Testing universality in Gauge Theories

Alberto Ramos <alberto.ramos@ific.uv.es> IFIC (CSIC/UV)

Guilherme Catumba, Nicolas Lang

IFIC
INSTITUT DE FÍSICA CORPUSCULAR

CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

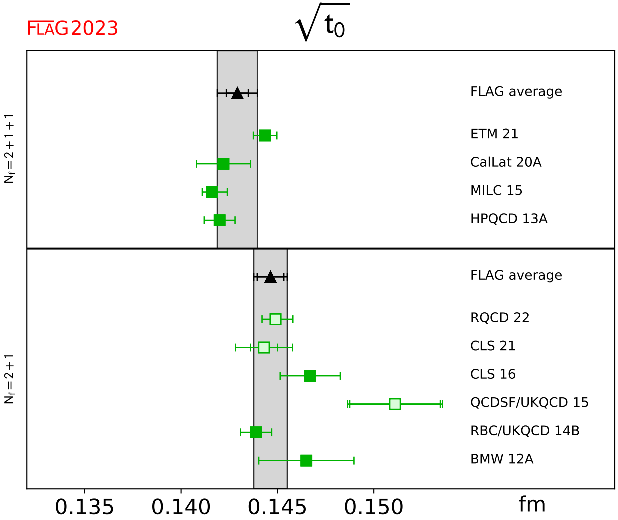
GENERALITAT VALENCIANA

Motivation: understanding scale setting

- t_0 Very precise, little systematic
- 1.5% in scale determinations
- Crucial for precision physics
- Results obtained with different actions
- $a \in [0.05 - 0.1]$ fm

Potential problems

- Continuum extrapolation
- Determination of physical quantity (i.e. $f_\pi, M_\Omega, \dots$)



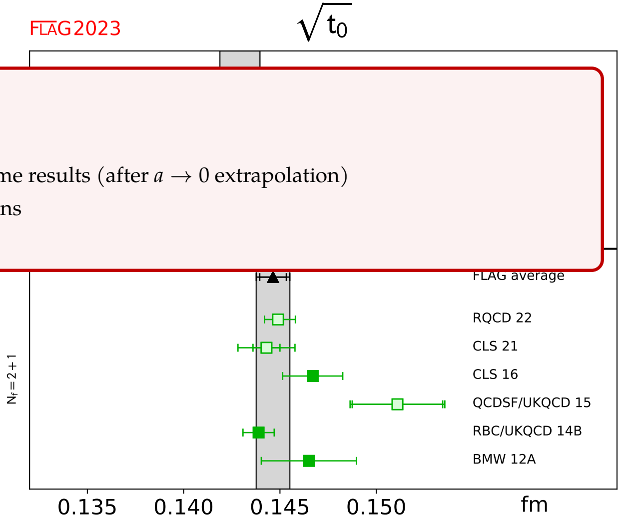
Motivation: understanding scale setting

This talk

- Which action has smaller cutoff effects?
- Is g_0 small enough?
- Universality: different actions give the same results (after $a \to 0$ extrapolation)
- Some points on checks on scaling violations
- Preliminary results on pure gauge

Potential problems

- Continuum extrapolation
- Determination of physical quantity (i.e. $f_\pi, M_\Omega, \ldots$)



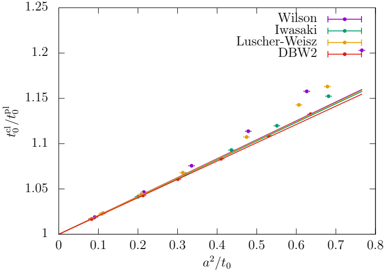
Testing scaling with flow quantities: The wrong way

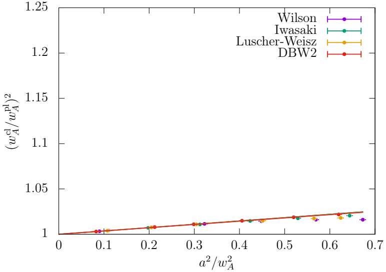
Ideal quantity: $t_0^{\rm pl}/t_0^{\rm cl}$

- We know the continuum limit

$$\lim_{a\to 0} \frac{t_0^{\rm pl}}{t_0^{\rm cl}} = 1$$

- Extremely precise (correlated numerator/denominator)
- Same game with $(w_A^{\rm pl}/w_A^{\rm cl})^2$





Testing scaling with flow quantities: The wrong way

Wrong conclusions

- Wilson, Iwasaki, LW, DB2 all have similar cutoff effects (see **[Husung, Fri]**).
- w_0 - like scales have much smaller cutoff effects
- Violations to a^2 scaling are below 8% at $a < 0.08$ fm for t_0 - like scales
- Violations to a^2 scaling are below 1% at $a < 0.08$ fm for w_0 - like scales

Testing scaling with flow quantities: The wrong way

Symanzik effective description for flow quantities

- $t^2\langle E(t)\rangle$ is a *non-local* observable (i.e. smeared over a distance $\sqrt{8t}$)
- Special care to interpret scaling violations of flow quantities

How to understand cutoff effects?

Symanzik effective theory

- Any lattice action that we simulate S_{latt} can be described by an effective action

$$S_{\text{latt}} \overset{a\to 0}{\sim} S_{\text{cont}} + a^2 S_2 + \dots$$

- Spectral quantities computed on the lattice have an asymptotic expansion

$$\langle O \rangle_{\text{latt}} \overset{a\to 0}{\sim} \langle O \rangle + a^2 \langle O S_2 \rangle_c + \dots$$

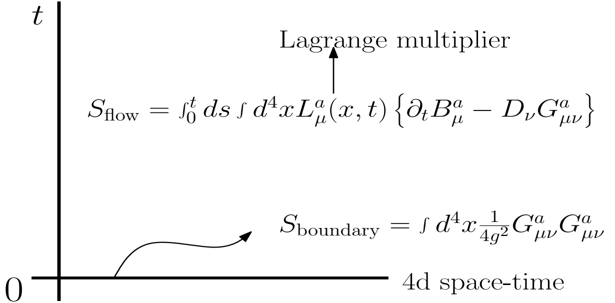
But they are difficult to compute (signal-to-noise, finding plateaus, ...)

Flow quantities as an alternative

- Symanzik expansion for flow quantities
- Lessons for QCD?

5d Local formulation [Lüscher, Weisz '11]

We can see the theory as a 5d local field theory [Zinn-Justin '86, Zinn-Justin, Zwanziger '88]



$$S_{\text{Total}} = S_{\text{flow}} + S_{\text{boundary}}$$

The important point

- No loops on the bulk $\Rightarrow$ "Classical theory" at $t > 0$

Symanzik effective theory for the gradient flow [A. Ramos, S. Sint '15]

Symanzik effective theory has several "parts"

$$S^{5d}_{\text{latt}} \overset{a\to 0}{\sim} S^{5d}_{\text{cont}} + a^2 S_{2,b} + a^2 S_{2,fl} + \dots$$

- "Usual" corrections
- Affects <u>all</u> quantities (i.e. $m_p, g-2, t_0, \dots$)
- Determined by the action that you simulate (i.e. Iwasaki/Wilson, Domain Wall/Clover)
- Affects <u>only</u> flow quantities
- Determined by *how you integrate the flow equations* (i.e. Wilson/Symanzik flow)

Symanzik expansion of a flow quantity $O \overset{a\to 0}{\sim} O_0 + a^2 O_2$

$$\langle O\rangle_{\text{latt}} \overset{a\to 0}{\sim} \langle O_0\rangle + a^2 \left\{ \langle O_2\rangle + \langle O_0 S_{2,b}\rangle + \langle O_0 S_{2,fl}\rangle + c_b t^2 \frac{\mathrm{d}}{\mathrm{d}t}\Big|_{t_0} \langle O_0\rangle \right\}$$

Theory "classical" at $t > 0$: Non-perturbative result/all improvement

Use Zeuthen flow $\Longrightarrow$ $S_{2,fl} = 0$

Use Classically improved observables (i.e. $(4E_{\text{pl}} - E_{\text{cl}})/3)$) $\Longrightarrow$ $O_2 = 0$

Understanding $t_0^{\rm pl}/t_0^{\rm cl}$

Apply Symanzik expansion for t_0

$$t_0^{\rm pl} \overset{a\to 0}{\sim} t_0 - \frac{a^2}{D}\left\{ t_0^2\langle E(t_0) S_{2,b}\rangle + t_0^2\langle E(t_0) S_{2,fl}\rangle + t_0^2 \langle E_2^{\rm pl}(t_0)\rangle + c_b \frac{\mathrm{d}}{\mathrm{d}t} t^2\langle E(t)\rangle \right\}$$

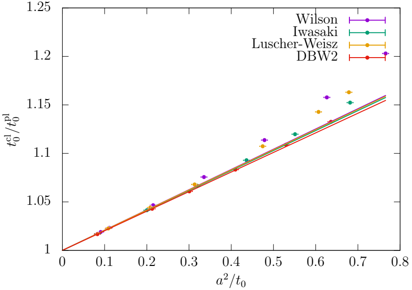
$$t_0^{\rm cl} \overset{a\to 0}{\sim} t_0 - \frac{a^2}{D}\left\{ t_0^2\langle E(t_0) S_{2,b}\rangle + t_0^2\langle E(t_0) S_{2,fl}\rangle + t_0^2 \langle E_2^{\rm cl}(t_0)\rangle + c_b \frac{\mathrm{d}}{\mathrm{d}t} t^2\langle E(t)\rangle \right\}$$

The ratio/difference does not say anything useful

$$\frac{t_0^{\rm pl}}{t_0^{\rm cl}} \overset{t\to 0}{\sim} 1 - \frac{a^2}{D}\left\{ t_0^2\langle E_2^{\rm pl}(t_0)\rangle - t_0^2\langle E_2^{\rm cl}(t_0)\rangle \right\}$$

- Insensitive to $S_{2,b}$
- Only sensitive to something that can be made zero explicitly: Choose

$$E^{\rm latt}(t) = \frac{4}{3}E^{\rm pl}(t) - \frac{1}{3}E^{\rm cl}(t)$$



Improvement of the flow [A. Ramos, S. Sint '15]

$$\langle O\rangle_{\text{latt}} \overset{a\to 0}{\sim} \langle O_0\rangle + a^2\left\{\langle O_2\rangle + \langle O_0 S_{2,b}\rangle + \langle O_0 S_{2,fl}\rangle\right\}$$

The Zeuthen flow: $S_{2,fl} = 0$

$$a^2 \frac{d}{dt} V_\mu(x,t) = -g_0^2 \left(1 + \frac{a^2}{12} D_\mu D_\mu^*\right) \frac{\delta S^{\text{LW}}[V]}{\delta V_\mu(x,t)} V_\mu(x,t)$$

Classically improved observables: $O_2 = 0$

$$E^{\text{latt}}(t) = \frac{4}{3} E^{\text{pl}}(t) - \frac{1}{3} E^{\text{cl}}(t)$$

Extra required improvement parameter: $c_b(g_0^2)$

- Shift in the initial condition (similar to τ-shift [Cheng et. al. '14])

$$V_\mu(t,x)\Big|_{t=0} = \exp\{c_b g_0^2 \partial_{x,\mu} S_g[U]\} U_\mu(x)$$

- Tree-level improvement requires $c_b^{(0)}(g_0^2) = 0$. Reasonable range $|c_b| < 0.03$

c_b DEPENDENCE

- t_0-like scales more sensitive to c_b than w_0-like scales

Another point of view for the c_b effect

A shift in the initial condition

$$V_\mu(t,x)\Big|_{t=0} = \exp\{c_b g_0^2 \partial_{x,\mu} S_g[U]\} U_\mu(x)$$

can be understood as a shift at some time $t > 0$

$$V'_\mu(t,x)\Big|_{t=t_s} = \exp\{c_b g_0^2 \partial_{x,\mu} S_g[V]\} V_\mu(t_s,x)$$

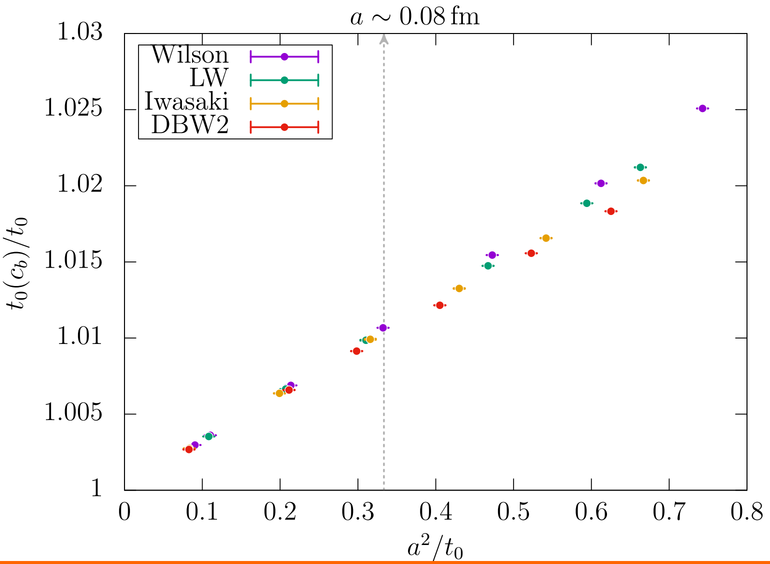
In particular if you use $t^2\langle E(t + c_b a^2)\rangle$ to determine t_0:

$$t_0(c_b) \overset{a\to 0}{\sim} t_0 - \frac{a^2}{D}\left\{ t_0^2 \langle E(t_0) S_{2,b}\rangle + c_b t_0^2 \frac{\mathrm{d}}{\mathrm{d}t}\Big|_{t_0} E(t)\right\}$$

c_b moved to positive flow time: Classical effect, pure a^2-term. Different c_b Does not give useful information

$$\frac{t_0(c_b)}{t_0} \overset{t\to 0}{\sim} 1 - \frac{a^2}{D}\left\{ c_b t_0^2 \frac{\mathrm{d}}{\mathrm{d}t}\Big|_{t_0} E(t)\right\}$$

c_b DEPENDENCE



Testing universality

It is difficult

- Changes in flow discretization, comparing t_0^{pl} and t_0^{cl}, comparing $t_0(c_b)$ with t_0 give no information!: Trivial "classical" a^2-effects.

Viable strategy

$$\langle O \rangle_{\mathrm{latt}} \overset{a\to 0}{\sim} \langle O_0 \rangle + a^2 \left\{ \cancel{\langle O_2 \rangle} + \langle O_0 S_{2,b} \rangle + \cancel{\langle O_0 S_{2,fl} \rangle} \right\}$$

- Use Zeuthen flow/classically improved observables
- Use ratios t_1/t_0 or w_A^2/w_B^2
- These quantities can be considered "spectral quantities": Probes of cutoff effects of your action

Data: pure gauge simulations with different actions $(L/a = T/a = 64)$

Action	β	a [fm]	MDU's	MSM	t_0/a^2	$\tau_{\rm int}(t_0)$
Plaquette	6.13	0.082	29400	700	4.6865(19)	1.08(31)
	6.25	0.068	44940	1070	6.7877(43)	1.92(57)
	6.35	0.059	86520	1030	9.0685(61)	1.20(30)
	6.42	0.053	204120	1944	11.0507(71)	1.34(27)
	6.52	0.046	404460	1926	14.5442(95)	0.79(13)
Lüscher-Weisz	4.45	0.100	10150	145	3.2287(14)	0.66(29)
	4.59	0.080	31500	450	4.8324(22)	0.81(24)
	4.71	0.068	50400	480	6.6988(35)	0.61(17)
	4.83	0.058	219240	1566	9.2193(53)	1.25(27)
	4.93	0.051	402360	2874	11.9463(69)	1.39(24)
	5.00	0.047	845040	3018	14.2632(73)	0.84(11)
Ïwasaki	2.79	0.079	48300	1150	4.9186(21)	1.68(47)
	2.91	0.067	100800	2400	6.9980(38)	3.21(84)
	3.00	0.059	122220	1455	8.9465(55)	1.48(35)
	3.11	0.051	673260	6412	12.0012(51)	1.78(24)
	3.18	0.047	580440	2764	14.4398(87)	1.07(17)
DBW2	1.111	0.081	77280	920	4.7300(13)	0.73(16)
	1.16	0.073	63840	760	5.7980(25)	1.02(28)
	1.24	0.063	245700	2925	7.9661(34)	1.93(38)
	1.35	0.051	103700	1250	11.678(10)	2.67(23)
	1.40	0.046	763980	9095	14.4996(84)	3.15(47)

Dataset: flow scales

Two natural candidates

- t_0 - like scales [Luscher '10]

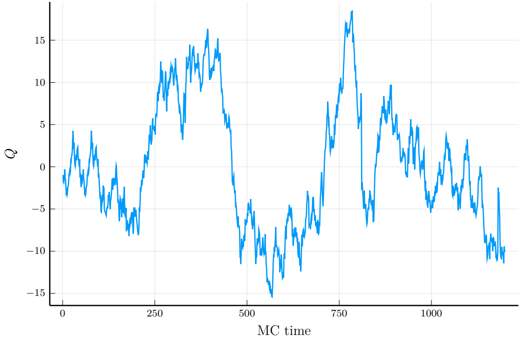
$$t^2 \langle E(t) \rangle \Big|_{t=t_c} = \begin{cases} 0.15 & (t_c = t_2) \\ 0.3 & (t_c = t_0) \\ 0.5 & (t_c = t_1) \end{cases}$$

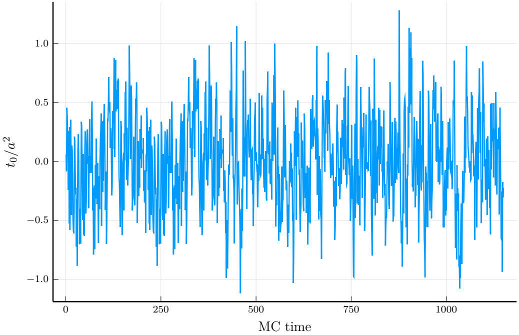
- w_0 - like scales [BMW '10]: Very similar conditions

$$t \frac{\mathrm{d}}{\mathrm{d}t} t^2 \langle E(t) \rangle \Big|_{t=w_c^2} = \begin{cases} 0.097 & (w_c = w_A) \\ 0.285 & (w_c = w_B) \\ 0.550 & (w_c = w_C) \end{cases}$$

- Spoiler: Conclusions practically identical for both set of quantities

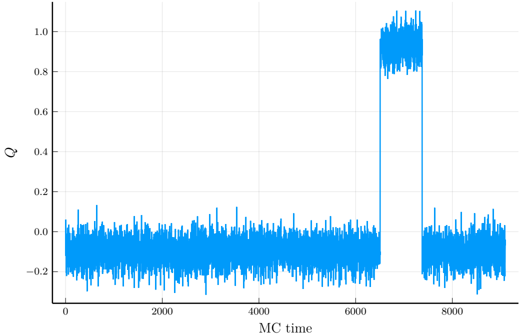
Dataset: Topology freezing for Iwasaki

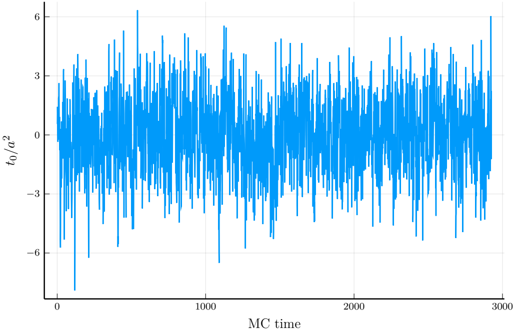




- Severe topology freezing
- No visible effect in t_0 (large volumes $L \approx 3.2 - 6.5$ fm)
- We need simulations with open boundaries

Dataset: Topology freezing for DBW2





- Severe topology freezing
- No visible effect in t_0 (large volumes $L \approx 3.2 - 6.5$ fm)
- We need simulations with open boundaries

Adaptive size integrators

- Flow equation on the Latttice

$$a^2 \frac{dV_\mu(x,t)}{dt} = Z(V)\, V_\mu(x,t)\,.$$

- Difficult to beat RK3 used with $\epsilon = 0.01$ [Lüscher, 2010]

$$\begin{aligned} W_0 &= V_\mu(x,t) \\ W_1 &= \exp\left\{\frac{1}{4} Z_0\right\} W_0 \\ W_2 &= \exp\left\{\frac{8}{9} Z_1 - \frac{17}{36} Z_0\right\} W_1 \\ V_\mu(x, t + a^2\epsilon) &= \exp\left\{\frac{3}{4} Z_2 - \frac{8}{9} Z_1 + \frac{17}{36} Z_0\right\} W_2 \end{aligned}$$

- Detailed study in [A. Bazavov, T. Chuna; arxiv:2101.05320]

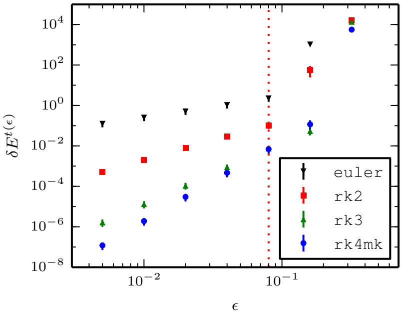


Figure: [Phys. Rev. D 92, 074502 (2015)]

Adaptive size integrators

- Flow equation on the Latttice

$$a^2 \frac{\mathrm{d}V_\mu(x,t)}{\mathrm{d}t} = Z(V)\, V_\mu(x,t)\,.$$

- Difficult to beat RK3 used with $\epsilon = 0.01$ [Lüscher, 2010]

$$\begin{aligned} W_0 &= V_\mu(x,t) \\ W_1 &= \exp\left\{\frac{1}{4}Z_0\right\} W_0 \\ W_2 &= \exp\left\{\frac{8}{9}Z_1 - \frac{17}{36}Z_0\right\} W_1 \\ V_\mu(x,t+a^2\epsilon) &= \exp\left\{\frac{3}{4}Z_2 - \frac{8}{9}Z_1 + \frac{17}{36}Z_0\right\} W_2 \end{aligned}$$

- Detailed study in [A. Bazavov, T. Chuna; arxiv:2101.05320]

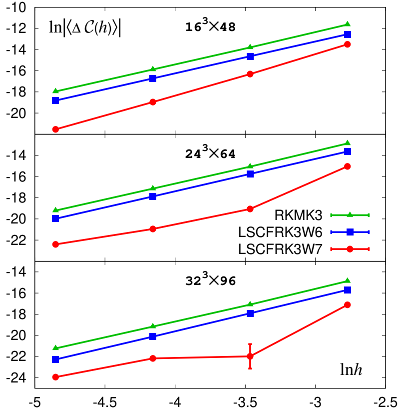


Figure: [A. Bazavov, T. Chuna; arxiv:2101.05320]

Adaptive step size integrators

- RK2 embeded in RK3

$$
\begin{aligned}
W_0 &= V_\mu(x,t) \\
W_1 &= \exp\left\{\frac{1}{4}Z_0\right\} W_0 \\
W_2 &= \exp\left\{\frac{8}{9}Z_1 - \frac{17}{36}Z_0\right\} W_1 \\
V_\mu(x,t+a^2\epsilon) &= \exp\left\{\frac{3}{4}Z_2 - \frac{8}{9}Z_1 + \frac{17}{36}Z_0\right\} W_2 \\
V'_\mu(x,t+a^2\epsilon) &= \exp\left\{Z_0 + 2Z_1\right\} W_0
\end{aligned}
$$

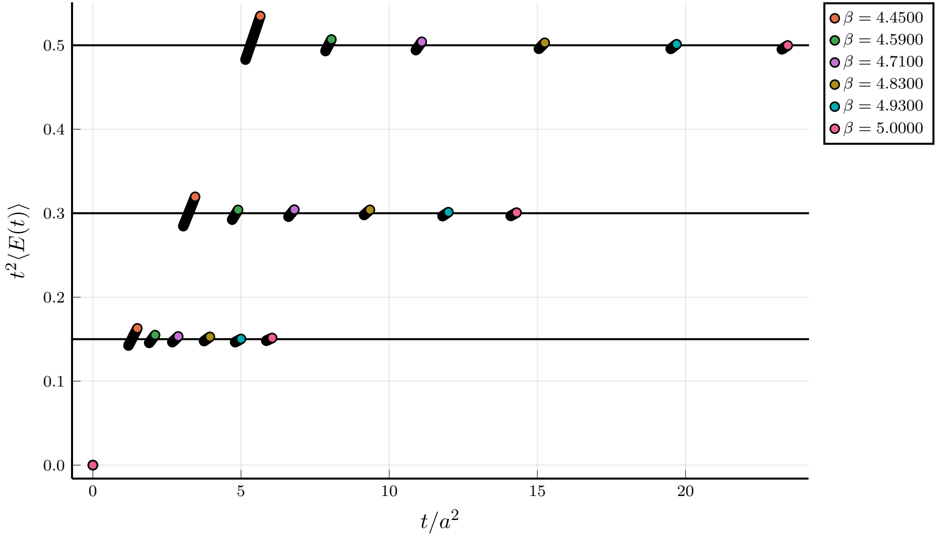
- Compare V and V'

$$\mathrm{dist}(V,V') = \frac{1}{9}\max_{x,\mu}\sqrt{V_\mu(x,t+a^2\epsilon) - V'_\mu(x,t+a^2\epsilon)}$$

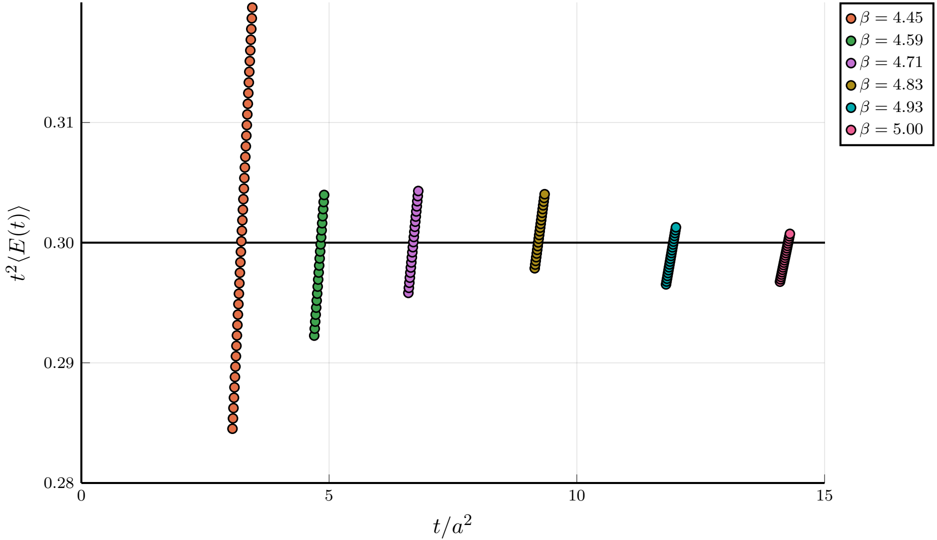
- Define tolerance δ and scale step size ($\delta = 10^{-8}$)

$$\epsilon \to \epsilon \times 0.95\sqrt[3]{\frac{\delta}{d}}$$

Adaptive size integrators: example with LW action



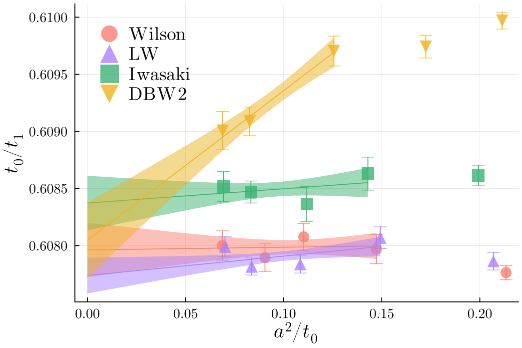
Adaptive size integrators: example with LW action



Adaptive size integrators: example with LW action

At finer lattice spacing: $\mathcal{O}(500)$ steps to reach $t/a^2 \approx 25$ ($5\times$ cheaper!)

Long distance ratio

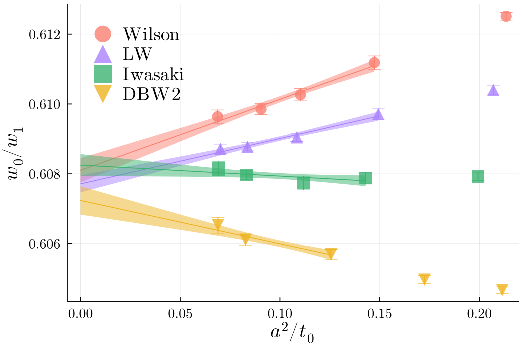


- Reasonable agreement for ratio of scales
- Good description of data (ex: t_0)

$$\frac{\chi^2}{\langle\chi^2\rangle} = \begin{cases} 1.18/2 & PL \\ 4.79/2 & LW \\ 1.42/2 & IW \\ 0.23/2 & DB \end{cases}$$

- Results very similar both for $t-$, $w-$like scales

Long distance ratio

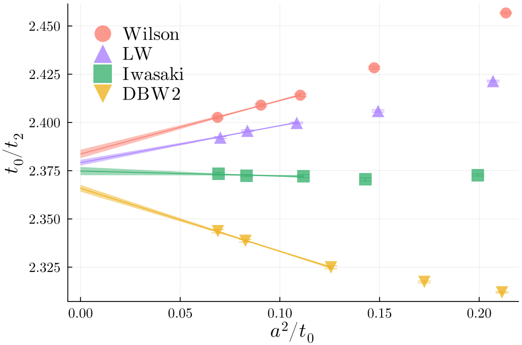


- Reasonable agreement for ratio of scales
- Good description of data (ex: t_0)

$$\frac{\chi^2}{\langle \chi^2 \rangle} = \begin{cases} 1.18/2 & PL \\ 4.79/2 & LW \\ 1.42/2 & IW \\ 0.23/2 & DB \end{cases}$$

- Results very similar both for $t-$, $w-$like scales

Short distance ratio

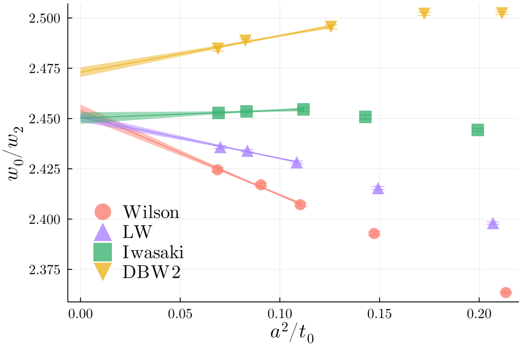


- Reasonable agreement for ratio of scales
- Good description of data (ex: t_0)

$$\frac{\chi^2}{\langle\chi^2\rangle} = \begin{cases} 0.14/1 & PL \\ 2.51/1 & LW \\ 0.69/1 & IW \\ 0.22/1 & DB \end{cases}$$

- Results very similar both for $t-, w-$like scales

Short distance ratio



- Reasonable agreement for ratio of scales
- Good description of data (ex: t_0)

$$\frac{\chi^2}{\langle\chi^2\rangle} = \begin{cases} 0.14/1 & PL \\ 2.51/1 & LW \\ 0.69/1 & IW \\ 0.22/1 & DB \end{cases}$$

- Results very similar both for $t-, w-$like scales

Conclusions

- Do not draw strong conclusions from plaquette = clover
- Flow quantities in improved setup
  - No essential difference between $t-$, $w-$ like (should explore both!)
  - Cutoff effects (mainly) come from action used
- Adaptive step size integrators: Can save significant time at fine lattices
-

Ratio of (improved) flow scales in pure gauge

- When short distance physics is involved, things can go wrong
- …**without being visible in the data**
- How well are we doing in QCD??