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Relaxed anomalous hydrodynamic

(an overview)

Holographic perspective on chiral transport and spin dynamics
ETC, Trento

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My group in Genoa…

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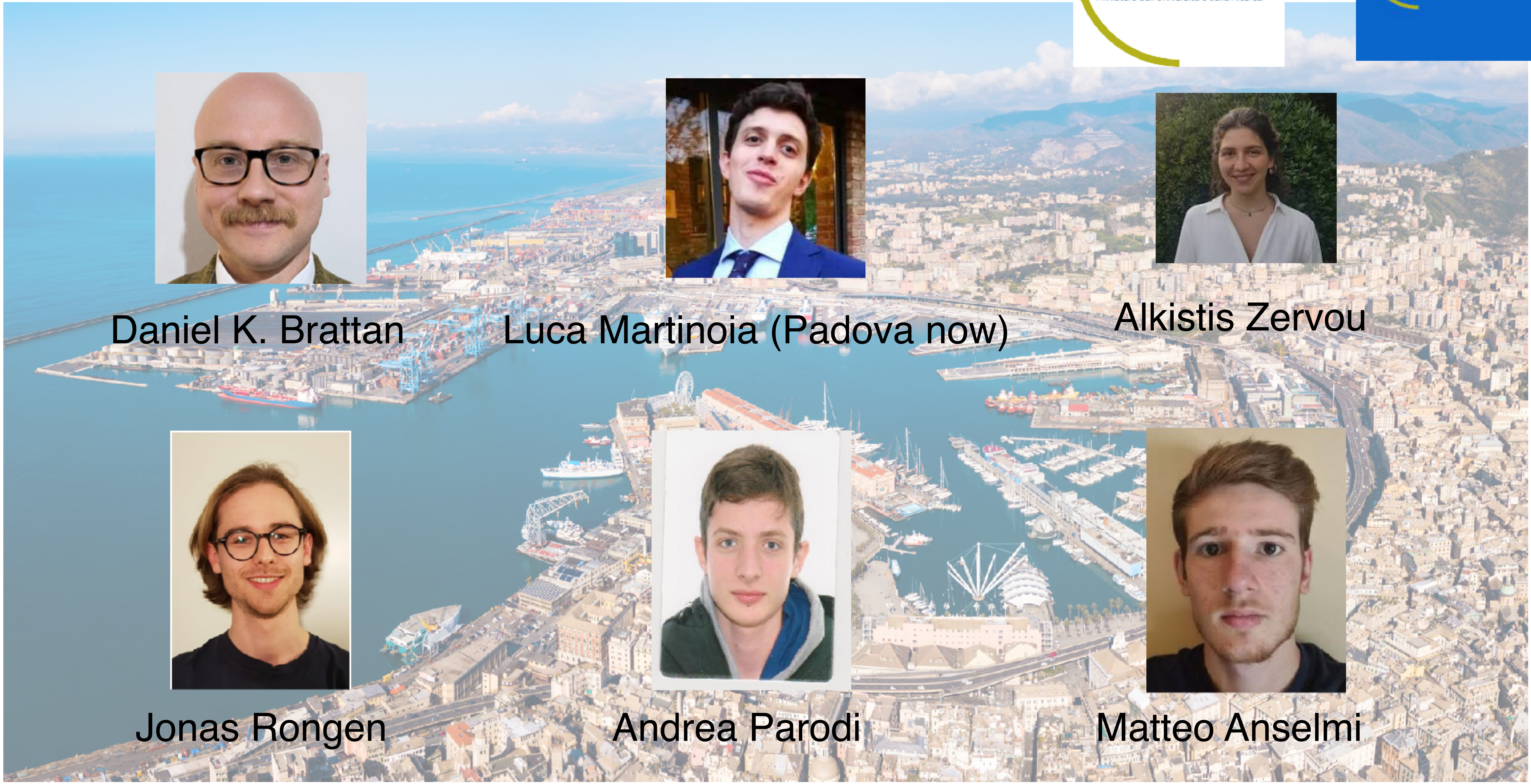

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Outline

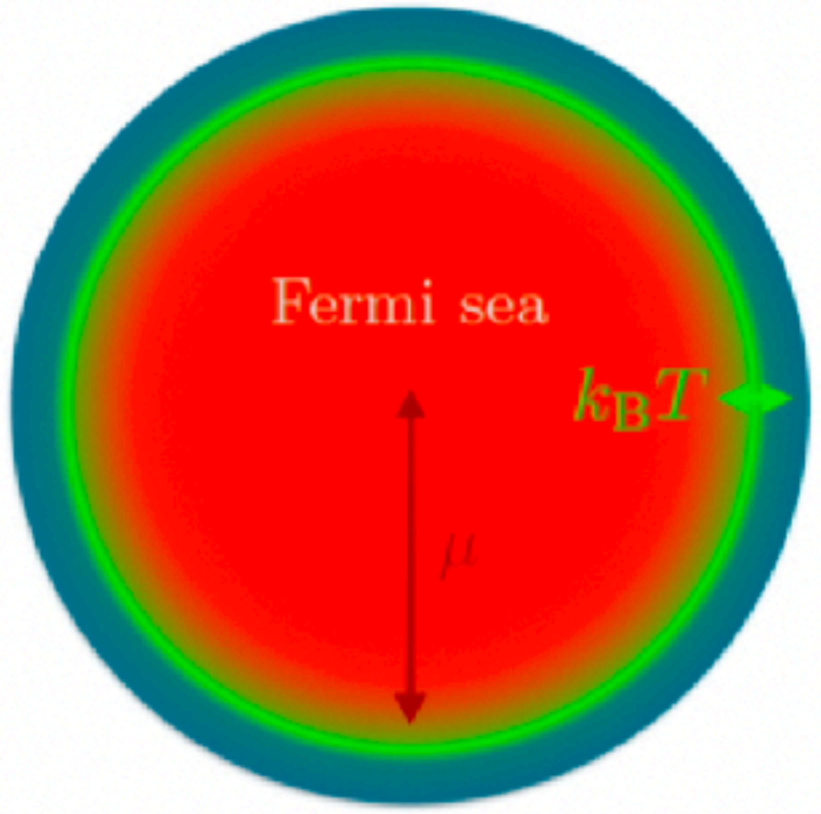
- Motivations and context
- Hydrodynamic setup
- Relaxations and DC transport
- Relaxations from kinetic theory

Context….

Fermi liquid: long-lived quasiparticles

$\tau_{ee} \gg \tau_{\text{imp}}, \tau_{e\gamma} \Rightarrow$ Wiedemann–Franz law

$$\frac{\kappa}{\sigma T} = L_0 = \frac{\pi^2}{3}$$

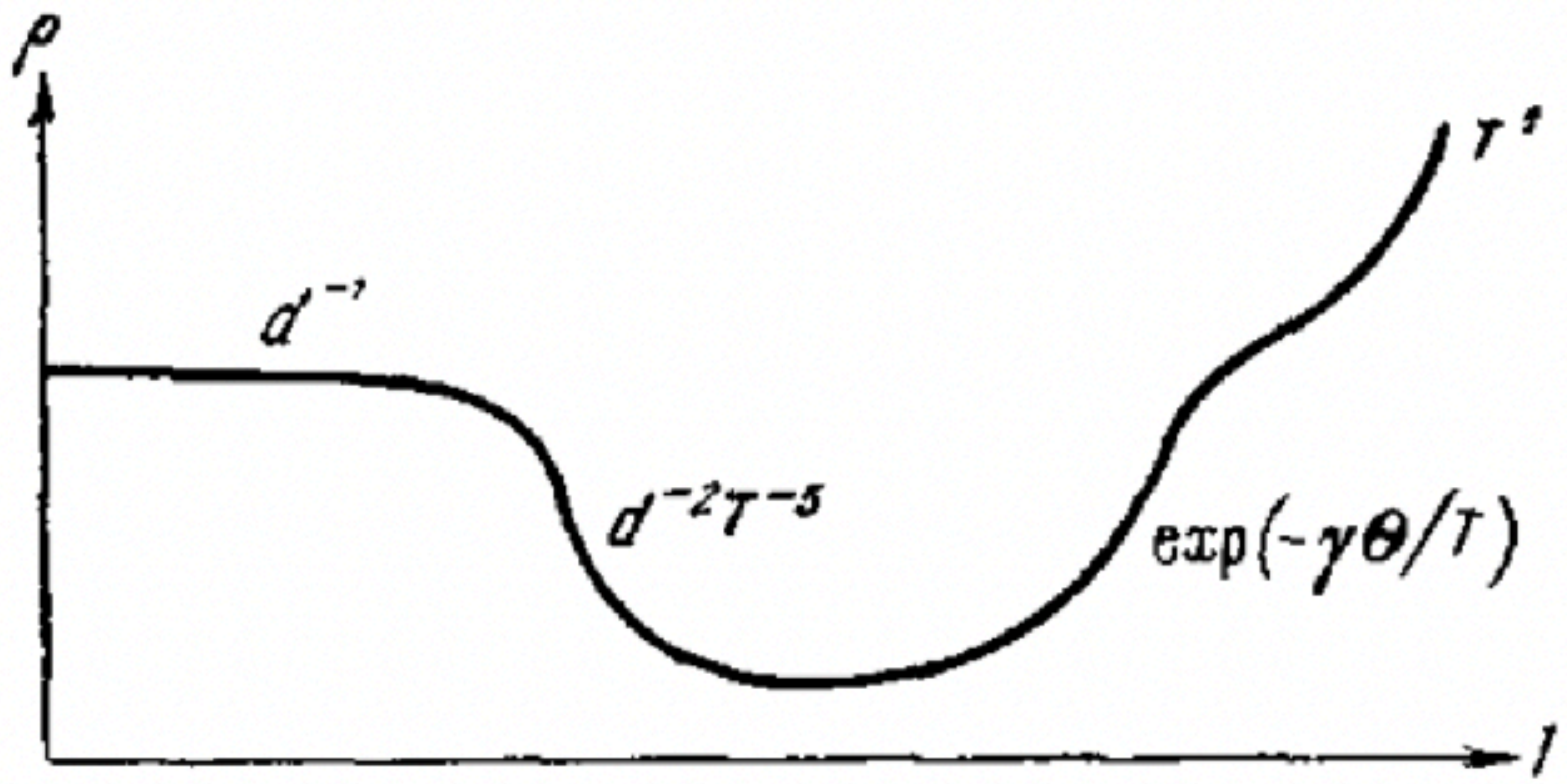


Clean, strongly-coupled materials

$\Rightarrow \quad \tau_{ee} \ll \tau_{\text{imp}}, \tau_{e\gamma}$ (no quasiparticles)

conserved momentum $\Rightarrow$ **emergent hydrodynamic transport**

[review, Narozhny (2022)].

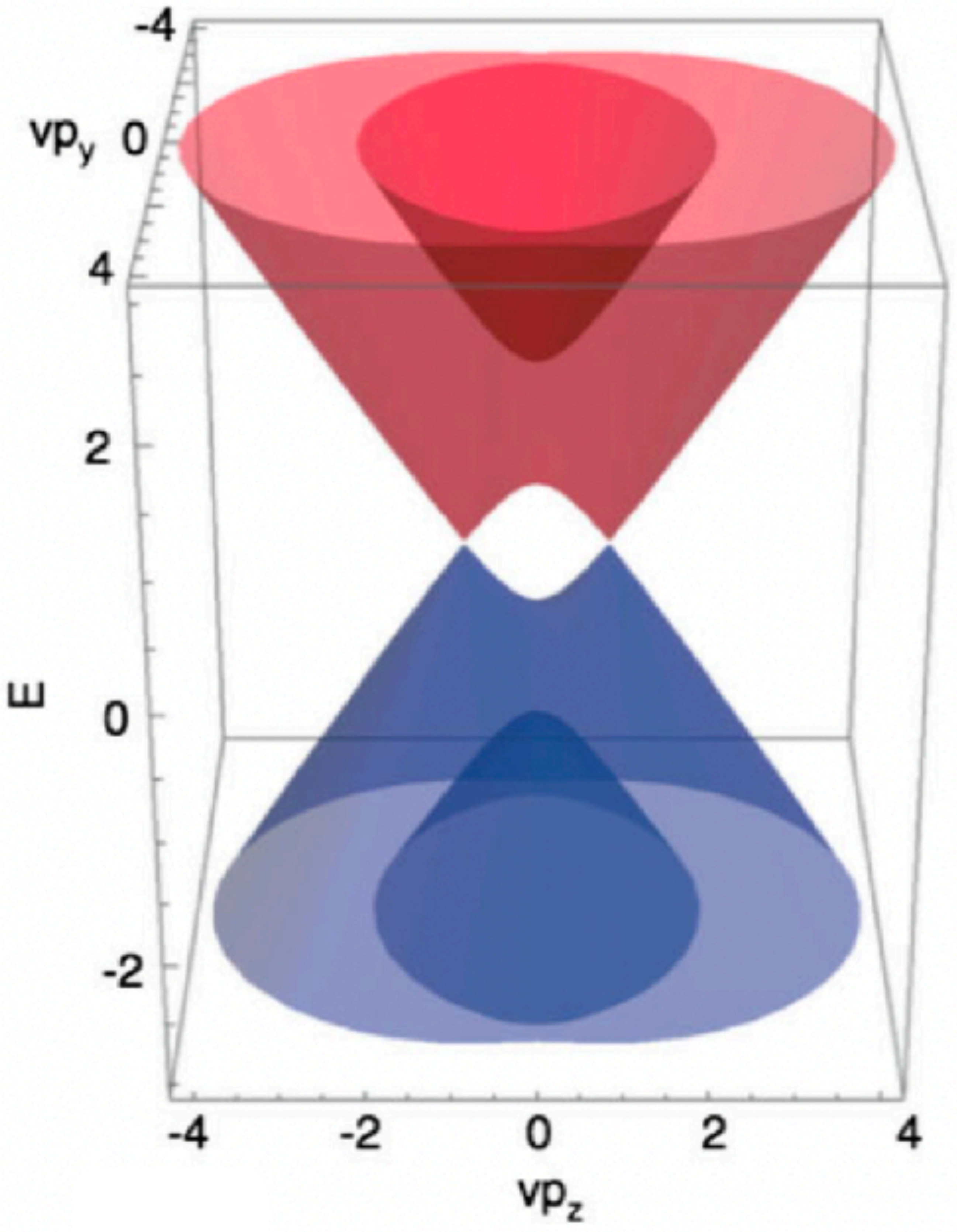


Features of transport:

- Gurzhi effect (minimum of resistivity),
- Breakdown of Wiedemann–Franz law,
- Non-local transport.

…and motivations

Typical band structure of Weyl semimetals [Armitage et al. (2018)]. Examples: NbP, TaAs, TaP, NbAs, WP_2.



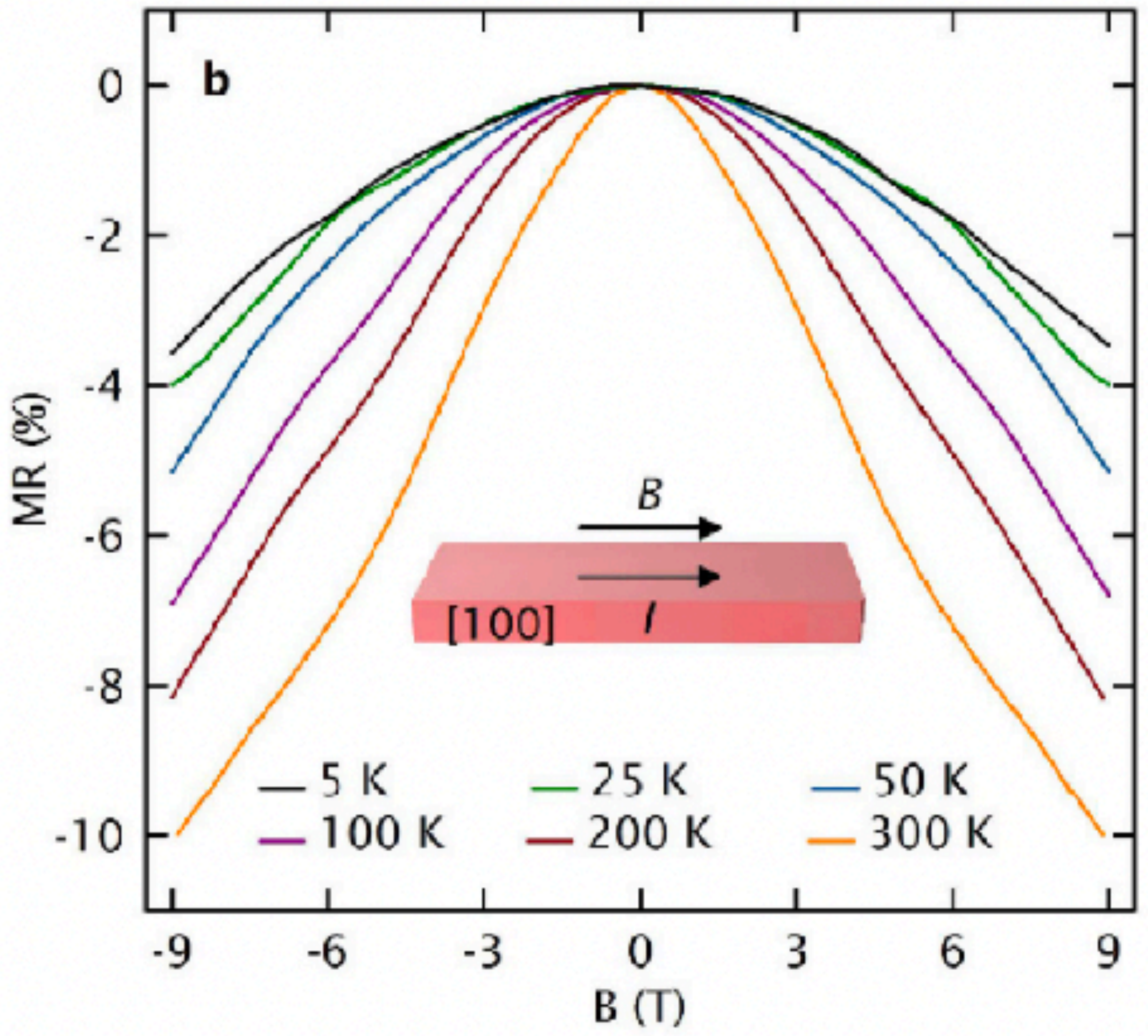
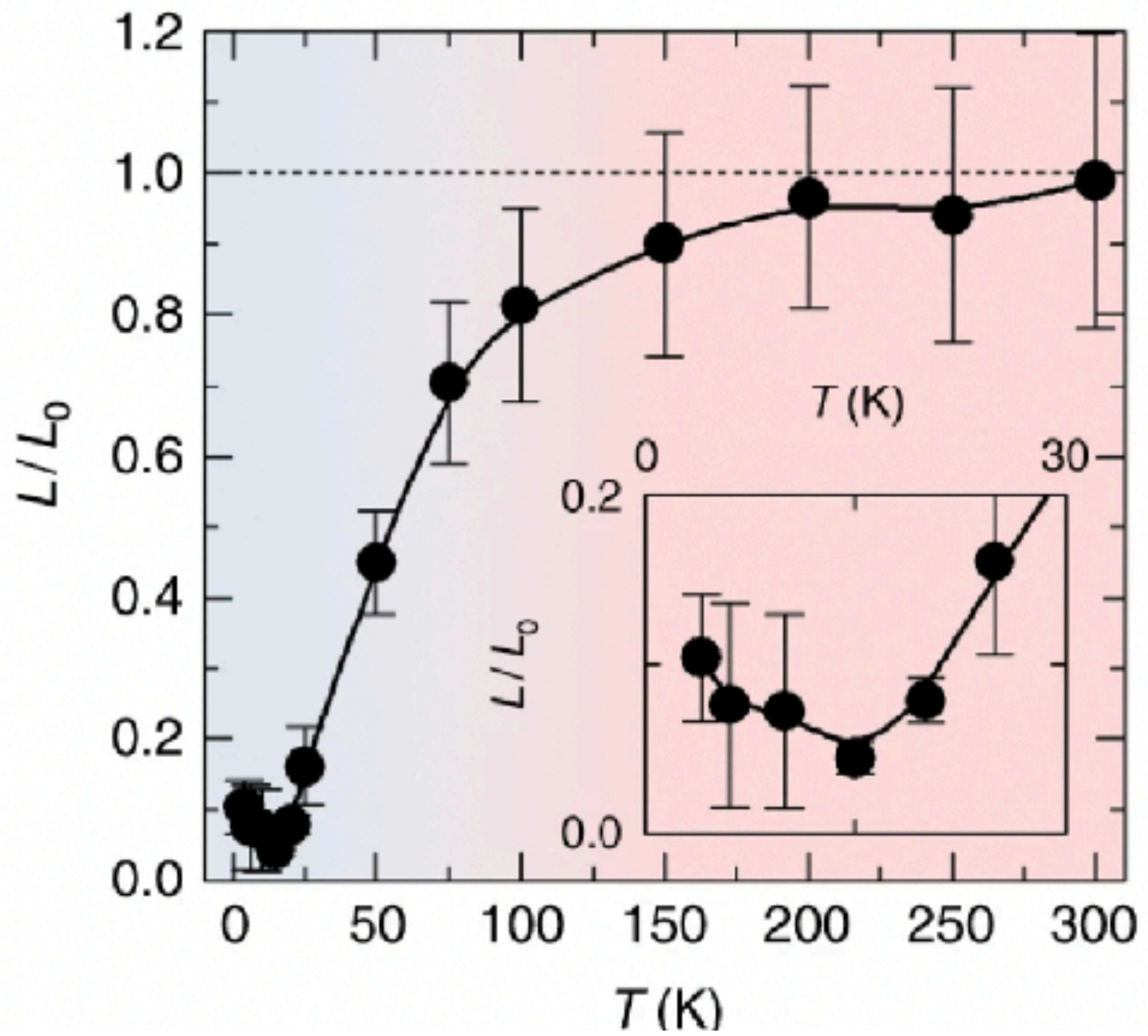


Figure: Negative longitudinal $(B \parallel E)$ magneto-resistance [Nielsen, Ninomiya (1981)]. $\sigma \propto B^2$ in NbP [Niemann et al. (2017)].

Figure: Breakdown of the Wiedemann-Franz law in WP_2 [Gooth et al. (2018)].



Setup

Conserved charges [Landsteiner et al., Lucas et al., Gorbar et al., Chernodub et al., . . .]

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \qquad \partial_\mu J^\mu = 0 \qquad \partial_\mu J_5^\mu = cE \cdot B$$

Constitutive relations:

- symmetries,
- derivative expansion,
- second law of thermodynamics $\partial_\mu S^\mu \geq 0$.

Relativistic hydrodynamics with $U(1)_V \times U(1)_A$ anomaly [Son, Surówka (2009)] and $B \sim \mathcal{O}(1)$

$$\begin{aligned} T^{\mu\nu} &= \varepsilon u^\mu u^\nu + P\Delta^{\mu\nu} + \xi^\varepsilon \left(u^\mu B^\nu + u^\nu B^\mu\right) + \mathcal{O}(\partial) \\ J^\mu &= n u^\mu + \xi B^\mu + \mathcal{O}(\partial) \\ J_5^\mu &= n_5 u^\mu + \xi_5 B^\mu + \mathcal{O}(\partial) \end{aligned}$$

with $\xi = c\mu_5$, $\xi_5 = c\mu$, and $\xi^\varepsilon = c\mu\mu_5$. Dissipative and hydrostatic terms are $\mathcal{O}(\partial)$.

Linear response and transport

Linear response theory [Martin, Kadanoff (1963)][1]:

$$\begin{pmatrix} \delta\mathbf{J} \\ \delta\mathbf{Q} \end{pmatrix} = \begin{pmatrix} \sigma(\omega) & \alpha(\omega) \\ T\bar{\alpha}(\omega) & \bar{\kappa}(\omega) \end{pmatrix} \begin{pmatrix} \delta\mathbf{E} \\ -\boldsymbol{\nabla}\delta T \end{pmatrix}$$

Compute *longitudinal DC transport* $\mathbf{E} \parallel \mathbf{B} \Rightarrow$ conductivities diverge as $\omega \to 0$.

We look for relaxations such that:

- conductivities are finite in DC,
- transport coefficients are Onsager-reciprocal $\alpha = \bar{\alpha}$,
- electric charge is conserved,
- (relaxations are independent).

DC transport I

Longitudinal DC conductivities are infinite ⇒ add momentum, energy and charge relaxations [Landsteiner et al. (2014), Abbasi et al. (2016)]

$$\partial_\mu \delta T^{\mu 0} = \delta(F^{0\lambda} J_\lambda) - \frac{\delta T^{00}}{\tau_{\varepsilon\varepsilon}}$$

$$\partial_\mu \delta T^{\mu i} = \delta(F^{i\lambda} J_\lambda) - \frac{\delta T^{0i}}{\tau_m}$$

$$\partial_\mu \delta J^{\mu} = -\frac{\delta J^0}{\tau_{nn}}$$

$$\partial_\mu \delta J_5^{\mu} = c\delta E \cdot B - \frac{\delta J_5^0}{\tau_{n_5 n_5}}$$

Onsager relations $\tau_{\varepsilon\varepsilon} = \tau_{nn} = \tau_{n_5 n_5} = \tau_m$ ⇒ unphysical solution.

DC transport II

First suggestion:
anomalous flow is *superfluid*-like [Sadofyev, Yin (2016), Stephanov, Yee (2016)] $\Rightarrow$ relax normal component only, e.g. $\delta J^0 = \delta n + c\mu_5 \mathbf{B}\cdot\delta\mathbf{v} \to \delta n$

$$\partial_\mu \delta T^{\mu 0} = \delta(F^{0\lambda} J_\lambda) - \frac{\delta\varepsilon}{\tau_{\varepsilon\varepsilon}}$$

$$\partial_\mu \delta T^{\mu i} = \delta(F^{i\lambda} J_\lambda) - \frac{\delta P^i}{\tau_m}$$

$$\partial_\mu \delta J^{\mu} = -\frac{\delta n}{\tau_{nn}}$$

$$\partial_\mu \delta J_5^{\mu} = c\delta E\cdot B - \frac{\delta n_5}{\tau_{n_5 n_5}}$$

Onsager relations $\tau_{\varepsilon\varepsilon} = \tau_{nn} = \tau_{n_5 n_5}$, while $\tau_m \geq 0$ is free $\Rightarrow$ still bad.

Generalized relaxations

$$\left.\begin{aligned}\partial_t\delta\varepsilon + \ldots &= -\frac{1}{\tau_{\varepsilon\varepsilon}}\delta\varepsilon - \frac{1}{\tau_{\varepsilon n}}\delta n - \frac{1}{\tau_{\varepsilon n_5}}\delta n_5\\ \partial_t\delta n + \ldots &= -\frac{1}{\tau_{n\varepsilon}}\delta\varepsilon - \frac{1}{\tau_{nn}}\delta n - \frac{1}{\tau_{nn_5}}\delta n_5\\ \partial_t\delta n_5 + \ldots &= -\frac{1}{\tau_{n_5\varepsilon}}\delta\varepsilon - \frac{1}{\tau_{n_5 n}}\delta n - \frac{1}{\tau_{n_5 n_5}}\delta n_5\end{aligned}\right\} = \hat{\tau}\cdot(\delta\varepsilon, \delta n, \delta n_5)$$

$$\partial_t\delta P^i + \cdots = -\frac{\delta v^i}{\tau_m}$$

Onsager relations imply $\hat{\chi}\cdot\hat{\tau} = \hat{\tau}^T\cdot\hat{\chi}^T$, explicitly

$$0 = \frac{\chi_{nn_5}}{\tau_{\varepsilon n_5}} + \frac{\chi_{nn}}{\tau_{\varepsilon n}} - \frac{\chi_{\varepsilon n_5}}{\tau_{nn_5}} + \frac{\chi_{\varepsilon n}}{\tau_{\varepsilon\varepsilon}} - \frac{\chi_{\varepsilon n}}{\tau_{nn}} - \frac{\chi_{\varepsilon\varepsilon}}{\tau_{n\varepsilon}} \qquad + \text{2 more}$$

Kinetic theory

Boltzmann equation (BE) for $f_{\mathbf{p}} = f(t, \mathbf{x}, \mathbf{p})$

$$\partial_t f_{\mathbf{p}} + \mathbf{p} \cdot \boldsymbol{\nabla} f_{\mathbf{p}} = I_{\text{coll}}[f_{\mathbf{p}}]$$

If $I_{\text{coll}} = I_{ee}$, then $I_{ee} = 0$ gives Detailed Balance $\Rightarrow$ Local Thermodynamic Equilibrium

$$f_{\mathbf{p}} = \frac{1}{1 + e^{(\varepsilon_{\mathbf{p}} - \mathbf{u} \cdot \mathbf{p} - \mu)/T}}$$

Integrate BE in momentum space against $\varepsilon_{\mathbf{p}}$, $\mathbf{p}$ and $1 \Rightarrow$ hydrodynamics

$$\int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} A \, I_{ee} = 0 \qquad \text{for} \qquad A = \{\varepsilon_{\mathbf{p}}, \mathbf{p}, 1\}$$

Collision integral

We take $I_{\text{coll}} = I_{ee} + I_{\text{imp}} + I_{e\gamma}$ such that

$$I_{\text{imp}} = \int d^3\mathbf{p}' W_{\mathbf{p}\to\mathbf{p}'} \left[f_{\mathbf{p}} - f_{\mathbf{p}'}\right] \delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}'}) \qquad \Rightarrow \qquad I_{\text{imp}} \approx -\frac{f_{\mathbf{p}} - f^{(0)}}{\tau_m}$$

and

$$I_{e\gamma} = \int d^3\mathbf{q} W_{\mathbf{p}',\mathbf{q}\to\mathbf{p}} \left[f_{\mathbf{p}'}(1 - f_{\mathbf{p}})n_{\mathbf{q}} - f_{\mathbf{p}}(1 - f_{\mathbf{p}'})(1 + n_{\mathbf{q}})\right] \delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}'} - \omega_{\mathbf{q}}) +$$
$$+ \int d^3\mathbf{q} W_{\mathbf{p}'\to\mathbf{p},\mathbf{q}} \left[f_{\mathbf{p}'}(1 - f_{\mathbf{p}})(1 + n_{\mathbf{q}}) - f_{\mathbf{p}}(1 - f_{\mathbf{p}'})n_{\mathbf{q}}\right] \delta(\varepsilon_{\mathbf{p}} + \omega_{\mathbf{k}} - \varepsilon_{\mathbf{p}'})$$

if phonons in thermal equilibrium

$$I_{e\gamma} \approx -\frac{f_{\mathbf{p}} - \bar{f}^{(0)}}{\tau_n} \qquad \text{with} \qquad \bar{f}^{(0)} = \frac{1}{1 + e^{(\varepsilon_p - \bar{\mu})/\bar{T}}}$$

Relaxation time approximation

Momentum relaxation: linearize [Gorbar et al. (2018)]

$$f_{\mathbf{p}} \approx f^{(0)} + (\mathbf{p}\cdot\mathbf{u})\frac{\partial f^{(0)}}{\partial \varepsilon_{\mathbf{p}}} \qquad \text{with} \qquad f^{(0)} = \frac{1}{1+e^{(\varepsilon_{\mathbf{p}}-\mu)/T}}$$

Considering $I_{\text{coll}} = I_{ee} + I_{\text{imp}}$ we have

$$I_{\text{imp}} \approx -\frac{f_{\mathbf{p}} - f^{(0)}}{\tau_m} \qquad \Rightarrow \qquad \partial_t P^i + \cdots = -\frac{P^i}{\tau_m}$$

Energy and charge relaxations: $I_{\text{coll}} = I_{ee} + I_{\text{imp}} + I_{e\gamma}$

$$I_{e\gamma} \approx -\frac{f_{\mathbf{p}} - \bar{f}^{(0)}}{\tau_n} \qquad \Rightarrow \qquad \begin{cases} \partial_t \varepsilon + \cdots = -\frac{\varepsilon - \bar{\varepsilon}}{\tau_n} \\ \partial_t n + \cdots + \cdots = -\frac{n-\bar{n}}{\tau_n} \end{cases}$$

Generalized relaxations from kinetic theory

Consider $\tau_n = \tau_n(\varepsilon_{\mathbf{p}})$ and expand

$$I_{e\gamma} = \sum_{j\geq -2} \varepsilon_{\mathbf{p}}^j \frac{f^{(0)} - \bar{f}^{(0)}}{\tau_{j+2}} = \frac{1}{\varepsilon_{\mathbf{p}}^2} \frac{f^{(0)} - \bar{f}^{(0)}}{\tau_0} + \frac{1}{\varepsilon_{\mathbf{p}}} \frac{f^{(0)} - \bar{f}^{(0)}}{\tau_1} + \frac{f^{(0)} - \bar{f}^{(0)}}{\tau_2} + \dots$$

Integrate

$$\partial_t \varepsilon + \dots = -\frac{M_1 - \bar{M}_1}{\tau_0} - \frac{n - \bar{n}}{\tau_1} - \frac{\varepsilon - \bar{\varepsilon}}{\tau_2} - \frac{M_3 - \bar{M}_3}{\tau_3} + \dots$$

$$\partial_t n + \dots = -\frac{M_0 - \bar{M}_0}{\tau_0} - \frac{M_1 - \bar{M}_1}{\tau_1} - \frac{n - \bar{n}}{\tau_2} - \frac{\varepsilon - \bar{\varepsilon}}{\tau_3} + \dots$$

Linearize and identify

$$\frac{1}{\tau_{nn}} = \frac{\partial M_0}{\partial n} \frac{1}{\tau_0} + \frac{\partial M_1}{\partial n} \frac{1}{\tau_1} + \frac{1}{\tau_2} + \dots \qquad \frac{1}{\tau_{n\varepsilon}} = \dots$$

Summary and outlook

- Hydrodynamic regime of Weyl semimetals $\Rightarrow$ anomalous relativistic two-components fluid.
- Longitudinal magneto-conductivities are divergent in DC $\Rightarrow$ need energy, momentum and axial charge relaxations.
- **Generalized relaxations** are necessary to satisfy fundamental considerations:
  - finite DC conductivities
  - Onsager relations
  - conservation of electric charge
- They can be justified from kinetic theory using energy-dependent RTA.
- **Entropy production** not positive definite.
- **Thermoelectric transport** not anomalous.
- BKG-like model to preserve charge conservation.

For the future:

- Explicit examples from microscopic physics?
- Other relaxations mechanisms?

Thank you