Perturbation vs. confinement for bound states

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Emergent mass and its consequences in the Standard Model
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"The most fundamental emergent phenomena in Quantum Chromodynamics (QCD), e.g. confinement, dynamical chiral symmetry breaking, mass generation for both gluons and quarks, and bound state formation, can only be tackled using non-perturbative methods."

Yes, but: Bound states can have both non-perturbative and perturbative aspects (QED).

PH 1807.05598 and refs. therein

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Example: Hyperfine splitting in Positronium

G. S. Adkins, Hyperfine Interact. **233** (2015) 59

$$\begin{aligned}\Delta\nu_{QED} \;=\; & m_e\alpha^4\left\{\frac{7}{12}-\frac{\alpha}{\pi}\left(\frac{8}{9}+\frac{\ln 2}{2}\right)\right. \\ & +\frac{\alpha^2}{\pi^2}\left[-\frac{5}{24}\pi^2\ln\alpha+\frac{1367}{648}-\frac{5197}{3456}\pi^2+\left(\frac{221}{144}\pi^2+\frac{1}{2}\right)\ln 2-\frac{53}{32}\zeta(3)\right] \\ & \left.-\frac{7\alpha^3}{8\pi}\ln^2\alpha+\frac{\alpha^3}{\pi}\ln\alpha\left(\frac{17}{3}\ln 2-\frac{217}{90}\right)+\mathcal{O}\left(\alpha^3\right)\right\} = 203.39169(41)\ \text{GHz}\end{aligned}$$

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- Binding energy is perturbative in α and $\log(\alpha)$ (measurable)
- Wave function $\psi(r) \propto \exp(-m\alpha r)$ is of $\mathcal{O}(\alpha^\infty)$ (gauge dependent)

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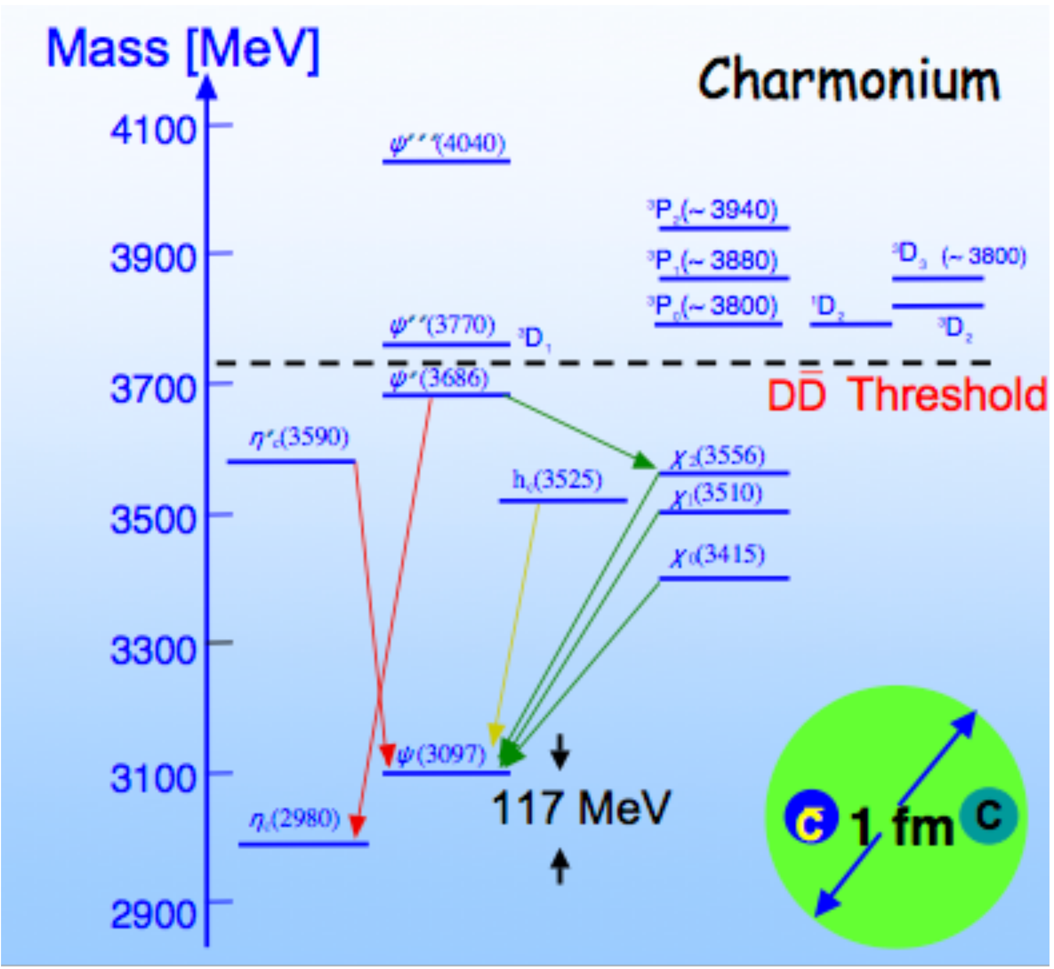

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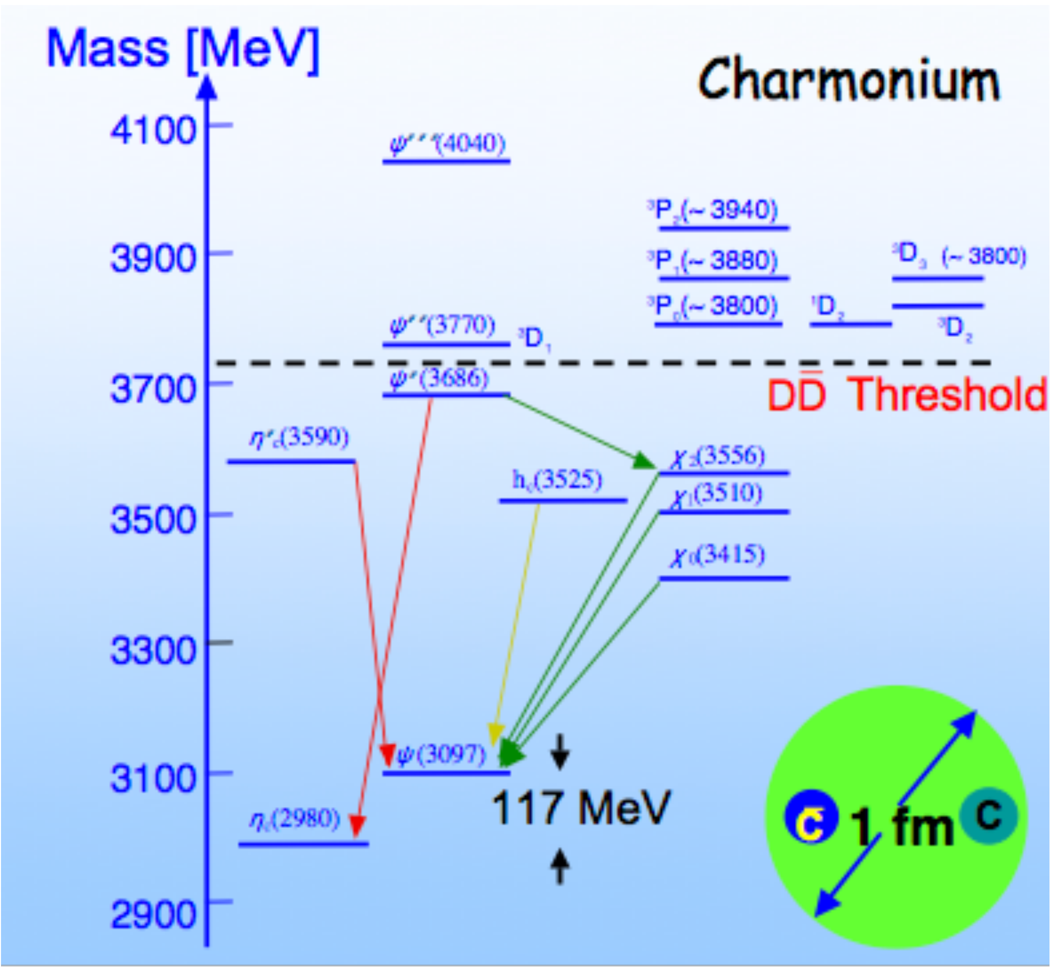

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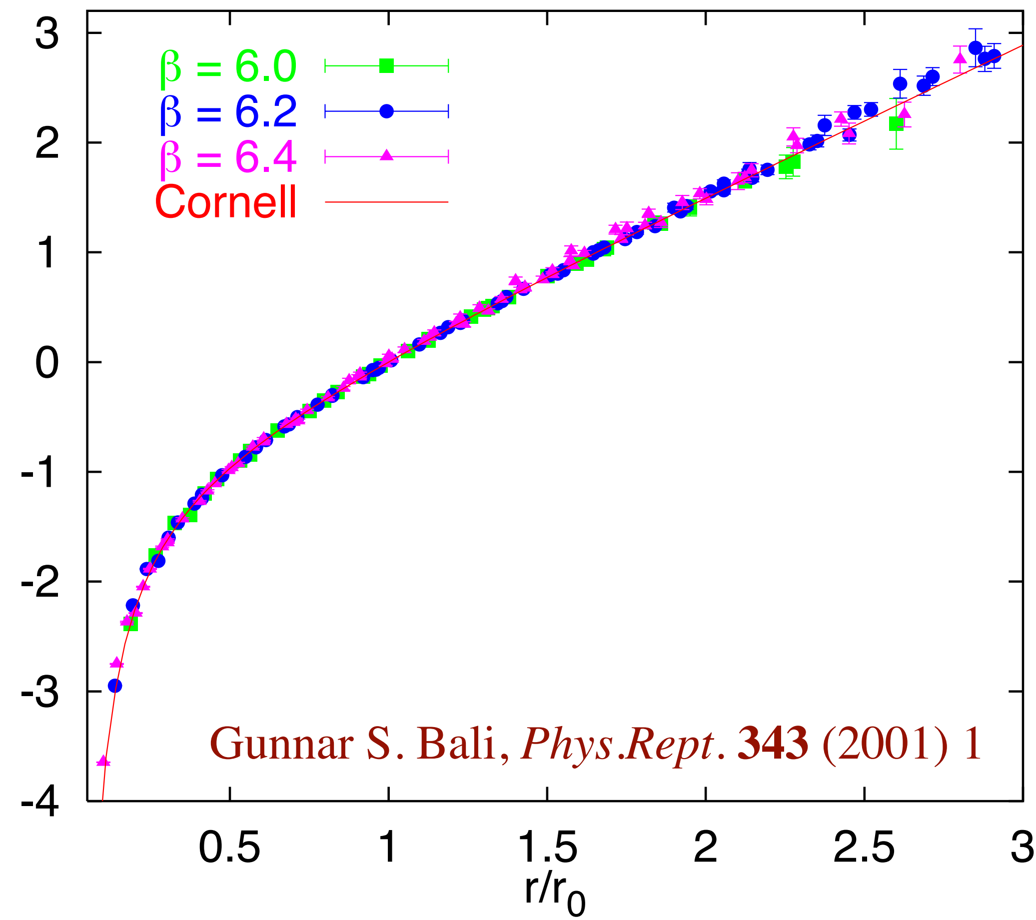
$V(r)$: Cornell vs. Lattice



Quoting Craig Roberts, et al.:

- Interpretation of given observable depends on the basis employed
- K. G. Wilson, Walhout, Harindranath, Zhang, Perry, Glazek: Phys. Rev. D **49** (1994) pp. 6720-6766 … Arguing for the use of quasiparticle operators:
  As is always the case, the division of the Hamiltonian into a free part and an interaction part is arbitrary; however, it is also true that the convergence of a perturbative expansion depends crucially on how this choice is made.
- **Clearest/simplest picture will likely change with the resolution scale**

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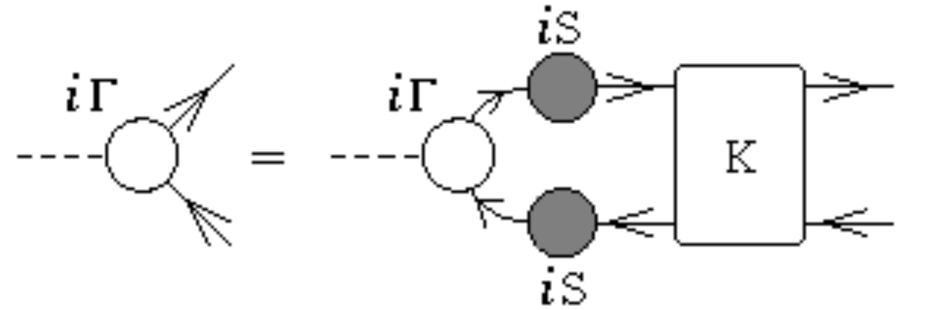

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Explicit Lorentz covariance

No analytic solution even at lowest order in *S* and *K*

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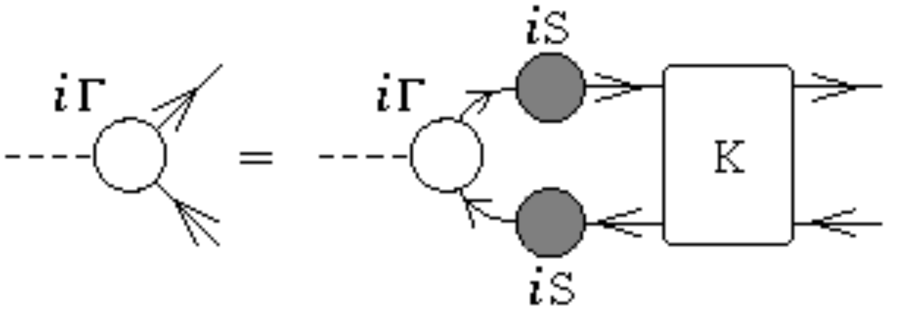

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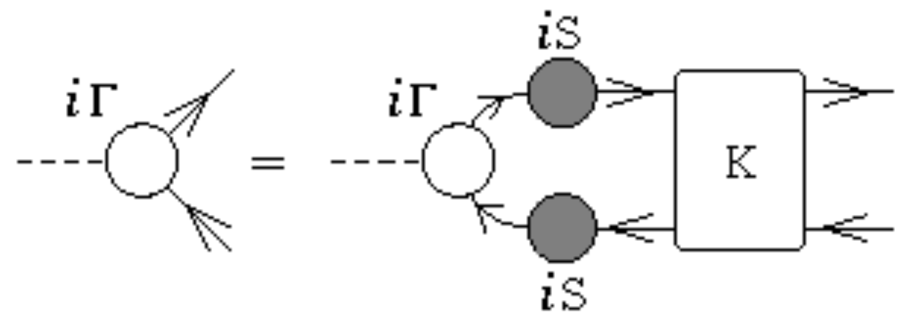

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- 1986: Caswell & Lepage NRQED: Effective NR field theory

Expand QED action in powers of ∇/m_e
Choose to start from Schrödinger atoms

Bound states in the S-matrix

Feynman diagrams are derived in the Interaction Picture:

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I \qquad\qquad \mathcal{H}_0 \left| i \right\rangle_{in} = E_i \left| i \right\rangle_{in}$$

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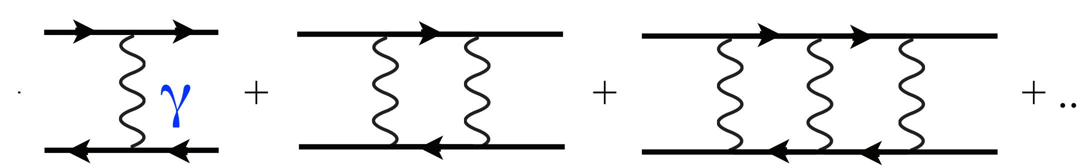
No Feynman diagram has a bound state pole.



Expanding around free states is inappropriate for bound states.

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The $\hbar \to 0$ limit:

- Selects an optimal perturbative expansion.
- Preserves symmetries and unitarity.
- Applies also to relativistic dynamics

Implementation: The "Potential Picture"

$$\mathcal{H} = \mathcal{H}_V + \mathcal{H}_I \qquad \qquad \mathcal{H}_V = \mathcal{H}_0 + \mathcal{H}_I(A_{cl})$$

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- Classical field vanishes for widely separated charges $\Rightarrow$ Standard P.T.
- For bound states, A_{cl} is not given by Feynman diagrams.
- Born level must give a fair approximation of bound states: $(\mathcal{H}_I)^0$, $\hbar^0$

The classical field for Positronium

$$\frac{\delta \mathcal{S}_{QED}}{\delta \hat{A}^0(t,\boldsymbol{x})} = 0 \qquad \Longrightarrow \qquad -\boldsymbol{\nabla}^2 \hat{A}^0(t,\boldsymbol{x}) = e\psi^\dagger(t,\boldsymbol{x})\psi(t,\boldsymbol{x})$$

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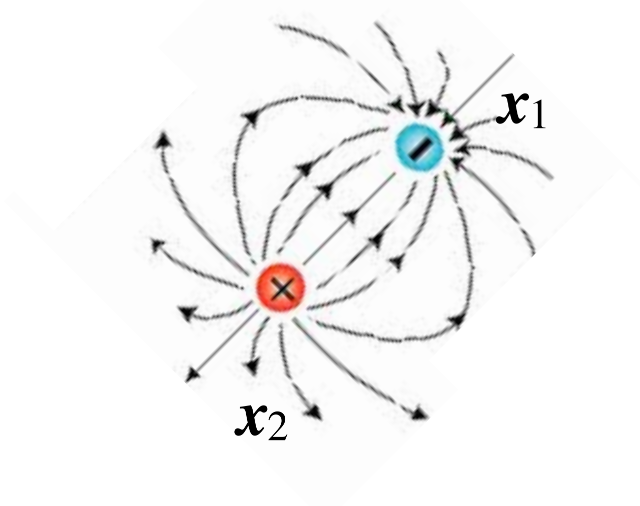

$$\frac{\langle \boldsymbol{x}_1,\boldsymbol{x}_2|\, e\hat{A}^0(\boldsymbol{x})\,|\boldsymbol{x}_1,\boldsymbol{x}_2\rangle}{\langle \boldsymbol{x}_1,\boldsymbol{x}_2|\,\boldsymbol{x}_1,\boldsymbol{x}_2\rangle} = \frac{\alpha}{|\boldsymbol{x}-\boldsymbol{x}_1|} - \frac{\alpha}{|\boldsymbol{x}-\boldsymbol{x}_2|}$$

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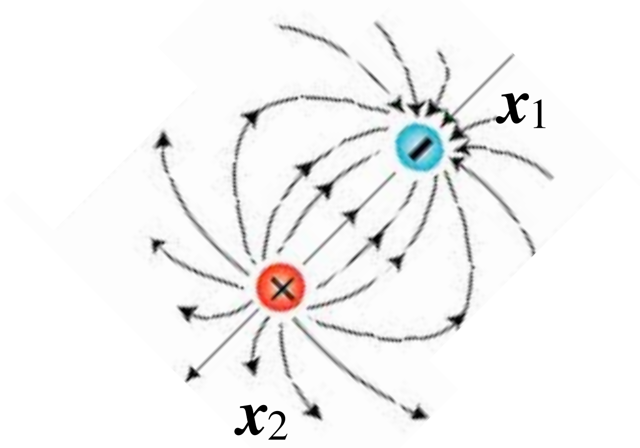

Note:
- A^0 is determined instantaneously for all $\boldsymbol{x}$
- It depends on $\boldsymbol{x}_1, \boldsymbol{x}_2$ $\Rightarrow$ The charges determine the field
- $eA^0(\boldsymbol{x}_1) = -eA^0(\boldsymbol{x}_2) = -\dfrac{\alpha}{|\boldsymbol{x}_1-\boldsymbol{x}_2|}$ is the classical $-\alpha/r$ potential

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$$|M\rangle_V = \int d\boldsymbol{x}_1\, d\boldsymbol{x}_2\, \bar{\psi}(\boldsymbol{x}_1)\, \Phi(\boldsymbol{x}_1 - \boldsymbol{x}_2)\, \psi(\boldsymbol{x}_2)\, |0\rangle$$

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This BSE reduces to the Schrödinger equation for non-relativistic kinematics.

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The Schrödinger equation should be derived from the QED action in courses on field theory!

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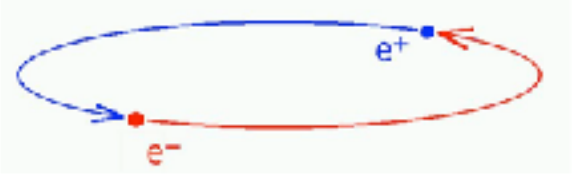

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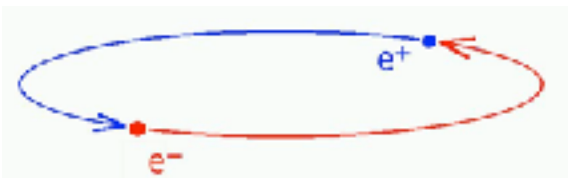

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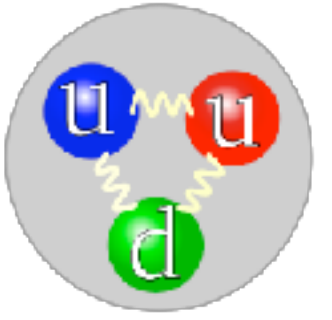

$$A_a^0(\boldsymbol{x}) = 0$$

Proton

QCD

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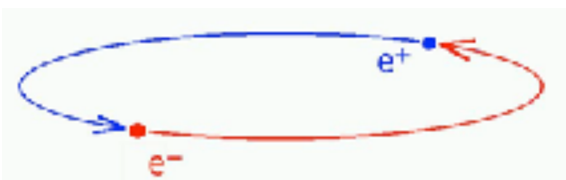

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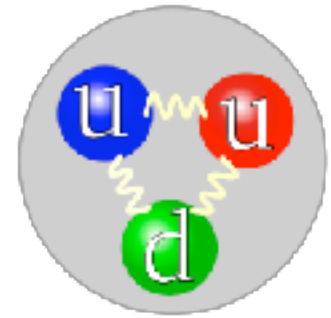

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However:
The classical gluon field is non-vanishing for each color component C of the state

$$A_a^0(\boldsymbol{x};C) \neq 0 \qquad \sum_C A_a^0(\boldsymbol{x};C) = 0$$

Three consequences of $\hbar \to 0$ in QCD

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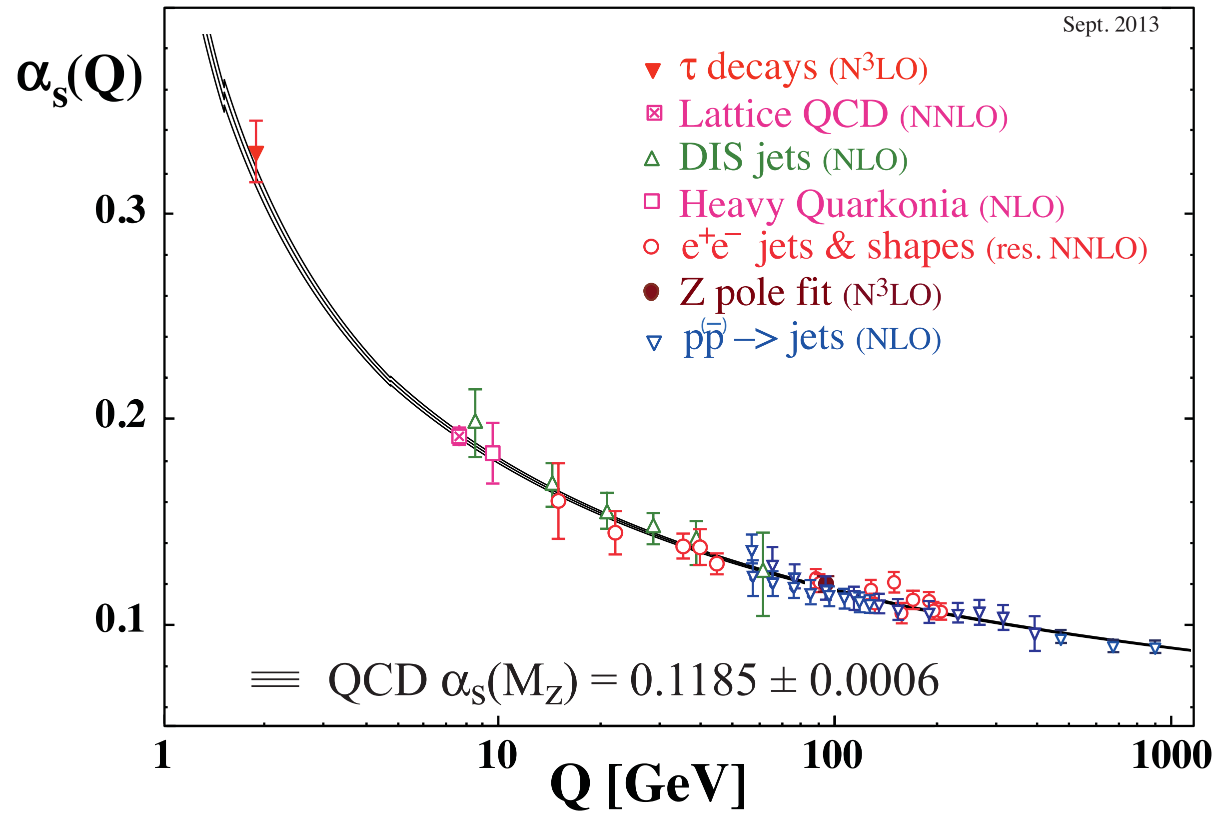
$\alpha_s(0)/\pi \approx 0.14$

$\Rightarrow$ PQCD corrections to $\mathcal{O}(\hbar^0)$ can be relevant.



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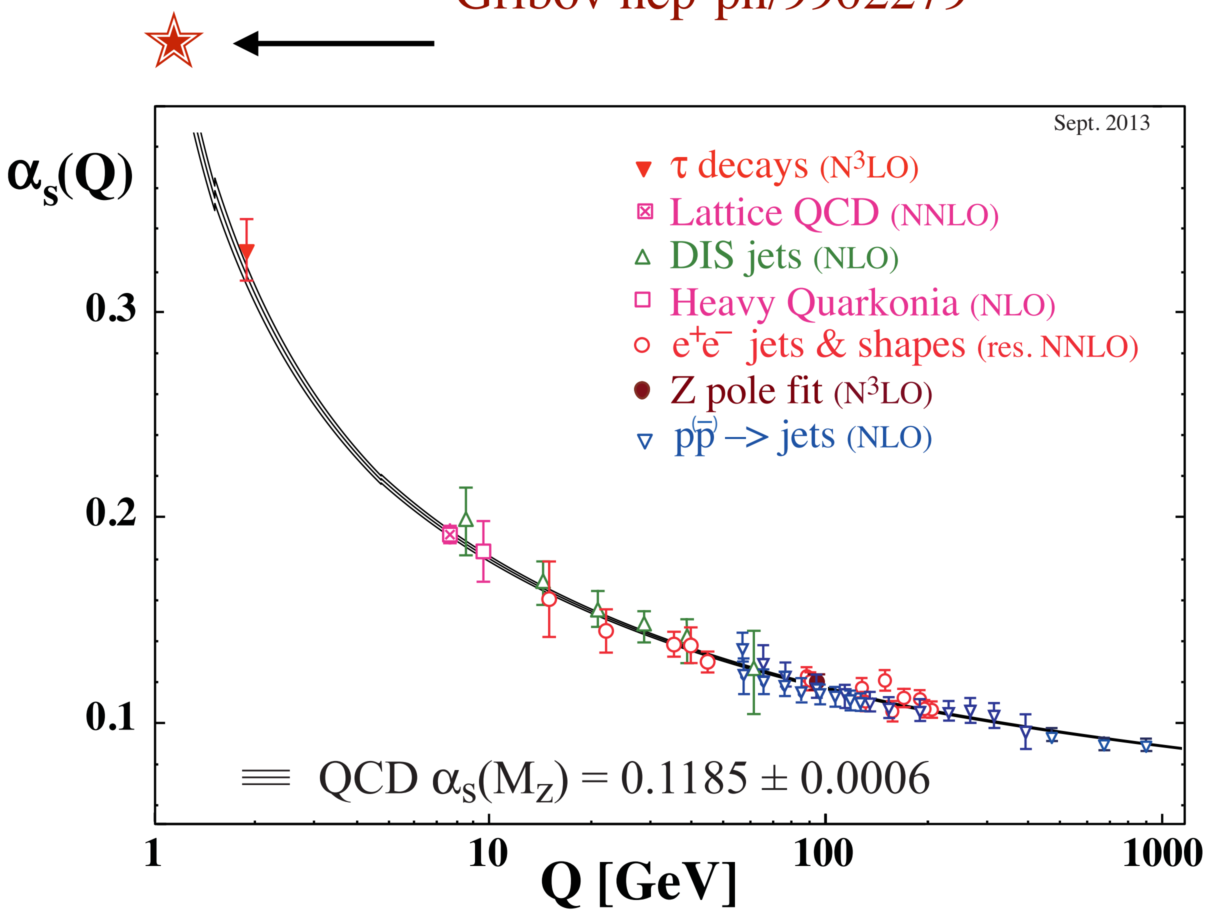

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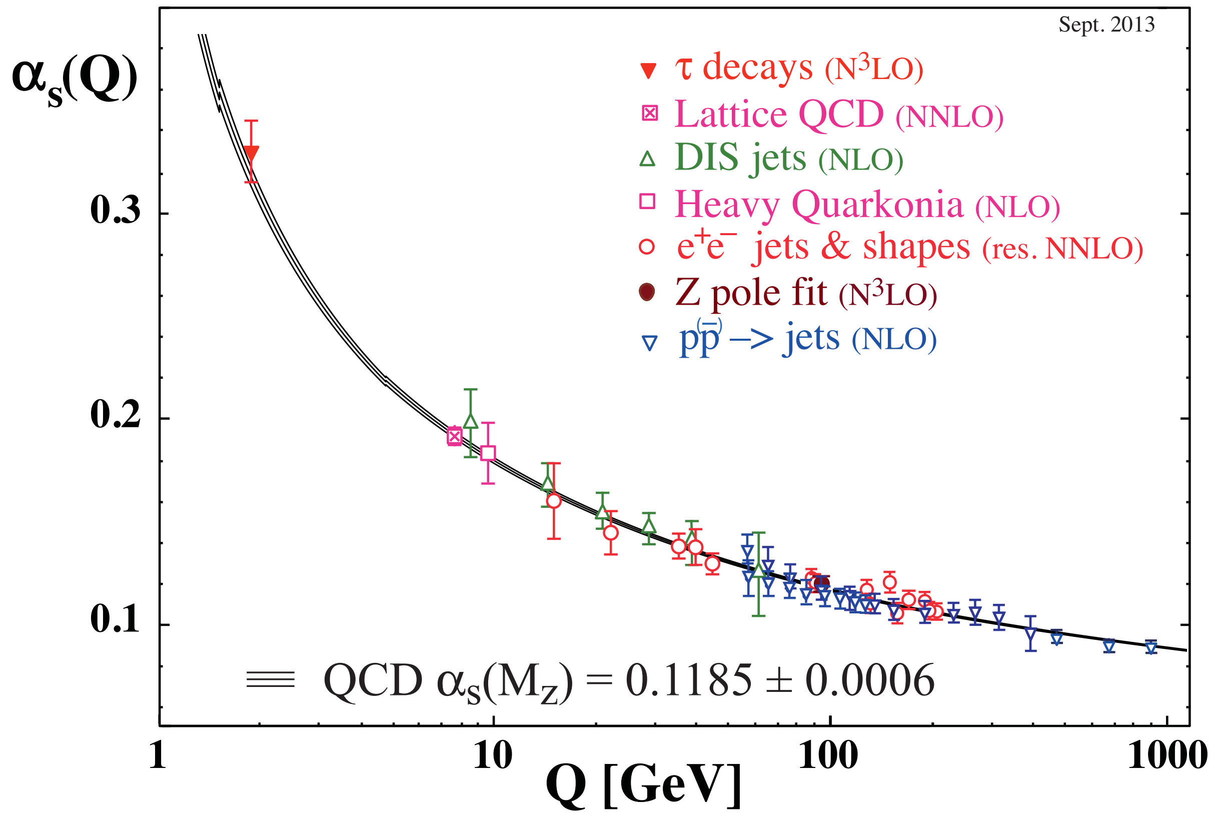

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3. Poincaré invariance, unitarity etc. should hold at each power of $\hbar$

The QCD scale Λ_{QCD}

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At $O(\hbar^0)$ (no loops) the QCD scale can only arise via a boundary condition

$$\frac{\delta}{\delta A_a^0} S_{QCD} = 0 \qquad \Rightarrow \qquad \partial_i F_a^{i0} = -g f_{abc} A_b^i F_c^{i0} + g \psi_A^\dagger T_a^{AB} \psi_B$$

The QCD scale Λ_{QCD}

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- $\boldsymbol{x}$-independent field energy density $\sum_a |\nabla \hat{A}_a^0(\boldsymbol{x})|^2$ must be universal

$\Rightarrow$ determines $\varkappa$ up to a scale Λ [GeV]

Classical color field for mesons

$$|M\rangle = \sum_{A,B} \int d\boldsymbol{x}_1\, d\boldsymbol{x}_2\, \bar{\psi}^A(\boldsymbol{x}_1)\, \Phi^{AB}(\boldsymbol{x}_1 - \boldsymbol{x}_2)\, \psi^B(\boldsymbol{x}_2)\, |0\rangle$$

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$$\sum_a \left[\boldsymbol{\nabla}_x A_a^0(\boldsymbol{x}; \boldsymbol{x}_1, \boldsymbol{x}_2, A)\right]^2 = 12\Lambda^4$$ Universal field energy density

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Universal field energy density

$$\sum_A A_a^0(\boldsymbol{x}; \boldsymbol{x}_1, \boldsymbol{x}_2, A) \propto \mathrm{Tr}\, T^{AA} = 0$$

Another hadron feels
no field at any $\boldsymbol{x}$

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$$V(\boldsymbol{x}_1 - \boldsymbol{x}_2) = \tfrac{1}{2} g \sum_a T_a^{AA} \left[A_a^0(\boldsymbol{x}_1; \boldsymbol{x}_1, \boldsymbol{x}_2, A) - A_a^0(\boldsymbol{x}_2; \boldsymbol{x}_1, \boldsymbol{x}_2, A)\right] = g\Lambda^2 |\boldsymbol{x}_1 - \boldsymbol{x}_2|$$

Linear potential, independent of quark color component A

Classical color field for baryons

$$|M\rangle = \sum_{A,B,C} \int d\boldsymbol{x}_1\, d\boldsymbol{x}_2\, d\boldsymbol{x}_3\, \psi_A^\dagger(\boldsymbol{x}_1)\psi_B^\dagger(\boldsymbol{x}_2)\psi_C^\dagger(\boldsymbol{x}_3)\, \Phi^{ABC}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3)\,|0\rangle \qquad \Phi^{ABC} = \epsilon^{ABC}\Phi$$

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in $\psi_A^\dagger(\boldsymbol{x}_1)\psi_B^\dagger(\boldsymbol{x}_2)\psi_C^\dagger(\boldsymbol{x}_3)\,|0\rangle$ $(A \neq B \neq C)$ determines the classical field:

$$A_a^0(\boldsymbol{x};\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3,ABC) = \big[\boldsymbol{x} - \tfrac{1}{3}(\boldsymbol{x}_1+\boldsymbol{x}_2+\boldsymbol{x}_3)\big]\cdot(T_a^{AA}\boldsymbol{x}_1 + T_a^{BB}\boldsymbol{x}_2 + T_a^{CC}\boldsymbol{x}_3)\frac{6\Lambda^2}{d(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3)}$$

where $d(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3) = \frac{1}{\sqrt{2}}\sqrt{(\boldsymbol{x}_1-\boldsymbol{x}_2)^2 + (\boldsymbol{x}_2-\boldsymbol{x}_3)^2 + (\boldsymbol{x}_3-\boldsymbol{x}_1)^2}$

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$$V(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3) = g\Lambda^2 d(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3)$$

Bound state equation for mesons (rest frame)

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$\mathcal{H}_V \left|M\right\rangle_V = M \left|M\right\rangle_V$ Bound state condition implies, with $\boldsymbol{x} = \boldsymbol{x}_1 - \boldsymbol{x}_2$

$$i\boldsymbol{\nabla}\cdot\left\{\gamma^0\boldsymbol{\gamma},\Phi(\boldsymbol{x})\right\} + m\left[\gamma^0,\Phi(\boldsymbol{x})\right] = \left[M - V(\boldsymbol{x})\right]\Phi(\boldsymbol{x})$$

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Expanding the 4 × 4 wave function in a basis of 16 Dirac structures $\Gamma_i(\boldsymbol{x})$

$$\Phi(\boldsymbol{x}) = \sum_i \Gamma_i(\boldsymbol{x})F_i(r)Y_{j\lambda}(\hat{\boldsymbol{x}})$$

we may use rotational, parity and charge conjugation invariance to determine which $\Gamma_i(\boldsymbol{x})$ may occur for a state of given j^{PC}:

0^{-+} trajectory $[s=0,\ \ell=j]$: $\quad -\eta_P = \eta_C = (-1)^j \quad \gamma_5,\ \gamma^0\gamma_5,\ \gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{x},\ \gamma_5\,\boldsymbol{\alpha}\cdot\boldsymbol{x}\times\boldsymbol{L}$

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$\Rightarrow$ There are no solutions for quantum numbers that would be exotic in the quark model (despite the relativistic dynamics)

Example: 0⁻⁺ trajectory wf's

$$\Phi_{-+}(\boldsymbol{x}) = \Big[\frac{2}{M-V}(i\boldsymbol{\alpha}\cdot\overrightarrow{\boldsymbol{\nabla}} + m\gamma^0) + 1\Big]\gamma_5\, F_1(r) Y_{j\lambda}(\hat{\boldsymbol{x}})$$

$\eta_P = (-1)^{j+1}$

$\eta_C = (-1)^j$

Radial equation: $F_1'' + \Big(\frac{2}{r} + \frac{V'}{M-V}\Big)F_1' + \Big[\frac{1}{4}(M-V)^2 - m^2 - \frac{j(j+1)}{r^2}\Big]F_1 = 0$

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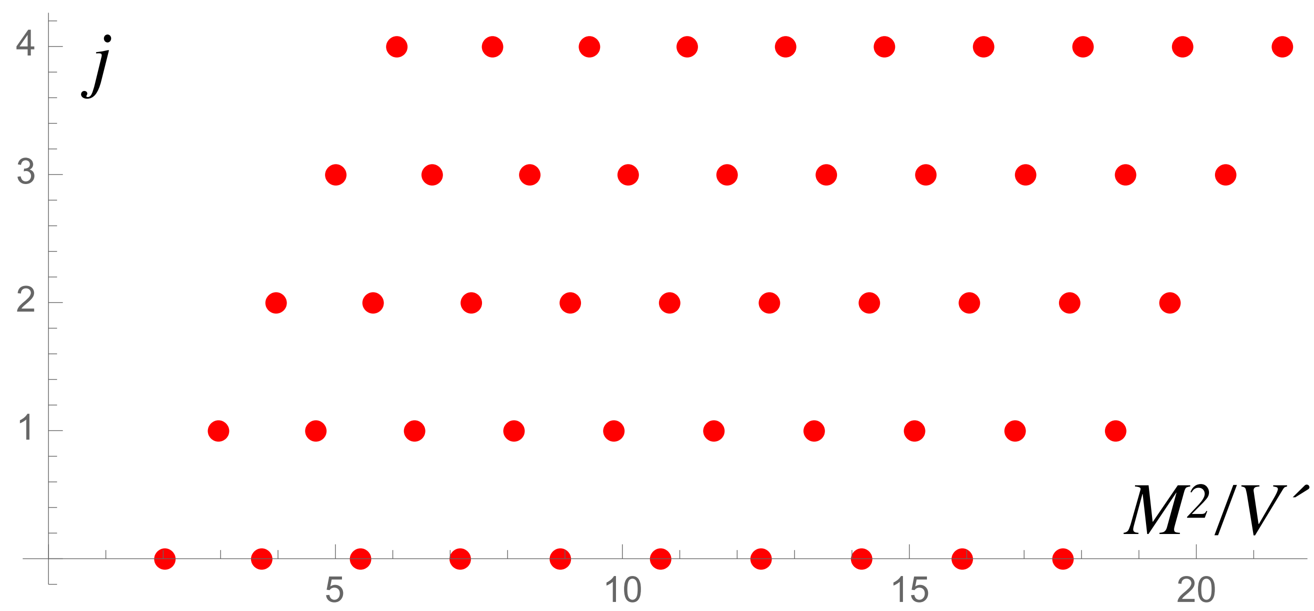
Mass spectrum:

Linear Regge trajectories with daughters

Spectrum similar to dual models

$m = 0$



Spin S^z contribution to $J^z = L^z + S^z = 1$

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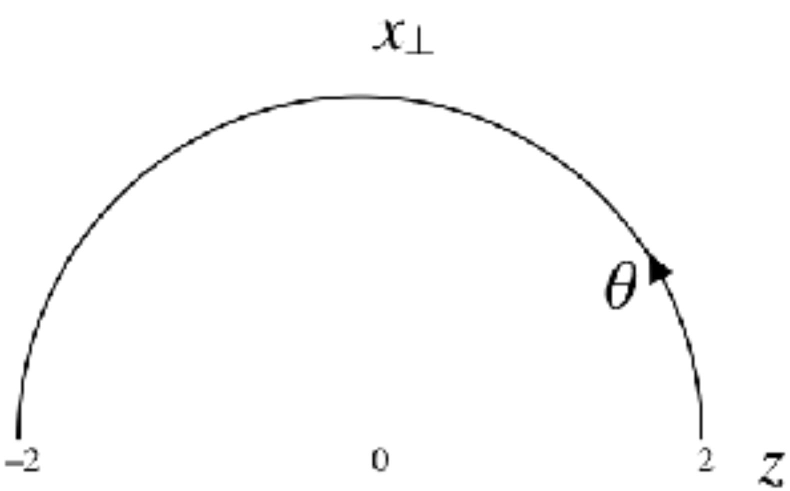

$|\Phi|^2$ and $\langle S^z \rangle$ along
a curve of radius $2/\sqrt{V'}$
in the $x_\perp - z$ plane,
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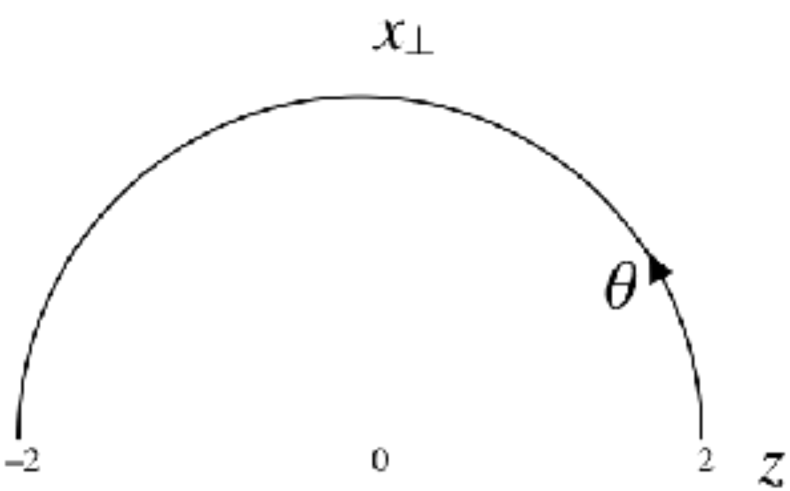

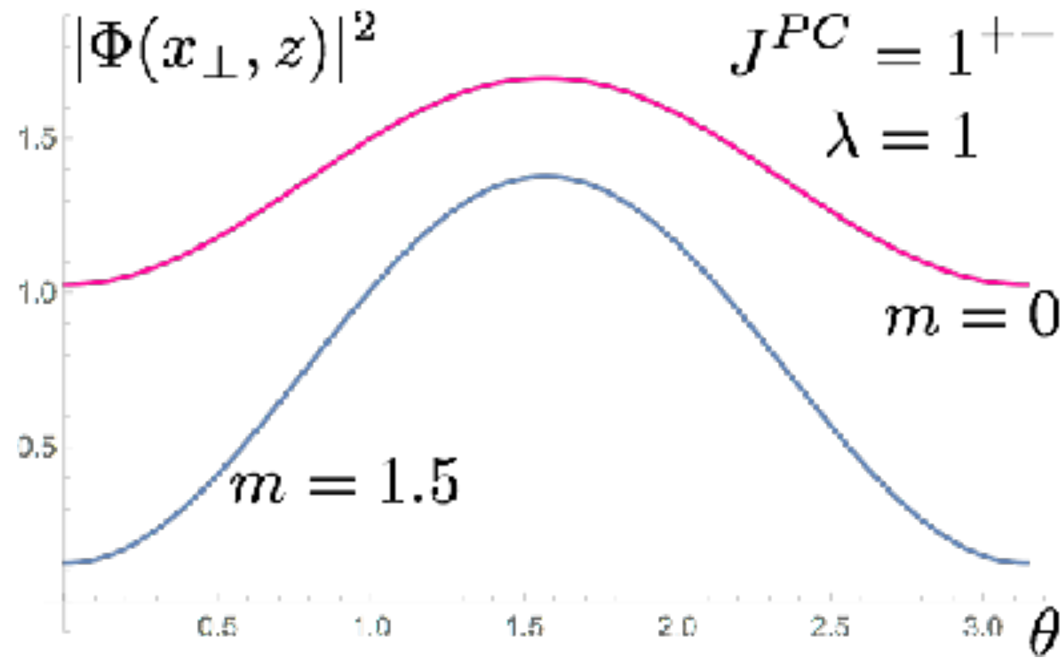

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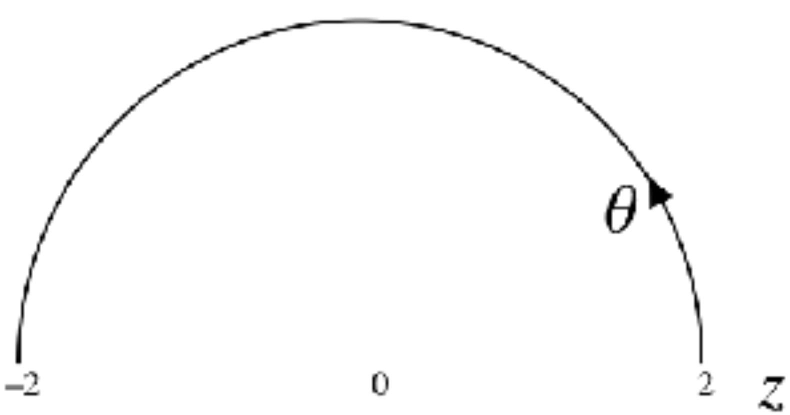
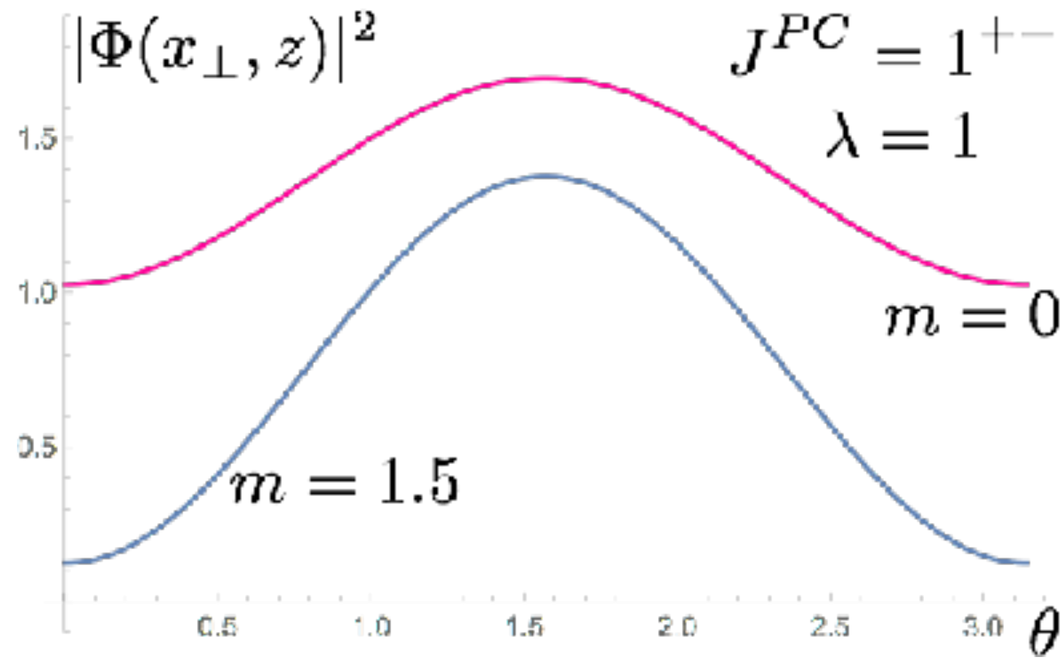
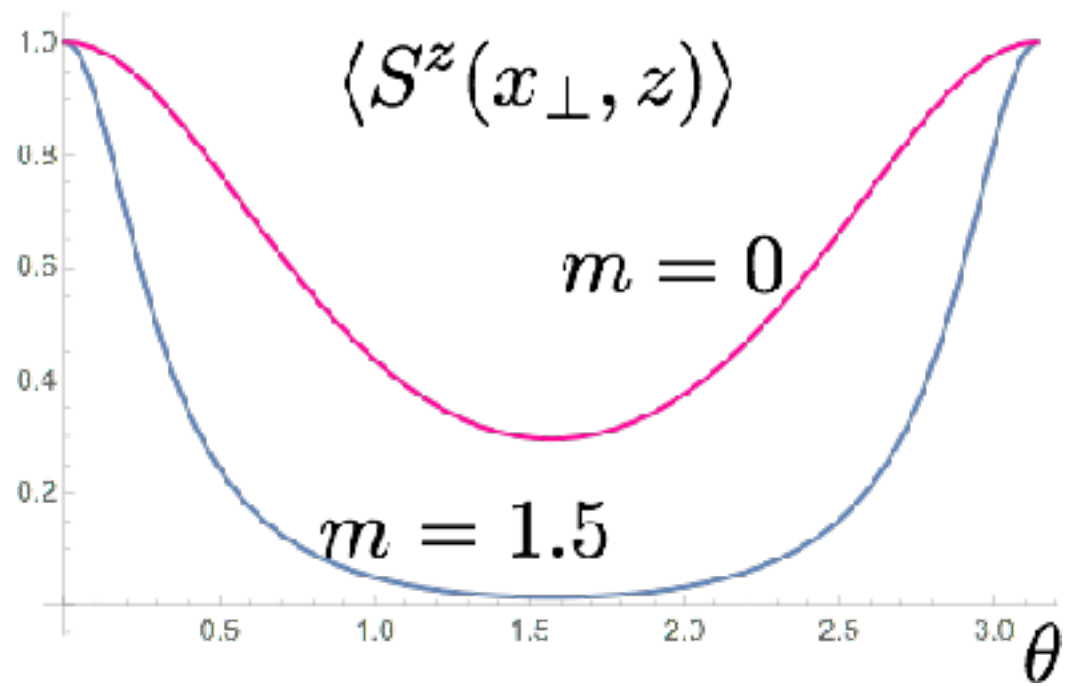


$|\Phi|^2$ and $\langle S^z \rangle$ along
a curve of radius $2/\sqrt{V'}$
in the $x_\perp - z$ plane,
for two quark masses,
$m = 0$ and $m = 1.5\sqrt{V'}$





Parton distributions have a sea component

In D=1+1 dimensions the sea component is prominent at low m/e :

$m/e = 0.1$

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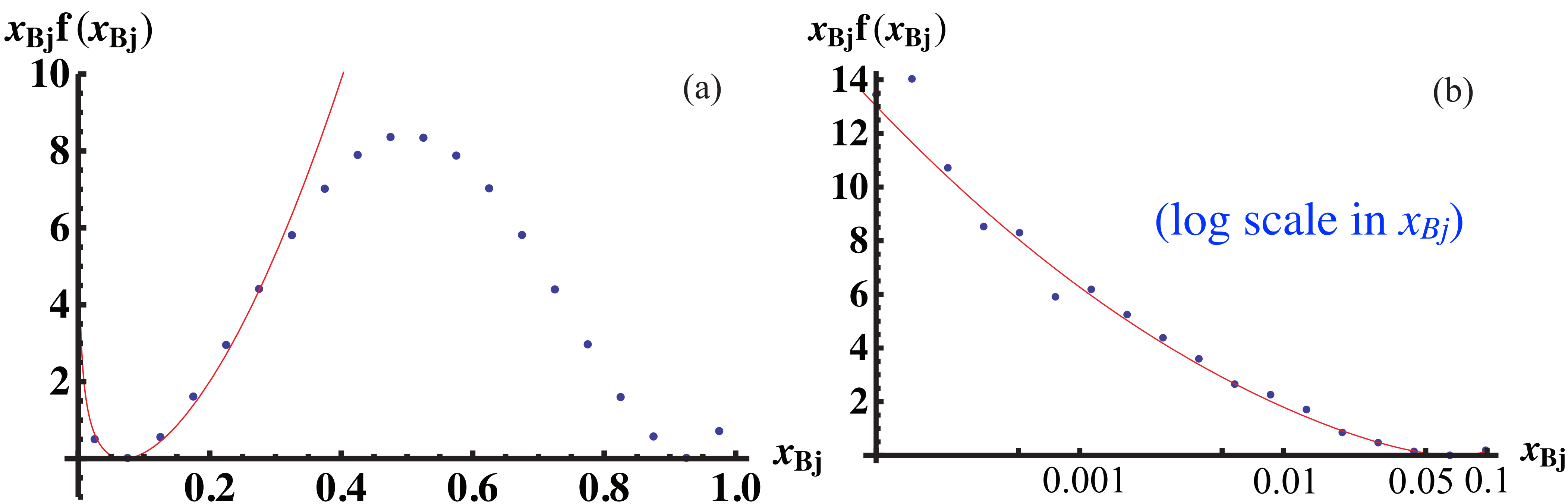

The red curve is an analytic approximation, valid in the $x_{Bj} \to 0$ limit.

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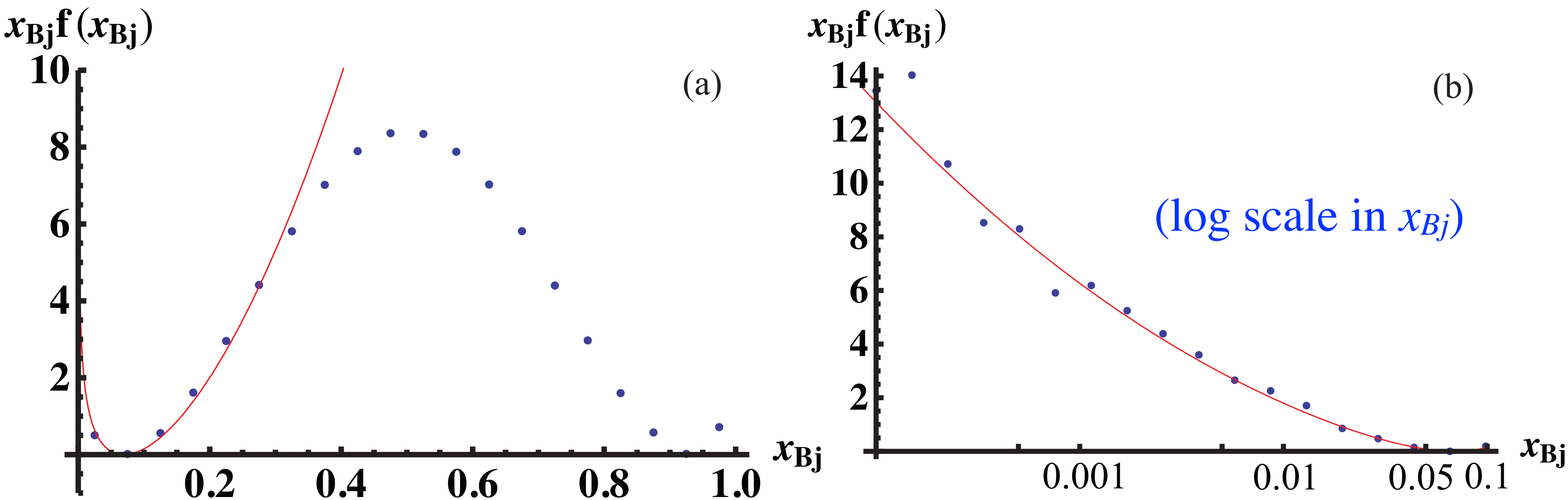

The red curve is an analytic approximation, valid in the $x_{Bj} \to 0$ limit.

Note: Enhancement at low x is due to bd (sea), not to $b^\dagger d^\dagger$ (valence) component.

String breaking at $O(1/N_c)$ is calculable, but is not included here.

Bound states in motion

A $q\bar{q}$ bound state with CM momentum $\boldsymbol{P}$ may be expressed as

$$|M,P\rangle_V \equiv \int d\boldsymbol{x}_1\, d\boldsymbol{x}_2\, \bar{\psi}(t=0,\boldsymbol{x}_1)\, e^{i\boldsymbol{P}\cdot(\boldsymbol{x}_1+\boldsymbol{x}_2)/2}\Phi^{(P)}(\boldsymbol{x}_1-\boldsymbol{x}_2)\psi(t=0,\boldsymbol{x}_2)\,|0\rangle$$

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What is the classical field $A^\mu_{(P)}$ in the potential Hamiltonian?

$$\mathcal{H}_V = \int d\boldsymbol{x}\,\psi^\dagger(t,\boldsymbol{x})\big[-i\boldsymbol{\alpha}\cdot\overrightarrow{\boldsymbol{\nabla}}+m\gamma^0+\tfrac{1}{2}\gamma^0 g\not{A}_{(P)}\big]\psi(t,\boldsymbol{x})$$

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The answer depends on the frame of the observer.

- Observer at rest, bound state is moving: $A^\mu_{(P)} = A^0$ $\quad$ P-independent
- Observer is moving, bound state at rest: $A^\mu_{(P)}$ is boosted A^0 field

States with $M = 0$

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The massless 0^{++} meson "σ"

$$|\sigma\rangle = \int d\boldsymbol{x}_1\, d\boldsymbol{x}_2\, \bar\psi(\boldsymbol{x}_1)\, \Phi_\sigma(\boldsymbol{x}_1 - \boldsymbol{x}_2)\, \psi(\boldsymbol{x}_2)\, |0\rangle \equiv \hat\sigma\, |0\rangle$$

For $m = 0$ and $V' = 1$:

$$\Phi_\sigma(\boldsymbol{x}) = N_\sigma \Big[J_0(\tfrac{1}{4}r^2) + \boldsymbol{\alpha}\cdot\boldsymbol{x}\frac{i}{r}\, J_1(\tfrac{1}{4}r^2) \Big]$$

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$$\hat P^\mu\,|\sigma\rangle = 0$$

State has *vanishing four-momentum* in any frame.
It may mix with the perturbative vacuum.
This spontaneously breaks chiral invariance.

A chiral condensate (m = 0)

Since $|\,\sigma\,\rangle$ has vacuum quantum numbers and zero momentum it can mix with the perturbative vacuum without violating Poincaré invariance

Ansatz: $|\chi\rangle = \exp(\hat{\sigma})\,|0\rangle$ implies $\langle\chi|\bar{\psi}\psi\,|\chi\rangle = 4N_\sigma$

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An infinitesimal chiral rotation of the condensate generates a pion

$$U_\chi(\beta) = \exp\left[i\beta\int d\boldsymbol{x}\,\psi^\dagger(\boldsymbol{x})\gamma_5\psi(\boldsymbol{x})\right] \qquad U_\chi(\beta)\,|\chi\rangle = (1-2i\beta\,\hat{\pi}\,|\chi\rangle$$

where $\hat{\pi}$ is the massless 0^{-+} state with wave function $\Phi_\pi = \gamma_5\Phi_\sigma$

Small quark mass: m > 0

When $m \neq 0$ the massless ($M_\sigma = 0$) sigma 0^{++} state has wave function

$$\Phi_\sigma(\boldsymbol{x}) = f_1(r) + i\,\boldsymbol{\alpha}\cdot\boldsymbol{x}\,f_2(r) + i\,\boldsymbol{\gamma}\cdot\boldsymbol{x}\,g_2(r)$$

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$$\Phi_\pi(\boldsymbol{x}) = \big[F_1(r) + i\,\boldsymbol{\alpha}\cdot\boldsymbol{x}\,F_2(r) + \gamma^0\,F_4(r)\big]\gamma_5$$

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$$\langle\chi|j_5^\mu(x)\hat{\pi}\,|\chi\rangle = iP^\mu f_\pi\,e^{-iP\cdot x} \quad\Longrightarrow\quad F_4(0) = \tfrac{1}{4}iM_\pi f_\pi$$

$$\langle\chi|\bar{\psi}(x)\gamma_5\psi(x)\,\hat{\pi}\,|\chi\rangle = -i\,\frac{M^2}{2m}\,f_\pi\,e^{-iP\cdot x} \quad\Longrightarrow\quad F_1(0) = i\,\frac{M^2}{8m}\,f_\pi$$

CSB relations are satisfied for any P.

Final remarks

Bound states can have both perturbative and non-perturbative aspects.

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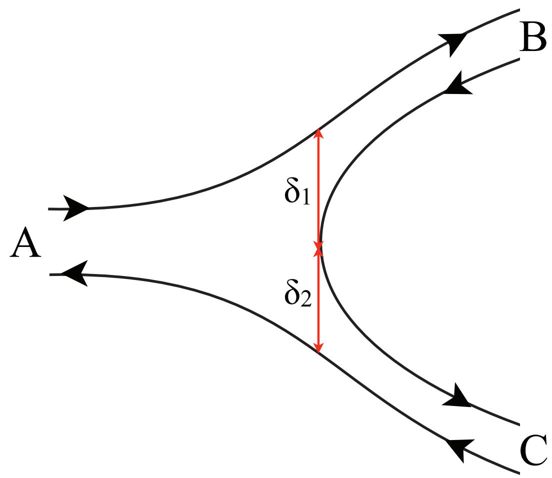

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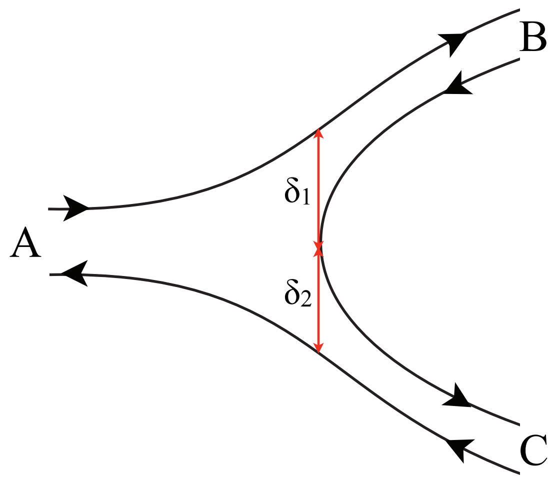

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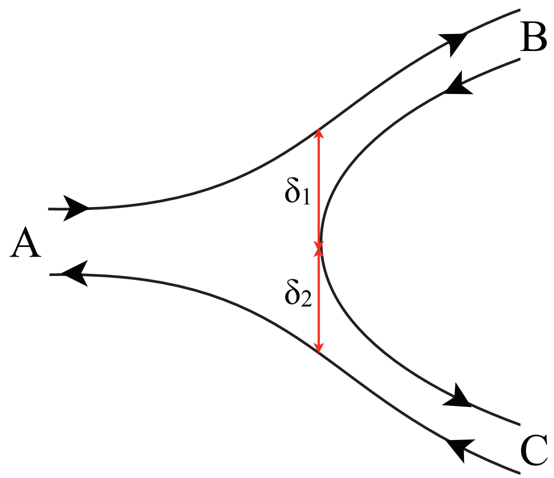

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Many interesting studies & tests are waiting to be done!