Consequences of anomaly in holographic QCD

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POSTECH
POHANG UNIVERSITY OF SCIENCE AND TECHNOLOGY

Holographic perspectives on chiral transport and spin dynamics

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in collaboration with

Jesús Cruz Rojas (UNAM Mexico), Tuna Demircik (Utrecht);
Niko Jokela, Aleksi Piispa (Helsinki) [2405.02392, 2405.02399]
Domingo Gallegos (UNAM Mexico), Eamonn Weitz (Bielefeld) [2406.07617]

Outline

1. The holographic V-QCD model
   - Implementation of anomalies
   - Fitting to lattice data
2. Spatial instability in V-QCD
3. Chiral separation effect in V-QCD
4. Conclusion

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The V-QCD model

V-QCD: A holographic bottom-up model for QCD in the Veneziano limit
(large N_f, N_c; fixed N_f/N_c)

- Bottom-up, but trying to follow principles from string theory as closely as possible
- Many parameters: effective description of QCD
- Comparison with QCD data essential
- Relatively complicated model (because QCD is complicated)
- Inclusion of axial and chiral anomalies worked out

The model is obtained through a fusion of two building blocks: [MJ, Kiritsis arXiv:1112.1261]

1. IHQCD: model for glue inspired by string theory (dilaton gravity) [Gürsoy, Kiritsis, Nitti; Gubser, Nellore]
2. Adding flavor and chiral symmetry breaking via space filling $D4 - \overline{D4}$ branes and tachyon condensation [Klebanov,Maldacena; Bigazzi,Casero,Cotrone,Iatrakis,Kiritsis,Paredes]

Full backreaction between the two sectors in the Veneziano limit

Chirally symmetric V-QCD

Dual fields: $\phi \leftrightarrow G_{\mu\nu}G^{\mu\nu}$, $\mathfrak{a} \leftrightarrow G_{\mu\nu}\widetilde{G}^{\mu\nu}$, $(A_\mu^{L/R})^{ij} \leftrightarrow \bar{\psi}^i(1 \pm \gamma_5)\gamma_\mu\psi^j$

$$\mathcal{S}_{\mathrm{V-QCD}} = S_g + S_{\mathrm{DBI}} + S_a + S_{\mathrm{CS}}$$

$$\begin{aligned}
S_g &= M^3 N_c^2 \int d^5x \sqrt{-g}\left[R - \frac{4}{3}(\partial\phi)^2 + V_g(\phi)\right] \\
S_{\mathrm{DBI}} &= -M^3 N_c \int d^5x\ V_f(\phi)\ \mathrm{Tr}\left[\sqrt{-\det(g_{\mu\nu} + w(\phi)F_{\mu\nu}^{(L)})} + (L \leftrightarrow R)\right] \\
S_a &= -\frac{M^3 N_c^2}{2}\int d^5x \sqrt{-g}\ Z(\phi)\left[\partial_\mu \mathfrak{a} - \mathrm{Tr}\left(A_\mu^L - A_\mu^R\right)/N_c\right]^2 \\
S_{\mathrm{CS}} &= \frac{iN_c}{24\pi^2}\int \mathrm{Tr}\Big[-iA_L \wedge F_L \wedge F_L + \frac{1}{2}A_L \wedge A_L \wedge A_L \wedge F_L + \\
&\quad + \frac{i}{10}A_L \wedge A_L \wedge A_L \wedge A_L \wedge A_L + (L \leftrightarrow R)\Big]
\end{aligned}$$

- A slight generalization of models discussed e.g. by Sebastian on Tuesday
- Most of the structure fixed by chiral symmetry, parity, and anomalies
- Many potentials V_g, V_f, w, Z – however need to be "simple" functions – determined by comparing to lattice data

Anomaly terms in V-QCD

1. Axial anomaly:

$$S_a = -\frac{N_c^2 M^3}{2} \int d^5x \sqrt{-g}\; Z(\phi) \left[\partial_\mu \mathfrak{a} - \mathrm{Tr}\left(A_\mu^L - A_\mu^R\right)/N_c\right]^2$$

is invariant under the $U(1)_A$ gauge transformation

$$(A_\mu^L)^{ij} \to (A_\mu^L)^{ij} + \delta^{ij}\partial_\mu\epsilon\;, \qquad (A_\mu^R)^{ij} \to (A_\mu^R)^{ij} - \delta^{ij}\partial_\mu\epsilon, \qquad \mathfrak{a} \to \mathfrak{a} + 2\frac{N_f}{N_c}\epsilon$$

- Symmetry implies the axial anomaly in QCD
- S_a includes a bulk mass for the axial gauge field

$$\partial_\mu J_A^\mu = \frac{N_f}{16\pi^2} G_{\mu\nu}\tilde{G}^{\mu\nu}$$

2. Global chiral anomalies:

The gauge transformation of S_{CS} (with parameters $\Lambda_{L/R}$) is a boundary term matching with field theory anomaly

[See Niko's talk]

$$\delta S_{CS} = \frac{iN_c}{24\pi^2} \int_\partial \mathrm{Tr}\left[\Lambda_L F_L \wedge F_L - \Lambda_R F_R \wedge F_R + \cdots\right]$$

N.B. chirally broken S_a and S_{CS} also available

[Casero, Paredes, Kiritsis hep-th/0702155; Arean, Iatrakis, MJ, Kiritsis 1309.2286, 1609.08922]
[MJ, Kiritsis, Nitti, Préau 2209.05868; 2212.06747]

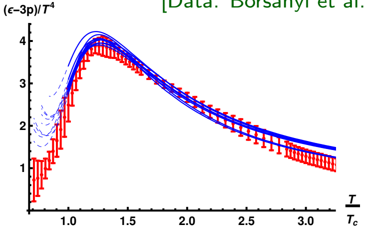
Comparing with lattice data

Potentials V_g, V_f, w (gravity + DBI) determined by fitting to lattice QCD data

[Jokela, MJ, Remes 1809.07770]

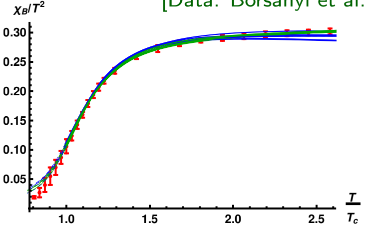
Interaction measure $\frac{\epsilon - 3p}{T^4}$,
2+1 flavors

[Data: Borsanyi et al. 1309.5258]



Baryon number susceptibility $\chi_2 = \frac{d^2 p}{d\mu^2}\Big|_{\mu=0}$

[Data: Borsanyi et al. 1112.4416]



Choice of $Z(\phi)$ in S_a less constrained – consider three possibilities

$$Z(\phi) = Z_0(1 + c_1 e^{\phi} + c_4 e^{4\phi}) \ , \quad Z(\phi) = Z_0(e^{\phi} + c_4 e^{4\phi}) \ , \quad Z(\phi) = Z_0(e^{2\phi} + c_4 e^{4\phi})$$

Fit c_i to lattice Yang-Mills data for pseudo-scalar glueballs and Z_0 to topological susceptibility

[Gallegos, MJ, Weitz 2406.07617]

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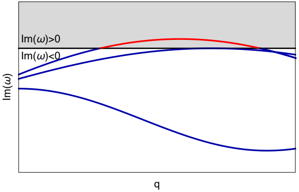
Spatially modulated instability in V-QCD?

The CS term could drive the Nakamura-Ooguri-Park instability in V-QCD

[Nakamura, Ooguri, Park 0911.0679; Ooguri, Park 1011.4144]

$$S_{CS} = \frac{iN_c}{24\pi^2}\int \mathrm{Tr}\left[-iA_L\wedge F_L\wedge F_L + iA_R\wedge F_R\wedge F_R+\cdots\right]$$

- Background: charged black hole in V-QCD
- CS term only affects "helicity-one" gauge field fluctuations



Two cases:

1. Non-Abelian fluctuations: decoupled fields $\sim (\delta A^x_{L/R}(r)\pm i\delta A^y_{L/R}(r))t^a e^{-i\omega t+iqz}$ where t^a is a generator of $SU(N_f)$
2. Abelian fluctuations (simpler in vector/axial basis due to parity)

$$\delta A^x \pm i\delta A^y \underset{\text{coupled (CS)}}{\longleftrightarrow} \delta V^x \pm i\delta V^y \underset{\text{coupled}}{\longleftrightarrow} \delta g_{zx}\pm i\delta g_{zy}$$

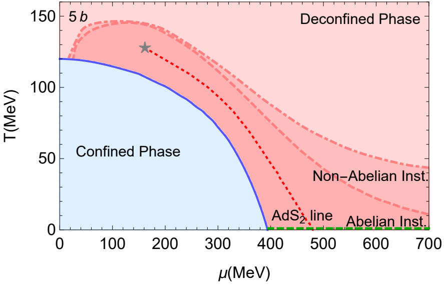
Check numerically the region where the instability exists

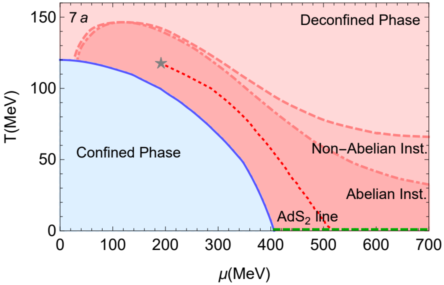
[Cruz Rojas, Demircik, MJ 2405.02399]

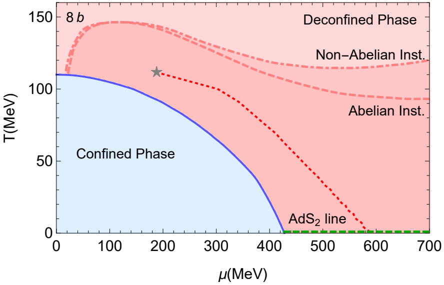
Results for instability

- Extends to surprisingly low μ/T!
- Non-Abelian instability stronger!
- Three variants 5b, 7a, 8b: uncertainty after lattice fit
- Result actually "universal"! [Niko's talk]

[Demircik, Jokela, MJ, Piispa 2405.02392]



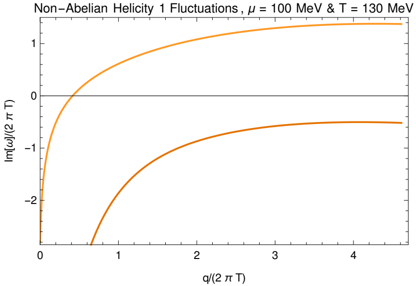


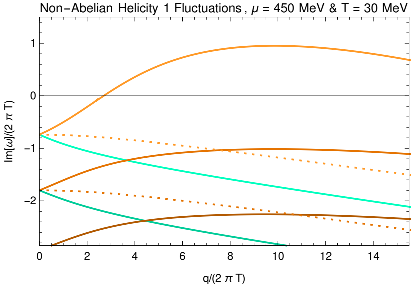


Dispersion relations

Dispersion relations of the unstable modes at low μ/T (upper plot) and high μ/T (lower plot)

- Details vary over the phase diagram
- Characteristic frequency and momentum roughly constant, $\omega \sim \Lambda_{\mathrm{QCD}} \sim q$
- Apart from minor mixing effects, results for Abelian and non-Abelian modes similar





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Anomalous transport in V-QCD

We did the basic computation of anomalous conductivities

[Gallegos, MJ, Weitz 2406.07617]

$$\delta\langle\mathcal{J}_V^k\rangle = \sigma_{VV}B^k + \sigma_{VA}B_A^k + \sigma_{V\Omega}\omega^k$$

- Background: black holes with vectorial and/or axial charge
- Turned on constant infinitesimal magnetic fields and vorticity $\omega^i = \epsilon^{ijk}\partial_j\delta g_{tk}|_{\mathrm{bdry}}$
- Computed variation of consistent vectorial current by solving the fluctuations
- In V-QCD, the consistent current is UV finite, whereas the covariant current is not
- In this basis, only σ_{VA} is nonzero – the chiral separation conductivity
- Result deviates from universal value $\sigma_{VA} = \frac{N_f N_c}{2\pi^2}\mu$ due to dynamical gauge fields, i.e. the mass of the axial gauge field in the bulk

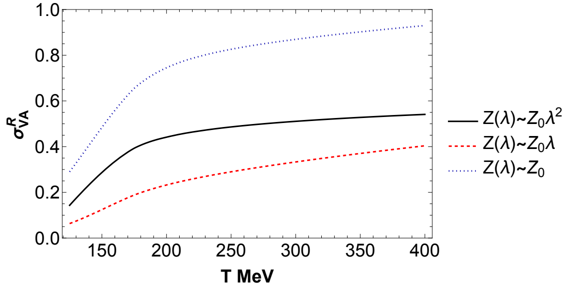
[Jimenez-Alba, Landsteiner, Melgar 1407.8162; Gallegos, Gürsoy 1806.07138]

Results at zero density

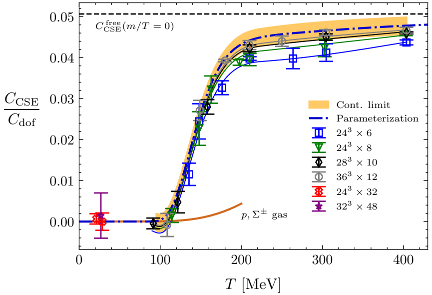
We show the conductivity normalized to the universal result in the limit of $\mu \to 0$

- Used three different variants of $Z(\phi)$ controling the mass of the axial gauge field
- Qualitative agreement with recent lattice analysis
  [Brandt, Endrodi, Garnacho-Velasco, Marko 2312.02945]
- Tuning $Z(\phi)$ to obtain more precise match with lattice seems possible

Our result



Lattice ($N_f = 2+1$)



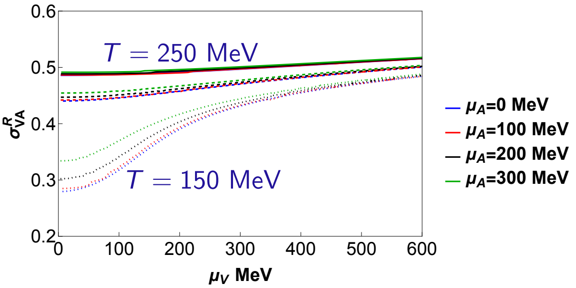
Results at finite density

Predictions at finite vectorial or axial density

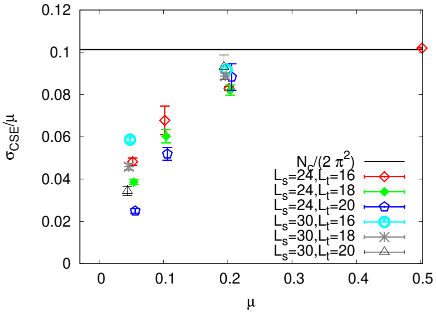
- ($N_c = 3$) lattice QCD results unavailable (sign problem)
- However can compare to $N_c = 2$ results

[Buividovich, Smith, von Smekal 2012.05184]

Our result



Lattice ($N_c = 2$)



Outline

Conclusions

- I presented a (chirally symmetric) holographic model which fits lattice data and includes QCD anomalies
- First application: Nakamura-Ooguri-Park instability at surprisingly low μ/T
  - Turns out to be a universal result (see Niko's talk)
- Second application: analyzed chiral separation effect – found qualitative agreement with lattice results

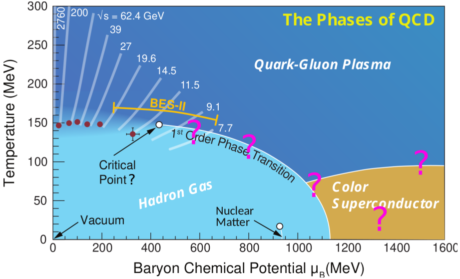
Thank you!

QCD phase diagram and the critical point

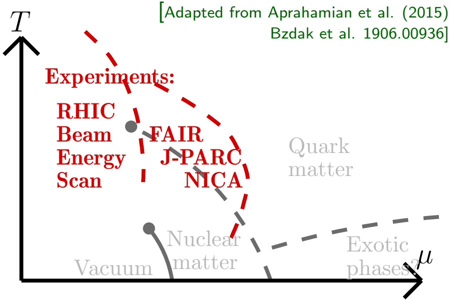
Search for the critical point: ongoing effort at RHIC

- Beam Energy Scan stage I results available
- Stage II finished, results being analyzed

Will be extended by future experiments at FAIR, J-PARC, NICA



[Adapted from Aprahamian et al. (2015)
Bzdak et al. 1906.00936]



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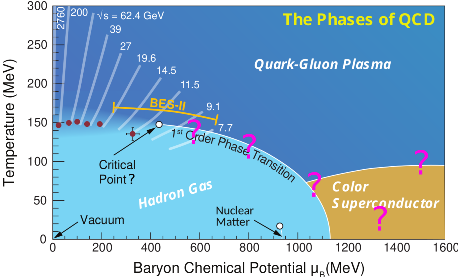
Neutron star observations give complementary information at high density



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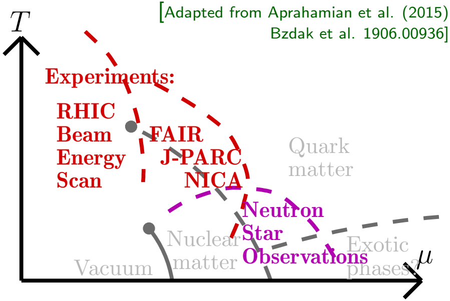
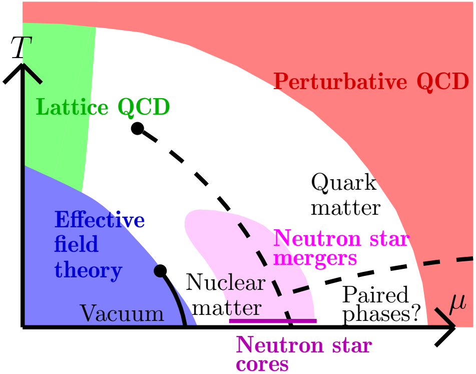


QCD phase diagram and the critical point

Theoretical approaches

- First-principles methods do not work in the region relevant for critical point
- Phase diagram or even relevant phases not known



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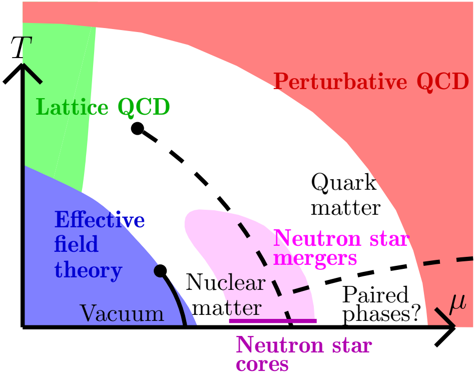

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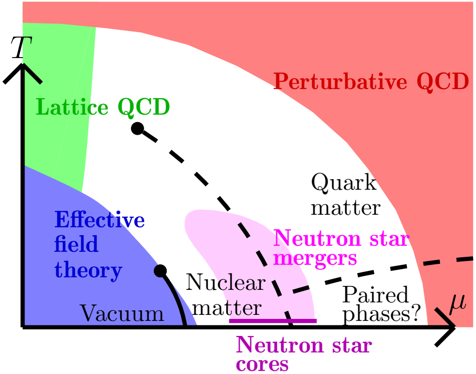

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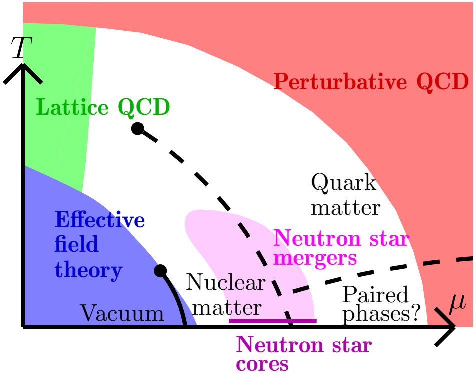

- Basic idea (bottom-up): use the gauge/gravity duality to extrapolate lattice (and other) data to higher density

[DeWolfe et al. 1012.1864; Knaute et al. 1702.06731; Critelli et al. 1706.00455 Jokela, MJ, Remes 1809.07770; Demircik, Ecker, MJ 2112.12157 Cai, He, Li, Wang 2201.02004; Li, Liang, He, Li 2305.13874 . . .]

[See also the talk by Mei Huang]

Outline

Generic holographic approach: fields

We want to describe holographically (chirally symmetric) QCD plasma (N_f massless flavors)

Most important (relevant and marginal) operators

- $T_{\mu\nu}$, dual to the metric $g_{\mu\nu}$
- Gluon operator $G^2_{\mu\nu}$, dual to a scalar (the dilaton) ϕ
- Flavor currents $\bar{\psi}_i\gamma_\mu(1 \pm \gamma_5)\psi_j$, dual to the gauge fields $(A_\mu^{L/R})_{ij}$ (with $i, j = 1 \ldots N_f$) – global $\mathrm{U}(N_f)_L \times \mathrm{U}(N_f)_R$ of QCD promoted to gauge symmetry
- Flavor bilinears $\bar{\psi}_i\psi_j$ dual to a complex scalar T_{ij}

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What are our options for the choice of 5D action?

Chern-Simons (CS) terms in holography

Chiral anomalies in QCD: consider the chiral $\mathrm{U}(N_f)_L \times \mathrm{U}(N_f)_R$ coupled to external fields A_L, A_R

- Under transformation with parameters $\Lambda_{L/R}$

$$S_{\mathrm{QCD}} \mapsto S_{\mathrm{QCD}} + \frac{iN_c}{24\pi^2} \int \mathrm{Tr}\left[\Lambda_L F_L \wedge F_L - \Lambda_R F_R \wedge F_R + \cdots\right]$$

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Holographic counterpart

- External fields promoted to 5D gauge fields
- Gauge variation at the boundary must agree with the anomaly
- 5D CS term – unique when chiral symmetry intact

[Witten hep-th/9802150]

$$S_{\mathrm{CS}} = \frac{iN_c}{24\pi^2} \int \mathrm{Tr}\Big[- iA_L \wedge F_L \wedge F_L + \frac{1}{2} A_L \wedge A_L \wedge A_L \wedge F_L + \\ + \frac{i}{10} A_L \wedge A_L \wedge A_L \wedge A_L \wedge A_L - (L \leftrightarrow R)\Big]$$

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- Generalizations (e.g. chirally broken) worked out

[Casero, Paredes, Kiritsis hep-th/0702155; Lau, Sugimoto 1612.09503; MJ, Kiritsis, Nitti, Préau 2209.05868]

- Note: $U(1)_A$ anomaly is a separate issue – not needed here

Generic holographic approach: actions

We write down expected (two-derivative) terms

$$S = S_{\mathrm{gr}} + S_{\mathrm{matter}} + S_{\mathrm{CS}}$$

where S_{CS} is fixed by anomalies, and

$$S_{\mathrm{gr}} = M_{\mathrm{p}}^3 N_c^2 \int d^5x \sqrt{-\det g} \left[R - \frac{4}{3}(\partial\phi)^2 + V_{\mathrm{g}}(\phi) \right]$$

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Choice of $S_{\rm matter}$ less obvious. Options: $S_{\rm matter} = S_{\rm DBI}$ or $S_{\rm matter} = S_{\rm YM}$, with

1. $S_{\rm DBI} = M_{\rm p}^3 N_c \int V_{\rm f}(\phi)\,{\rm Tr}\left[\sqrt{-\det\left[g_{\mu\nu} + w(\phi)(F_L)_{\mu\nu}\right]} + (L \leftrightarrow R)\right]$
2. $S_{\rm YM} = M_{\rm p}^3 N_c \int Z(\phi)\,{\rm Tr}\left[F_L^2 + F_R^2\right]$

Generic holographic approach: actions

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2. $$S_{\text{YM}} = M_{\text{p}}^3 N_c \int Z(\phi)\,\text{Tr}\left[F_L^2 + F_R^2\right]$$

- Background gauge fields sourced by $\mu_B \Rightarrow$ at small density, $F_{L/R}$ small $\Rightarrow$ DBI and YM reduce to the same choice
- Potentials (V_g, V_f, w or V_g, Z) to be fixed by QCD data

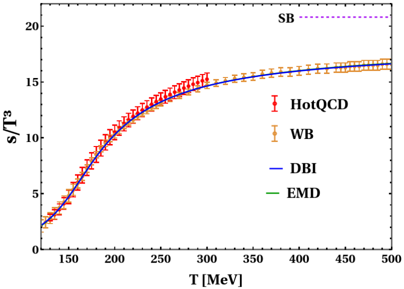
Fitting the potentials to data

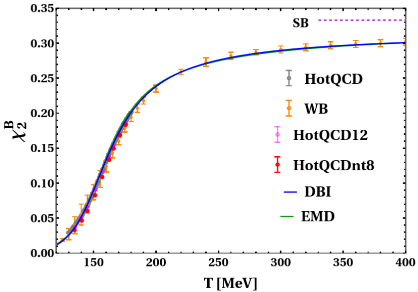
Potentials determined by comparison to lattice data

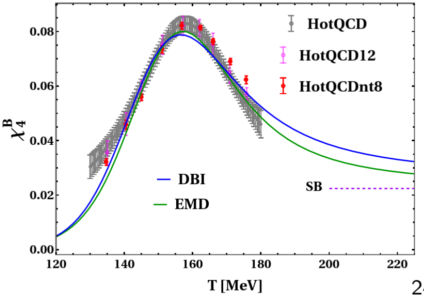
- Data for Yang-Mills (V_g)
- Data for full QCD (other potentials):
  equation of state, $\chi_2^B = \frac{d^2p}{d\mu_B^2}\Big|_{\mu_B=0}$ ...

In case of DBI action we use two approaches

1. With confinement and phase transition (V-QCD)
2. Without confinement, direct fit to data







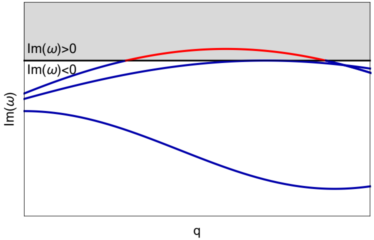
Outline

Inhomogeneity in holographic plasma?

Spatially modulated instability

[Nakamura, Ooguri, Park 0911.0679;
Ooguri, Park 1011.4144]

- Exponentially growing perturbation at $q \neq 0$: a quasi-normal mode with $\mathrm{Im}\,\omega > 0$
- The Chern-Simons term can drive such a modulated instability at finite density



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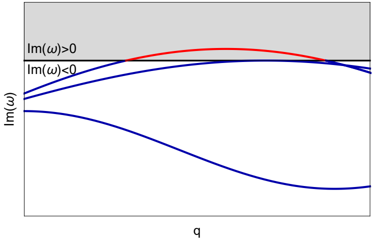

Schematic fluctuation equation

$$\psi''(r) + \left(A' + \frac{f'}{f}\right)\psi'(r) + \underbrace{\frac{qn}{M_p^3 f e^{2A} Z(\phi)^2}}_{\text{From CS term}}\psi(r) + \left(\frac{\omega^2}{f^2} - \frac{q^2}{f}\right)\psi(r) = 0$$

$\psi = \delta A^x_{L/R} \pm i\delta A^y_{L/R}$ $\qquad$ $r =$ holographic coord.

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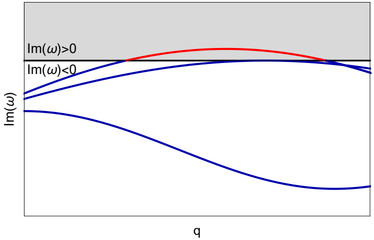

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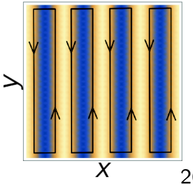
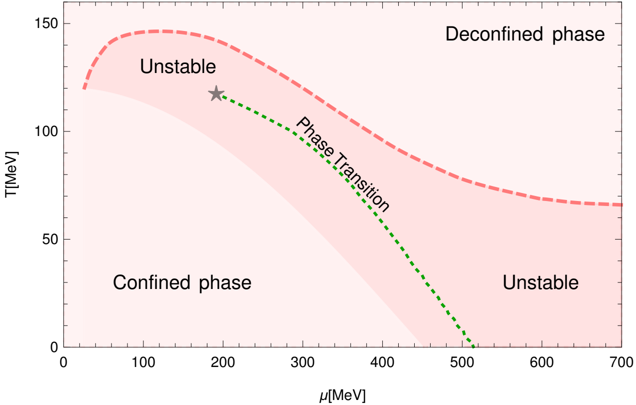


- Ground state: Modulated 5D gauge fields dual to modulated persistent chiral currents in field theory

Modulated instability in V-QCD

The region where instability exists in V-QCD

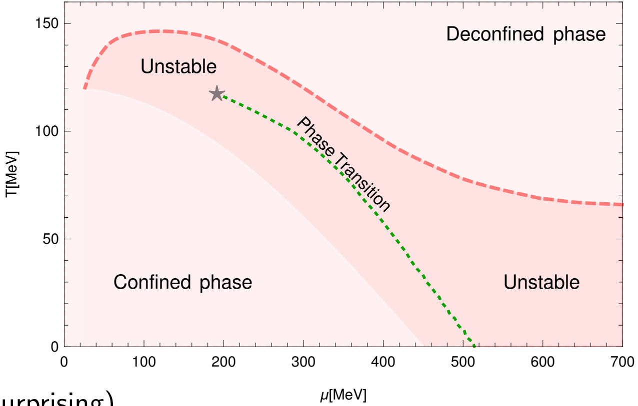
[Cruz Rojas, Demircik, MJ 2405.02399]



Modulated instability in V-QCD

The region where instability exists in V-QCD

[Cruz Rojas, Demircik, MJ 2405.02399]



- The Chern-Simons term is strong enough to create an instability in V-QCD (unsurprising)
- Instability is found at low T and large density (expected)

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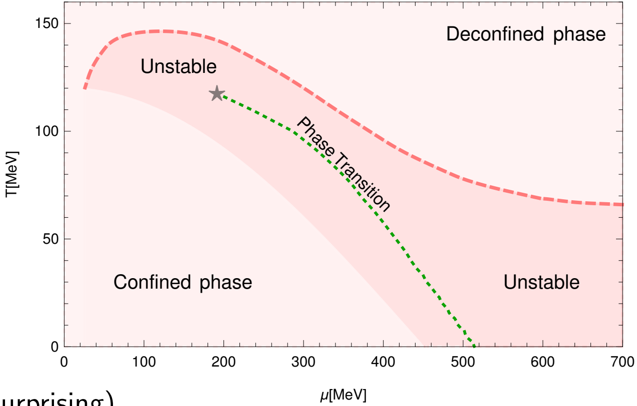

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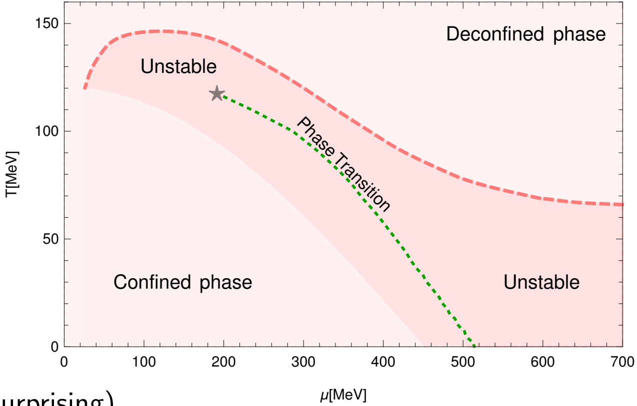

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- Instability is found at low T and large density (expected)
- Instability is also found at higher T, near the regime with critical point?! (a big surprise)
- Estimate for transition and critical point from earlier work

[Demircik, Ecker, MJ 2112.12157]

Model dependence: fitting uncertainty

Low-density instability would be phenomenologically highly interesting and potentially testable

- There may be caveats and uncertainties (choices in fitting the data, model dependence and reliability. . .)
- However, at low densities, expect that models strictly fixed by lattice data
- Important to check this! [Demircik, Jokela, MJ, Piispa 2405.02392]

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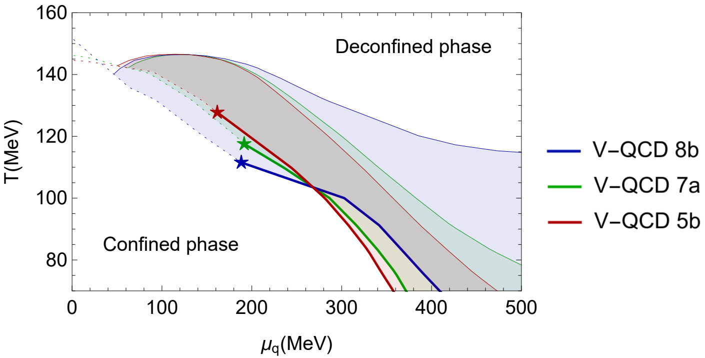
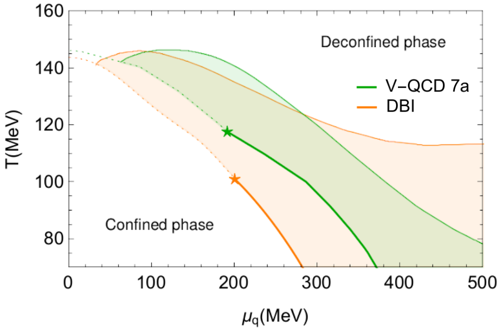
Parameter dependence in V-QCD: rather weak

- Onset of instability solidly determined by lattice fit



Model dependence: other checks

Vary fitting strategy



- Only minor changes

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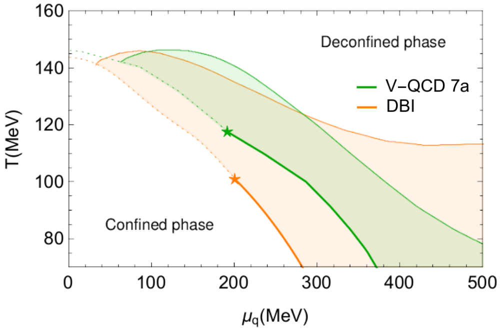

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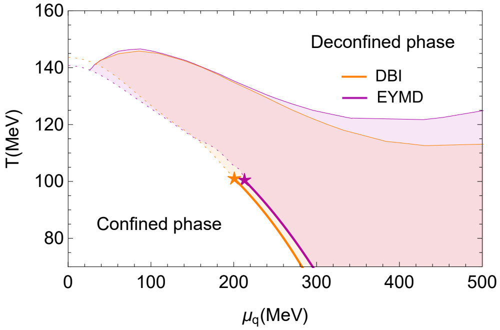

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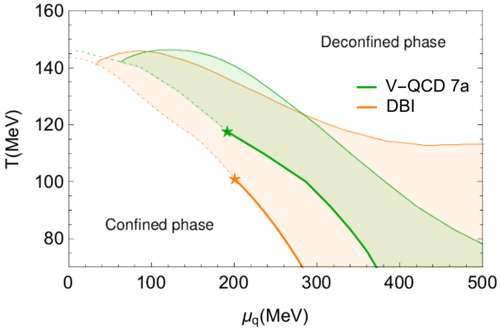

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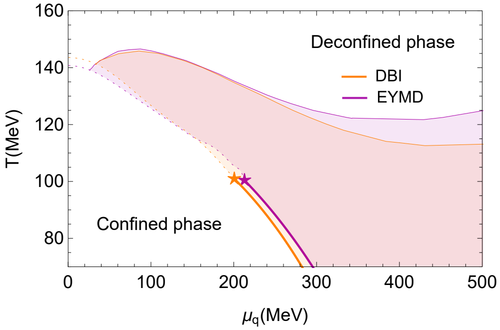
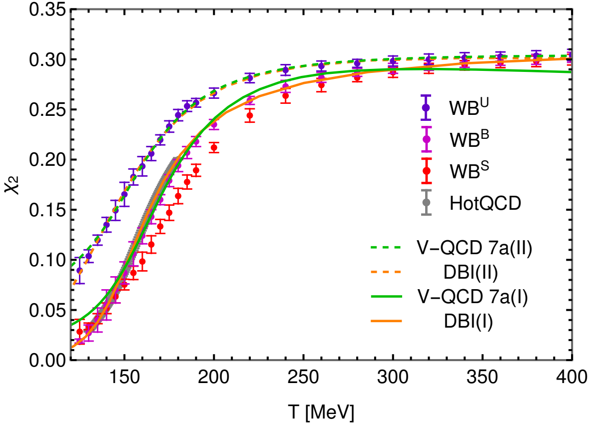


- Only minor changes – in particular, DBI and Yang-Mills actions give essentially identical results
- This means that the instability appears in a wide class of models in the literature

Model dependence: strange quark mass

- Instability potentially sensitive to fit to $\chi_2 = \frac{d^2 p}{d\mu^2}\Big|_{\mu=0}$
- Lattice data shows mild flavor dependence [Borsanyi et al. 1112.4416]
- Naive test: fit instead of the full χ_2 the light quark χ_2 (dashed curves) of the $N_f = 2+1$ lattice result $\Rightarrow$ isolate the instability in the light quark sector



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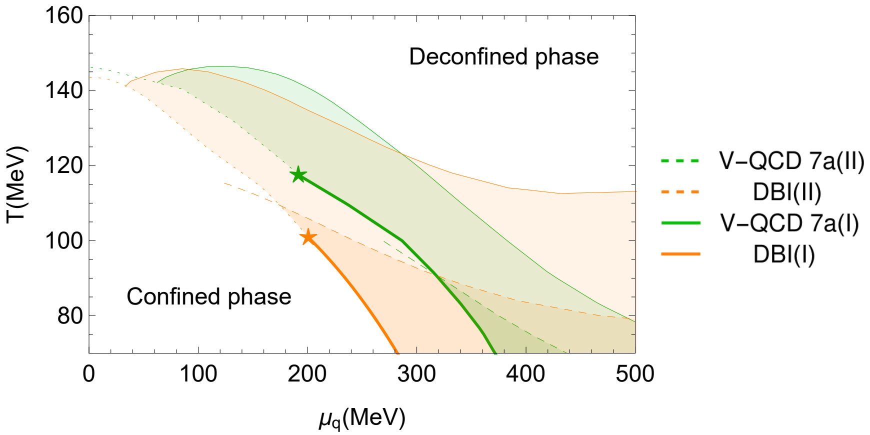

- Rather strong suppression of the instability!
- However, not a consistent check due to strange quark effects in lattice data
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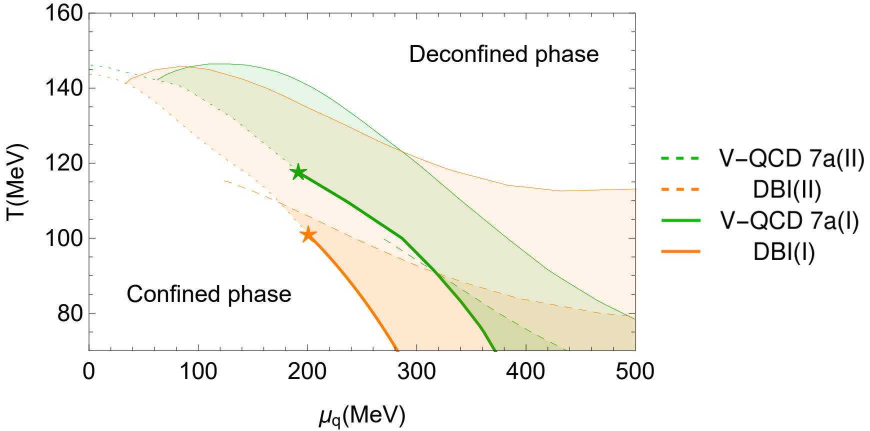

- Rather strong suppression of the instability!
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- Moreover, fit to strange quark χ_2 would instead enhance instability
- Therefore further careful study is required

Outline

1. Introduction
2. Holographic models
3. Spatial Instability
4. Conclusion

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- Dependence on fitting procedure and choice of flavor action small at low density – affects ALL models fitted to equation of state and χ_2^B
- Flavor effects, in particular dependence on strange quark mass, expected to be significant
- Next step, therefore: add separate flavors and strange quark mass – in progress with Toshali Mitra – fitting already done

Thank you!

Generic holographic approach: fitting strategies

Potentials are determined by comparing with lattice results for QCD thermodynamics.
Two main strategies:

Strategy I: Include confined phase, with $S_{\text{on-shell}} = \mathcal{O}(N_c^0)$, and the transition to a deconfined phase, with $S_{\text{on-shell}} = \mathcal{O}(N_c^2)$

- Used in Improved Holographic QCD and V-QCD models
  [Gürsoy, Kiritsis 0707.1324; Gürsoy, Kiritsis, Nitti 0707.1349; MJ, Kiritsis 1112.1261]
- Fit lattice data above $T = T_c$
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Strategy II: Only deconfined black holes: no phase transition at low density

- Fit lattice data at all temperatures
  [Gubser, Nellore, Pufu, Rocha 0804.1950; Gubser, Nellore 0804.0434; DeWolfe, Gubser, Rosen 1012.1864; . . .]
- Follows the behavior in the phase diagram of QCD (crossover at low density)

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We study both approaches

Fitting the models: setup

Solve numerically black hole geometries

$$ds^2 = e^{2A(r)} \left(\frac{1}{f(r)} dr^2 - f(r) dt^2 + d\vec{x}^2 \right)$$

with a horizon $f(r = r_h) = 0$ and a background gauge field

$$A_L^t(r) = A_R^t(r) = \Phi(r)\mathbb{I}$$

Black hole thermodynamics $\Rightarrow$ equation of state

$$T = \frac{1}{4\pi} |f'(r_h)| \qquad s = 4\pi M_p^2 N_c^2 e^{3A(r_h)}$$

Relation between quark number n and chemical potential (for YM action)

$$\mu = \Phi(r = 0) = n \int_0^{r_h} \frac{1}{e^A Z(\phi)}$$

Numerical expansion $\Rightarrow$ susceptibilities

$$\chi_k(T, \mu) = \frac{\partial^k p(T, \mu)}{\partial \mu^k} = \frac{\partial^{k-1} n(T, \mu)}{\partial \mu^{k-1}}$$

Constraining the potentials

In the UV ($\lambda \to 0$):

- UV expansions of potentials matched with perturbative QCD beta functions $\Rightarrow$ asymptotic freedom and logarithmic flow of the coupling and quark mass, as in QCD

[Gürsoy, Kiritsis 0707.1324; MJ, Kiritsis 1112.1261]

In the IR ($\lambda \to \infty$): various qualitative constraints

- Linear confinement, discrete glueball & meson spectrum, linear radial trajectories
- Existence of a "good" IR singularity
- Correct behavior at large quark masses
- Working potentials often string-inspired power-laws, multiplied by logarithmic corrections (i.e, first guesses usually work!)

[Gürsoy, Kiritsis, Nitti 0707.1349; MJ, Kiritsis 1112.1261; Arean, Iatrakis, MJ, Kiritsis 1309.2286, 1609.08922; MJ 1501.07272]

Final task: determine the potentials in the middle, $\lambda = \mathcal{O}(1)$

- Qualitative comparison to lattice/experimental data

Ansatz for potentials, $(x = 1)$

$$V_g(\lambda) = 12\left[1 + V_1\lambda + \frac{V_2\lambda^2}{1+\lambda/\lambda_0} + V_{\rm IR}\, e^{-\lambda_0/\lambda}(\lambda/\lambda_0)^{4/3}\sqrt{\log(1+\lambda/\lambda_0)}\right]$$

$$V_{f0}(\lambda) = W_0 + W_1\lambda + \frac{W_2\lambda^2}{1+\lambda/\lambda_0} + W_{\rm IR}\, e^{-\lambda_0/\lambda}(\lambda/\lambda_0)^2$$

$$\frac{1}{w(\lambda)} = w_0\left[1 + \frac{w_1\lambda/\lambda_0}{1+\lambda/\lambda_0} + \bar{w}_0\, e^{-\lambda_0/\lambda w_s}\frac{(w_s\lambda/\lambda_0)^{4/3}}{\log(1+w_s\lambda/\lambda_0)}\right]$$

$$V_1 = \frac{11}{27\pi^2}\ , \qquad V_2 = \frac{4619}{46656\pi^4}$$

$$W_1 = \frac{8 + 3\,W_0}{9\pi^2}\ ; \qquad W_2 = \frac{6488 + 999\,W_0}{15552\pi^4}$$

Fixed UV/IR asymptotics $\Rightarrow$ fit parameters only affect details in the middle

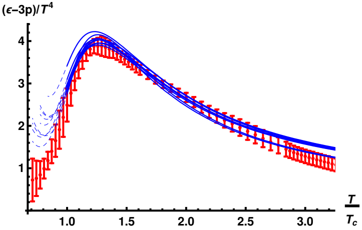
Fitting example: V-QCD (strategy I)

Fit to lattice data near $\mu = 0$ with DBI action and fitting strategy I (with transition): the V-QCD model (in the chirally symmetric phase) [MJ, Jokela, Remes, 1809.07770]

- Choose suitable Ansätze for the potentials, many parameters
- Parameters adjusted "by hand"
- Good description of lattice data – nontrivial result!
- Flat direction in the fit $\Rightarrow$ a one-parameter family of models

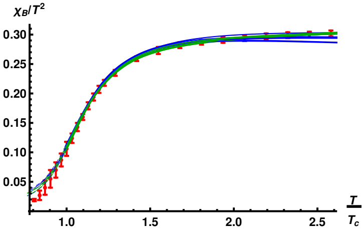
Interaction measure $\frac{\epsilon-3p}{T^4}$, 2+1 flavors

[Data: Borsanyi et al. 1309.5258]



Baryon number susceptibility $\chi_2 = \frac{d^2p}{d\mu^2}\Big|_{\mu=0}$

[Data: Borsanyi et al. 1112.4416]



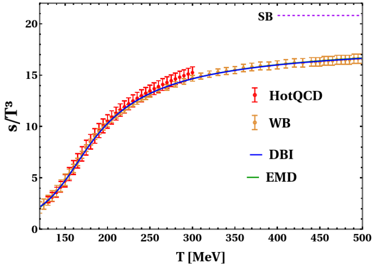
Fitting example: direct fit (strategy II)

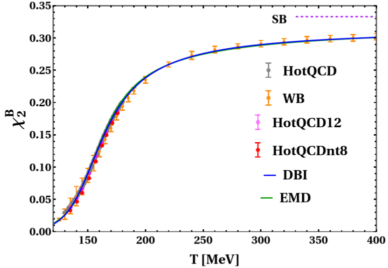
Use strategy II (no phase transition) with both DBI and YM [Jokela, MJ, Piispa 2405.02394]

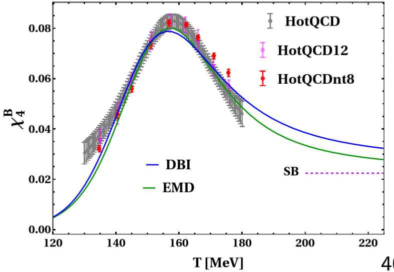
Systematic statistical fit to

1. Equation of state
   (through entropy density)
2. Cumulants χ_2 and χ_4

- (Here YM $\to$ EMD:
  for Abelian background,
  Yang-Mills=Maxwell)





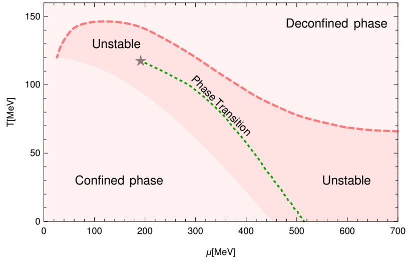


How does the instability arise?

Looks quite different from Nakamura-Ooguri-Park, where the onset was at fixed μ/T . . . what is going on?

- Also differs from result in Witten-Sakai-Sugimoto

[Ooguri, Park 1011.4144]



- Look at the fluctuation equation

$$\psi'' + \left(A' + \frac{f'}{f}\right)\psi' + \frac{qn}{M_p^3 f e^{2A} Z(\phi)^2}\psi + \left(\frac{\omega^2}{f^2} - \frac{q^2}{f}\right)\psi = 0$$

- Values of ϕ largest near horizon, and grow for smaller black holes
- Smallest black holes found near the deconfinement transition [Alho, MJ, Kajantie, Kiritsis, Rosen, Tuominen 1312.5199]
- $Z(\phi)$ determined by fit to χ_2: fast increase of χ_2 with T
  $\Rightarrow$ fast decrease of Z with ϕ
- Enhances instability strongly for small black holes