Spacetime dynamics of chiral magnetic currents in a hot non-Abelian plasma

Holographic perspectives on chiral transport and spin dynamics

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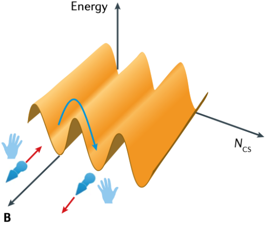
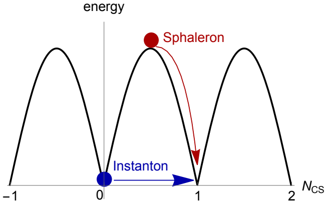
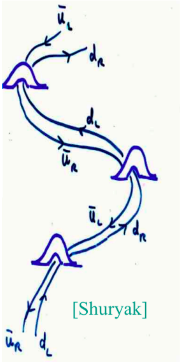
The Chiral Magnetic Effect in QCD

[Kharzeev, Jinfeng Liao; '21]



sfaleros ($\sigma\varphi\alpha\lambda\varepsilon\rho\varsigma$: ready to fall) [Klinkhamer, Manton; '84]

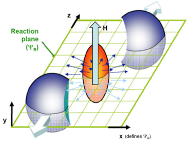




- QCD vacuum has periodic structure; minima different CS #
- instanton/sphaleron transition between such energy-degenerate but topologically distinct vacuum sectors ⇒ change of chirality of the chiral fermions [Fukushima, Kharzeev, Warringa] [Vilenkin; '80], [Alekseev, Chaianov, Fröhlich], [Giovaninni, Shaposhnikov],...
- In magnetic field: Change in chirality ⇒ change of direction of momentum ⇒ Charge separation (measurable as CME)

Axial Charge

[Kharzeev; '14] Prog.Part.Nucl.Phys. 75 (2014)



- Abelian anomaly: $\partial_\mu J_5^\mu = C\,\epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} F_{\rho\lambda}$
- CME current $\vec{J} = 8C\,\mu_5\,\vec{B}$

In Heavy-ion collisions

1. μ_5 (and n_5) is generated dynamically and not put in by hand in form of an axial chemical potential
2. n_5 is not a conserved quantity since axial symmetry is broken explicitly by quark masses and gluonic effects

- Experimental observable directly linked to flucs of electric current $\cos(\Delta\phi_\alpha + \Delta\phi_\beta) \propto \frac{\alpha\beta}{N_\alpha N_\beta}(J_\perp^2 - J_\parallel^2)$ [Voloshin],[Fukushima, Kharzeev, Warringa]
- **Correlations of electric current sensitive to topology at large distances (at strong coupling)?**

Holographic Model: wish list

Dictionary

Field Theory in $d = 3 + 1$	$\Leftrightarrow$	Gravity Theory in asymp. AdS_5
finite T, temp. flucs	$\Leftrightarrow$	black hole, flucs of metric
external magnetic field	$\Leftrightarrow$	$F_{xy} = B$ in $U(1)_V$ gauge field
fluctuations electric current	$\Leftrightarrow$	fluctuations of vector gauge field
fluctions of axial charge n_5	$\Leftrightarrow$	fluctuations of axial gauge field
abelian anomaly	$\Leftrightarrow$	$\alpha\cdot$ (CS-term+mixed CS-term)
non-abelian anomaly, top. ef	$\Leftrightarrow$	axial gauge field massive in bulk

[Klebanov, Ouyang, Witten; '02], [Gürsoy, Jansen; '14], [Jimenez-Alba, Landsteiner, Melgar; '14], [Jimenez, Landsteiner, Liu, Sun; '15] [Iatrakis, Lin, Yin; '14 + '15], [Gallegos, Gürsoy; '18]

Holographic Model: wish list

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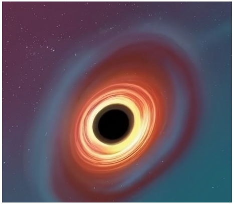
Anomalous dimension

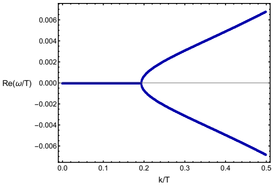
$$\dim[\langle J_5^\mu \rangle] = 3 + \Delta(m_s)$$

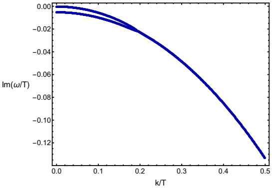
Ward identities

$$\partial_\mu J^\mu = 0,\ \partial_\mu J_5^\mu = m_s \,\mathrm{tr} G \wedge G + \alpha \Big(3F \wedge F + F^{(5)} \wedge F^{(5)} \Big)$$

Setup



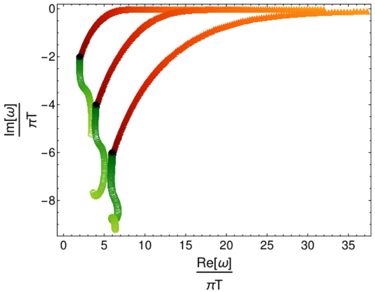


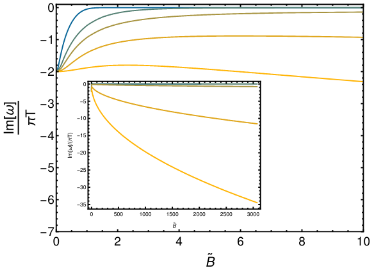


- Background: magnetic brane i.e. anisotropy enters thermodynamics of QFT [Ammon, **SG**, Hernandez, Kaminski, Koirala, Leiber, Wu; '20], [Adhikari, . . . , SG, . . . , Hernandez, . . . , Kaminski, . . . ; '24] $(\langle T_\mu{}^\mu \rangle \sim B^2)$
- Consider time and space dependent fluctuations of gauge fields, scalar, and metric about this background (in Fourier space)
- $m_s = 0$: axial charge and electric current can oscillate into each other $\to$ Chiral magnetic wave [Kharzeev, Yee; '10]
- With $m_s \neq 0$: axial charge pulled into black hole $\to$ **Chiral magnetic wave overdamped, finite lifetime of axial charge**
- Special cases $\boldsymbol{k} = k_{||} : \{a_t, a_z, v_t, v_z, \theta\}$, $\boldsymbol{k} = k_\perp : \{h_{yz}, a_t, a_\perp, v_z, \theta\}$
- Also important: anisotropy of the background

Preliminary: $m_s = 0$, $\boldsymbol{k} = 0$

[Ammon, **SG**, Jimenez-Alba, Macedo, Melgar; '16]





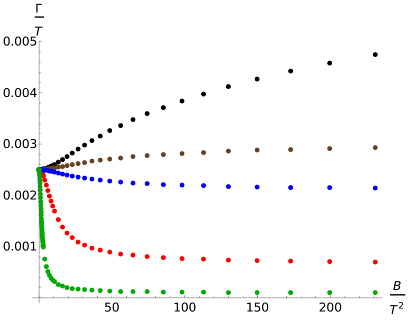
Left: Magnetic field increases from dark to light.

Right: Strength of abelian anomaly (CS coupling) increases from yellow to blue.

see also [Haack, Sarkar, Yarom; '17], [Meiring, Shyovitz, Waeber, Yarom; '24], [Waeber, Yarom; '24] + talk by Amos Yarom

Axial charge relaxation rate in strong B

$\alpha = 0$, $\alpha = 1/15$, $\alpha = 1/10$, $\alpha = 6/19$, $\alpha = 2$ (strength of ab. anomaly)



- $m_s L$ is fixed
- All curves behave like $\Gamma/T \sim c_1 \pm c_2(B/T^2)^2$ initially
- sufficiently large $B, \alpha \Rightarrow$ Abelian anomaly overpowers non-Abelian one
- Red and green curve decay as $\Gamma/T \sim c_1 - c_2(T^2/B)$ at large B/T^2.

Chern-Simons Diffusion rate

$$\frac{\mathrm{d}n_5}{\mathrm{d}t} = -2q_{\text{top}} = -\frac{2\Gamma_{CS}}{\chi_5\, T} n_5 = -\frac{n_5}{\tau_{\text{sph}}} = -\Gamma n_5$$

Chern-Simons rate increases quadratically for small B/T^2 and linearly at large B/T^2 (matches scaling of [Kharzeev, Basar; '12])

Procedure

- Prepare background: magnetic brane at finite background magnetic field, no charges
- Compute electric current two-point function as a function of $\boldsymbol{k}$ at fixed $B = B\, e_z$ and ω and consider the subtracted correlator

$$\Delta G^{\text{ret}}_{J^z J^z}(\omega, \boldsymbol{k}) \equiv G^{\text{ret}}_{J^z J^z}(\omega, \boldsymbol{k}, m_s) - G^{\text{ret}}_{J^z J^z}(\omega, \boldsymbol{k}, m_s = 0),$$

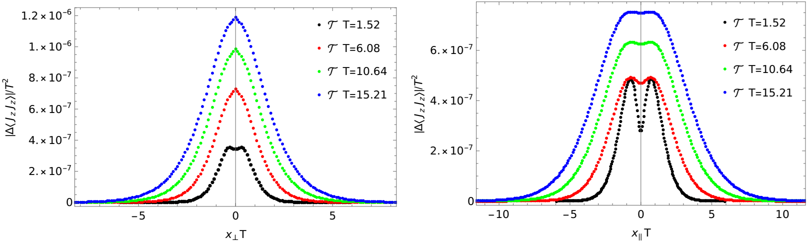
  which isolates the contributions coming from topological fluctuations
- Perform inverse (discrete) Fourier transform to real space
- Extend is given by root mean square

$$x_{\text{rms}} = \sqrt{\frac{\int \mathrm{d}x\, x^2\, |\Delta G^{\text{ret}}_{J^z J^z}(x)|}{\int \mathrm{d}x\, |\Delta G^{\text{ret}}_{J^z J^z}(x)|}}$$

- Encodes the information about spatial profile of induced axial charge by topological fluctuations for a given magnetic field and time interval

Initial spatial distributions

Fix $B/T^2 = 0.22$, $\alpha = 6/19$, $m_s = 0.04$, $\tau_{5,\mathrm{rel}} = 2501/T (\Rightarrow \tau_{5,\mathrm{rel}} = 1645$ fm)



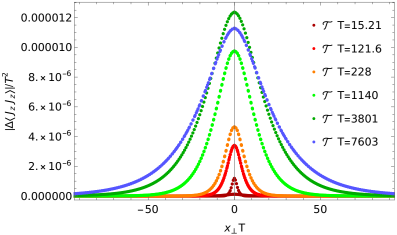
$\perp$ and $\parallel$ with respect to B; $\mathcal{T} = \frac{2\pi}{\omega}$ is time interval ($\sim$ inverse Fourier frequency). $\mathcal{T}T = 15.21$ corresponds to $\mathcal{T} = 10$ fm in dimensionful units.

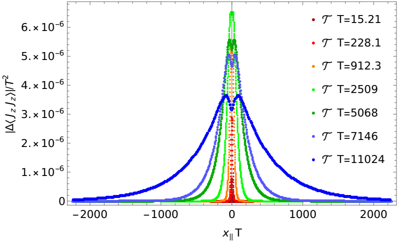
Interpretation

Two peaks (black): strong coupling analog of the 2 chiral fermions in weak coupling picture. $\mathcal{T} \uparrow \rightarrow$ spatial extent + magnitude $\uparrow$; area between two off axis peaks fills up corresponding to filling up the sphaleron shell.

Spatial distributions late times

Fix $B/T^2 = 0.22$, $\alpha = 6/19$, $m_s = 0.04$, $\tau_{5,\mathrm{rel}} = 2501/T (\Rightarrow \tau_{5,\mathrm{rel}} = 1645$ fm)





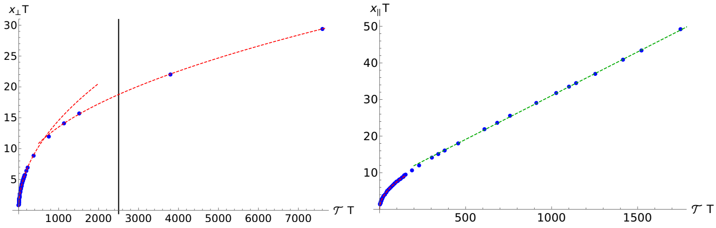
$\perp$ with respect to B; $\mathcal{T} = \frac{2\pi}{\omega}$ is time interval ($\sim$ inverse Fourier frequency). $\mathcal{T}T = 15.21$ corresponds to $\mathcal{T} = 10$ fm in dimensionful units.

Interpretation

After reaching the axial charge relaxation time magnitude of the distributions starts to decrease while their spatial extent continues to increase. 2 peaks start appearing again in longitudinal distribution (sphaleron explosion).

Transverse and longitudinal extent

Reminder: $x_{\perp,\mathrm{rms}} = \sqrt{\frac{\int \mathrm{d}x_\perp\, x_\perp^2\, |\Delta G^{\mathrm{ret}}_{J^z J^z}(x_\perp,\omega)|}{\int \mathrm{d}x_\perp\, |\Delta G^{\mathrm{ret}}_{J^z J^z}(x_\perp,\omega)|}}$; black line is τ_{rel} of axial charge



- Compare to: $\rho = 0.3$ fm [Ostrovsky, Carter, Shuryak; '02], [Shuryak, Zahed; '21]
- Only diffusive in transverse direction (exponent $1/2$)
- For $k \| B$ ballistic behavior for sufficiently large time (linear growth)
- Size enhanced along magnetic field (no backscattering)
- Velocity: $\Delta x_\| / \Delta \mathcal{T} = 0.08 \ll 1$

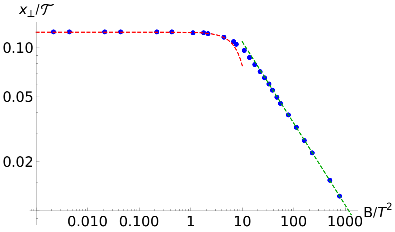
Dimensionful units

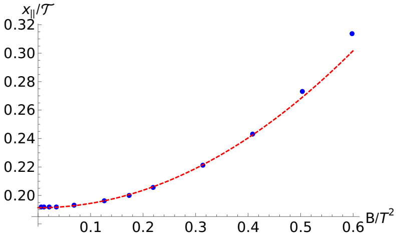
Let's put $T = 300\mathrm{MeV}$, $B = 1 m_\pi^2$, $\mathcal{T} = 10\,\mathrm{fm}$ (for $m_s L = 0.04$)

$x_\perp = 1.25\,\mathrm{fm}$ and $x_\| = 1.94\,\mathrm{fm}$

Dependence on magnetic field

Fix $\mathcal{T}T \approx 15$, $\alpha = 6/19$, $m_s = 0.04$.





Observation

Size perpendicular to magnetic field drops $\Rightarrow$ effective 1+1 dimensional dynamics at large B (LLL). Significant enhancement in $x_\parallel$ $\Rightarrow$ become more elongated

$x_\parallel/\mathcal{T} \sim a_1 + a_2(B/T^2)^3$; $x_\perp/\mathcal{T} \sim a_3 + a_4 T/\sqrt{B}$ for large B/T^2

Axial charge dynamics in expanding plasma

[Ammon, **SG**, Jimenez, Macedo, Melgar; '16], [Ghosh, **SG**, Landsteiner, Morales-Tejera; '21], [Cartwright, Kaminski, Schenke; '22], [**SG**, Morales-Tejera, Garcia-Romeu; '25]

[**SG**, Morales-Tejera; '23] $T_0 \approx 300$ MeV, $B = m_\pi^2$ (homogeneous setup)

Consider expanding black hole with bdry metric: $\mathrm{d}s^2_{\text{boundary}} \sim -\mathrm{d}\tau^2 + (\tau^2 \mathrm{d}\eta^2 + \mathrm{d}x_\perp^2)$

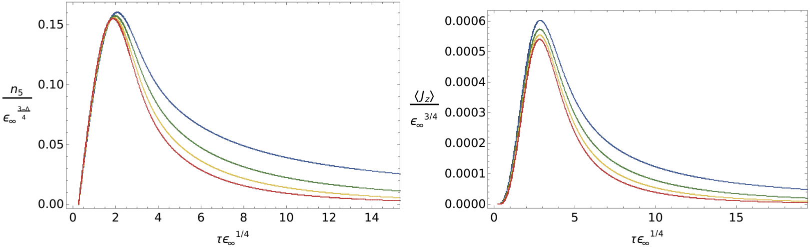
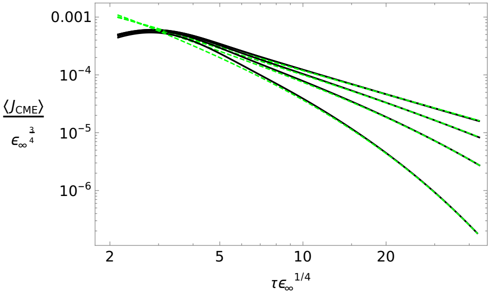


Figure: Axial charge density (left) and chiral magnetic current (right) as a function of time corresponding to $\sqrt{s} = 200\,\text{GeV}$ initial conditions. The coupling m_s increases from blue ($\Delta = 1.25 \times 10^{-7}$) to red ($\Delta = 0.3$).

Small axial charge relaxation rate, Large axial charge relaxation rate

Horizon formula for chiral magnetic current



Late time behavior of the chiral magnetic current (in Bjorken regime) for increasing values of m_s (black lines). Green dashed line:

Chiral magnetic current

$$\underbrace{\frac{19\,\kappa_5^2}{24\pi^2}}_{=1}\langle J_{\mathrm{CME}}\rangle = \frac{\alpha}{3(1-\Delta)}A_v(\tau,1)B(\tau).$$

Conclusions and Outlook

Conclusions

- Insights into spatial profile of axial charge induced by top. flucs
- Correlations of el. currents sensitive to topology at large distances
- Range grows with time: diffusive in $\perp$, ballistic in $\parallel$
- At large B the $\perp$ size decreases with $1/\sqrt{B}$ (consistent with LL picture). $\parallel$ size grows with B^3
- Shows that sphalerons are large objects even at strong coupling

Outlook

- Derive formula for CME in QCD
- Competition between expansion and growth of CP odd bubbles?
- Improved holographic models closer to phenomenology
- Full non-linear, 3+1 dimensional dynamics with time-dependent magnetic fields

Thank you for your attention!

Holographic Stückelberg Model

Gravitational Action [Jimenez-Alba, Landsteiner, Melgar; '14]

$$S = \frac{1}{2\,\kappa_5^2}\int_{\mathcal{M}} d^5x\sqrt{-g}\left[R + \frac{12}{L^2} - \frac{1}{4}F^2 - \frac{1}{4}F_{(5)}^2 + \frac{m_s^2}{2}(A_m - \partial_m\theta)^2 + \frac{\alpha}{3}\epsilon^{mnklp}(A_m - \partial_m\theta)\left(3F_{nk}F_{lp} + F^{(5)}_{nk}F^{(5)}_{lp}\right)\right] + S_{bdy} + S_{ct}$$

with $F = \mathrm{d}V,\ F^{(5)} = \mathrm{d}A$

Ward identities

$$\partial_\mu J^\mu = 0,\ \partial_\mu J_5^\mu = m_s\,\mathrm{tr}G \wedge G + \alpha\left(3F \wedge F + F^{(5)} \wedge F^{(5)}\right)$$

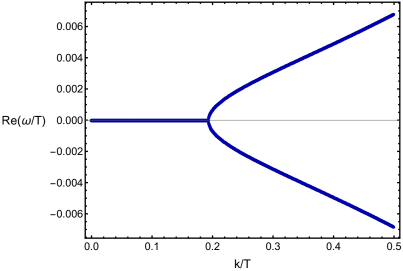
Two contributions: non-abelian anomaly + abelian QED anomaly

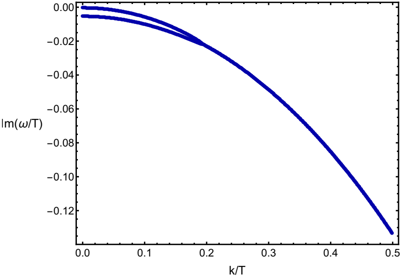
CMW and axial charge dissipation

$$\partial_\mu J^\mu = 0, \quad J^z = \frac{\alpha\,\rho_5 B}{\chi_5} - D\,\partial_z \rho; \qquad \partial_\mu J_5^\mu = -\Gamma \rho_5;\; J_5^z = \frac{\alpha\,\rho\,B}{\chi} - D\,\partial_z \rho_5$$

Chiral magnetic wave is gapped!

$$\omega_\pm = -\frac{i\Gamma}{2} - iDk^2 \pm \sqrt{\frac{B^2 k^2 \alpha^2}{\chi_5 \chi} - \frac{\Gamma^2}{4}}$$

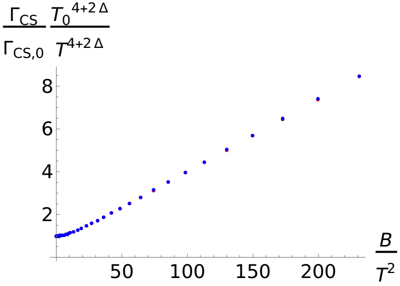


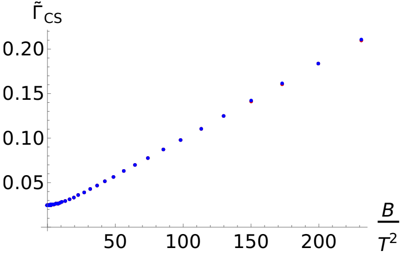


momentum gap: critical k above which propagating behavior is restored
matching to hydro possible; similar idea: [Ammon, Areán, Baggioli, Gray, SG; '21]

Chern-Simons Diffusion rate

$\alpha = 0 \equiv \alpha_0$, $\alpha = 6/19 \equiv \alpha_1$, $\alpha = 2$



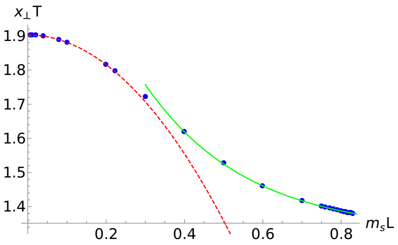


Coupling dependence

Chern-Simons rate increases quadratically for small B/T^2 and linearly at large B/T (matches scaling of [Kharzeev, Basar; '12])

Dependence on mass/coupling

Fix $B/T^2 = 0.22$, $\alpha = 6/19$, $T\mathcal{T} = 15.21$, $m_s L < \sqrt{3}$

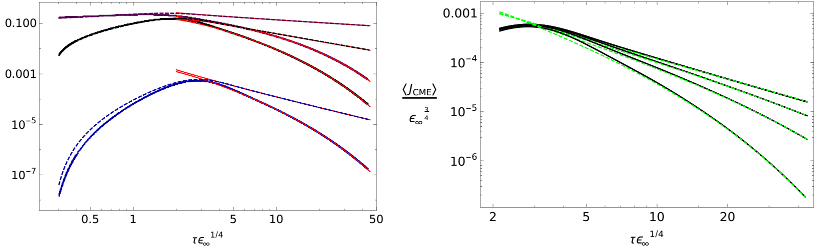


$x_{\parallel,\perp} T \sim a_{\parallel,\perp} + b_{\parallel,\perp} (m_s L)^2$ (red dashed); $\frac{x_\parallel(\sqrt{3})}{x_\parallel(0)} = 0.83$; $\frac{x_\perp(\sqrt{3})}{x_\perp(0)} = 0.70$.

Coupling dependence

Size decreases for increasing the coupling strength; ratio $x_\perp/x_\parallel$ roughly independent of $m_s L$ for small $m_s L$.
Note: Gap Γ has to be small for quasi-hydro to be applicable $\Rightarrow$ $m_s L \ll 1$ for $B/T^2 < 1$.

Horizon formula for chiral magnetic current



Left: $\langle J_{\rm CME}\rangle/\epsilon_\infty^{3/4}$ (blue), $n_5/\epsilon_\infty^{3/4+\Delta/4}$ (black) and $A_v(1)$ (purple). The dashed lines correspond to $m_s \approx 0$ and the solid lines to $m_s \neq 0 \Rightarrow$ Late time power laws modified (B decays, time-dep. CS diffusion rate)

Right: Late time behavior of the chiral magnetic current for increasing values of m_s (black lines). Green dashed line:

$$\underbrace{\frac{19\,\kappa_5^2}{24\pi^2}}_{=1}\langle J_{\rm CME}\rangle = \frac{\alpha}{3(1-\Delta)}A_v(\tau,1)B(\tau).$$