Holographic driven steady states

based on 2404.05568 with M. Matsumoto, A. Amoretti and M. Baggioli

Daniel Keith Brattan
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Università di Genova

INFN
Istituto Nazionale di Fisica Nucleare

NESS and Cattaneo

How to get a steady state using relaxation

- Driven steady states are hard…
- Interested in NESS that are a balance of driving and loss.
- Classic example: thermal gradients and steady flow - Cattaneo-Christov phenomenological model (thermal gradients propagate at finite speed, see Navid's talk)

$$\left[1 + \tau\left(\partial_t + \vec{v}_{\text{drift}} \cdot \overrightarrow{\nabla}\right)\right] \overrightarrow{\mathcal{Q}} = -\kappa \overrightarrow{\nabla} T\,.$$

- Decay of currents resolve a lot of physical problems e.g. finite DC conductivity.
- What about an analogue with electric fields and conduction?

Perturbations without a background electric field

- Modulate expectations: the (suggested) effective linearised hydrodynamic equations:

$$\partial_t \delta\rho + \overrightarrow{\nabla} \cdot \delta\vec{J} = 0 \, ,$$
$$\partial_t \delta J^i + \partial_j \mathcal{T}^{ij} + \frac{1}{\tau}\delta J^i = \chi \delta E^i \, .$$

- Lowest order constitutive relation: $\mathcal{T}^{ij} = \rho \delta^{ij}$.

- One finds near the origin in complex frequency (realised in holographic models at large charge density)

$$\omega = -\, iDk^2 + \mathcal{O}(k^4) \, , \qquad \omega = -\frac{i}{\tau} + iDk^2 + \mathcal{O}(k^4) \, .$$

- An aside: in the ultra-high density limit we get “holographic zero sound” [Chen & Lucas - 1709.01520]

$$\omega = \pm \frac{1}{\sqrt{d}} k - i\Gamma k^2 + \mathcal{O}(k^4) \, .$$

Perturbations with a background electric field

- The (suggested) effective hydrodynamic equations:

$$\partial_t \delta\rho + \vec{\nabla} \cdot \delta\vec{J} = 0 \ ,$$
$$\partial_t \delta J^i + \partial_j \mathcal{T}^{ij} - \alpha E^i \delta\rho + \frac{1}{\tau}\delta J^i = \chi \delta E^i \ .$$

- The constitutive relations are <u>bloody</u> complicated! Take E^i small in amplitude but not derivatives, and again large charge density.

- Now three types of mode due to broken spatial rotation invariance:

$$\omega_{\text{gapless}} = \alpha\,\tau\,\vec{k} \cdot \vec{E} + \mathcal{O}(\vec{k}^2, \vec{E}^2) \ ,$$
$$\omega_{\text{gapped}} = -\frac{i}{\tau} + \left(\eta_1 + \eta_2 - \alpha\,\tau\right)\vec{k} \cdot \vec{E} + \mathcal{O}(\vec{k}^2, \vec{E}^2) \ ,$$
$$\omega_{\perp} = -\frac{i}{\tau} + \eta_3\,\vec{k} \cdot \vec{E} + \mathcal{O}(\vec{k}^2, \vec{E}^2) \ .$$

Correlation functions

and the driven steady state

- Perturbing the electric field allows us to show

$$\sigma_{\mathrm{DC}} = \tau\chi \ .$$

- Now, consider time independent equations:

$$\partial_i \delta J^i = 0 \ , \qquad \chi \partial_i \delta\rho = \alpha E_i \delta\rho \ .$$

- What do the time independent correlation functions look like?

$$\langle \rho\rho \rangle_{\mathrm{R}}(0,\vec{k}) = \frac{|\vec{k}|\chi}{v^2|\vec{k}| + i\alpha|\vec{E}|\cos\varphi} \ .$$

- There is a quasinormal mode at complex momentum.

An old story

Probe branes with a constant electric field

- D3-D7 probe brane action:

$$S_{D7} = -T_{D7} \int d^8\xi \sqrt{-\det\left(g_{ab} + 2\pi\alpha' F_{ab}\right)} \, .$$

- Now you do the standard things:
  - Place the D7 brane in the black brane background (finite T),
  - We take a trivial embedding (quark mass is zero),
  - Set $A_t = \mu$ at the AdS boundary (non-zero charge density).
- And one "non-standard" thing: $A_x = E_x t$ at the boundary.

The background solution

Standard Karch-O'Bannon

- There exists an exact solution to our BCS [Karch & O'Bannon 0705.3870]. Asymptotically

$$A_t = \mu + \rho/r^2 + \dots \ , \qquad A_x = -E_x t + J_x/r^2 + \dots \ .$$

- Plug back into the action to determine the free energy - square root becomes imaginary unless J_x has a specific form i.e.

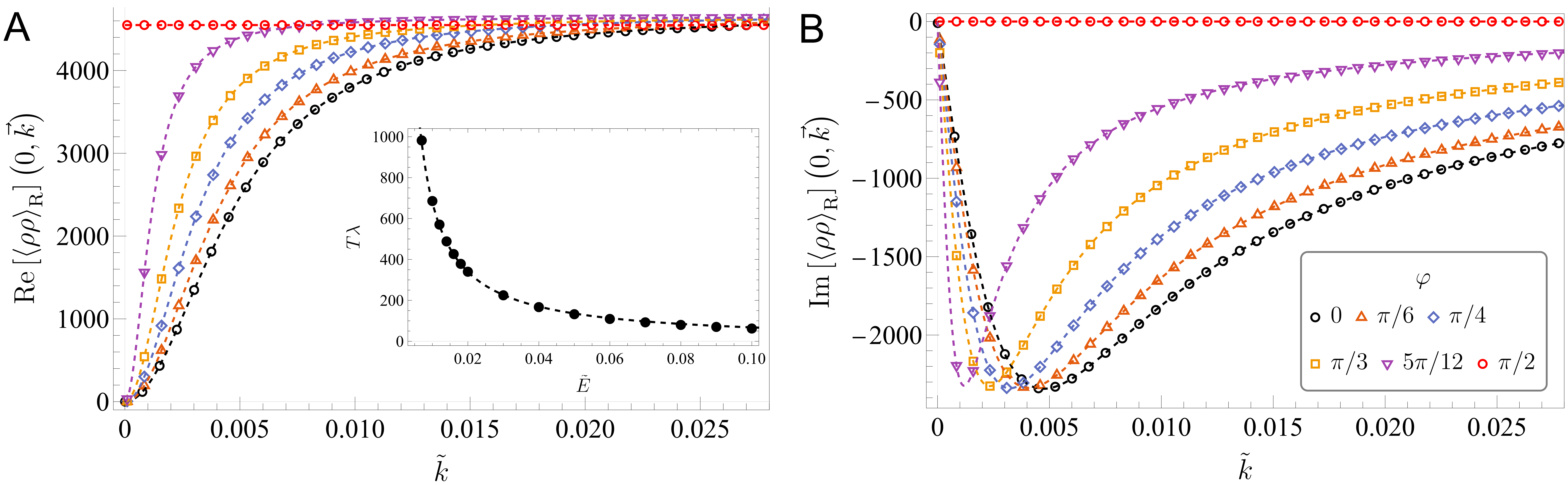
$$\frac{\sigma_{\rm DC}}{\pi T} = \frac{\tilde{J}}{\tilde{E}} = \left(\frac{\tilde{\rho}^2}{1+\tilde{E}^2} + \sqrt{1+\tilde{E}^2} \right)^{1/2} .$$

- Related to appearance of a pseudo-horizon for gauge perturbations:

$$z_* \sim z_{\rm H} \left(\sqrt{1+\tilde{E}^2} - \tilde{E} \right)^{\frac{1}{2}} .$$

Gauge field perturbations

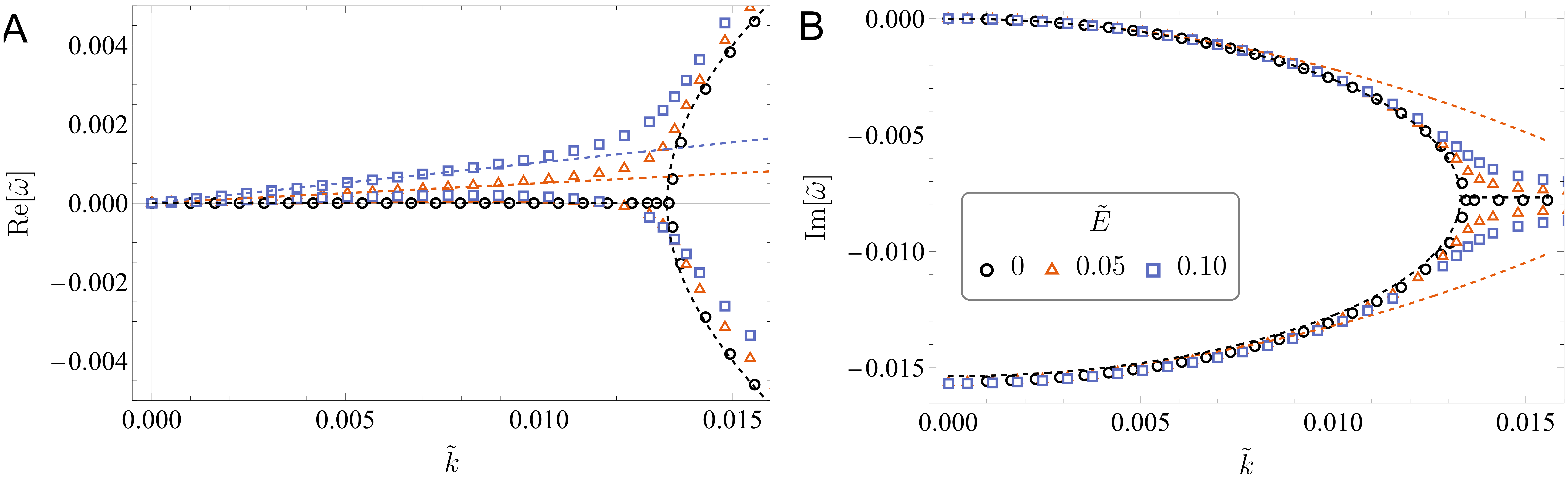
Does the (static) effective theory match the model? (Yes.)



Static, relaxed hydrodynamic theory vs. microscopic model. Charge-charge static response for different values of the angle with $\tilde{\rho} = 10^5$ and $\tilde{E} = 0.1$. All lines are the predictions of RHT while points are holographic data from the microscopic model. The inset shows the renormalized screening length.

Gauge field perturbations

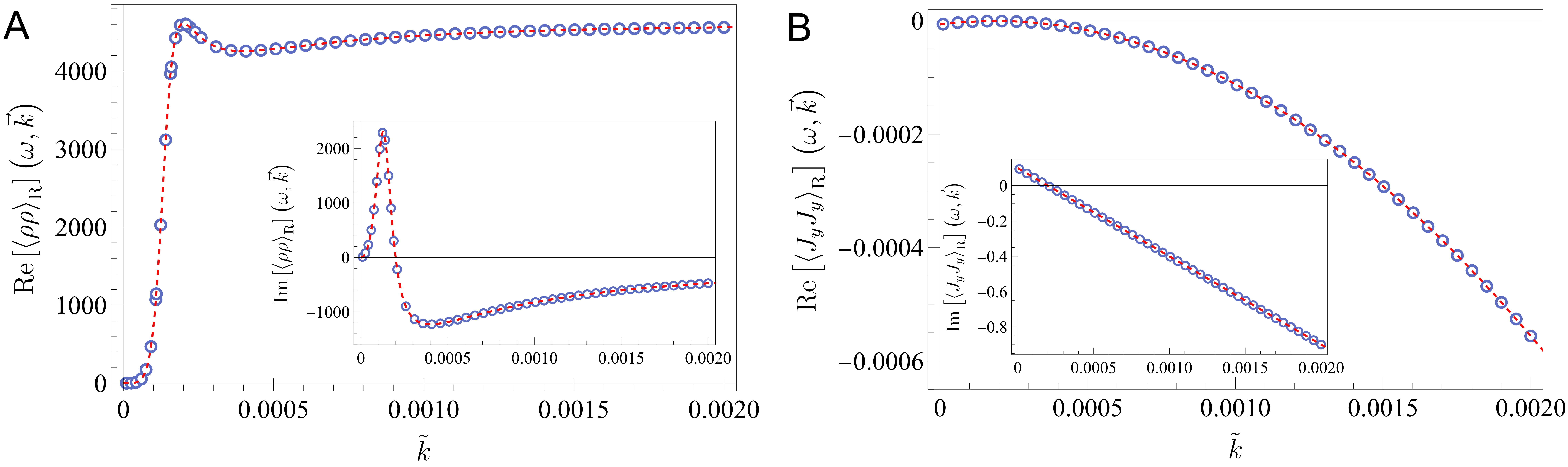
Does the dynamic effective theory match the model? (Yes.)



Collective excitations around a non-equilibrium steady state. The dispersion of the lowest excitations in the NESS in the collinear limit. (A) The real part and (B) imaginary part of the frequency are shown. Different colors correspond to different strengths of the dimensionless electric field.

Gauge field perturbations

Hydrodynamics is not just the modes.



Time dependent dynamics and retarded correlators. The retarded correlators at finite frequency as a function of wavevector. (**A**) The charge-charge correlator and (**B**) perpendicular current-current correlator. The insets show the imaginary part of the corresponding correlator. The points are the data from the microscopic model and the dashed lines denote our predictions.

Future work

- I want to understand structure of low energy theory: SL(2,Z), higher corrections in charge density and derivatives, non-linear solutions etc.
- Make your master's student cry (a guide):
  - We don't really know what the stationarity conditions are:

$$\delta J^i = \sigma_{DC}\delta E^i \qquad \rightarrow \qquad J^i = \sigma_{DC}E^i + O(\partial E)?$$

  - Make gullible student try to compute derivative corrections to the stationarity condition.
  - Cackle evilly like a Disney villain!!!

Thanks!