Chiral dynamics and gravitational form factors

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Outline

- Effective action of chiral EFT in curved spacetime;
- Energy-momentum-tensor;
- Gravitational form factors of the nucleon;
- Gravitational form factors of the deuteron;
- Spatial densities;
- Summary;

I will not talk about ...

“Virtual photons in the pion form-factors and the energy momentum tensor,”
B. Kubis and U.-G. Meißner, Nucl. Phys. A **671** (2000), 332-356.

“Chiral theory of ρ-meson gravitational form factors,”
E. Epelbaum, J. G., U.-G. Meißner and M. V. Polyakov, Phys. Rev. D **105** (2022) no.1, 016018.

“Gravitational form factors of the delta resonance in chiral EFT,”
H. Alharazin, E. Epelbaum, J. G., U.-G. Meißner and B. D. Sun, Eur. Phys. J. C **82** (2022) no.10, 907.

“Gravitational $p \to \Delta^+$ transition form factors in chiral perturbation theory,”
H. Alharazin, B. D. Sun, E. Epelbaum, J. G. and U.-G. Meißner, JHEP **03** (2024), 007.

Effective action of chiral EFT in curved spacetime

Effective Lagrangian for pions and nucleons in Minkowski metric:

J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) **158**, 142 (1984).

N. Fettes, U.-G. Meißner, M. Mojžiš, and S. Steininger, Ann. Phys. (N.Y.) **283**, 273 (2000); **288**, 249 (2001).

For the purpose of obtaining the EMT one needs to consider the coupling to the gravitational field.

J. F. Donoghue and H. Leutwyler, Z. Phys. C **52**, 343 (1991).

The LO action of pseudoscalar mesons in curved spacetime:

$$S_\pi = \int d^4x \sqrt{-g} \left\{ \frac{F^2}{4} g^{\mu\nu} \mathrm{Tr}(D_\mu U (D_\nu U)^\dagger) + \frac{F^2}{4} \mathrm{Tr}(\chi U^\dagger + U \chi^\dagger) \right\},$$

$\chi = 2B_0(s + ip)$, $D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu$, $U = u^2$ represents the pion fields, B_0 is related to the vacuum condensate of quark fields and s, p, l_μ and r_μ are external sources.

H. Alharazin, D. Djukanovic, J. G. and M. V. Polyakov, Phys. Rev. D **102**, no.7, 076023 (2020).

LO + NLO action of nucleons interacting with pions in curved spacetime:

$$
\begin{aligned}
S_{\pi \mathrm{N}} &= \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \bar{\Psi}\, i e_a^\mu \gamma^a \nabla_\mu \Psi - \frac{1}{2} \nabla_\mu \bar{\Psi}\, i e_a^\mu \gamma^a \Psi - m \bar{\Psi}\Psi \right. \\
&+ \frac{g_A}{2} \bar{\Psi} e_a^\mu \gamma^a \gamma_5 u_\mu \Psi + c_1 \langle \chi_+ \rangle \bar{\Psi}\Psi \\
&- \frac{c_2}{8m^2} g^{\mu\alpha} g^{\nu\beta} \langle u_\mu u_\nu \rangle \left(\bar{\Psi} \{\nabla_\alpha, \nabla_\beta\} \Psi + \{\nabla_\alpha, \nabla_\beta\} \bar{\Psi}\Psi \right) \\
&+ \frac{c_3}{2} g^{\mu\nu} \langle u_\mu u_\nu \rangle \bar{\Psi}\Psi + \frac{ic_4}{4} \bar{\Psi}\, e_a^\mu e_b^\nu \sigma^{ab} \left[u_\mu, u_\nu \right] \Psi + c_5 \bar{\Psi} \hat{\chi}_+ \Psi \\
&+ \frac{c_6}{8m} \bar{\Psi}\, e_a^\mu e_b^\nu \sigma^{ab} F^+_{\mu\nu} \Psi + \frac{c_7}{8m} \bar{\Psi}\, e_a^\mu e_b^\nu \sigma^{ab} \langle F^+_{\mu\nu} \rangle \Psi \\
&+ \left. \frac{c_8}{8} R \bar{\Psi}\Psi + \frac{ic_9}{m} R^{\mu\nu} \left(\bar{\Psi} e^a_\mu \gamma_a \nabla_\nu \Psi - \nabla_\nu \bar{\Psi} e^a_\mu \gamma_a \Psi \right) \right\},
\end{aligned}
$$

where $g^{\mu\nu}$ and e_a^μ are the metric and vielbein gravitational fields.

NLO action contains two new LECs, c_8 and c_9.

Building blocks:

$$
\begin{aligned}
u_\mu &= i\left[u^\dagger \partial_\mu u - u\partial_\mu u^\dagger - i(u^\dagger v_\mu u - u v_\mu u^\dagger)\right] \,, \\
F^+_{\mu\nu} &= u^\dagger F_{R\mu\nu} u + u F_{L\mu\nu} u^\dagger \,, \\
F_{R\mu\nu} &= \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu , r_\nu] \,, \\
F_{L\mu\nu} &= \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\mu , l_\nu] \,, \\
\chi_+ &= u^\dagger \chi u^\dagger + u\chi^\dagger u \,, \\
\hat{\chi}_+ &= \chi_+ - \frac{1}{2}\langle\chi_+\rangle \,, \\
\nabla_\mu \Psi &= \partial_\mu \Psi + \frac{i}{2}\,\omega_\mu^{ab}\sigma_{ab}\Psi + \left(\Gamma_\mu - i v_\mu^{(s)}\right)\Psi, \\
\nabla_\mu \bar{\Psi} &= \partial_\mu \bar{\Psi} - \frac{i}{2}\,\bar{\Psi}\,\sigma_{ab}\,\omega_\mu^{ab} - \bar{\Psi}\left(\Gamma_\mu - i v_\mu^{(s)}\right) \,, \\
\Gamma_\mu &= \frac{1}{2}\left[u^\dagger \partial_\mu u + u\partial_\mu u^\dagger - i(u^\dagger v_\mu u + u v_\mu u^\dagger)\right] \,, \\
\omega_\mu^{ab} &= -g^{\nu\lambda} e^a_\lambda \left(\partial_\mu e^b_\nu - e^b_\sigma \Gamma^\sigma_{\mu\nu}\right) \,, \\
\Gamma^\lambda_{\alpha\beta} &= \frac{1}{2} g^{\lambda\sigma}\left(\partial_\alpha g_{\beta\sigma} + \partial_\beta g_{\alpha\sigma} - \partial_\sigma g_{\alpha\beta}\right) \,, \\
R^\rho{}_{\sigma\mu\nu} &= \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma} \,, \\
R &= g^{\mu\nu} R^\lambda{}_{\mu\lambda\nu} \,.
\end{aligned}
$$

Energy-momentum-tensor

Using the definition of the EMT for bosonic matter fields

$$T_{\mu\nu}(g,\psi) = \frac{2}{\sqrt{-g}}\frac{\delta S_{\rm m}}{\delta g^{\mu\nu}}\,,$$

we obtain in flat spacetime

$$\begin{aligned} T_{\mu\nu}^{(\pi)} &= \frac{F^2}{4}\,{\rm Tr}(D_\mu U (D_\nu U)^\dagger + D_\nu U (D_\mu U)^\dagger) \\ &- \eta_{\mu\nu}\left\{\frac{F^2}{4}\,{\rm Tr}(D^\alpha U (D_\alpha U)^\dagger) + \frac{F^2}{4}\,{\rm Tr}(\chi U^\dagger + U\chi^\dagger)\right\}, \end{aligned}$$

where $\eta_{\mu\nu}$ is the Minkowski metric tensor.

For the fermion fields we use

$$T_{\mu\nu}(g,\psi) = \frac{1}{2e}\left[\frac{\delta S}{\delta e^{a\mu}}\, e^a_\nu + \frac{\delta S}{\delta e^{a\nu}}\, e^a_\mu\right],$$

where e is the determinant of e^a_μ, and obtain:

$$\begin{aligned}
T^{(\pi N)}_{\mu\nu} &= \frac{i}{4}\left(\bar\Psi\,\gamma_\mu D_\nu\Psi + \bar\Psi\,\gamma_\nu D_\mu\Psi - D_\mu\bar\Psi\,\gamma_\nu\Psi - D_\nu\bar\Psi\,\gamma_\mu\Psi\right) \\
&+ \frac{g_A}{4}\left(\bar\Psi\,\gamma_\mu\gamma_5 u_\nu\Psi + \bar\Psi\,\gamma_\nu\gamma_5 u_\mu\Psi\right) \\
\dots & \\
&+ \frac{c_8}{4}(\eta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)\bar\Psi\Psi \\
&+ \frac{ic_9}{2m}\left(\eta_{\mu\alpha}\eta_{\nu\beta}\partial^2 + \eta_{\mu\nu}\partial_\alpha\partial_\beta - \eta_{\mu\alpha}\partial_\nu\partial_\beta - \eta_{\nu\alpha}\partial_\mu\partial_\beta\right) \\
&\times \left(\bar\Psi\gamma^\alpha D^\beta\Psi - D^\beta\bar\Psi\gamma^\alpha\Psi + \bar\Psi\gamma^\beta D^\alpha\Psi - D^\alpha\bar\Psi\gamma^\beta\Psi\right),
\end{aligned}$$

where

$$\begin{aligned}
D_\mu\Psi &= \partial_\mu\Psi + \left(\Gamma_\mu - iv^{(s)}_\mu\right)\Psi, \\
D_\mu\bar\Psi &= \partial_\mu\bar\Psi - \bar\Psi\left(\Gamma_\mu - iv^{(s)}_\mu\right).
\end{aligned}$$

Gravitational form factors of the nucleon

At chiral order four there are tree and one-loop contributions to the nucleon matrix element of the EMT.

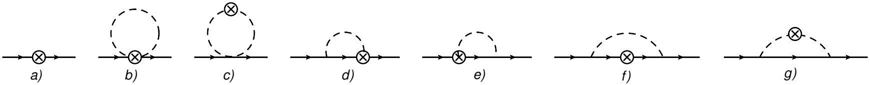

Figure: Diagrams contributing to GFFs of the nucleon. Solid and dashed lines correspond to nucleons and pions, respectively. Crosses stand for EMT insertions.

Standard power counting rules apply to these diagrams:

- The pion lines $\sim p^{-2}$;
- The nucleon lines $\sim p^{-1}$;
- Interaction vertices from the Lagrangian of order $N \sim p^{N}$;
- Vertices generated by EMT have orders corresponding to the number of quark mass factors and derivatives acting on the pion fields;
  Derivatives acting on the nucleon fields $\sim p^{0}$;
- The momentum transfer between initial and final nucleons $\sim p$;
- Integration over loop momenta $\sim p^{4}$.

Power counting is realized in the results only after renormalization!

M. V. Polyakov, P. Schweitzer, Int. J. Mod. Phys. A **33**, 26, 1830025 (2018).

The one-nucleon matrix element of the EMT is parameterised as

$$\langle p_f, s_f | T_{\mu\nu} | p_i, s_i \rangle = \bar{u}(p_f, s_f) \left[A(t) \frac{P_\mu P_\nu}{m_N} + iJ(t) \frac{P_\mu \sigma_{\nu\alpha}\Delta^\alpha + P_\nu \sigma_{\mu\alpha}\Delta^\alpha}{2m_N} + D(t) \frac{\Delta_\mu \Delta_\nu - \eta_{\mu\nu}\Delta^2}{4m_N} \right] u(p_i, s_i)\,,$$

(p_i, s_i) and (p_f, s_f) correspond to incoming and outgoing nucleons, and $P = (p_i + p_f)/2$, $\Delta = p_f - p_i$, $t = \Delta^2$.

The tree-order diagrams up to chiral order four lead to

$$\begin{aligned} A_{\rm tree}(t) &= 1 - \frac{2c_9}{m_N}\, t + x_1 M_\pi^2 t + x_2 t^2\,, \\ J_{\rm tree}(t) &= \frac{1}{2} - \frac{c_9}{m_N}\, t\,, \\ D_{\rm tree}(t) &= c_8 m_N + y_1 t + y_2 M_\pi^2\,. \end{aligned}$$

y_i and x_i parameterize tree-order contributions of the third and fourth chiral orders.

We renormalize loop diagrams by applying the EOMS scheme

J. G. and G. Japaridze, Phys. Rev. D **60**, 114038 (1999),

T. Fuchs, J. G., G. Japaridze and S. Scherer, Phys. Rev. D **68**, 056005 (2003).

Power counting violating contribution to $A(t)$ is absorbed into c_9.

c_8 cancels the divergent and power counting violating parts of $D(t)$.

$D(0)$ expanded in powers of the pion mass:

$$\frac{D(0)}{m_N} = c_8 + \frac{g_A^2}{16\pi F^2} M_\pi + \frac{2\left(c_2 + 2c_3 - 4c_1\right) - \frac{3g_A^2}{m_N}}{8\pi^2 F^2} M_\pi^2 \ln\left(\frac{M_\pi}{m_N}\right) + \frac{(8c_3 - 16c_1) - g_A^2\left(3c_8 + \frac{14}{m_N}\right)}{32\pi^2 F^2} M_\pi^2 + \frac{y_2}{m_N} M_\pi^2 + \mathcal{O}(M_\pi^3)\,.$$

Next we define the slopes of GFFs by writing the form factors as:

$$\begin{aligned} A(t) &= 1 + s_A t + \mathcal{O}(t^2)\,, \\ J(t) &= \frac{1}{2} + s_J t + \mathcal{O}(t^2)\,, \\ D(t) &= D(0) + s_D t + \mathcal{O}(t^2)\,. \end{aligned}$$

Chiral expansion of the loop contributions to the slopes:

$$
\begin{aligned}
s_A &= -\frac{7g_A^2}{128\pi F^2 m_N}\, M_\pi + \frac{c_2 m_N - 4g_A^2}{16\pi^2 F^2 m_N^2}\, M_\pi^2 \ln\left(\frac{M_\pi}{m_N}\right) \\
&- \frac{3g_A^2\,(2c_9 m_N + 1)}{32\pi^2 F^2 m_N^2}\, M_\pi^2 + \mathcal{O}(M_\pi^3)\,, \\
s_J &= -\frac{g_A^2}{32\pi^2 F^2}\, \ln\left(\frac{M_\pi}{m_N}\right) + \frac{g_A^2\,(4c_9 m_N - 5)}{64\pi^2 F^2} \\
&+ \frac{7g_A^2}{128\pi F^2 m_N}\, M_\pi + \mathcal{O}(M_\pi^2)\,, \\
s_D &= -\frac{g_A^2 m_N}{40\pi F^2}\,\frac{1}{M_\pi} - \frac{(5g_A^2 + 4\,(c_2 + 5c_3)\,m_N)}{80\pi^2 F^2}\, \ln\left(\frac{M_\pi}{m_N}\right) \\
&+ \frac{g_A^2\,(24 + (15c_8 + 40c_9)m_N)}{480\pi^2 F^2} \\
&+ \frac{(4c_1 - c_2 - 7c_3)\,m_N}{40\pi^2 F^2} + \mathcal{O}(M_\pi)\,.
\end{aligned}
$$

These expressions can be used for the analysis of lattice data.

M. V. Polyakov, Phys. Lett. B **555**, 57 (2003),

M. V. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A **33**, no. 26, 1830025 (2018),

$A(t), J(t)$ and $D(t)$ can be related to the energy and spin densities via

$$\begin{aligned}\rho_E(r) &= m_N \int \frac{d^3\Delta}{(2\pi)^3}\, e^{-ir\Delta} \left[A(-\Delta^2) \right.\\ &+ \left. \frac{\Delta^2}{4m_N^2}\left[A(-\Delta^2) - 2J(-\Delta^2) + D(-\Delta^2)\right]\right],\\ \rho_J(r) &= \int \frac{d^3\Delta}{(2\pi)^3}\, e^{-ir\Delta} \left[J(-\Delta^2) + \frac{2}{3}\Delta^2 \frac{dJ(-\Delta^2)}{d\Delta^2}\right].\end{aligned}$$

The distribution of the pressure $p(r)$ and shear forces $s(r)$:

$$s(r) = -\frac{1}{4m_N}\, r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\widetilde{D}(r), \quad p(r) = \frac{1}{6m_N}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\widetilde{D}(r),$$

$$\widetilde{D}(r) = \int \frac{d^3\Delta}{(2\pi)^3}\, e^{-i\Delta r}\, D(-\Delta^2).$$

Behavior of these quantities in the region $1/\Lambda_{\text{strong}} \ll r \ll 1/M_\pi$ can be obtained from GFFs for small t in chiral limit.

Expansion at $t = 0$ of GFFs in the chiral limit:

$$\begin{aligned}
A(t) &= 1 - \frac{2c_9}{m_N}\, t + \frac{3g_A^2}{512F^2 m_N}(-t)^{\frac{3}{2}} - \frac{(c_2 m_N - 10g_A^2)}{320\pi^2 F^2 m_N^2}\, t^2 \ln\left(\frac{-t}{m_N^2}\right) \\
&- \frac{\left(25g_A^2\,(12c_9 m_N - 7) - 62c_2 m_N\right)}{9600\pi^2 F^2 m_N^2}\, t^2 + O(t^{\frac{5}{2}})\,, \\
J(t) &= \frac{1}{2} - \frac{c_9}{m_N}\, t - \frac{g_A^2}{64\pi^2 F^2}\, t \ln\left(\frac{-t}{m_N^2}\right) + \frac{g_A^2\,(12c_9 m_N - 7)}{192\pi^2 F^2}\, t \\
&- \frac{3g_A^2}{512F^2 m_N}(-t)^{\frac{3}{2}} + O(t^2)\,, \\
D(t) &= m_N c_8 + \frac{3g_A^2 m_N}{128F^2}\sqrt{-t} - \frac{\left(5g_A^2 + 4\,(c_2 + 5c_3)\, m_N\right)}{160\pi^2 F^2}\, t \ln\left(\frac{-t}{m_N^2}\right) \\
&+ \frac{\left(5g_A^2\,(40c_9 m_N + 15c_8 m_N + 28) + 94c_2 m_N + 200c_3 m_N\right)}{2400\pi^2 F^2}\, t \\
&+ O(t^{\frac{3}{2}})\,.
\end{aligned}$$

F. T. gives the large distance behavior for $1/\Lambda_{\rm strong} \ll r \ll 1/M_\pi$:

$$
\begin{aligned}
\rho_E(r) &= \frac{9g_A^2}{64\pi^2F^2}\frac{1}{r^6} - \frac{3\left(10g_A^2/m_N + (c_2+10c_3)\right)}{16\pi^3F^2}\frac{1}{r^7} + O\left(\frac{1}{r^8}\right),\\
\rho_J(r) &= \frac{5g_A^2}{64\pi^3F^2}\frac{1}{r^5} - \frac{9g_A^2}{64\pi^2F^2m_N}\frac{1}{r^6} + O\left(\frac{1}{r^7}\right),\\
\widetilde{D}(r) &= -\frac{3g_A^2m_N}{128\pi^2F^2}\frac{1}{r^4} + \frac{3\left(5g_A^2 + 4(c_2+5c_3)\,m_N\right)}{160\pi^3F^2}\frac{1}{r^5} + O\left(\frac{1}{r^6}\right),\\
p(r) &= -\frac{3g_A^2}{64\pi^2F^2}\frac{1}{r^6} + \frac{\left(5g_A^2/m_N + 4(c_2+5c_3)\right)}{16\pi^3F^2}\frac{1}{r^7} + O\left(\frac{1}{r^8}\right),\\
s(r) &= \frac{9g_A^2}{64\pi^2F^2}\frac{1}{r^6} - \frac{21\left(5g_A^2/m_N + 4(c_2+5c_3)\right)}{128\pi^3F^2}\frac{1}{r^7} + O\left(\frac{1}{r^8}\right).
\end{aligned}
$$

The above expressions satisfy the general stability conditions - $\rho_E(r) > 0$ and $\frac{2}{3}s(r) + p(r) > 0$.

Gravitational form factors of the deuteron

Matrix elements of conserved EMT operator for spin-1 systems:

M. V. Polyakov and B.-D. Sun, Phys. Rev. D **100** (2019) no.3, 036003

$$
\begin{aligned}
t_{\mu\nu} &= \langle p', \sigma' | \hat{T}_{\mu\nu}(0) | p, \sigma \rangle = 2 P_\mu P_\nu \left[-\epsilon' \cdot \epsilon A_0(q^2) + \frac{P\cdot\epsilon' P\cdot\epsilon}{M^2} A_1(q^2) \right] \\
&+ 2\Big[P_\mu \left(\epsilon'_\nu P\cdot\epsilon + \epsilon_\nu P\cdot\epsilon'\right) + P_\nu \left(\epsilon'_\mu P\cdot\epsilon + \epsilon_\mu P\cdot\epsilon'\right) \Big] J(q^2) \\
&+ \frac{1}{2}\left(q_\mu q_\nu - g_{\mu\nu} q^2\right) \left[\epsilon'\cdot\epsilon D_0(q^2) + \frac{P\cdot\epsilon' P\cdot\epsilon}{M^2} D_1(q^2) \right] - \Big[4 g_{\mu\nu} P\cdot\epsilon' P\cdot\epsilon \\
&- \frac{1}{2} q^2 \left(\epsilon_\mu \epsilon'_\nu + \epsilon'_\mu \epsilon_\nu\right) + \left(\epsilon'_\nu q_\mu + \epsilon'_\mu q_\nu\right) \epsilon\cdot P - \left(\epsilon_\nu q_\mu + \epsilon_\mu q_\nu\right) \epsilon'\cdot P \Big] E(q^2),
\end{aligned}
$$

$q = p' - p$, $P = (p + p')/2$, and $\epsilon'^\beta \equiv \epsilon^{\star\beta}(p', \sigma')$, $\epsilon^\beta \equiv \epsilon^\beta(p, \sigma)$ are polarization vectors.

The one-particle states $|p, \sigma\rangle$ satisfy the normalization condition

$$\langle p', \sigma' | p, \sigma \rangle = 2\, p^0\, (2\,\pi)^3\, \delta^3(\mathbf{p} - \mathbf{p}')\delta_{\sigma\sigma'} .$$

J. Y. Panteleeva, E. Epelbaum, J. G. and U.-G. Meißner, JHEP **07**, 237 (2023)

In static approximation for the kinematics with $P^i = 0$, we obtain:

$$
\begin{aligned}
t^{00} &= 2m^2\left[\delta_{\sigma'\sigma}\left(\mathcal{E}_0(-\mathbf{q}^2)+\frac{\mathbf{q}^2}{3M^2}\mathcal{E}_2(-\mathbf{q}^2)\right)-\frac{q_{\sigma'}q_\sigma}{M^2}\mathcal{E}_2(-\mathbf{q}^2)\right],\\
t^{0i} &= -m\mathcal{J}(-\mathbf{q}^2)\left(\delta_{i\sigma'}q_\sigma-\delta_{i\sigma}q_{\sigma'}\right),\\
t^{ij} &= 2\Bigg[\mathcal{D}_2(-\mathbf{q}^2)\bigg(\delta_{ij}q_\sigma q_{\sigma'}-\frac{1}{2}q^i(q_\sigma\delta_{j\sigma'}+q_{\sigma'}\delta_{j\sigma})-\frac{1}{2}q^j(q_\sigma\delta_{i\sigma'}+q_{\sigma'}\delta_{i\sigma})\\
&+\frac{\mathbf{q}^2}{2}\left(\delta_{i\sigma}\delta_{j\sigma'}+\delta_{i\sigma'}\delta_{j\sigma}\right)\bigg)+\left(\mathbf{q}^2\delta_{ij}-q_iq_j\right)\bigg\{-\frac{q_{\sigma'}q_\sigma}{M^2}\mathcal{D}_3(-\mathbf{q}^2)\\
&+\delta_{\sigma'\sigma}\left(\mathcal{D}_0(-\mathbf{q}^2)-\frac{2}{3}\mathcal{D}_2(-\mathbf{q}^2)+\frac{\mathbf{q}^2}{3M^2}\mathcal{D}_3(-\mathbf{q}^2)\right)\bigg\}\Bigg].
\end{aligned}
$$

The combinations of the form factors are given by

$$
\begin{aligned}
\mathcal{E}_0(-\mathbf{q}^2) &= A_0(-\mathbf{q}^2) - \frac{\mathbf{q}^2}{12M^2} A_1(-\mathbf{q}^2)\,, \\
\mathcal{E}_2(-\mathbf{q}^2) &= \frac{A_1(-\mathbf{q}^2)}{4}\,, \\
\mathcal{J}(-\mathbf{q}^2) &= J(-\mathbf{q}^2)\,, \\
\mathcal{D}_0(-\mathbf{q}^2) &= \frac{D_0(-\mathbf{q}^2)}{4} + \frac{\mathbf{q}^2}{48M^2} D_1(-\mathbf{q}^2) - \frac{E(-\mathbf{q}^2)}{3}\,, \\
\mathcal{D}_2(-\mathbf{q}^2) &= -\frac{E(-\mathbf{q}^2)}{2}\,, \\
\mathcal{D}_3(-\mathbf{q}^2) &= -\frac{D_1(-\mathbf{q}^2)}{16}\,.
\end{aligned}
$$

We extract the GFFs from the three-point function of the EMT operator and two interpolating fields of the deuteron:

$$G^{\mu\nu}_{\sigma'\sigma}(p',p) = \int d^4x d^4y\, e^{-ip'\cdot y} e^{ip\cdot x} \langle 0|T\left[\mathcal{D}^\dagger_\sigma(x)\,\hat{T}^{\mu\nu}(0)\,\mathcal{D}_{\sigma'}(y)\right]|0\rangle,$$

using the LSZ reduction (D. B. Kaplan, M. J. Savage and M. B. Wise, Phys. Rev. C **59**, 617 (1999)):

$$\langle p',\sigma'|\hat{T}^{\mu\nu}|p,\sigma\rangle = -\frac{1}{Z}\left[\left(p^2-M_d^2\right)\left(p'^2-M_d^2\right)G^{\mu\nu}_{\sigma'\sigma}(p',p)\right]_{p^2,\,p'^2\to M_d^2}.$$

$M_d = 2m_N - E_b$ is the deuteron mass, E_b - its binding energy, and Z - the residue of the propagator.

The deuteron interpolating field:

$$\mathcal{D}_i \equiv N^T\mathcal{P}_i N = \sum_{\alpha,\beta,a,b=1}^{2} N_{\alpha,a}\mathcal{P}^{\alpha\beta}_{i,ab}N_{\beta,b}, \quad \mathcal{P}_i \equiv \frac{1}{\sqrt{8}}\sigma_2\sigma_i\tau_2,$$

where α, β and a, b are the spin and isospin indices, respectively.

The two-point function of the deuteron interpolating fields:

$$G_{\mathcal{D}}(p)\delta_{\sigma'\sigma} = \int d^4x e^{-ipx}\langle 0|T\left[\mathcal{D}^\dagger_{\sigma'}(x)\mathcal{D}_\sigma(0)\right]|0\rangle = \delta_{\sigma'\sigma}\frac{i\,2M_d\,Z}{p^2-M_d^2+i\,\epsilon} + \text{N.P.}.$$

Vertex function $G^{\mu\nu}_{\sigma'\sigma}(p', p)$ represented diagrammatically:

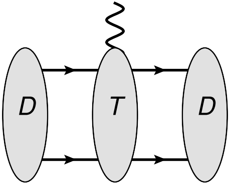

Figure: *D*-s represent the amplitudes of the deuteron interpolating field interacting with two nucleon fields, *T* stands for the two-nucleon-irreducible part of the vertex function. Solid and wavy lines denote the nucleons and the EMT insertion, respectively.

The deuteron equation

The amplitude of the deuteron field interacting with a pair of nucleon fields in the rest frame of the deuteron:

$$D_j(\mathbf{p}') = \mathcal{P}_j + m_N \int \frac{d^3\mathbf{k}}{(2\,\pi)^3}\, \frac{\mathcal{P}_j\, T(\mathbf{p}', \mathbf{k})}{m_N E - \mathbf{k}^2 + i\,\epsilon}\,,$$

where the NN scattering amplitude T satisfies:

$$T = V + VGT\,.$$

We consider the regulated LO NN potential of chiral EFT given by

$$\begin{aligned} V_0\,(\mathbf{p}', \mathbf{p}) &= (C_S + C_T\, \sigma_\mathbf{1} \cdot \sigma_\mathbf{2})\, \frac{\Lambda^4}{(\mathbf{p}'^2 + \Lambda^2)(\mathbf{p}^2 + \Lambda^2)} \\ &- \frac{g_A^2}{4\,F_\pi^2}\, \tau_\mathbf{1} \cdot \tau_\mathbf{2}\, \frac{\sigma_\mathbf{1} \cdot (\mathbf{p}' - \mathbf{p})\, \sigma_\mathbf{2} \cdot (\mathbf{p}' - \mathbf{p})}{(\mathbf{p}' - \mathbf{p})^2 + M_\pi^2}\, \frac{\Lambda^2 - M_\pi^2}{(\mathbf{p}' - \mathbf{p})^2 + \Lambda^2}\,, \end{aligned}$$

where C_S and C_T are LECs of the LO contact interactions.

We introduced smooth cutoff Λ to regularize UV divergences.

Our regularization violates EMT conservation, however such effects appear to be rather small.

We set the cutoff parameter $\Lambda \sim \Lambda_b \sim 400 - 600$ MeV.

A more systematic approach can be developed by applying a symmetry-preserving regularization:

D. Djukanovic, M. R. Schindler, J. G. and S. Scherer, Phys. Rev. D **72**, 045002 (2005),

H. Krebs and E. Epelbaum, Phys. Rev. C **110**, no.4, 044004 (2024).

Calculation of the form factors

Matrix element of the EMT in the Breit frame for the initial and final deuteron states with quantum numbers $(-\mathbf{q}/2, \sigma)$ and $(\mathbf{q}/2, \sigma')$

$$\langle \mathbf{q}/2, \sigma' | \hat{T}_{\mu\nu} | -\mathbf{q}/2, \sigma \rangle = m_N^2 \int \frac{d^3 k_1}{(2\pi)^3} \frac{d^3 k_2}{(2\pi)^3} D^{\gamma\delta}_{\sigma', cd}(\mathbf{k}_2)$$
$$\times \frac{T^{cd,\gamma\delta;ab,\alpha\beta}_{\mu\nu}\left(\frac{\mathbf{q}}{4} + \mathbf{k}_2, \frac{\mathbf{q}}{4} - \mathbf{k}_2; \mathbf{k}_1 - \frac{\mathbf{q}}{4}, -\mathbf{k}_1 - \frac{\mathbf{q}}{4}\right) D^{\dagger,\alpha\beta}_{\sigma,ab}(\mathbf{k}_1)}{(\mathbf{k}_1^2 + p_B^2)(\mathbf{k}_2^2 + p_B^2)},$$

$p_B = \sqrt{m_N E_b}$ and the arguments of the amplitude in the integrand correspond to individual momenta of both nucleons.

We apply the standard Weinberg power counting for the few-body sector of chiral EFT.

In our calculation, for EMT we include diagrams:

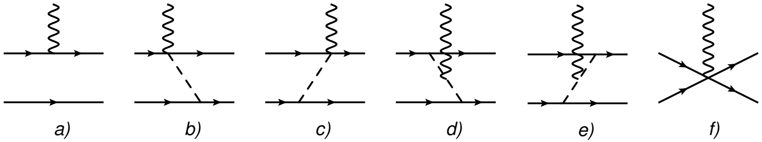

Figure: Diagrams where the graviton couples to the second nucleon are not shown. Solid, dashed and wavy lines correspond to nucleons, pions and gravitons, respectively.

One-loop corrections to the single-nucleon EMT merely renormalize the nucleon mass, nucleon field and c_8 and c_9.

OPE two-nucleon irreducible diagrams where the EMT couples to a single nucleon are canceled by $1/m_N$ corrections to OPE potential.

LO contribution to GFFs is given by diagram a). It has the form:

$$\langle \mathbf{q}/2, \sigma' | \hat{T}^{\rm LO}_{\mu\nu} | -\mathbf{q}/2, \sigma \rangle = 4m_N^2 \int \frac{d^3k}{(2\pi)^3} D_{\sigma'}(\mathbf{k}+\mathbf{q}/4) \times \frac{T_{\mu\nu,a}(\mathbf{k}+\mathbf{q}/2, \mathbf{k}-\mathbf{q}/2) D^\dagger_\sigma(\mathbf{k}-\mathbf{q}/4)}{\left[(\mathbf{k}+\mathbf{q}/4)^2 + p_B^2\right]\left[(\mathbf{k}-\mathbf{q}/4)^2 + p_B^2\right]},$$

where, up to the accuracy of our calculation

$$\begin{aligned} T_{00,a}(\mathbf{k}+\mathbf{q}/2, \mathbf{k}-\mathbf{q}/2) &= m_N + \frac{\mathbf{k}^2}{2m_N} - \frac{i\epsilon^{lmn}\sigma^l k^m q^n}{4m_N} + c_8\frac{\mathbf{q}^2}{4} + 2c_9\,\mathbf{q}^2, \\ T_{0i,a}(\mathbf{k}+\mathbf{q}/2, \mathbf{k}-\mathbf{q}/2) &= k_i + \frac{i\epsilon_{ilm}\sigma^l q^m}{4} + \frac{c_9}{m_N}\left[\mathbf{q}^2 k_i + (\mathbf{k}\cdot\mathbf{q})q_i\right], \\ T_{ij,a}(\mathbf{k}+\mathbf{q}/2, \mathbf{k}-\mathbf{q}/2) &= \frac{k_i k_j}{m_N} + \frac{i\sigma^l q^m}{4m_N}(k_i\epsilon_{jlm} + k_j\epsilon_{ilm}) - \frac{c_8}{4}\left[\mathbf{q}^2\delta_{ij} - q_i q_j\right]. \end{aligned}$$

$\mathbf{k}-\mathbf{q}/2$ and $\mathbf{k}+\mathbf{q}/2$ are the momenta of the incoming and outgoing nucleons, respectively.

The regularized contributions of diagrams b) and c):

$$
\begin{aligned}
T_{00,b+c} &= -\frac{g_A^2\,\tau_1\cdot\tau_2}{F_\pi^2}\frac{\sigma_1\cdot\tilde{\mathbf{k}}\,\sigma_2\cdot\tilde{\mathbf{k}}}{\tilde{\mathbf{k}}^2+M_\pi^2}\frac{\Lambda^2-M_\pi^2}{\tilde{\mathbf{k}}^2+\Lambda^2}\,,\\
T_{0i,b+c} &= 0\,, \qquad (1)\\
T_{ij,b+c} &= -\frac{g_A^2\,\tau_1\cdot\tau_2}{4F_\pi^2}\left[\frac{\sigma_1\cdot\tilde{\mathbf{k}}}{\tilde{\mathbf{k}}^2+M_\pi^2}\left(-2\delta_{ij}\,\sigma_2\cdot\tilde{\mathbf{k}}+\tilde{k}^i\sigma_2^j+\tilde{k}^j\sigma_2^i\right)\right.\\
&+\left.\frac{\sigma_2\cdot\tilde{\mathbf{k}}}{\tilde{\mathbf{k}}^2+M_\pi^2}\left(-2\delta_{ij}\,\sigma_1\cdot\tilde{\mathbf{k}}+\tilde{k}^i\sigma_1^j+\tilde{k}^j\sigma_1^i\right)\right]\frac{\Lambda^2-M_\pi^2}{\tilde{\mathbf{k}}^2+\Lambda^2}\,,
\end{aligned}
$$

The regularized contributions of diagrams d) and e) have the form:

$$
\begin{aligned}
T_{00,d+e} &= \frac{g_A^2\,\tau_1\cdot\tau_2}{4F_\pi^2}\frac{\sigma_1\cdot\bar{\mathbf{k}}\,\sigma_2\cdot\tilde{\mathbf{k}}}{(\bar{\mathbf{k}}^2+M_\pi^2)(\tilde{\mathbf{k}}^2+M_\pi^2)}\left[M_\pi^2+\bar{\mathbf{k}}\cdot\tilde{\mathbf{k}}\right]\frac{\Lambda^2-M_\pi^2}{\bar{\mathbf{k}}^2+\Lambda^2}\frac{\Lambda^2-M_\pi^2}{\tilde{\mathbf{k}}^2+\Lambda^2}\,,\\
T_{0i,d+e} &= 0\,,\\
T_{ij,d+e} &= \frac{g_A^2\,\tau_1\cdot\tau_2}{4F_\pi^2}\frac{\sigma_1\cdot\bar{\mathbf{k}}\,\sigma_2\cdot\tilde{\mathbf{k}}}{(\bar{\mathbf{k}}^2+M_\pi^2)(\tilde{\mathbf{k}}^2+M_\pi^2)}\left[\bar{k}^i\tilde{k}^j+\bar{k}^j\tilde{k}^i-\delta_{ij}(M_\pi^2+\bar{\mathbf{k}}\cdot\tilde{\mathbf{k}})\right]\\
&\times\frac{\Lambda^2-M_\pi^2}{\bar{\mathbf{k}}^2+\Lambda^2}\frac{\Lambda^2-M_\pi^2}{\tilde{\mathbf{k}}^2+\Lambda^2}\,, \qquad (2)
\end{aligned}
$$

$\bar{\mathbf{k}}=\mathbf{k}_1-\mathbf{k}_2-\mathbf{q}/2$ and $\tilde{\mathbf{k}}=\mathbf{k}_1-\mathbf{k}_2+\mathbf{q}/2$.

Result of diagram with EMT of the LO NN contact interaction:

$$T_{\mu\nu,f}=-2(C_S+C_T)g^{\mu\nu}\,.$$

We obtained the GFFs of the deuteron by calculating the matrix elements of EMT numerically.

We fixed the coupling constant of the S-wave NN contact interaction by reproducing the binding energy of the deuteron.

We show results for $\Lambda = 500$ MeV together with the results of (HZ)
F. He and I. Zahed, Phys. Rev. C **110**, no.1, 014312 (2024)
for the deuteron GFFs in our parameterization.

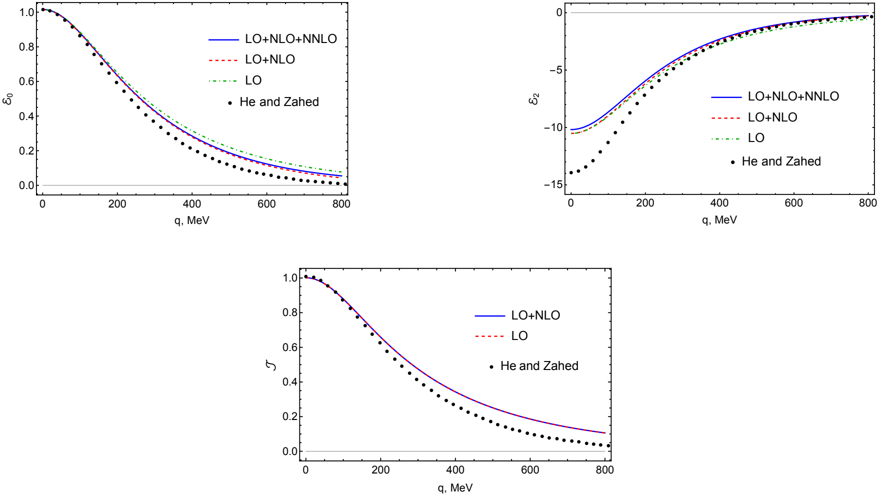

Figure: GFFs $\mathcal{E}$ and $\mathcal{J}$ of the deuteron for $\Lambda = 500$ MeV. We compare our results with the GFFs by HZ. Our two curves for $\mathcal{J}$ coincide due to vanishing NLO contribution to this GFF.

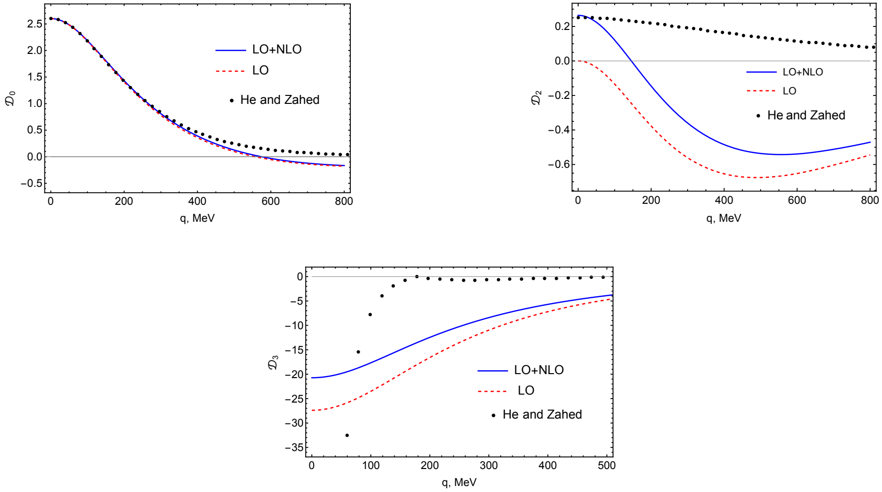

Figure: GFFs $\mathcal{D}_i$ for $\Lambda = 500$ MeV. We compare our results with the GFFs by HZ.

While for comparison we plot the figures up to $800\,\mathrm{MeV}$, our results cannot be trusted at such large values.

Our calculated $\mathcal{E}_0$, $\mathcal{E}_2$ and $\mathcal{J}$ show similar behavior to those of HZ.

Deviation of $\mathcal{E}_0$ from 1 at $q = 0$ is tiny, indicating that the effect of the non-invariant regularization is rather small.

We fixed $c_8 = -2.77\ \mathrm{GeV}^{-1}$ such that our $\mathcal{D}_0(0)$ coincides with HZ.

The resulting q-dependence of our $\mathcal{D}_0$ is very similar to that of HZ.

Our $\mathcal{D}_2$ has a different shape and $\mathcal{D}_3$ shows qualitatively different behavior.

Our numerical results show a very mild cutoff dependence for Λ between ~ 400 and 600 MeV.

Obtained GFFs demonstrate a reasonable convergence rate of the chiral expansion for the deuteron GFFs.

Definition of spatial densities

E.Epelbaum, J.Gegelia, N.Lange., U.-G.Meißner., M.V.Polyakov,
Phys. Rev. Lett. **129**, 012001 (2022), [arXiv:2201.02565 [hep-ph]].

J.Y.Panteleeva, E.Epelbaum, J.Gegelia, U.-G.Meißner,
Phys. Rev. D **106**, no.5, 056019 (2022), [arXiv:2205.15061 [hep-ph]].

J.Y.Panteleeva, E.Epelbaum, J.Gegelia, U.-G.Meißner,
[arXiv:2211.09596 [hep-ph]].

We revisited the definition of the charge density and other spatial densities ...

Disclaimer:

$$\frac{\textit{What I know about internal structure of hadrons}}{\textit{What I do not know}} \approx 0\,!$$

Definition of spatial distributions has attracted much attention.

To give examples ...

The light-front approach allows one to define purely intrinsic spatial densities, which have probabilistic interpretation

M. Burkardt, Phys. Rev. D **62** (2000), 071503(R).
G. A. Miller, Phys. Rev. Lett. **99**, 112001 (2007).
G. A. Miller, Phys. Rev. C **79**, 055204 (2009).
G. A. Miller, Ann. Rev. Nucl. Part. Sci. **60** (2010), 1-25.

Y. Guo, X. Ji and K. Shiells, Nucl. Phys. B **969**, 115440 (2021).

These densities are obtained as two-dimensional distributions.

Alternatively, the phase-space approach allows one to define fully relativistic and unambiguous three-dimensional spatial densities.

C. Lorcé, Phys. Rev. Lett. **125**, no.23, 232002 (2020).
C. Lorcé, P. Schweitzer, K. Tezgin, Phys. Rev. D **106**, no.1, 1 (2022).

C. Lorcé, H. Moutarde, A. P. Trawiński, Eur. Phys. J. C **79**, no.1, 89 (2019).

These densities depend on both coordinates and momenta.

I present definition of spatial densities via sharply localized states.

We use spherically symmetric wave packets, corresponding to ZAMF.

Thanks Cedric Lorcé for pointing out that similar results for spin-0 systems have been published long ago in

G. N. Fleming, *Charge Distributions from Relativistic Form Factors.* Physical Reality & Math. Descrip., 357 (1974).

Localized states

Localized states:

$$|\Phi, \mathbf{X}, s\rangle = \int \frac{d^3p}{\sqrt{2E(2\pi)^3}}\, \phi(s, \mathbf{p})\, e^{-i\mathbf{p}\cdot\mathbf{X}} |p, s\rangle,$$

where $\mathbf{X}$ is the position of the system, and $|p, s\rangle$ are normalized as

$$\langle p', s' | p, s\rangle = 2E(2\pi)^3 \delta_{s's}\, \delta^{(3)}(\mathbf{p}' - \mathbf{p})\,, \qquad p = (E, \mathbf{p})\,.$$

Profile function $\phi(s, \mathbf{p}) = \phi(|\mathbf{p}|)$ corresponds to ZAMF and

$$\int d^3p\, |\phi(s, \mathbf{p})|^2 = 1\,.$$

It is convenient to define:

$$\phi(\mathbf{p}) = R^{3/2}\, \tilde{\phi}(R\mathbf{p})\,,$$

Sharp localizations correspond to small R.

The charge density of a spin-0 system in ZAMF

Matrix elements of $\hat{\rho}(\mathbf{r},0)$ for momentum eigenstates ($Q=1$):

$$\langle p'|\hat{\rho}(\mathbf{r},0)|p\rangle = e^{i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{r}}(E+E')F(q^2)\,,$$

$F(q^2)$ is the electric FF and $q = p' - p$.

The charge density distribution:

$$\langle\Phi,\mathbf{X}|\hat{\rho}(\mathbf{r},0)|\Phi,\mathbf{X}\rangle = \int \frac{d^3p\, d^3p'(E+E')}{(2\pi)^3\sqrt{4EE'}} F\left(q^2\right)\phi^\star(\mathbf{p}')\phi(\mathbf{p})e^{i\mathbf{q}\cdot(\mathbf{X}+\mathbf{r})}.$$

Without loss of generality we choose $\mathbf{X}=0$.

Introducing the total and relative momenta via $\mathbf{p} = \mathbf{P} - \mathbf{q}/2$ and $\mathbf{p}' = \mathbf{P} + \mathbf{q}/2$, the charge density is written as

$$\begin{aligned}\rho_\phi(\mathbf{r}) \;&\equiv\; \langle\Phi,\mathbf{0}|\hat{\rho}(\mathbf{r},0)|\Phi,\mathbf{0}\rangle = \int \frac{d^3P\, d^3q}{(2\pi)^3\sqrt{4EE'}}\,(E+E')\,F\left[(E-E')^2-\mathbf{q}^2\right]\\ &\times\; \phi\left(\mathbf{P}-\frac{\mathbf{q}}{2}\right)\phi^\star\left(\mathbf{P}+\frac{\mathbf{q}}{2}\right)e^{i\mathbf{q}\cdot\mathbf{r}}.\end{aligned}$$

Traditional interpretation of the charge density emerges by first taking the static limit $E = E' = m$ in the integrand:

$$\rho_{\phi,\,\text{naive}}(\mathbf{r}) = \int \frac{d^3P\, d^3q}{(2\pi)^3}\, \phi\left(\mathbf{P} - \frac{\mathbf{q}}{2}\right) \phi^{\star}\left(\mathbf{P} + \frac{\mathbf{q}}{2}\right) F\left(-\mathbf{q}^2\right)\, e^{i\mathbf{q}\cdot\mathbf{r}},$$

and then taking the limit $R \to 0$.

This can be done without specifying $F\left(q^2\right)$ and $\phi(\mathbf{p})$ using the method of

J.G., G.Japaridze, K.Turashvili, Theor. Math. Phys. **101**, 1313 (1994).

The only non-vanishing contribution for $R \to 0$ is obtained from the region of large $\mathbf{P}$:

$$\begin{aligned} \rho_{\text{naive}}(r) &= \int \frac{d^3\tilde{P}\, d^3q}{(2\pi)^3}\, F\left(-\mathbf{q}^2\right)\, |\tilde{\phi}(\tilde{\mathbf{P}})|^2\, e^{i\mathbf{q}\cdot\mathbf{r}} \\ &= \int \frac{d^3q}{(2\pi)^3}\, F\left(-\mathbf{q}^2\right)\, e^{i\mathbf{q}\cdot\mathbf{r}}. \end{aligned}$$

This expression corresponds to ZAMF in states with $R \gg \frac{1}{m}$.

New definition in ZAMF

Taking the $R \to 0$ limit without the static approximation results in

$$\rho_\phi(\mathbf{r}) = \int \frac{d^3\tilde{P}\, d^3q}{(2\pi)^3} F\left[\frac{(\tilde{\mathbf{P}}\cdot\mathbf{q})^2}{\tilde{\mathbf{P}}^2} - \mathbf{q}^2\right] |\tilde{\phi}(\tilde{\mathbf{P}})|^2 e^{i\mathbf{q}\cdot\mathbf{r}}. \qquad (3)$$

Using $\tilde{\phi}(\tilde{\mathbf{P}}) = \tilde{\phi}(|\tilde{\mathbf{P}}|)$ and switching to spherical coordinates we get

$$\rho(r) = \frac{1}{4\pi}\int d^2\hat{n}\,\delta(r_{\|})\rho_{\hat{\mathbf{n}}}(r_\perp)\,,$$

where

$$\rho_{\hat{\mathbf{n}}}(r_\perp) = \int \frac{d^2q_\perp}{(2\pi)^2} F(-\mathbf{q}_\perp^2)\, e^{i\mathbf{q}_\perp\cdot\mathbf{r}_\perp}\,,$$

$\hat{\mathbf{n}}$ is a unit vector, $\mathbf{a}_\perp = \hat{\mathbf{n}} \times (\mathbf{a} \times \hat{\mathbf{n}})$, $a_{\|} = \mathbf{a}\cdot\hat{\mathbf{n}}$, $a_\perp \equiv |\mathbf{a}_\perp|$.

The ZAMF expression $\rho(r)$ is given by a continuous (isotropic) superposition of the two-dimensional "images" of the system.

The full image of a three-dimensional object can be reconstructed by putting together all possible two-dimensional projections.

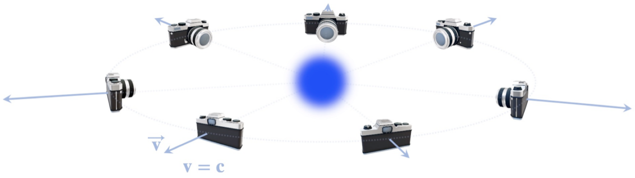

Figure: 3D image as a composition of 2D pictures

$\rho_{\text{naive}}(r)$ is valid only for $\Delta \gg 1/m$, because we have to take $\Delta \gg R \gg 1/m$.

The validity of ZAMF definition does not depend on the relation between Δ and $1/m$.

$\rho_{\text{naive}}(r)$ does *not* emerge from $\rho(r)$ by taking the static limit:

$$\rho_{\text{naive}}(r) \neq \lim_{m\to\infty} \rho(r).$$

Reason: the $R \to 0$ and $m \to \infty$ limits of $\rho_\phi(\mathbf{r})$ do not commute.

The dependence of $\rho(r)$ on $F(-\mathbf{q}_\perp^2)$ rather than on $F(-\mathbf{q}^2)$ affects the radial profile of the charge density.

We compare $\rho(r)$ and $\rho_{\rm naive}(r)$ for a charged and a neutral particles.

We employ form factors

$F_{\rm p}(q^2) = G_{\rm D}(q^2) = (1 - q^2/\Lambda^2)^{-2}$

with $\Lambda^2 = 0.71$ GeV^2,

and

$F_{\rm n}(q^2) = A\tau/(1 + B\tau)\, G_{\rm D}(q^2)$,

where $\tau = -q^2/(4m_p^2)$, $A = 1.70$, $B = 3.30$.

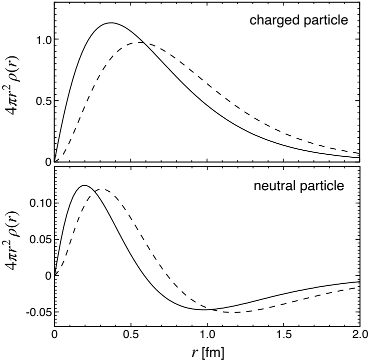

Figure: Radial charge density distributions $4\pi r^2 \rho(r)$ (solid lines) and $4\pi r^2 \rho_{\rm naive}(r)$ (dashed lines) for a charged and a neutral particles.

EMT spatial densities for spin-0 particles

Matrix element of EMT in a localized state:

$$
\begin{aligned}
t_{\phi}^{\mu\nu}(\mathbf{r}) &= \langle \Phi, \mathbf{0} | \hat{T}^{\mu\nu}(\mathbf{r}, 0) | \Phi, \mathbf{0} \rangle \\
&= \int \frac{d^3p' \, d^3p}{(2\pi)^3 \sqrt{4E'E}} \, \phi^\star(\mathbf{p}') \, \phi(\mathbf{p}) \langle p' | \, \hat{T}^{\mu\nu}(\mathbf{x}, 0) | p \rangle \\
&= \int \frac{d^3P \, d^3q}{(2\pi)^3 \sqrt{4EE'}} \left[\left(q^2 g^{\mu\nu} - q^\mu q^\nu \right) \Theta_1 \left(q^2 \right) + 2P^\mu P^\nu \Theta_2 \left(q^2 \right) \right] \\
&\times \phi \left(\mathbf{P} - \frac{\mathbf{q}}{2} \right) \phi^\star \left(\mathbf{P} + \frac{\mathbf{q}}{2} \right) e^{-i\mathbf{q}\cdot(\mathbf{r})} \, .
\end{aligned}
$$

$q = p' - p$ and $P = (p + p')/2$.

Static approximation

Local densities in terms of FFs in the Breit frame emerge by expanding the integrand in $1/m$ and localizing the wave packet:

$$
\begin{aligned}
t^{00}_{\text{naive}}(\mathbf{r}) &= \int \frac{d^3q}{(2\pi)^3}\, m\,\Theta_2\left(-\mathbf{q}^2\right)\, e^{-i\mathbf{q}\cdot\mathbf{r}}\,, \\
t^{0i}_{\text{naive}}(\mathbf{r}) &= 0\,, \\
t^{ij}_{\text{naive}}(\mathbf{r}) &= \frac{4\pi}{mR^2}\int d\tilde{P}\,\tilde{P}^2\,\tilde{P}^i\tilde{P}^j|\tilde{\phi}(\tilde{\mathbf{P}})|^2 \int \frac{d^3q}{(2\pi)^3}\,\Theta_2\left(-\mathbf{q}^2\right) e^{-i\mathbf{q}\cdot\mathbf{r}} \\
&+ \frac{1}{2m}\int \frac{d^3q}{(2\pi)^3}\left(\mathbf{q}^2\delta^{ij} - q^i q^j\right)\Theta_1\left(-\mathbf{q}^2\right) e^{-i\mathbf{q}\cdot\mathbf{r}}\,.
\end{aligned}
$$

t^{00}_{naive} and the second term of t^{ij}_{naive} coincide with the spatial densities defined via the gravitational FFs in the Breit frame.

M. V. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A **33** (2018) no.26, 1830025.

Both of these terms do not depend on the packet.

Spatial densities in sharply localized states ($R \to 0$):

$$t^{\mu\nu}(\mathbf{r}) = N_{\phi,R} \int \frac{d^2\hat{\tilde{P}}\, d^3q}{(2\pi)^3}\, \hat{\tilde{P}}^\mu \hat{\tilde{P}}^\nu\, \Theta_2\left[-\mathbf{q}_\perp^2\right] e^{-i\mathbf{q}\cdot\mathbf{r}}$$
$$- N_{\phi,R,2} \int \frac{d^2\hat{\tilde{P}}\, d^3q}{(2\pi)^3} \left(\tilde{q}^\mu \tilde{q}^\nu + \mathbf{q}_\perp^2 g^{\mu\nu}\right) \Theta_1\left[-\mathbf{q}_\perp^2\right] e^{-i\mathbf{q}\cdot\mathbf{r}},$$

where $\tilde{q}^\mu = (\hat{\tilde{\mathbf{P}}} \cdot \mathbf{q}, \mathbf{q})$, $\tilde{P}^\mu = (\tilde{P}, \tilde{\mathbf{P}})$, $\hat{\tilde{P}}^\mu = \left(1, \frac{\tilde{\mathbf{P}}}{\tilde{P}}\right)$, $\tilde{P} = |\tilde{\mathbf{P}}|$, $\mathbf{q}_\perp = \hat{\tilde{\mathbf{P}}} \times \left(\mathbf{q} \times \hat{\tilde{\mathbf{P}}}\right)$, $\mathbf{q}_\perp^2 \equiv -\tilde{q}^2$ and

$$N_{\phi,R} = \frac{1}{R} \int d\tilde{P}\, \tilde{P}^3 |\tilde{\phi}(|\tilde{\mathbf{P}}|)|^2,$$
$$N_{\phi,R,2} = \frac{R}{2} \int d\tilde{P}\, \tilde{P} |\tilde{\phi}(|\tilde{\mathbf{P}}|)|^2.$$

The dependence on the profile function remains in $N_{\phi,R}$ and $N_{\phi,R,2}$.

Interpretation

Separate interpretation of different contributions:

A. Freese and G. A. Miller, Phys. Rev. D **105**, no.1, 014003 (2022), [arXiv:2108.03301 [hep-ph]].

There are two types of contributions:

1. Depending on the velocity - characterizing the system as a whole.
2. Contributions which are related to internal properties.

In ZAMF the term

$$t_2^{ij}(\mathbf{r}) = N_{\phi,R,2} \int \frac{d^2\hat{n}\, d^3q}{(2\pi)^3} \left(-q^i q^j + \mathbf{q}_\perp^2 \delta^{ij}\right) \Theta_1 \left[-\mathbf{q}_\perp^2\right] e^{-i\mathbf{q}\cdot\mathbf{r}}$$

is interpreted as characterizing the distribution of internal forces.

While normalization depends on the profile function spatial distribution is uniquely determined by the EMT form factor.

We identify the traceless and the trace parts via

$$t_2^{ij}(\mathbf{r}) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\,\delta^{ij}\right) s(r) + \delta^{ij} p(r)\,, \tag{4}$$

Quantities $s(r)$ and $p(r)$ have been interpreted as the shear force and the pressure, respectively.

M. V. Polyakov, Phys. Lett. B **555**, 57 (2003).

M. V. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A **33** (2018) no.26, 1830025.

This interpretation has been questioned in

X. Ji and Y. Liu, Phys. Rev. D **106**, no.3, 034028 (2022).

Breit-frame expressions correspond to systems in ZAMF in states with characteristic scales of packets much larger than $1/m$.

Such packet is dominated by eigenstates of the energy with $E \approx m$.

$t^{00}(\mathbf{r})$ can be interpreted in this case as the spatial distribution of the mass.

Sharp localization of the system requires huge amount of energy, therefore the normalization factor for the energy and the momentum distributions $N_{\phi,R}$ explodes. Again, the functional form of these densities is uniquely determined by the corresponding form factors.

Summary

- Presented the effective chiral Lagrangian of pions and nucleons up to the second chiral order in curved spacetime.
- Derived the corresponding EMT of pions and nucleons in flat spacetime.
- Presented the one-loop contributions to GFFs of the nucleon.
- Discussed the calculation of the deuteron GFFs in chiral EFT, the formalism and numerical results.
- Very briefly discussed spatial densities.