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Quark TMDs from back-to-back dijet production at forward rapidities in pA collisions beyond eikonal accuracy in the CGC

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Probing the CGC and QCD matter at hadron colliders
Galileo Galilei Institute for Theoretical Physics, Florence
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T. Altinoluk, G. Beuf, E. Blanco, S. Mulani - arXiv: 2412.08485

Happy 60th birthday Edmond!

Sto lat!

Dilute-dense scattering within CGC and Eikonal approximation

High energy scattering within the CGC relies on two pillars:

Semi-classical approximation → dense target is represented by strong semiclassical gluon field $\mathcal{A}^\mu_a(x) = O\left(1/g\right)$ at weak coupling g

Eikonal approximation → keeping only the leading power terms in the high energy limit

High energy limit can be achieved by boosting the target along x^-:

$$\mathcal{A}^\mu_a(x) \mapsto \begin{cases} \gamma_t\, \mathcal{A}^-_a\left(\gamma_t\, x^+,\, \frac{x^-}{\gamma_t},\, \mathbf{x}\right) \\ \frac{1}{\gamma_t}\mathcal{A}^+_a\left(\gamma_t\, x^+,\, \frac{x^-}{\gamma_t},\, \mathbf{x}\right) \\ \mathcal{A}^i_a\left(\gamma_t\, x^+,\, \frac{x^-}{\gamma_t},\, \mathbf{x}\right) \end{cases}$$

The Eikonal approximation can be understood as the limit of infinite boost of $\mathcal{A}^\mu_a(x)$

(i) Background field is independent of x^- due to Lorentz time dilation (static limit) → no p^+ transfer from the target

(ii) Background field is Lorentz contracted (shockwave limit) → no transverse motion within the target

(iii) Only the largest component of the background field is accounted for during the interaction

In the Eikonal limit the background field → $\boxed{\mathcal{A}^\mu(x^+, x^-, \mathbf{x}) \simeq \delta^{\mu -}\mathcal{A}^-_a(x^+\mathbf{x}) \propto \delta(x^+)}$

$\left(g\mathcal{A}^-(x^+, \mathbf{x})\right)^n$ are resumed to all orders which leads to Wilson lines along x^+

Next-to-Eikonal corrections to the CGC

Next-to-Eikonal (NEik) corrections are $\mathcal{O}(1/\gamma_t)$ at the level of the boosted background field.

NEik corrections arise from relaxing either of the three approximations:

1. Interactions with the suppressed components of background field (transverse component)
2. Finite longitudinal width of the target — transverse motion of the parton in the medium
3. x^- dependence of the background field beyond infinite Lorentz dilation

NEik corrections to quark and gluon propagators in a gluon background have been computed with applications to — Altinoluk *et al.* (2014-2025)

- forward parton-nucleus scattering at NEik (both dilute and dense limits)
- DIS dijet production at NEik (both dilute and dense limits)

An extra source of NEik corrections: interaction with the quark background of the target — Altinoluk *et al.* (2023)

- interaction between the projectile parton and the target occurs via *t-channel quark exchange*
- application to quark-gluon dijet production in DIS

see also

quark and gluon helicity evolutions & single and/or double spin asymmetries	Kovchegov *et al.* (2016-2025)
NEik corrections to quark and gluon propagators in high energy OPE formalism	Chirilli (2018-2021)
subeikonal corrections via allowing longitudinal momentum exchange between projectile and target	Jalilian-Marian (2017-2020)
formulation of inclusive DIS and exclusive Compton scattering that interpolates between small and moderate x	Boussarie *et al.* (2020-2023)
NEik corrections in the CGC via an effective Hamiltonian approach	Li (2023-2024)

Power counting for quark background field

Under a boost of the target of parameter γ_t along the “-” direction, a current associated with the target should behave as

$$J^{-}(z) \propto \gamma_t\,, \quad J^{j}(z) \propto (\gamma_t)^{0}\,, \quad J^{+}(z) \propto (\gamma_t)^{-1}$$

The quark background field of the target can be split into good and bad components as

$$\Psi(z) = \Psi^{(-)}(z) + \Psi^{(+)}(z)$$ with $$\Psi^{(-)}(z) \equiv \frac{\gamma^{+}\gamma^{-}}{2}\,\Psi(z)\,, \quad \Psi^{(+)}(z) \equiv \frac{\gamma^{-}\gamma^{+}}{2}\,\Psi(z)$$

Then the components of quark background current reads

$$\boxed{\begin{aligned}
\overline{\Psi}(z)\,\gamma^{-}\,\Psi(z) &= \overline{\Psi^{(-)}}(z)\,\gamma^{-}\,\Psi^{(-)}(z),\\
\overline{\Psi}(z)\,\gamma^{j}\,\Psi(z) &= \overline{\Psi^{(-)}}(z)\,\gamma^{j}\,\Psi^{(+)}(z) + \overline{\Psi^{(+)}}(z)\,\gamma^{j}\,\Psi^{(-)}(z),\\
\overline{\Psi}(z)\,\gamma^{+}\,\Psi(z) &= \overline{\Psi^{(+)}}(z)\,\gamma^{-}\,\Psi^{(+)}(z)\,.
\end{aligned}}$$

Under a boost of the target, the projections $\Psi^{(-)}(z)$ and $\Psi^{(+)}(z)$, should scale as

$$\Psi^{(-)}(z) \propto (\gamma_t)^{\frac{1}{2}}\,, \quad \Psi^{(+)}(z) \propto (\gamma_t)^{-\frac{1}{2}}$$

We keep only the leading component the quark background field: $\Psi^{(-)}(z)$

Forward jet/hadron production at Eikonal order

Frequent observable used to test the compatibility of saturation phenomena with the high energy pA data:
“single inclusive hadron/jet production in the forward rapidity region”

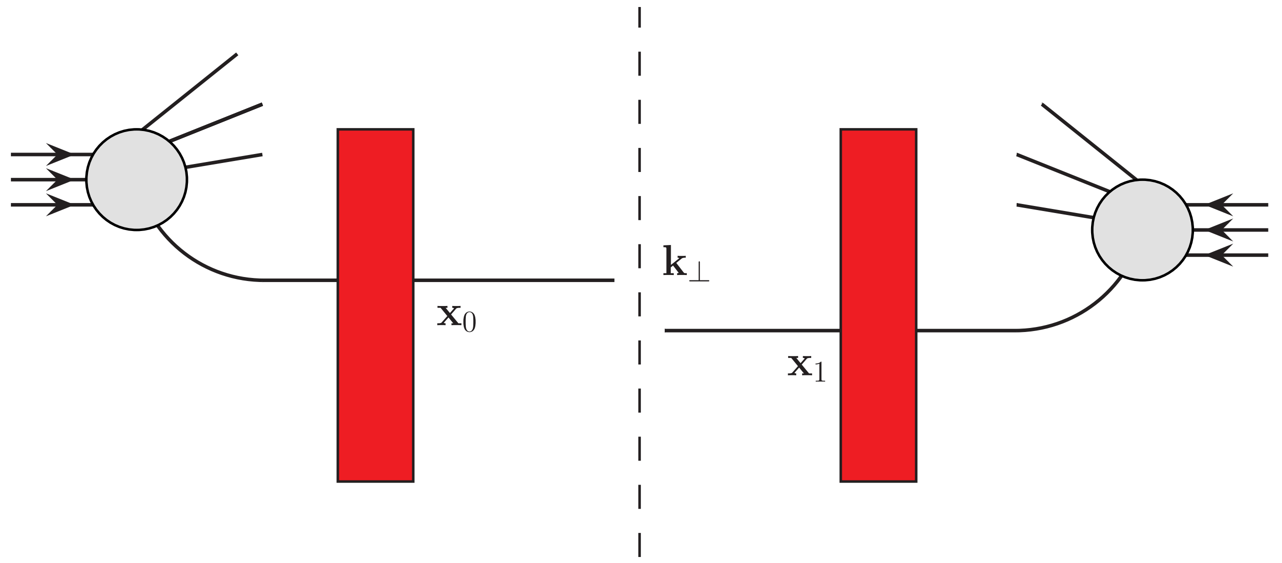
Dumitru, Hayashigaki, Jalilian-Marian (2005)

At LO the state-of-the-art calculation framework for forward production in pA collisions: **Hybrid factorization**

Projectile proton is treated within collinear factorization
(an assembly of partons with zero intrinsic transverse momenta)

Perturbative corrections to projectile wave function via DGLAP evolution

Target is defined via strong color fields - transfer of transverse momentum from target to projectile (CGC like treatment)

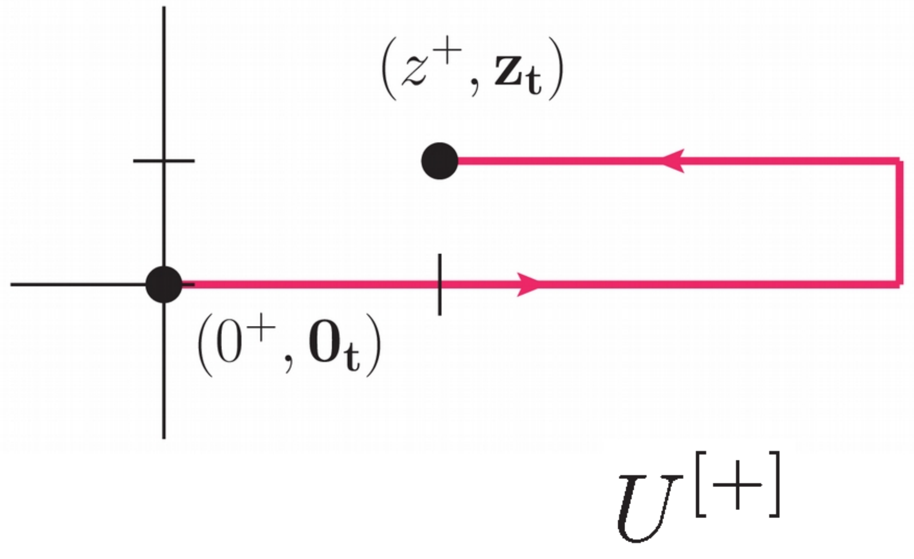


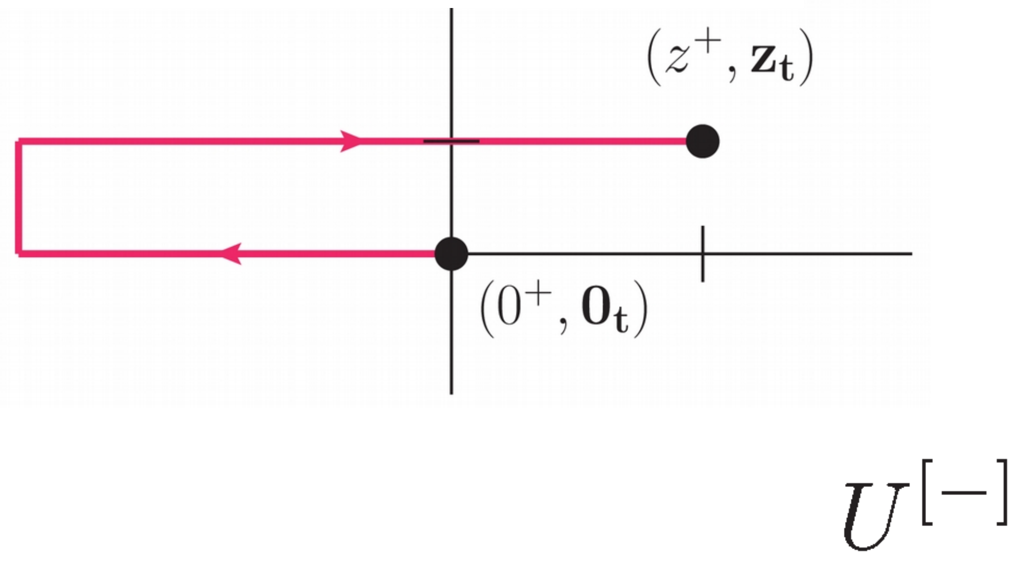
$$\frac{d\sigma^{pA\to q+X}}{dk^+ d^2k_\perp} \propto \int dx_p f_q(x_p,\mu^2) \int e^{ik_\perp(x_0-x_1)} \langle d(x_0,x_1)\rangle$$

Unpolarized TMDs are defined as the FT of forward matrix elements of bilocal products of gluon field strength tensor

$$\boxed{\mathcal{F}(x_2,k_t) \propto \int dz^+\, d^2z_\perp e^{ix_2 p_A^- z^+ - ik_t\cdot z_\perp} \langle p_A|\mathrm{tr}\big[F_0^{i-}\, U^{[C]}_{(0,z)}\, F_z^{i-}\, U^{[C']}_{(z,0)}\big]|p_A\rangle}$$

$U^{[C]}_{(0,z)}$: gauge staples connecting the points $(0^+,0_\perp)$ and $(z^+,z_\perp)$ to ensure gauge invariance





Gauge links and correlation limit of CGC

The Wilson line operator

$$\boxed{U(-\infty,+\infty;x)=\mathcal{P}\exp\left[ig\int_{-\infty}^{+\infty}dx^+A^-(x^+,x)\right]}$$

Derivative of the Wilson line operator

$$\boxed{\partial^i U(x)=ig\int dx^+\,U(-\infty,x^+,x)F^{i-}(x^+,x)U(x^+,+\infty;x)}$$

$$\boxed{\partial^i U^\dagger(x)=-ig\int dx^+\,U(+\infty,x^+,x)F^{i-}(x^+,x)U(x^+,-\infty,x)}$$

How do we get derivatives of Wilson lines?

Dominguez, Marquet, Xiao, Yuan (2011)

Average over the target state is replaced with CGC averaging

$$\frac{\langle p_A|\cdots|p_A\rangle}{\langle p_A|p_A\rangle}\to\langle\cdots\rangle_{x_2}$$

Weizsacker-Williams TMD

$$\mathcal{F}^{(3)}_{gg}(x_2,k_t)\propto\int dz^+d^2z\,e^{ix_2p_A^-z^++ik_tz}\left\langle \mathrm{tr}\left[F^{i-}(\tfrac{z}{2})U^{[+]\dagger}F^{i-}(-\tfrac{z}{2})U^{[+]}\right]\right\rangle$$

in the small-x limit:

$$\mathcal{F}^{(3)}_{gg}(x_2,k_t)\to\int d^2z\,e^{ik_tz}\left\langle \mathrm{tr}\left\{\left[\partial^iU(\tfrac{z}{2})\right]U^\dagger(-\tfrac{z}{2})\left[\partial^iU(-\tfrac{z}{2})\right]U^\dagger(\tfrac{z}{2})\right\}\right\rangle$$

Dijet momentum imbalance $\mathbf{k}=\mathbf{k}_1+\mathbf{k}_2$

Relative momentum $\mathbf{P}=(z_2\mathbf{k}_1-z_1\mathbf{k}_2)$

Back-to-back correlation limit: $|\mathbf{k}|\ll|\mathbf{P}|$

in coordinate space: $\mathbf{b}=(z_1\mathbf{x}_1+z_2\mathbf{x}_2)$ conjugate to $\mathbf{k}$

$\mathbf{r}=(\mathbf{x}_1-\mathbf{x}_2)$ conjugate to $\mathbf{P}$

Back-to-back correlation limit: $|\mathbf{r}|\ll|\mathbf{b}|$

Taylor expand the Wilson lines in the correlation limit $\longrightarrow$ $U(\mathbf{x}_1)=U(\mathbf{b}+z_2\mathbf{r})\simeq U(\mathbf{b})+z_2\mathbf{r}^i\partial^iU(\mathbf{b})+O(\mathbf{r}^2)$ $\longrightarrow$ get access to TMDs

small-x limit of TMD factorization ≡ correlation limit of the CGC

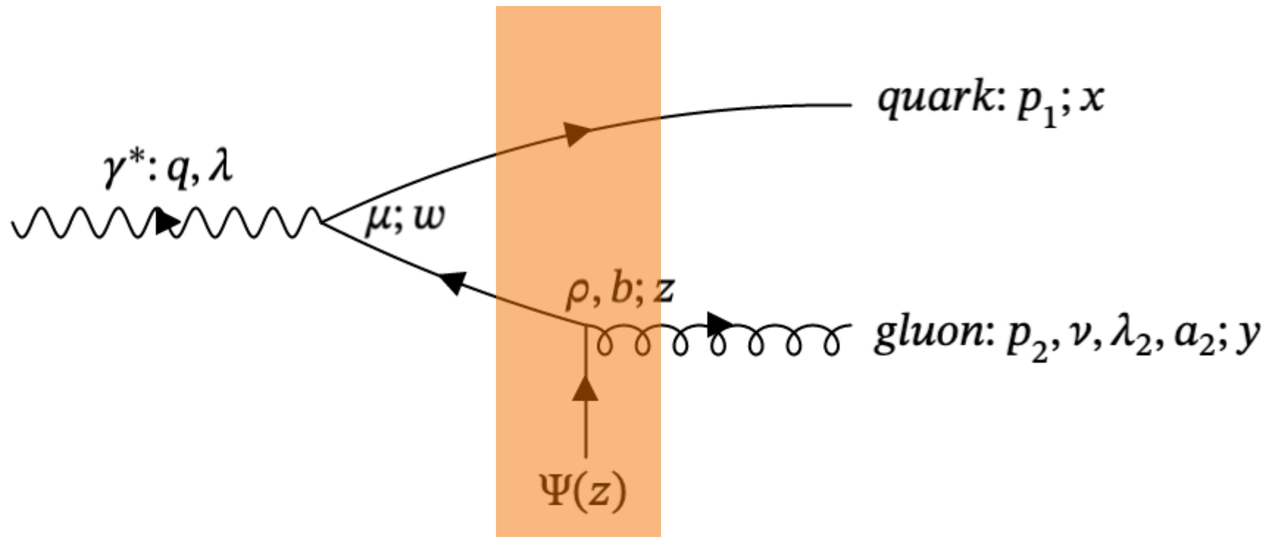
Quark TMDs from back-to-back dijet production in DIS

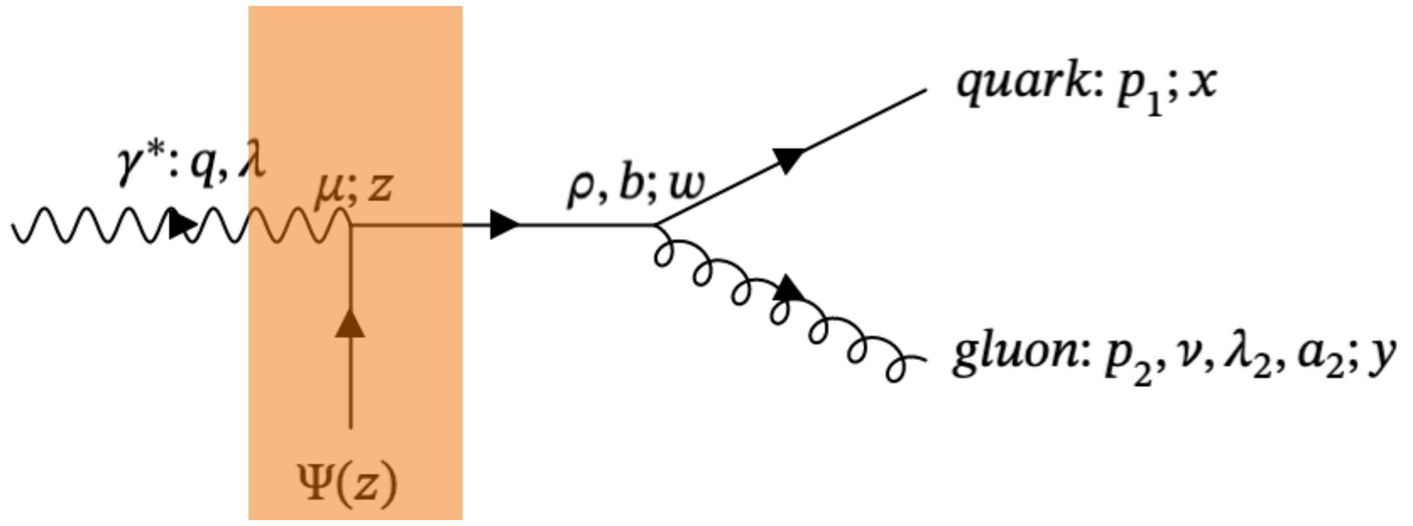
Altinoluk, Armesto, Beuf (2023)

Gluon TMDs are dominant in the eikonal CGC.

Beyond eikonal accuracy quark background field is included in the CGC ➡ **possibility to probe quark TMDs from non-eikonal CGC!**

First study: quark-gluon dijet production in DIS





In the back-to-back limit, the cross section

$$(2\pi)^6(2p_1^+)(2p_2^+)\,\frac{d\sigma^{\gamma_{T,L}\to q_1 g_2}}{dp_1^+ d^2\mathbf{p}_1 dp_2^+ d^2\mathbf{p}_2}\bigg|_{\text{corr.lim.}} = 2\pi\delta(p_1^+ + p_2^+ - q^+)\,(4\pi)^2\alpha_{\text{em}}\alpha_s C_F e_f^2\,\mathcal{H}_{T,L}(\mathbf{P},z,Q)\mathcal{T}(\mathbf{k})$$

with the hard factors

$$\mathcal{H}_L = \frac{4Q^2z^3(1-z)^2}{[\mathbf{P}^2+\bar{Q}^2]^2} \quad \text{and} \quad \mathcal{H}_T = z\left\{\frac{(1+z^2)\mathbf{P}^2+(1-z)^4m^2}{[\mathbf{P}^2+(1-z)^2m^2]^2} + \frac{[z^2+(1-z)^2]\mathbf{P}^2+m^2}{[\mathbf{P}^2+\bar{Q}^2]^2} - \frac{2z^2\mathbf{P}^2}{[\mathbf{P}^2+\bar{Q}^2][\mathbf{P}^2+(1-z)^2m^2]}\right\}$$

target averaged color operator

$$\mathcal{T}(\mathbf{k}) = \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}\Big\langle\bar{\Psi}(z'^+,\mathbf{b}')\gamma^- U_F^\dagger(+\infty,z'^+;\mathbf{b}')U_F(+\infty,z^+;\mathbf{b})\Psi(z^+,\mathbf{b})\Big\rangle$$

unpolarized quark TMD

$$f_1^q(x,\mathbf{k}) = \frac{1}{(2\pi)^3}\int_{\mathbf{b}} e^{i\mathbf{k}\cdot\mathbf{b}}\int_{z^+} e^{-iz^+xP^-_{tar}}\langle P_{tar}|\bar{\Psi}(z^+,\mathbf{b})\frac{\gamma^-}{2}U_F^\dagger(+\infty,z^+;\mathbf{b})U_F(+\infty,0;\mathbf{0})\Psi(0,\mathbf{0})|P_{tar}\rangle$$

CGC average ↔ quantum expectation value in target state

$$\langle\mathcal{O}\rangle = \lim_{P'_{tar}\to P_{tar}}\frac{\langle P'_{tar}|\hat{\mathcal{O}}|P_{tar}\rangle}{\langle P'_{tar}|P_{tar}\rangle} = \lim_{P'_{tar}\to P_{tar}}\frac{\langle P'_{tar}|\hat{\mathcal{O}}|P_{tar}\rangle}{2P^-_{tar}\,(2\pi)^3\,\delta(P'^-_{tar}-P^-_{tar})\,\delta^{(2)}(\mathbf{P}'_{tar}-\mathbf{P}_{tar})}$$

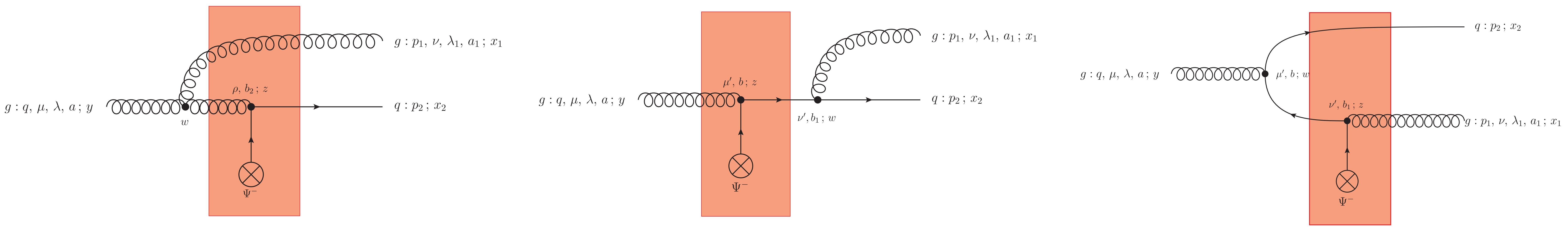
➡ $$\boxed{\mathcal{T}(\mathbf{k}) = \frac{(2\pi)^3}{P^-_{tar}}f_1^q(x=0,\mathbf{k})}$$

Quark TMDs from back-to-back dijet production in forward pA collisions

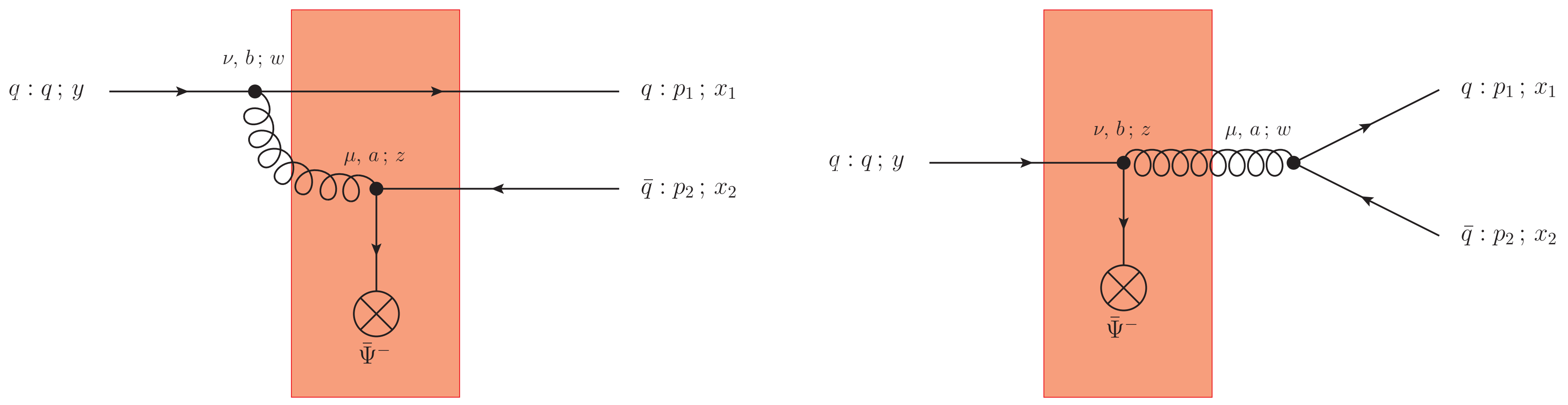
Various channels contribute to dijet production via t-channel quark exchange with the target

Altinoluk, Beuf, Blanco, Mulani (2024)

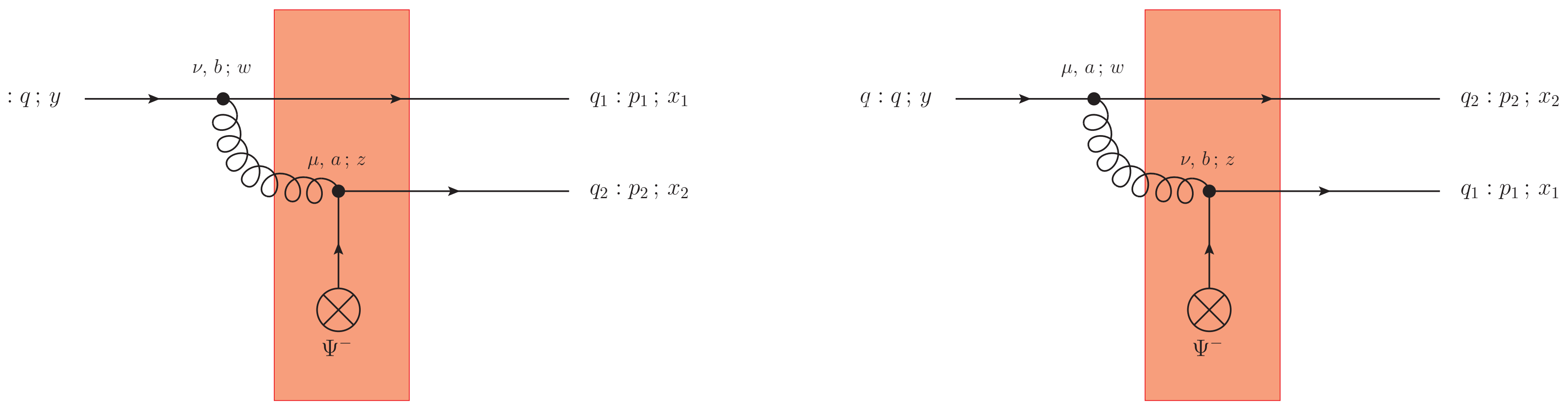
$g \to gq$ channel



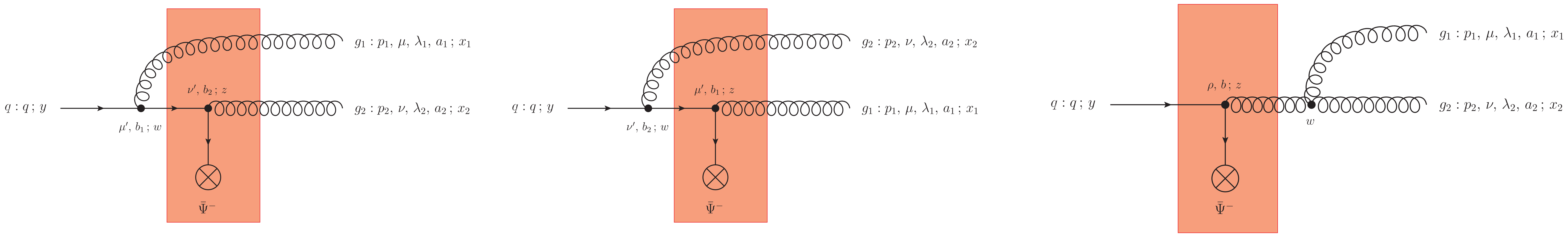
$q \to q\bar{q}$ channel



$q \to qq$ channel



$q \to gg$ channel

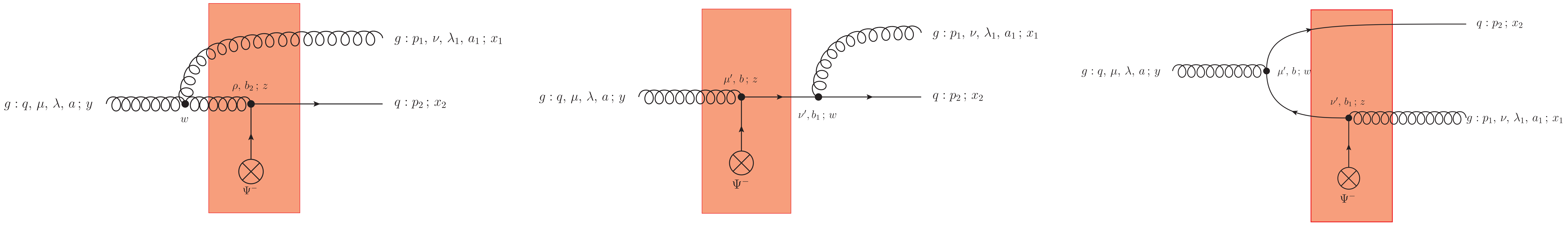


plus $\mathbf{q \to \bar{q}}$

$g \to gq$ channel

incoming gluon ➡ 4-momenta q, polarization λ, color a & produced gluon ➡ 4-momenta p_1, polarization λ_1, color a_1

produced quark ➡ 4-momenta p_2, helicity h



Total scattering amplitude for this channel ➡ $\mathcal{M}_{g\to gq,\,\mathrm{tot.}} = \mathcal{M}_{g\to gq,\,1} + \mathcal{M}_{g\to gq,\,2} + \mathcal{M}_{g\to gq,\,3},$

Altinoluk, Beuf, Mulani (2024)

The first mechanism, S-matrix element reads

$$\mathcal{S}_{g\to gq,\,1} = \lim_{y^+\to-\infty}\ \lim_{x_1^+,x_2^+\to\infty} \int_{\mathbf{y},\mathbf{x_1},\mathbf{x_2}} \int_{y^-,x_1^-,x_2^-} \int_{\mathbf{w},\mathbf{z}} \int_{w^-,z^-} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \int_{-\infty}^{-\frac{L^+}{2}} dw^+$$
$$\times\, e^{ix_1\cdot\check{p}_1}\ e^{ix_2\cdot\check{p}_2}\ e^{-iy\cdot\check{q}}(-2q^+)\ \epsilon^{\mu}_{\lambda}(q)\ (-2p_1^+)\ \epsilon^{\nu}_{\lambda_1}(p_1)$$
$$\times \Big[G^{\mu'\mu}_{0,\mathrm{F}}(w,y)\Big]_{a'a} \Big[G^{\nu\nu'}_{\mathrm{F}}(x_1,w)\Big|^{\mathrm{BA}}_{\mathrm{Eik.}}\Big]_{a_1b_1} \Big[G^{\rho\rho'}_{\mathrm{F}}(z,w)\Big|^{\mathrm{BI}}_{\mathrm{Eik.}}\Big]_{b_2b} V^{a'b_1b}_{\mu'\nu'\rho'}$$
$$\times\, \overline{u}(\check{p}_2,h)\gamma^+ \Big[S_{\mathrm{F}}(x_2,z)\Big|^{\mathrm{IA},q}_{\mathrm{Eik.}}(-ig)t^{b_2}\Big]_{\alpha_2\beta} \gamma_\rho \Psi^-_\beta(z),$$

Before-to-after gluon propagator

$$G^{\mu\nu}_{\mathrm{F}}(x,y)\Big|^{\mathrm{BA}}_{\mathrm{Eik.}} = \int \frac{d^3\underline{k_1}}{(2\pi)^3}\,\theta(k_1^+)\,e^{-ix\cdot\check{k}_1} \int \frac{d^3\underline{k_2}}{(2\pi)^3}\,\frac{\theta(k_2^+)}{k_1^++k_2^+}\,e^{iy\cdot\check{k}_2}$$
$$\times \left[-g^{\mu\nu} + \frac{\check{k}_2^{\mu}\eta^{\nu}}{k_2^+} + \frac{\eta^{\mu}\check{k}_1^{\nu}}{k_1^+} - \frac{\eta^{\mu}\eta^{\nu}}{k_1^+k_2^+}\check{k}_1\cdot\check{k}_2\right] \int_{\mathbf{z}} e^{-i\mathbf{z}\cdot(\mathbf{k_1}-\mathbf{k_2})} \int_{z^-} e^{iz^-(k_1^+-k_2^+)}\,\mathcal{U}_{\mathrm{A}}(x^+,y^+,\mathbf{z})$$

Before-to-inside gluon propagator

$$G^{\mu\nu}_{\mathrm{F}}(x,y)\Big|^{\mathrm{BI}}_{\mathrm{Eik.}} = \int \frac{d^3\underline{k}}{(2\pi)^3}\,\frac{\theta(k^+)}{2k^+}\,e^{iy\cdot\check{k}}e^{-ix\cdot\underline{k}}$$
$$\times \left[-g^{\mu}{}_{j}\,g^{j\nu} + g^{\mu}{}_{j}\eta^{\nu}\frac{\mathbf{k}^j}{k^+} + i\left(\frac{\eta^{\mu}g^{\nu}{}_{j}}{k^+} - \eta^{\mu}\eta^{\nu}\frac{\mathbf{k}^j}{(k^+)^2}\right)\left(\overrightarrow{\mathcal{D}}^{A}_{\mathbf{x}^j} + i\mathbf{k}^j\right)\right]\mathcal{U}_{\mathrm{A}}(x^+,y^+,\mathbf{x}).$$

Inside-to-after quark propagator

$$S_{\mathrm{F}}(x,y)\Big|^{\mathrm{IA},q}_{\mathrm{Eik.}} = \int \frac{d^3\underline{k}}{(2\pi)^3}\,\frac{\theta(k^+)}{2k^+}\,\mathcal{U}_{\mathrm{F}}(x^+,y^+,\mathbf{y})\ (\check{\not{k}}+m)\ \left[1 - i\frac{\gamma^+\gamma^j}{2k^+}\overleftarrow{\mathcal{D}}^{F}_{\mathbf{y}^j}\right]\ e^{-ix\cdot\check{k}}\ e^{iy\cdot\underline{k}}.$$

$g \to gq$ channel: back-to-back limit (I)

Scattering amplitude can be obtained from the S-matrix element

$$\mathcal{S}_{g\to gq} = (2q^+)(2\pi)\,\delta\left(p_1^+ + p_2^+ - q^+\right) i\mathcal{M}_{g\to gq}.$$

The scattering amplitude for the first mechanism in general kinematics

$$\begin{aligned}
i\mathcal{M}_{g\to gq,\,1} =& -ig^2 \frac{1}{(2p_2^+)(2q^+)} f^{ab_1b} \int_{\mathbf{z},\mathbf{z_1}} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \int_{-\infty}^{-\frac{L^+}{2}} dw^+ \int \frac{d^2\mathbf{k_1}}{(2\pi)^2} \\
&\times e^{-iw^+\left(\frac{\mathbf{q}^2}{2q^+} - \frac{\mathbf{k_1}^2}{2p_1^+} - \frac{(\mathbf{k_1}-\mathbf{q})^2+m^2}{2p_2^+}\right)} e^{-i\mathbf{z}\cdot(\mathbf{p_2}+\mathbf{k_1}-\mathbf{q})}\; e^{-i\mathbf{z_1}\cdot(\mathbf{p_1}-\mathbf{k_1})} \\
&\times \overline{u}(\check{p}_2,h)\, \mathcal{U}_{\mathrm{F}}(+\infty,z^+;\mathbf{z})\, t^{b_2}\, \mathcal{U}_{\mathrm{A}}(z^+,w^+;\mathbf{z_1})_{b_2b}\, \mathcal{U}_{\mathrm{A}}(+\infty,w^+;\mathbf{z})_{a_1b_1}\; \gamma^l\gamma^+\gamma^-\; \Psi(\underline{z}) \\
&\times \varepsilon_\lambda^i \varepsilon_{\lambda_1}^{j*}\left[g^{ij}\left(\frac{p_1^+}{p_2^+}\mathbf{q}^l - \frac{q^+}{p_2^+}\mathbf{k_1}^l\right) + g^{il}\left(\mathbf{q}^j - \frac{q^+}{p_1^+}\mathbf{k_1}^j\right) - g^{jl}\left(\frac{p_1^+}{q^+}\mathbf{q}^i - \mathbf{k_1}^i\right)\right]
\end{aligned}$$

Back-to-back limit: perform the change of variables $(\mathbf{z},\mathbf{z_1}) \to (\mathbf{b},\mathbf{r})$ and $(\mathbf{p_1},\mathbf{p_2}) \to (\mathbf{P},\mathbf{k})$ with back-to-back limit $\mathbf{k} \ll \mathbf{P}$ or equivalently $\mathbf{r} \ll \mathbf{b}$

- Taylor expand the Wilson line structure around $\mathbf{r} = 0$ and keep only the first non-trivial term
- $\mathbf{r}$ and $\mathbf{k_1}$ dependence remains in the phase and integrated trivially

$$\begin{aligned}
i\mathcal{M}^{\mathrm{b2b}}_{g\to gq,\,1} =& \frac{ig^2}{(1-z)(2q^+)^2} f^{ab_1b} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \int_{-\infty}^{-\frac{L^+}{2}} dw^+ e^{i\frac{w^+}{2z(1-z)q^+}[\mathbf{P}^2 - zm^2]} \\
&\times \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot(\mathbf{k}-\mathbf{q})}\, \mathcal{U}_{\mathrm{A}}(+\infty,w^+;\mathbf{b})_{a_1b_1}\, \mathcal{U}_{\mathrm{A}}(z^+,w^+;\mathbf{b})_{b_2b}\, \mathcal{U}_{\mathrm{F}}(+\infty,z^+;\mathbf{b})\; t^{b_2} \\
&\times \overline{u}(\check{p}_2,h)\; \varepsilon_\lambda^i\; \varepsilon_{\lambda_1}^{j*}\left[\frac{1}{1-z}g^{ij}\mathbf{P}^l + \frac{1}{z}g^{il}\mathbf{P}^j - g^{jl}\mathbf{P}^i\right]\gamma^l\gamma^+\gamma^-\Psi(z^+;\mathbf{b})
\end{aligned}$$

- ω^+ is before the medium and is integrated up to the edge of the medium
- since background fields vanish outside the medium, take $\omega^+ \to \infty$ in Wilson lines
- ω^+ integral $\Rightarrow$ L^+ phase
- L^+ phase can be approximated by 1 at NEik accuracy

$g \to gq$ channel: back-to-back limit (II)

In the back-to-back and massless limits one gets

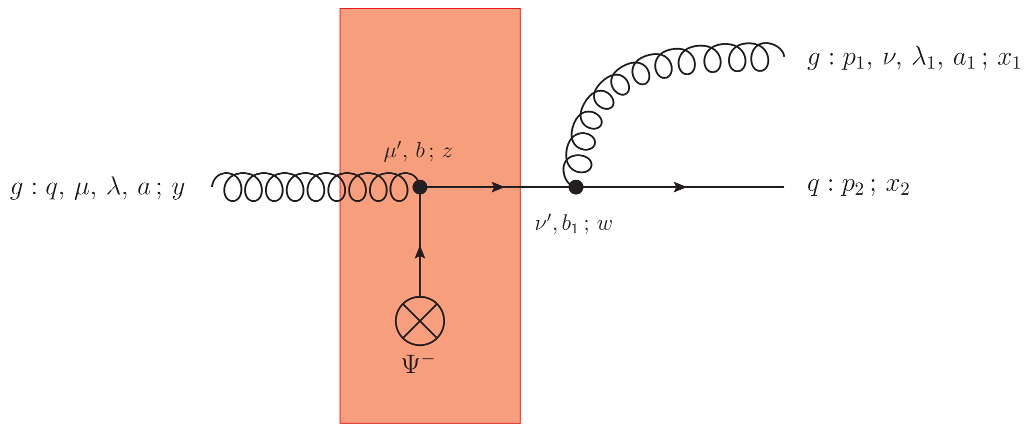
$$i\mathcal{M}^{\text{b2b},\, m=0}_{g\to gq,\,1} = g^2 \frac{z}{(2q^+)}\frac{1}{\mathbf{P}^2} f^{ab_1b} \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \int_{\mathbf{b}} e^{-i\mathbf{p}\cdot(\mathbf{k}-\mathbf{q})}$$
$$\times\, \mathcal{U}_{\text{A}}(+\infty,-\infty;\mathbf{b})_{a_1b_1}\, \mathcal{U}_{\text{A}}(z^+,-\infty;\mathbf{b})_{b_2b}\, \mathcal{U}_{\text{F}}(+\infty,z^+;\mathbf{b})\; t^{b_2}$$
$$\times\, \overline{u}(\check{p}_2,h)\; \varepsilon^i_\lambda\; \varepsilon^{j*}_{\lambda_1} \left[g^{ij}\frac{\mathbf{P}^l}{(1-z)} + g^{il}\frac{\mathbf{P}^j}{z} - g^{jl}\mathbf{P}^i\right] \gamma^l\gamma^+\gamma^-\Psi(z^+;\mathbf{b})$$

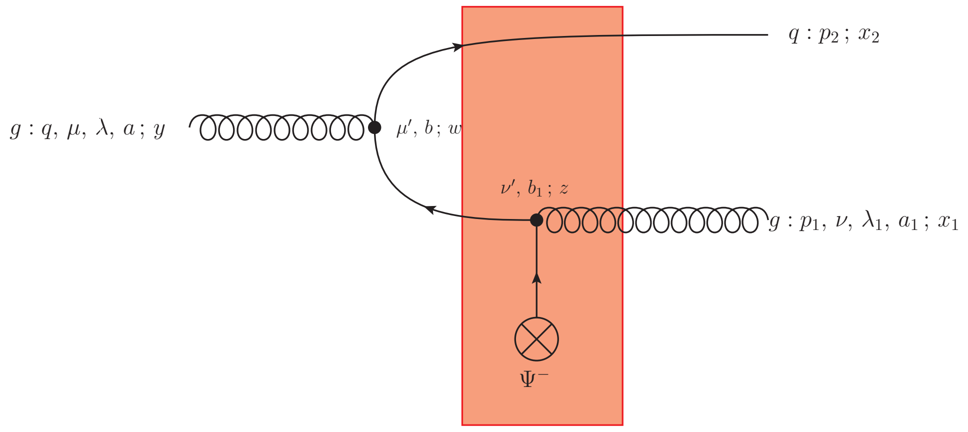
Color structure can be simplified using Fierz identities:

$$\mathcal{U}_{\text{A}}(+\infty,-\infty;\mathbf{b})_{a_1b_1}\mathcal{U}_{\text{A}}(z^+,-\infty;\mathbf{b})_{b_2b}\left[\mathcal{U}_{\text{F}}(+\infty,z^+;\mathbf{b})t^{b_2}f^{ab_1b}\Psi(z^+;\mathbf{b})\right]$$
$$= -i\; \mathcal{U}_{\text{A}}(+\infty,-\infty;\mathbf{b})_{ca}\left[[t^c,t^{a_1}]\,\mathcal{U}_{\text{F}}(+\infty,z^+;\mathbf{b})\Psi(z^+;\mathbf{b})\right]$$

Back-to-back masses production amplitude for 1st mechanism

$$i\mathcal{M}^{\text{b2b},\, m=0}_{g\to gq,\,1} = -i\, g^2 \frac{1}{(2q^+)}\frac{1}{\mathbf{P}^2} \int_{-\frac{L}{2}}^{\frac{L}{2}} dz^+ \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot(\mathbf{k}-\mathbf{q})}\, \mathcal{U}_{\text{A}}(+\infty,-\infty;\mathbf{b})_{ca}\,[t^c,t^{a_1}]\,\mathcal{U}_{\text{F}}(+\infty,z^+;\mathbf{b})$$
$$\times\, \overline{u}(\check{p}_2,h)\; \varepsilon^i_\lambda\; \varepsilon^{j*}_{\lambda_1}\left[\frac{z}{1-z}g^{ij}\mathbf{P}^l + g^{il}\mathbf{P}^j - zg^{jl}\mathbf{P}^i\right]\gamma^l\gamma^+\gamma^-\Psi(z^+;\mathbf{b})$$





$$i\mathcal{M}^{\text{b2b},\, m=0}_{g\to gq,\,2} = -\frac{i}{2}\, g^2 \frac{1}{(2q^+)}\frac{1}{\mathbf{P}^2} \int_{-\frac{L}{2}}^{\frac{L}{2}} dz^+ \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot(\mathbf{k}-\mathbf{q})}\; \overline{u}(\check{p}_2,h)\; \varepsilon^i_\lambda\; \varepsilon^{j*}_{\lambda_1}\left[z\gamma^l\gamma^j\mathbf{P}^l - 2(1-z\right.$$
$$\times\, \gamma^i\gamma^+\gamma^-\mathcal{U}_{\text{A}}(+\infty,-\infty;\mathbf{b})_{ca}\; t^{b_1}\; t^c\; \mathcal{U}_{\text{F}}(+\infty,z^+;\mathbf{b})\; \Psi(z^+,\mathbf{b})$$

$$i\mathcal{M}^{\text{b2b},\, m=0}_{g\to gq,\,3} = \frac{i}{2}\, g^2 \frac{1}{(2q^+)}\frac{1}{\mathbf{P}^2} \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot(\mathbf{k}-\mathbf{q})}\, \mathcal{U}_{\text{A}}(+\infty,-\infty;\mathbf{b})_{ca}\; t^c\; t^{a_1}\; \mathcal{U}_{\text{F}}(+\infty,z^+;\mathbf{b})$$
$$\times\, \overline{u}(\check{q}_2,h)\; \varepsilon^i_\lambda\; \varepsilon^{j*}_{\lambda_1}\left[-z\mathbf{P}^l\gamma^l\gamma^i + (1-z)\mathbf{P}^l\gamma^i\gamma^l\right]\gamma^j\gamma^+\gamma^-\Psi(z^+;\mathbf{b})$$

$g \to gq$ channel: back-to-back production cross section

sum of three mechanisms

$$i\mathcal{M}^{\text{b2b},\, m=0}_{g\to gq,\,\text{tot.}} = \frac{i}{2}\, g^2 \frac{1}{2q^+}\frac{1}{\mathbf{P}^2}\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot(\mathbf{k}-\mathbf{q})}\, \mathcal{U}_{\text{A}}(+\infty,-\infty;\mathbf{b})_{ba}$$
$$\times\, \overline{u}(\check{q}_2,h)\,\left[t^{a_1}t^b \mathfrak{h}^{(1)}_{g\to gq} + t^b t^{a_1}\mathfrak{h}^{(2)}_{g\to gq}\right]\gamma^+\gamma^-\, \mathcal{U}_{\text{F}}(+\infty,z^+;\mathbf{b})\Psi(z^+;\mathbf{b})$$

$$\mathfrak{h}^{(1)}_{g\to gq} = \varepsilon^i_\lambda\, \varepsilon^{j*}_{\lambda_1}\left(-z\mathbf{P}^l\,\gamma^l\gamma^j\gamma^i + 2(2-z)\mathbf{P}^j\,\gamma^i - 2z\mathbf{P}^i\gamma^j + 2\frac{z}{1-z}\mathbf{P}^l\, g^{ij}\gamma^l\right)$$
$$\mathfrak{h}^{(2)}_{g\to gq} = \varepsilon^i_\lambda\, \varepsilon^{j*}_{\lambda_1}\left(-2z\mathbf{P}^l\,\gamma^l\gamma^i\gamma^j + (1-2z)\mathbf{P}^l\,\gamma^i\gamma^l\gamma^j - 2\mathbf{P}^j\gamma^i - 2\frac{z}{1-z}\mathbf{P}^l g^{ij}\gamma^l\right)$$

partonic cross section in the back-to-back and massless limits:

$$\frac{d\sigma^{\text{b2b},\, m=0}_{g\to gq}}{d\text{P.S.}} = (2q^+)\; 2\pi\delta\left(p_1^+ + p_2^+ - q^+\right)\frac{1}{2(N_c^2-1)}\sum_{\lambda,\lambda_1}\sum_h\sum_{a,a_1}\left\langle\left|i\mathcal{M}^{\text{b2b},\, m=0}_{g\to gq,\,\text{tot.}}\right|^2\right\rangle, \qquad \text{with} \qquad d\text{P.S.} = \frac{d^2\mathbf{k}}{(2\pi)^2}\frac{dq^+}{(2\pi)2q^+}\frac{d^2\mathbf{P}}{(2\pi)^2}\frac{dz}{(2\pi)2z(1-z)}$$

(after some simplifications) cross section takes a factorized form:

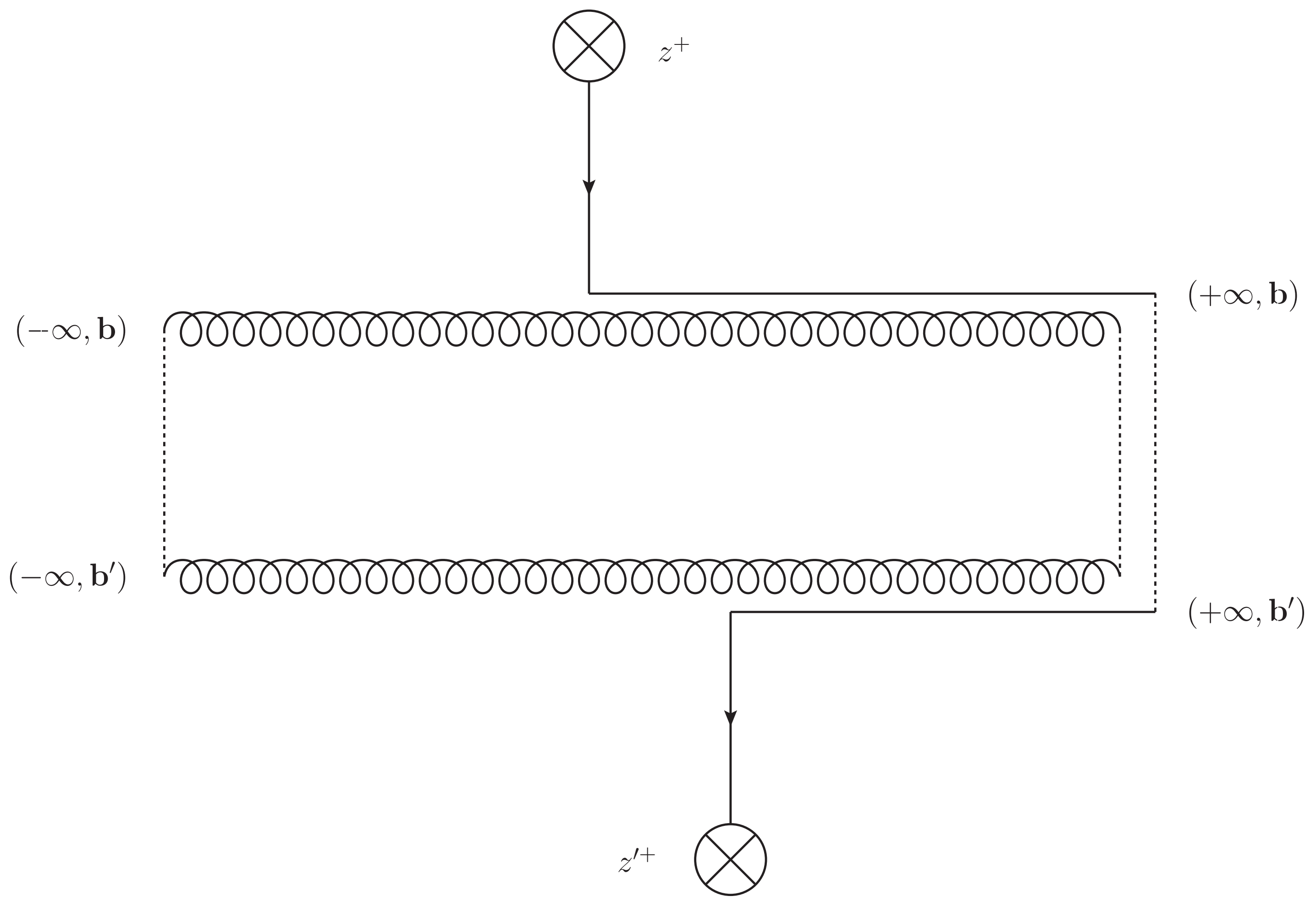
CGC averaged color structures

$$\frac{d\sigma^{\text{b2b},\, m=0}_{g\to gq}}{d\text{P.S.}} = g^4\,(2\pi)\delta\left(p_1^+ + p_2^+ - q^+\right)\int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz^+ \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dz'^+ \int_{\mathbf{b},\mathbf{b}'} e^{-i(\mathbf{b}-\mathbf{b}')\cdot(\mathbf{k}-\mathbf{q})}\left[\mathcal{H}^{+g}_{g\to gq}\,\mathcal{C}^{+g} + \mathcal{H}^{+\Box_g}_{g\to gq}\,\mathcal{C}^{+\Box_g}\right]$$

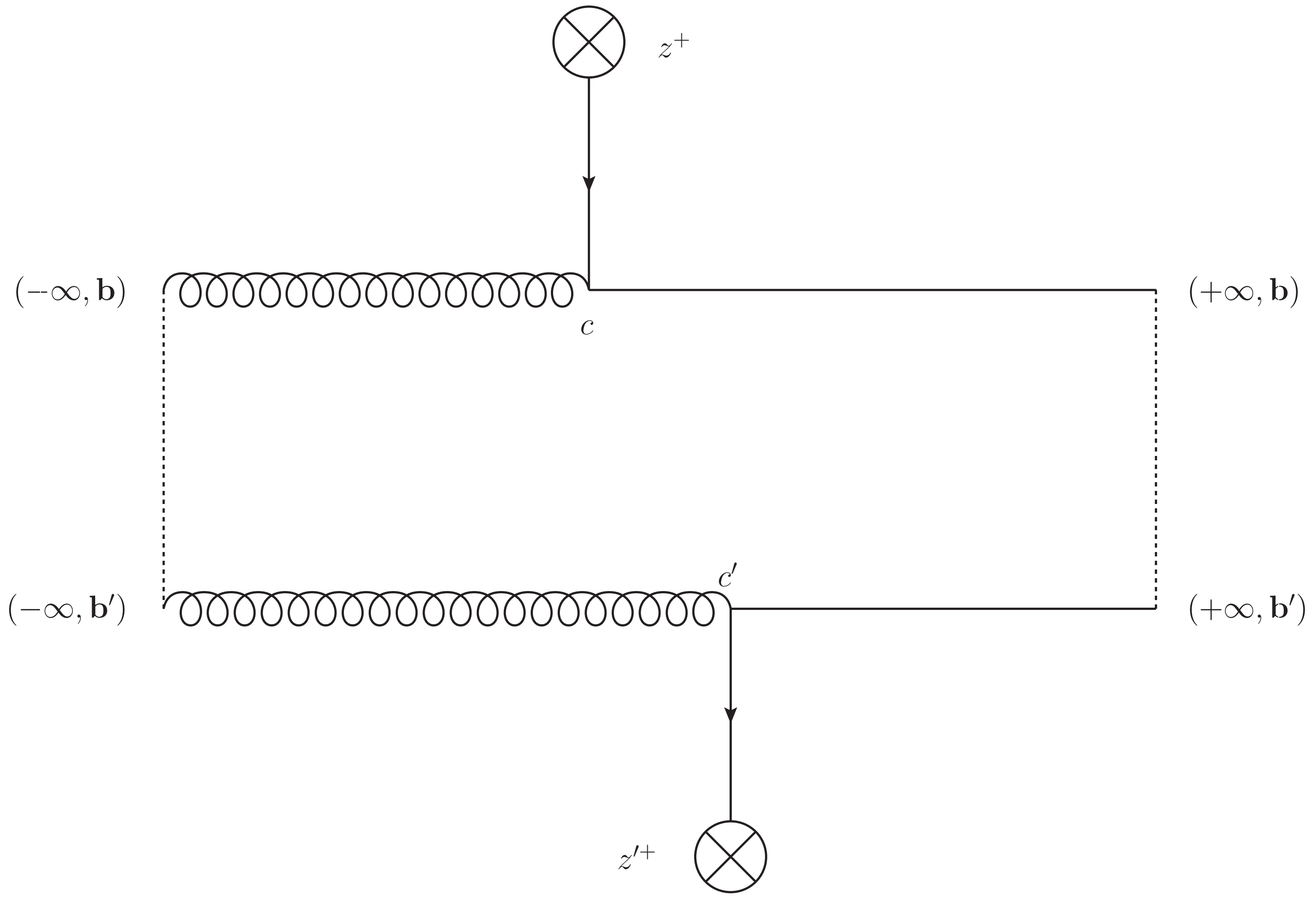
Hard factors

CGC averaged color structures

$$\mathcal{C}^{+\Box g} \equiv \Big\langle \overline{\Psi}(z'^{+};\mathbf{b}')\,\gamma^{-}\,\mathcal{U}_{\mathrm{F}}^{\dagger}(+\infty,z'^{+};\mathbf{b}')\,\mathcal{U}_{\mathrm{F}}(+\infty,z^{+};\mathbf{b})\;\Psi(z^{+},\mathbf{b}) \times \mathcal{U}_{\mathrm{A}}(+\infty,-\infty;\mathbf{b}')_{ba}\,\mathcal{U}_{\mathrm{A}}(+\infty,-\infty;\mathbf{b})_{ba}\Big\rangle$$



$$\mathcal{C}^{+g} \equiv \Big\langle \overline{\Psi}(z'^{+};\mathbf{b}')\,\gamma^{-}\;t^{c'}\;\mathcal{U}_{\mathrm{F}}^{\dagger}(+\infty,z'^{+};\mathbf{b}')\,\mathcal{U}_{\mathrm{F}}(+\infty,z^{+};\mathbf{b})\;t^{c}\;\Psi(z^{+},\mathbf{b}) \times \mathcal{U}_{\mathrm{A}}(z'^{+},-\infty;\mathbf{b}')_{c'a}\,\mathcal{U}_{\mathrm{A}}(z^{+},-\infty;\mathbf{b})_{ca}\Big\rangle$$



Relation to the quark TMDs

The color structure appear in the form of $\mathcal{T}^{(\cdots)}(\mathbf{k}) \equiv \int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{z^+,z'^+} \left\langle \mathcal{C}^{(\cdots)} \right\rangle$ ⟶ target averaging in the spirit of CGC

various color structures

Dominguez ,Marquet, Xiao, Yuan (2011)

from $\langle \mathcal{O} \rangle$ to quantum expectation value $\langle \mathcal{O} \rangle = \lim_{P'_{tar} \to P_{tar}} \frac{\langle P'_{tar} | \hat{\mathcal{O}} | P_{tar} \rangle}{\langle P'_{tar} | P_{tar} \rangle}$, with the normalization $\langle P'_{tar} | P_{tar} \rangle = 2P^-_{tar}\,(2\pi)^3\,\delta(P'^-_{tar} - P^-_{tar})\,\delta^{(2)}(\mathbf{P}'_{tar} - \mathbf{P}_{tar}).$

any local operator obeys $\hat{\mathcal{O}}(x) = e^{ia^\mu \hat{P}_\mu} \hat{\mathcal{O}}(x-a) e^{-ia^\mu \hat{P}_\mu}$ with $\hat{P}_\mu$ being the momentum operator

Matrix element of non-local operator:

$$\begin{aligned}\langle P'_{tar} | \hat{\mathcal{O}}_1(x_1) \ldots \hat{\mathcal{O}}_n(x_n) | P_{tar} \rangle &= \langle P'_{tar} | e^{ia^\mu \hat{P}_\mu} \hat{\mathcal{O}}_1(x_1 - a) \ldots \hat{\mathcal{O}}_n(x_n - a) e^{-ia^\mu \hat{P}_\mu} | P_{tar} \rangle \\ &= e^{ia^\mu[(P'_{tar})_\mu - (P_{tar})_\mu]} \langle P'_{tar} | \hat{\mathcal{O}}_1(x_1 - a \ldots \hat{\mathcal{O}}_n(x_n - a) | P_{tar} \rangle\end{aligned}$$

ex. $\mathcal{C}^+ \equiv \left\langle \overline{\Psi}(z'^+, \mathbf{b}') \gamma^- \mathcal{U}^\dagger_{\mathrm{F}}(+\infty, z'^+; \mathbf{b}') \mathcal{U}_{\mathrm{F}}(+\infty, z^+; \mathbf{b}) \Psi(z^+, \mathbf{b}) \right\rangle$

then $\mathcal{T}^+(\mathbf{k}) = \lim_{P'_{tar} \to P_{tar}} \int_{\Delta\mathbf{b}} \frac{e^{-i\mathbf{k}\cdot\Delta\mathbf{b}}}{2P^-_{tar}} \int_{\Delta z^+} \left\langle P'_{tar} | \overline{\Psi}(\Delta z^+; \Delta\mathbf{b}) \gamma^- \mathcal{U}^\dagger_{\mathrm{F}}(+\infty, \Delta z^+; \Delta\mathbf{b}) \mathcal{U}_{\mathrm{F}}(+\infty, 0; \mathbf{0}) \Psi(0, \mathbf{0}) | P_{tar} \right\rangle$

compare with the unpolarized quark TMD $f^+_q(\mathrm{x}, \mathbf{k}) = \frac{1}{(2\pi)^3} \int_{\mathbf{b}} e^{i\mathbf{k}\cdot\mathbf{b}} \int_{z^+} e^{-i\mathrm{x}z^+ P^-_{tar}} \left\langle P_{tar} | \overline{\Psi}(z^+; \mathbf{b}) \frac{\gamma^-}{2} \mathcal{U}^\dagger_{\mathrm{F}}(+\infty, z^+; \mathbf{b}) \mathcal{U}_{\mathrm{F}}(+\infty, 0; \mathbf{0}) \Psi(0, \mathbf{0}) | P_{tar} \right\rangle$

$\mathcal{T}^+(\mathbf{k}) = \frac{(2\pi)^3}{P^-_{tar}} f^+_q(\mathrm{x} = 0, \mathbf{k}).$ can be generalized to

$$\boxed{\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')} \int_{z^+,z'^+} \mathcal{C}^{(\cdots)}(\mathbf{k}) = \frac{(2\pi)^3}{P^-_{tar}} f^{(\cdots)}_q(\mathrm{x} = 0, \mathbf{k})}$$

Factorized cross section

using these relations $g \to gq$ channel:

$$\boxed{\frac{d\sigma^{\mathrm{b2b},\, m=0}_{g\to gq}}{d^2\mathbf{k}\, d^2\mathbf{P}\, dz} = \frac{\alpha_s^2}{2\pi}\frac{1}{W^2}\left[\overline{\mathcal{H}}^{+g}_{g\to gq}\; f^{+g}(\mathrm{x}=0,\mathbf{k}-\mathbf{q}) + \overline{\mathcal{H}}^{+\Box_g}_{g\to gq}\; f^{+\Box_g}(\mathrm{x}=0,\mathbf{k}-\mathbf{q})\right]}$$

with W being the centre of mass energy of the parton-target scattering $W^2 = (q+P_{tar})^2 \simeq 2q^+P^-_{tar}$

$1/W^2$ suppression is the characteristic of NEik corrections

associated quark TMDs

$$f_q^{+g}(\mathrm{x},\mathbf{k}) = \frac{1}{(2\pi)^3}\int_{\mathbf{b}} e^{i\mathbf{k}\cdot\mathbf{b}}\int_{z^+} e^{-i\mathrm{x}P^-_{tar}z^+}\left[\mathcal{U}_{\mathrm{A}}(z^+,-\infty;\mathbf{b})_{c'a}\mathcal{U}_{\mathrm{A}}(0,-\infty;\mathbf{0})_{ca}\right]$$
$$\times\left\langle P_{tar}\left|\overline{\Psi}(z^+;\mathbf{b})\frac{\gamma^-}{2}t^{c'}\mathcal{U}^\dagger_{\mathrm{F}}(\infty,z^+;\mathbf{b})\mathcal{U}_{\mathrm{F}}(\infty,0;\mathbf{0})t^c\Psi(0,\mathbf{0})\right|P_{tar}\right\rangle$$

$$f_q^{+\Box_g}(\mathrm{x},\mathbf{k}) = \frac{1}{(2\pi)^3}\int_{\mathbf{b}} e^{i\mathbf{k}\cdot\mathbf{b}}\int_{z^+} e^{-i\mathrm{x}P^-_{tar}z^+}\left[\mathcal{U}_{\mathrm{A}}(\infty,-\infty;\mathbf{b})_{ba}\mathcal{U}_{\mathrm{A}}(\infty,-\infty;\mathbf{0})_{ba}\right]$$
$$\times\left\langle P_{tar}\left|\overline{\Psi}(z^+;\mathbf{b})\frac{\gamma^-}{2}\mathcal{U}^\dagger_{\mathrm{F}}(\infty,z^+;\mathbf{b})\mathcal{U}_{\mathrm{F}}(\infty,0;\mathbf{0})\Psi(0,\mathbf{0})\right|P_{tar}\right\rangle$$

Factorized cross section

the other channels are also computed

$$\frac{d\sigma^{\mathrm{b2b},\,m=0}_{q_f\to q_{f_1}\bar{q}_{f_2}}}{d^2\mathbf{k}\,d^2\mathbf{P}\,dz} = \frac{\alpha_s^2}{2\pi}\frac{1}{W^2}\Big[\overline{\mathcal{H}}^{-}_{q_f\to q_{f_1}\bar{q}_{f_2}}\; f^{-}(\mathrm{x}=0,\mathbf{k}-\mathbf{q}) + \overline{\mathcal{H}}^{+\Box}_{q_f\to q_{f_1}\bar{q}_{f_2}}\; f^{+\Box}(\mathrm{x}=0,\mathbf{k}-\mathbf{q})\Big]$$

$$\frac{d\sigma^{\mathrm{b2b},\,m=0}_{q\to gg}}{d^2\mathbf{k}\,d^2\mathbf{P}\,dz} = \frac{\alpha_s^2}{2\pi}\frac{1}{W^2}\Big[\overline{\mathcal{H}}^{-}_{q\to gg}\; f^{-}(\mathrm{x}=0,\mathbf{k}-\mathbf{q}) + \overline{\mathcal{H}}^{-g}_{q\to gg}\; f^{-g}(\mathrm{x}=0,\mathbf{k}-\mathbf{q})\Big]$$

$$\frac{d\sigma^{\mathrm{b2b},\,m=0}_{q_f\to q_{f_1}q_{f_2}}}{d^2\mathbf{k}\,d^2\mathbf{P}\,dz} = \frac{\alpha_s^2}{2\pi}\frac{1}{W^2}\Big[\overline{\mathcal{H}}^{+\Box}_{q_f\to q_{f_1}q_{f_2}}\; f^{+\Box}(\mathrm{x}=0,\mathbf{k}-\mathbf{q}) + \overline{\mathcal{H}}^{+-+}_{q_f\to q_{f_1}q_{f_2}}\; f^{+-+}(\mathrm{x}=0,\mathbf{k}-\mathbf{q})\Big]$$

full list of the quark TMDs and associated hard factors can be found in arXiv: 2412.08485 [hep-ph]

Summary and outlook

We studied the dijet production in forward pA at NEik accuracy $\longrightarrow$ contributions from incoming parton scattering on the target via a t-channel quark exchange

Scattering amplitudes and production cross sections are computed for channels: $g \to gq, \quad q \to q\bar{q}, \quad q \to gg$ and $q \to qq \quad + (q \leftrightarrow \bar{q})$

For all channels, the back-to-back cross sections are obtained in a **factorized form**: a quark TMD times associated hard factor

on going work — Altinoluk, Beuf, Blanco, Mulani

back-to-back dijet production in forward pA at NEik accuracy in a pure gluon background field $\longrightarrow$ possibility of recovering non-zero value of momentum fraction x in twist 2 gluon TMDs from NEik corrections

on going work — Altinoluk, Armesto, Beuf, Favrel

photon + jet production in forward pA collisions at NEik accuracy $\longrightarrow$

- contributions both from gluon and quark background fields
- probe both gluon and quark TMDs of the target
- at Eik order no need to for expansion to probe dipole gluon TMD, is it true at NEik order?