BChPT x 1/Nc
masses, currents and πN scattering

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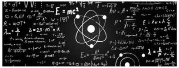
MECHANICAL PROPERTIES OF HADRONS: STRUCTURE, DYNAMICS, VISUALIZATION



31 March 2025 — 04 April 2025

Outline

- 1/Nc expansion and baryons
- Baryons and the spin-flavor symmetry
- 1/Nc expansion in effective theory
- BChPT x 1/Nc
- Baryon masses and σ terms
- SU(3) Vector currents
- Axial currents
- πN scattering
- Comments

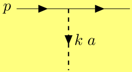
Collaborators
Alvaro Calle-Cordon, Ishara Fernando, Rubén Flores-Mendieta, Jose Alarcón, Dulitha Jayakodige

$1/N_c$ expansion and baryons

Large N_c limit - 'tHooft framework:
$SU(N_c)$, quarks in fundamental irrep, N_f fixed
Setting scales: M_ρ, M_π, $M_{K's}$

Hadronic level N_c scalings:

$F_\pi \sim \sqrt{N_c}$; $m_{\rm baryon} \sim N_c$; $g_A \sim N_c$; $g_{\pi NN} \sim N_c^{3/2}$



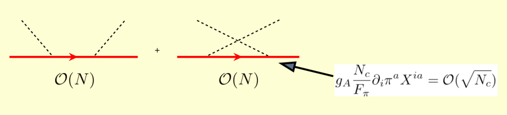
$$= \frac{\mathring{g}_A}{F_\pi} k^i G^{ia} = \mathcal{O}(\sqrt{N_c})$$

@ large N_c: meson sector weakly coupled, meson-baryon coupling increasingly strong

emergent dynamical constraints in baryon sector dictated by unitarity

Baryons and spin-flavor dynamical symmetry

[Gervais & Sakita (1984); Dashen & Manohar (1993)]

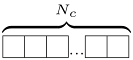


$$\sim \frac{k^i k'^j}{k^0} \frac{g_A N_c^2}{F_\pi^2} \langle B' \mid [X^{ia}, X^{jb}] \mid B \rangle = \mathcal{O}(N_c^0)$$

$\Rightarrow [X^{ia}, X^{jb}] = \mathcal{O}(1/N_c)$ key requirement for large N_c consistency

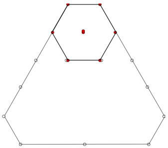
S^i, T^a, X^{ia} generate contracted spin-flavor group $SU(2N_f)$

$SU(2N_f)$: organizing tool for $1/N_c$ expansion in baryons

S: 

ground state baryons: tower with $S = \frac{1}{2} \cdots \frac{N_c}{2}$

$N_f = 3$ states in $SU(2) \times SU(3)$: $[S, R] = [S, (2S, \frac{1}{2}(N - 2S))]$



Spin-flavor Symmetry

- symmetry of spectrum at large N_c
- dynamical symmetry: not a Noether symmetry!
- imposes constraints in effective Lagrangians: relations between LECs

- mass splitting for $S = \mathcal{O}(N_c^0)$ are $\mathcal{O}(1/N_c)$ e.g. $m_\Delta - m_N$
- $m_B = \mathcal{O}(N_c) \Rightarrow$ @ low energy NR expansion $\sim 1/N_c$ expansion

1/Nc in effective theory

Low energy light hadron sector:

ChPT consistent with requirements of $1/N_c$ expansion

Goldstone Boson sector: fully implemented long ago - includes η' for consistency: $M^2_{\eta'} \sim \mathcal{O}(1/N_c)$ if $m_q = 0$.

Baryon sector: more recently implemented -
must include full baryon SF multiplet as degrees of freedom ($N_c = 3$ must include Δ)

BChPT x 1/Nc [Jenkins; Flores-Mendieta; Calle-Cordon & JLG]

- $m_B = \mathcal{O}(N_c) \Rightarrow$ HB expansion is a $1/N_c$ expansion

- Lagrangians built with chiral and spin-flavor tensor operators:

$$\mathcal{L} = \mathbf{B}^\dagger T_\chi \otimes T_{SF} \quad \mathbf{B} \qquad\qquad \mathbf{B} = \begin{pmatrix} B_{S=1/2} \\ B_{S=3/2} \\ \vdots \end{pmatrix} \quad \text{GS baryons}$$

T_χ chiral tensor, T_{SF} spin-flavor tensor $\otimes$ of $\mathrm{SU}(2N_f)$ generators

- $1/N_c$ power counting: factor $1/N_c^{n-1}$ for n-body SF tensor

- Systematic construction using bases of SF tensors

- LECs: $\mathcal{O}\left(N_c^0\right)$, have a $1/N_c$ expansion themselves

- Lagrangian terms have well defined leading chiral and $1/N_c$ power

- Small mass scale: $\Delta_{HF} = m_{3/2} - m_{1/2}$ - need to link chiral and $1/N_c$ expansions: ξ expansion: $\xi = \mathcal{O}(1/N_c) = \mathcal{O}(p)$

Baryon chiral Lagrangians to $\mathcal{O}(\xi^3)$

$$\mathcal{L}_B^{(1)} = \mathbf{B}^\dagger \left(iD_0 - \overset{\circ}{g}_A \; u^{ia} G^{ia} - \frac{C_{\rm HF}}{N_c} \hat{S}^2 + \frac{c_1}{2\Lambda} \hat{\chi}_+ \right) \mathbf{B} \qquad \overset{\circ}{g}_A = \frac{6}{5} g_A^N$$

$$\mathcal{L}_{\mathbf{B}}^{(2)} = \mathbf{B}^\dagger \left(\frac{c_2}{\Lambda} \chi_+^0 + \frac{C_1^A}{N_c} u^{ia} S^i T^a + \frac{\tau_1}{N_c} \left(u_0^a G^{ia} D_i + \overleftarrow{D}_i u_0^a G^{ia} \right) \right.$$

$$\left. + \frac{1}{m} \left(\vec{B}_+^0 + \vec{B}_+^a T^a \right) \cdot \vec{S} + \frac{1}{2m} \left(2 \left(\kappa_0 \vec{B}_+^0 + \kappa_1 \vec{B}_+^a T^a \right) \cdot \vec{S} + \frac{6}{5} \kappa_2 B_+^{ia} G^{ia} \right) + \cdots \right) \mathbf{B}$$

$$\mathcal{L}_{\mathbf{B}}^{(3)} = \mathbf{B}^\dagger \left(\frac{1}{2m} D^\mu D_\mu + \frac{c_3}{N_c \Lambda^3} \hat{\chi}_+^2 + \frac{h_1}{N^3} \hat{S}^4 + \frac{h_2}{N_c^2 \Lambda} \hat{\chi} + \hat{S}^2 + \frac{h_3}{N_c \Lambda} \chi_+^0 \hat{S}^2 + \frac{h_4}{N_c \Lambda} \chi_+^a \left\{ S^i, G^{ia} \right\} \right.$$

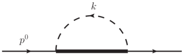
$$+ \frac{C_2^A}{N_c^2} u^{ia} \left\{ \hat{S}^2, G^{ia} \right\} + \frac{C_4^A}{N^2} u^{ia} S^i S^j G^{ja} + \frac{D_1^A}{\Lambda^2} \chi_+^0 u^{ia} G^{ia} + \frac{D_2^A}{\Lambda^2} \chi_+^a u^{ia} S^i$$

$$+ \frac{D_3^A(d)}{\Lambda^2} d^{abc} \chi_+^a u^{ib} G^{ic} + \frac{D_3^A(f)}{\Lambda^2} f^{abc} \chi_+^a u^{ib} G^{ic}$$

$$\left. + \left(\frac{1}{8m^2} + \frac{g_0}{\Lambda^2} \right) \partial_i E_{+i}^0 + \left(\frac{1}{8m^2} + \frac{g_1}{\Lambda^2} \right) (D_i E_{+i})^a \, T^a + \cdots \right) \mathbf{B}$$

[Jenkins; Flores-Mendieta et al; Calle-Cordon & JLG; Fernando & JLG]

Combined chiral and $1/Nc$ expansions



$$= \int \frac{d^dk}{(2\pi)^d} \frac{i}{k^2 - M_\pi^2} \frac{i}{p^0 + k^0 - \underbrace{(m_{B'} - m_B)}_{\mathcal{O}(1/N_c)}} \times \text{vertex factors}$$

contains non-analytic terms:

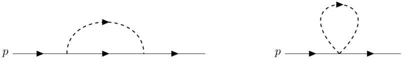
$$(M_\pi^2 - (m_\Delta - m_N)^2)^{\frac{3}{2}}, \; tanh^{-1}\left(\frac{(m_\Delta - m_N)}{\sqrt{1/(-M_\pi^2 + (m_\Delta - m_N)^2}}\right)$$

link $1/N_c$ and chiral expansions:

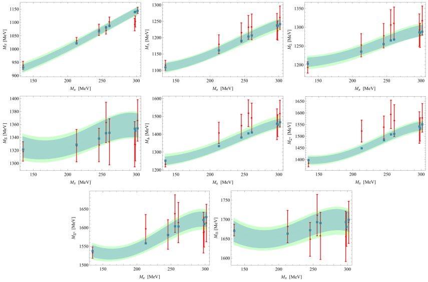
$$\xi - \text{expansion}: \quad \mathcal{O}(p) = \mathcal{O}(1/N_c) = \mathcal{O}(\xi)$$

equivalent to not expanding non-analytic terms

Baryon masses and σ terms [Calle-Cordon & JLG, Fernando, Alarcon & JLG]



- WF renormalization factor is $\mathcal{O}(N_c)$ - plays key role in N_c power counting consistency in loops
- mass corrections are $\mathcal{O}(N_c)$ (terms proportional to M_{GB}^3)
- $\mathrm{SU}(3)$ mass splitting remain $\mathcal{O}\left(N_c^0\right)$
- M_π dependency from LQCD



GMO $\Delta_{GMO} = \mathrm{Th}: \left(\frac{g_A^N(LO)}{g_A^N}\right)^2 44 \pm 5\mathrm{MeV}$ vs Exp: $25.6 \pm 1.5\mathrm{MeV}$

$$\Delta_{GMO} = -\left(\frac{g_A}{4\pi F_\pi}\right)^2 \left(\frac{2\pi}{3}\left(M_K^3 - \frac{1}{4}M_\pi^3 - \frac{2}{\sqrt{3}}\left(M_K^2 - \frac{1}{4}M_\pi^2\right)^{\frac{3}{2}}\right)\right.$$
$$\left. + \frac{2C_{HF}}{N_c}\left(-M_K^2 \log M_K^2 + \frac{1}{4}M_\pi^2 \log M_\pi^2 + \left(M_K^2 - \frac{1}{4}M_\pi^2\right)\log\left(\frac{4}{3}M_K^2 - \frac{1}{3}M_\pi^2\right)\right)\right)$$
$$= 37\mathrm{MeV} + \mathcal{O}\left(1/N_c^3\right)$$

ES $\Delta_{ES} = m_{\Xi^*} - 2m_{\Sigma^*} + m_\Delta =$

$\mathrm{Th}: -\left(\frac{g_A^N(LO)}{g_A^N}\right)^2 6.5\mathrm{MeV}$ vs Exp: $-4 \pm 7\mathrm{MeV} = \mathcal{O}(1/N_c)$

GR $\Delta_{GR} = m_{\Xi^*} - m_{\Sigma^*} - (m_\Xi - m_\Sigma) = 0,$ Exp: $21 \pm 7\mathrm{MeV}$

$$\Delta_{GR} = \frac{h_2}{\Lambda}\frac{12}{N_c}\left(M_K^2 - M_\pi^2\right) + \underbrace{\mathrm{O}\left(1/N_c\right)\ \text{UV finite no-analytic terms}}_{\sim 68\mathrm{MeV}\times\left(\frac{g_A^N(LO)}{g_A^N}\right)^2}$$

all deviations from LO mass relations are $\mathcal{O}(1/N_c)$

σ-terms: quark mass contributions to baryon masses: Feynman-Hellmann theorem

$$\sigma_{\pi N} = \langle N \mid \hat{m}(\bar{u}u + \bar{d}d) \mid N \rangle = \hat{m}\frac{\partial m_N}{\partial \hat{m}} \quad \hat{m} = \frac{1}{2}(m_u + m_d)$$

$$\sigma_s = \langle N \mid m_s \bar{s}s) \mid N \rangle$$

$$\hat{\sigma} = \hat{m}\langle N \mid \hat{m}(\bar{u}u + \bar{d}d - 2\bar{s}s) \mid N \rangle = \frac{\hat{m}}{m_8}\sigma_8 \quad m_8 = \frac{1}{\sqrt{3}}(\hat{m} - m_s)$$

$$\sigma_{\pi N} = \hat{\sigma} + 2\frac{\hat{m}}{m_s}\sigma_s$$

The σ puzzle:
i) expect $\sigma_{\pi N} \sim \hat{\sigma}$
ii) LO ChPT: $\sigma_8 = \frac{1}{3}(\frac{5N_c-3}{2}m_N - (2N_c - 3)m_\Sigma - \frac{N_c+3}{2}m_\Xi)$
gives $\hat{\sigma} \sim 23\ \mathrm{MeV}$ - need for very large σ_s to agree with $\sigma_{\pi N}|_{\mathrm{exp}} > 50\ \mathrm{MeV}$
iii) further puzzle: $\sigma_{\pi N} = \mathcal{O}(N_c)$ vs $\Delta_{GMO} = \mathcal{O}(1/N_c)$!

BChPT x 1/Nc: **8** masses to NNLO - combine results for Δ_{GMO} and σ_8 giving parameter free relation.

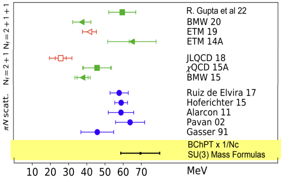
Use value of Δ_{GMO} to determine correction to $\hat{\sigma}$

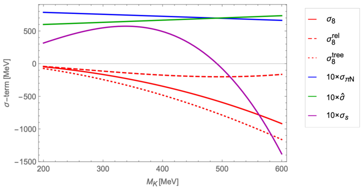
$$\sigma = \sqrt{3}\frac{\hat{m}}{m_8}\sigma_8 + \Delta\hat{\sigma} \qquad \Delta\hat{\sigma} \sim 40 \text{ MeV}$$

key role of **10**: $\Delta\hat{\sigma}$: 40% from baryon **8** and 60% from **10**

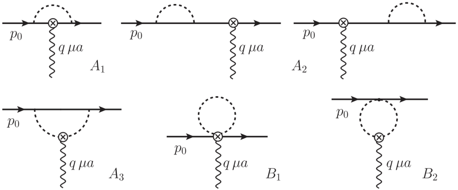
[Fernando, Alarcon & JLG]

Fit	Δ_{GMO}^{phys} MeV	σ_8 MeV	$\Delta\sigma_8$ MeV	$\hat{\sigma}$ MeV	$\sigma_{\pi N}$ MeV	σ_s MeV	σ_3 MeV	$\sigma_{u+d}(p-n)$ MeV
1	25.6(1.1)	−583(24)	−382(13)	70(3)(6)	–	–	−1.0(3)	−1.6(6)
2	25.5(1.5)	−582(55)	−381(20)	70(7)(6)	69(8)(6)	−3(32)	−1.0(4)	−1.6(8)
3	25.8*	−615(80)	−384(2)	74(1)(6)	65(15)(6)	−121(15)	–	–





Vector currents [Flores-Mendieta & JLG; Fernando & JLG]



• Diagrams A: different spin baryons in loop give $\mathcal{O}(N_c)$ terms - key cancellations give N_c consistency.

• Magnetic moments: Isovector is $\mathcal{O}(N_c)$, Isoscalar is $\mathcal{O}(N_c^0)$.

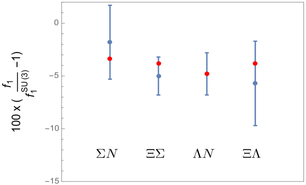
$$\mu_0 = \frac{1}{2}(\mu_p + \mu_n) = 0.44\mu_N \quad \mu_3 = \frac{1}{2}(\mu_p - \mu_n) = 2.35\mu_N$$

• Charge radii: $\mathcal{O}(N_c^0)$

• SU(3) breaking in charges: Ademollo-Gatto theorem $\mathcal{O}(\xi^2)$ by non-analytic calculable loop terms $\mathcal{O}(N_c^0)$.

SU(3) breaking in vector charges of baryon octet

Calculable correction - finite loop contributions only: Ademollo-Gatto



LQCD [Shanahan et al (2015)]

	$\frac{\delta f_1}{f_1}$ One-loop	$\frac{\delta f_1}{f_1}$ LQCD
Λp	$-0.067(15)$	$-0.05(2)$
$\Sigma^- n$	$-0.025(10)$	$-0.02(3)$
$\Xi^- \Lambda$	$-0.053(10)$	$-0.06(4)$
$\Xi^- \Sigma^0$	$-0.068(17)$	$-0.05(2)$

		BChPT$\times 1/N_c$	**HBChPT; 8+10**	**HBChPT: 8 only**	RBChPT: 8+10
Λp	0.952	-0.048	-0.080	-0.097	-0.031
$\Sigma^- n$	0.966	-0.034	-0.024	0.008	-0.022
$\Xi^- \Lambda$	0.953	-0.047	-0.063	-0.063	-0.029
$\Xi^- \Sigma^0$	0.962	-0.038	-0.076	-0.094	-0.030

Charge radii of baryon octet

	$\langle r^2\rangle[\mathrm{fm}^2]$		
	Full	CT	Exp
p	0.707	0.596	0.7071(7)
n	−0.116	−0.049	−0.116(2)
Λ	−0.029	−0.024	…
Σ^+	0.742	0.596	…
Σ^0	0.029	0.024	…
Σ^-	0.683	0.548	0.608(156)
Ξ^0	−0.016	−0.049	…
Ξ^-	0.633	0.548	…

2 LECs fitted

Magnetic moments

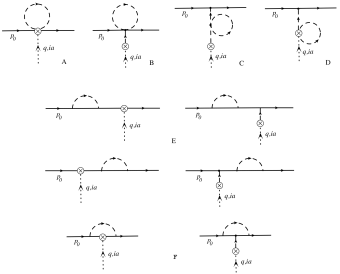
	μ_{LO}	μ_{NNLO}	μ_{Exp}
p	2.691	Input	2.792847356(23)
n	−1.794	Input	−1.9130427(5)
Σ^+	2.691	2.367	2.458(10)
Σ^0	0.897	0.869	…
Σ^-	−0.897	−0.629	−1.160(25)
Λ	−0.897	−0.611	−0.613(4)
Ξ^0	−1.794	−1.275	−1.250(14)
Ξ^-	−0.897	−0.652	−0.6507(25)
$\Delta^+ p$	2.537	3.65	3.58(10)
$\Sigma^0\Lambda$	1.553	1.57	1.61(8)
$\Sigma^{*0}\Lambda$	2.197	2.68	2.73(25)[a]
$\Sigma^{*+}\Sigma^+$	−2.537	−2.35	−3.17(36)[b]

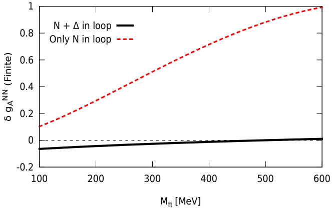
	μ_{LO}	μ_{NNLO}	μ_{Exp}
Δ^{++}	5.381	5.962	3.7 − 7.5
Δ^+	2.691	3.049	2.7(3.6)
Δ^0	0	0.136	…
Δ^-	−2.691	−2.777	…
Σ^{*+}	2.691	3.151	…
Σ^{*0}	0	0.343	…
Σ^{*-}	−2.691	−2.465	…
Ξ^{*0}	0	0.490	…
Ξ^{*-}	−2.691	−2.208	…
Ω	−2.691	−2.005	−2.02(5)

7 LECs fitted

Predicts charge radii and magnetic moments of all SU(3) vector currents relevant for precision semileptonic weak decays

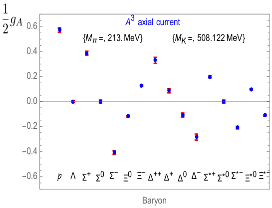
Axial currents [A. Calle-Cordon & JLG, I. Fernando & JLG]

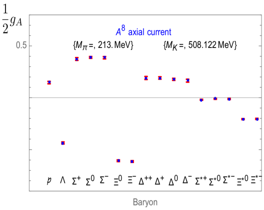


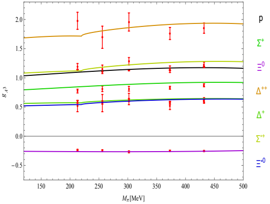


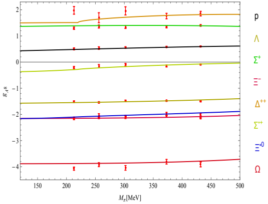
Cancellation of N_c counting violating terms between N and Δ in loop contributions
general feature for all loop contributions in BChPT x 1/Nc

NNLO $g_A^{3,8}$ for **8** and **10** & LQCD   LQCD [Alexandrou et al (2016)]







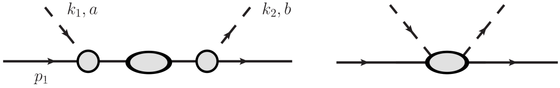


- NNLO SU(3) $g_A's$: 7 LECs
- Fit LQCD varying M_π
- Very small M_π dependence

πN scattering [D. Jayakodige & JLG]

$$T^{ba} = \frac{E_N + m_N}{2m_N}(\delta^{ab}(g^+(s,t) - i(\vec{k}_1 \wedge \vec{k}_2)\cdot\vec{\sigma}h^+(s,t)) + i\epsilon^{bac}\tau^c(g^-(s,t) - i(\vec{k}_1 \wedge \vec{k}_2)\cdot\vec{\sigma}h^-(s,t)))$$

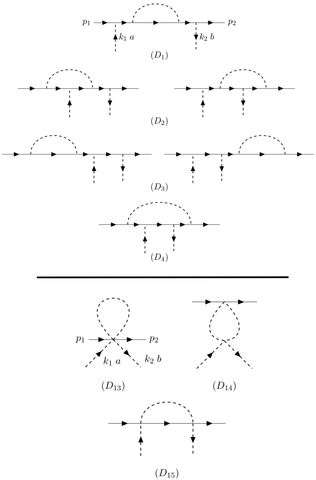
$$Re\ f_\ell = \frac{1}{2k}\sin(2\delta_\ell) \qquad Im\ f_\ell = \frac{1}{k}\sin^2\delta_\ell$$

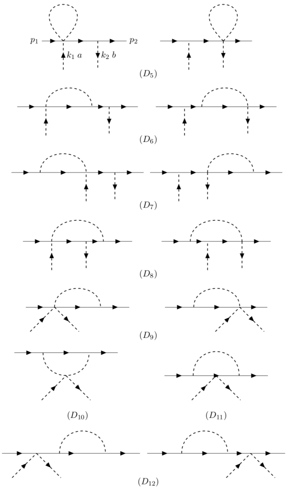


NNLO contributions

g_A^4

g_A^0





g_A^2

• NNLO calculation for general N_c in ξ-expansion - describes general $\pi B \to \pi B'$

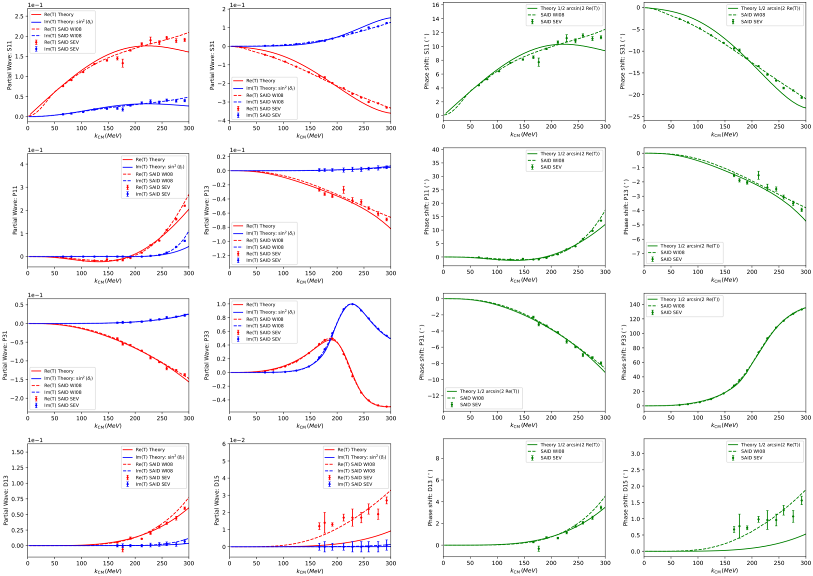
• renormalization implemented - check consistency with large N_c

• Fits to the SAID data base for elastic πN scattering

• Fit Re parts of partial wave amplitudes up to D-waves, and also include Im part of the P_{33} amplitude.

• Obtain phase shifts and determine Im parts via unitarity

• Use the S-wave scattering lengths as constraints



Partial Wave: S11
1e−1
Re(T) Theory
Im(T) Theory: sin² (δ_I)
Re(T) SAID WI08
Im(T) SAID WI08
Re(T) SAID SEV
Im(T) SAID SEV
k_CM (MeV)
Partial Wave: S31
Phase shift: S11 (°)
Phase shift: S31 (°)
Theory 1/2 arcsin(2 Re(T))
SAID WI08
SAID SEV
Partial Wave: P11
Partial Wave: P13
Phase shift: P11 (°)
Phase shift: P13 (°)
Partial Wave: P31
Partial Wave: P33
Phase shift: P31 (°)
Phase shift: P33 (°)
Partial Wave: D13
Partial Wave: D15
1e−2
Phase shift: D13 (°)
Phase shift: D15 (°)

Comments

- EFTs of QCD MUST be consistent with a $1/N_c$ expansion.
- In mesons it allows to include η' in ChPT.
- Huge impact in baryons: SF symmetry and Δ as active degree of freedom.
- Well defined tools to implement the expansion at hadronic level
- @ low energy the chiral and $1/N_c$ expansion must be linked- most obvious way is the $\xi-$expansion
- BChPT x 1/Nc improves range of convergence of low energy expansion - tames the loop contributions
- More applications: SSA in electron-nucleon scattering [Weiss, Willemyns & JLG], Compton scattering, EMT, ...