MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES
Financiado por la Unión Europea NextGenerationEU
Plan de Recuperación, Transformación y Resiliencia
AGENCIA ESTATAL DE INVESTIGACIÓN

Edinburgh Consensus

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Antonin Portelli, Nazario Tantalo (**FLAG:** The low-energy limit of the Standard Model). Special thanks to M. Golterman

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GENERALITAT VALENCIANA

CIDEGENT/2019/040 ASFAE/2022/020 CNS2022-136005

$\mathrm{QCD}_{N_\mathrm{f}}$ on the lattice

$N_\mathrm{f} + 1$ dimensionless inputs

$$S = -\frac{\beta}{6}\sum_{\mathcal{W}}(U_{\mathcal{W}}) + \sum_{\mathrm{f}}\overline{\psi}D(\hat{m}_\mathrm{f})\psi\,, \qquad (\hat{m} = am_0)\ .$$

- $\beta = \frac{1}{g_0^2}$, $\hat{m}_\mathrm{f} = am_{0,\mathrm{f}}$ are dimensionless

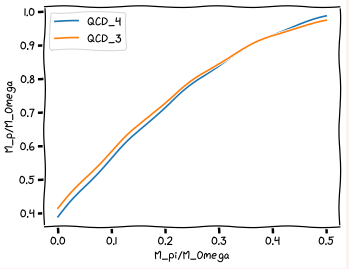
Only dimensionless predictions in the continuum are unambiguous

- N_f **dimensionless** input
- Different theories, different curves
- Since the whole curve is unambiguous, also derivatives are

$$\frac{\partial}{\partial(M_\pi/M_\Omega)}\frac{M_p}{M_\Omega}$$

in contrast to

$$\frac{\partial}{\partial M_\pi}M_p$$



$(\mathrm{QCD} + \mathrm{QED})_{N_\mathrm{f}}$ on the lattice

$N_\mathrm{f} + 2$ dimensionless input

- $g_0^2, \hat{m}_\mathrm{f}, \alpha_\mathrm{EM}$
- If working to leading order in α_EM (i.e. ignoring $\mathcal{O}(\alpha_\mathrm{EM}^2)$)

$$\alpha_\mathrm{EM} = \alpha_0 \approx 1/137$$

In principle only dimensionless predictions in the continuum are unambiguous

- N_f **dimensionless** input
- Different theories, different curves
- Since the whole curve is unambiguous, also derivatives are

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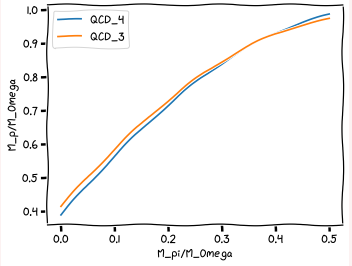
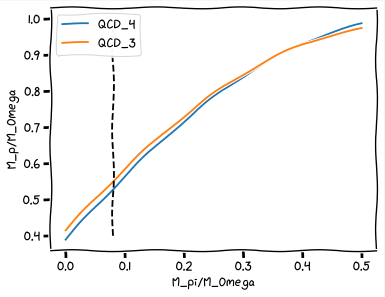


Differences[1] between QCD_{N_f} and $(\mathrm{QCD}+\mathrm{QED})_{N_f}$

QCD_{N_f}



- What to choose for $\frac{M_\pi}{M_\Omega} = ??$
- What input to use (i.e. $M_\Omega, f_\pi, \dots$) to give final results?

$(\mathrm{QCD}+\mathrm{QED})_{N_f}$

- Real world, up to irrelevant precision
- Clear option

$$\frac{M_\pi^+}{M_\Omega} = \frac{[M_\pi^+]^{\mathrm{exp}}}{[M_\Omega]^{\mathrm{exp}}}$$

- Input choice is irrelevant:

$$[M_\Omega]^{\mathrm{exp}} \times \frac{M_p}{M_\Omega}\Big|_{\frac{M_\pi^+}{M_\Omega}=\mathrm{exp}} = [M_\Xi]^{\mathrm{exp}} \times \frac{M_p}{M_\Xi}\Big|_{\frac{M_\pi^+}{M_\Xi}=\mathrm{exp}}$$

[1] We assumme $N_f \geq 3$, and up to some "irrelevant" precision

Edinburgh consensus [FLAG '24]

	QCD	isoQCD
$M_{\pi+}$	135.0 MeV	135.0 MeV
M_{K+}	491.6 MeV	494.6 MeV
M_{K^0}	497.6 MeV	494.6 MeV
$M_{D_s^+}$	1967 MeV	1967 MeV
$M_{B_s^0}$	5367 MeV	5367 MeV
$f_{\pi+}$	130.5 MeV	130.5 MeV

	QCD	isoQCD
$M_{\pi+}/f_{\pi+}$	1.034	1.034
$M_{K+}/f_{\pi+}$	3.767	3.790
$M_{K^0}/f_{\pi+}$	3.813	3.790
$M_{D_s^+}/f_{\pi+}$	15.07	15.07
$M_{B_s^0}/f_{\pi+}$	41.13	41.13

Table: Edinburgh Consensus for the definition of pure QCD and isospin-symmetric QCD.

- f_π cannot be directly measured by experiments, instead we measure the photon-inclusive decay rate

$$\Gamma_{\pi l2} = \Gamma(\pi \to l\nu) + \Gamma(\pi \to l\nu\gamma)$$

- In (iso)QCD$_{N_f}$ we have access to (i.e. $f_\pi = f_{\pi+}$ no strong isospin breaking!!):

$$\Gamma(\pi \to l\nu)\Big|_{\alpha=0, m_u=m_d} = \frac{G_F |V_{ud}|^2 f_\pi^2}{8\pi} M_\pi m_l^2 \left[1 - \frac{m_l^2}{M_\pi^2}\right]^2 .$$

We need CKM elements V_{ud}

Choosing f_π as scale

$$\Gamma_{\pi l2} = \Gamma(\pi \to l\nu) \times [1 + \delta_{\rm EW}] \times \left(1 + \delta^p_{\rm EM}\right).$$

- Universal electro-weak correction (2%): Short distance quantity (γ attached to EW vertex2)

$$\delta_{\rm EW} = 0.0232 \approx \frac{2\alpha}{\pi} \log\left(\frac{m_Z}{m_\rho}\right),$$

- Electromagnetic corrections are universal ($F(x)$) and structure dependent ($c_i^\pi; \tilde{c}_2^\pi$)

$$\delta^\pi_{\rm EM} = -\frac{\alpha}{\pi}\left\{-F\left(\frac{m_l^2}{m_\pi^2}\right) + \frac{3}{2}\log\left(\frac{m_\rho}{m_\pi}\right) + c_1^\pi + \frac{m_l^2}{m_\rho^2}\left[c_2^\pi \log\left(\frac{m_\rho}{m_l}\right) + c_3^\pi + c_4^\pi\left(\frac{m_l}{m_\pi}\right)\right] - \tilde{c}_2^\pi \frac{m_\pi^2}{m_\rho^2}\log\left(\frac{m_\rho^2}{m_l^2}\right)\right\}.$$

- Best model based on $SU(3) - \chi$PT and $N \to \infty$ arguments **[Cirigliano, Rosell; '07]**

$$c_1^\pi = -2.56 \quad c_2^\pi = 5.2 \quad c_3^\pi = -10.5 \quad c_4^\pi = 1.69 \quad \tilde{c}_2^\pi = 0\,.$$

- Compare with point-particle approximation $c_i^\pi = \tilde{c}_2^\pi = 0$: 0.3% effect in f_π
- Agreement with RM123 (electroquenched) computation **[Di Carlo et. al. '19]**

Choices for meson mases

	QCD ($\hat{M}$)	isoQCD ($\bar{M}$)
$M_{\pi+}$	135.0 MeV	135.0 MeV
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- Any physics dimensionless quantity $X(\alpha, m_l/\mathcal{S}, M/\mathcal{S})$ at the physical point can be decomposed

$$X(\alpha^\phi, m_l^\phi/\mathcal{S}^\phi, M^\phi/\mathcal{S}^\phi) = \bar{X} + X_\gamma + X_{SU(2)}$$

  - $\bar{X}$: isoQCD value ($\alpha = m_u - m_d = 0$)
  - X_γ: Electromagnetic corrections
  - $X_{SU(2)}$: Strong isospin corrections

- Decomposition at physical point **unique** once we fix $\hat{M}$ and $\bar{M}$

$$X_\gamma = X(\alpha^\phi, M^\phi/\mathcal{S}^\phi) - X(0, \hat{M}/\hat{\mathcal{S}}); \quad X_{SU(2)} = X(0, \hat{M}/\hat{\mathcal{S}}) - X(0, \bar{M}/\bar{\mathcal{S}})$$

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- Particular choices for $\hat{M}, \bar{M}$ from preious (electroquenched!) experience **[RM123,BMW,]**

Conclusions

- Choice of scale: f_π
  - Easy to compute on the lattice (lomg plateaus)
  - Ambiguities well below a 1%.
  - $|V_{ud}|$ well known experimentally[a]
- Choices for (iso)QCD masses: $\hat{M}, \bar{M}$ based on electroquenched experience

[a] There are discussions on V_{ud}, but even PIBETA has a 0.3% result free of nuclear uncertainties **[Phys.Rev.Lett.93:181803,2004]**

- Discrepancies with choice $M_\Omega = 1672.45$ MeV??
- We need to understand "scheme dependence" better