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Vortical waves in a chiral fluid

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Holographic perspectives on chiral transport and spin dynamics
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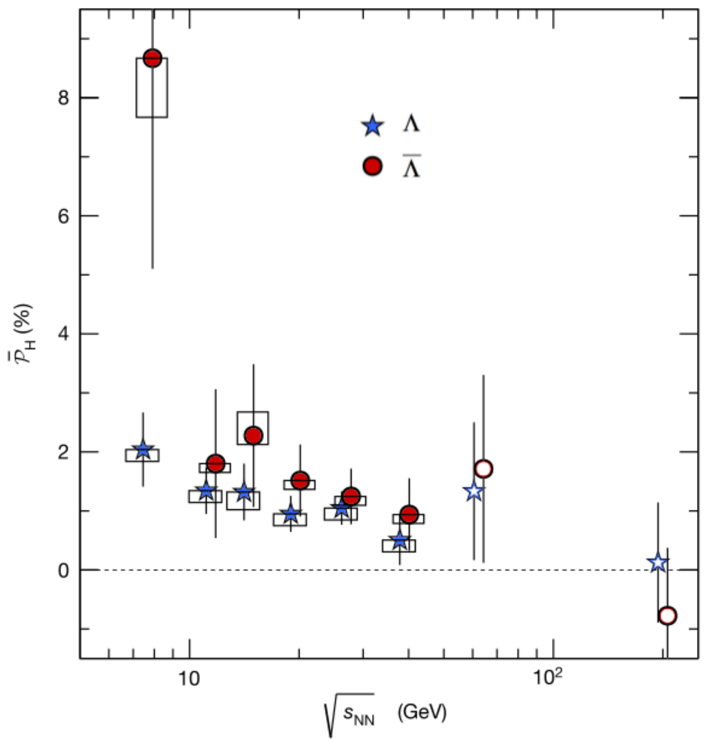
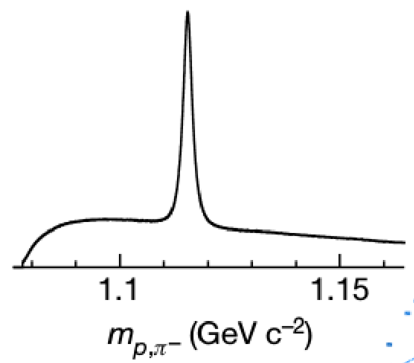
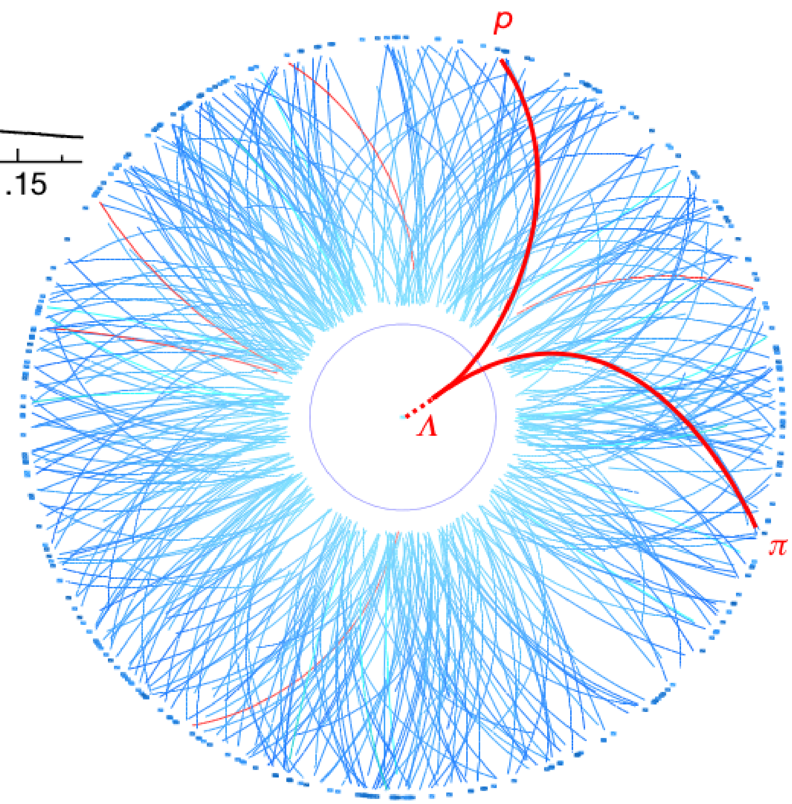
Outline

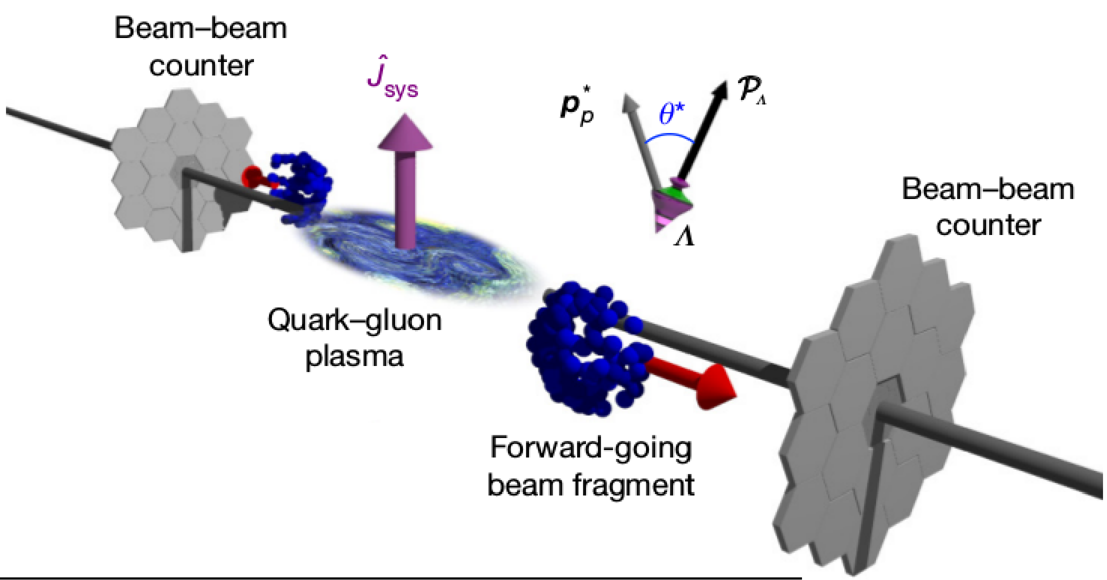
Quark-gluon plasma: Polarisation of Λ-hyperons[3]



- $\overline{\mathcal{P}}_H \equiv$ polarization along $\hat{J}_{\text{sys}}$.
- $|\omega| \approx k_B T(\overline{\mathcal{P}}_{\Lambda'} + \overline{\mathcal{P}}_{\overline{\Lambda}'})/\hbar$.

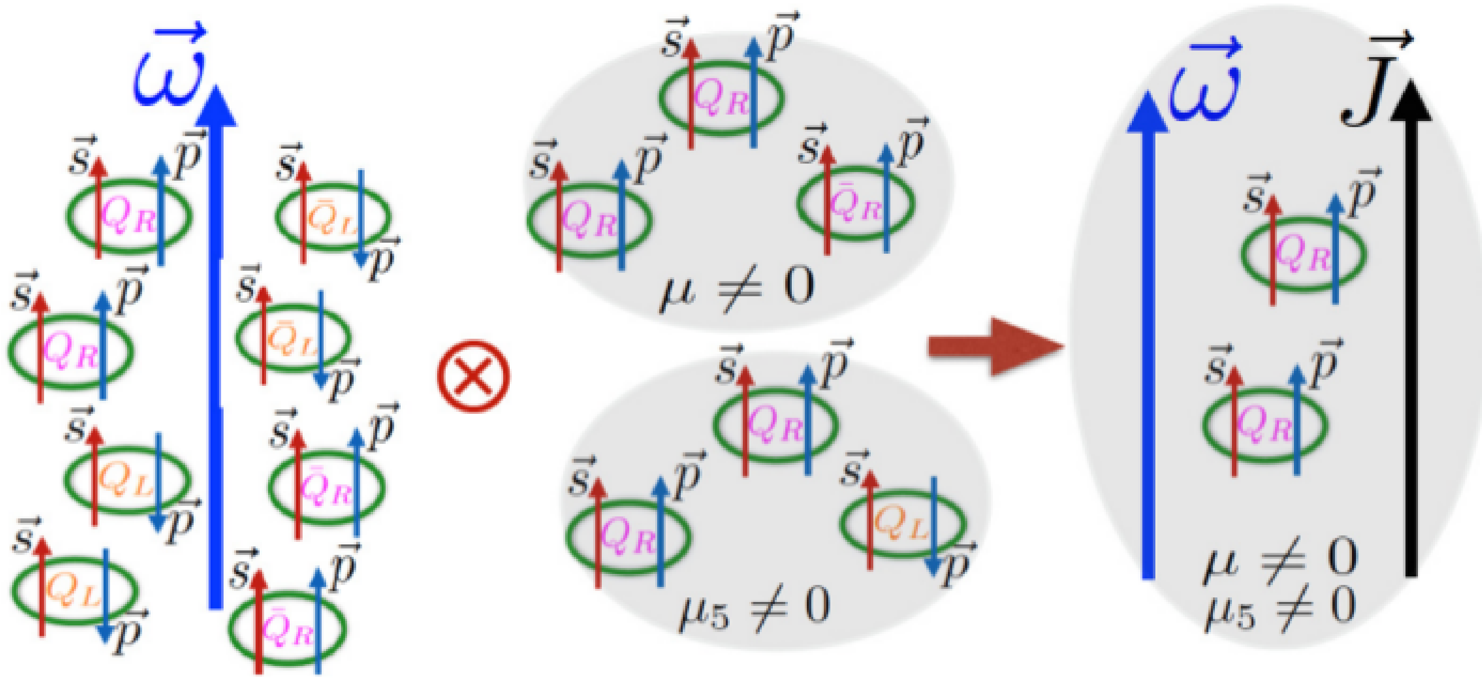






[3] STAR Collaboration, Nature **548** (2017) 62–65.

Polarisation mechanism: Chiral vortical effect (CVE)[1]



Non-vanishing vorticity $\boldsymbol{\omega}$ induces electric $(\mathbf{J}_V)$ and chiral $(\mathbf{J}_A)$ currents via CVE:

$$\mathbf{J}_V = \sigma_V \boldsymbol{\omega}, \qquad \sigma_V = \frac{\mu_V \mu_A}{\pi^2},$$

$$\mathbf{J}_A = \sigma_A \boldsymbol{\omega}, \qquad \sigma_A = \frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2}{2\pi^2}.$$

$\mathbf{J}_A \neq 0$ even when $\mu_A = 0$!

[1] D. E. Kharzeev *et al.*, Nucl. Phys. **88** (2016) 1.

V/A/H charges for (not so) free fermions

- The Dirac field is described by

$$\mathcal{L}_{\text{free}} = \frac{i}{2}(\overline{\psi}\gamma^\mu\partial_\mu\psi - \partial_\mu\overline{\psi}\gamma^\mu\psi), \qquad i\partial\!\!\!/\psi = 0. \tag{1}$$

- The (free) theory supports (at least) three conserved currents:

$$J^\mu_V = \overline{\psi}\gamma^\mu\psi, \qquad J^\mu_A = \overline{\psi}\gamma^\mu\gamma^5\psi, \qquad J^\mu_H = \overline{\psi}\gamma^\mu h\psi + \overline{h\psi}\gamma^\mu\psi. \tag{2}$$

- The vector current is always conserved:

$$\partial_\mu J^\mu_V = 0. \tag{3}$$

- The axial current cons. is broken by (effective) mass and by anomaly:

$$\partial_\mu J^\mu_A = 2im\bar{\psi}\gamma^5\psi - \frac{e^2}{16\pi^2}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} \tag{4}$$

- The helicity current cons. is broken by HVPA (helicity-violating pair annihilation) processes:

$$\partial_\mu J^\mu_H = -\frac{1}{\tau_H}Q_H + ??, \tag{5}$$

where we leave ?? for possible (unknown) anomalies.

Helicity number violation (HVPA)

- $e^+_R e^-_L \to e^+_L e^-_R$ violates helicity: $Q^i_A = Q^f_A = 0$ vs. $Q^i_H = -2 = -Q^f_H$.
- In QED, the HVPA cross section is [Peskin & Schroeder]

$$\frac{d\sigma}{d\Omega}(e^+_R e^-_L \to e^+_L e^-_R) = \frac{\alpha^2}{4E^2_{\text{cm}}}(1-\cos\theta_{cm})^2, \tag{6}$$

where $\alpha = e^2/4\pi$ while E_{cm} and θ_{cm} are related to Mandelstam s and t via

$$s = (p+k)^2 = E^2_{cm}, \qquad t = (p-p')^2 = -\frac{1}{2}E^2_{cm}(1-\cos\theta_{cm}), \tag{7}$$

with (p,k) and (p',k') the incoming and outgoing momenta, respectively.
- In QCD, the HVPA processes take place via gluon exchange:

$$\frac{d\sigma}{d\Omega}(q^i_R \bar{q}^j_L \to q^{i'}_L \bar{q}^{j'}_R) = \frac{\alpha^2_{QCD}}{4E^2_{\text{cm}}}(1-\cos\theta_{cm})^2 \sum_{a,b} t^a_{ji} t^a_{j'i'} t^b_{ij} t^b_{i'j'}, \tag{8}$$

where t^a_{ij} $(1 \le a \le 8)$ are the SU(3) generators, while (i,j) and (i',j') are the colour indices of the initial and final quarks.

Kinetic model: Collision term

[VEA & MC, EPJC **82** (2023)]

- Consider a helically-imballanced state described by

$$f^{\text{eq};\varsigma}_{\mathbf{p},\lambda} = \left[\exp\left(\frac{p\cdot u - \boldsymbol{q}_{\varsigma,\lambda}\cdot\boldsymbol{\mu}}{T}\right) + 1\right]^{-1}, \quad \boldsymbol{q}_{\varsigma,\lambda}\cdot\boldsymbol{\mu} = \varsigma\mu_V + 2\lambda\mu_A + 2\varsigma\lambda\mu_H. \tag{9}$$

- We seek to compute the helicity relaxation time, defined via

$$\frac{dQ_H}{dt} \simeq -\frac{Q_H}{\tau_H}, \quad Q_H = g\sum_{\varsigma,\lambda}\int dP(u\cdot p) f^{\text{eq};\varsigma}_{\mathbf{p},\lambda}, \tag{10}$$

with $dP = d^3p/[(2\pi)^3 E_{\mathbf{p}}]$ and $g = N_c N_f$.

- Assuming $p^\mu \partial_\mu f^\varsigma_{\mathbf{p},\lambda} = C[f]$, dQ_H/dt selects just the HVPA $2\to 2$ processes:

$$\begin{aligned}\frac{dQ_H}{dt} = g\sum_{\lambda,\varsigma} 2\varsigma\lambda \int dP dK dP' dK' \delta^4(p+k-p'-k') s (2\pi)^6 \\ \times [f^\varsigma_{\mathbf{p}',-\lambda} f^{-\varsigma}_{\mathbf{k}',\lambda} \tilde{f}^\varsigma_{\mathbf{p},\lambda} \tilde{f}^{-\varsigma}_{\mathbf{k},-\lambda} - f^\varsigma_{\mathbf{p},\lambda} f^{-\varsigma}_{\mathbf{k},-\lambda} \tilde{f}^\varsigma_{\mathbf{p}',-\lambda} \tilde{f}^{-\varsigma}_{\mathbf{k}',\lambda}] \\ \times N_f \sum_{i',j,j'} \frac{d\sigma}{d\Omega}(q^{\varsigma,i}_{\mathbf{p},\lambda} q^{-\varsigma,j}_{\mathbf{k},-\lambda} \to q^{\varsigma,i'}_{\mathbf{p}',-\lambda} q^{-\varsigma,j'}_{\mathbf{k}',\lambda}),\end{aligned} \tag{11}$$

where $\tilde{f}^\varsigma_{\mathbf{p},\lambda} = 1 - f^\varsigma_{\mathbf{p},\lambda}$ etc are the Pauli blocking factors.

Helicity relaxation time τ_H [VEA & MC, EPJC **82** (2023)]

- We now consider that the plasma is charge-neutral ($\mu_V = 0$) and slightly polarized, (small μ_H), such that

$$f^{\varsigma}_{\mathbf{p},\lambda} \simeq f_{0\mathbf{p}} + 2\lambda\varsigma\beta\mu_H f_{0\mathbf{p}}\tilde{f}_{0\mathbf{p}}, \qquad f_{0\mathbf{p}} = [e^{\beta E_{\mathbf{p}}} + 1]^{-1}, \quad \tilde{f}_{0\mathbf{p}} = [1 + e^{-\beta E_{\mathbf{p}}}]^{-1}. \tag{12}$$

- In this case, $Q_H \simeq g\mu_H/3\beta^2$ and $\frac{dQ_H}{dt} = -\frac{Q_H}{\tau_H}$, where

$$\tau_H^{-1} = \frac{8}{3}(2\pi)^6 g\alpha^2_{\text{QCD}}\beta^3 \int dPdKdP'dK'(1-\cos\theta_{cm})^2\delta^4(p+k-p'-k') \times f_{0\mathbf{p}}f_{0\mathbf{k}}\tilde{f}_{0\mathbf{p}'}\tilde{f}_{0\mathbf{k}'}(\tilde{f}_{0\mathbf{p}} + f_{0\mathbf{p}'}). \tag{13}$$

- The momentum integrals appearing above can be performed, eventually leading to

$$\tau_H = 0.392 \times \frac{\pi^3\beta}{N_f\alpha^2_{\text{QCD}}} \simeq \left(\frac{250\ \text{MeV}}{k_B T}\right)\left(\frac{1}{\alpha_{\text{QCD}}}\right)^2\left(\frac{2}{N_f}\right) \times 4.80\ \text{fm}/c. \tag{14}$$

- Unexpectedly, $\tau_H \simeq 4.80\,\text{fm}/c \gg \tau_A \simeq 0.25\ \text{fm}/c$, giving helicity imbalance a chance to survive in the hot QGP. [N. Astrakhantsev et al, PRD **102** (2020) 054516]

V/A/H charges in QCD

- Chiral and helicity imbalance can be modelled in LSM_q:

$$\mathcal{L} = \mathcal{L}_q + \mathcal{L}_{\mathcal{M}}, \quad \mathcal{L}_q = \bar{\psi}(i\partial\!\!\!/ - g\sigma + \mu_V \gamma^0 + \mu_A \gamma^0 \gamma^5 + 2\mu_H \gamma^0 h)\psi, \tag{15}$$

with $\mathcal{L}_{\mathcal{M}} = \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - V(\sigma)$ and $V(\sigma) = \frac{\lambda}{4}(\sigma - v^2)^2 - h\sigma$.

- The energy eigenvalues read

$$p_{0,\lambda}^{(\varsigma)}(\mathbf{p}) = -\mu_V - 2\lambda\mu_H + \varsigma\sqrt{m_q^2 + (|\mathbf{p}| - 2\lambda\mu_A)^2}, \quad m_q = g\sigma. \tag{16}$$

- The grand potential of the model reads

$$\Phi = -\frac{T}{V}\ln\mathcal{Z} = V(\sigma) + \Phi_q^{\mathrm{zp}} + \Phi_q^{\beta},$$

$$\Phi_q^{\mathrm{zp}} = -\frac{N_c N_f}{2}\sum_{\sigma,\lambda}\int\frac{d^3p}{(2\pi)^3}\varsigma p_{0,\lambda}^{(\varsigma)}(\mathbf{p}) = -N_c N_f \int \frac{dp\, p^2}{2\pi^2}\sum_{\lambda=\pm\frac{1}{2}}\sqrt{m_q^2 + (p - 2\lambda\mu_A)}$$

$$\Phi_q^{\beta} = -TN_c N_f \sum_{\sigma,\lambda}\int\frac{d^3p}{(2\pi)^3}\ln(1 + e^{-\varsigma p_{0,\lambda}^{(\varsigma)}(\boldsymbol{p})/T}). \tag{17}$$

- When $\mu_A \neq 0$, Φ_q^{zp} is medium-dependent!

QCD with chiral chemical potential

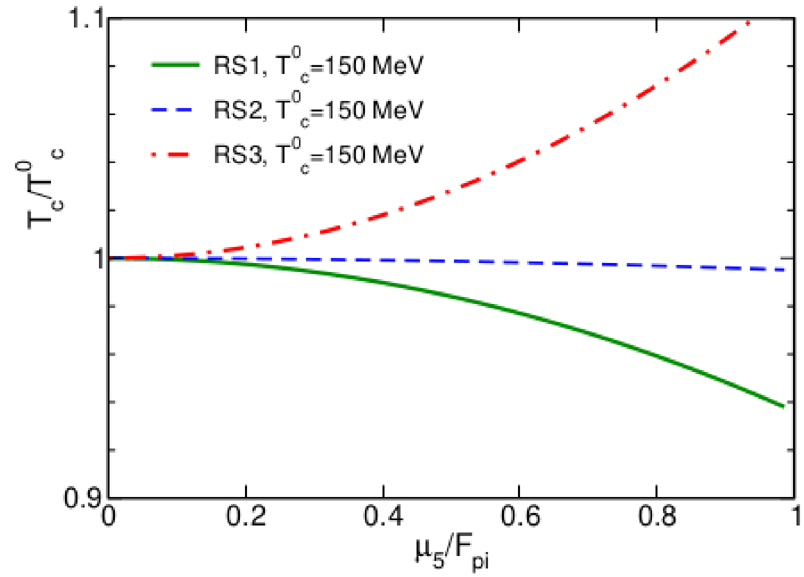
- Due to its μ_5 dependence, the infinite $\Phi_q^{\rm zp}$ must be regularized in a consistent manner:

$$\Omega_q^{\rm zp} = \Omega_0^{\rm zp}(s) + \Omega_5^{\rm zp}(\xi) + \delta\Omega_{\rm fin}^{\rm zp},$$

$$\Omega_0(s) = -\frac{N_c N_f}{2\pi^2}\mathcal{I}_1,$$

$$\Omega_5(\xi) = -m_q^2\mu_5^2\frac{N_c N_f}{2\pi^2}\mathcal{I}_2,$$

$$\Omega_{\rm fin}^{\rm zp} = -\mu_5^4\frac{N_c N_f}{12\pi^2},$$



where the divergent integrals $\mathcal{I}_1$ and $\mathcal{I}_2$ can be obtained using dimensional regularization:

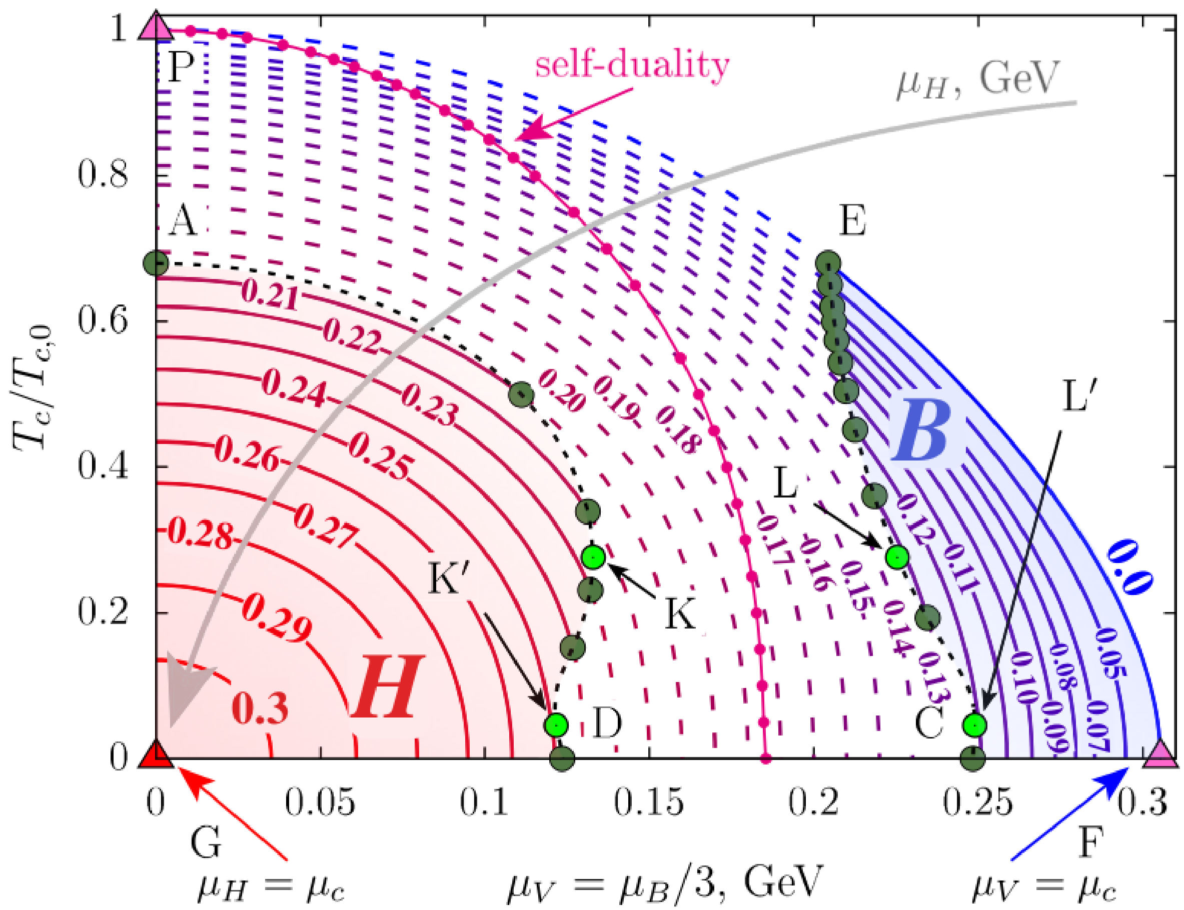
$$\mathcal{I}_1 = 2\mu^{2s}\int\frac{p^2dp}{(p^2+m_q^2)^{s-\frac{1}{2}}} = -\frac{m_q^4}{8s} + \frac{m_q^4}{16}\left[-3 + 2\gamma_E + 2\psi(-\tfrac{1}{2}) + 4\ln\frac{m_q}{\mu}\right],$$

$$\mathcal{I}_2 = \mu^{2\xi}\int\frac{p^2dp}{(p^2+m_q^2)^{\xi+\frac{3}{2}}} = \frac{1}{2\xi} - \left[\frac{\gamma_E}{2} + \frac{1}{2}\psi(\tfrac{3}{2}) + \ln\frac{m_q}{\mu}\right].$$

- Renormalizing the LSM model using 3 schemes gives 3 different results

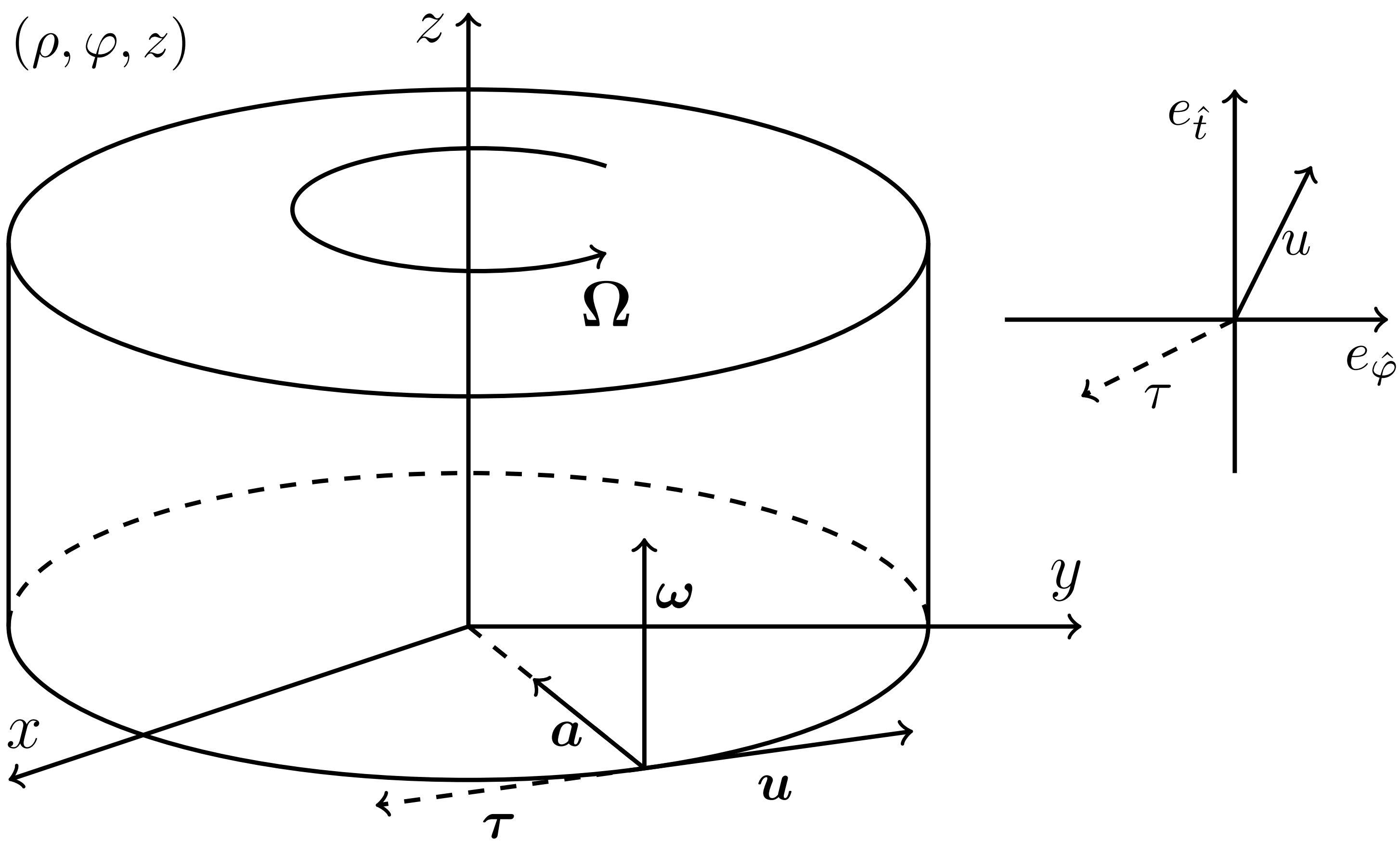
[M. Ruggieri, G. X. Peng, arXiv:1602.03651]

QCD with helical chemical potential



- When $\mu_A = 0$, Ω_q^{zp} is independent of T, μ_V, μ_H.
- μ_H plays a role dual to that of μ_V, the PD exhibiting a *self-duality* with respect to $\mu_V \leftrightarrow \mu_H$.

Kinematic frame for rigid rotation



A "kinematic" orthogonal tetrad is given by: [Becattini, Grossi, PRD 2015]

Velocity : $u = \Gamma(e_{\hat t} + \rho\Omega e_{\hat\varphi}), \qquad \Gamma = (1-\rho^2\Omega^2)^{-1/2},$

Acceleration : $a = \nabla_u u = -\rho\Omega^2\Gamma^2 e_{\hat\rho},$

Vorticity : $\omega = \frac{1}{2}\varepsilon^{\hat\alpha\hat\beta\hat\gamma\hat\sigma} e_{\hat\alpha} u_{\hat\beta}(\nabla_{\hat\gamma} u_{\hat\sigma}) = \Gamma^2\Omega e_{\hat z},$

Fourth vector : $\tau = -\,\varepsilon^{\hat\alpha\hat\beta\hat\gamma\hat\sigma} e_{\hat\alpha}\omega_{\hat\beta} a_{\hat\gamma} u_{\hat\sigma} = -\rho\Omega^3\Gamma^5(\rho\Omega e_{\hat t} + e_{\hat\varphi}).$

Rigidly-rotating thermal states

	Q_V	Q_A	Q_H	$\boldsymbol{J}_V$	$\boldsymbol{J}_A$	$\boldsymbol{J}_H$	$\boldsymbol{\omega}$
C	$-$	$+$	$-$	$-$	$+$	$-$	$+$
P	$+$	$-$	$-$	$-$	$+$	$+$	$+$
T	$+$	$+$	$+$	$-$	$-$	$-$	$-$

- Rotating thermal states can be constructed using

$$\langle \hat{A} \rangle = Z^{-1} \mathrm{Tr}(\hat{\varrho} \hat{A}), \qquad Z = \mathrm{Tr}(\hat{\varrho}),$$
$$\hat{\varrho} = \exp\left[-\beta_0 \left(\hat{H} - \Omega \widehat{M}^z - \sum_\ell \mu_{\ell;0} \widehat{Q}_\ell \right)\right],$$

  where $\beta_0 = T_0^{-1}$ and $\mu_{V/A/H;0}$ are measured on the rotation axis.
- The currents $J_\ell^\mu = Q_\ell u^\mu + \sigma_\ell^\omega \omega_\ell^\mu + \sigma_\ell^\tau \tau^\mu$ have vortical conductivities:

$$\sigma_V \simeq \frac{2\mu_H T}{\pi^2} \ln 2 + \frac{\mu_V \mu_A}{\pi^2}, \quad \sigma_A \simeq \frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2 + \mu_H^2}{2\pi^2}, \quad \sigma_H \simeq \frac{2\mu_V T}{\pi^2} \ln 2 + \frac{\mu_H \mu_A}{\pi^2}.$$

- In rigid rotation, $T = T_0 \Gamma$ and $\mu_\ell = \mu_{\ell;0} \Gamma$, with $\Gamma = (1 - \rho^2 \Omega^2)^{-1/2}$.

Neutral isothermal plasma at rest

- At leading order w.r.t. T, $\sigma^{\omega}_{V/H}$ are given by

$$\sigma^{\omega}_{V} = \frac{2\mu_H T}{\pi^2}\ln 2, \qquad \sigma^{\omega}_{H} = \frac{2\mu_V T}{\pi^2}\ln 2.$$

- Close to the rotation axis,

$$J_\ell \simeq Q_\ell \partial_t + \Omega\sigma^{\omega}_{\ell} + O(\rho\Omega).$$

- Considering a neutral plasma ($\mu_V = \mu_A = \mu_H = 0$), the charge densities and vortical conductivities are

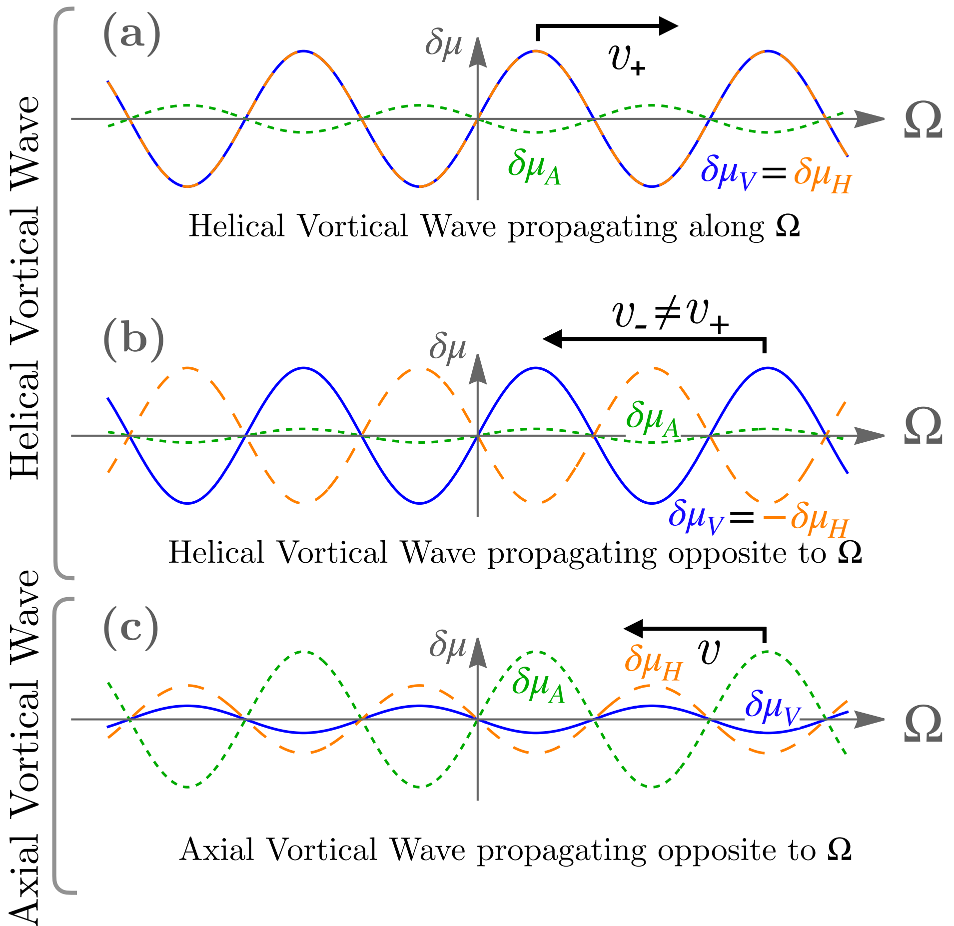
$$Q_\ell = \frac{T^2}{3}\delta\mu_\ell, \qquad \sigma_V = \frac{2T\ln 2}{\pi^2}\delta\mu_H, \qquad \sigma_H = \frac{2T\ln 2}{\pi^2}\delta\mu_V.$$

- Considering $\delta\mu_{V/H} = \delta\mu_{V/H;0}e^{-ik(vt-z)}$, imposing $\partial_\mu J^{\mu}_{V/H} = 0$ and neglecting fluctuations in T and u^μ, the velocity of the HVW is

$$v_{\rm HVW} = \frac{6\ln 2}{\pi}\frac{\hbar|\Omega|}{k_B T}c.$$

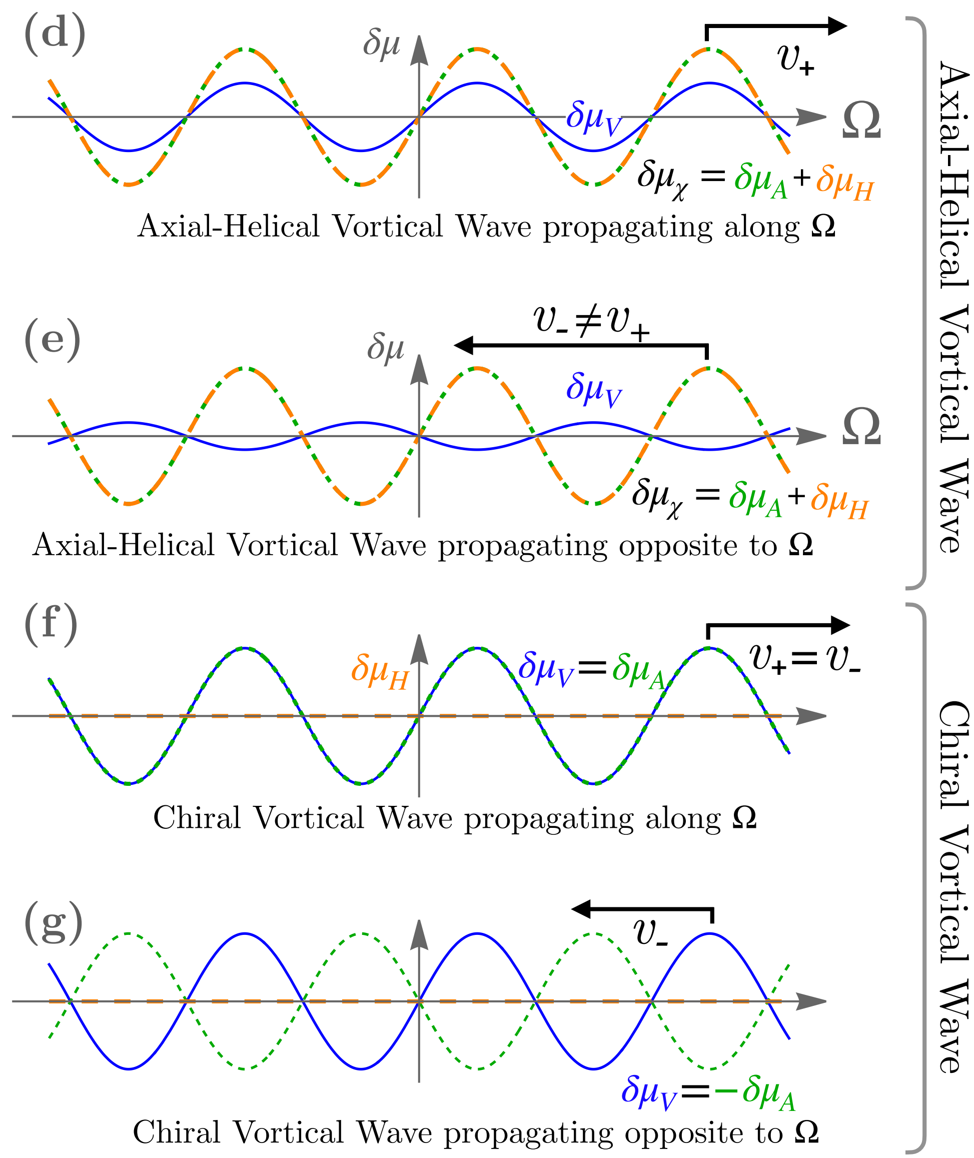
- The V and H (not A) dofs naturally combine in the HVW!
- In the context of the QGP, $\hbar\Omega \simeq 6.6$ MeV, $k_B T \simeq 150$ MeV and $v_{\rm HVW} \simeq 2\times 10^{-2}c$.

Vortical waves overview



oscillations in the vector, axial and helical chemical potentials assuming hierarchy of the background:

$$\mu_V > \mu_H > 0, \quad \mu_A > 0$$



From β to Landau frame

- Neglecting $O(\Omega^2)$ terms, $J_\ell^\mu = \langle \widehat{J}_\ell^\mu \rangle$ and $T^{\mu\nu} = \langle \widehat{T}^{\mu\nu} \rangle$ read

$$J_\ell^\mu = Q_\ell^\beta u_\Omega^\mu + \sigma_{\ell;\beta}^\mu, \quad T^{\mu\nu} = E_\beta u_\Omega^\mu u_\Omega^\nu - P_\beta \Delta_\Omega^{\mu\nu} + \sigma_{\varepsilon;\beta}^\omega (\omega_\Omega^\mu u_\Omega^\nu + \omega_\Omega^\nu u_\Omega^\mu), \quad (18)$$

with $\Delta^{\mu\nu} = g^{\mu\nu} - u_\Omega^\mu u_\Omega^\nu$ and $E_\beta = 3P_\beta$.

- The study is simpler in the Landau frame, $T^\mu{}_\nu u_L^\nu = E_L u_L^\mu$, with

$$T^{\mu\nu} = E_L u_L^\mu u_L^\nu - P_L \Delta_L^{\mu\nu}, \quad J_\ell^\mu = Q_\ell^L u_L^\mu + \sigma_{\ell;L}^\omega \omega_L^\mu, \quad \sigma_{\ell;L}^\omega = \sigma_{\ell;\beta}^\omega - \frac{Q_\ell \sigma_{\varepsilon;\beta}^\omega}{E+P}. \quad (19)$$

- u_L^μ has a vorticity correction that can be eliminated via a Lorentz boost L:

$$u_L^\mu = u_\Omega^\mu + \frac{\sigma_{\varepsilon;\beta}^\omega}{E+P} \omega_\Omega^\mu \xrightarrow[L]{} u_\Omega^\mu, \quad L^{\mu\nu} = g^{\mu\nu} - \frac{\sigma_{\varepsilon;\beta}^\omega}{E+P} (u_\Omega^\mu \omega_\Omega^\nu - u_\Omega^\nu \omega_\Omega^\mu). \quad (20)$$

- In what follows, we work in the boosted Landau frame, where

$$P \simeq -\frac{T^4}{\pi^2} \sum_{\sigma,\lambda} \mathrm{Li}_4(-e^{\mu_{\sigma,\lambda}/T}), \quad Q_\ell = \frac{\partial P}{\partial \mu_\ell}, \quad \sigma_\ell^\omega = \frac{1}{2} \frac{\partial^2 P}{\partial \mu_\ell \partial \mu_A} - \frac{Q_A Q_\ell}{E+P}. \quad (21)$$

Conservation equations

- We work in the boosted Landau frame ("L" subscript dropped).
- The energy-momentum conservation $\partial_\mu T^{\mu\nu} = 0$ leads to

$$DE + (E+P)\theta = 0, \quad (E+P)Du^\mu - \nabla^\mu P = 0, \tag{22}$$

  with $D = u^\mu \partial_\mu$, $\nabla^\mu = \Delta^{\mu\nu}\partial_\nu$ and $\theta = \partial_\mu \theta^\mu$.
- Non-conservation of J^μ_A, J^μ_H can be modelled as a relaxation process, in two ways:

$$\partial_\mu J^\mu_\ell = -\frac{Q_\ell}{\tau_\ell} \qquad \text{(leads to instabilities)}$$

$$\partial_\mu J^\mu_\ell = -\frac{\mu_\ell T^2}{3\tau_\ell} \qquad \text{(compatible with GCE)} \tag{23}$$

- We take the second approach and solve

$$DQ_V + Q_V\theta + \partial_\mu(\sigma^\omega_V \omega^\mu) = 0, \tag{24}$$

$$DQ_A + Q_A\theta + \partial_\mu(\sigma^\omega_A \omega^\mu) = -\frac{\mu_A T^2}{3\tau_A}, \tag{25}$$

$$DQ_H + Q_H\theta + \partial_\mu(\sigma^\omega_H \omega^\mu) = -\frac{\mu_H T^2}{3\tau_H}. \tag{26}$$

- The limit $\tau_A, \tau_H \to \infty \Leftrightarrow$ ideal plasma;
- The limit $\tau_A \to \infty, 0 < \tau_H < \infty \Leftrightarrow$ QED plasma;
- The case $0 < \tau_A, \tau_H < \infty \Leftrightarrow$ QCD plasma.

Kinetic dissipation

- In real fluids, interparticle collisions happending on a timescale τ_R lead both to the equilibration of the fluid and to viscous dissipation.
- The "perfect fluid" limit means $\tau \to 0$, which should imply also $\tau_A, \tau_H \to 0$ (frozen axial and helical dofs).
- For consistency, when $\tau_A, \tau_H > 0$, one should consider the effects of τ_R.
- In the simplest model, dissipation is added as a first order term:

$$T^{\mu\nu} = (E+P)u^\mu u^\nu - P g^{\mu\nu} + \pi_d^{\mu\nu}, \quad J_\ell^\mu = Q_\ell u^\mu + \sigma_\ell^\omega \omega^\mu + V_{\ell;d}^\mu. \tag{27}$$

- In the RTA by Anderson-Witting,

$$\pi_d^{\mu\nu} = 2\eta\sigma^{\mu\nu}, \quad V_{\ell;d}^\mu = \tau_R \left(\frac{1}{3} \nabla^\mu Q_\ell - \frac{Q_\ell \nabla^\mu P}{E+P} \right), \tag{28}$$

  with $\eta = \frac{4}{5}\tau_R P$ and $\sigma^{\mu\nu} = \nabla^{(\mu} u^{\nu)} - \frac{1}{3}\Delta^{\mu\nu}\theta$.
- The value of τ_R can be estimated from the ratio η/s:

$$\tau_R = \frac{5\eta}{sT} = 0.26 \text{ fm}/c \times (4\pi\eta/s) \times \left(\frac{300 \text{ MeV}}{T} \right). \tag{29}$$

- First-order dissipation is acausal. More rigorous frameworks include Chiral kinetic theory

[E.V. Gorbar, D.O. Rybalka, I.A. Shovkovy, PRD **95** (2017) 096010]

Energy-momentum sector

- In the following, we consider perturbations around rigid rotation, localized in the vicinity of the rotation axis ($\rho\Omega \to 0$).
- A perturbed quantity $\bar{f}$ is split into background and perturbations:

$$\bar{f} = f + \delta\bar{f}, \quad \delta\bar{f} = \int_{-\infty}^{\infty} dk\, e^{ikz} \sum_{\omega} e^{-i\omega(k)t} \delta f_{\omega}(k), \tag{30}$$

- The conservation of $T^{\mu\nu}$ (dissipation included) gives rise to a closed set of equations:

$$\begin{pmatrix} -3\omega & 4kP \\ k & -4P\omega - \frac{4}{3} i\eta k^2 \end{pmatrix} \begin{pmatrix} \delta P_{\omega} \\ \delta u^z_{\omega} \end{pmatrix} = 0, \tag{31}$$

where we assume the perturbations are purely longitudinal.
- Imposing vanishing determinant reveals the sound modes:

[E.V. Gorbar, D.O. Rybalka, I.A. Shovkovy, PRD **95** (2017) 096010]

$$\omega^{\pm}_{\text{ac.}} = \pm k c_s(\eta) - \frac{ik^2\eta}{6P}, \quad c_s(\eta) = \frac{1}{\sqrt{3}}\sqrt{1 - \frac{k^2\eta^2}{12P^2}}. \tag{32}$$

- A small discrepancy in $\omega^{\pm}_{\text{ac.}}$ w.r.t. Gorbar et al is due to neglecting $\delta\mathbf{u}_{\perp}$ in Ref. [Gorbar et al].
- For the charge modes, the determinant no longer vanishes and it follows that $\delta P_{\omega} = \delta u^z_{\omega} = 0$!

Charge current modes

- For the charge modes, $\delta P_\omega = \delta u^\mu_\omega = 0$.
- When charge ℓ is not conserved, its chemical potential also becomes infinitesimal: $\bar{\mu}_\ell = \delta\bar{\mu}_\ell$:

$$\left(\omega + \frac{ik^2\tau_R}{3}\right)\delta Q_{\ell;\omega} - k\Omega\delta\sigma^\omega_{\ell;\omega} = -\frac{T^2}{3\tau_\ell}\delta\mu_\ell. \tag{33}$$

- Expressing now $\tilde{\omega} = \omega + \frac{ik^2\tau_R}{3}$ and writing $\delta Q_{\ell;\omega}$ and $\delta\sigma^\omega_{\ell;\omega}$ in terms of δT_ω and $\delta\mu_{\ell;\omega}$ gives

$$\mathbb{M}_{\ell\ell'}\delta\mu_{\ell';\omega} = 0, \quad \frac{1}{T^2}\mathbb{M} = \tilde{\omega}\mathbb{M}_\omega - \kappa_\Omega\mathbb{M}_\Omega + \frac{i}{3\tau_A}\mathbb{I}_A + \frac{i}{3\tau_H}\mathbb{I}_H, \tag{34}$$

with $\kappa_\Omega = k\Omega/T$, $(\mathbb{I}_A)_{\ell\ell'} = \delta_{\ell A}\delta_{\ell' A}$ and $(\mathbb{I}_H)_{\ell\ell'} = \delta_{\ell H}\delta_{\ell' H}$, as well as

$$\mathbb{M}^\omega_{\ell\ell'} = \frac{1}{T^2}\left(\frac{\partial Q_\ell}{\partial\mu_{\ell'}} - \frac{3Q_\ell Q_{\ell'}}{sT} + \frac{Q_{\ell'}\vec{\mu}}{sT}\cdot\frac{\partial Q_\ell}{\partial\vec{\mu}}\right), \tag{35}$$

$$\mathbb{M}^\Omega_{\ell\ell'} = \frac{1}{T}\left(\frac{\partial\sigma^\omega_\ell}{\partial\mu_{\ell'}} - \frac{2\sigma^\omega_\ell Q_{\ell'}}{sT} + \frac{Q_{\ell'}\vec{\mu}}{sT}\cdot\frac{\partial\sigma^\omega_\ell}{\partial\vec{\mu}}\right). \tag{36}$$

Conserved A and H charges: large T limit

- Let us consider first $\tau_A, \tau_H \to \infty$.
- At large temperatures, the pressure becomes

$$P = \frac{7\pi^2 T^4}{180} + \frac{\vec{\mu}^2 T^2}{6} + \frac{4\mu_\times^3 T}{\pi^2}\ln 2 + \frac{(\vec{\mu}^2)^2}{12\pi^2} + \frac{\mu_A^2\mu_H^2 + \mu_V^2\mu_H^2 + \mu_V^2\mu_A^2}{3\pi^2} + O(T^{-1}). \quad (37)$$

- The matrices $\mathbb{M}_\omega$ and $\mathbb{M}_\Omega$ become:

$$\mathbb{M}_\omega = \frac{1}{3}\mathbb{I} + \frac{4\ln 2}{\pi^2 T}\begin{pmatrix} 0 & \mu_H & \mu_A \\ \mu_H & 0 & \mu_V \\ \mu_A & \mu_V & 0 \end{pmatrix} + O(T^{-2}), \quad (38)$$

$$\mathbb{M}_\Omega = \frac{2\ln 2}{\pi^2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} + \frac{2}{7\pi^2 T}\begin{pmatrix} \mu_A & \mu_V & 0 \\ \mu_V & -4\mu_A & \mu_H \\ 0 & \mu_H & \mu_A \end{pmatrix} + O(T^{-2}). \quad (39)$$

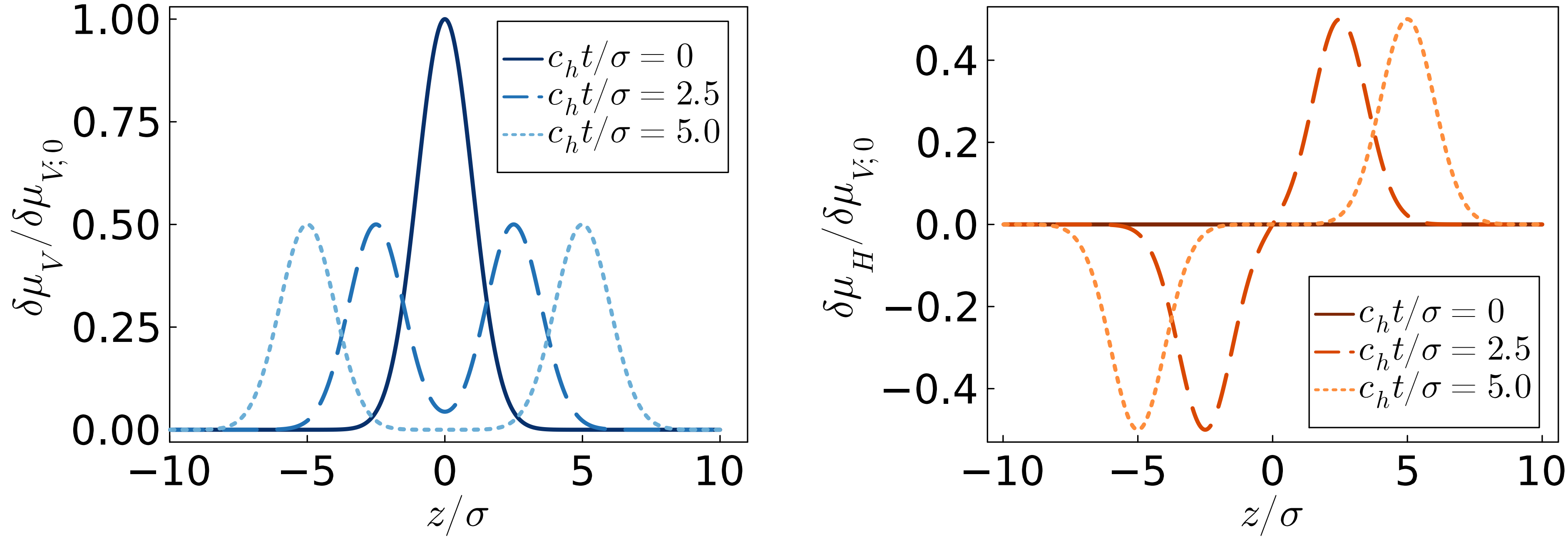
- Writing $\tilde{\omega} = \tilde{\omega}_0 + T^{-1}\tilde{\omega}_1 + T^{-2}\tilde{\omega}_2 + O(T^{-3})$ reveals:

$$\text{HVW}: \quad \frac{\omega_h^\pm}{k} = \pm\frac{6\ln 2}{\pi^2}\frac{\Omega}{T} - \frac{6}{7\pi^2}\left[\frac{84}{\pi^2}(\ln 2)^2 - 1\right]\frac{\Omega\mu_A}{T^2} + O(T^{-3}), \quad (40)$$

$$\text{AVW}: \quad \frac{\omega_{\mathfrak{a}}}{k} = -\frac{24}{7\pi^2}\frac{\mu_A\Omega}{T^2} + O(T^{-3}). \quad (41)$$

- The AVW is non-reciprocal (only travels in one direction).
- Non-reciprocity seen in the HVW, in the subleading term.

HVW propagation: split of helical signal

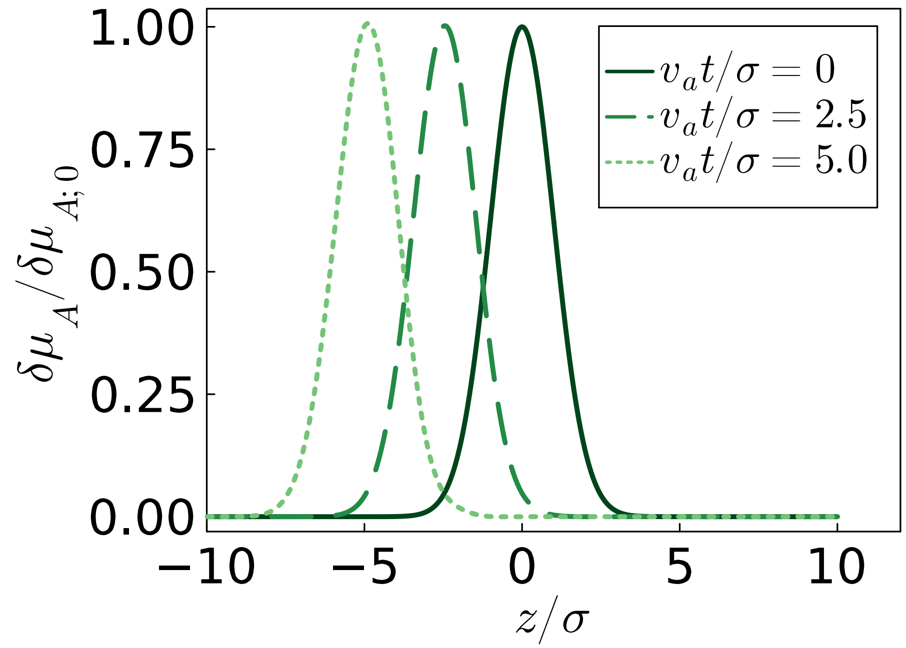


- We take $\bar{\mu}_V(0, z) = \delta\bar{\mu}_{V;0} e^{-z^2/2\sigma^2}$.
- The exact solution for the propagation of the initial Gaussian lump reads:

$$\delta\bar{\mu}_V(t, z) = \frac{\delta\bar{\mu}_{V;0}}{2} \left[e^{-(z-c_h t)^2/2\sigma^2} + e^{-(z+c_h t)^2/2\sigma^2} \right], \tag{42}$$

$$\delta\bar{\mu}_H(t, z) = \frac{\delta\bar{\mu}_{V;0}}{2} \left[e^{-(z-c_h t)^2/2\sigma^2} - e^{-(z+c_h t)^2/2\sigma^2} \right]. \tag{43}$$

AVW propagation: non-reciprocity



- The AVW propagates only at finite μ_A, which we take as $\mu_T = 0.1T$.
- For the initial profile $\bar{\mu}_A(0, z) = \mu_A + \delta\bar{\mu}_A e^{-z^2/2\sigma^2}$, we find

$$\bar{\mu}_A(t, z) = \mu_A + \delta\bar{\mu}_A \, e^{-(z-v_a t)^2/2\sigma^2}. \tag{44}$$

Charged, unpolarized plasma: $\mu_A = \mu_H = 0$

- The pressure of the unpolarized plasma reads

$$P = \frac{7\pi^2 T^4}{180} + \frac{\mu_V^2 T^2}{6} + \frac{\mu_V^4}{12\pi^2}. \tag{45}$$

- The matrix $T^{-2}\mathbb{M} = \tilde{\omega}\mathbb{M}_\omega - \kappa_\Omega \mathbb{M}_\Omega$ becomes

$$\mathbb{M}_\omega = \frac{2}{T^2}\begin{pmatrix} \sigma_A^\omega - \frac{T^2}{3}\Delta H & 0 & 0 \\ 0 & \sigma_A^\omega & \sigma_H^\omega \\ 0 & \sigma_H^\omega & \sigma_A^\omega \end{pmatrix}, \qquad \mathbb{M}_\Omega = \begin{pmatrix} 0 & \frac{1}{H}A & \frac{1}{H}B \\ A & 0 & 0 \\ B & 0 & 0 \end{pmatrix}, \tag{46}$$

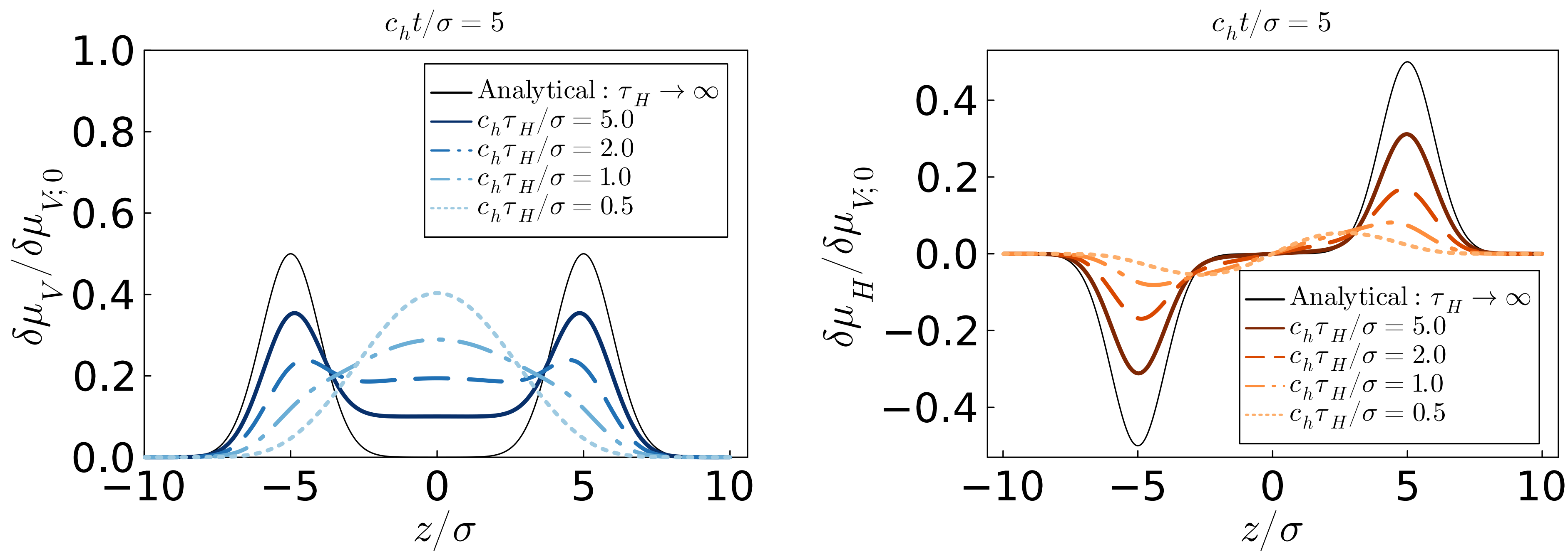
where $\Delta H = H - 1$, $H = (E+P)/sT$, while A and B are

$$A = \frac{\alpha_V}{\pi^2} - \frac{Q_V}{3s}, \qquad B = \frac{HL}{\pi^2} - \frac{2Q_V}{sT^2}\sigma_H^\omega. \tag{47}$$

- The corresponding characteristic equation reads

$$\det\left(\tilde{\omega}\mathbb{M}_\omega - \kappa_\Omega\mathbb{M}_\Omega + \frac{i}{3\tau_H}\mathbb{I}_H + \frac{i}{3\tau_A}\mathbb{I}_A\right)$$
$$= \frac{2\omega}{T^2}\left\{\left(\frac{2\omega}{T^2}\right)^2\left(\sigma_A^\omega - \frac{T^2}{3}\Delta H\right)[(\sigma_A^\omega)^2 - (\sigma_H^\omega)^2] - \frac{\kappa_\Omega^2}{H}[(A^2+B^2)\sigma_A^\omega - 2AB\sigma_H^\omega]\right\}$$
$$+ \frac{i}{3}\left[\left(\frac{2\omega}{T^2}\right)^2\sigma_A^\omega\left(\sigma_A^\omega - \frac{T^2}{3}\Delta H\right)\left(\frac{1}{\tau_H} + \frac{1}{\tau_A}\right) - \frac{\kappa_\Omega^2}{H}\left(\frac{A^2}{\tau_H} + \frac{B^2}{\tau_A}\right)\right]$$
$$- \frac{2\omega}{9T^2\tau_A\tau_H}\left(\sigma_A^\omega - \frac{T^2}{3}\Delta H\right) = 0. \tag{48}$$

Damping of the HVW

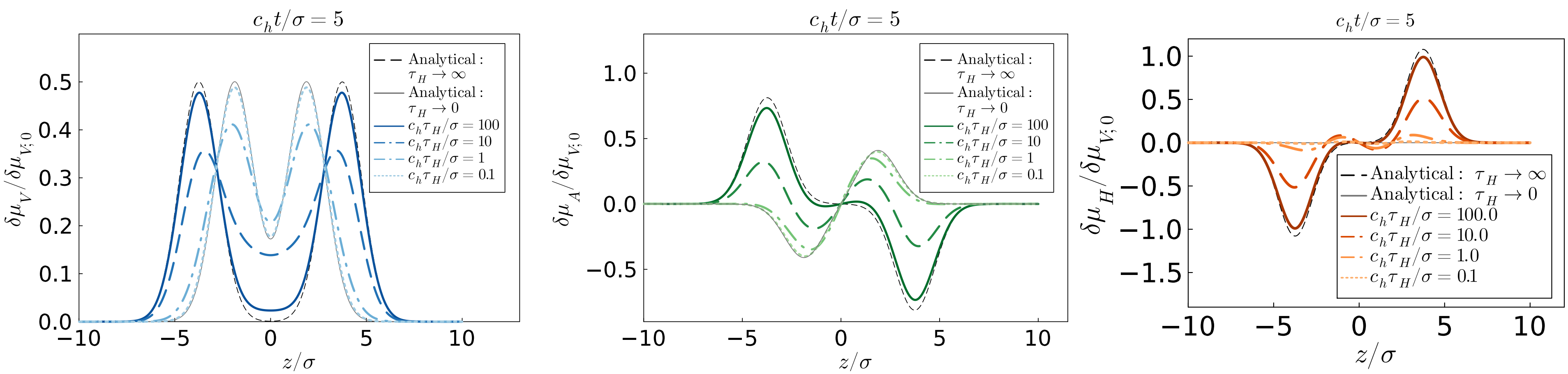


- Consider the $|\mu_V| \ll T$ limit ($\mu_A = \mu_H = 0$). The HVW behaves as

$$\omega_{\mathrm{HVW}} = -\frac{i}{2\tau_H} \pm kc_h\sqrt{1-\frac{k_{\mathrm{th}}^2}{k^2}}, \quad k_{\mathrm{th}} = \frac{1}{2\tau_H c_h} = \frac{\pi^2}{12\ln 2}\frac{T}{\Omega\tau_H}. \tag{49}$$

- τ_H prevents the propagation of large wavelengths.

Freezing the H mode: resurgence of AVW



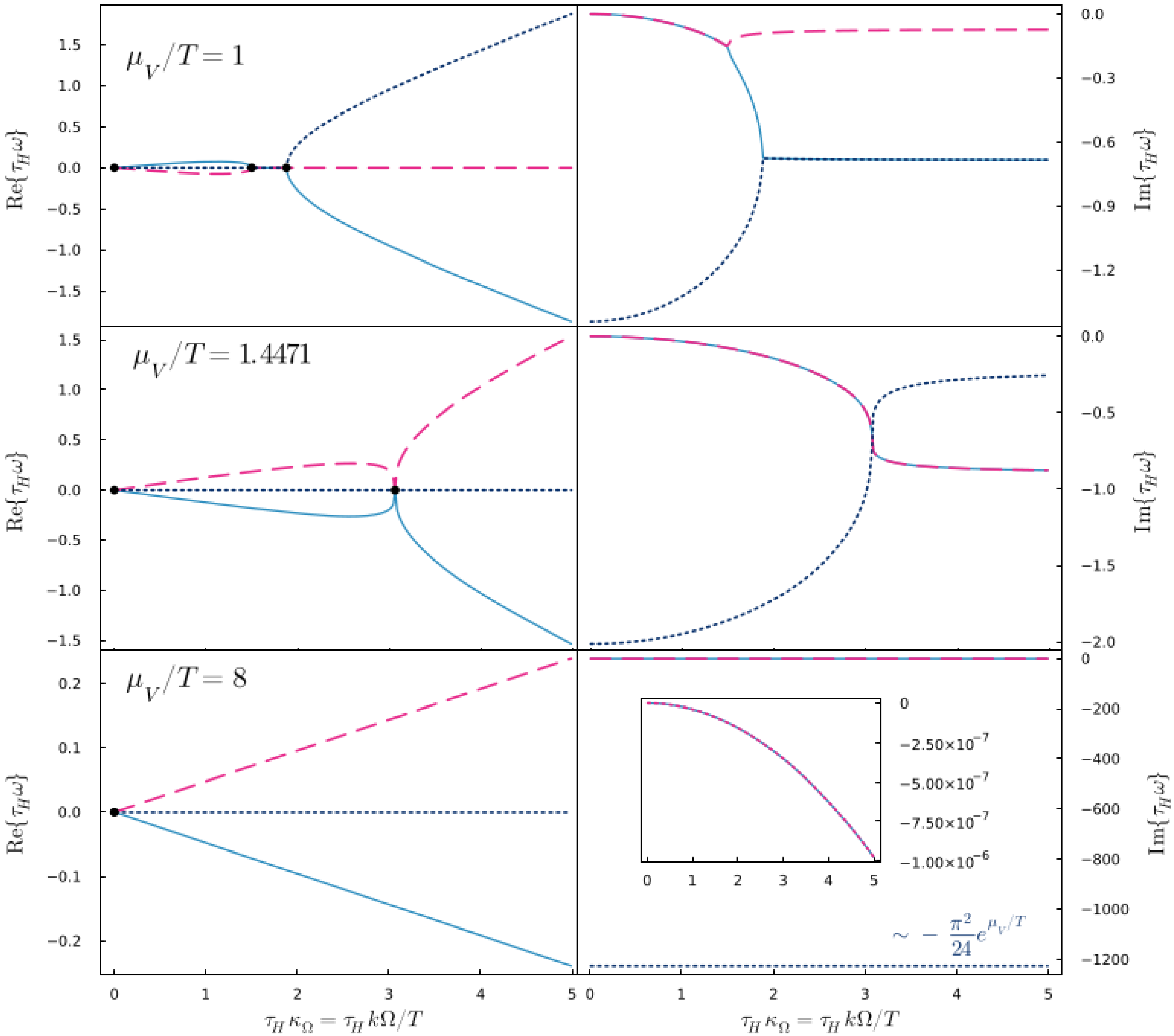
- When $\tau_H = 0$, the $\delta\mu_H \to 0$ instantaneously $\Rightarrow$ the helical dof doesn't propagate and

$$\omega_h = -\frac{iT^2\sigma_A^\omega}{6\tau_H[(\sigma_A^\omega)^2 - (\sigma_H^\omega)^2]}. \tag{50}$$

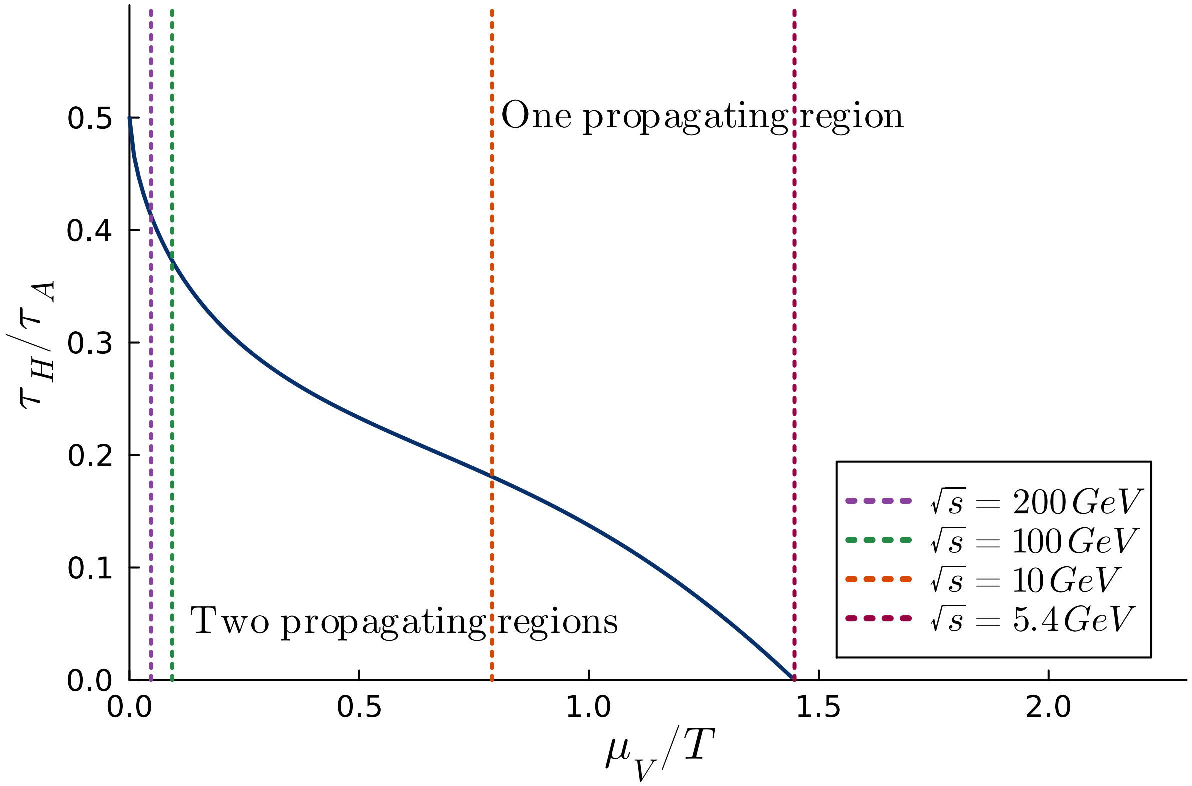
- In this regime, the A and V dofs couple and give rise to a propagating wave:

$$\omega_{\mathfrak{a}}^{\pm} = \pm\frac{AT^2}{2\sigma_A^\omega\sqrt{H}}\left(1 - \frac{T^2\Delta H}{3\sigma_A^\omega}\right)^{-1/2}\frac{k\Omega}{T}. \tag{51}$$

Wave spectrum



Damping of the AVW

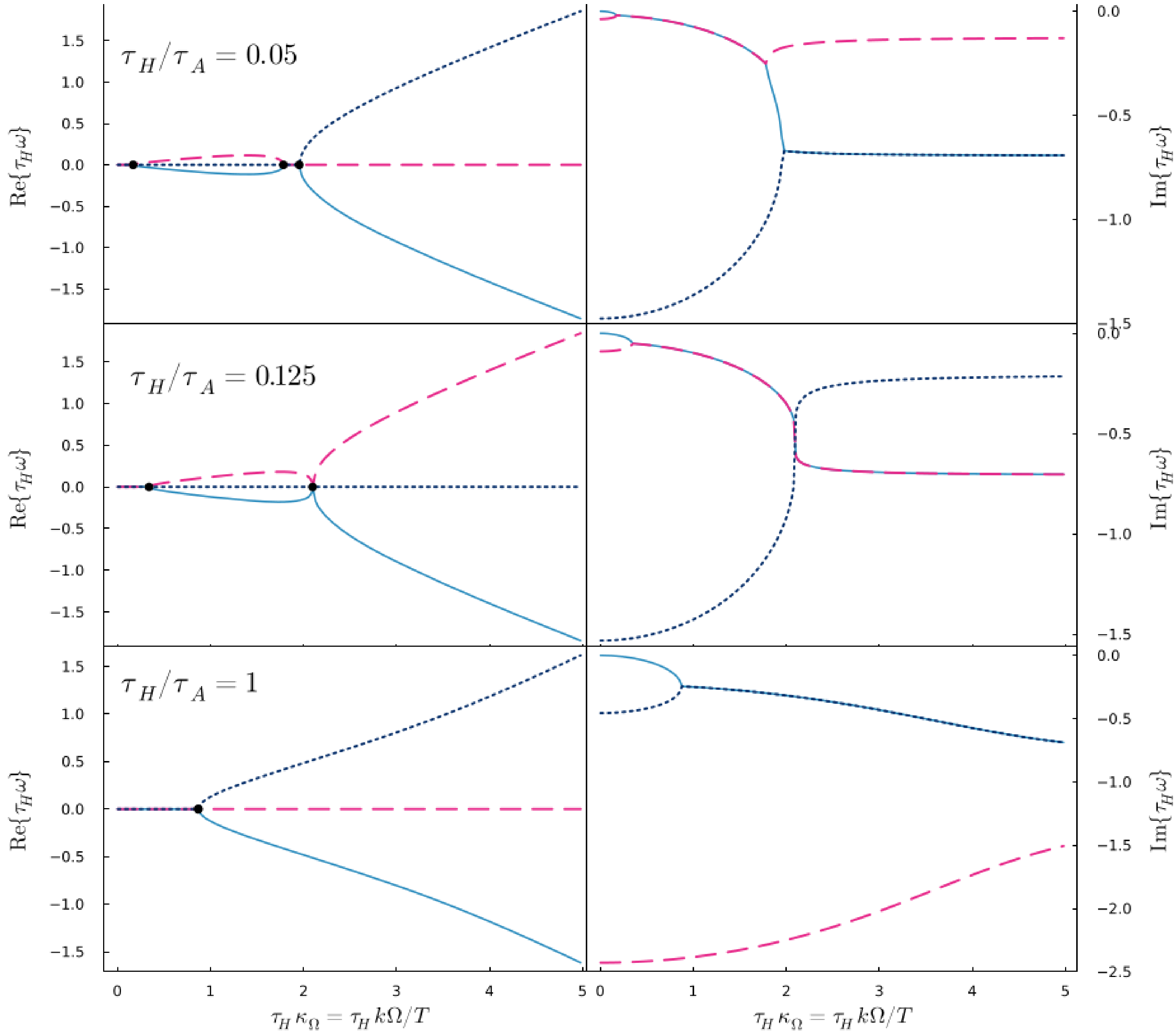


- Consider the $|\mu_V| \ll T$ limit ($\mu_A = \mu_H = 0$). The AVW behaves as

$$\omega_{\mathfrak{a}} = \frac{3i\alpha_V^2}{\pi^2 \tau_A (\tau_A - \tau_H - \tau_H \tau_A^2 k^2 c_h^2)} \times \left[(\tau_A - \tau_H)\left(1 + \frac{12}{49\pi^2}\kappa_\Omega^2 \tau_A^2\right) + \tau_H \frac{48}{\pi^2}(\ln 2)^2 \left(1 + \frac{3}{28\pi^2}\kappa_\Omega^2\tau_A^2\right)\right]. \quad (52)$$

- τ_A inhibits the propagation of the AVW for large wavenumbers.

Wave spectrum



Conclusion

- The Dirac theory allows for (at least) three conserved currents: J_V^μ, J_A^μ and J_H^μ.
- Associated to Q_H is μ_H, which accounts for helicity imbalance.
- For $m = 0$, (μ_V, μ_A, μ_H) form a triad, contributing non-trivially to anomalous transport (vortical effects).
- At $m \neq 0$, Q_H remains conserved and μ_H is well defined.
- The μ_H dof gives rise to the helical vortical wave (HVW), which propagates faster than the CVW.
- While μ_A is incompatible with the LSM_q, μ_H appears to be dual to μ_V.
- Neither J_H^μ nor J_A^μ are conserved in interacting theories $\Rightarrow$ τ_H and τ_A inhibit the propagation of the HVW and AVW at large wavelengths.

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