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D(t) form factor of the neutron in the classical model and the comparison to the proton.

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Motivation

- Calculate hadronic D-term using classical model
- D(t) divergent for t→0 in Proton (QED effect for charged particles)
- D(t) of Proton comparable to Neutron in the experimentally measurable t-range
- Comparing results for EMT of Proton and Neutron
- Summary

Definition of EMT form factor for a nucleon

$$< p'|\,\hat{T}^{a}_{\mu\nu}|p> = \bar{u}\left[\frac{\gamma_\mu P_\nu + \gamma_\nu P_\mu}{2}\boldsymbol{A^a(t)} + \frac{i(\sigma_{\mu\rho}P_\nu + \sigma_{\nu\rho}P_\mu)\Delta^\rho}{2m}\boldsymbol{B^a(t)} + \frac{\Delta_\mu\Delta_\nu + g_{\mu\nu}\Delta^2}{4m}\boldsymbol{D^a(t)} + m\boldsymbol{\bar{c}^a(t)}g_{\mu\nu}\right]u\, e^{iqx}$$

$$t = q^2 = \Delta^2, \qquad P = \frac{1}{2}(p' + p), \qquad \Delta = (p' - p), \qquad \partial^\mu \hat{T}_{\mu\nu} = 0, \qquad \hat{T}_{\mu\nu} = \sum_q \hat{T}^q_{\mu\nu} + \hat{T}^g_{\mu\nu}$$

- $\sum_a A^a(t), \sum_a B^a(t), \sum_a D^a(t), \sum_a \bar{c}^a(t) \rightarrow renormalization\ scale\ independent$
- $\sum_a \bar{A}^a(0)$ = 1, $\sum_a B^a(0)$ = 0, $\sum_a \bar{c}^a(t)$ = 0 $\quad \boldsymbol{A^a(t) + B^a(t) = 2\,J^a(t)},$ (X. Ji, PRL 78 (1997) 610)
- Constraints:

mass $\leftrightarrow \boldsymbol{A(0)} = 1$ $(i.e.\ quarks\ and\ gluons\ carry\ 100\%\ of\ nucleon\ momentum)$

spin $\leftrightarrow \boldsymbol{B(0)} = 0 \leftrightarrow J(0) = \frac{1}{2}$*(i.e. quarks and gluons carry 100% of nucleon spin)*

D-term $\leftrightarrow \boldsymbol{D(0)} \equiv D \leftrightarrow$ *not fixed by spacetime symmetries* (Polyakov, Weiss PRD 60:114017,1999)

Interpretation of EMT

- In Breit frame where $\Delta^{\mu} = (0, \vec{\Delta})$ defines the static EMT

$$T^{\mu\nu}(\vec{r}) = \int \frac{d^3\vec{\Delta}}{(2\pi)^3 2E} < p' \left| \hat{T}^{\mu\nu} \right| p > e^{-i\vec{\Delta}\cdot\vec{r}}$$

- Components of the static EMT (same as in classical theory):
  - $T^{00} \rightarrow energy\ density$
  - $T^{0i} \rightarrow momentum\ density$
  - $T^{i0} \rightarrow energy\ flux$
  - $\boldsymbol{T^{ij} \rightarrow stress\ tensor}$

(M.V. Polyakov, PLB 555 (2003) 57-62)

Interpretation of EMT (cont'd)

- $\int T^{00}(\vec{r})d^3(r) = M\ (mass\ of\ particle)$
- $\hat{T}^{ij}(\vec{r}) = s(r)\left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p(r)\delta_{ij}$ (stress tensor)

  $s(r) \rightarrow shear\ distribution$

  $p(r) \rightarrow pressure\ distribution$
- $D = D_p = D_s$ → related to stress tensor
- $D_p = m\int d^3r\, r^2 p(r)$
- $D_s = -\frac{4}{15} m \int d^3r\, r^2 s(r)$

← Important!! Will come back to this

Illustration: Pressure and Shear Forces

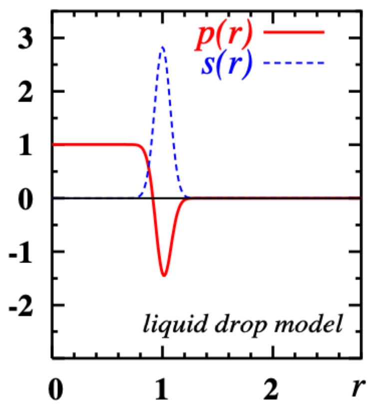
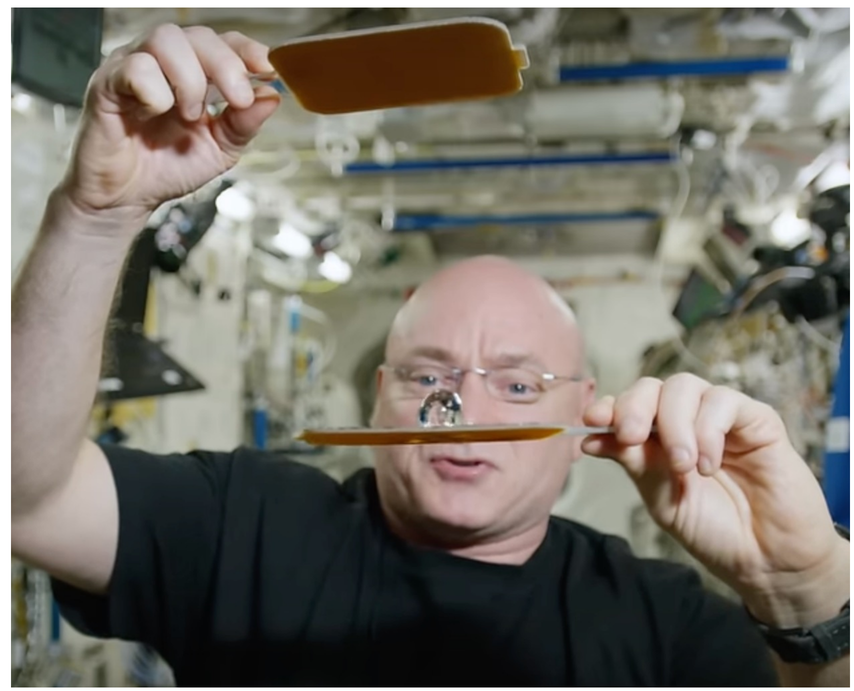


Figure 3. The pressure and shear forces of nuclei (in units of p_0) as functions of r (in units of nuclear radius R_A) in the liquid drop model.

$$p(r) = p_0\theta(r-R) - \frac{p_0 R}{3}\delta(r-R), \qquad s(r) = \gamma\delta(r-R)$$

$p_0 = \frac{2\gamma}{R}$ → Kelvin relation



(Polyakov, PLB 555 (2003) 57-62)

Why a classical model?

- Since the discovery of electron in 1897 people have tried to develop classical models of a charged particle.
- First consistent relativistic model of an extended charged particle (Bialynicki-Birula PLA 182:346-352,1993).
- All previous model studies of hadronic D-terms included only short-range forces (em forces are “negligible” in hardons).
- Only one model was studied using hadronic short-range forces and the Coulomb long-range force (Varma, Schweitzer PRD 102, 014047 (2020)).
- Classical model reproduces QED for D(t), $t \rightarrow 0$ (Metz et al PLB 820 (2021)).

New study

- Construct classical model of neutron and calculate D-term.

Classical Model of the Proton

- Used to study EMT form factors of proton with long range EM forces.
- D-term becomes positive and divergent due to the long-range force for $t \rightarrow 0$ (Varma, Schweitzer PRD 102, 014047 (2020)).
- Divergence of D(t) visible at very small t and agrees with QED.
- Divergence due to QED effects for charged particles already known in literature (Kubis, Meissner NPA 671, 332 (2000) ; Donoghue et al PLB 529, 132 (2002)).

Classical Model

- Dust: non-interacting pressure-less distribution
- Dust particles described by density normalized as $\int_0^\infty d^3r\rho(r) = 1$
- Dust bound by interplay of classical fields: ϕ, V^μ, A^μ
- Field equations in static case where $V^\mu = (V^0, \vec{0})$, $A^\mu = (A^0, \vec{0})$:

$$(-\boldsymbol{\Delta} + m_V{}^2)V_0 = g_v\,\rho$$
$$(-\boldsymbol{\Delta} + m_s{}^2)\phi = g_s\,\rho$$
$$-\boldsymbol{\Delta}\mathrm{A}_0 = e\rho$$
$$\rho\boldsymbol{F} \equiv -\rho\nabla(eA_0 - g_s\phi + g_v V_o) = 0 \quad \text{(equilibrium condition)}$$

$$\frac{g_S^2}{\hbar c} = 91.64 \qquad \frac{g_V^2}{\hbar c} = 136.2 \qquad \alpha = \frac{e^2}{4\pi\hbar c} = \frac{1}{137} \qquad m_S c^2 = 550\ MeV, \quad m_V c^2 = 783\ MeV$$

parameters from nuclear models (see Bialynicki-Birula (1993) for details)

Proton Model: Static solution to the classical equations

r ≤ R region:

- $\rho(r) = f_+(r) - f_-(r)$
- $eA_{0(r)} = e^2\left(\frac{f_+(r)}{k_+^2} - \frac{f_-(r)}{k_-^2}\right) + 2E_B$
- $g_s\phi(r) = g_s^2\left(\frac{f_+(r)}{k_+^2+m_s^2} - \frac{f_-(r)}{k_-^2+m_s^2}\right)$
- $g_v V_0(r) = g_v^2\left(\frac{f_+(r)}{k_+^2+m_v^2} - \frac{f_-(r)}{k_-^2+m_v^2}\right)$
- $f_\pm(r) = \frac{d_\pm}{4\pi}\frac{\sin(k_\pm r)}{r}, k_\pm = \frac{B \pm \sqrt{D}}{2Q^2}$
- $B = (g_s^2 - e^2)\, m_v^2 - (g_v^2 + e^2) m_s^2$
- $D = B^2 - 4e^2Q^2\, m_s^2\, m_v^2, Q^2 = e^2 - g_s^2 + g_v^2$

r > R region:

- $\rho(r) = 0$
- $eA_0(r) = \frac{e^2}{4\pi r}$
- $g_s\phi(r) = \frac{b_s}{4\pi r} e^{-m_s(r-R)}$
- $g_v V_0(r) = \frac{b_v}{4\pi r} e^{-m_v(r-R)}$

With b_v, b_s, d_+, d_-, $2E_B$, R_p fixed from boundary conditions (No free parameters)

(Bialynicki-Birula PLA 182:346-352,1993)

Neutron Model: Static solution to the classical equations

r ≤ R region:

- $\rho(r) = f_+(r) - f_-(r)$
- $eA_{0}(r) = e^2\left(\frac{f_+(r)}{k_+^2} - \frac{f_-(r)}{k_-^2}\right) + 2E_B$ $\mathbf{= 0,\ e \to 0}$
- $g_s\phi(r) = g_s^2\left(\frac{f_+(r)}{k_+^2+m_s^2} - \frac{f_-(r)}{k_-^2+m_s^2}\right)$
- $g_v V_0(r) = g_v^2\left(\frac{f_+(r)}{k_+^2+m_v^2} - \frac{f_-(r)}{k_-^2+m_v^2}\right)$
- $f_\pm(r) = \frac{d_\pm}{4\pi}\frac{\sin(k_\pm r)}{r}, k_\pm = \frac{B \pm \sqrt{D}}{2Q^2} \to$ $\mathbf{(k_- \to 0, neutron)}$
- $B = (g_s^2 - e^2)\, m_v^2 - (g_v^2 + e^2) m_s^2$
- $D = B^2 - 4e^2Q^2\, m_s^2\, m_v^2, Q^2 = e^2 - g_s^2 + g_v^2$

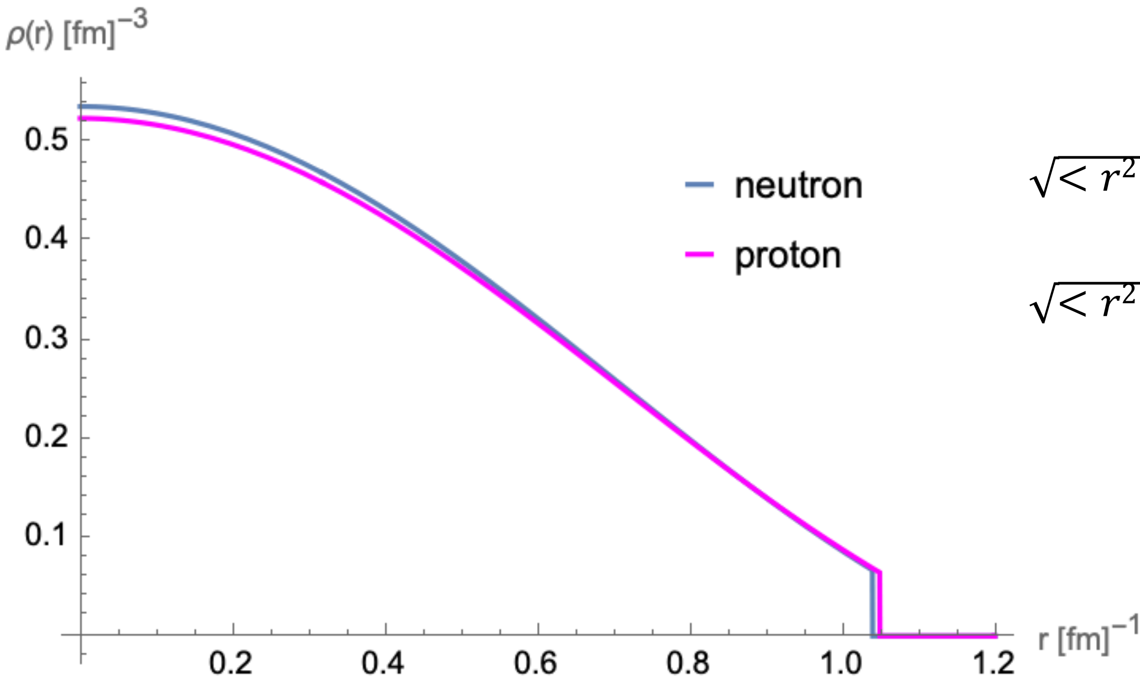
r > R region:

- $\rho(r) = 0$
- $eA_0(r) = \frac{e^2}{4\pi r}$ $\mathbf{= 0, e \to 0}$
- $g_s\phi(r) = \frac{b_s}{4\pi r} e^{-m_s(r-R)}$
- $g_v V_0(r) = \frac{b_v}{4\pi r} e^{-m_v(r-R)}$

With b_v, b_s, d_+, d_-, $2E_B$, R_p fixed from boundary conditions (No free parameters)

(Bialynicki-Birula PLA 182:346-352,1993)

Proton vs. Neutron: A Comparison



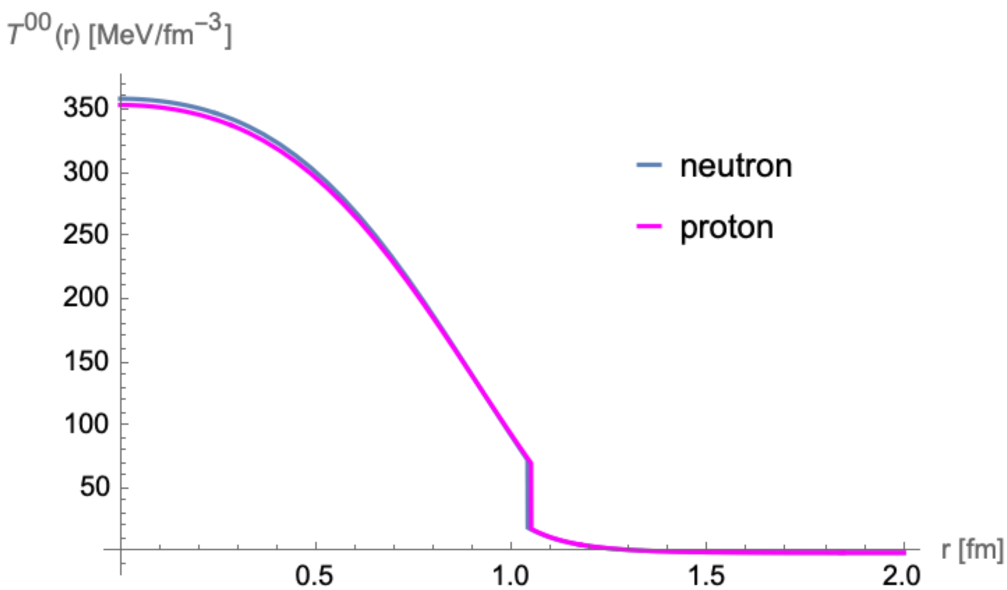
(Mejía et al 2025 (in prep))

$$\sqrt{< r^2 >_{\mathrm{dust,P}}} = \mathbf{0.710\ fm} \rightarrow \sqrt{< r^2 >_{\mathrm{ch,P}}} = \mathbf{0.84\ fm}$$

$$\sqrt{< r^2 >_{\mathrm{dust},N}} = \mathbf{0.704\ fm}$$

(Mejía et al 2025 (in prep))

Energy Density and Mass



$$M = \int T^{00}(\vec{r}) d^3(r) = m + E_b$$

$$= 940MeV - 15.7MeV$$

Correct value for EM contribution to mass difference between proton and neutron:

$$((M_P c^2 - M_N c^2)_{EM} = 0.95MeV$$

$$Lattice\ QCD + QED: 1.00(07)(14)MeV$$

(Borsanyi et al, Science 347 (2015) 1452)

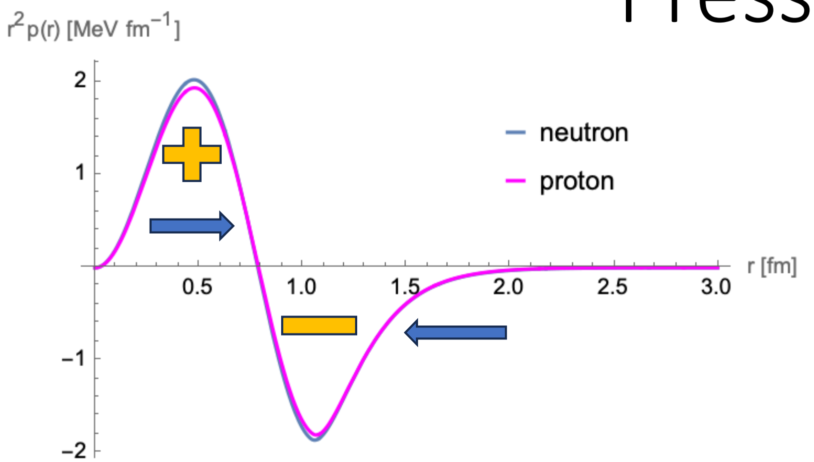
$$(M_N c^2 - M_P c^2)_{nature} = 1.29333236(46)MeV$$

↳ *due to $m_d > m_u$ isospin violation*

(Navas et al. (Particle Data Group), PRD 110, 030001 (2024))

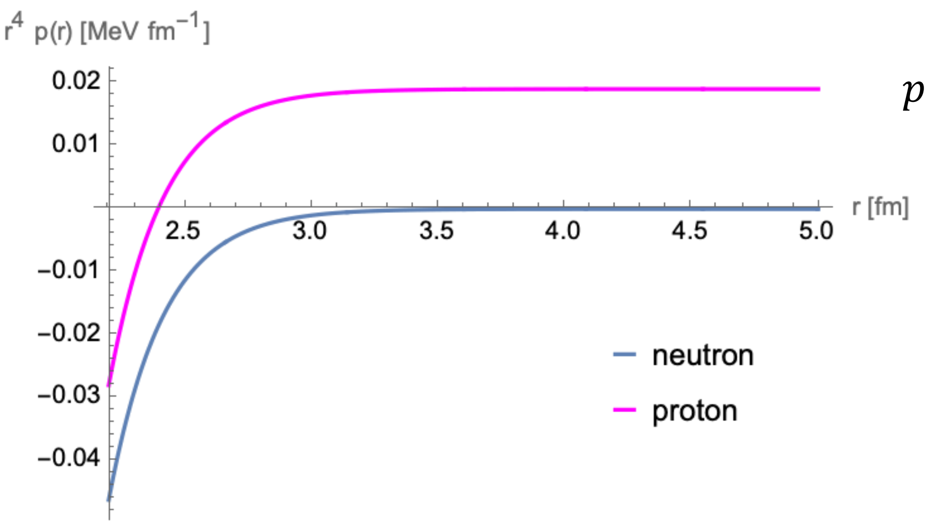
No isospin violation in Bialynicki-Birula model

Pressure Distribution



$$\int_0^\infty dr\, r^2 p(r) = 0$$

(Von Laue Condition, 1911)



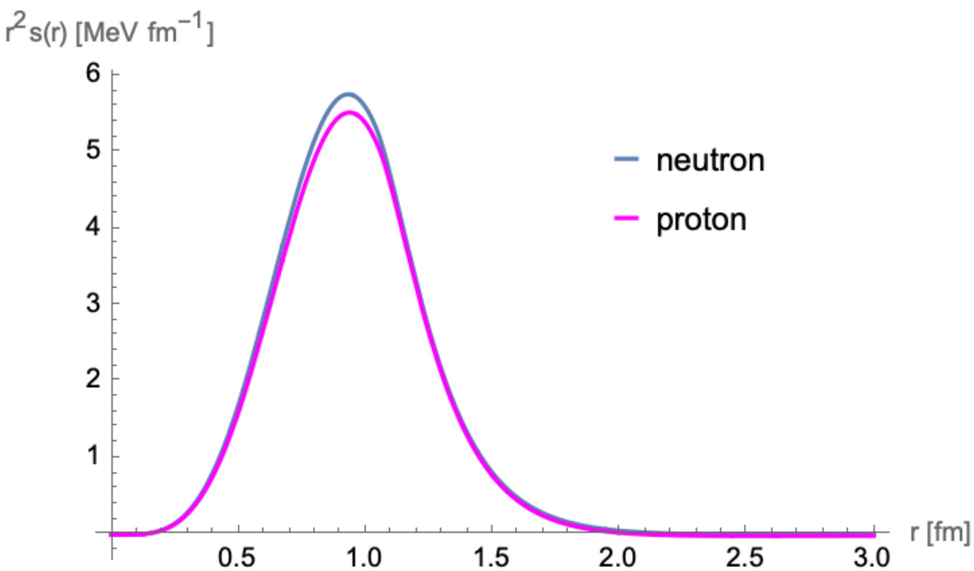
$$p(r) = \frac{1}{6}\frac{\alpha}{4\pi r^4}\ at\ large\ r \quad (exact\ and\ agrees\ with\ QED)$$

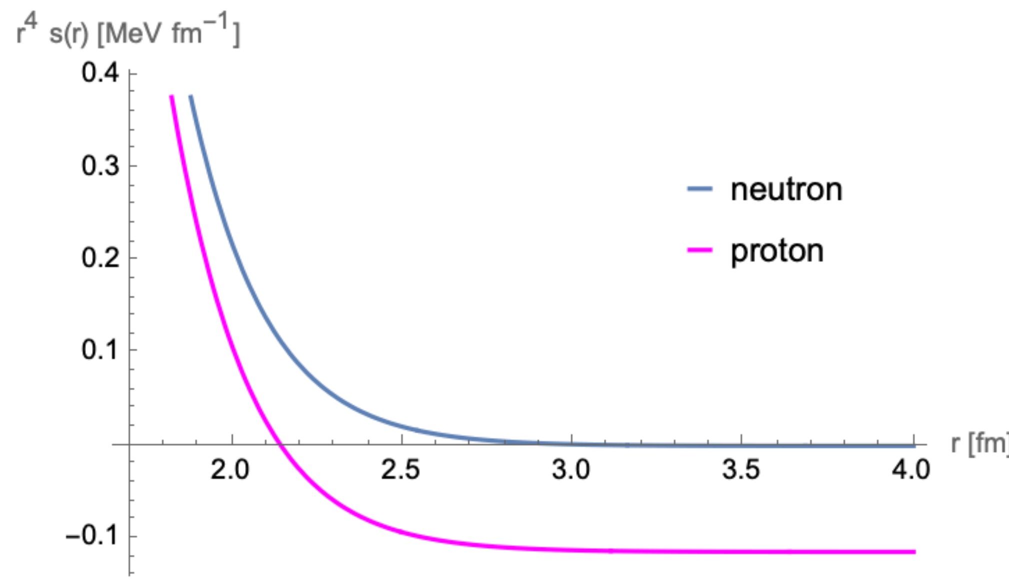
$$D_p = M \int d^3r\, r^4 p(r) \rightarrow +\infty$$

(Mejía et al, 2025 (in prep))

(Mejía et al, 2025 (in prep))

Shear Force



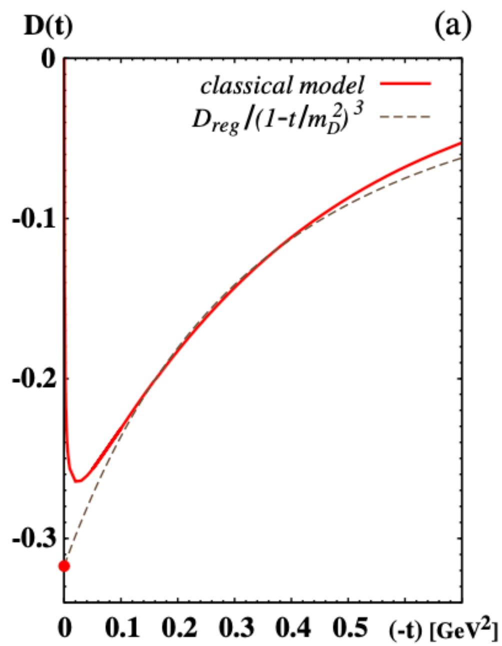


$s(r) = -\frac{1}{4\pi}\frac{\alpha}{r^4}$ *at large* r, $\quad D_s = -\frac{16\pi}{15} M \int dr\, r^4 s(r) \to +\infty$

- Proton: $D_p \to \infty$, $D_s \to \infty$ (due to em)
- If Integrals convergent: $D_p = D_s = D$ → always finite and equal

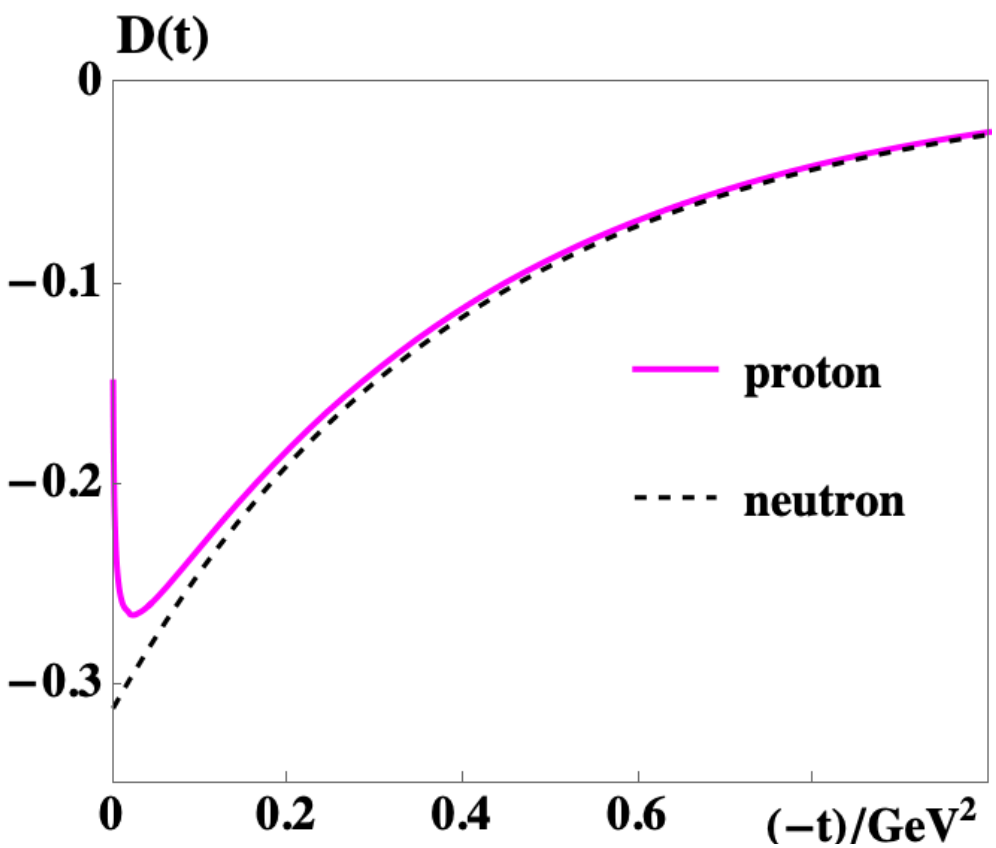
  i.e. $D(\varsigma) = \varsigma D_p + (1-\varsigma) D_s$ = always the same for all ς
- For proton, exists one value: $\varsigma = \frac{8}{3}$ such that

$$D_{reg,prot} = D\left(\varsigma_{reg}\right) = M \int d^3r\, r^2 \frac{4}{9}[6p(r) + s(r)] = -0.317(\hbar c)^2$$



$$D(t)_{exp.,proton} \approx \frac{D_{reg}}{\left(1 - \frac{t}{m_D^2}\right)^3}$$

Dterm



$$D_{neutron} = -0.312(\hbar c)^2$$

$$D_{reg,proton} = -0.317(\hbar c)^2$$

Regularization = unique way of removing EM tails in p(r) proposed in Varma, Schweitzer 2020

Is this a reasonable regularization? Yes, because for proton and neutron we get almost same result (meets expectations) for $|t| > 0.05\ GeV^2$

(Mejía et al., 2025 (in prep))

Summary

- Constructed a classical model of the neutron (Mejia et al., 2025 (in prep)).
- Strong forces simulated by classical fields + classical EM field (Bialynicki-Birula, 1993)
- Classical models for both proton and neutron capture particle size and EM-mass difference.
- Proton D-term divergent at (probably) unmeasurably small |t|.
- Neutron D-term finite, agrees numerically; confirms “reg. method” for |t|> 0.05 GeV^2
- In DVCS and other experiments, proton and neutron will appear with nearly the same D(t) form factor
- Could EIC measure the QED effect at small |t| for proton?

Thank You!

Support Slides

$$M_p c^2 + E_B = 940MeV - 15MeV\ (mass\ of\ bound\ dust)$$

$$D_s = -\frac{2(n-1)}{n(n+2)} M \int d^n r \, r^2 s(r), \qquad (36)$$

$$D_p = M \int d^n r \, r^2 p(r), \qquad (37)$$

$$D = D_p = D_s$$

Diverge due to asymptotic behavior:

$$s(r) = -\frac{\alpha}{4\pi}\frac{\hbar c}{r^4} + \ldots$$

$$p(r) = \frac{1}{6}\frac{\alpha}{4\pi}\frac{\hbar c}{r^4} + \ldots$$

regularization method removes the divergences from Dp and Ds

Varma, Schweitzer 2020

Proton Model: Static solution to the classical equations

- $eA_{0(r)} = e^2\left(\frac{f_+(r)}{k_+^2} - \frac{f_-(r)}{k_-^2}\right) + 2E_B(= 0,\ e \to 0\ neutron)$
- $f_\pm(r) = \frac{d_\pm}{4\pi}\frac{\sin(k_\pm r)}{r}$, $k_\pm = \frac{B \pm \sqrt{D}}{2Q^2} \to (k_- = 0, neutron)$
- $B = (g_s^2 - e^2)\, m_v^2 - (g_v^2 + e^2)m_s^2$
- $D = B^2 - 4e^2Q^2\, m_s^2\, m_v^2,\ Q^2 = e^2 - g_s^2 + g_v^2$ Type equation here.

$$\lim_{e\to 0} eA_{0(r)} = [e^2\left(\frac{f_+(r)}{k_+^2} - \frac{f_-(r)}{k_-^2}\right)] + 2E_B$$

$$0 = \lim_{e\to 0}\left[-e^2\frac{f_-(r)}{k_-^2}\right] + 2E_B$$

$$f_-(r) = \frac{d_-}{4\pi}\frac{\sin(k_- r)}{r}$$

Series Expansion

(Bialynicki-Birula 1993)

$$k_+ = \frac{B + \sqrt{D}}{2Q^2} = \frac{B + \sqrt{B^2}}{2Q^2} = \frac{B}{Q^2}$$

$$k_- = \frac{B - \sqrt{D}}{2Q^2} = \frac{B - \sqrt{B^2}}{2Q^2} = 0 \to Indeterminate!$$

$$k_-^2 = B - B\sqrt{1 - \frac{4e^2Q^2m_v^2m_s^2}{B^2}}$$

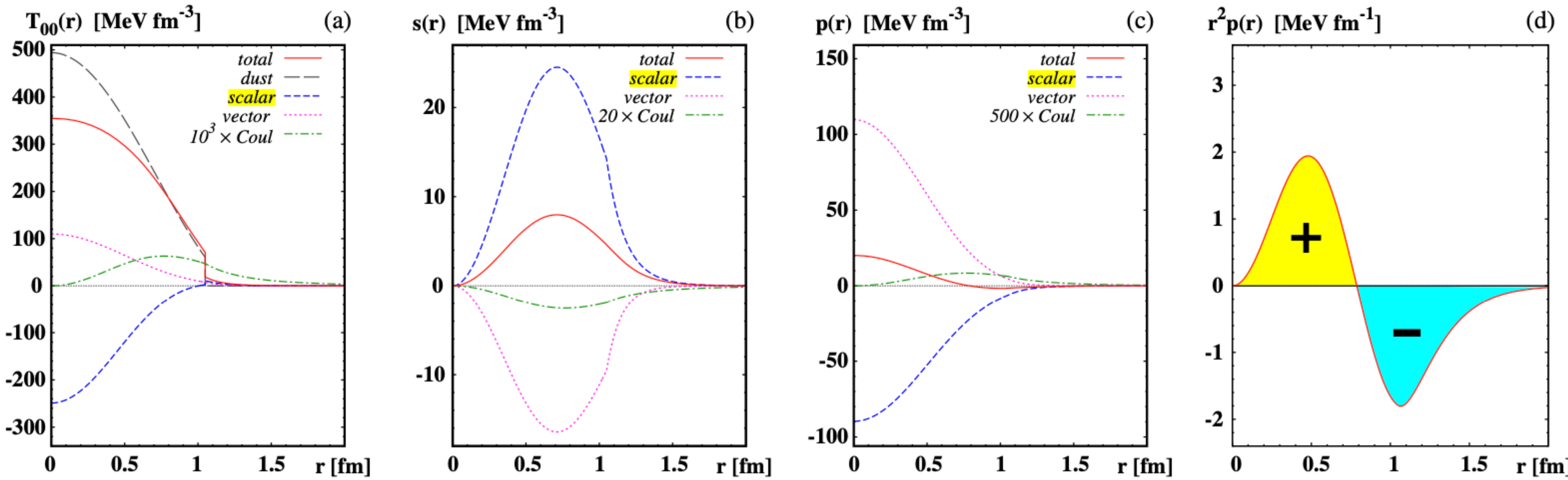
Series Expansion

$$k_-^2 = \frac{e^2 m_v^2 m_s^2}{B} \quad = \chi^2, \qquad k_-^2 = e^2\chi^2$$

$$2E_B = \lim_{e\to 0}\left[\frac{d_-}{4\pi\chi}\right] \to 8\pi\chi E_B = \lim_{e\to 0}[e * d_-]$$

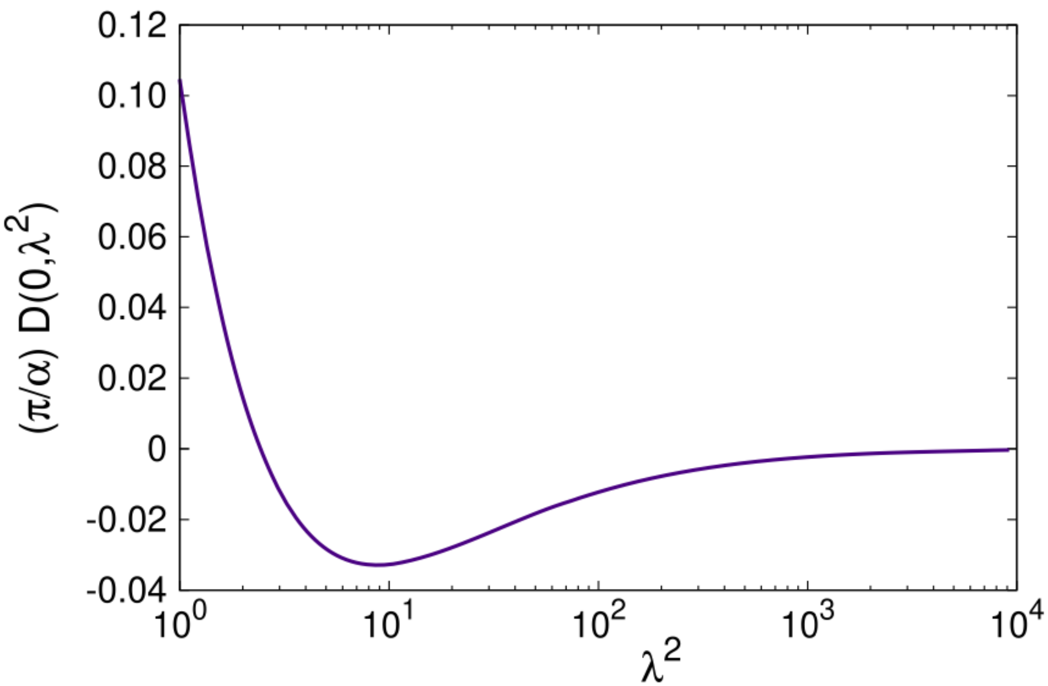
$$\lim_{e\to 0}[f_-] = \lim_{e\to 0}\left[\frac{d_-}{4\pi}\frac{\sin(k_- r)}{r}\right] = \lim_{e\to 0}[\frac{8\pi\chi E_B}{e4\pi r} * e\chi r]$$

$$\therefore\ f_-(r) = constant$$



$T_{00}(r)$ [MeV fm^{-3}]
(a)
total
dust
scalar
vector
$10^3 \times$ Coul
r [fm]
$s(r)$ [MeV fm^{-3}]
(b)
total
scalar
vector
$20 \times$ Coul
r [fm]
$p(r)$ [MeV fm^{-3}]
(c)
total
scalar
vector
$500 \times$ Coul
r [fm]
$r^2 p(r)$ [MeV fm^{-1}]
(d)
+
-
r [fm]

QED study on GFF D(t) of electron



Metz et al PLB 820 (2021)