Finanțat de Uniunea Europeană
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Planul Național de Redresare și Reziliență

Chiral magnetic effect with time dependent magnetic fields in holography

Holographic perspectives on Chiral Transport and Spin dynamics

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25 March 2025

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[2503.10593]

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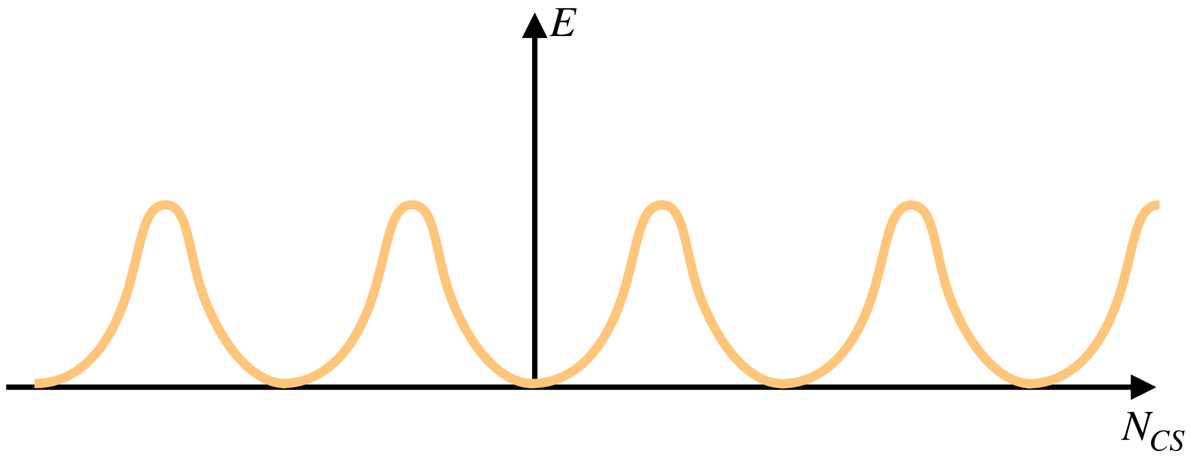
1. Chiral Magnetic Effect (CME)

- Generation of an electric current in the presence of a magnetic field and chiral imbalance.
- For a non-expanding abelian plasma $J_{CME} = 8\alpha\mu_5 B$
- The magnetic field is naturally present in the quark-gluon plasma (QGP), although large uncertainties in its dynamics.
- Chiral imbalance can be generated through the chiral anomaly

$$\partial_\mu J^\mu_A = \frac{1}{32\pi^2} Tr\{F\tilde{F}\} \quad \Rightarrow \quad \Delta n_5 \propto \Delta N_{CS}$$

[Kharzeev, McLerran, Warringa (2007)]

Interest:
Its measurement in heavy-ion collisions would serve as a evidence for the non-trivial topology of non-abelian gauge fields



2. CME in holography

Chern-Simons $U(1)_A \times U(1)_V$ model

$$S = \frac{1}{2\kappa^2}\int_{\mathcal{M}} d^5x\sqrt{-g}\left[R + \frac{12}{\ell^2} - \frac{1}{4}F^2 - \frac{1}{4}F_5^2 + \frac{\alpha}{3}\epsilon^{\mu\nu\rho\sigma\tau}A_\mu\left(3F_{\nu\rho}F_{\sigma\tau} + F^5_{\nu\rho}F^5_{\sigma\tau}\right)\right]$$

- It incorporates the vector and axial abelian symmetries, as well as the abelian contribution to the anomaly through the Chern-Simons term.
- It is conformal, so it does not describe the confined chirally-broken phase of QCD.
- The parameters of the model α and κ are matched to the entropy density and anomaly coefficient of 3 flavour QCD.

$$\alpha = \frac{6}{19}, \qquad \kappa^2 = \frac{24\pi^2}{19}.$$

- Non-conformal effects become more important as the temperature of the plasma is decreased.

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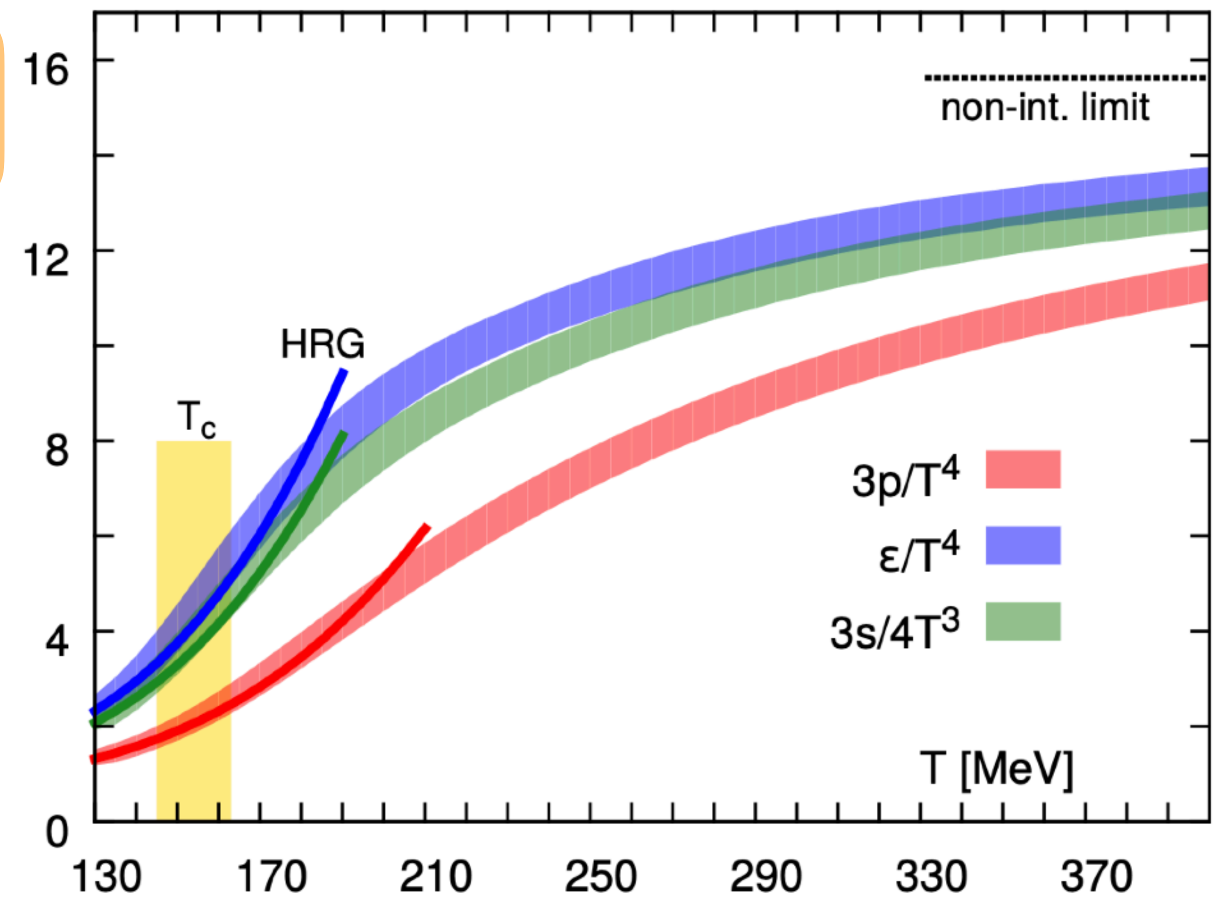

[Bazavov et al (2014)]

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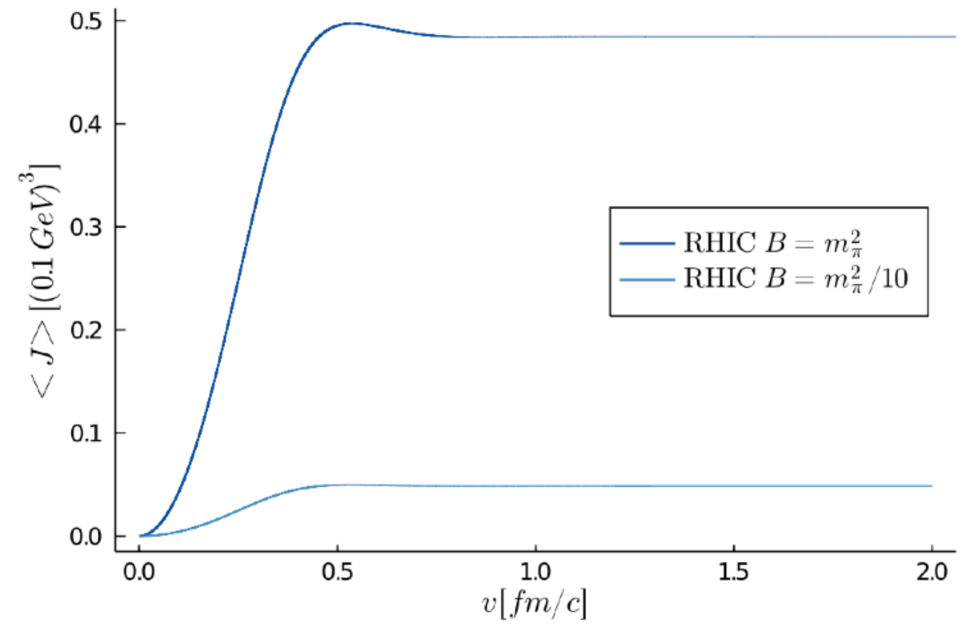
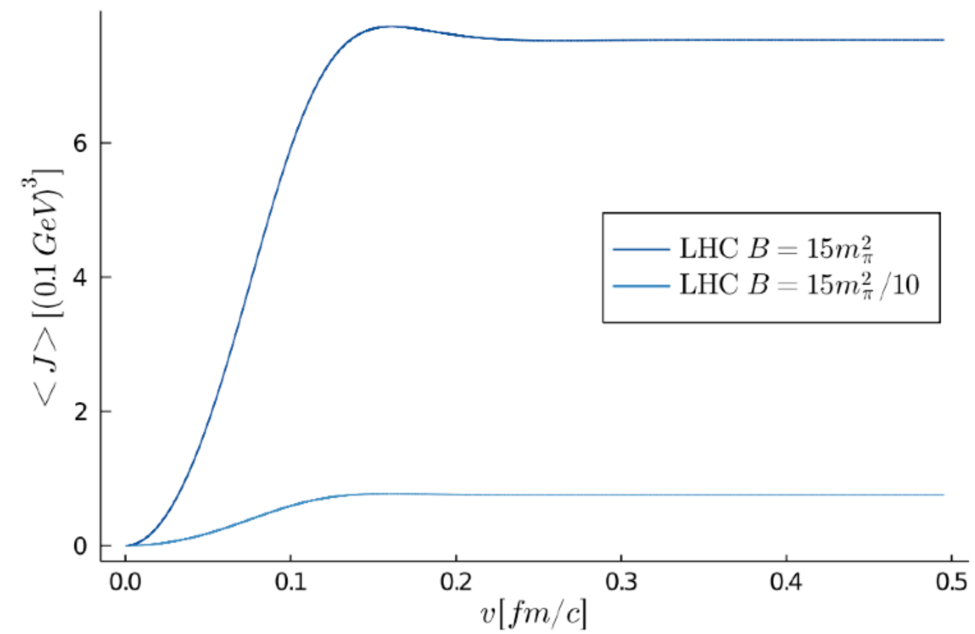
2. CME in holography

Previous results

- Out-of-equilibirum CME in a non-expanding plasma for constant magnetic field and axial charge. Concludes that lower energies are preferred (based on lifetime of magnetic field)

$$\tau_B(200\ \text{GeV}) = 0.6\ \text{fm/c} \quad \tau_B(5.2\ \text{TeV}) = 0.02\ \text{fm/c}$$





[Ghosh, Grieninger, SMT, Landsteiner (2021)]

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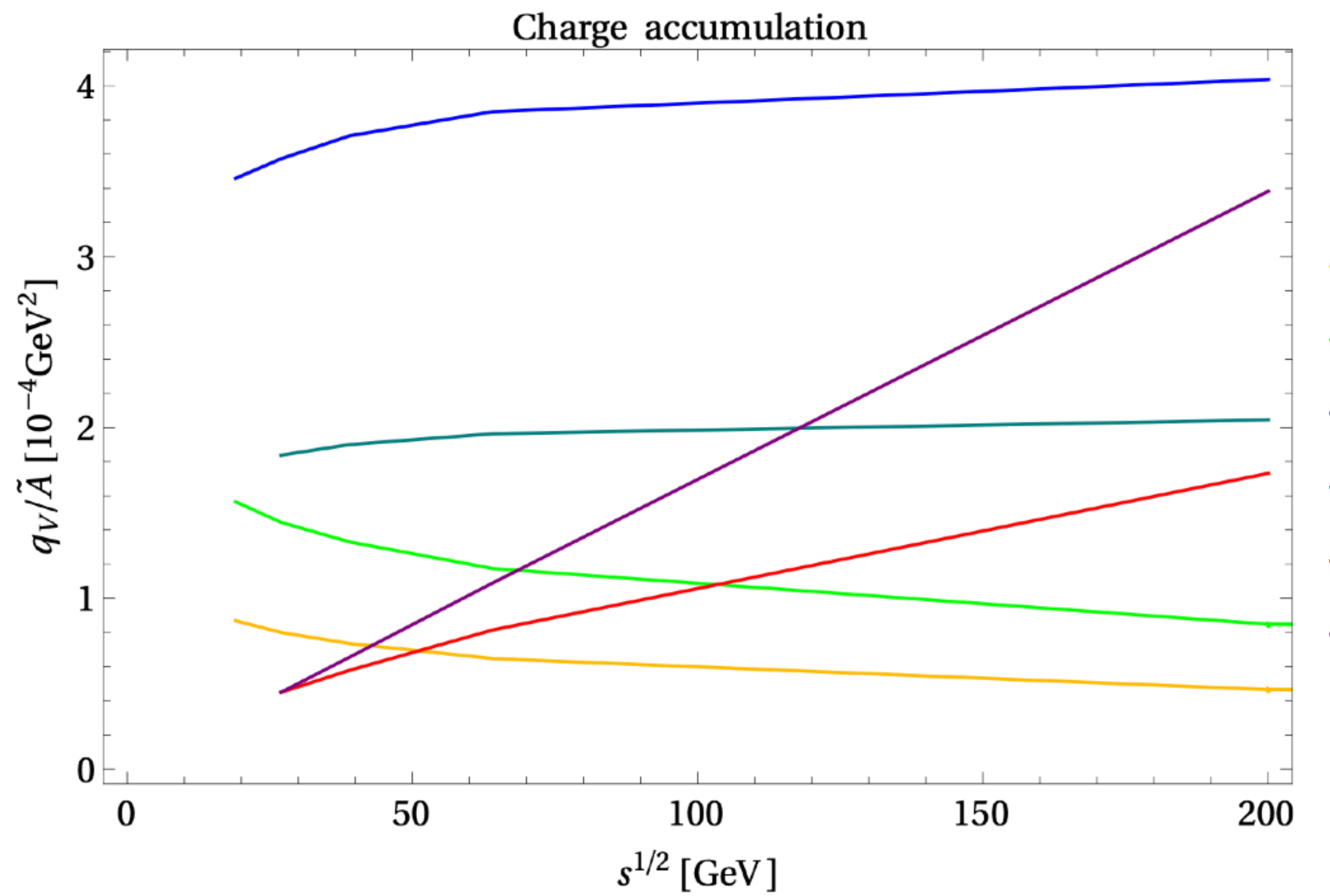
$\tau_B(200\text{ GeV}) = 0.6\text{ fm/c} \quad \tau_B(5.2\text{ TeV}) = 0.02\text{ fm/c}$

- Out-of-equilibrium CME for the same model in an expanding plasma. Obtains a larger CME signal for higher collision energies in 4 out of the 6 scenarios.



[Cartwright,Kaminski,Schenke (2021)]

2. CME in holography

"Weak" magnetic field

- The previous studies consider a full back-reacted geometry at "constant" magnetic field.

We extend the construction to consider time-dependent magnetic fields, under the assumption that it is sufficiently small.

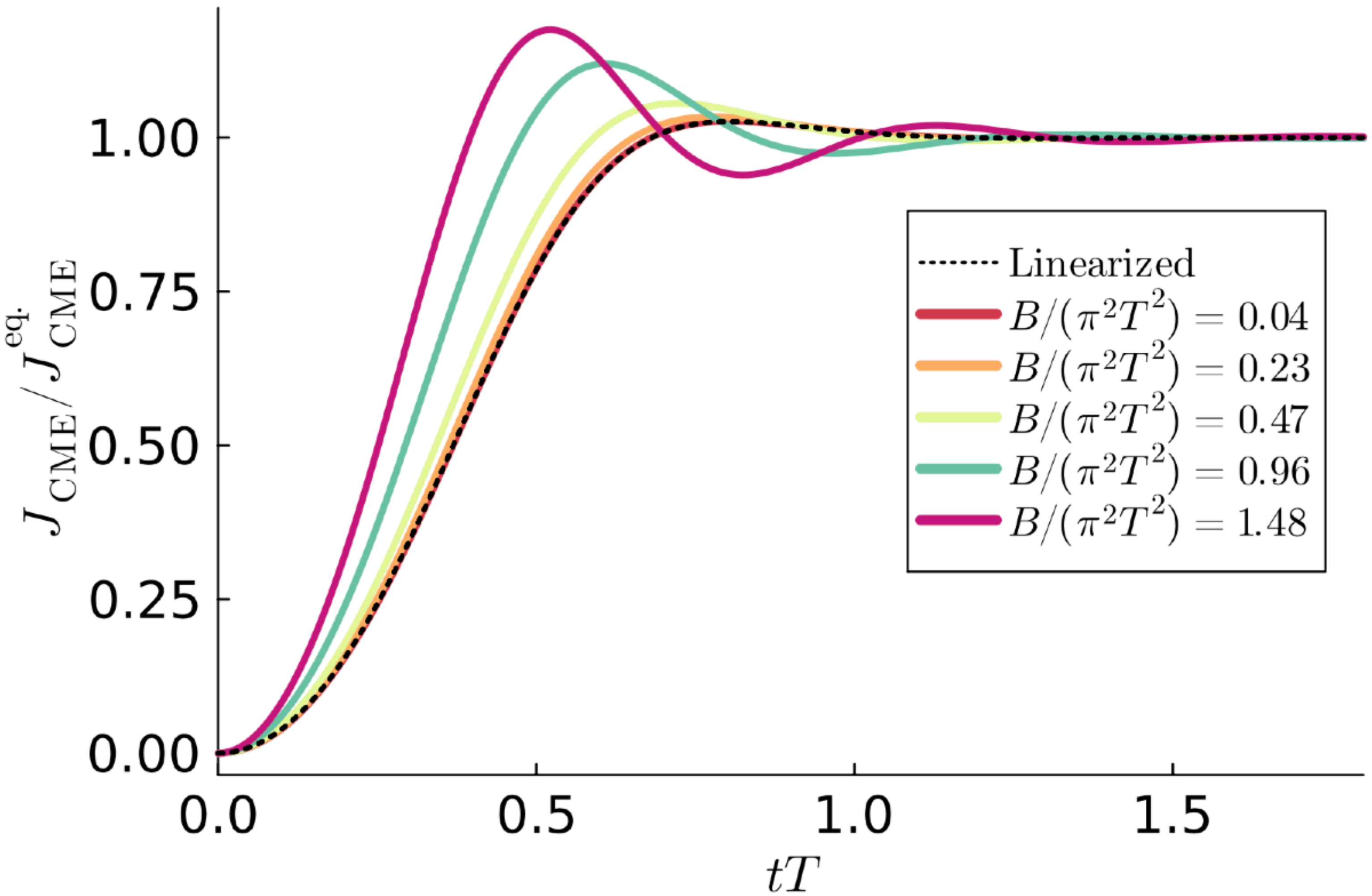
- The magnitude of the magnetic field at $\sqrt{s} = 200$ GeV is about $B_{max} \sim m_\pi^2$, while the *temperature* of the plasma is about $T \sim 300$ MeV. The ratio that controls a small magnetic field expansion is, in this case,

$$\frac{B_{max}}{(\pi T)^2} \sim 0.022$$

2. CME in holography

"Weak" magnetic field

Advantage:
Neglect transverse plane dynamics to leading order.



$\frac{B_{max}}{(\pi T)^2} \sim 0.02$

2. CME in holography

Expanding background

- We construct a background metric that is chiral imbalanced and at finite temperature. Our ansatz is

$$ds^2 = -fdv^2 - \frac{2}{u^2}dudv + \Sigma^2\left(e^{-2\xi}d\eta^2 + e^{\xi}dy^2 + e^{\xi}dz^2\right)$$

$$A = A_v dv \qquad V = \epsilon\ \eta V_\perp(v,u)dy + \epsilon\ V_z(v,u)dz$$

and we demand that the metric asymptotes to

$$ds^2 \to \frac{1}{u^2}\left(-d\tau^2 + \tau^2 d\eta^2 + dy^2 + dz^2\right)$$

- We follow **[Kalaydzhyan,Kirsch(2011)]** and construct the boost-invariant plasma with axial chemical potential at late-times. To this end, we define the scaling coordinate.

$$\zeta = \frac{u}{v^{1/3}}$$

2. CME in holography

Expanding background

- The leading order solution for the background is:

$$ds^2 = \frac{1}{u^2}\left[-\left(1 + \frac{g_4}{v^{4/3}}u^4 + \frac{1}{12}\frac{\tilde{q}_5^2}{v^2}u^6\right)dv^2 - 2du dv + \left(v^2 d\eta^2 + dy^2 + dz^2\right)\right] \qquad A_v = \frac{1}{2}\frac{\tilde{q}_5}{v}u^2$$

where g_4 and $\tilde{q}_5$ are integration constants controlling the energy density and axial charge of the solution.

- The resemblance to the RN solution is suggestive to the following definitions of effective temperature and chemical potentials:

$$T \equiv \frac{1}{\tau^{1/3}}\frac{1}{\pi\zeta_h}\left(1 - \frac{1}{24}\tilde{q}_5^2\zeta_h^6\right) \qquad \mu_5 \equiv \frac{\tilde{q}_5}{2\tau}u_h^2 = \frac{1}{2}\frac{\tilde{q}_5\zeta_h^2}{\tau^{1/3}}$$

$$\zeta_h = \frac{u_h(v)}{v^{1/3}} = \text{constant}$$

2. CME in holography

Expanding background

$$ds^2 = \frac{1}{u^2}\left[-\left(1 + \frac{g_4}{v^{4/3}}u^4 + \frac{1}{12}\frac{\tilde{q}_5^2}{v^2}u^6\right)dv^2 - 2dudv + \left(v^2 d\eta^2 + dy^2 + dz^2\right)\right] \qquad A_v = \frac{1}{2}\frac{\tilde{q}_5}{v}u^2$$

$$T \equiv \frac{1}{\tau^{1/3}}\frac{1}{\pi\zeta_h}$$

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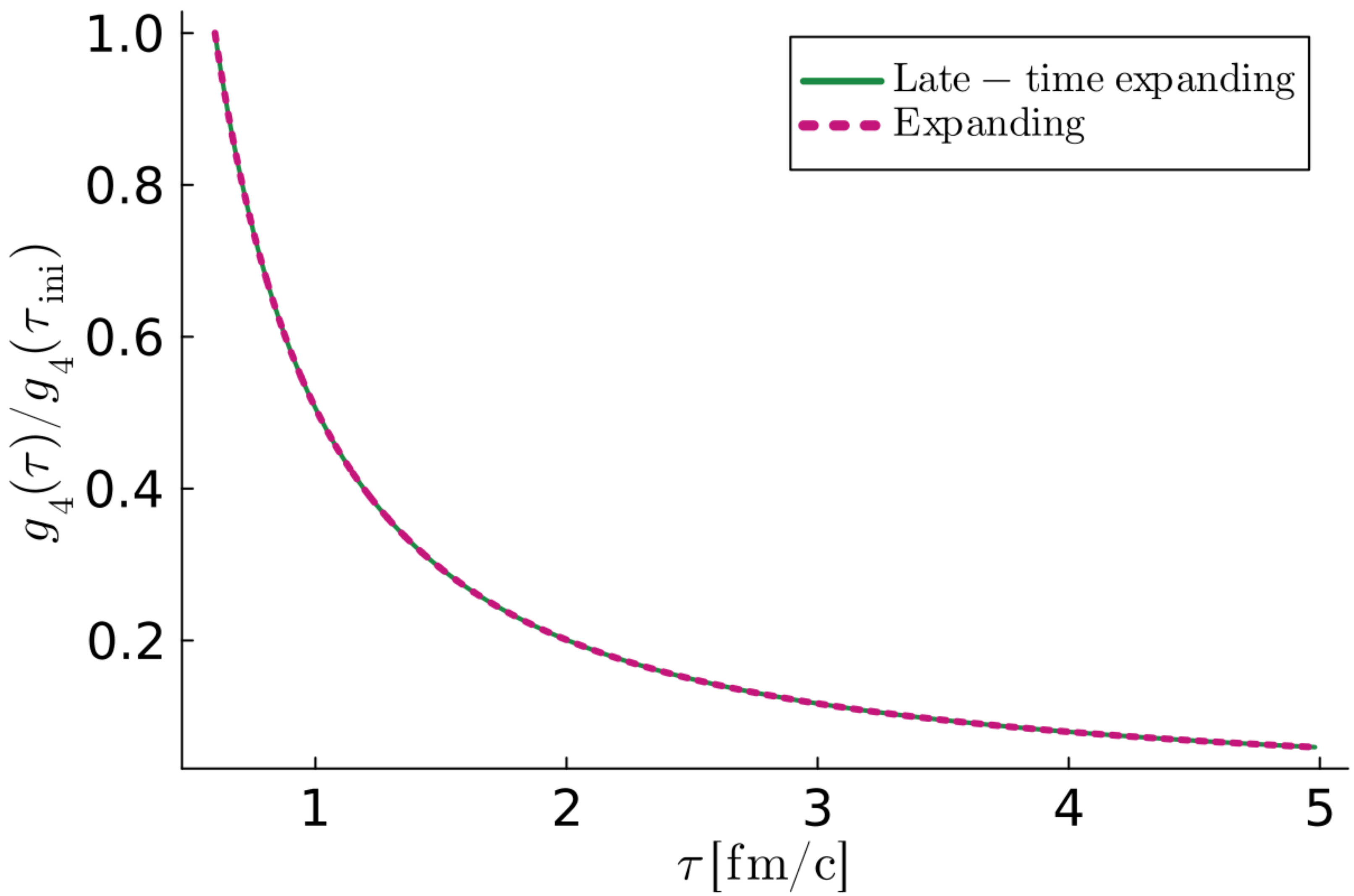

2. CME in holography

Expanding background

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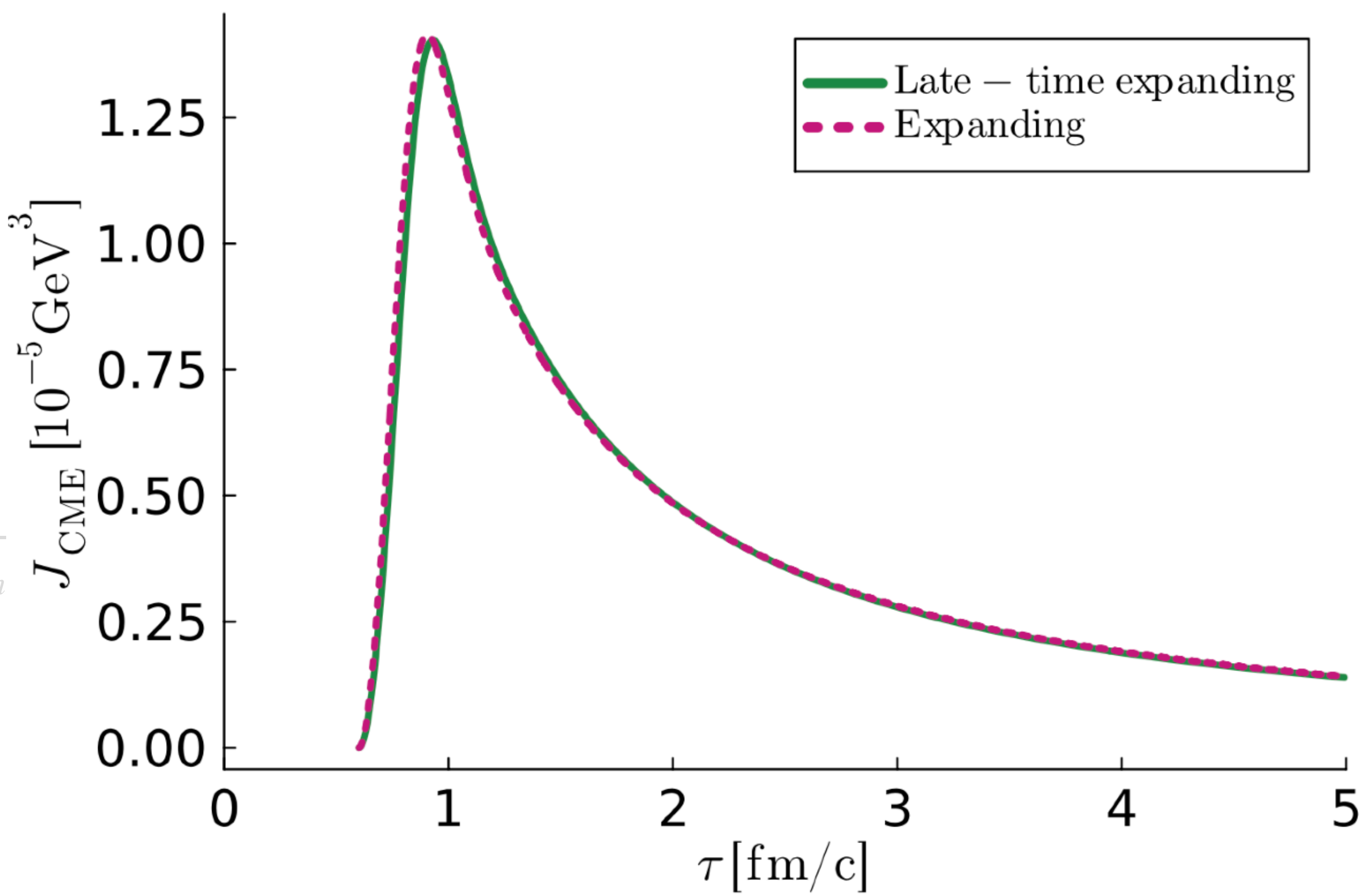

constant

2. CME in holography

Expanding background

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We solve the gauge-field e.o.m. and obtain the CME current, on top of this approximate expanding solution

$$T \equiv \frac{1}{\tau^{1/3}}\frac{1}{\pi\zeta_h}\left(1 - \frac{1}{24}\tilde{q}_5^2\zeta_h^6\right) \qquad \mu_5 \equiv \frac{\tilde{q}_5}{2\tau}u_h^2 = \frac{1}{2}\frac{\tilde{q}_5\zeta_h^2}{\tau^{1/3}}$$

$$\zeta_h = \frac{u_h(v)}{v^{1/3}} = \text{constant}$$

2. CME in holography

Late-time chiral magnetic effect

- Similarly, we can find the late time solution for the vector gauge fields. The ansatz $V = \epsilon\ \eta V_\perp(v,u)dy + \epsilon\ V_z(v,u)dz$ leads to

$$\left(\partial_u - \frac{1}{2u}\right) dV_\perp + \left(\frac{1}{2v} + \frac{u}{4}f\right) V'_\perp = 0$$

$$\left(\partial_u - \frac{1}{2u}\right) dV_z + \left(\frac{1}{2v} + \frac{u}{4}f\right) V'_z = -\frac{4\alpha\tilde{q}_5 u^2 V_\perp}{v^2}$$

$$f = \frac{1}{u^2}\left(1 + \frac{g_4}{v^{4/3}}u^4 + \frac{1}{12}\frac{\tilde{q}_5^2}{v^2}u^6\right)$$

- We change variables $(v,u) \to (v,\zeta)$ and expand the previous equations at late times.

$$V_\perp(v,u) = \tilde{V}_\perp(\zeta)b_z(v) + \dots \quad \Rightarrow \quad v^{2/3}b_z(v)\left[\partial_\zeta \tilde{V}_\perp(\zeta\partial_z - 1)G + \zeta G\partial_\zeta^2\tilde{V}_\perp\right] + v\dot{b}_z(v)\left(1 - 2\zeta\partial_\zeta\right)\tilde{V}_\perp + \dots = 0$$

$$V_z(v,u) = \tilde{V}_z(\zeta)g(v) + \dots \quad \Rightarrow$$

$$\Rightarrow \quad v^{2/3}g(v)\left[\partial_\zeta \tilde{V}_z(\zeta\partial_\zeta - 1)G + \zeta G\partial_\zeta^2\tilde{V}_z\right] + v\dot{g}(v)\left(1 - 2\zeta\partial_\zeta\right)\tilde{V}_z + 8\tilde{q}_5\alpha\zeta^3\tilde{V}_\perp b_z(v) + \dots = 0$$

2. CME in holography

Late-time chiral magnetic effect

$$J_{CME} \equiv \langle J_z \rangle = \frac{1}{2\kappa^2}\left(\partial_u^2 V_z\right)_{u=0}$$

- If $v^{2/3} b_z > v|\dot{b}_z|$, then the non-trivial solution is

$$\tilde{V}_\perp = 1$$

$$g(v) = \frac{b_z(v)}{v^{2/3}}$$

$$\partial_\zeta \tilde{V}_z = -\frac{4\alpha\tilde{q}_5\zeta(\zeta^2-\zeta_h^2)}{1+g_4\zeta^4+\tilde{q}_5^2\zeta^6/12} \quad \Rightarrow \quad \tilde{V}_z = 2\alpha\tilde{q}_5\zeta_h^2\zeta^2 + O(\zeta^3)$$

$$V_z = 4\alpha\left(\frac{1}{2v^{1/3}}\tilde{q}_5\zeta_h^2\right)\frac{b_z(v)}{v}u^2 + O(u^3)$$

$$J_{CME} = 8\alpha\mu_5(\tau)B_z(\tau)$$

$$V_\perp(v,u) = \tilde{V}_\perp(\zeta)b_z(v) + \dots \quad \Rightarrow \quad v^{2/3}b_z(v)\left[\partial_\zeta\tilde{V}_\perp(\zeta\partial_z - 1)G + \zeta G\partial_\zeta^2\tilde{V}_\perp\right] + v\dot{b}_z(v)\left(1-2\zeta\partial_\zeta\right)\tilde{V}_\perp + \dots = 0$$

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3. Thermodynamic parameters

- Temperature
- Axial charge
- Magnetic field
- Initial time

3. Thermodynamic parameters

Temperature

$$T_0\left(\sqrt{s}\right) = 96.50\, s^{0.23/2} - 23.51 \quad [\mathrm{MeV}]$$

- We fit the temperatures used in [Cartwright,Kaminski, Schenke(2021)] to the formula given above.

$\sqrt{s}$	19	27	39	64	200	2750
T_0	165	181	200	225	300	577

$$T \equiv \frac{1}{\tau^{1/3}}\frac{1}{\pi\zeta_h}\left(1 - \frac{1}{24}\tilde{q}_5^2\zeta_h^6\right)$$

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Axial charge

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T_0	165	181	200	225	300	577

$$T \equiv \frac{1}{\tau^{1/3}} \frac{1}{\pi \zeta_h} \left(1 - \frac{1}{24} \tilde{q}_5^2 \zeta_h^6\right)$$

- For axial charge, we use the estimation $n_5^0(200\, GeV) = 0.0027$ GeV^3 and assume that it scales as $n_5^0 \sim \left(\sqrt{s}\right)^{1/3}$.
- Different estimations, e.g [Shi,Jiang,Lilleskov,Liao(2018)]. $n_5^0/s_0 \sim 0.065$ are about 10 times larger, but lead to a trivial multiplicative factor in the CME signal.

$$n_5 \equiv \langle J_5^0 \rangle = \frac{1}{2\kappa^2} \frac{\tilde{q}_5}{\tau}$$

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Magnetic field

- Peak value of magnetic field (given at $\tau = 0$ except exceptions)

$$B_{max} = \frac{\sqrt{s}}{200} m_\pi^2$$

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3. Thermodynamic parameters

Magnetic field

- Peak value of magnetic field (given at $\tau = 0$ except exceptions)

$$B_{max} = \frac{\sqrt{s}}{200} m_\pi^2$$

Profile A: $$B(\tau) = \frac{B_{max}\tau_{ini}}{\tau}.$$ **(Bjorken expansion)**

Profile B: $$B = \frac{B_{\mathrm{max.}}}{1 + \left(\tau/\tau_B\right)^2}, \quad \tau_B = \frac{92\,\mathrm{GeV}\cdot\mathrm{fm/c}}{\sqrt{s}}.$$

Profile C: $$B = \frac{B_{\mathrm{max.}}}{\left[1 + \left(\tau/\tau_B\right)^2\right]^{3/2}}, \quad \tau_B = \frac{125\,\mathrm{GeV}\cdot\mathrm{fm/c}}{\sqrt{s}}.$$

Profile D: $$B = B_{\mathrm{max.}} e^{-\tau/\tau_B}, \quad \tau_B = \frac{128\,\mathrm{GeV}\cdot\mathrm{fm/c}}{\sqrt{s}}.$$

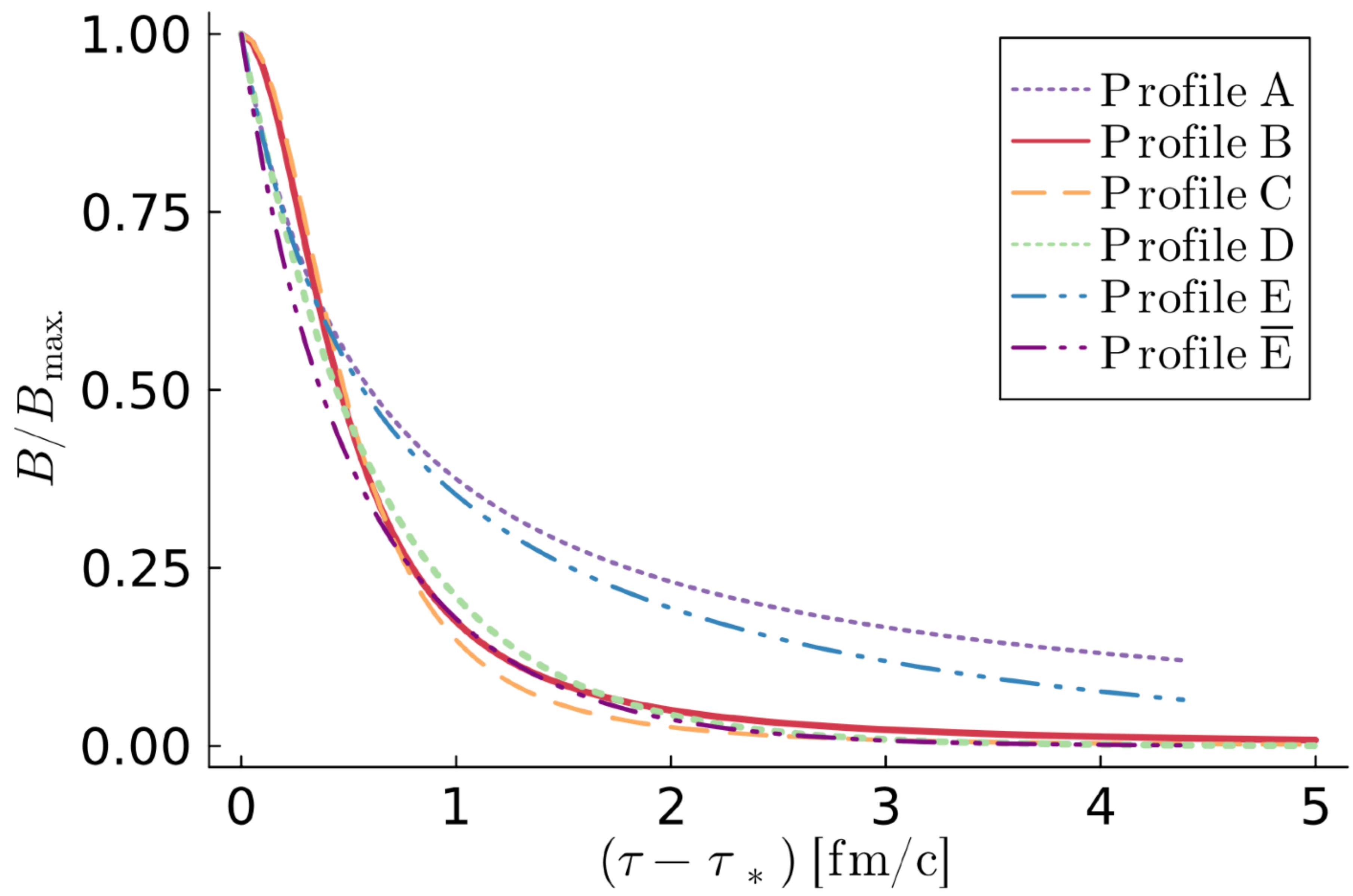
[Guo,Shi,Feng, Liao(2019)]

BHAC-QGP: $B(\tau)$ from **[Mayer,Dash,Inghirami,Elfner, Rezzolla,Rischke (2024)]**

Profile E: $$B(\tau) = B_{\mathrm{max.}} \frac{\sinh(\tau_{\mathrm{ini.}}/\tau_B)}{\sinh(\tau/\tau_B)}, \quad \tau_B = \frac{484\,\mathrm{GeV}\cdot\mathrm{fm/c}}{\sqrt{s}}.$$

3. Thermodynamic parameters

Magnetic field



$$B_{max} = \frac{\sqrt{s}}{200} m_\pi^2$$

3. Thermodynamic parameters

Initial time

- We consider two different initial times for the expanding plasma: $\tau_{ini} = 0.1$ fm/c and $\tau_{ini} = \tau_0 = 0.6$ fm/c.
- In the first scenario, the initial stages of the collision are gluon dominated. The magnetic field is evolving but there is no response from the quarks degrees of freedom, and no CME. [Huang,She,Shi,Huang,Liao(2023)].
- The second scenario is more conservative with respect to the approximate expanding background that we use.
- At initial time, we assume no CME signal. The initial state is specified by

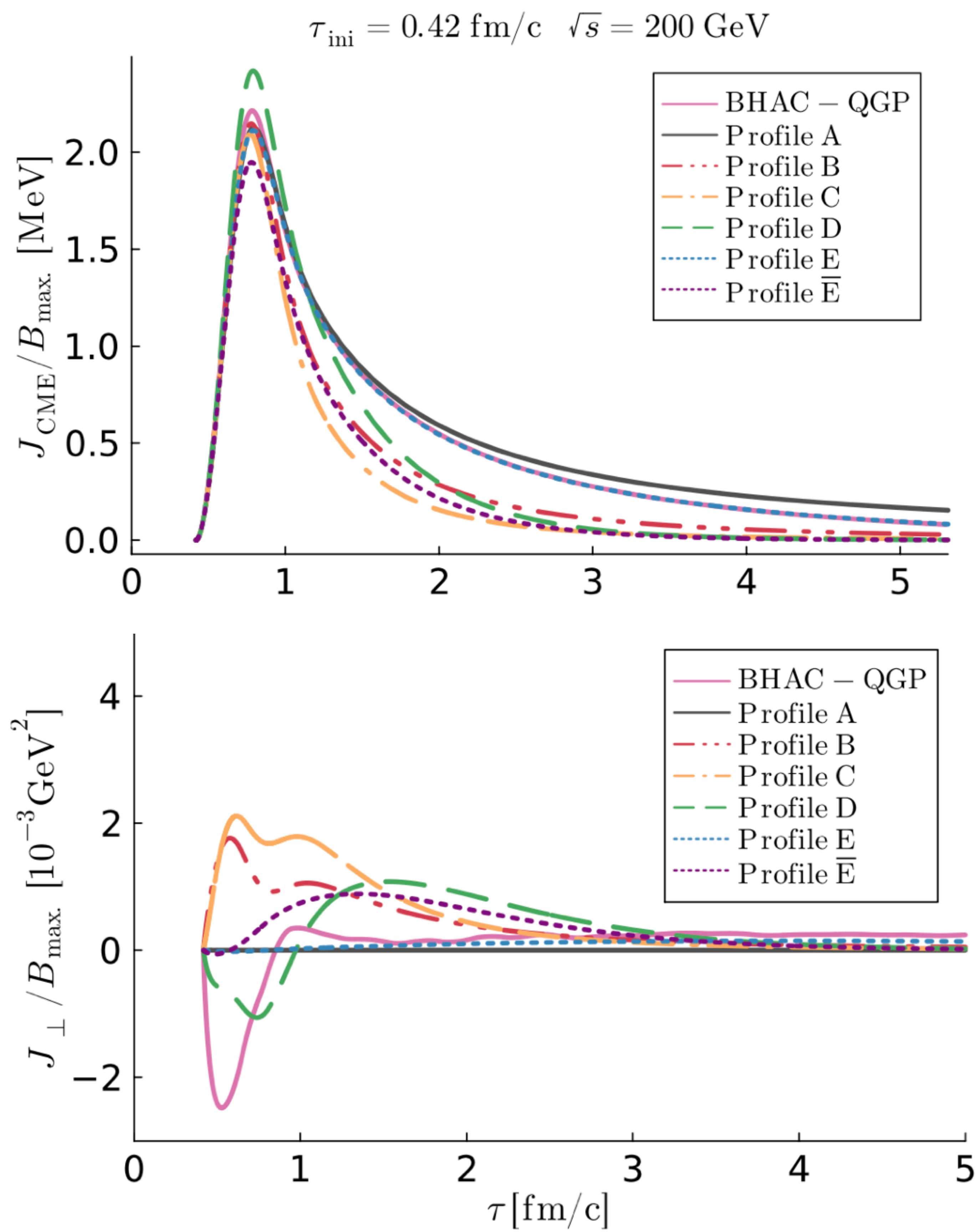
$$V_z^{ini} = 0\,, \quad V_\perp^{ini} = b_z(0) + \dot{b}_z(0) u\,.$$

4. Energy scan

Results at $\sqrt{s} = 200$ GeV

- Qualitative features are similar for the chiral magnetic current in all cases.
- The time-dependence of the magnetic field triggers a circular current in the transverse plane.

$$J_\perp \equiv \partial_y J^x - \partial_x J^y = \frac{1}{4\kappa^2}\left(\partial_u^2 V_\perp\right)_{u=0}$$



4. Energy scan

Results at $\sqrt{s} = 200$ GeV

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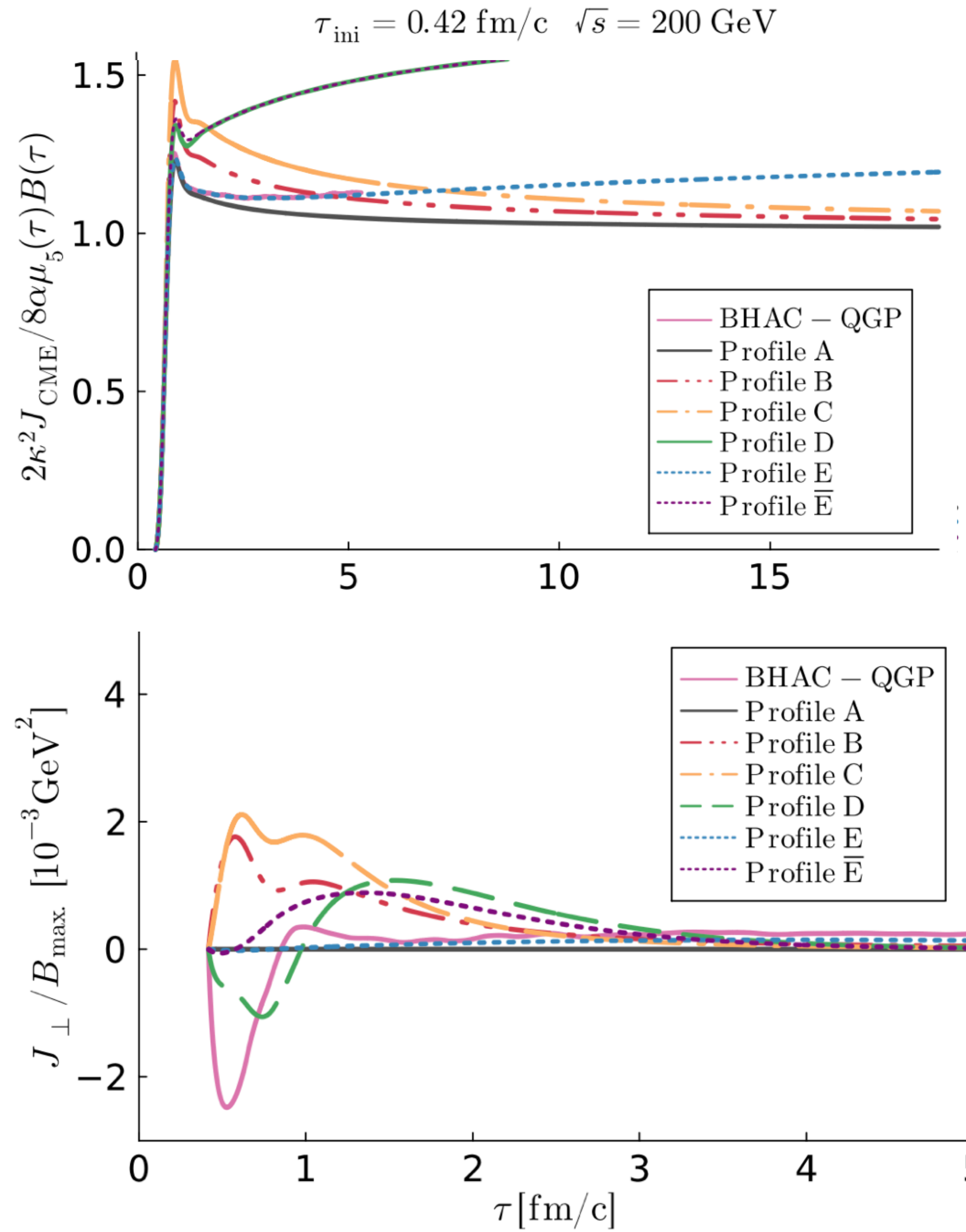
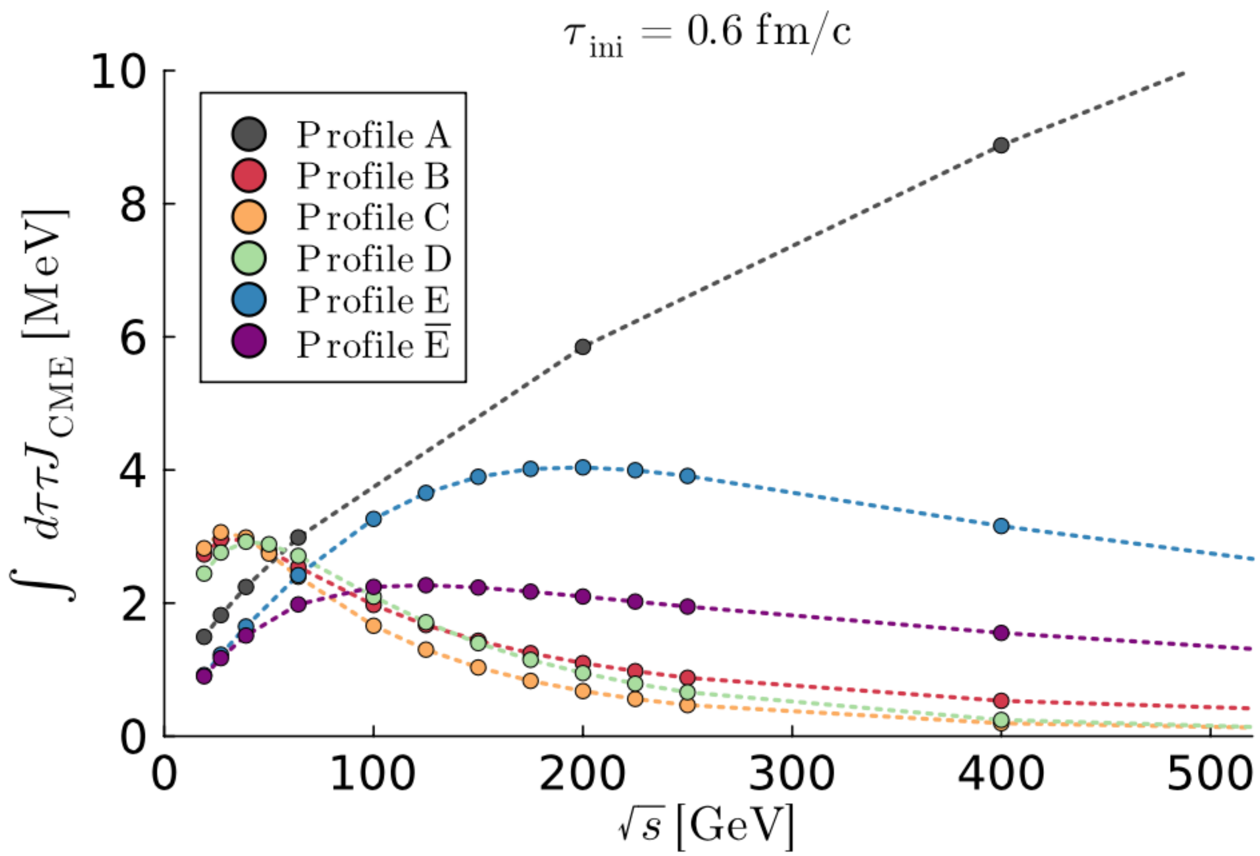
- The time-dependence of the magnetic field triggers a circular current in the transverse plane.
- The late-time CME formula applies for the "slowly decaying" profiles



4. Energy scan

Exploring collision energies



- The Bjorken expanding magnetic field (profile A) predicts a monotonic increase with the signal with collision energy. In agreement with [Cartwright,Kaminski, Schenke(2021)] (Case VI).
- Profiles E and $\bar{E}$ suggest an optimal collision energy slightly below 200 GeV. (Different from the other two scenarios).
- Profiles B, C and D give very similar qualitative predictions, with an optimal collision energy of about 40 GeV. This conclusion aligns with the results of [Ghosh,Grieninger,SMT,Landsteiner(2021)].

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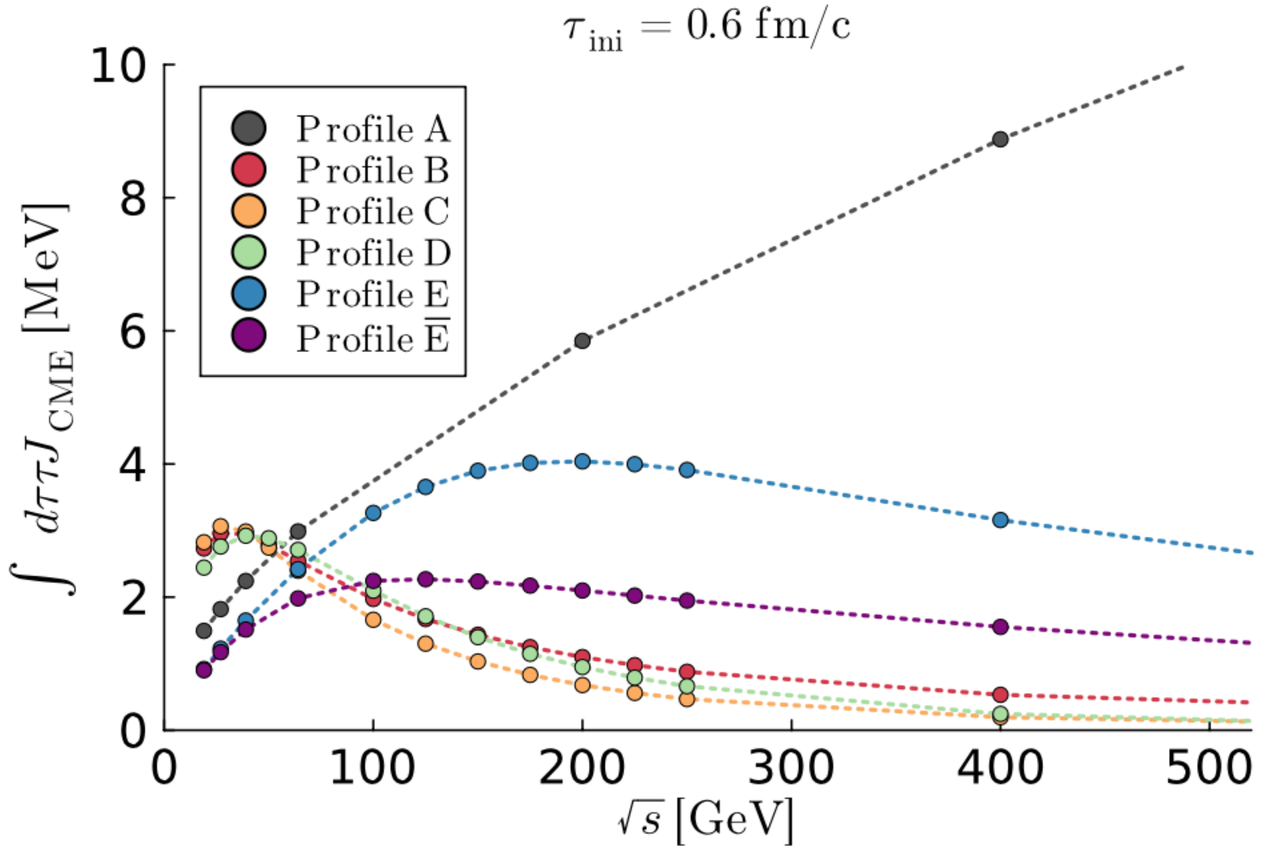
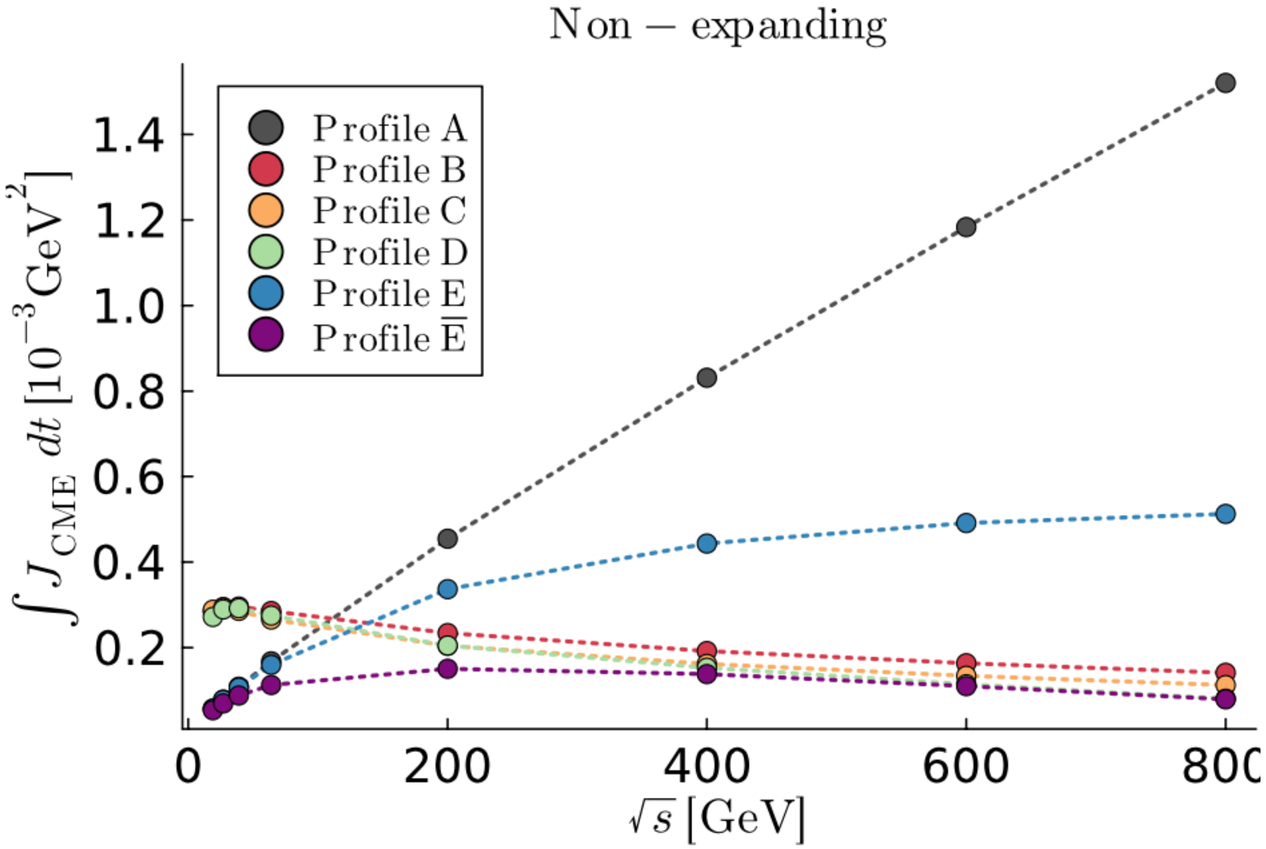
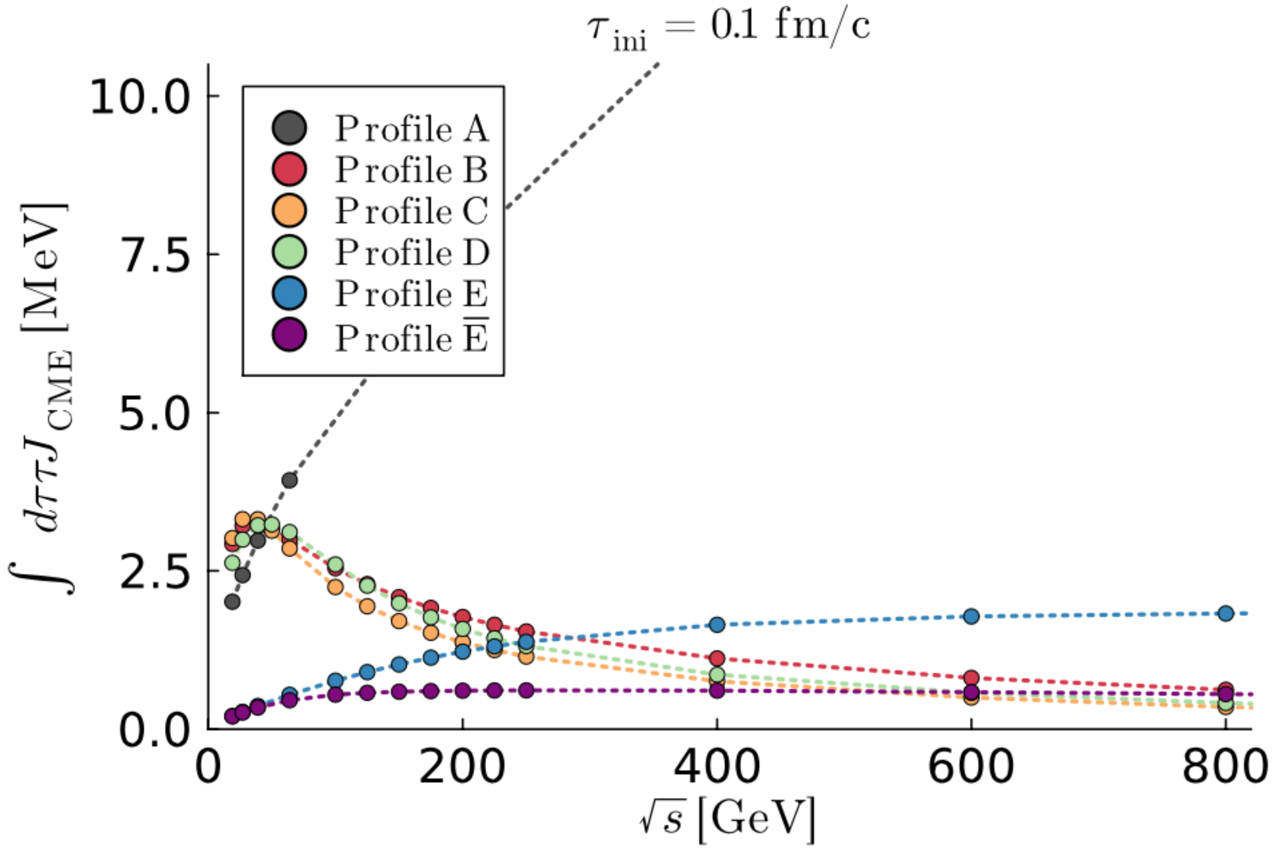
Exploring collision energies



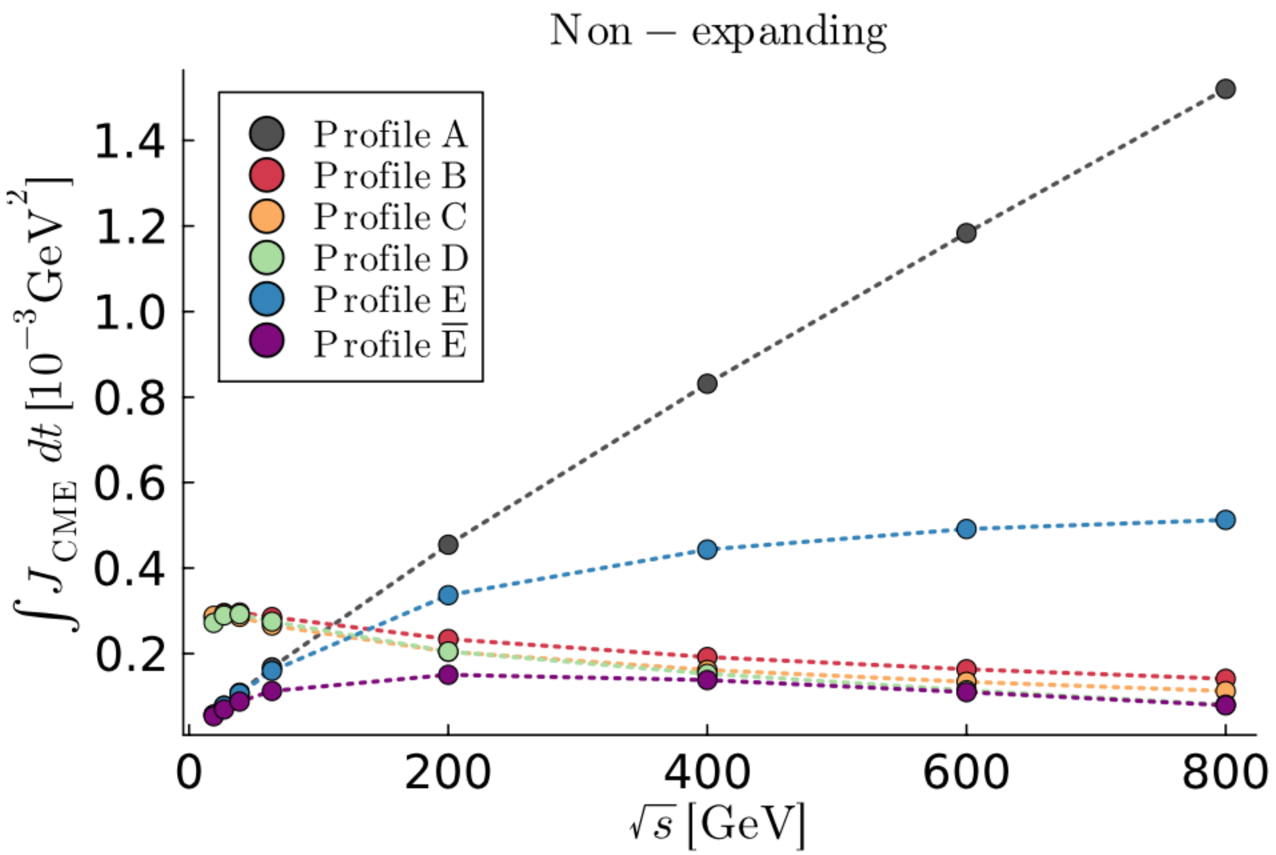


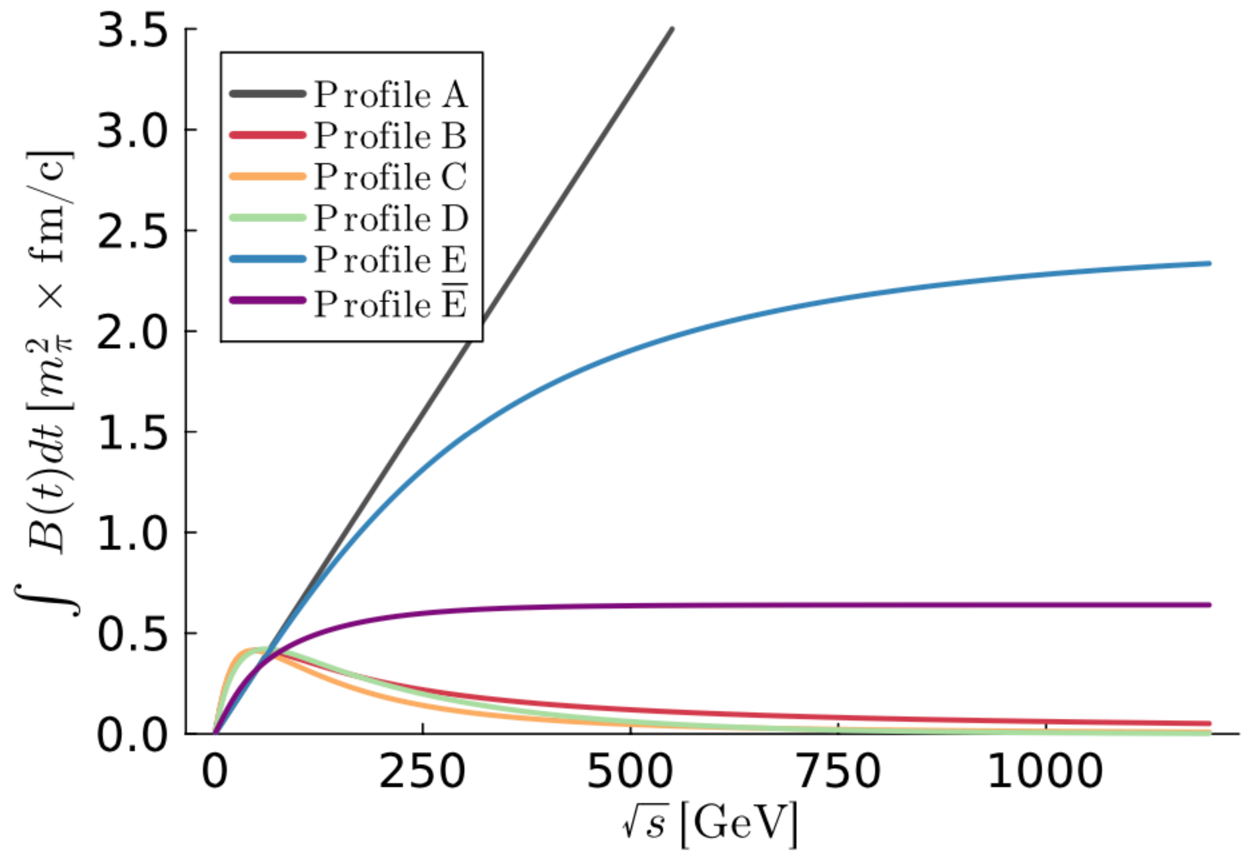


4. Energy scan

Exploring collision energies

- The integrated CME signal in the non-expanding case is essentially controlled by the integrated magnetic field.





5. Conclusions & Outlook

The small magnetic field limit seems to be a good approximation to model the CME in holography.

The CME satisfies the quasi-equilibrium formula also in the expanding plasma, provided that the decay of the magnetic field is sufficiently slow.

The dynamics of the magnetic field highly influences the response of the CME.

The magnetic field profiles obtained from a fit to different energies predict an enhancement of the CME at energies below 200 GeV.

5. Conclusions & Outlook

Take into account non-conformal effects. More important at lower temperatures.

Study transverse dynamics and possible oscillatory behaviour in arbitrarily strong magnetic fields.

Regard magnetic field as dynamical and obtain its time-dependence from holography.

This work is supported by the European Union - NextGenerationEU through grant No. 760079/23.05.2023, funded by the Romanian ministry of research, innovation and digitalization through Romania's National Recovery and Resilience Plan, call no. PNRR-III-C9-2022-I8.

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Thanks!