Is holographic quark-gluon plasma homogeneous?

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Holographic perspectives on chiral transport and spin dynamics

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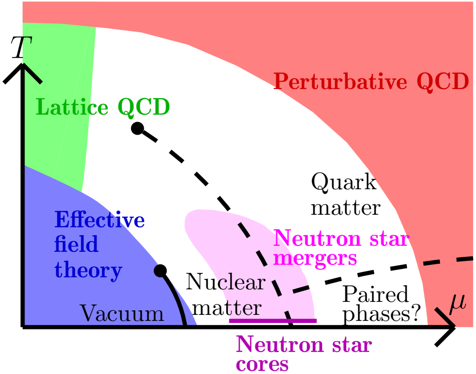
Research premise

Theoretical approaches

- First-principles methods do not work in the region relevant for critical point
- Phase diagram or even relevant phases not known
- Can be accessed via the gauge/gravity duality?
- Basic idea (bottom-up): use the gauge/gravity duality to extrapolate lattice (and other) data to higher density



[lots of work to cite here. . .]

- Likely include spatially modulated phases: effect e.g. on transport (see Matti's talk)
- This talk, main references:

[Demircik–NJ–Järvinen–Piispa 2405.02392; NJ–Järvinen–Piispa 2405.02394; CruzRojas–Demircik–Järvinen 2405.02399]

Generic holographic approach: fields

We want to describe holographically (chirally symmetric) QCD plasma (N_f massless flavors)

Most important (relevant and marginal) operators

- $T_{\mu\nu}$, dual to the metric $g_{\mu\nu}$
- Gluon operator $G^2_{\mu\nu}$, dual to a scalar (the dilaton) ϕ
- Flavor currents $\bar{\psi}_i \gamma_\mu (1 \pm \gamma_5) \psi_j$, dual to the gauge fields $(A^{L/R}_\mu)_{ij}$ (with $i, j = 1 \ldots N_f$) – global $\mathrm{U}(N_f)_L \times \mathrm{U}(N_f)_R$ of QCD promoted to gauge symmetry
- ~~Flavor bilinears $\bar{\psi}_i \psi_j$ dual to a complex scalar T_{ij}~~ – irrelevant in chirally symmetric phase

Chern-Simons (CS) terms in holography

Chiral anomalies in QCD: consider $\mathrm{U}(N_f)_L \times \mathrm{U}(N_f)_R$ coupled to external fields A_L, A_R

- Under gauge transformation with parameters $\Lambda_{L/R}$ [Fujikawa'79]

$$S_{\mathrm{QCD}} \mapsto S_{\mathrm{QCD}} + \frac{iN_c}{24\pi^2} \int \mathrm{Tr}\left[\Lambda_L F_L \wedge F_L - \Lambda_R F_R \wedge F_R + \ldots\right]$$

- 't Hooft: low energy EFT should have the same anomaly
- slide lifted from talk by S. Sugimoto:

- Original derivation of the WZW term

Witten: Nuclear Physics B223 (1983) 422-432

If (23) is obeyed, a gauge invariant generalization of Γ can be constructed somewhat tediously by trial and error. It is useful to define $U_{\nu\mathrm{L}} = (\partial_\nu U)U^{-1}$ and $U_{\nu\mathrm{R}} = U^{-1}\partial_\nu U$. The gauge invariant functional then turns out to be

$$\tilde{\Gamma}(A_\mu, U) = \Gamma(U) + \frac{1}{48\pi^2}\int \mathrm{d}^4x\, \varepsilon^{\mu\nu\alpha\beta} Z_{\mu\nu\alpha\beta},$$

where

$$\begin{aligned} Z_{\mu\nu\alpha\beta} = & -\mathrm{Tr}\left[A_{\mu\mathrm{L}} U_{\nu\mathrm{L}} U_{\alpha\mathrm{L}} U_{\beta\mathrm{L}} + (\mathrm{L}\to\mathrm{R})\right] \\ & + i\,\mathrm{Tr}\left[\left[(\partial_\mu A_{\nu\mathrm{L}})A_{\alpha\mathrm{L}} + A_{\mu\mathrm{L}}(\partial_\nu A_{\alpha\mathrm{L}})\right]U_{\beta\mathrm{L}} + (\mathrm{L}\to\mathrm{R})\right] \\ & + i\,\mathrm{Tr}\left[(\partial_\mu A_{\nu\mathrm{R}})U^{-1}A_{\alpha\mathrm{L}}\,\partial_\beta U + A_{\mu\mathrm{L}}U^{-1}(\partial_\nu A_{\alpha\mathrm{R}})\,\partial_\beta U\right] \\ & - \tfrac{1}{2} i\,\mathrm{Tr}\left(A_{\mu\mathrm{L}} U_{\nu\mathrm{L}} A_{\alpha\mathrm{L}} U_{\beta\mathrm{L}} - (\mathrm{L}\to\mathrm{R})\right) \\ & + i\,\mathrm{Tr}\left[A_{\mu\mathrm{L}} U A_{\nu\mathrm{R}} U^{-1} U_{\alpha\mathrm{L}} U_{\beta\mathrm{L}} - A_{\mu\mathrm{R}} U^{-1} A_{\nu\mathrm{L}} U U_{\alpha\mathrm{R}} U_{\beta\mathrm{R}}\right] \\ & - \mathrm{Tr}\left[\left[(\partial_\mu A_{\nu\mathrm{R}})A_{\alpha\mathrm{R}} + A_{\mu\mathrm{R}}(\partial_\nu A_{\alpha\mathrm{R}})\right]U^{-1}A_{\beta\mathrm{L}}U \right. \end{aligned}$$

$\vdots$

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Holographic counterpart

- External fields promoted to 5D gauge fields

$$S_{\mathrm{CS}} = \frac{iN_c}{24\pi^2} \int \mathrm{Tr}\Big[- iA_L \wedge F_L \wedge F_L + \frac{1}{2} A_L \wedge A_L \wedge A_L \wedge F_L + \frac{i}{10} A_L \wedge A_L \wedge A_L \wedge A_L \wedge A_L - (L \leftrightarrow R) \Big]$$

- This term is not gauge invariant in the presence of a boundary
- Gauge variation at the boundary must agree with the anomaly: S_{CS} unique when chiral symmetry intact

[Witten hep-th/9802150]

Generic holographic approach: actions

We write down expected (two-derivative) terms

$$S = S_{\rm gr} + S_{\rm matter} + S_{\rm CS}$$

where $S_{\rm CS}$ is fixed by anomalies, and

$$S_{\rm gr} = M_{\rm p}^3 N_c^2 \int d^5x \sqrt{-\det g}\left[R - \frac{4}{3}(\partial\phi)^2 + V_{\rm g}(\phi)\right]$$

Choice of $S_{\rm matter}$ less obvious. Options: $S_{\rm matter} = S_{\rm DBI}$ or $S_{\rm matter} = S_{\rm YM}$, with

1. $$S_{\rm DBI} = M_{\rm p}^3 N_c \int V_{\rm f}(\phi)\,{\rm Tr}\left[\sqrt{-\det\left[g_{\mu\nu} + w(\phi)(F_L)_{\mu\nu}\right]} + (L \leftrightarrow R)\right]$$
2. $$S_{\rm YM} = M_{\rm p}^3 N_c \int Z(\phi)\,{\rm Tr}\left[F_L^2 + F_R^2\right]$$

- Background gauge fields sourced by μ_B $\Rightarrow$ at small density, $F_{L/R}$ small $\Rightarrow$ DBI and YM reduce to the same choice
- Potentials (V_g, V_f, w or V_g, Z) to be fixed by QCD data

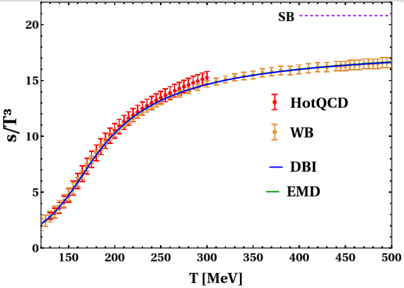
Fitting the potentials to data

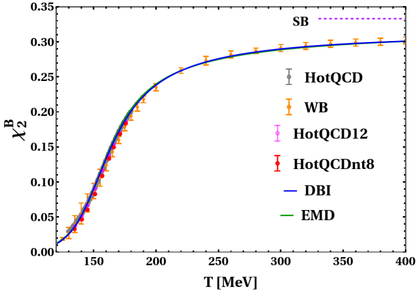
Potentials determined by comparison to lattice data

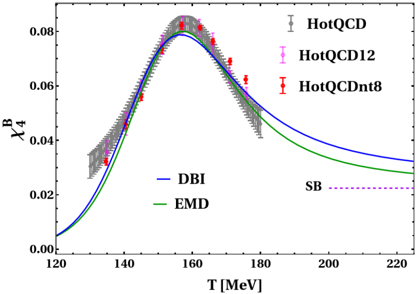
- Data for Yang-Mills (V_g)
- Data for full QCD (other potentials):
  equation of state, $\chi_2^B = \frac{d^2 p}{d\mu_B^2}\Big|_{\mu_B=0}$ ...

In case of DBI action we use two approaches

1. With confinement and phase transition (V-QCD)
2. Without confinement, direct fit to data



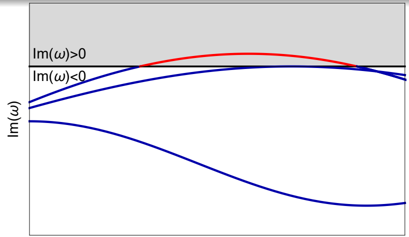


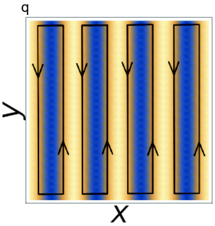


Spatially modulated phases

[(Nakamura)–Ooguri–Park 0911.0679,1011.4144,. . .]

- Exponentially growing perturbation at $q \neq 0$: a quasi-normal mode with $\mathrm{Im}\,\omega > 0$
- Chern–Simons term drives a modulated instability at finite density n
- Modulated 5D gauge fields dual to modulated persistent chiral currents $\bar{\psi}\gamma^y(1-\gamma_5)t^a\psi(x)$





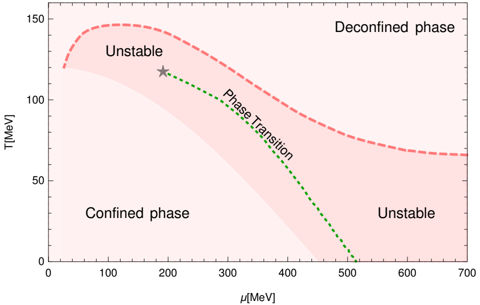
Schematic fluctuation equation

$$\delta\varphi''(r) + \left(A' + \frac{f'}{f}\right)\delta\varphi'(r) + \underbrace{\frac{q \times n}{M_p^3 f e^{2A} w(\phi)^2}}_{\text{From CS term}}\delta\varphi(r) + \left(\frac{\omega^2}{f^2} - \frac{q^2}{f}\right)\delta\varphi(r) = 0$$

$\delta\varphi = \delta A^y_{L/R} \pm i\delta A^z_{L/R}$ $\quad$ $r =$ holographic coord.

The region where instability exists

[CruzRojas–Demircik–Järvinen 2405.02399; Demircik–NJ–Järvinen–Piispa 2405.02392]



- Holographic QCD is unstable to forming inhomogeneous ground state:

  Unsurprising

- Instability is found at low T and large density – region relevant for neutron stars:

  Expected

- Instability is also found at higher T, near the regime with critical point:

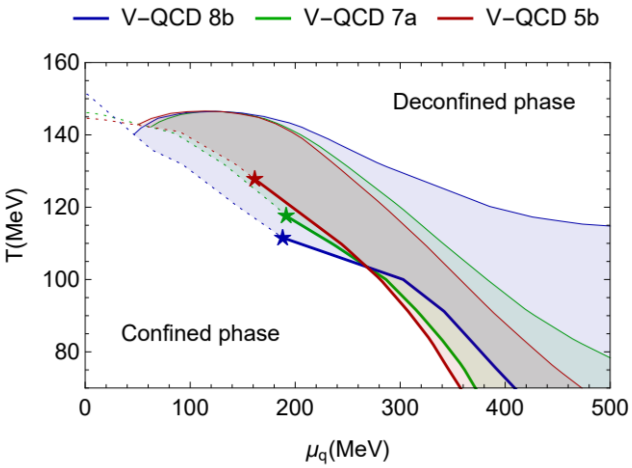
  A big surprise

Model dependence: strange quark mass

- Model dependence is really mild:

  [Demircik–NJ–Järvinen–Piispa 2405.02392]

  - varied model parameters ↔ freedom in fitting to lattice data



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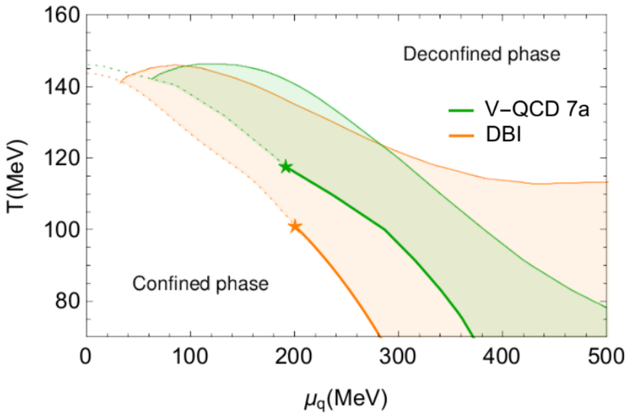

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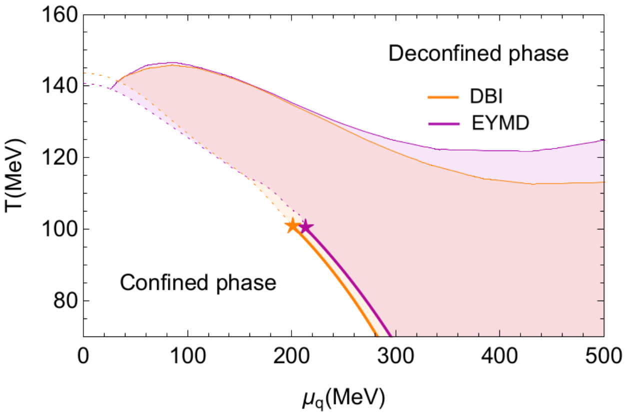

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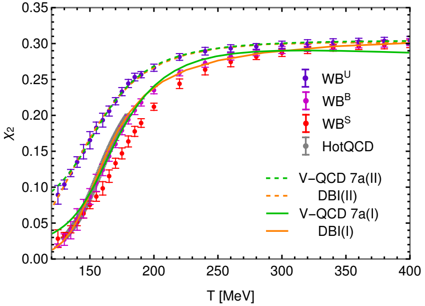
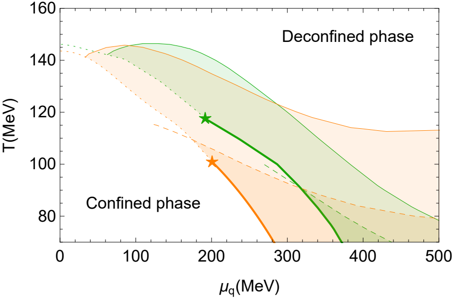
  - varied model parameters ↔ freedom in fitting to lattice data
  - varied fitting model w/ confinement and phase transition (V-QCD) ↔ without
  - varied the flavor action DBI ↔ Yang–Mills truncation
- All holographic massless QM models fitted to lattice data has instability at high-T
- Flavor dependence in susceptibilities, visible in lattice data? [Borsanyi et al. 1112.4416]
- Naive test: fit instead light quark χ_2 to the $N_f = 2 + 1$ lattice result





Conclusion and outlook

- ALL holographic bottom-up QCD models anchored to lattice data suffer from strong Nakamura–Ooguri–Park instability
  - Model dependence weak, so perhaps also a feature of real QCD?
- Appears at high density, region potentially reached in neutron star cores and neutron star mergers
- A surprise: also found at low density and high temperature, region reachable by lattice or experiments
- Flavor effects, in particular dependence on strange quark mass, expected to be significant

Grazie a tutti!