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Recent theoretical results on spin polarization in relativistic heavy ion collisions

OUTLINE

- Theory summary
- Second order corrections in the expansion of local equilibrium
- Numerical calculations and bulk viscosity
- Spin polarization and spin hydrodynamics
- Conclusions

ECT* Trento Holography workshop March 2025

Introduction: a theory summary

F. B., Lecture Notes in Physics 987, 15 (2021) arXiv:2004.04050

There are two methods to calculate the spin polarization of final particles
1- A quantum statistical method built on local equilibrium density operator
2- An extension of relativistic kinetic theory to particles with spin

Spin polarization vector for spin ½ particles:

$$S^\mu(p) = \sum_{i=1}^{3} [p]_i^\mu \mathrm{tr}(D^{1/2}(J_i)\Theta(p)) \qquad \Theta(p)_{sr} = \frac{\mathrm{Tr}(\widehat{\rho}\widehat{a}^\dagger(p)_r \widehat{a}(p)_s)}{\sum_t \mathrm{Tr}(\widehat{\rho}\widehat{a}^\dagger(p)_t \widehat{a}(p)_t)}$$

$$S^\mu(p) = \frac{1}{2}\frac{\int \mathrm{d}\Sigma \cdot p \, \mathrm{tr}_4(\gamma^\mu\gamma^5 W_+(x,p))}{\int \mathrm{d}\Sigma \cdot p \, \mathrm{tr}_4 W_+(x,p)}$$

$$\begin{aligned}\widehat{W}(x,k)_{AB} &= -\frac{1}{(2\pi)^4}\int \mathrm{d}^4y \, \mathrm{e}^{-ik\cdot y} :\Psi_A(x-y/2)\overline{\Psi}_B(x+y/2): \\ &= \frac{1}{(2\pi)^4}\int \mathrm{d}^4y \, \mathrm{e}^{-ik\cdot y} :\overline{\Psi}_B(x+y/2)\Psi_A(x-y/2):\end{aligned}$$

$$W(x,k) = \mathrm{Tr}(\widehat{\rho}\widehat{W}(x,k))$$

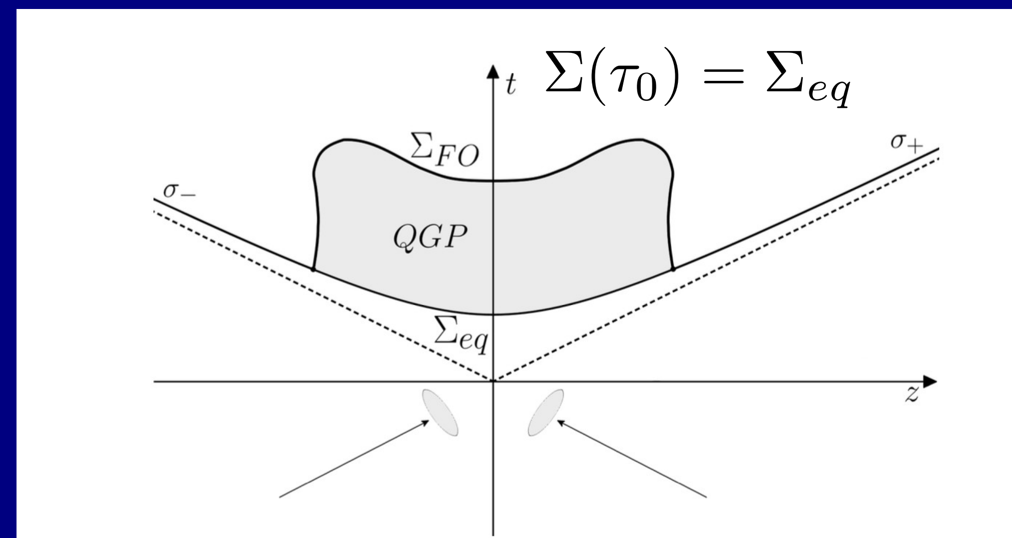
Density operator

$$\widehat{\rho} = \frac{1}{Z}\exp\left[-\int_{\Sigma(\tau_0)} \mathrm{d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu}\beta_\nu - \zeta\widehat{j}^\mu\right)\right].$$

Density operator: local equilibrium at the initial time

T_B is the Belinfante stress-energy tensor

$$\beta = \frac{1}{T}u \quad \zeta = \frac{\mu}{T}$$



With the Gauss theorem: calculate at Freeze-out

$$\widehat{\rho} = \frac{1}{Z}\exp\left[-\int_{\Sigma(\tau)} \mathrm{d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu}\beta_\nu - \zeta\widehat{j}^\mu\right) + \int_\Theta \mathrm{d}\Theta \left(\widehat{T}_B^{\mu\nu}\nabla_\mu\beta_\nu - \widehat{j}^\mu\nabla_\mu\zeta\right)\right],$$

Local equilibrium, non-dissipative terms

Dissipative terms

Local equilibrium and hydrodynamic limit

$$W(x,k)_{\rm LE} = \frac{1}{Z} {\rm Tr} \left(\exp \left[- \int_{\Sigma_{FO}} {\rm d}\Sigma_\mu(y) \widehat{T}_B^{\mu\nu}(y) \beta_\nu(y) - \zeta(y) \widehat{j}^\mu(y) \right] \widehat{W}(x,k) \right)$$

Expand the β and ζ fields from the point *x* where the Wigner operator is to be evaluated

$$\beta_\nu(y) = \beta_\nu(x) + \partial_\lambda \beta_\nu(x) (y-x)^\lambda + \ldots$$

$$\int_\Sigma {\rm d}\Sigma_\mu T_B^{\mu\nu}(y) \beta_\nu(x) = \beta_\nu(x) \int_\Sigma {\rm d}\Sigma_\mu T_B^{\mu\nu}(y) = \beta_\nu(x) \widehat{P}^\nu$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z} \exp \left[-\beta_\mu(x) \widehat{P}^\mu - \frac{1}{4} (\partial_\mu \beta_\nu(x) - \partial_\nu \beta_\mu(x)) \widehat{J}_x^{\mu\nu} - \frac{1}{4} (\partial_\mu \beta_\nu(x) + \partial_\nu \beta_\mu(x)) \widehat{Q}_x^{\mu\nu} + \ldots \right]$$

$$\widehat{J}_x^{\mu\nu} = \int {\rm d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) - (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

$$\widehat{Q}_x^{\mu\nu} = \int {\rm d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) + (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z} \exp[-\beta_\mu(x)\widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}^{\mu\nu}_x - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}^{\mu\nu}_x + \ldots]$$

$$\varpi_{\mu\nu} = -\frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu)$$

Thermal vorticity
Adimensional in natural units

$$\xi_{\mu\nu} = \frac{1}{2}(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu)$$

Thermal shear
Adimensional in natural units

At global equilibrium the thermal shear vanishes because of the Killing equation

Linear response theory

$$\mathrm{e}^{\widehat{A}+\widehat{B}} = \mathrm{e}^{\widehat{A}} + \int_0^1 \mathrm{d}z \; \mathrm{e}^{z(\widehat{A}+\widehat{B})} \widehat{B} \mathrm{e}^{-z\widehat{A}} \mathrm{e}^{\widehat{A}} \simeq \mathrm{e}^{\widehat{A}} + \int_0^1 \mathrm{d}z \; \mathrm{e}^{z\widehat{A}} \widehat{B} \mathrm{e}^{-z\widehat{A}} \mathrm{e}^{\widehat{A}}$$

$$\widehat{A} = -\beta_\mu(x)\widehat{P}^\mu$$

$$\widehat{B} = \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}^{\mu\nu}_x - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}^{\mu\nu}_x + \ldots]$$

$$W(x,k) \simeq \frac{1}{Z}\mathrm{Tr}(\mathrm{e}^{\widehat{A}+\widehat{B}}\widehat{W}(x,k)) \simeq \ldots$$

CORRELATORS

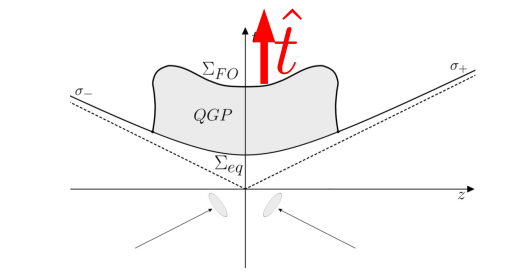
$$\langle \widehat{Q}^{\mu\nu}_x \widehat{W}(x,p) \rangle \qquad \langle \widehat{J}^{\mu\nu}_x \widehat{W}(x,p) \rangle$$

Spin mean vector at leading order

$$\widehat{\rho}_{\rm LE} \simeq \frac{1}{Z} \exp[-\beta_\mu(x)\widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}^{\mu\nu}_x - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}^{\mu\nu}_x + \ldots]$$

$$S^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\sigma \frac{\int_\Sigma {\rm d}\Sigma_\tau p^\tau n_F(1-n_F)\partial_\nu\beta_\rho}{\int_\Sigma {\rm d}\Sigma_\tau p^\tau \; n_F}$$

Neglected by “prejudice” until 2021



See also

R. H. Fang, L. G. Pang, Q. Wang, X. N. Wang, Phys. Rev. C 94 (2016) 024904
W. Florkowski, A. Kumar and R. Ryblewski, Phys. Rev. C 98 (2018) 044906
Y. C. Liu, L. L. Gao, K. Mameda and X. G. Huang, Phys. Rev. D 99 (2019) 085014
N. Weickgenannt, X. L. Sheng, E. Speranza, Q. Wang and D. H. Rischke, Phys. Rev. D 100 (2019) 056018

$$S^\mu_\xi(p) = -\frac{1}{4m}\epsilon^{\mu\nu\sigma\tau}\frac{p_\tau p^\rho}{\varepsilon}\frac{\int_\Sigma {\rm d}\Sigma\cdot p\; n_F(1-n_F)\hat{t}_\nu\xi_{\sigma\rho}}{\int_\Sigma {\rm d}\Sigma\cdot p\; n_F},$$

$$\xi_{\mu\nu} = \frac{1}{2}\left(\partial_\mu\beta_\nu + \partial_\nu\beta_\mu\right).$$

F. B., M. Buzzegoli, A. Palermo, Phys. Lett. B 820 (2021) 136519
S. Liu, Y. Yin, JHEP 07 (2021) 188
Confirmed by C. Yi, S. Pu, D. L. Yang, Phys.Rev.C 104 (2021) 6, 064901
Y. C. Liu, X. G. Huang, Sci.China Phys.Mech.Astron. 65 (2022) 7, 272011

Second order terms in LE expansion

X. L. Sheng, F. B., X. G. Huang, Z. H. Zhang, Phys. Rev. C 110 (2024) 064908

$$\widehat{A}_x = -\beta(x) \cdot \widehat{P}$$
$$\widehat{B}_x = \frac{1}{2}\varpi(x) : \widehat{J}_x + \frac{1}{2}\xi : \widehat{Q}_x$$

Quadratic in the gradients

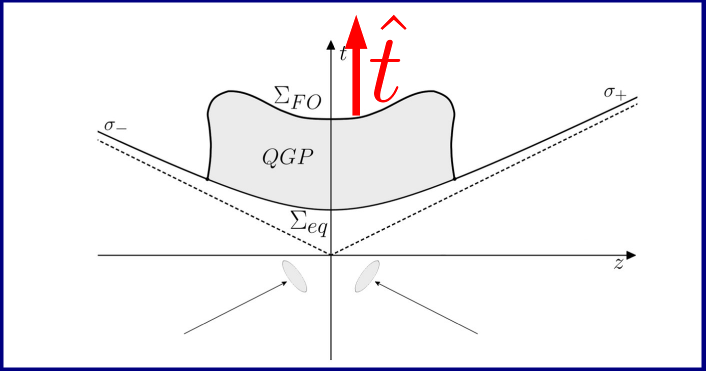
$$\mathrm{e}^{\widehat{A}_x + \widehat{B}_x} \approx \mathrm{e}^{\widehat{A}_x} + \int_0^1 \mathrm{d}z\, \mathrm{e}^{z\widehat{A}_x} \widehat{B}_x \mathrm{e}^{-z\widehat{A}_x} + \int_0^1 \mathrm{d}z_1 \int_0^{z_1} \mathrm{d}z_2\, \mathrm{e}^{z_1\widehat{A}_x} \widehat{B}_x \mathrm{e}^{(z_1-z_2)\widehat{A}_x} \widehat{B}_x \mathrm{e}^{-z_1\widehat{A}_x} + \mathcal{O}(\widehat{B}^3)\,.$$

Linear in the second-order derivatives

$$\widehat{\rho}_{\mathrm{LE}} \simeq \frac{1}{Z} \exp\left[-\beta_\mu(x)\widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}(x)\widehat{J}_x^{\mu\nu} - \frac{1}{2}\xi_{\mu\nu}(x)\widehat{Q}_x^{\mu\nu} + \partial_\lambda\partial_\mu\beta^\nu(x)\widehat{\Theta}_x^{\lambda\mu\nu} + \ldots\right]$$

Linear terms

Calculation with canonical stress-energy tensor and spin potential



$$\widehat{\rho}_{\rm LE} = \frac{1}{Z}\exp\left[-\int_{\Sigma_{FO}} {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}_C\beta_\nu - \zeta\widehat{j}^\mu(y) + \frac{1}{2}\Omega_{\lambda\nu}\widehat{\mathcal{S}}^{\mu\lambda\nu}\right)\right]$$

$$\begin{aligned} S^{(1)\mu}(p) \;&=\; -\frac{1}{8mN}\int {\rm d}\Sigma\cdot p_+\, n_F(x,p)\left[1-n_F(x,p)\right] \\ &\times\left\{\epsilon^{\mu\nu\lambda\sigma}\Omega_{\nu\lambda}p_\sigma - \frac{2}{p\cdot\hat{t}}\hat{t}_\nu\epsilon^{\mu\nu\lambda\sigma}p_\lambda\left[\left(\xi_{\sigma\rho}+\Omega_{\sigma\rho}-\varpi_{\sigma\rho}\right)p^\rho - \partial_\sigma\zeta\right]\right\}\,, \end{aligned}$$

Confirms previous findings, with an extension to the canonical spin potential (M. Buzzegoli PRC 105 (2022) 4, 044907)

$$\begin{aligned} S^{(2)\mu}_{\rm lin}(p) \;&=\; \frac{1}{4m(p\cdot\hat{t})^2N}\int {\rm d}\Sigma\cdot p_+\, n_F(x,p)\left[1-n_F(x,p)\right]\left(y_\Sigma(0)-x\right)\cdot\hat{t} \\ &\times\hat{t}_\alpha p_\rho\left[\epsilon^{\mu\sigma\alpha\rho}p^\lambda p^\nu\partial_\sigma\xi_{\nu\lambda} + \left(\frac{1}{2}p^\alpha\epsilon^{\mu\nu\lambda\rho} - \epsilon^{\mu\alpha\lambda\rho}p^\nu\right)p^\sigma\partial_\sigma\varpi_{\nu\lambda}\right. \\ &\left.-\epsilon^{\mu\sigma\alpha\rho}p^\lambda\partial_\sigma\partial_\lambda\zeta + \frac{1}{2}\epsilon^{\alpha\nu\lambda\sigma}\partial^\rho(\Omega_{\nu\lambda}-\varpi_{\nu\lambda})(p^\mu p_\sigma - m^2 g^\mu_\sigma)\right]\,, \end{aligned}$$

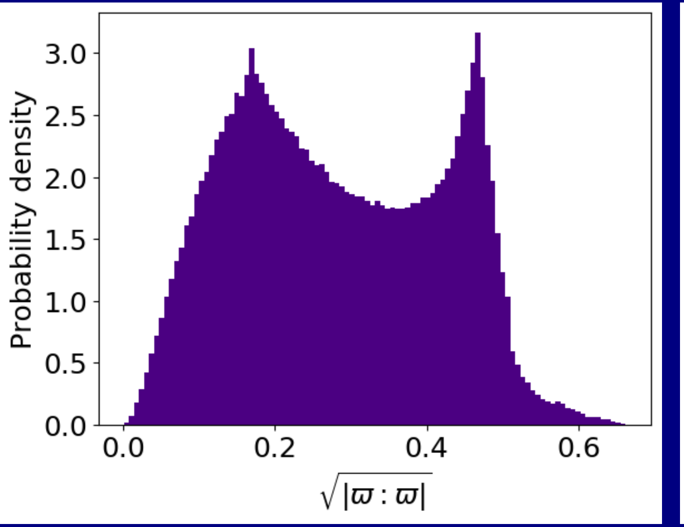
Quadratic terms

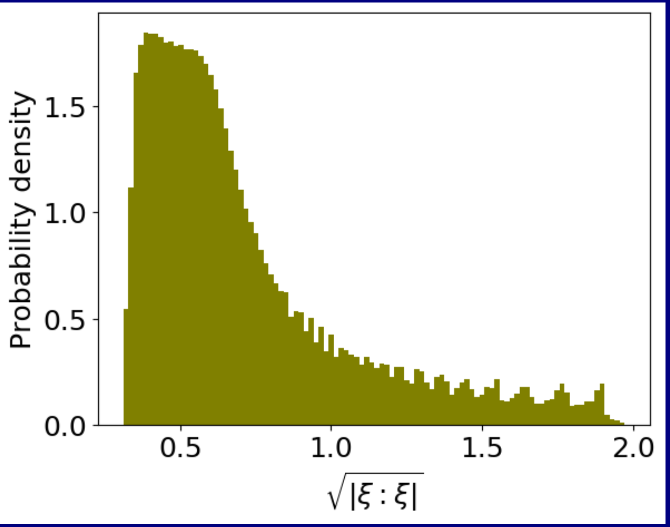
$$S^{(2)\mu}_{\rm quad}(p) = \frac{1}{2}\frac{\int \mathrm{d}\Sigma\cdot p_+\,\mathrm{tr}\left[\gamma^\mu\gamma^5 W_{\rm quad}(x,p)\right]}{\int \mathrm{d}\Sigma\cdot p_+\,\mathrm{tr}[W^{(0)}(x,p)]} - S^{(1)\mu}(p)\frac{\int \mathrm{d}\Sigma\cdot p_+\,\mathrm{tr}[W^{(1)}(x,p)]}{\int \mathrm{d}\Sigma\cdot p_+\,\mathrm{tr}[W^{(0)}(x,p)]},$$

$$\mathrm{tr}\left[\gamma^\mu\gamma^5 W_{\rm quad}(x,p)\right] = \frac{[1-2n_F(x,p)]\,\mathrm{tr}\left[\gamma^\mu\gamma^5 W^{(1)}(x,p)\right]\mathrm{tr}\left[W^{(1)}(x,p)\right]}{[1-n_F(x,p)]\,\mathrm{tr}\left[W^{(0)}(x,p)\right]}.$$

$$W_0(x,p) = \frac{\delta(p^2-m^2)\mathrm{sgn}(p^0)}{(2\pi)^3}(\not{p}+m)n_F(x,p),$$

$$\mathrm{tr}\left[W^{(1)}(x,p)\right] = -\frac{\delta(p^2-m^2)}{(2\pi)^3|p^0|}n_F(x,p)\left[1-n_F(x,p)\right](y^0_\Sigma - x^0)4mp^\lambda\partial_\lambda[p^\sigma\beta_\sigma(x)-\zeta(x)],$$





Expectation: should be small

Amplitude distribution (one entry per cell) at the freeze-out

Numerical computation: sensitivity to initial conditions and bulk viscosity

A. Palermo, F.B., E. Grossi, I. Karpenko, Eur. Phys. J. 84 (2024) 9, 920

Recent hydro calculations of Λ polarization in relativistic heavy ion collisions

S. Alzhrani, S. Ryu, and C. Shen, Phys. Rev. C **106**, 014905 (2022), arXiv:2203.15718 [nucl-th].
F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, and A. Palermo, Phys. Rev. Lett. **127**, 272302 (2021), arXiv:2103.14621 [nucl-th].
B. Fu, S. Y. F. Liu, L. Pang, H. Song, and Y. Yin, Phys. Rev. Lett. **127**, 142301 (2021), arXiv:2103.10403 [hep-ph].
X.-Y. Wu, C. Yi, G.-Y. Qin, and S. Pu, Phys. Rev. C **105**, 064909 (2022), arXiv:2204.02218 [hep-ph].
Z.-F. Jiang, X.-Y. Wu, H.-Q. Yu, S.-S. Cao, and B.-W. Zhang, Acta Phys. Sin. **72**, 072504 (2023).
Z.-F. Jiang, X.-Y. Wu, S. Cao, and B.-W. Zhang, Phys. Rev. C **108**, 064904 (2023), arXiv:2307.04257 [nucl-th].
V. H. Ribeiro, D. Dobrigkeit Chinellato, M. A. Lisa, W. Matioli Serenone, C. Shen, J. Takahashi, and G. Torrieri, Phys. Rev. C **109**, 014905 (2024), arXiv:2305.02428 [hep-ph].

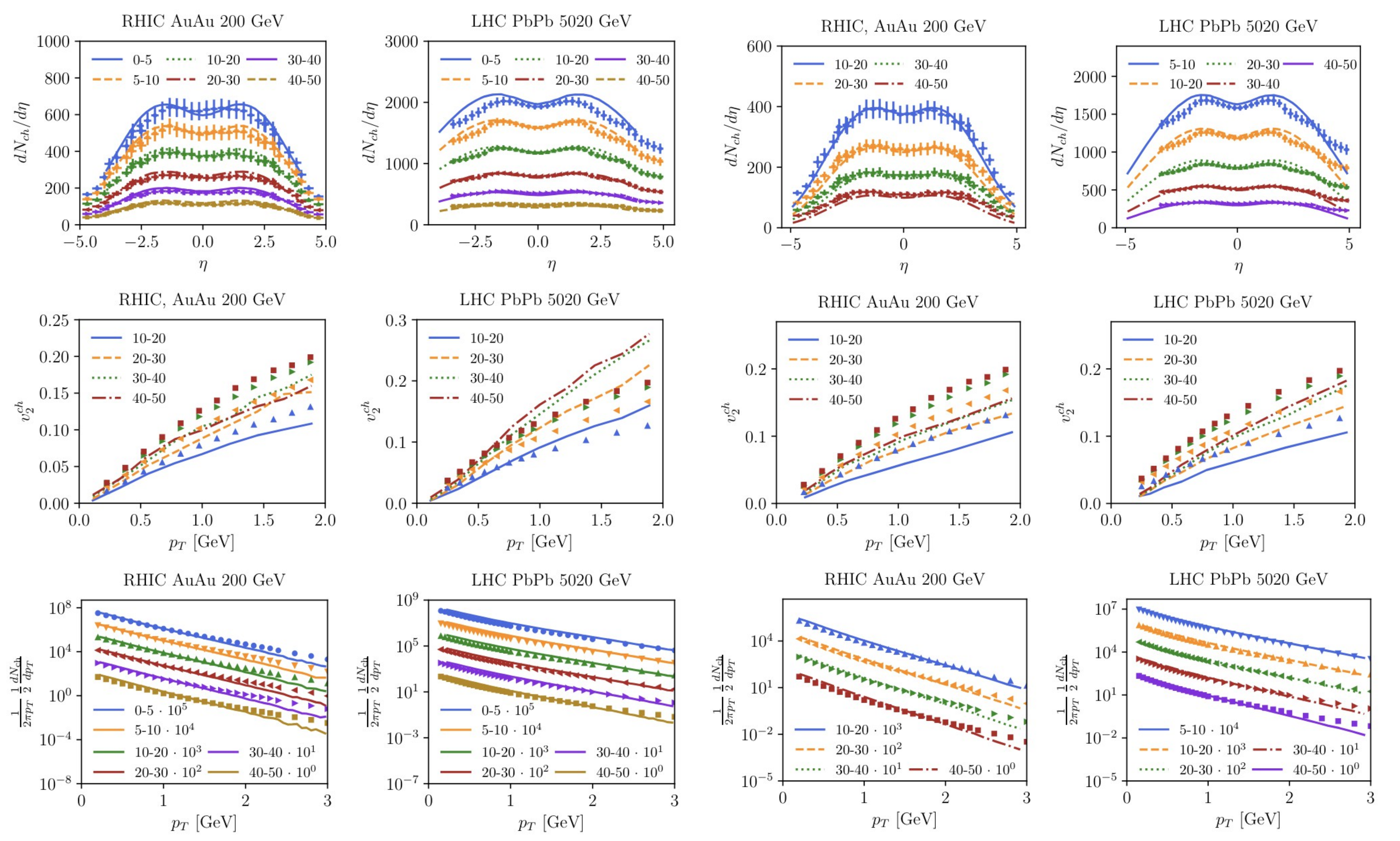
Numerical implementation of 3+1 D causal viscous hydrodynamics (VHLLE) with statistical hadronization and particle rescattering (afterburner SMASH)

Initial state model: SUPERMC (C. Shen et al.), GLISSANDO (Monte-Carlo Glauber)

Polarization transferred to Λ in secondary decays of Σ^0 and Σ^* taken into account

Qualification of the code

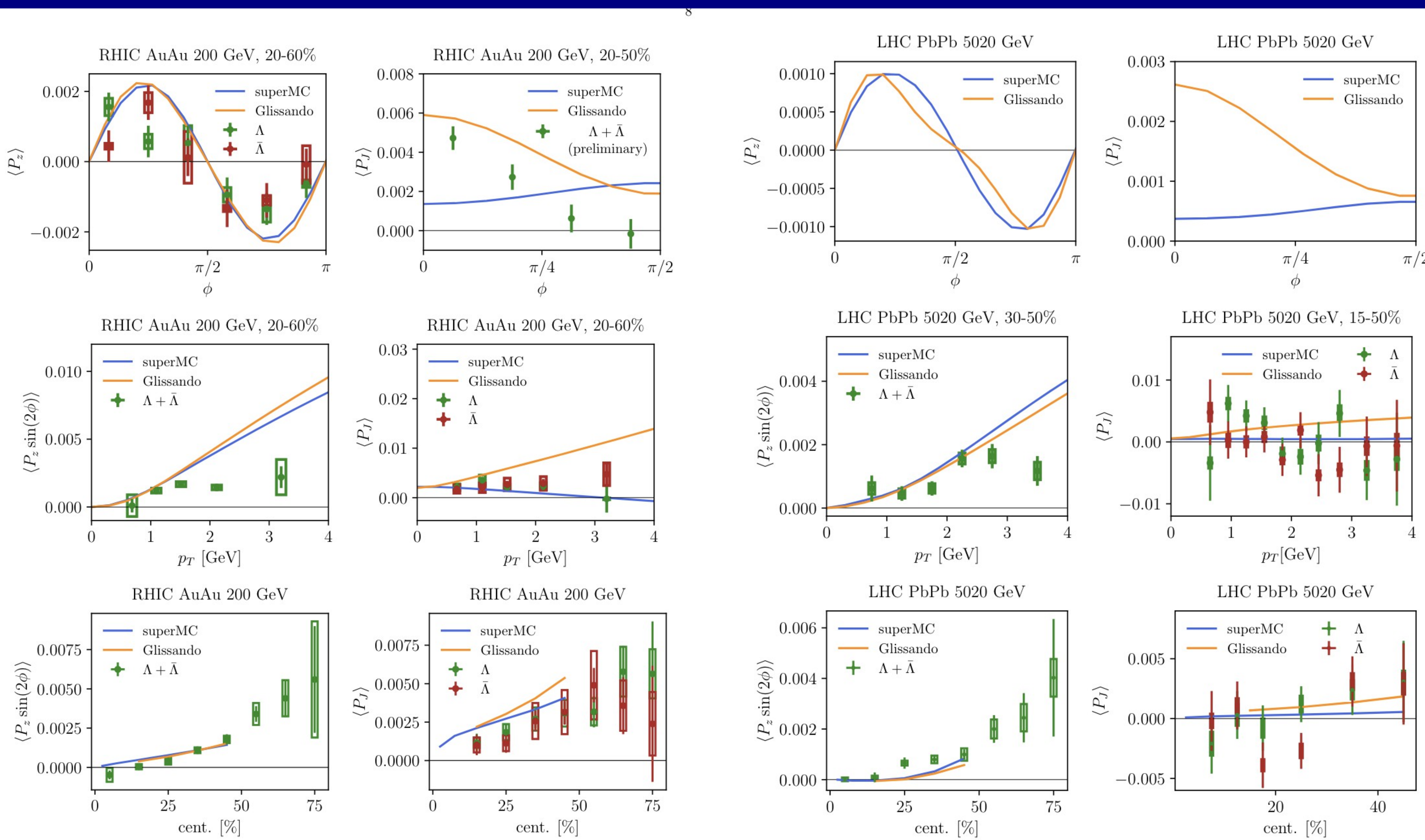
Benchmark distributions



SUPERMC

GLISSANDO

RESULTS



8
RHIC AuAu 200 GeV, 20-60%
superMC
Glissando
Λ
Λ̄
⟨P_z⟩
ϕ
RHIC AuAu 200 GeV, 20-50%
superMC
Glissando
Λ + Λ̄
(preliminary)
⟨P_J⟩
ϕ
LHC PbPb 5020 GeV
superMC
Glissando
⟨P_z⟩
ϕ
LHC PbPb 5020 GeV
superMC
Glissando
⟨P_J⟩
ϕ
RHIC AuAu 200 GeV, 20-60%
superMC
Glissando
Λ + Λ̄
⟨P_z sin(2ϕ)⟩
p_T [GeV]
RHIC AuAu 200 GeV, 20-60%
superMC
Glissando
Λ
Λ̄
⟨P_J⟩
p_T [GeV]
LHC PbPb 5020 GeV, 30-50%
superMC
Glissando
Λ + Λ̄
⟨P_z sin(2ϕ)⟩
p_T [GeV]
LHC PbPb 5020 GeV, 15-50%
superMC
Glissando
Λ
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⟨P_J⟩
p_T[GeV]
RHIC AuAu 200 GeV
superMC
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RHIC AuAu 200 GeV
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LHC PbPb 5020 GeV
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⟨P_z sin(2ϕ)⟩
cent. [%]
LHC PbPb 5020 GeV
superMC
Glissando
Λ
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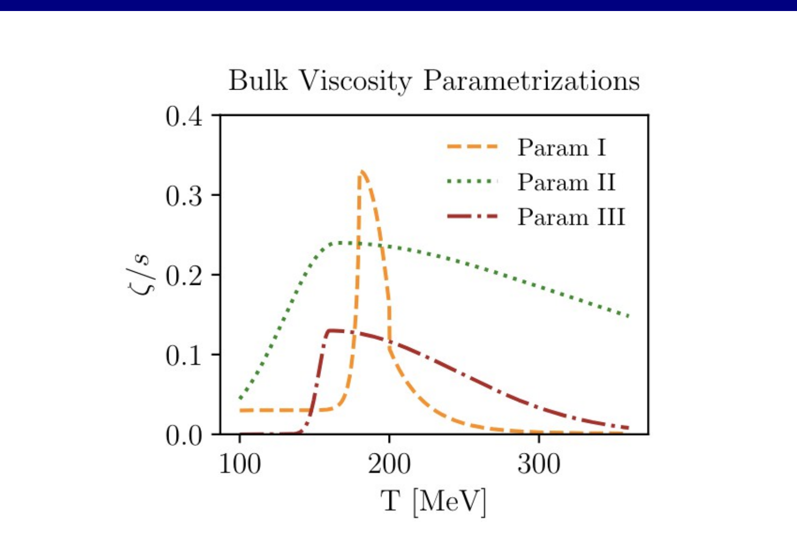
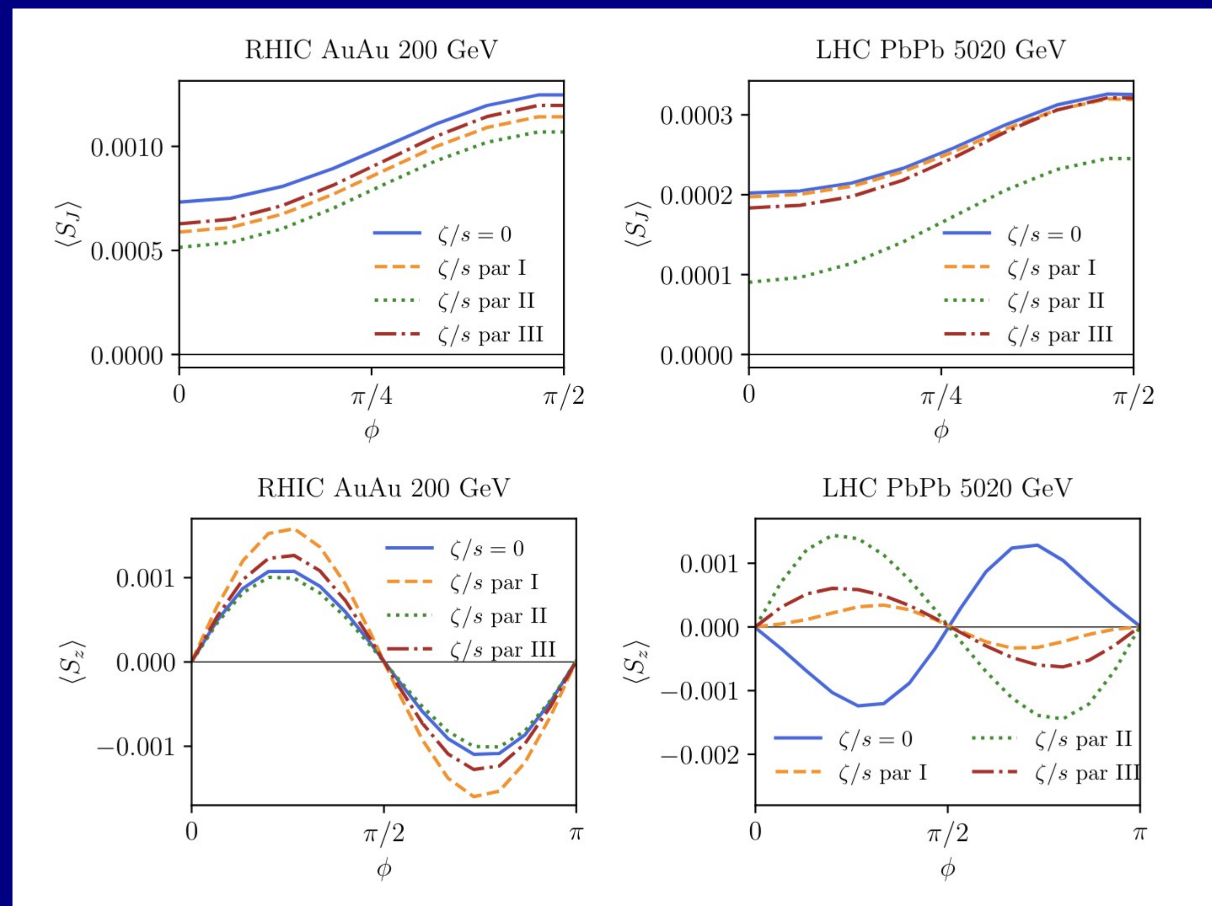
cent. [%]

Sensitivity of polarization to bulk viscosity

While polarization seems not to depend much on shear viscosity, it turns out to be very sensitive to bulk viscosity at the highest LHC energy





Spin-hydro and pseudo-gauge transformations

F. W. Hehl, Rept. Math. Phys. 9 (1976) 55 (see also F. B., L. Tinti Phys. Rev. D 84 (2011) 025013)

$$\widehat{T}'^{\mu\nu} = \widehat{T}^{\mu\nu} + \frac{1}{2}\partial_\alpha \left(\widehat{\Phi}^{\alpha,\mu\nu} - \widehat{\Phi}^{\mu,\alpha\nu} - \widehat{\Phi}^{\nu,\alpha\mu} \right)$$

$$\widehat{\mathcal{S}}'^{\lambda,\mu\nu} = \widehat{\mathcal{S}}^{\lambda,\mu\nu} - \widehat{\Phi}^{\lambda,\mu\nu}$$

They leave the conservation equations and spacial integrals (=generators, or total energy, momentum and angular momentum operators) invariant

EXAMPLE: Belinfante symmetrization

$$\widehat{T}'^{\mu\nu} = \widehat{T}^{\mu\nu} + \frac{1}{2}\partial_\alpha \left(\widehat{\mathcal{S}}^{\alpha,\mu\nu} - \widehat{\mathcal{S}}^{\mu,\alpha\nu} - \widehat{\mathcal{S}}^{\nu,\alpha\mu} \right)$$

$$\widehat{\mathcal{S}}'^{\lambda,\mu\nu} = 0$$

Morale: cannot uniquely separate orbital from spin angular momentum

Free Dirac field:

$$\widehat{T}^{\mu\nu} = \frac{1}{2}\overline{\Psi}\gamma^\mu \overleftrightarrow{\partial}^\nu \Psi$$

$$\widehat{\mathcal{S}}^{\lambda,\mu\nu} = \frac{1}{2}\overline{\Psi}\{\gamma^\lambda, \Sigma^{\mu\nu}\}\Psi = \frac{i}{8}\overline{\Psi}\{\gamma^\lambda[\gamma^\mu,\gamma^\nu]\}\Psi$$

Canonical pseudo-gauge

$$\widehat{T}'^{\mu\nu} = \frac{i}{4}\left[\overline{\Psi}\gamma^\mu \overleftrightarrow{\partial}^\nu \Psi + \overline{\Psi}\gamma^\nu \overleftrightarrow{\partial}^\mu \Psi\right]$$

$$\widehat{\mathcal{S}}'^{\lambda,\mu\nu} = 0$$

Belinfante pseudo-gauge

Spin polarization and spin hydrodynamics

There is no direct relation between spin tensor and spin of the particles.
Spin polarization can be non-vanishing even if there is no spin tensor contributing to the angular momentum current (that is, Belinfante PG).

The spin polarization vector operator does NOT depend on the pseudo-gauge

$$S^\mu(p) = \sum_{i=1}^{3} [p]_i^\mu \mathrm{tr}(D^{1/2}(J_i)\Theta(p)) \qquad \Theta(p)_{sr} = \frac{\mathrm{Tr}(\widehat{\rho}\widehat{a}^\dagger(p)_r \widehat{a}(p)_s)}{\sum_t \mathrm{Tr}(\widehat{\rho}\widehat{a}^\dagger(p)_t \widehat{a}(p)_t)}$$

Spin polarization acquire a dependence on the pseudo-gauge choice because the quantum state CAN depend on the pseudo-gauge: this is the problem

$$\widehat{\rho}_{\mathrm{LE}} = \frac{1}{Z} \exp\left[-\int_\Sigma \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \frac{1}{2}\Omega_{\lambda\nu}\widehat{\mathcal{S}}^{\mu,\lambda\nu} - \zeta\widehat{j}^\mu\right)\right].$$

$$\Omega_{\lambda\nu} \equiv \text{spin potential}$$

Examples: leading order expressions at LTE

M. Buzzegoli, Phys. Rev. C 105 (2022) 4, 044907

1) Belinfante PG

$$S^\mu_{\rm B}(k) \simeq S^\mu_\varpi(k) + S^\mu_\xi(k);$$

$$S^\mu_\varpi(k) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}k_\tau \frac{\int_\Sigma {\rm d}\Sigma\cdot k\, n_{\rm F}\,(1-n_{\rm F})\,\varpi_{\rho\sigma}}{\int_\Sigma {\rm d}\Sigma\cdot k\, n_{\rm F}},$$

$$S^\mu_\xi(k) = -\frac{1}{4m}\epsilon^{\mu\lambda\sigma\tau}\frac{k_\tau k^\rho}{\varepsilon_k} \frac{\int_\Sigma {\rm d}\Sigma\cdot k\, n_{\rm F}\,(1-n_{\rm F})\,\hat{t}_\lambda \xi_{\rho\sigma}}{\int_\Sigma {\rm d}\Sigma\cdot k\, n_{\rm F}},$$

2) Canonical PG

$$S^\mu_{\rm C}(k) \simeq S^\mu_\varpi(k) + S^\mu_\xi(k) + \Delta^{\rm C}_\Theta S^\mu(k).$$

$$\Delta^{\rm C}_\Theta S^\mu(k) = \frac{\epsilon^{\lambda\rho\sigma\tau}\hat{t}_\lambda(k^\mu k_\tau - \eta^\mu_\tau m^2)}{8m\varepsilon_k} \times \frac{\int_\Sigma {\rm d}\Sigma(x)\cdot k\, n_{\rm F}\,(1-n_{\rm F})\,(\varpi_{\rho\sigma}-\Omega_{\rho\sigma})}{\int_\Sigma {\rm d}\Sigma\cdot k\, n_{\rm F}}.$$

3) GLW-HW PG (spin tensor conserved)

$$S^\mu_{\rm GLW,HW}(k) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau}k_\tau \frac{\int_\Sigma {\rm d}\Sigma\cdot k\, n_{\rm F}\,(1-n_{\rm F})\,\Omega_{\rho\sigma}}{\int_\Sigma {\rm d}\Sigma\cdot k\, n_{\rm F}}.$$

Pseudo-gauge transformations of currents

$$\widehat{j}'^{\mu} = \widehat{j}^{\mu} + \partial_\lambda \widehat{M}^{\lambda\mu}$$

$$n_\mu j^\mu = \mathrm{Tr}\left(\widehat{\rho}_{\mathrm{LE}} \widehat{j}^{\mu}\right)$$

The LE quantum state depends on the PG

Example for the Dirac field

$$\bar{\Psi}\gamma^\mu\Psi \to \bar{\Psi}\gamma^\mu\Psi + C\partial_\lambda\left(\bar{\Psi}[\gamma^\lambda,\gamma^\mu]\Psi\right)$$

Why has nobody raised such an issue? Because QED is a gauge theory which is not invariant under a PG transformation of the currents. For instance, the Hamiltonian, integrating the gauge-invariant stress-energy tensor, is not PG invariant

$$\widehat{T}^{\mu\nu} = \frac{i}{4}\bar{\Psi}\gamma^\mu\overleftrightarrow{\partial}^\nu\Psi - \frac{q}{2}\bar{\Psi}\gamma^\mu\Psi\widehat{A}^\nu + (\mu\leftrightarrow\nu) \equiv \widehat{T}_F^{\mu\nu} - \frac{1}{2}\left(\widehat{j}^\mu\widehat{A}^\nu + \widehat{j}^\nu\widehat{A}^\mu\right)$$

In fact, there is a measurable field which is not invariant under a PG transformation of the current:

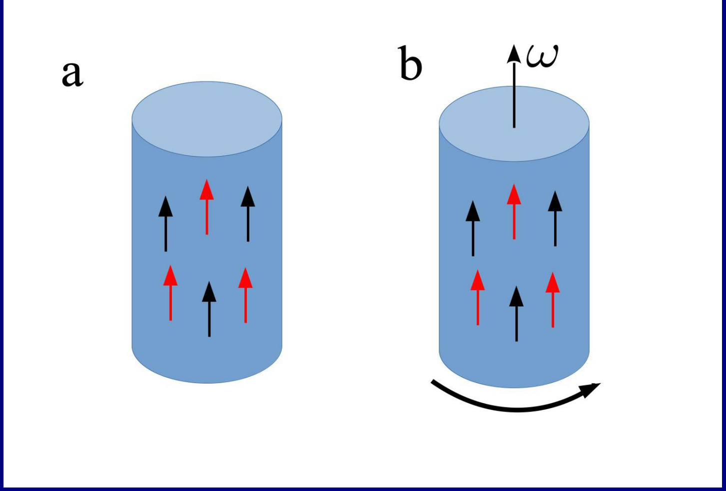
$$\partial_\lambda \widehat{F}^{\lambda\nu} = \widehat{j}^\nu$$

Does the same apply to the stress-energy tensor PG transformations?

In minimal GR is it believed that Belinfante SET is the source of geometry

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = 8\pi G \langle \widehat{T}_B^{\mu\nu} \rangle$$

Spin tensor is not the source of a field, only in extended theories of gravity



a) Non-rotating globally neutral meta-stable state with both particles and anti-particles polarized and zero velocity

b) Global equilibrium state

F. B., W. Florkowski and E. Speranza, Phys. Lett. B 789 (2019), 419-425

M. Hongo, X. G. Huang, M. Kaminski, M. Stephanov and H. U. Yee, JHEP 11 (2021), 150

For a quantum state to represent a), a spin tensor is needed.

Question: can we prepare a state like a *???*

Summary

• Calculation of spin polarization corrections at local equilibrium (non-dissipative contribution) at first and second-order in the gradient expansion

• Numerical results in hydrodynamic simulation up to 1st order terms show the high sensitivity of spin polarization to bulk viscosity

• Remarks on relativistic spin hydrodynamics

Shear and bulk viscosity of the QGP

Measuring the shear and bulk viscosity of the Quark Gluon Plasma is one of the most important objectives


nature physics
LETTERS
https://doi.org/10.1038/s41567-019-0611-8

Bayesian estimation of the specific shear and bulk viscosity of quark–gluon plasma

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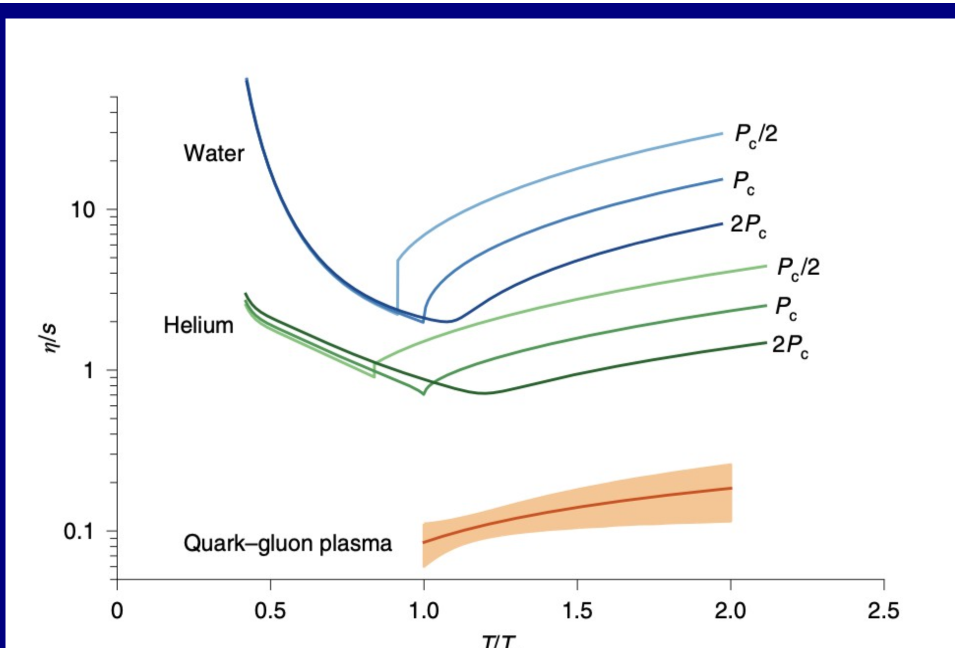
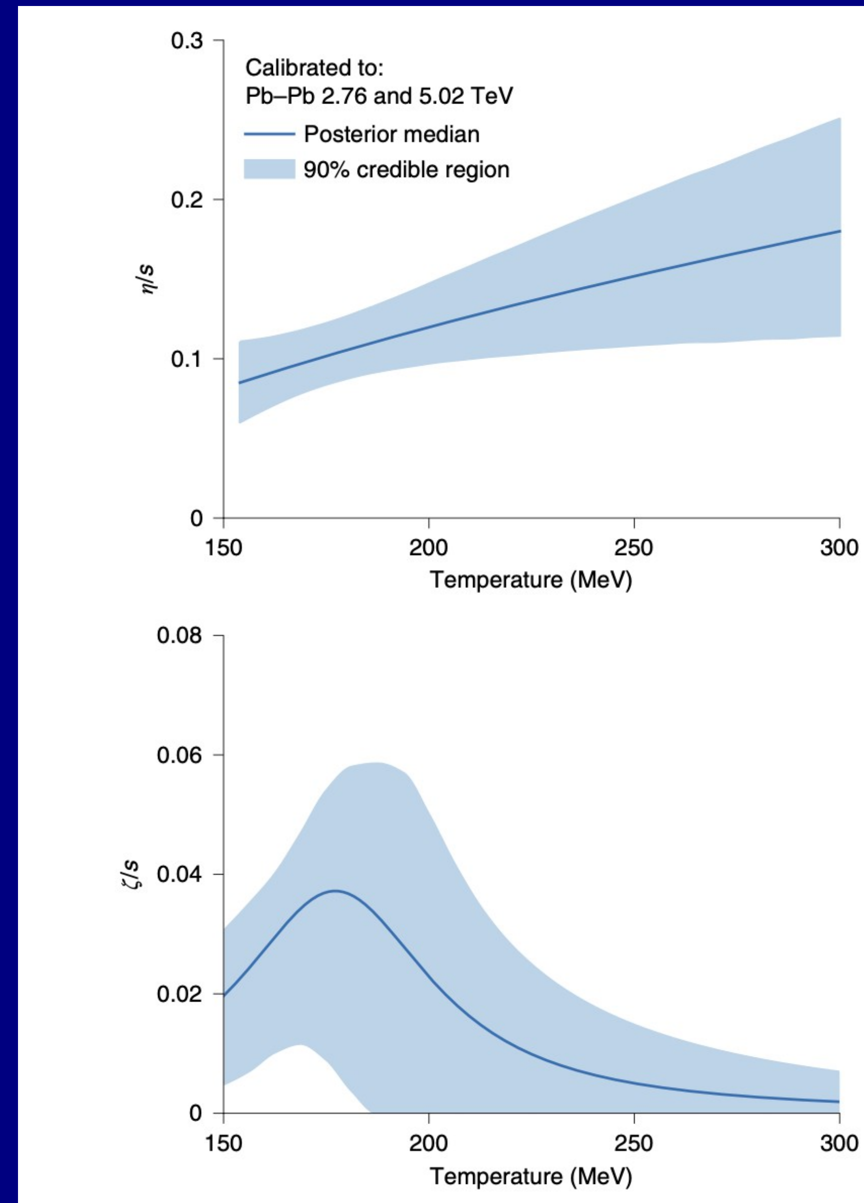


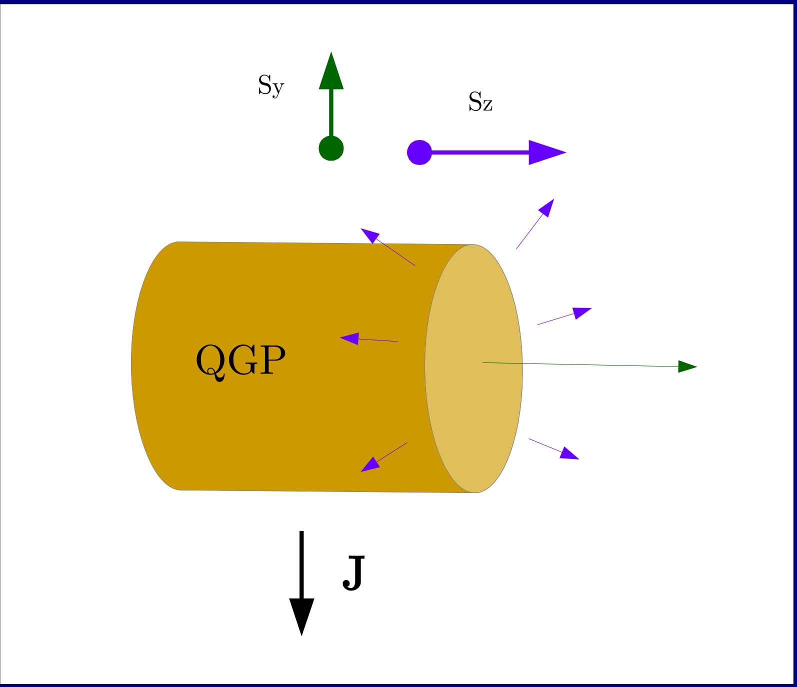


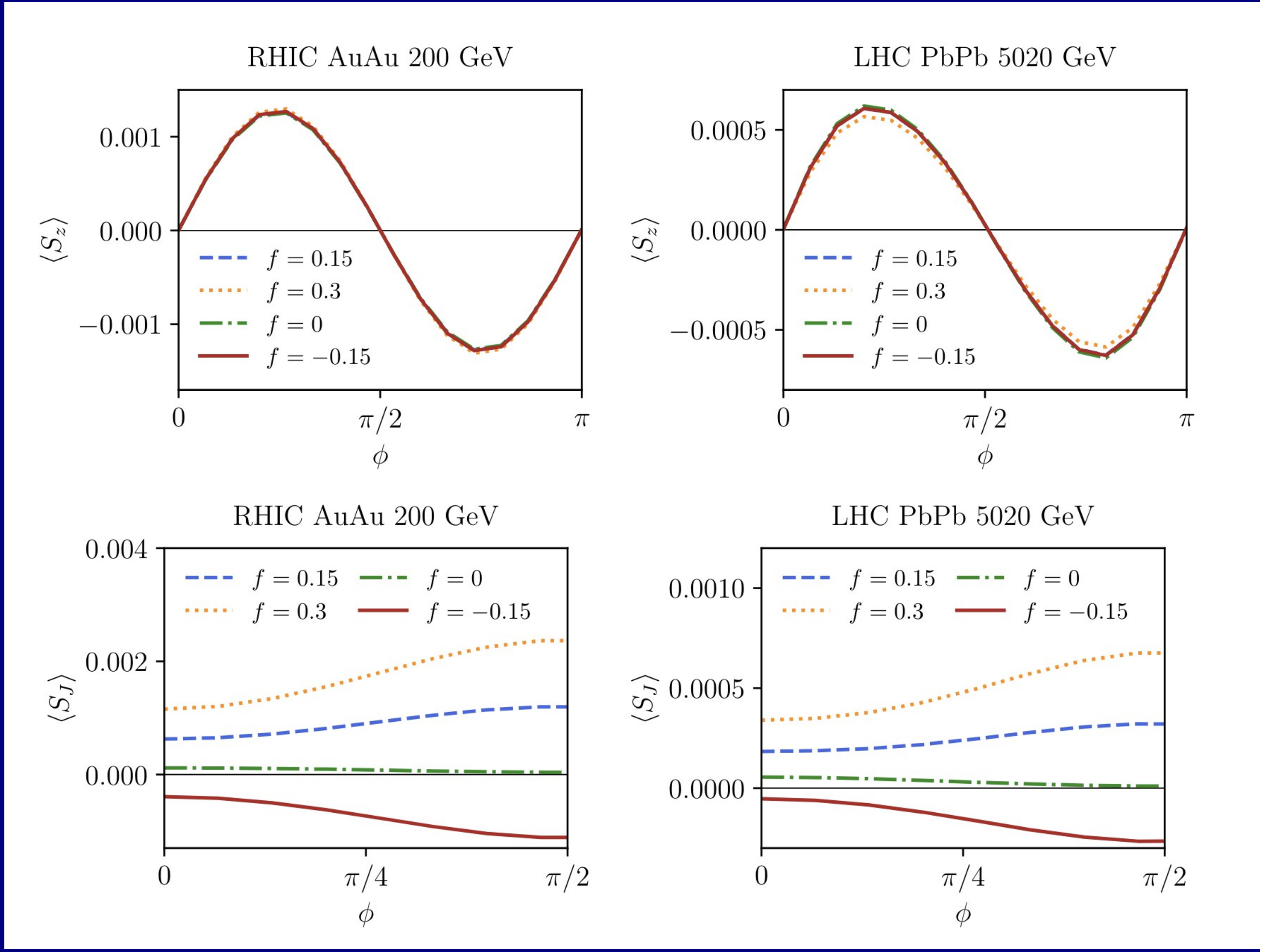
Fit by using momentum-related observables

Sensitivity to initial longitudinal flow

Variation of SUPERMC flow parameter

$$T^{\tau\tau} = \rho \cosh(f\, y_{CM})$$
$$T^{\tau\eta} = \frac{\rho}{\tau} \sinh(f\, y_{CM})$$





Pseudo-gauge dependence analysis

If the spin tensor is non-zero (non-Belinfante) angular momentum constraints must be additionally implemented

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z} \exp\left[- \int_\Sigma {\rm d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu - \frac{1}{2}\Omega_{\lambda\nu}\widehat{\mathcal{S}}^{\mu,\lambda\nu} - \zeta \widehat{j}^\mu \right)\right].$$

$$\Omega_{\lambda\nu} \equiv \text{spin potential}$$

Can to Bel transformation

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z} \exp\left[- \int_\Sigma {\rm d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu}\beta_\nu - \frac{1}{2}(\Omega_{\lambda\nu} - \varpi_{\lambda\nu})\widehat{\mathcal{S}}^{\mu,\lambda\nu} + \frac{1}{2}\xi_{\lambda\nu}\left(\widehat{\mathcal{S}}^{\lambda,\mu\nu} + \widehat{\mathcal{S}}^{\nu,\mu\lambda}\right) - \zeta \widehat{j}^\mu \right)\right],$$

which is the same as the previous (in form)

$$\widehat{\rho}_{\rm LE} = \frac{1}{Z} \exp\left[- \int_\Sigma {\rm d}\Sigma_\mu \left(\widehat{T}_B^{\mu\nu}\beta_\nu - \zeta \widehat{j}^\mu \right)\right],$$

only if

$$\varpi = \Omega$$

$$\xi_{\mu\nu} = \frac{1}{2}(\nabla_\mu\beta_\nu + \nabla_\nu\beta_\mu) = 0$$

GLOBAL THERMODYNAMIC EQUILIBRIUM

Spin polarization and spin hydrodynamics

In QFT in flat space-time, from translational and Lorentz invariance one obtains two conserved Noether currents:

$$\widehat{T}^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi^a)} \partial^\nu \Psi^a - g^{\mu\nu} \mathcal{L}$$

$$\widehat{\mathcal{S}}^{\lambda,\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\lambda \Psi^a)} D^A (J^{\mu\nu})^a_b \Psi^b$$

$$\partial_\mu \widehat{T}^{\mu\nu} = 0$$

$$\partial_\lambda \widehat{\mathcal{J}}^{\lambda,\mu\nu} = \partial_\lambda \left(\widehat{\mathcal{S}}^{\lambda,\mu\nu} + x^\mu \widehat{T}^{\lambda\nu} - x^\nu \widehat{T}^{\lambda\mu} \right) = \partial_\lambda \widehat{\mathcal{S}}^{\lambda,\mu\nu} + \widehat{T}^{\mu\nu} - \widehat{T}^{\nu\mu} = 0$$

However, the Lagrangian density can be changed and so, are those tensors objectively defined?
(well known problem already for the EM stress-energy tensor)

Why do we have a dependence on Σ ?

$$\widehat{J}_x^{\mu\nu} = \int \mathrm{d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) - (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

$$\widehat{Q}_x^{\mu\nu} = \int_{\Sigma_{FO}} \mathrm{d}\Sigma_\lambda (y-x)^\mu \widehat{T}_B^{\lambda\nu}(y) + (y-x)^\nu \widehat{T}_B^{\lambda\mu}(y)$$

The divergence of the integrand of $J^{I\ K}$ vanishes, therefore it does not depend on the integration hypersurface (it is a constant of motion) and

$$\widehat{\Lambda} \widehat{J}_x^{\mu\nu} \widehat{\Lambda}^{-1} = \Lambda_\alpha^{-1\mu} \Lambda_\beta^{-1\nu} \widehat{J}_x^{\alpha\beta}$$

The divergence of the integrand of $Q^{I\ K}$ does not vanish, therefore it does depend on the integration hypersurface and

$$\widehat{\Lambda} \widehat{Q}^{\mu\nu} \widehat{\Lambda}^{-1} \neq \Lambda_\alpha^{-1\mu} \Lambda_\beta^{-1\nu} \widehat{Q}^{\alpha\beta}$$

Exact formulae at global equilibrium

Resummation of the power series expansion in (constant) thermal vorticity
A. Palermo, F.B., Eur. Phys. J. Plus 138 (2023) 6, 547

Global equilibrium density operator

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-b_\mu \widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\widehat{J}^{\mu\nu}\right]$$

$$\langle \widehat{O} \rangle = \mathrm{Tr}\left(\widehat{\rho}\widehat{O}\right)$$

$$\beta^\mu(x) = b^\mu + \varpi^{\mu\nu}x_\nu \equiv \frac{u^\mu}{T}$$

1) Analytic continuation to imaginary thermal vorticity

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-b_\mu \widehat{P}^\mu - \frac{i}{2}\phi_{\mu\nu}\widehat{J}^{\mu\nu}\right]$$

3) Factorization of the density operator

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-\widetilde{b}_\mu(\phi)\widehat{P}^\mu\right] \exp\left[-i\frac{\phi_{\mu\nu}}{2}\widehat{J}^{\mu\nu}\right] \equiv \frac{1}{Z} \exp\left[-\widetilde{b}_\mu(\phi)\widehat{P}^\mu\right]\widehat{\Lambda}$$

$$\widetilde{b}^\mu(\phi) = \sum_{k=0}^{\infty} \frac{1}{(k+1)!} (\underbrace{\phi^\mu{}_{\alpha_1}\phi^{\alpha_1}{}_{\alpha_2}\ldots\phi^{\alpha_{k-1}}{}_{\alpha_k}}_{k \text{ times}})b^{\alpha_k}$$

4) TEV of creation/annihilation quadratic combination obtained by iterations

$$\langle \widehat{a}_s^\dagger(p)\widehat{a}_t(p')\rangle = 2\varepsilon' \sum_{n=1}^{\infty} (-1)^{2S(n+1)}\delta^3(\Lambda^n \boldsymbol{p} - \boldsymbol{p}')D^S(W(\Lambda^n, p))_{ts}\mathrm{e}^{-\widetilde{b}\cdot\sum_{k=1}^n \Lambda^k p}$$

$$W(\Lambda, p) = [\Lambda p]^{-1}\Lambda[p]$$

5) Calculate the Wigner function

$$W(x,k) = \frac{1}{(2\pi)^3}\int \frac{\mathrm{d}^3p}{2\varepsilon}\sum_{n=1}^{\infty}(-1)^{n+1}\mathrm{e}^{-n\tilde{\beta}(in\phi)\cdot p}\times$$
$$\left[\mathrm{e}^{-in\frac{\phi:\Sigma}{2}}(m+\not{p})\delta^4\left(k-(\Lambda^n p + p)/2\right) + (m-\not{p})\mathrm{e}^{in\frac{\phi:\Sigma}{2}}\delta^4\left(k+(\Lambda^n p+p)/2\right)\right]$$

Spin vector

$$S^\mu(p) = \frac{1}{2}\frac{\int \mathrm{d}\Sigma\cdot p\ \mathrm{tr}\left(\gamma^\mu\gamma_5 W_+(x,p)\right)}{\int \mathrm{d}\Sigma\cdot p\ \mathrm{tr}\left(W_+(x,p)\right)}$$

$$S^\mu(p) = \frac{1}{2m}\frac{\sum_{n=1}^\infty (-1)^{n+1}\mathrm{e}^{-n\widetilde{b}(in\phi)\cdot p}\mathrm{tr}\left(\gamma^\mu\gamma_5\mathrm{e}^{-in\frac{\phi:\Sigma}{2}}\not{p}\right)\delta^3(\Lambda^n p - p)}{\sum_{n=1}^\infty (-1)^{n+1}\mathrm{e}^{-n\widetilde{b}(in\phi)\cdot p}\mathrm{tr}\left(\mathrm{e}^{-in\frac{\phi:\Sigma}{2}}\right)\delta^3(\Lambda^n p - p)}$$

The series can be resummed:

$$S^\mu(p) = \frac{-i\xi^\mu}{2\sqrt{-\xi^2}}\frac{\sin\left(\sqrt{-\xi^2}/2\right)}{\cos\left(\sqrt{-\xi^2}/2\right) + \mathrm{e}^{-b\cdot p+\zeta}}$$

and analytically continued

$$S^\mu(p) = \frac{1}{2}\frac{\theta^\mu}{\sqrt{-\theta^2}}\frac{\sinh\left(\frac{\sqrt{-\theta^2}}{2}\right)}{\cosh\left(\frac{\sqrt{-\theta^2}}{2}\right) + e^{-b\cdot p+\zeta}}$$

$$\xi^\mu \mapsto \theta^\mu = -\frac{1}{2m}\epsilon^{\mu\nu\rho\sigma}\varpi_{\nu\rho}p_\sigma$$

Extending the formula to local equilibrium with ϖ(x)

$$S^\mu(p) = -\frac{1}{4m}\epsilon^{\mu\nu\rho\sigma} p_\sigma \frac{\int \mathrm{d}\Sigma \cdot p\, n_F \frac{\varpi_{\nu\rho}}{\sqrt{-\theta^2}} \frac{\sinh\left(\sqrt{-\theta^2}/2\right)}{\cosh\left(\sqrt{-\theta^2}/2\right) + \mathrm{e}^{-b\cdot p+\zeta}}}{\int \mathrm{d}\Sigma \cdot p\, n_F}$$