Chiral Vortical Instability

in collaboration with S. Wang, K. Hattori, and X.-G. Huang

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LIP, Lisbon

Holographic perspectives on chiral transport and spin dynamics, ECT*, 2025

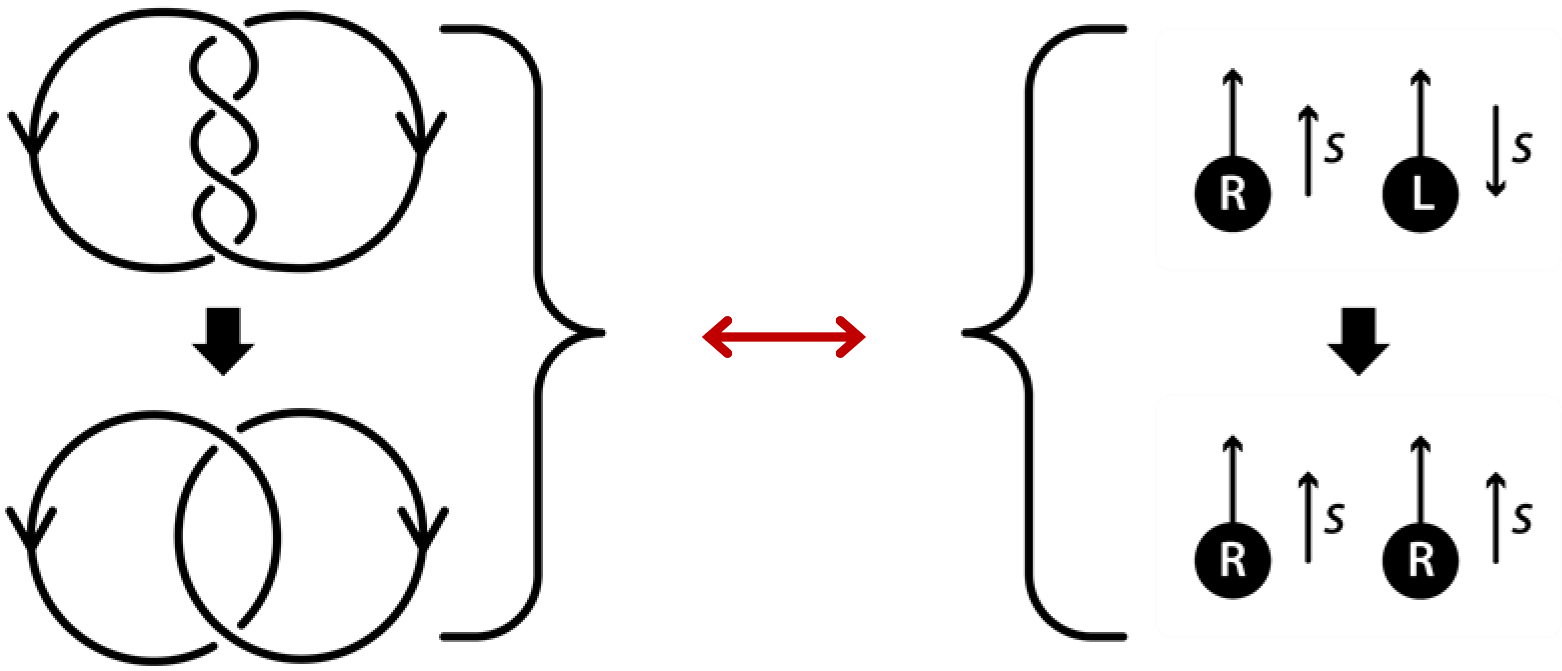
Generalized axial charge

$$\partial_\alpha J_5^\alpha = C\mathbf{E}\cdot\mathbf{B}$$

$$\downarrow$$

$$\partial_t\left(N_5 + \frac{C}{2}\int_{\mathbf{x}} \mathbf{A}\cdot\mathbf{B}\right) = 0$$

Generalized axial charge



J. Erdmenger, M. Haack, M. Kaminski, A. Yarom, JHEP, 2009
N. Banerjee et. al., JHEP, 2011
D.T. Son, P. Surowka, PRL, 2009

Generalized axial charge

EOMs

$$\begin{cases} \partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda \\ \partial_\mu J^\mu = C E \cdot B \end{cases}$$

Constitutive relations

$$T^{\lambda\alpha} = (\epsilon + P) u^\lambda u^\alpha + P \eta^{\lambda\alpha} + \tau^{\lambda\alpha}$$

$$J^\lambda = n u^\lambda + \nu^\lambda$$

$$\downarrow$$

$\tilde{F}^{\mu\nu} u_\nu$

$$J^\lambda = n u^\lambda + \sigma E^\lambda - \sigma T P^{\lambda\alpha} \partial_\alpha \frac{\mu}{T} + \xi_\omega \omega^\lambda + \xi_B B^\lambda$$

$\frac{1}{2} \epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta$

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$\frac{1}{2}\epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta$

$\downarrow$

$$\partial_t \left(N_5 + \int_{\mathbf{x}} \xi_\omega \, \mathbf{v} \cdot \omega + \int_{\mathbf{x}} \xi_B \, \mathbf{v} \cdot \mathbf{B} + \frac{C}{2} \int_{\mathbf{x}} \mathbf{A} \cdot \mathbf{B} \right) = 0$$

ideal MHD limit

A. Avdoshkin, V. Kirilin, AS, V.I. Zakharov, PLB, 2016
N. Yamamoto, PRD, 2016
A. Avkhadiev, AS, PRD, 2017
V. Kirilin, AS, PRD, 2017
P. Wiegmann, A. Abanov, JHEP, 2022
C. Manuel, J. Torres-Rincon, PRD, 2023

Generalized axial charge

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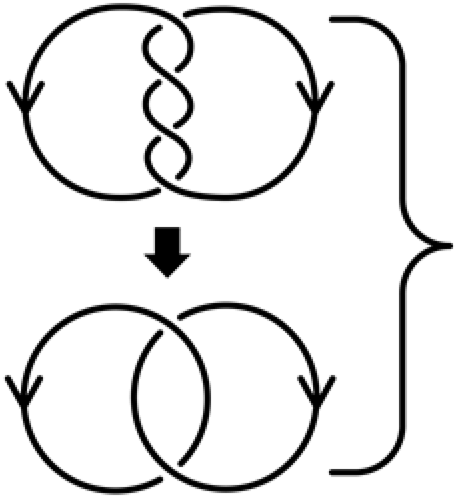

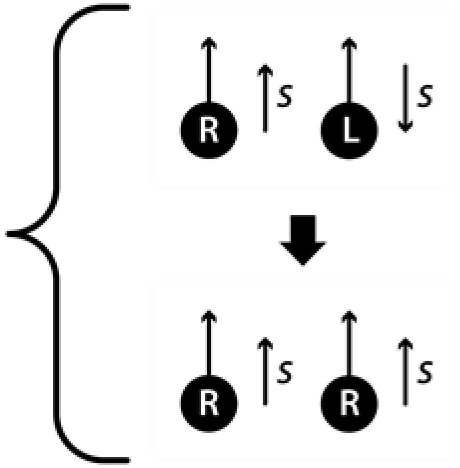

Rubakov (1986); M. Joyce, M. E. Shaposhnikov (1997); … Y. Akamatsu, N. Yamamoto, (2013)
A. Avdoshkin, V. Kirilin, AS, V.I. Zakharov, PLB, 2016
S. Wang, X.-G. Huang, PRD, 2024

Chiral instabilities

$$\partial_t \left(N_5 + \int_{\mathbf{x}} \xi_\omega \overset{\text{???}}{\mathbf{v} \cdot \omega} + \int_{\mathbf{x}} \xi_B \overset{\text{CMVI}}{\mathbf{v} \cdot \mathbf{B}} + \frac{C}{2} \int_{\mathbf{x}} \overset{\text{CPI}}{\mathbf{A} \cdot \mathbf{B}} \right) = 0$$

$$\updownarrow$$

$$N_5 \sim \mathcal{H}_{fh} \sim \mathcal{H}_{mfh} \sim \mathcal{H}_{mh}$$

A. Avkhadiev, AS, PRD, 2017
N. Yamamoto, PRD, 2017
M. N. Chernodub, A. Cortijo, K. Landsteiner, PRD, 2018
Y. Hirono, D. Kharzeev, Y. Yin, PRL, 2016
V. Kirilin, AS, PRD, 2017

Chiral instabilities

- If the generalized axial charge picture works, the helicities and chirality could mutually transform
- How could that happen microscopically? (e.g. CVE for light)
- This picture not only constraints the dynamics but also indicates that the system can exhibit various instabilities and non-linear chiral effects (and it is not necessary to start with μ_5)

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Chiral instabilities

- If the generalized axial charge picture works, the helicities and chirality could mutually transform
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- This picture not only constraints the dynamics but also indicates that the system can exhibit various instabilities and non-linear chiral effects (and it is not necessary to start with μ_5)
- **A simpler bit: Is there a vortical channel of chiral instability when you start at $\mu_5 \neq 0$?**

Linearized hydrodynamics

$$T_{\mu\nu} = w u_\mu u_\nu + P g_{\mu\nu} - \eta \sigma_{\mu\nu} + \sum \lambda_i \Sigma^{(i)}_{\mu\nu}$$

$$J^\alpha = n u^\alpha - \kappa P^{\alpha\beta} \partial_\beta \frac{\mu}{T} + \xi_\omega \omega^\alpha + \sum \zeta_i \Theta^{(i)}_\alpha$$

$$\Sigma^{(1)}_{\mu\nu} = P_{\mu\alpha} P_{\nu\beta} u^\lambda \partial_\lambda \sigma^{\alpha\beta} - \frac{1}{3} P_{\mu\nu} P^{\alpha\beta} u^\lambda \partial_\lambda \sigma_{\alpha\beta} \,,$$

$$\Sigma^{(2)}_{\mu\nu} = P_{\mu\alpha} P_{\nu\beta} \partial^\alpha \partial^\beta \frac{\mu}{T} - \frac{1}{3} P_{\mu\nu} P^{\alpha\beta} \partial_\alpha \partial_\beta \frac{\mu}{T} \,,$$

$$\Sigma^{(3)}_{\mu\nu} = \frac{1}{2} P_{\mu\alpha} P_{\nu\beta} (\partial^\alpha \omega^\beta + \partial^\beta \omega^\alpha) - \frac{1}{3} P_{\mu\nu} P^{\alpha\beta} \partial_\alpha \omega_\beta \,,$$

$$\Theta^{(1)}_\alpha = P_{\alpha\beta} \partial_\lambda \sigma^{\lambda\beta} \,, \qquad \Theta^{(2)}_\alpha = P_{\alpha\beta} \partial_\lambda \omega^{\lambda\beta}$$

No instability in LF at 1st order

$$\left(\omega - i \frac{\eta}{w} k^2 - \frac{\lambda_1}{w} \omega k^2 \right) \delta \mathbf{v}_\perp + \frac{k^2}{4w} \lambda_3 (\mathbf{k} \times \delta \mathbf{v}_\perp) = 0$$

B. Sahoo, H.-U. Yee, PLB, 2010
D. Kharzeev, H.-U. Yee, PRD, 2011

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$$\omega = ik^2\frac{\eta \pm \frac{k}{4}\lambda_3}{w - \lambda_1 k^2}$$

Chiral Shear Wave

Chiral shear waves

CSWs: $$\omega = ik^2 \frac{\eta \pm \frac{k}{4}\lambda_3}{w - \lambda_1 k^2} \longrightarrow k > \frac{4\eta}{|\lambda_3|}$$

unstable for

Chiral shear waves

unstable for

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Other frames: $$u_\nu \to u_\nu + \alpha_1 P_{\nu\gamma}\partial^\gamma \frac{\mu}{T} + \alpha_2 \omega_\nu + \alpha_3 \Theta_\nu^{(1)} + \alpha_4 \Theta_\nu^{(2)}$$

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$$\sigma^{\mu\nu} = P^{\mu\alpha}P^{\nu\beta}(\partial_\alpha u_\beta + \partial_\beta u_\alpha) - \frac{2}{3}P^{\mu\nu}\partial^\alpha u_\alpha$$

$$\downarrow$$

$$\Sigma^{(3)}_{\mu\nu} = \frac{1}{2}P_{\mu\alpha}P_{\nu\beta}(\partial^\alpha\omega^\beta + \partial^\beta\omega^\alpha) - \frac{1}{3}P_{\mu\nu}P^{\alpha\beta}\partial_\alpha\omega_\beta$$

Chiral shear waves

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$\downarrow$

$$\omega = ik^2 \frac{\eta\left(1 \mp \frac{1}{2}\alpha_2 k\right) \pm \frac{k}{4}\lambda_3}{w(1 \mp \frac{1}{2}\alpha_2 k) - (\lambda_1 + \alpha_3 w + \frac{1}{2}\alpha_4 w)k^2}$$

Chiral vortical instability

- For sufficiently large k, CSWs are unstable
- Their leading behavior is frame-independent
- λ_3 can be fixed by the entropy current analysis $\longrightarrow$ $$\lambda_3 = -\frac{4\eta}{w}\left(\frac{1}{3}C\mu^3 + \beta\mu T^2\right)$$

  controls mixed anomaly
- Without a microscopic theory, one can only argue that there are situations where $k = 4\eta/|\lambda_3|$ remains within the hydrodynamic regime:

  a. $$2\sqrt[4]{6\pi^2/N_f} < k/w^{1/4} \ll 1$$

  b. $$4\eta < \frac{1}{3}C_\chi\mu^3 \Rightarrow w > \max(\eta k, |\lambda_3|k^2)$$
- Is it related to the spatially modulated phase for a large CS coupling in holographic plasma?

Chiral vortical instability

- We see that the modes of second-order chiral hydrodynamics are consistent with the presence of CVI, though it is pushed beyond the hydrodynamic regime (large k modes grow faster)
- CSWs can become unstable within the hydrodynamic regime under specific conditions, e.g. low viscosity and large anomalous coefficient
- CVI indicates the system's tendency to form spatially modulated flow configurations, likely consisting of self-linked vortices
- Apparently, CVI enhances the development of the turbulent cascade

Chiral vortical instability

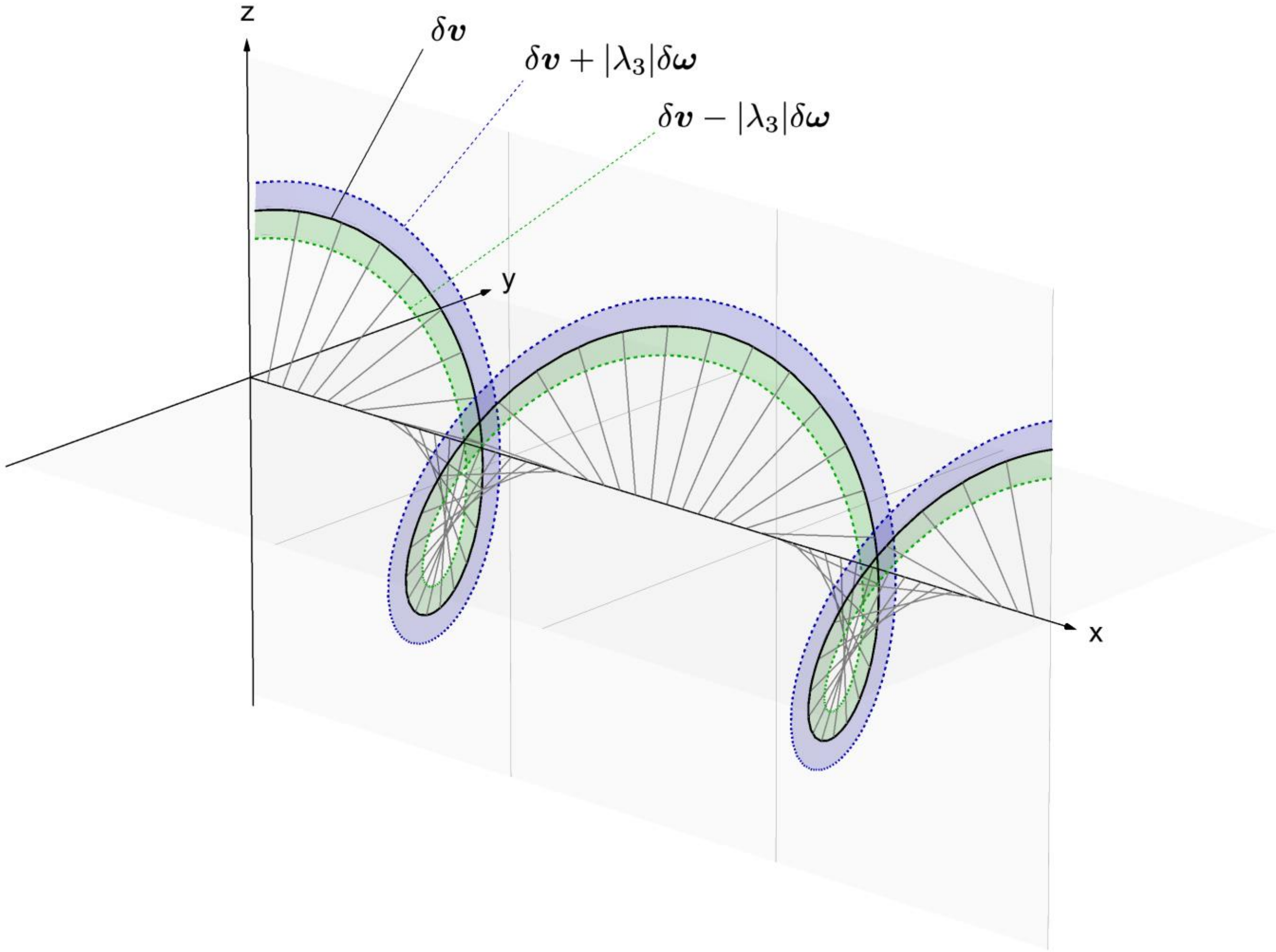
$$\left(\omega - i\frac{\eta}{w}k^2 - \frac{\lambda_1}{w}\omega k^2\right)\delta\mathbf{v}_\perp + \frac{k^2}{4w}\lambda_3(\mathbf{k}\times\delta\mathbf{v}_\perp) = 0$$

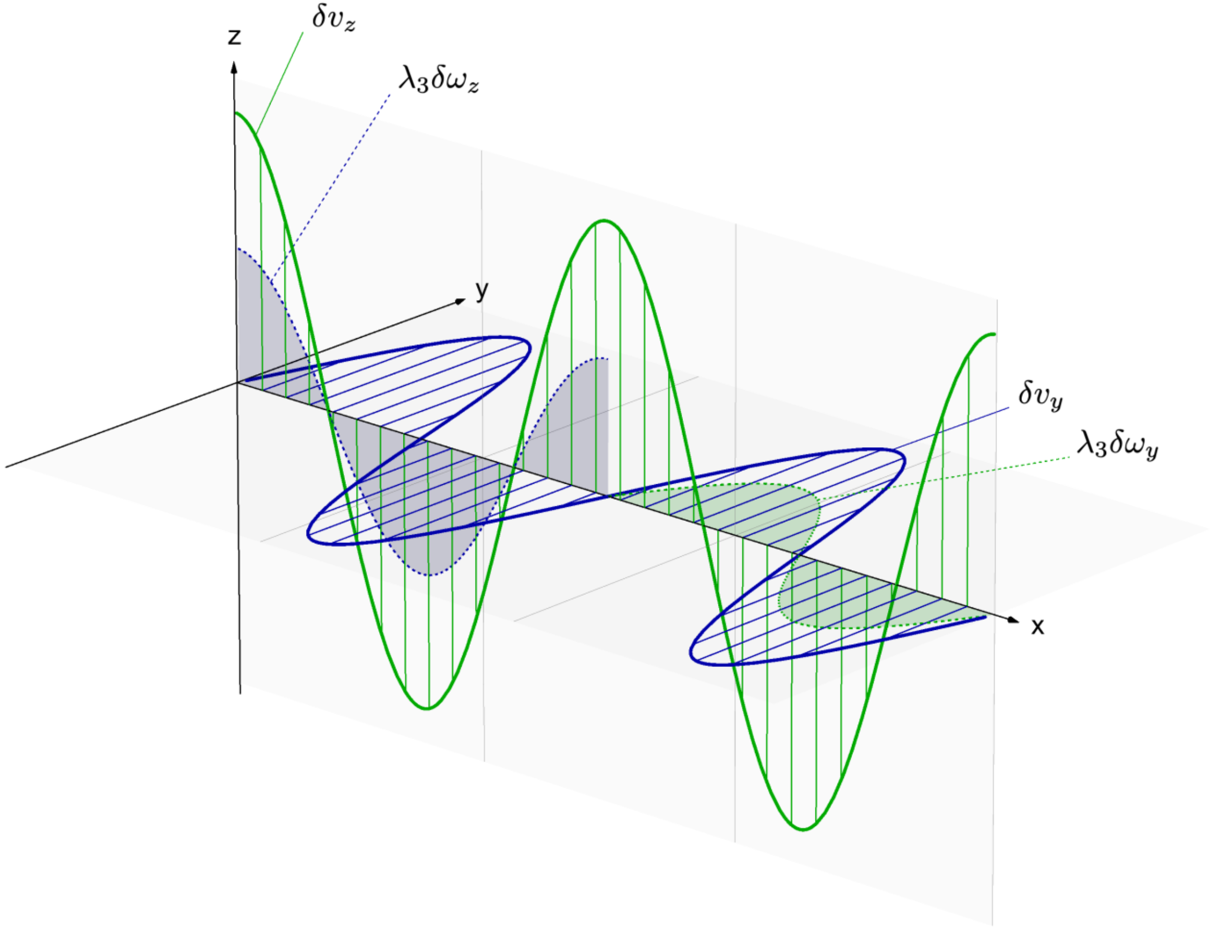
$$\downarrow$$

GABC flow

$$\delta\mathbf{v} = v_0 \begin{pmatrix} 0 \\ \sin kx_1 \\ \cos kx_1 \end{pmatrix} e^{-t\,k^2\,\frac{\eta - \frac{k}{4}\lambda_3}{w - \lambda_1 k^2}}$$

Chiral vortical instability





Chiral vortical instability

linearized

GABC: $$\partial_\alpha T^{\alpha\beta} = 0$$

Global rotation: $$\partial_\alpha T^{\alpha\beta} \sim \lambda_3 \boldsymbol{\omega}^2 \omega^\beta$$

Chiral vortical instability

$$\partial_\mu \left(-\beta_\nu T^{\mu\nu} + \frac{1}{2} S^{\mu\lambda\nu} \omega_{\lambda\nu} + \alpha J^\mu \right) = 0$$

$$\downarrow$$

$$u_\mu / T$$

$$\partial_\alpha T^{\alpha\beta} = 0 \longrightarrow \begin{array}{l} \partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0 \\ \partial_\mu \alpha = 0 \end{array}$$

$$\mu / T$$

Chiral vortical instability

$$k = \frac{4\eta}{|\lambda_3|}\sqrt{1 - v_0^2}$$

$$\mathbf{v} = v_0 \begin{pmatrix} 0 \\ \sin kx_1 \\ \cos kx_1 \end{pmatrix}$$

a solution

$$\mathbf{v} = \begin{pmatrix} -\omega x_2 \\ \omega x_1 \\ 0 \end{pmatrix}$$

not anymore

$$T^{\mu\nu}(\partial_\mu \beta_\nu + \partial_\nu \beta_\mu) = 0$$

Chiral vortical instability

CPI: $$i\mathbf{k} \times \mathbf{B} = \xi_B \mathbf{B}$$ $$k = |\xi_B|$$

- self-similar GABC fields
- $k < \xi_B$ (inversed cascade)
- $N_5 \rightarrow H_{mh}$

CVI: $$\lambda_3(\mathbf{k} \times \delta\mathbf{v}_\perp) = 4i\eta\delta\mathbf{v}_\perp$$ $$k = \frac{4\eta}{|\lambda_3|}\sqrt{1 - v_0^2}$$

- self-similar GABC fields
- $k > 4\eta/\gamma|\lambda_3|$ (direct cascade)
- $N_5 \rightarrow H_{fh}$

Summary

- Chiral media tend to generate helical flows at short distances, and the initial homogeneous state is unstable due to CVI

- CSWs may become unstable within the hydrodynamic regime, suggesting that CVI persists at higher orders

- Global rotation of chiral matter destabilizes, favoring helical solutions

- Elementary static GABC flows solve the full nonlinear dynamics, shedding light on CVE development

- Phenomenological implications: spin polarization in HIC, rotation of astrophysical and cosmological systems, etc.