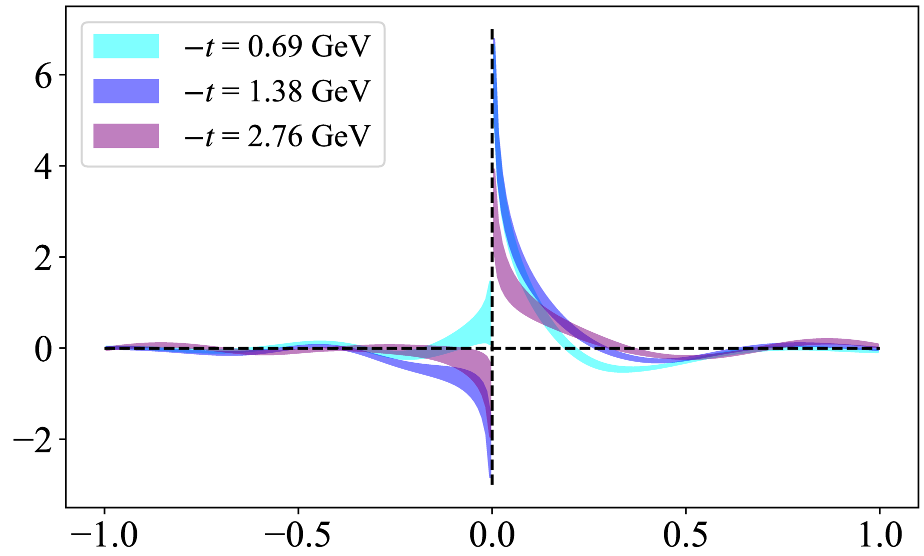


S. Bhattacharya et al.
PRD108(2023)054501



S. Bhattacharya et al.
PRD108(2023)054501





Consistency checks

Burkhardt-Cottingham-type sum rules:

$$G_P(t) = \int_{-1}^{1} dx \left(\widetilde{E}(x,\xi,t) + \widetilde{G}_1(x,\xi,t)\right) = \int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t)$$
$$G_A(t) = \int_{-1}^{1} dx \left(\widetilde{H}(x,\xi,t) + \widetilde{G}_2(x,\xi,t)\right) = \int_{-1}^{1} dx\, \widetilde{H}(x,\xi,t)$$
$$\Rightarrow \int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0$$

GPD	$P_3 = 0.83$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.67$ [GeV] $-t = 0.69$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 1.38$ [GeV^2]	$P_3 = 1.25$ [GeV] $-t = 2.76$ [GeV^2]
$\widetilde{H}$	0.741(21)	0.712(27)	0.802(48)	0.499(21)	0.281(18)
$\widetilde{H} + \widetilde{G}_2$	0.719(25)	0.750(33)	0.788(70)	0.511(36)	0.336(34)

- satisfied for $\widetilde{H} + \widetilde{G}_2$ – same local limit and norm as $\widetilde{H}$,
- cannot be tested for $\widetilde{E} + \widetilde{G}_1$ – $\widetilde{E}$ inaccessible at $\xi = 0$.
- norms of $\widetilde{G}_2$ and $\widetilde{G}_4$ close to vanishing.

Efremov-Leader-Teryaev-type sum rules:

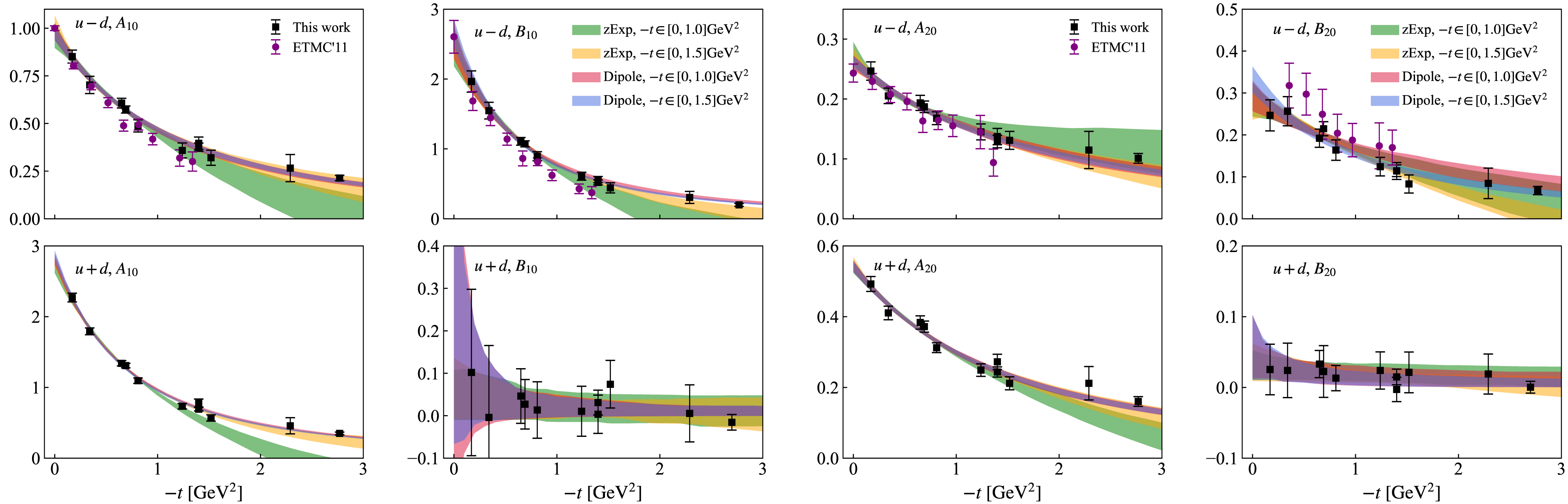
$$\int dx\, x\, \widetilde{G}_3(x,\xi,t) = \frac{\xi}{4} G_E(t)\,, \qquad \int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,\xi,t) = \frac{1}{4} G_E(t)\,.$$

- $\widetilde{G}_3$ indeed vanishes at $\xi = 0$,
- $\widetilde{G}_4$ non-vanishing and small.

Short-distance factorization of ratio-renormalized H/E:

$$\mathcal{F}^{\overline{\rm MS}}(z,P,\Delta) = \sum_{n=0} \frac{(-izP)^n}{n!} C_n^{\overline{\rm MS}}(\mu^2 z^2)\langle x^n\rangle + \mathcal{O}(\Lambda_{\rm QCD}^2 z^2),$$

$C_n^{\overline{\rm MS}}(\mu^2 z^2)$ – Wilson coefficients (NNLO for $u-d$, NLO for $u+d$)



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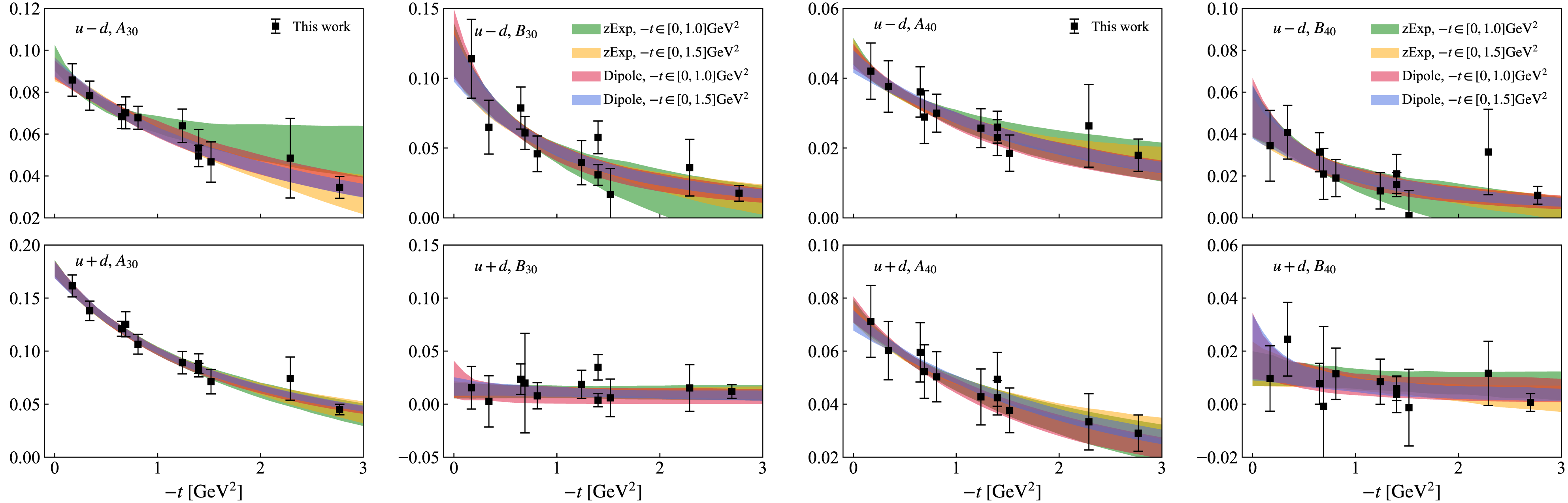
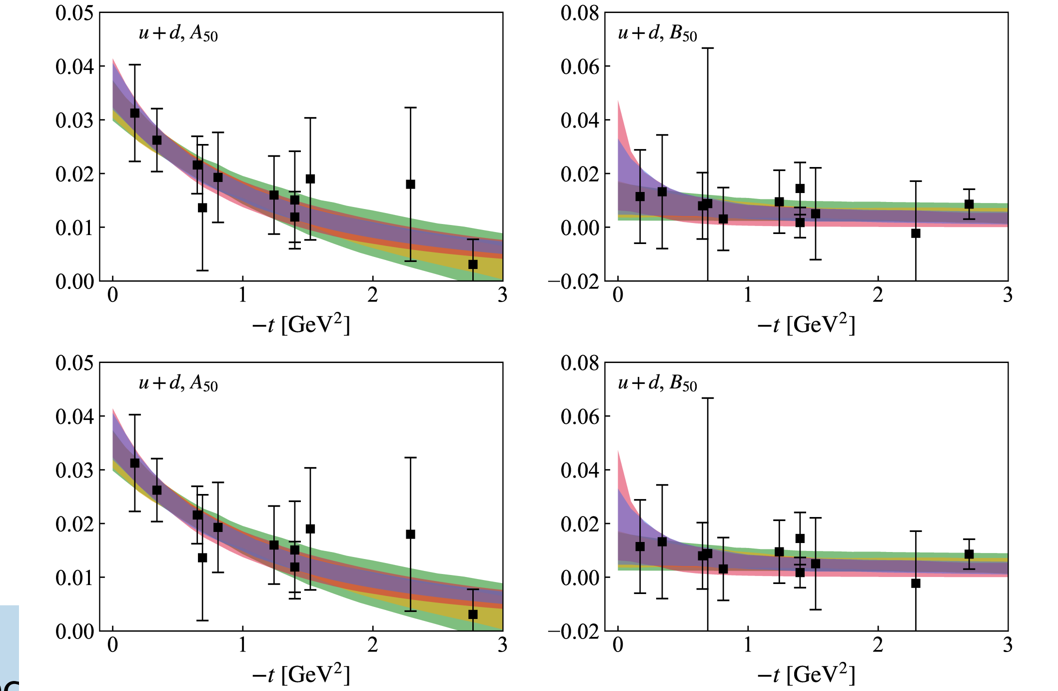
GPDs moments from OPE of non-local operators



Also
higher moments!



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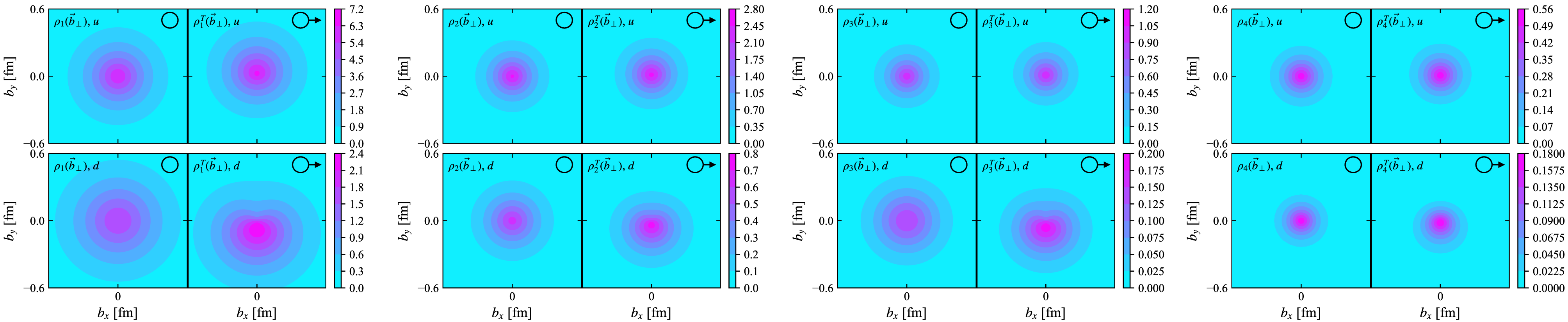
PRD 108(2023)014507

Moments of impact parameter parton distributions in the transverse plane:

$$\rho_{n+1}(\vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} A_{n+1,0}(-\vec{\Delta}_\perp^2) e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp},$$

$$\rho^T_{n+1}(\vec{b}_\perp) = \int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} [A_{n+1,0}(-\vec{\Delta}_\perp^2) + i\frac{\Delta_y}{2M} B_{n+1,0}(-\vec{\Delta}_\perp^2)] e^{-i\vec{b}_\perp\cdot\vec{\Delta}_\perp}.$$

1st moment | 2nd moment | 3rd moment | 4th moment



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