Meson dominance and gravitational form factors

Enrique Ruiz Arriola [1] and Wojciech Broniowski[2]

[1]Departamento de Física Atómica, Molecular y Nuclear,
Universidad de Granada, Spain.
[2]Jan Kochanowski U., Kielce and Inst. of Nuclear Physics PAN, Cracow, Poland

Mechanical properties of hadrons: Structure, dynamics, visualization
ECT* Trento, 31 March - 4 April 2025

Outline

- Energy momentum tensor
- Form factors
- Meson dominance
- Pion
- Nucleon
- Lattice
- Conclusions

Based on recent work

- Gravitational form factors of the pion and meson dominance
  Wojciech Broniowski, Enrique Ruiz Arriola
  Phys.Lett.B 859 (2024) 139138
  e-Print: 2405.07815 [hep-ph]
- Scalar and tensor meson dominance and gravitational form factors of the pion
  Enrique Ruiz Arriola, Wojciech Broniowski
  PoS QNP2024 (2025) 068
  e-Print: 2411.10354 [hep-ph]
- Transverse densities of the energy-momentum tensor and the gravitational form factors the pion
  Wojciech Broniowski, Enrique Ruiz Arriola
  Acta Physical Polonica B (in press)
  e-Print: 2412.00848 [hep-ph]
- Gravitational form factors and mechanical properties of the nucleon in a meson dominance approach
  Wojciech Broniowski, Enrique Ruiz Arriola
  e-Print: 2503.09297 [hep-ph]

... and older papers

- Meson dominance of hadron form factors and large-Nc phenomenology
  Pere Masjuan, Enrique Ruiz Arriola, Wojciech Broniowski
  Phys.Rev.D 87 (2013) 1, 014005
  e-Print: 1210.0760 [hep-ph]
- Scalar-isoscalar states in the large-N(c) Regge approach
  Enrique Ruiz Arriola, Wojciech Broniowski
  Phys.Rev.D 81 (2010) 054009
  e-Print: 1001.1636 [hep-ph]
- Gravitational and higher-order form factors of the pion in chiral quark models
  Wojciech Broniowski, Enrique Ruiz Arriola
  Phys.Rev.D 78 (2008) 094011
  e-Print: 0809.1744 [hep-ph]
- The Energy momentum tensor of chiral quark models at low energies
  E. Megias, E. Ruiz Arriola, L.L. Salcedo
  Phys.Rev.D 72 (2005) 014001
  e-Print: hep-ph/0504271 [hep-ph]
- Low-energy chiral Lagrangian in curved space-time from the spectral quark model
  E. Megias, E. Ruiz Arriola, L.L. Salcedo, W. Broniowski
  Phys.Rev.D 70 (2004) 034031
  e-Print: hep-ph/0403139 [hep-ph]

Motivation and Outline

The energy momentum tensor $\Theta_{\mu\nu}$ is the conserved Noether current corresponding to the symmetry under space-time translations

$$x^\mu \to x'^\mu = x^\mu + \epsilon^\mu \implies \phi'(x') = \phi(x) \implies \delta\phi(x) = \epsilon^\mu \partial_\mu \phi$$

The invariance of the Lagrangian gives

$$\delta\mathcal{L}(x) = \epsilon^\mu \partial_\mu \mathcal{L} = \frac{\partial \mathcal{L}}{\partial \phi}\delta\phi + \frac{\partial \mathcal{L}}{\partial \partial^\mu \phi}\delta\partial^\mu \phi = \partial^\nu \left[\frac{\partial \mathcal{L}}{\partial \partial^\nu \phi}\right]\delta\phi + \frac{\partial \mathcal{L}}{\partial \partial^\mu \phi}\delta\partial^\mu\phi \implies \epsilon^\nu \partial^\mu \Theta_{\mu\nu} = 0\,,$$

For example for scalar theory

$$\mathcal{L} = \frac{1}{2}(\partial^\mu \phi)^2 - U(\phi) \implies \Theta^{\mu\nu} = \partial^\mu \phi \partial^\nu \phi - g^{\mu\nu}\mathcal{L}$$

The canonical of Noether EMT is NOT always symmetric.
How to measure $\Theta^{\mu\nu}$? Natural way coupling to gravity via a curved space time.
We take the Hilbert or metric EMT

$$\Theta^{\mu\nu} = \frac{-2}{\sqrt{-g}}\frac{\delta S}{\delta g_{\mu\nu}}\Big|_{g^{\mu\nu}=\eta^{\mu\nu}}\,, \qquad \eta^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1) \implies \Theta^{\mu\nu} = \Theta^{\nu\mu}$$

Because of derivatives the quantum operator is badly divergent The improved EMT (Coleman+Callan+)

$$\bar\Theta^{\mu\nu} = \Theta^{\mu\nu} - \frac{1}{6}\left[\partial^\mu\partial^\nu - g^{\mu\nu}\partial^2\right]\phi^2 \implies \Theta = \Theta^\mu_\mu$$

has the property that for $U(\phi) = m^2\phi^2/2 + g\phi^4/4!$ with $m = 0$ one has scale invariance and a trace anomaly after quantization

$$\bar\Theta = 0 \implies \partial^\mu D_\mu = \Theta^\mu_\mu = \beta(g)\frac{1}{4!}\phi^4$$

Lorentz properties

$$x^\mu \to \Lambda^\mu_\alpha x^\alpha \implies \Theta^{\mu\nu} \to \Lambda^\mu_\alpha \Lambda^\nu_\beta \Theta^{\alpha\beta}$$

The (Hilbert) EMT is conserved and symmetric but not irreducible.

$$\Theta^{\mu\nu} = \Theta^{\nu\mu} . \qquad \partial_\mu \Theta^{\mu\nu} = 0 \, , \implies \text{6 independent componentes.}$$

The trace is a scalar

$$\Theta \equiv \Theta^\mu_\mu$$

A naive decomposition

$$\Theta^{\mu\nu} = \Theta^{\mu\nu}_S + \Theta^{\mu\nu}_T \equiv \frac{1}{4} g^{\mu\nu} \Theta + \left[\Theta^{\mu\nu} - \frac{1}{4} g^{\mu\nu} \Theta \right] \implies \partial_\mu \Theta^{\mu\nu}_S = \partial^\nu \Theta \neq 0$$

A consistent decomposition where two tensor components are conserved separately.

$$\Theta^{\mu\nu} = \Theta^{\mu\nu}_S + \Theta^{\mu\nu}_T$$

with

$$\Theta^{\mu\nu}_S = \frac{1}{6} \left[g^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{\partial^2} \right] \Theta \implies \partial_\mu \Theta^{\mu\nu}_S = 0$$

We will analyze lattice data using the consistent decomposition.

QCD

SEM tensor

$$\Theta^{\mu\nu} \quad = \quad \frac{i}{4}\bar{\Psi}\left[\gamma^{\mu}\overleftrightarrow{D}^{\mu}+\gamma^{\nu}\overleftrightarrow{D}^{\mu}\right]\Psi - F^{\mu\lambda a}F^{\nu}_{\lambda a}+\frac{1}{4}g^{\mu\nu}F^{\sigma\lambda a}F_{\sigma\lambda a}+\Theta^{\mu\nu}_{\mathrm{GF-EOM}}, \qquad (1)$$

Trace Anomaly

$$\partial^{\mu}D_{\mu}=\Theta^{\mu}_{\mu}\equiv\Theta=\frac{\beta(\alpha)}{2\alpha}G^{\mu\nu a}G^{a}_{\mu\nu}+\sum_{q}m_{q}\left[1+\gamma_{m}(\alpha)\right]\bar{q}q. \qquad (2)$$

Here $\beta(\alpha)=\mu^2 d\alpha/d\mu^2$ denotes the beta function, $\alpha=g^2/(4\pi)$ is the running coupling constant, $\gamma_m(\alpha)=d\log m/d\log\mu^2$ is the anomalous dimension of the current quark mass m_q, and $G^a_{\mu\nu}$ is the field strength tensor of the gluon field.
Breakup of hadron mass: (Ji 1995)

$$\Theta^{\mu\nu}=\Theta^{\mu\nu}_{q}+\Theta^{\mu\nu}_{g}$$

Scale dependent decomposition.

$$\langle p|\Theta^{\mu\nu}|p\rangle = 2p^{\mu}p^{\nu}\left[\langle x\rangle_{\mathrm{q}}+\langle x\rangle_{\mathrm{g}}\right] \implies \langle x\rangle_{\mathrm{q}}+\langle x\rangle_{\mathrm{g}}=1$$

In Deep Inelastic Scattering we have

$$\langle x\rangle^{\pi}_{\mathrm{val}}=\langle x\rangle^{N}_{\mathrm{val}}\sim 0.6 \qquad \mu=2\mathrm{GeV}$$

We will not analyze the separate contributions here

Gravitational Form factors

- The EMT has matrix elements between hadronic states (helicity-normality basis) $|pJ\lambda N$

$$\langle p' j' \lambda' N' | \Theta^{\mu\nu}(0) | p j \lambda N \rangle = \sum_i \chi^\dagger_{j'\lambda'} O_i^{\mu\nu}(p', p) \chi^\dagger_{j\lambda} F_i(q^2)$$

The invariant functions $F_i(q^2)$ are the corresponding gravitational form factors.

- Mechanical interpretation: M. Polyakov 2003, Polyakov, Schweitzer 2018, Ji 2021, Lorce, Metz, Pasquini, Rodini 2021, ...

$$T_H^{\mu\nu}(x) = \langle H | \Theta^{\mu\nu}(x) | H \rangle \tag{3}$$

where $|H\rangle$ is a general wave packet,

$$|H\rangle = \sum_s \int d^4 p \Psi_s(p) \delta_+(p^2 - M^2) |p, s\rangle \,, \qquad \delta_+(p^2 - M^2) = \begin{cases} \theta(p^+)\delta(p^2 - M^2) \\ \theta(p_0)\delta(p^2 - M^2) \end{cases}$$

- Space-like $x = (x_0, \vec{r}) = (x^+, x^-, \vec{b})$, $x^2 = x_0^2 - \vec{r}^2 = x^+ x^- - \vec{b}^2 < 0$ one may use two popular choices

$$\text{WB TALK} \qquad \begin{cases} x^+ = 0 \,, & x^2 = -\vec{b}^2 \,, & T^{++}(\vec{b}), T^{+-}(\vec{b}), T^{ij}(\vec{b}) \,, & \text{transverse} \\ x_0 = 0 \,, & x^2 = -r^2 \,, & T^{00}(\vec{r}), T^{0i}(\vec{r}), T^{ij}(\vec{r}) \,, & \text{3D} \end{cases}$$

- D - Druck term= Intrinsic hadronic property (Polyakov+Weiss, 1999)

$$O_D^{\mu\nu}(p', p) = q^\mu q^\nu - g^{\mu\nu} q^2 \implies D(q^2) \,, \qquad D(0)$$

Pion GFF

- The spin-0 particle like the pion is the simplest case

$$\boxed{\langle \pi^a(p')|\Theta^{\mu\nu}(0)|\pi^b(p)\rangle = \delta_{ab}\left[2P^\mu P^\nu A(t) + \frac{1}{2}\left(q^\mu q^\nu - g^{\mu\nu}q^2\right)D(t)\right]}$$

a, b - isospin, $P = \frac{1}{2}(p'+p)$, $q = p'+p$, $t = q^2 = -Q^2$

- Trace form factor

$$\Theta^\mu_\mu \equiv \Theta(q^2) = 2\left(m_\pi^2 - \frac{q^2}{4}\right)A(q^2) - \frac{3}{2}q^2 D(q^2). \qquad (4)$$

- Raman decomposition (Raman:1971jg) conserved irreducible tensors corresponding to well-defined total angular momentum, $J^{PC} = 0^{++}$ (scalar) and 2^{++} (tensor)

$$\Theta^{\mu\nu} = \Theta_S^{\mu\nu} + \Theta_T^{\mu\nu}, \qquad \begin{cases} \Theta_S^{\mu\nu} = \frac{1}{3}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\Theta \\ \Theta_T^{\mu\nu} = 2\left[P^\mu P^\nu - \frac{P^2}{3}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\right]A \end{cases} \underbrace{\Longrightarrow}_{q^2\to 0} 2A(0)m_\pi^2 = \Theta(0)$$

- Since Θ and A carry the information on good J^{PC} channels, they should be regarded as the primary objects, whereas the D-term form factor mixes the quantum numbers, with the explicit formula

$$D = -\frac{2}{3t}\left[\Theta - \left(2m_\pi^2 - \frac{1}{2}t\right)A\right], \qquad D_\pi(0) = -1 + \mathcal{O}(m_\pi^2), \qquad \text{(chiral theorem)}$$

Nucleon GFF

- Matrix elements

$$\langle p', s'|\Theta_{\mu\nu}(0)|p, s\rangle = \bar{u}(p', s')\Gamma_{\mu\nu}u(p, s)$$

- Gordon identity

$$2m\bar{u}'\gamma^{\alpha}u = \bar{u}'(2P^{\alpha} + i\sigma^{\alpha\rho}q_{\rho})u,$$

- Three representations

$$\begin{aligned}
\Gamma_{\mu\nu} &= A(t)\,\gamma_{\{\mu}P_{\nu\}} + B(t)\,\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}q^{\rho}}{2m_N} + D(t)\,\frac{q_\mu q_\nu - g_{\mu\nu}q^2}{4m_N} \\
&= A(t)P_\mu P_\nu + J(t)\,i\,P_{\{\mu}\sigma_{\nu\}\rho}q^{\rho} + D(t)\,\frac{q_\mu q_\nu - g_{\mu\nu}q^2}{4} \\
&= 2J(t)\,\gamma_{\{\mu}P_{\nu\}} - B(t)\frac{P_\mu P_\nu}{m_N} + D(t)\,\frac{q_\mu q_\nu - g_{\mu\nu}q^2}{4m_N}
\end{aligned}$$

- Relations and normalizations

$$J(t) = \frac{1}{2}(A(t) + B(t))\,, \qquad A(0) = 1\,, \qquad B(0) = 1\,, \qquad J(0) = \frac{1}{2}\,, \qquad D(0) = ?$$

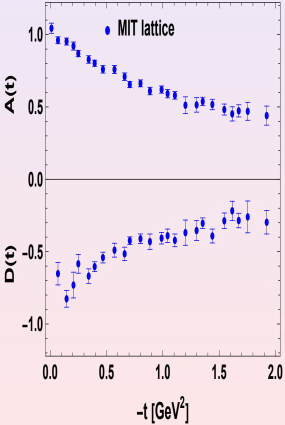
- Raman decomposition: Trace

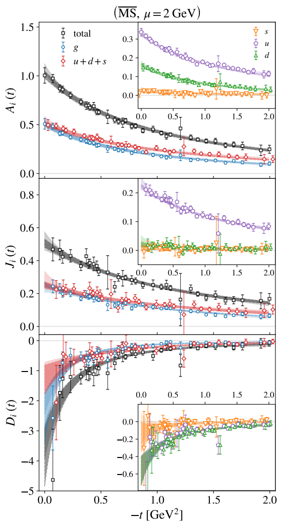
$$\Theta(t) = \frac{1}{m_N}\left[(m_N^2 - \frac{t}{4})A(t) - \frac{3}{4}tD(t) + \frac{1}{2}tJ(t)\right]\,, \quad \Theta(0) = m_N\,, \qquad D(0) = \frac{4m_N}{3}\left[m_N A'(0) - \Theta'(0)\right]$$

$$m_N\Gamma_T^{\mu\nu} = \left[P^\mu P^\nu - \frac{P^2}{3}Q^{\mu\nu}\right]A(t) + \left[i\,P^{\{\mu}\sigma^{\nu\}\rho}q_\rho - \frac{t}{6}Q^{\mu\nu}\right]J(t),$$

MIT data

[Phys.Rev.D 108 (2023) 11, 114504 & D. Pefkou, PhD Thesis]
Unprecedented accuracy, both quarks and gluons, $m_\pi = 170$ MeV (SPACE-LIKE RESULTS)
(below the total q+g used, as it corresponds to the conserved current $\to$ renorm invariant)





PION VECTOR FORM FACTOR

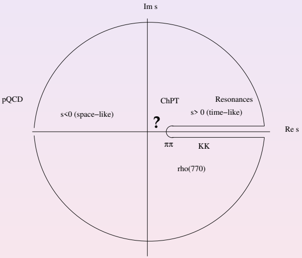
Dispersion relations and sum rules

Example: pion vector form factor

$$e^-\pi^+ \to e^-\pi^+ \,, \qquad \langle \pi^+(p')|J_3^\mu(0)|\pi^+(p)\rangle = F_\pi(q^2)(p'^\mu + p^\mu)\,, \qquad q^2 < 0 \qquad \text{space-like}$$

$$e^+e^- \to \pi^+\pi^- \,, \qquad \langle \pi^+(-p')\pi^-(p)|J_3^\mu(0)|0\rangle = F_\pi(q^2)(p'^\mu + p^\mu)\,, \qquad q^2 > 4m_\pi^2 \qquad \text{time-like}$$

Analyticity: the two processes correspond to the same function in different domains



$$F(q^2) = F(q^2)^* \,, \quad q^2 < 0 \implies F(z^*) = F(z)^* \implies \mathrm{Disc} F(q^2) = 2i \mathrm{Im} F(q^2 + i\epsilon)\,, \qquad q^2 > 4m_\pi^2$$

Unitarity cuts: line $q^2 > 4m_\pi^2 \implies$ Two Riemann sheets $F_{\rm I}(s)$ and $F_{\rm II}(s)$

Resonances:

$$F_{\rm II}(s) = S_{\rm II}(s) F_{\rm I}(s) \implies F_{\rm II}(s) \to \frac{Z_R}{s - m_R^2 + i m_R \Gamma_R} + \ldots$$

Large momentum behaviour (pQCD)

$$F(-Q^2) = \frac{16\pi F_\pi^2 \alpha_s(Q^2)}{Q^2} \sim \frac{1}{Q^2 \log Q^2} \underbrace{\rightarrow}_{Q^2 \to e^{-i\pi} s} \frac{1}{s(\log s - i\pi)} \implies \mathrm{Im} F(s) = -\frac{\pi}{s(\log s^2 + \pi^2)} < 0 \quad (6)$$

Unsubtracted Dispersion relations

$$F(-Q^2) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\mathrm{Im} F(s)}{s + Q^2}$$

Normalization

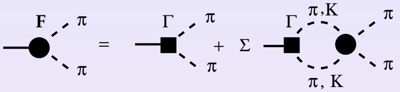
$$\boxed{F(0) = 1 = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\mathrm{Im} F(s)}{s}}$$

Superconvergent sum rule (Donoghue:1996bt)

$$Q^2 F(-Q^2) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds \mathrm{Im} F(s) + \mathcal{O}\left(\frac{1}{\log Q^2}\right) \implies \boxed{\int_{4m_\pi^2}^{\infty} ds \mathrm{Im} F(s) = 0} \implies \mathrm{Im} F(s) \quad \text{changes sign}$$

t-channel unitarity

- Bethe-Salpeter (coupled channel)



- Watson's theorem: $\pi\pi$ scattering in $J = I = 1$ channel

$$F(s) = |F(s)| e^{i\delta_{11}(s)} \implies \boxed{\mathrm{Im} F(s) = |F(s)| \sin \delta_{11}(s) > 0} \, , \qquad 4m_\pi^2 < s < 4m_K^2$$

- Threshold behaviour

$$\delta_{11}(s) \sim a_{11}(s/4 - m_\pi^2)^{\frac{3}{2}} \implies \mathrm{Im} F(s) \sim |F(4m_\pi^2)| a_{11}(s/4 - m_\pi^2)^{\frac{3}{2}}$$

- Omnes-Mushkelisvili solution in the spacelike region

$$F(-Q^2) = \exp \left[-\frac{1}{\pi} \int_{4m_\pi^2}^\infty \frac{Q^2}{s} \frac{\delta_{11}(s)}{s+Q^2} \right] \sim \underbrace{\rightarrow}_{\Gamma_\rho \to 0} = \frac{m_\rho^2}{m_\rho^2 + Q^2}$$

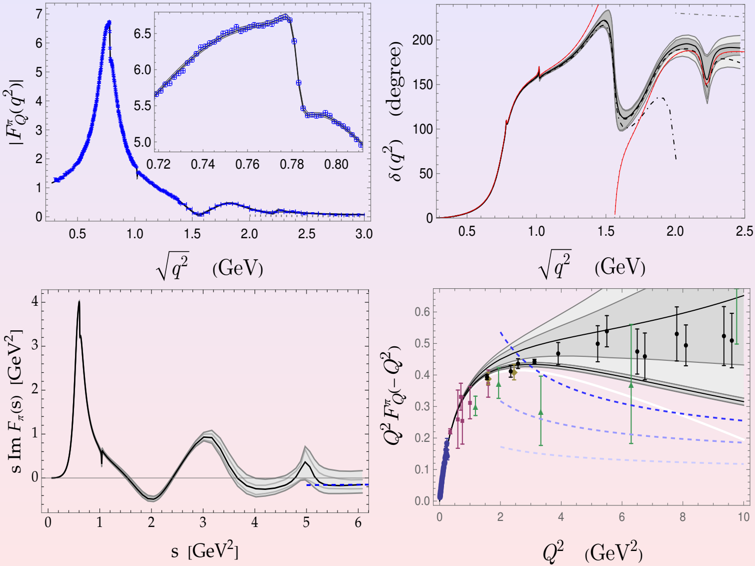
- Question of modeling/using the spectral density

$$\rho(s) = \frac{1}{\pi} \mathrm{Im} F(s) = \begin{cases} \rho_{\rm ChPT}(s) & 4m_\pi^2 \le s \le 16 m_\pi^2 \, , & \text{threshold region} \\ \rho_{\rm R}(s) & 16 m_\pi^2 \le s \le \Lambda_{\rm pQCD}^2 \, , & \text{resonance region} \\ \rho_{\rm pQCD}(s) & \Lambda_{\rm pQCD}^2 \le s \le \infty \, , & \text{pQCD region} \end{cases}$$

- What would be a reasonable $\Lambda_{\rm pQCD}$?

Analysis of Babar

ERA, Pablo Sanchez-Puertas (RuizArriola:2024gwb) for $\sim 3m_\pi \leq \sqrt{s} \leq 3\mathrm{GeV}$, DR: $|F(s)| \to \arg F(s)$



The incompleteness problem

ERA, Pablo Sanchez-Puertas, Christian Weiss (2025 Transverse dist)

- The maximum Babar $s_{\rm max} = 9{\rm GeV}^2$

$$\frac{1}{\pi}\int_{s_0}^{s_{\rm max}} ds \frac{{\rm Im}\, F(s)}{s}\Big|_{\rm Data} = 1.01(1)_{\rm st}(^{+2}_{-1})_{\rm syst}\,,$$

$$\frac{1}{\pi}\int_{s_0}^{s_{\rm max}} ds\, {\rm Im}\, F(s)\Big|_{\rm Data} = 0.63(2)_{\rm st}(^{+7}_{-4})_{\rm syst}\ {\rm GeV}^2. \qquad m_\rho^2 = 0.6{\rm GeV}^2$$

- The pQCD part extrapolated

$$\frac{1}{\pi}\int_{s_{\rm max}}^{\infty} ds \frac{{\rm Im}\, F(s)}{s}\Big|_{\rm pQCD} = -\underbrace{0.0025}_{\rm LO} - \underbrace{0.0011}_{\rm NLO} - \underbrace{0.0006}_{\rm NNLO}\,,$$

$$\frac{1}{\pi}\int_{s_{\rm max}}^{\infty} ds\, {\rm Im}\, F(s)\Big|_{\rm pQCD} = -\underbrace{0.114}_{\rm LO} - \underbrace{0.030}_{\rm NLO} - \underbrace{0.013}_{\rm NNLO}\ {\rm GeV}^2.$$

- Superconvergence is a theorem but pQCD is far away
- Solution: subtractions (but need constants independently)

$$F(-Q^2) = 1 - Q^2 F'(0) + \frac{1}{\pi}\left[\int_{4m_\pi^2}^{s_{\rm max}} + \int_{s_{\rm max}}^{\infty}\right] ds \frac{Q^4}{s^2}\frac{{\rm Im} F(s)}{s+Q^2}\,, \qquad \text{Last term} \quad \mathcal{O}(Q^4/s_{\rm max}^2)$$

- Space-like looks very much as Vector-Meson Dominance

$$F(-Q^2) = \frac{m_\rho^2}{m_\rho^2 + Q^2} \Longrightarrow J_3^\mu = f_\rho m_\rho^2 \rho_3^\mu\,, \qquad \text{current-field identity (Sakurai)}$$

- Space-like physics is INDEPENDENT of time-like details.

Extended meson dominance

- Generalized Current field identity

$$J_V^\mu = \sum_{\rho,\rho',\dots} f_V M_V^2 V^\mu \implies F_V(t) = \sum_V c_V \frac{M_V^2}{M_V^2 - t}, \qquad c_V = f_V g_{V\pi\pi}$$

- Short distance constraints

$$F_V(t) \sim \frac{\sum_T c_T m_V^2}{Q^2} + \dots$$

- Normalization

$$F_V(0) = 1 = \sum_T c_T$$

- Minimal hadronic ansatz

$$F_V(t) = \frac{m_\rho^2}{m_\rho^2 - t}$$

- Improved hadronic ansatz

$$F_V(t) = (1 + at) \frac{m_\rho^2}{m_\rho^2 - t} \frac{m_{\rho'}^2}{m_{\rho'}^2 - t}$$

Current-field identities for conserved SEM tensor

- Saturation with O^{++} and 2^{++} isoscalar states (Krolikowski:1967ryy,Raman:1970wq,Raman:1971ur) (Raman decomposition manifest)

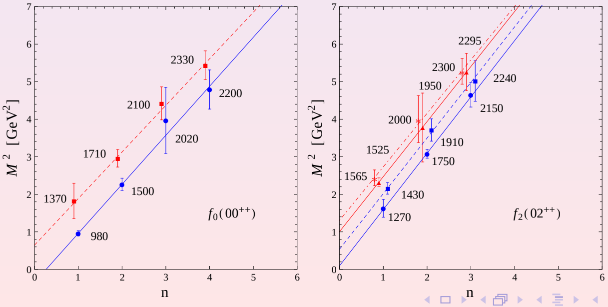
$$\Theta^{\mu\nu} = \sum_S \frac{1}{3} f_S \left(\partial^\mu \partial^\nu - g^{\mu\nu} \partial^2 \right) S + \sum_T f_T m_T^2 T^{\mu\nu} \,,$$

- Matrix elements

$$\langle A | \Theta^{\mu\nu} | B \rangle = \sum_S \frac{f_S}{3} \frac{g^{\mu\nu} q^2 - q^\mu q^\nu}{m_S^2 - q^2 - i\epsilon} \langle A | J_S | B \rangle + \sum_T f_T \frac{m_T^2}{m_T^2 - q^2 - i\epsilon} \langle A | \sum_\lambda \epsilon_\lambda^{\mu\nu} \epsilon_{\alpha\beta}^{\lambda} J_T^{\alpha\beta} | B \rangle$$

- PDG resonances follow radial regge trajectories (Masjuan:2012gc)

$$M_{nJ}^2 = a(n + J) + b$$



PION GRAVITATIONAL FORM FACTOR

Spectral Properties

- pQCD

$$A(t) = -3D(t)\,(1+\mathcal{O}(\alpha)) = -\frac{48\pi\alpha(t) f_\pi^2}{t}\,(1+\mathcal{O}(\alpha))\,,$$

- Watson's theorem implies $4m_\pi^2 < s < 4m_K^2$

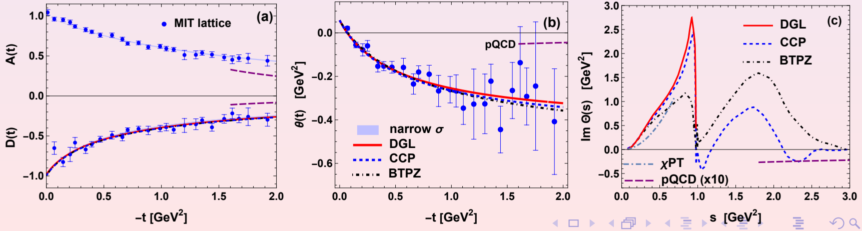
$$\mathrm{Im}\Theta(s) = |\Theta(s)|\sin\delta_{00}(s)\,, \qquad \mathrm{Im}A(s) = |A(s)|\sin\delta_{02}(s)$$

Question of modeling/using the spectral density

$$\rho(s) = \begin{cases} \rho_{\rm ChPT}(s) & 4m_\pi^2 \le s \le 16m_\pi^2 \\ \rho_{\rm R}(s) & 16m_\pi^2 \le s \le \Lambda^2_{pQCD} \\ \rho_{\rm pQCD}(s) & \Lambda^2_{pQCD} \le s \end{cases}$$

- Meson dominance ($m_\pi = 170\mathrm{MeV}$)

$$A^*(-Q^2) = \frac{m_{f_2}^{*2}}{m_{f_2}^{*2} + Q^2}\,, \qquad \Theta^*(-Q^2) = 2m_\pi^{*2} - \frac{m_\sigma^{*2} Q^2}{m_\sigma^{*2} + Q^2}\,.$$



Finite widths at space-like momenta

Energy dependent Breit-Wigner parametrization

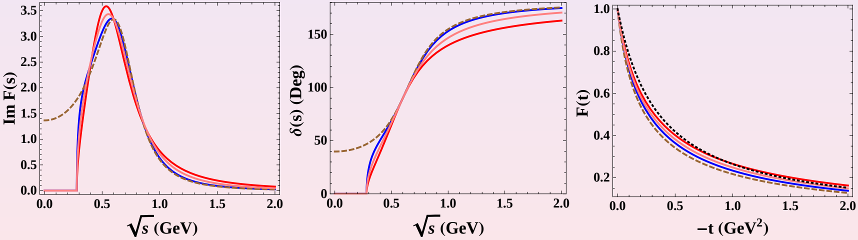
$$N(s) = M^2 - s + i\Gamma M \Gamma(s) \qquad S(s) = e^{2i\delta(s)} = \frac{N(s)}{N(s)^*} \implies \delta(M^2) = \frac{\pi}{2}$$

Resonance = Pole in the second Riemann sheet

$$1/S_{II}(s_R) = S_I(s_R) = 0$$

Omnes representation complies with Watson's theorem

$$F(t) = \exp\left[\frac{t}{\pi}\int_{4m_\pi^2} \frac{ds}{s}\frac{\delta(s)}{s-t}\right] \qquad F(0) = 1 \implies \frac{F(t+i0)}{F(t-i0)} = e^{2i\delta(s)}$$



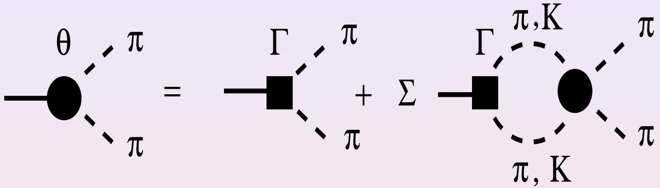
Even for a broad S-wave resonance the Form Factor resembles a monopole for space-like momenta

$$F(t) \sim \frac{M^2}{M^2 - t}$$

Hadronic representacion

For two coupled channels

$$\begin{pmatrix} \Theta_\pi(s) \\ \Theta_K(s) \end{pmatrix} = \begin{pmatrix} \Gamma_\pi(s) \\ \Gamma_K(s) \end{pmatrix} + \begin{pmatrix} T_{\pi\pi\to\pi\pi}(s) & T_{\pi\pi\to KK}(s) \\ T_{KK\to\pi\pi}(s) & T_{KK\to KK}(s) \end{pmatrix} \begin{pmatrix} \Delta_{\pi\pi}(s) & 0 \\ 0 & \Delta_{KK}(s) \end{pmatrix} \begin{pmatrix} \Gamma_\pi(s) \\ \Gamma_K(s) \end{pmatrix}$$



Watson's final state theorem

$$F = \Gamma + VG_0F = \Gamma + TG_0\Gamma \implies \mathrm{Im}F(s) = \mathrm{Im}\left[T(s)G_0(s)\right]\Gamma(s) \implies F(t) = F(0) + \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{t}{s}\frac{\mathrm{Im}F(s)}{s-t}$$

The poles of the FF in the second Riemann sheet coincide with the resonances of the S-matrix.

$$\Theta_{\rm II}(s) = S_{\rm II}(s)\Theta_{\rm I}(s)$$

Chiral extrapolation from $m_\pi = 170$MeV to physical

- In order to relate *different* pion masses a mass independent renormalization scheme is needed, such as $\overline{\mathrm{MS}}$ in chiral perturbation theory

$$\theta^{(0)}_{\mu\nu} = -\eta_{\mu\nu}\mathcal{L}^{(0)}, \quad (7)$$

$$\theta^{(2)}_{\mu\nu} = \frac{f^2}{4}\langle D_\mu U^\dagger D_\nu U\rangle - \eta_{\mu\nu}\mathcal{L}^{(2)}, \quad (8)$$

$$\begin{aligned}\theta^{(4)}_{\mu\nu} &= -\eta_{\mu\nu}\mathcal{L}^{(4)} + 2L_4\langle D_\mu U^\dagger D_\nu U\rangle\langle\chi^\dagger U + U^\dagger\chi\rangle + L_5\langle D_\mu U^\dagger D_\nu U + D_\nu U^\dagger D_\mu U\rangle\langle\chi^\dagger U + U^\dagger\chi\rangle \\ &- 2L_{11}\left(\eta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu\right)\langle D_\alpha U^\dagger D^\alpha U\rangle - 2L_{13}\left(\eta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu\right)\langle\chi^\dagger U + U^\dagger\chi\rangle \\ &- L_{12}\left(\eta_{\mu\alpha}\eta_{\nu\beta}\partial^2 + \eta_{\mu\nu}\partial_\alpha\partial_\beta - \eta_{\mu\alpha}\partial_\nu\partial_\beta - \eta_{\nu\alpha}\partial_\mu\partial_\beta\right)\langle D^\alpha U^\dagger D^\beta U\rangle\,, \quad (9)\end{aligned}$$

- We compute Θ and A in ChPT and obtain from MIT lattice

$$10^3\cdot L_{11}(m_\rho^2) = 1.06(15)\,, \qquad 10^3\cdot L_{12}(m_\rho^2) = -2.2(1)\,, \qquad 10^3\cdot L_{13}(m_\rho^2) = -0.7(1.1).$$

- This implies for $m_\pi = 140$ MeV yields

$$m_\sigma^* = 0.65(3) \to m_\sigma = 0.63(6)\,, \qquad m_{f_2}^* = 1.24(3) \to m_{f_2} = 1.27(4)$$

- Druck term at $m_\pi = 140$MeV.

$$D(0) = -0.95(3)$$

NUCLEON GRAVITATIONAL FORM FACTOR

Spectral Properties

- Large Q^2

$$A(t) \sim +\frac{\alpha(t)^2}{(-t)^2}\,, \qquad J(t) \sim +\frac{\alpha(t)^2}{(-t)^2}\,, \qquad B(t) \sim -\frac{\alpha(t)^2}{(-t)^3}\,, \qquad , D(t) \sim -\frac{\alpha(t)^2}{(-t)^3}\,.$$

- Large s

$$\mathrm{Im}\, A(s) \sim +\frac{1}{s^2 L^3}\,, \qquad \mathrm{Im}\, J(s) \sim +\frac{1}{s^2 L^3}\,, \qquad \mathrm{Im}\, B(s) \sim +\frac{1}{s^3 L^3}\,, \,, \qquad \mathrm{Im}\, D(s) \sim +\frac{1}{s^3 L^3}\,,$$

- Watson's theorem $4m_\pi^2 < s < 4m_K^2$ (Raman decomposition: helicity-flip $\pi\pi \to N\bar{N}$) where $\sigma_\pi = \sqrt{1 - 4m_\pi^2/t}$.

$$\mathrm{Im}\,\Theta(t) = \frac{3\sigma_\pi |f_{0,+}(t)||\Theta_\pi(t)|}{2(4m_N^2 - t)} > 0,$$

$$\mathrm{Im}\, J(t) = \frac{3t^2\sigma_\pi^5}{64\sqrt{6}}|f_{2,-}(t)||A_\pi(t)| > 0,$$

$$\mathrm{Im}\, A(t) + \frac{2t\mathrm{Im}J(t)}{4m_N^2 - t} = \frac{3t^2 m_N \sigma_\pi^5}{32\sqrt{6}}|f_{2,+}(t)||A_\pi(t)| > 0,$$

- Unsubtracted disperion relations
- Superconvergence sum rules

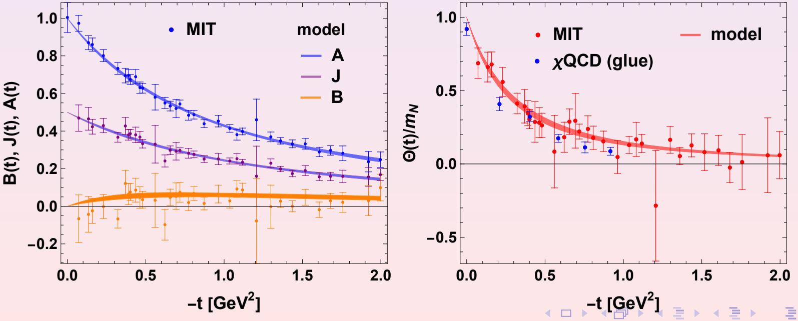
Meson dominance

$$\Theta(t) = \frac{m_N}{(1 - t/m_\sigma^2)(1 - t/m_{f_0}^2)},$$

$$A(t) = \frac{1 - c_A t + c_2 t^2}{(1 - t/m_{f_2}^2)(1 - t/m_{f_2'}^2)(1 - t/m_{f_2''}^2)(1 - t/m_{f_2'''}^2)},$$

$$J(t) = \frac{1 - c_J t + c_2 t^2}{2(1 - t/m_{f_2}^2)(1 - t/m_{f_2'}^2)(1 - t/m_{f_2''}^2)(1 - t/m_{f_2'''}^2)}. \quad (10)$$

We use PDG and fit c_J, c_A, c_2 and $m_\sigma = 650(50)\mathrm{MeV}$ (Consistent with pion)



Conclusions

1. Lattice results for gravitational ff of the pion and nucleon fully compatible with meson dominance at "intermediate" values of Q^2
2. Important to look at the data in good spin channels - all expected features satisfied
3. Matter radius larger due to small $\sigma-$mass $m_\sigma = 0.64(4)$GeV.

$$\langle r^2 \rangle_{\theta,\pi} = \frac{6}{m_\sigma^2} = \qquad \langle r^2 \rangle_{\theta,N} = \frac{6}{m_\sigma^2} + \frac{6}{m_{f_0}^2} = [0.90(4)\mathrm{fm}]^2$$

4. $D(t)$ (the Druck term) is a combination of good spin form factors

$$D_\pi(0) = -0.95(3) \qquad D_N(0) = -3.0(4)$$

5. Higher Q^2 desired approach pQCD ... Modeling involves the broad σ meson!
6. This was already expected [Masjuan, ERA, WB, 2013]

One sees mesons all over the lattice!