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$M^{\rm latt} = R_i^{-1}$ ($m^{\rm phys} = r_i^{-1}$) are such examples: r_i at physical quark masses or at some other well-defined point in the quark mass plane.

Some questions

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- At short distances $\alpha_s^{(N_f=4)}(r^{-1}) > \alpha_s^{(N_f=3)}(r^{-1})$ if these theories are matched in the infrared. (The $N_f = 3$ coupling runs faster towards small values.)
  This means $f^{(N_f=4)}(r) > f^{(N_f=3)}(r)$ and, therefore, $r_i^{(N_f=4)} < r_i^{(N_f=3)}$.
  This effect should be bigger for r_1 than for r_0? How big is it?