Fluctuations, Diffraction and Initial states

Probing the CGC and QCD matter at hadron colliders

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Edmond…. well before 60


Edmond.... well before 60
...on his path to "wisdom"...

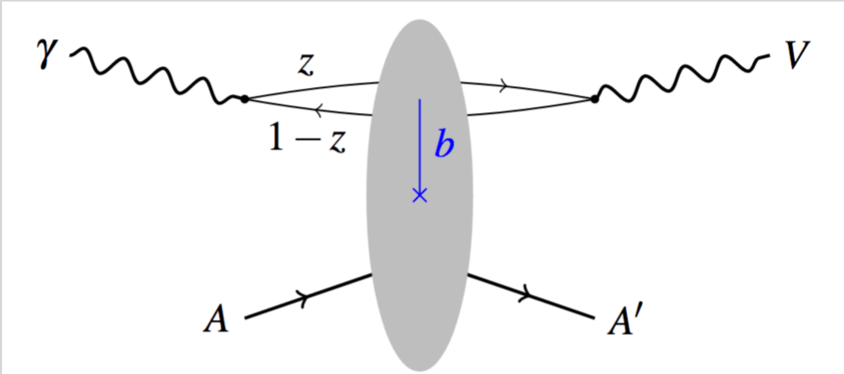
Diffraction

Good and Walker (1960)

Caldwell and Kowalski (2009)

Dipole picture

Diffraction (1)



Basic amplitude

$$\mathcal{A}^{\gamma p\to Vp'}(x_{I\!P},\boldsymbol{\Delta}) = i\int d^2\mathbf{r}\int_0^1 \frac{\mathrm{d}z}{4\pi}\,[\Psi_V^*\Psi_\gamma]_{(r,z)}\,\mathcal{A}_{q\bar{q}}(x_{I\!P},\boldsymbol{r},\boldsymbol{\Delta}).$$

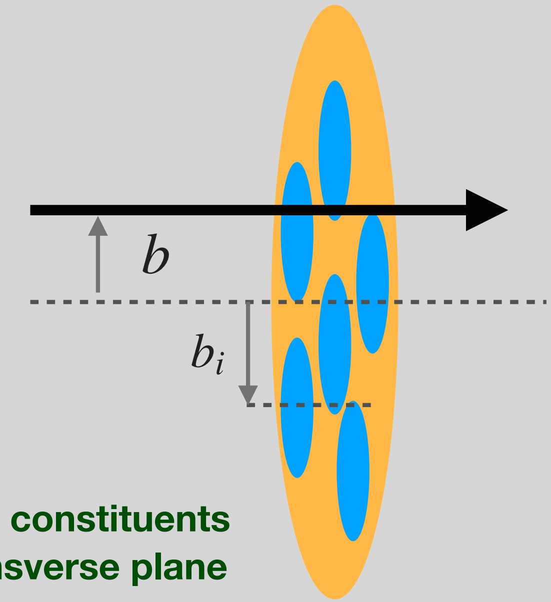
Cross section

$$\frac{d\sigma_{T,L}^{\gamma^* p\to Vp'}}{dt} = \frac{(1+\beta^2)}{16\pi}\left|\mathcal{A}_{T,L}^{\gamma^* p\to Vp}(x_{I\!P},Q^2,\boldsymbol{\Delta})\right|^2$$

Relation to dipole cross section

$$\mathcal{A}_{q\bar{q}}(\boldsymbol{b}) = 2\,[1-\mathcal{S}_{q\bar{q}}(\boldsymbol{r},\boldsymbol{b})] \equiv \frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\boldsymbol{b}}.$$

Diffraction (2)



Diffraction on a composite target (nucleon, nucleus)

$$\hat{T}_A(\boldsymbol{b}) = \sum_{i=1}^{A} T_G(\boldsymbol{b} - \hat{\boldsymbol{b}}_i) = \int \mathrm{d}^2\boldsymbol{x}\, \hat{\rho}(\boldsymbol{x}) T_G(\boldsymbol{x} - \boldsymbol{b})$$

$$\hat{\rho}(\boldsymbol{x}) = \sum_{i=1}^{A} \delta(\boldsymbol{x} - \hat{\boldsymbol{b}}_i)$$

Density of constituents in the transverse plane

$$T_G(\boldsymbol{b}) = \frac{1}{2\pi B_G}\, \mathrm{e}^{-\boldsymbol{b}^2/(2B_G)}$$

"Constituent" profile

Coherent cross section (square of elastic amplitude)

$$\int \mathrm{d}^2\boldsymbol{b}\, \mathrm{d}^2\boldsymbol{b}' \mathrm{e}^{-i\boldsymbol{\Delta}\cdot(\boldsymbol{b}-\boldsymbol{b}')} \left\langle \frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\boldsymbol{b}} \right\rangle \left\langle \frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\boldsymbol{b}'} \right\rangle = \langle \Sigma_{q\bar{q}}(\boldsymbol{\Delta}) \rangle^2$$

$$\varSigma_{q\bar{q}}(\varDelta) \equiv \int \mathrm{d}^2\boldsymbol{b}\, \mathrm{e}^{-i\varDelta\cdot\boldsymbol{b}} \frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\boldsymbol{b}} = \sigma_{\mathrm{dip}}\, T_G(\boldsymbol{\Delta})\, \hat{\rho}(\boldsymbol{\Delta})$$

$$\langle \Sigma_{q\bar{q}}(\Delta) \rangle = \sigma_{\mathrm{dip}} \int \mathrm{d}^2 b\, \mathrm{e}^{-i\Delta\cdot b} \langle \Psi_0 | \hat{T}_A(b) | \Psi_0 \rangle$$

Ground state expectation value of a (local) one-body operator

Diffraction (3)

Total (diffractive) cross section

$$\int \mathrm{d}^2\boldsymbol{b}\,\mathrm{d}^2\boldsymbol{b}'\mathrm{e}^{-i\boldsymbol{\Delta}\cdot(\boldsymbol{b}-\boldsymbol{b}')}\,\sigma_{\mathrm{dip}}^2\sum_n\left|\langle\Psi_n|\hat{T}_A(\boldsymbol{b})|\Psi_0\rangle\right|^2=\langle\Sigma_{q\bar{q}}^2(\boldsymbol{\Delta})\rangle.$$

Incoherent cross section

$$\langle\Sigma_{q\bar{q}}^2(\boldsymbol{\Delta})\rangle-\langle\Sigma_{q\bar{q}}(\boldsymbol{\Delta})\rangle^2=\sigma_{\mathrm{dip}}^2\int\mathrm{d}^2\boldsymbol{x}\,\mathrm{d}^2\boldsymbol{y}\,\mathrm{e}^{-i\boldsymbol{\Delta}\cdot(\boldsymbol{x}-\boldsymbol{y})}\,S(\boldsymbol{x},\boldsymbol{y})$$

Gives access to the density-density correlation function

$$\begin{aligned}S(\boldsymbol{x},\boldsymbol{y})&=\langle\hat{\rho}(\boldsymbol{x})\hat{\rho}(\boldsymbol{y})\rangle-\langle\hat{\rho}(\boldsymbol{x})\rangle\langle\hat{\rho}(\boldsymbol{y})\rangle\\&=A\rho^{(1)}(\boldsymbol{x})\delta(\boldsymbol{x}-\boldsymbol{y})+A(A-1)\rho^{(2)}(\boldsymbol{x},\boldsymbol{y})-A^2\rho^{(1)}(\boldsymbol{x})\rho^{(1)}(\boldsymbol{y})\end{aligned}$$

$$\int\mathrm{d}^2\boldsymbol{b}_1\cdots\mathrm{d}^2\boldsymbol{b}_A\,|\langle\boldsymbol{b}_1,\cdots,\boldsymbol{b}_A|\Psi_0\rangle|^2\sum_{i=1}^{A}\delta(\boldsymbol{x}-\boldsymbol{b}_i)=A\rho^{(1)}(\boldsymbol{x})$$

$$\int\mathrm{d}^2\boldsymbol{b}_1\cdots\mathrm{d}^2\boldsymbol{b}_A\,|\langle\boldsymbol{b}_1,\cdots,\boldsymbol{b}_A|\Psi_0\rangle|^2\sum_{i,j=1}^{A}\delta(\boldsymbol{x}-\boldsymbol{b}_i)\delta(\boldsymbol{y}-\boldsymbol{b}_j)=A\rho^{(1)}(\boldsymbol{x})+A(A-1)\rho^{(2)}(\boldsymbol{x},\boldsymbol{y}).$$

In the absence of correlations

$$\rho^{(2)}(\boldsymbol{x},\boldsymbol{y})=\rho^{(1)}(\boldsymbol{x})\rho^{(1)}(\boldsymbol{y})\;\longrightarrow\;S(\boldsymbol{x},y)=A\rho^{(1)}(\boldsymbol{x})\delta(\boldsymbol{x}-\boldsymbol{y})-A\rho^{(1)}(\boldsymbol{x})\rho^{(1)}(\boldsymbol{y})$$

Local density fluctuation

Fluctuations

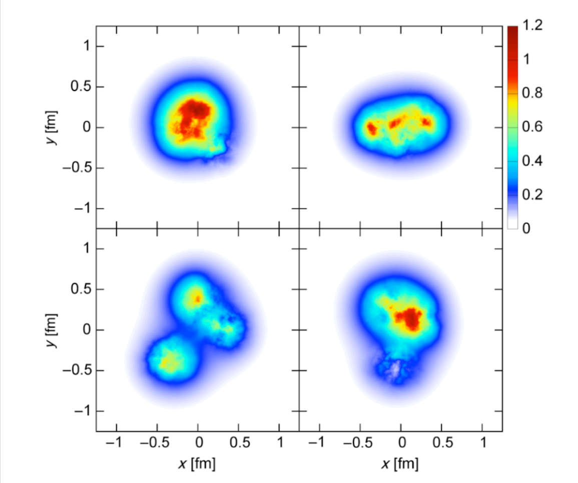
Fluctuations

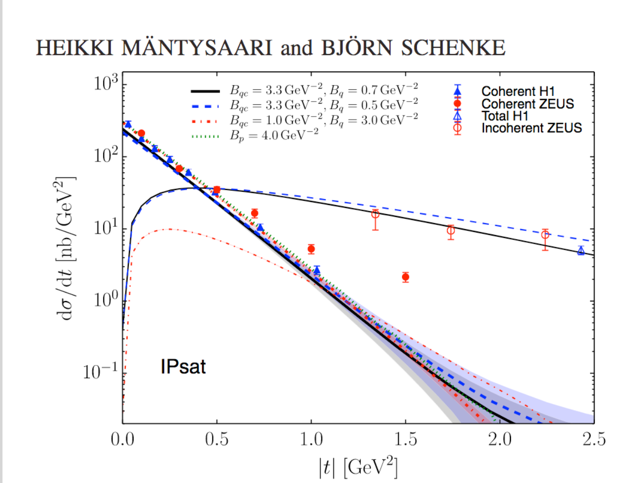
- short interaction time, frozen configurations of positions
- event by event measurements -> probe one configuration at a time
- configurations fluctuate from event to event
- interactions are local -> fluctuations are controlled by the density-density correlation function (and higher n-point functions)

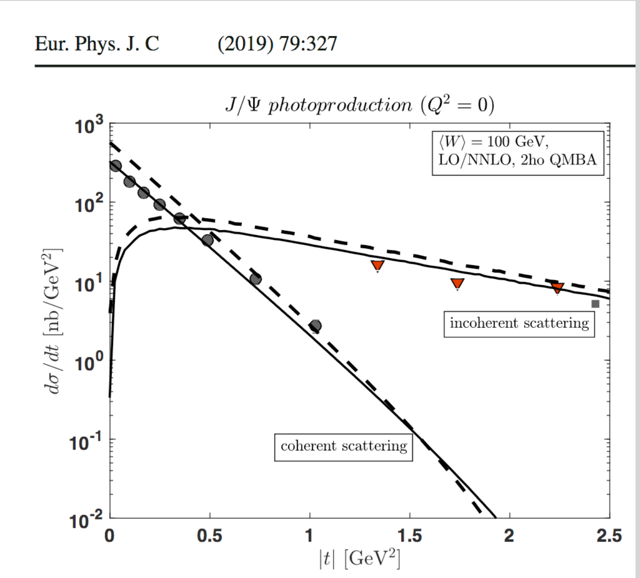
Diffractive production of vector mesons

For a review see Heikki Mäntysaari, Rep. Prog. Phys. 83 (2020) 082201

Sampling of the density, proton with three valence quarks (event-by event)







Sampling made from a model wave function of the proton (M. Traini and JPB)

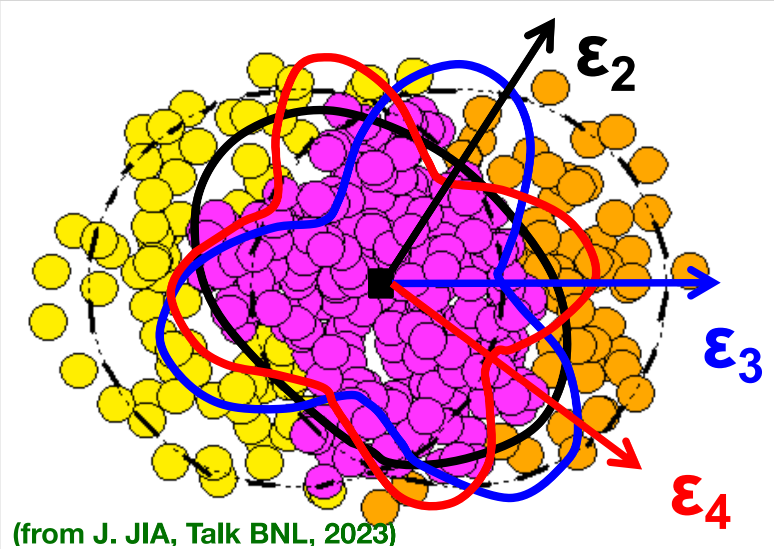
$$P(b_1, \cdots, b_A) = |\langle b_1, \cdots b_A | \psi_0 \rangle|^2$$

Initial states
in heavy ion collisions

"GLAUBER" INITIAL FLUCTUATIONS

The initial energy density (in the transverse plane) is not smooth

Typical distribution of "participants" in a Glauber calculation at some finite impact parameter



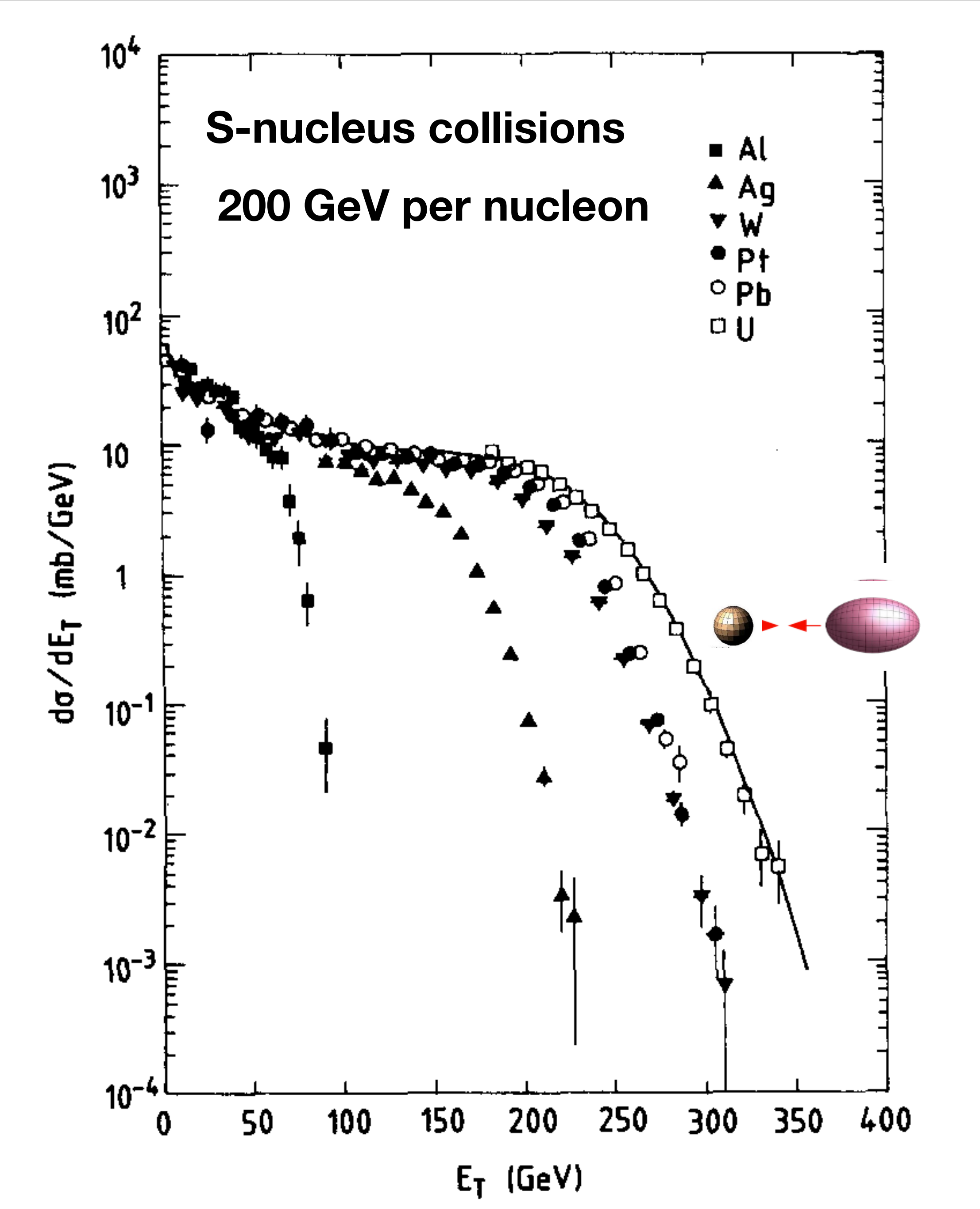
There are potentially many sources of fluctuations. The dominant ones seem to be those associated to the nucleon positions at the instant of the collision. Hence the connection to nuclear structure.

Shape of nuclei matters

The tail of the transverse energy distribution depends on the orientation of the Uranium nucleus

HELIOS collaboration, (CERN SPS)
Phys. Lett. B 214 (1988) 295

NB. Analogous finding in electron scattering



Heavy ion collisions and nuclear structure

Low-energy structure of nuclei affects outcome of high-energy collisions between nuclei

Numerous evidences for the influence of "intrinsic" nuclear shapes, e.g Ru/Zr ratios

Observations made at colliders impact our knowledge of nuclear structure

The large sensitivity to initial configurations of nucleons allows for a precise determination of deformation parameters, neutron skin, etc

This raises the question of why (and how) fine details of nuclear structure survive the complexity of a nucleus-nucleus collision at high energy?

Connection to nuclear structure

Is deformation "real" ?
Can it be observed ?

- **A deformed nucleus is characterised (for example) by a non vanishing quadrupole moment of the one-body density**

$$Q = r^2 P_2(\cos\theta)$$

$$\langle Q\rangle = \int \mathrm{d}^3 r\, \rho(\vec{r})\, Q(\vec{r}) \neq 0 \qquad \rho(\vec{r}) = \int \mathrm{d}^3 r_2 \cdots \mathrm{d}^3 r_N\, |\Phi(\vec{r}, \vec{r}_2, \cdots, \vec{r}_N)|^2$$

where $\Phi(\vec{r}_1, \vec{r}_2, \cdots, \vec{r}_N)$ **is the "deformed" independent particle wave function.**

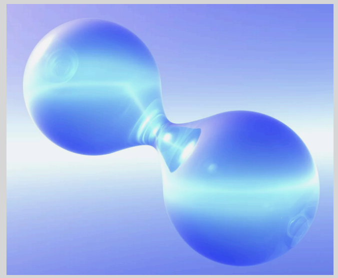
- **But** Φ **cannot be the ground state of the nuclear Hamiltonian since the ground state carries zero angular momentum**

$$\langle \Psi_{J=0} | Q | \Psi_{J=0} \rangle = 0$$

- **Way out:** Φ **is to be considered as an "intrinsic" state, function of intrinsic coordinates. The full wave function contains a factor that describe the collective rotation of the system.**

- **This remains a model though, or at best an approximation.**

[Some analogy with a diatomic molecule]



Conceptual issues

- In the intrinsic state the nucleons are essentially uncorrelated (mean field picture), but the average potential has some "orientation"

$$P_\Omega(r_1, r_2, \cdots, r_N) = \left|\Phi_\Omega^{\rm int}(r_1, r_2, \cdots, r_N)\right|^2$$

- Averaging over the collective wave function generates specific angular correlations (of all orders)

$$P(r_1, r_2, \cdots, r_N) = \int \frac{{\rm d}\Omega}{4\pi} \left|\Phi_\Omega^{\rm int}(r_1, r_2, \cdots, r_N)\right|^2$$
$$= |\Psi_0(r_1, \cdots r_A)|^2$$

NB. This average projects onto a spherical state

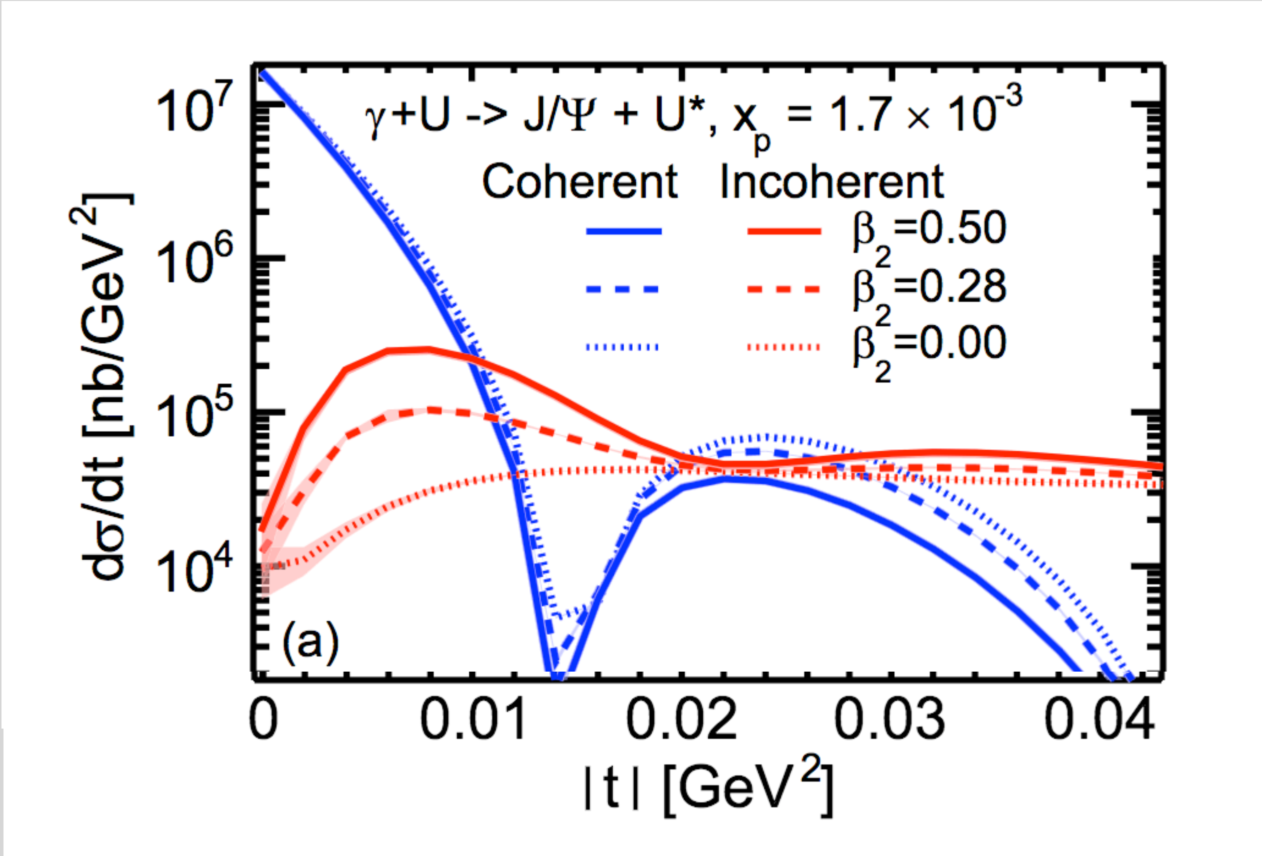
- Measurements (diffraction, HI collisions) involve "snapshots" (event-by-event) of the configurations of positions, i.e. they probe the exact distribution

$$|\Psi_0(r_1, \cdots r_A)|^2$$

- Characterization of the correlation functions without referring to an intrinsic state is work in progress [with Giulinao Giacalone]

Example of a prediction at the EIC

(Mäntysaari, PRL, 2023)



Why is that interesting ?

- Heavy ion collisions may offer us the possibility to capture the shapes of deformed nuclei in a more direct way than any other previous experiment.
- Not only does one "see" the deformed shapes, but the values of deformation parameters can be determined with surprisingly high precision.
- The sensitivity of observables (in diffraction and heavy ion collisions) to the exact locations of the nucleons at the instant of the collision (i.e. event-by-event) is intriguing…and challenging (e.g. for description of initial states).