Strongly Interacting High Baryon Density Matter

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GGI Florence, March 2025

Experiment:
Many measurements of neutron star masses
Radii
Collision and quadrupole deformability

Very good determinations of equations of state
Annal, Gorda, Kurkela and Vuorinen, Phy. Rev. Lett. 120, (2018) 17,172703

Consensus: At a few times nuclear matter density:

$$v^2_{sound} \sim \frac{1}{3} c^2$$

P. Bedaque and A. Steiner. Phys. Rev. Lett. 114 (2015) 3,031103

This is very interesting, because.....

Scale Invariant Behavior (masses are not important):

$$P = -\frac{dE}{dV}$$

$$E \sim (N/V)^{4/3} V \sim N^{4/3} V^{-1/3}$$

In scale invariant theory at zero temperature:

$$P = \frac{1}{3}\frac{E}{V} = \frac{1}{3}e$$

$$v_s^2 = \frac{dP}{de} = \frac{1}{3}$$

$$\Delta = \frac{1}{3} - \frac{p}{e} = 0$$

Fujimoto, Fukushima, Mclerran and Praszalowicz, Phys. Rev. Lett. 129, 25, 252702

In a change of typical length scale by 30-60%, 1.3-1.5 nuclear matter changes from a non-relativistic gas to an almost scale invariant relativistic system!

The trace of the stress energy tensor is taken to be a measure of scale invariance. It is anomalous in QCD.

$$T^{\mu}_{\mu} = -\frac{\beta(g)}{g}(E^2 - B^2) + m_q(1+\gamma_q)\overline{\psi}\psi$$

In this equation, the beta function of QCD is negative, and the fermion term is from quarks. The fermion term vanishes in the chiral limit.

If we take matrix elements of single particle states

$$< p \mid T^{\mu}_{\mu} \mid p > \sim p^2 = m^2 \geq = 0$$

In the chiral limit, this implies E > B, as we expect for massive quarks, except for the pion, which is very tightly. bound

For dilute systems, the trace anomaly is positive, as it is at high density for a quark gas. In general, we expect it to be positive, except possibly for small effects due to a pion condensate, if it exists.

What does a large sound velocity tell us about quarks?
First or second order second phase transition would have sound velocity 0!

For zero temperature Fermi gas:

$$\frac{n_B}{\mu_B dn_B/d\mu_B} = v_s^2$$

where the baryon chemical potential includes the effects of nucleon mass

$$\frac{\delta\mu_B}{\mu_B} \sim v_s^2 \ \frac{\delta n_B}{n_B}$$

So if the sound velocity is of order one, an order one change in the baryon density generates a change in the baryon number chemical potential of order the nucleon mass
For nuclear matter densities

$$\mu_B - M \sim \frac{\Lambda_{QCD}^2}{2M} \sim 100 \ MeV$$

For an ordinary Fermi gas

$$\rho \sim k_F^3$$

Large sound velocities will require very large intrinsic energy scales, and a partial occupation of available nucleon phase space because density is not changing much while Fermi energy changes a lot

Useful to think in terms of number of color of quarks, Nc

$$k_F^{quark} = k_F^{nucleon}/N_c$$

$$\rho_{quark} = \rho_{nucleon}/N_c^3$$

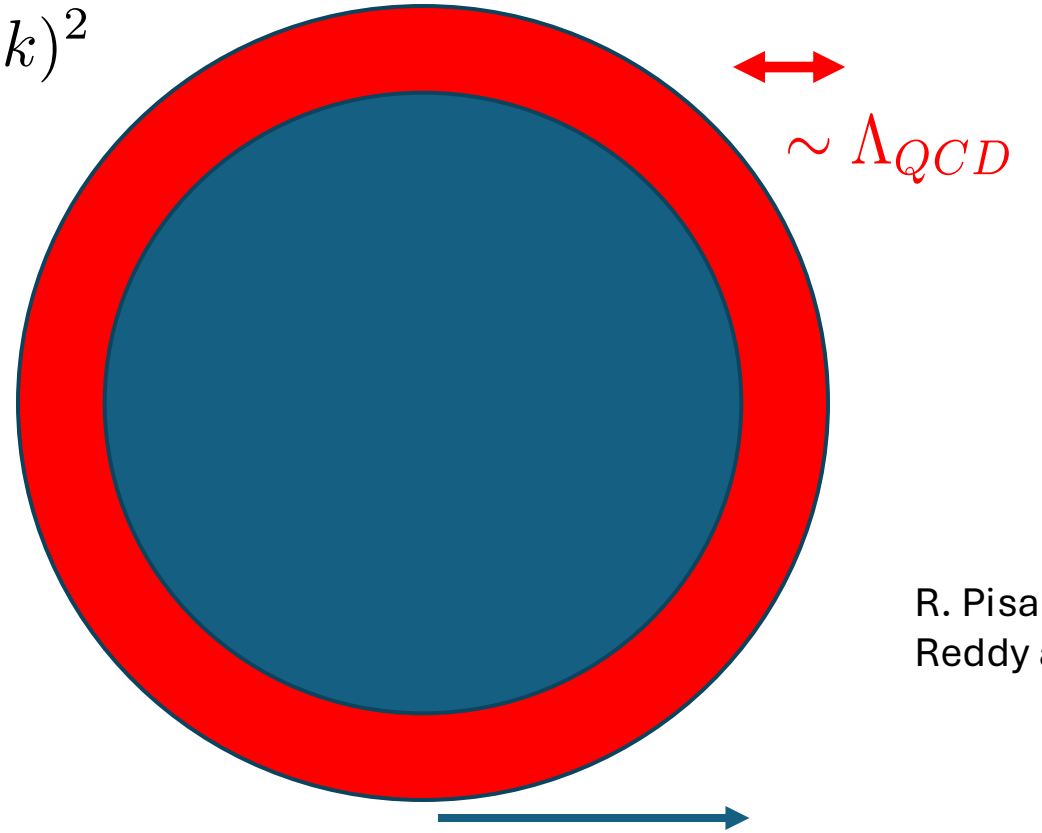
Solution is simple:

Put nucleons in a shell at the Fermi surface. Hold this density fixed by making shell thinner as baryon number increases. Dump baryon number into quarks while increasing Fermi momentum without much increasing baryon number density

Fermi Surface is Non-perturbative

$$n^Q_{Baryon} \sim k_Q^3 \sim \frac{1}{N_c^3} k_N^3 = \frac{1}{N_c^3} n^N_{baryon}$$

$$\rho^N_{surface} \sim k_F (\Delta k)^2$$



Fermi Surface
Interactions sensitive to infrared
Degrees of freedom: baryons, mesons and glueballs

Fermi Sea: Dominated by exchange interactions which are less sensitive to IR. Degrees of freedom are quarks

R. Pisarski and LDM, Nucl. Phys/ A796 (2007) 83-100;
Reddy and LDM, Phys. Rev. Lett. 122 (2019) 12,122701

Origin of the Ideas: Quarkyonic Matter

Pisarski, LDM

Hidaka, Pisarksi, LDM

Large Nc:

At finite T, gluons Debye screen potential.

At finite density, only quarks and they are suppressed by 1/Nc

Finite density world confines until a density which is parametrically large:

$$\rho_{conf} \sim N_c^{3/2} \Lambda_{QCD}^3$$

Inside the Fermi sea of quarks, interactions are perturbative because energy scales are large compared to QCD scale

At Fermi surface long distance interactions are important: Fermi surface of nucleon. Antiquarks and gluons are all bound into mesons

T. Kojo;
Y. Fujimoto., LM and T. Kojo

An Explicit Quantum Mechanical Theory of Quarkyonic Matter

Let occupation number density for nucleons and quarks be

$$f_q, f_N \qquad 0 \leq f_q, f_N \leq 1$$

Saturation criteria due to Kojo (KFC?)

A duality relation (nucleons are composed of quarks)

$$f_q(k) = \int [dp]\ K(k - p/N_c) f_N(p)$$

$$\int [dk]\ K(k) = 1$$

First: Free theory of nucleon and quarks (except for duality relation)

This is a solvable theory with non-trivial solution with two phases:
A nucleonic phase and a quarkyonic phase

A simple choice of kernel and explicit duality

$$f_q(k) = \int [dp]\ K(k - p/N_c) f_N(p)$$

$$K(k) = \frac{1}{4\pi\Lambda^2} \frac{e^{\frac{-|k|}{\Lambda}}}{|\ k\ |}$$

$$\left\{ -\nabla_K^2 + \frac{1}{\Lambda^2} \right\} K(k) = \frac{1}{\Lambda^2} \delta(k)$$

$$(-\nabla_k^2 + \frac{1}{\Lambda^2}) f_q(k) = \frac{N_c^3}{\Lambda^2} f_N(N_c k)$$

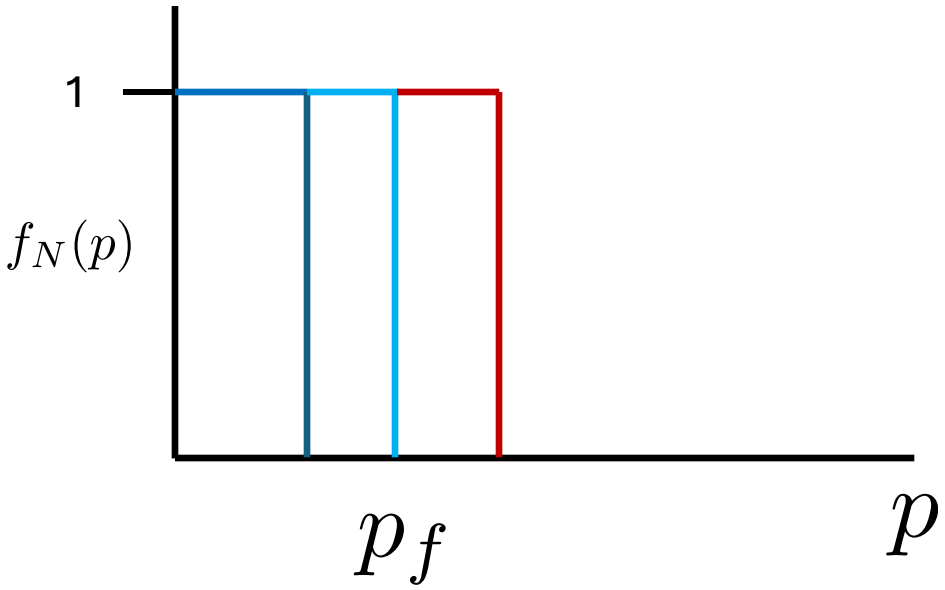
$$\epsilon = \int [dp] \sqrt{p^2 + M_n^2} f_N(p) = N_c \int [dk]\ E_q(k) f_q(k)$$

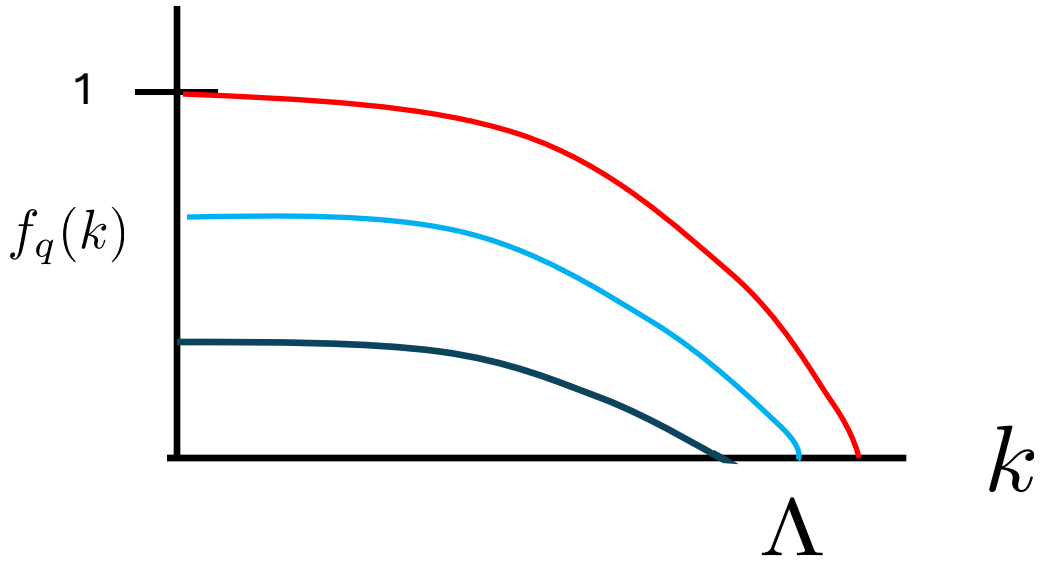
$$E_q(k) = \sqrt{k^2 + M_q^2} - \frac{M_q^2 \Lambda^2}{(k^2 + M_q^2)^{3/2}}$$

Free theory of quarks with modified kinetic energy term

Solution looks like:

Low density:





$$1 = \int [dp]\ K(k - p/N_c) f_N(p)$$

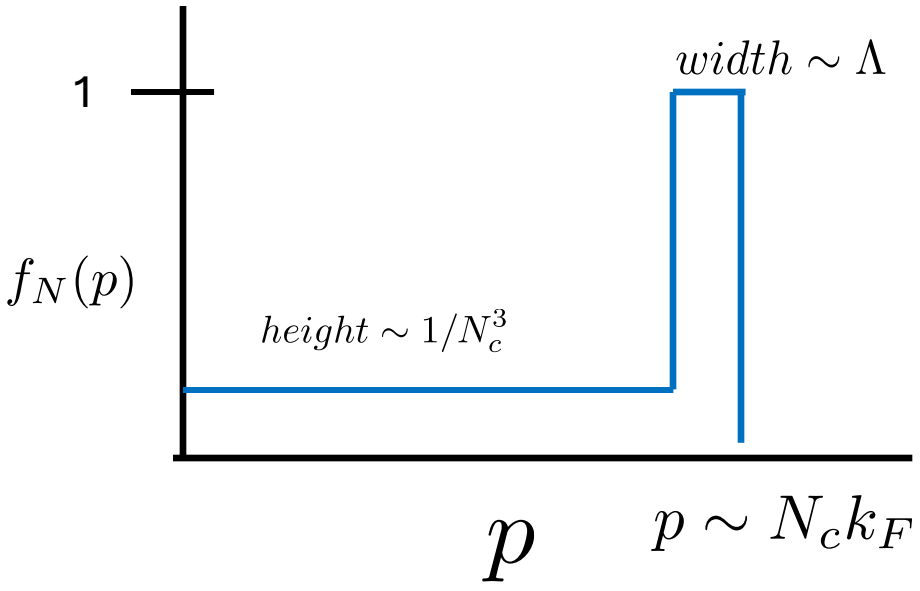
$$\sim K(0) \int [dp]\ f_N(p) = \kappa \frac{n_n}{\Lambda^3}$$

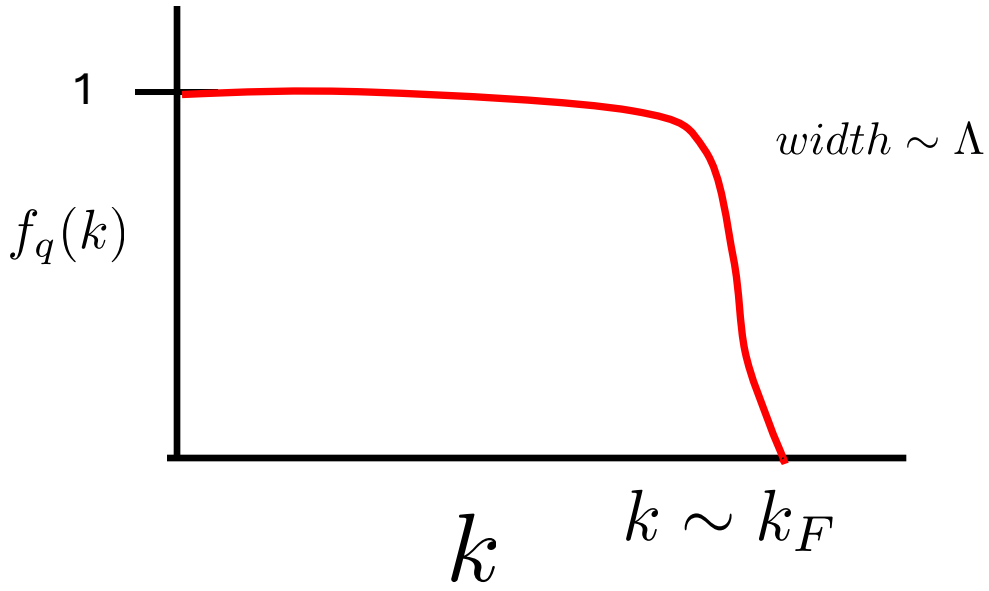
Width of quark distribution determined by intrinsic confinement scale of quarks inside of nucleons

$$\Lambda$$

$$n_N^{crit} \sim \Lambda^3$$

High Density:





At high densities, a thin shell of saturated baryon matter forms surrounded by an underoccupied distribution of nucleons

The quarks make a filled sea of nucleons with an underoccupied tail where the shell of nucleons are not Pauli blocked

Might Quarkyonic Matter be Relevant for Nuclear Matter?

General Considerations on the Transmutation Density to Quarkyonic Matter

Transmutation Criterium:

Saturation criteria due to Kojo

$$1 = f_Q(0) = \int^{k_f} \frac{d^3p}{(2\pi)^3}\ K(p/N_c)\ f_N(p) \sim \frac{1}{4} n_N\ K(0)$$

Using a Gaussian model for the distribution of quarks inside a hadron, and only including valence quarks

$$\phi_{gauss} = 8\pi^{3/2} R^3 e^{-p^2 R^2}$$

Using the RMS charge radius as R

$$R = \sqrt{\frac{2}{3}} r_{RMS}$$

Find transmutation density of twice nuclear matter for measured charge radius
But this ignores the contribution of quark-antiquark pairs: the meson cloud

Determination from observed quark distributions in. deep inelastic scattering:

LDM and Miller

Transverse momentum distribution functions were determined by de Teramond et. al.

These distributions functions describe measured integrated valence and sea quark distribution functions, and electromagnetic form factors

$$\frac{dn_Q(k)}{d^3k} = \frac{x}{E} \mid \psi(x, k_T) \mid^2$$

$$\int \frac{dx d^2k_T}{(2\pi)^3} \mid \psi(x, k_T) \mid^2 = 1$$

$$x = \frac{k^+ + E_k}{p^+ + E_p}$$

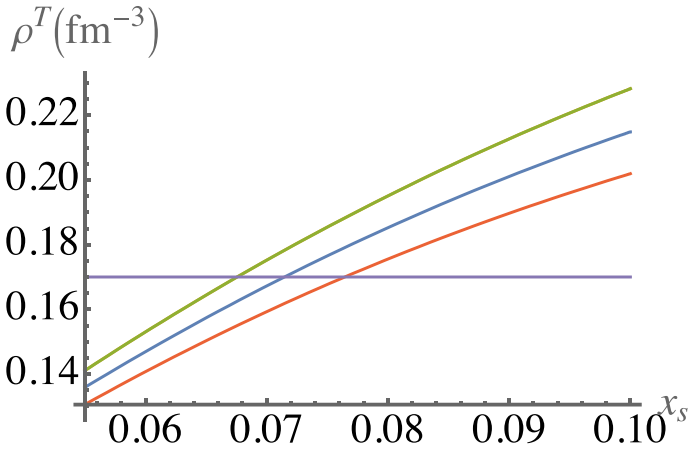
So that for a hadron at rest and a parton with zero momentum

$$x = E_0/M$$

$$\frac{dn_Q(0)}{d^3k} = \mid \psi(x_0, 0_T) \mid^2 \frac{1}{M}$$

Typical x for valence quarks is about 1/6, because glue carries ½ the momentum, and less than 1/10 for quark-antiquark pairs

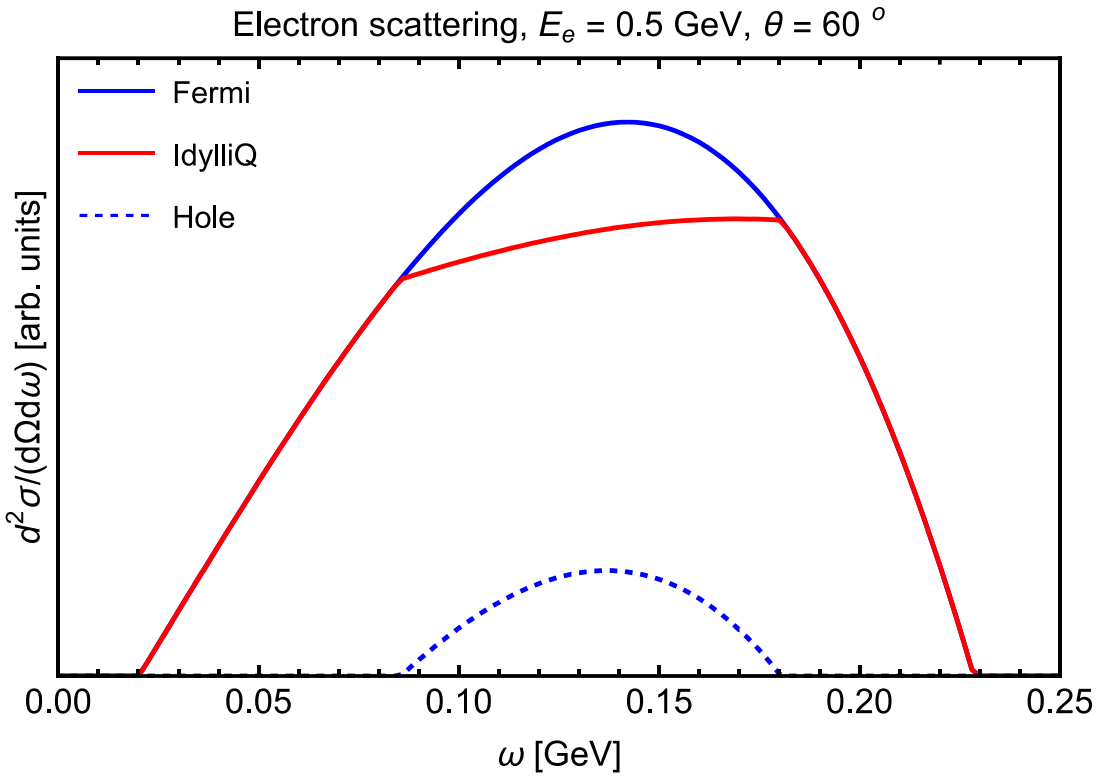
Find that for valence quarks alone the transmutation density would be about twice nuclear matter. Sea quark contribution has large uncertainties since it is sensitive to the value of x used.



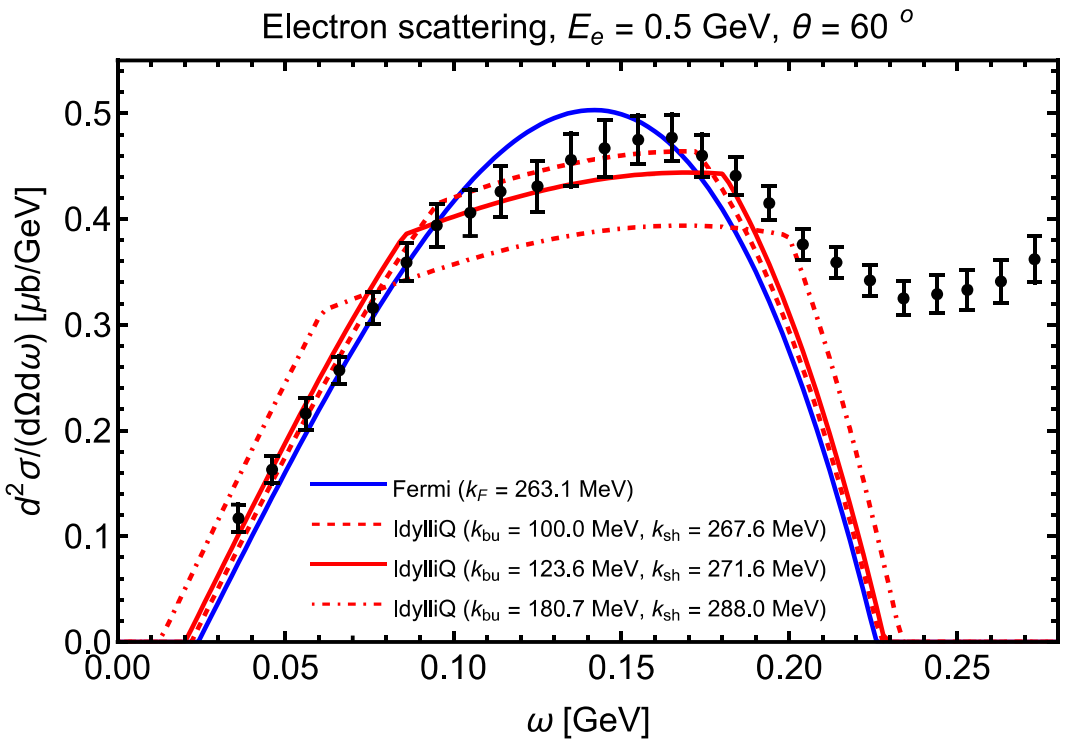
Plausible that the transmutation density is below or less than that of nuclear matter

Koch,LM, Miller, Vovchenko

Experimental Constraint: Electron Scattering:
Quarkyonic Matter has a Fully Occupied Fermi Surface of Nucleons and an underoccupied sea



Hole in Fermi sea lowers cross section near maximum. Peak would be delta function if there was no Fermi momentum



Data is Coulomb corrected and extrapolated to nuclear matter. Energy chosen minimizes final state interacs

T. Bluhm, Y. Fujimoto, M. Nahrgang and LM

What About Finite Temperature?

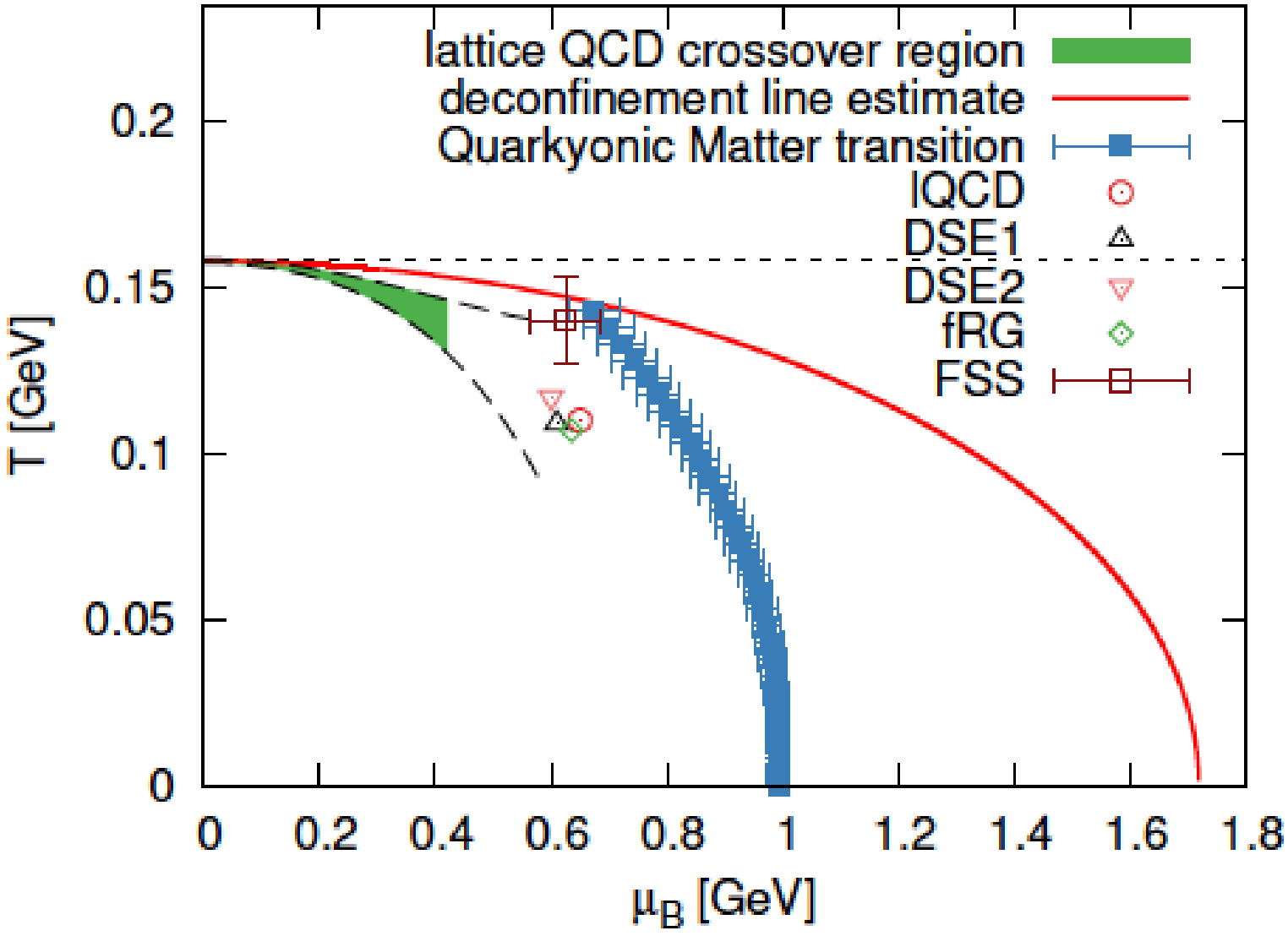
One can determine the Quarkyonic boundary by the saturation condition.

One most include the contributions form delta and baryon resonances, and a small contribution from mesons. The finite temperature Quarkyonic region is limited by the confinement boundary, which we is approximately by Debye screening:

$$m_D^2(T,\mu) = c_T N_c T^2 + c_\mu \mu^2$$

$$T_c(\mu)^2 = T_c^2 - \frac{c_\mu}{c_T}\frac{\mu^2}{N_c}$$

Our biggest uncertainty is the Quarkyonic transition density at zero temperature, which we take to be 1-3 times the density of nuclear matter



It is interesting that the intersections of the quarkyonic line with that of deconfinement occurs close to values estimated for the critical end point of chiral transition

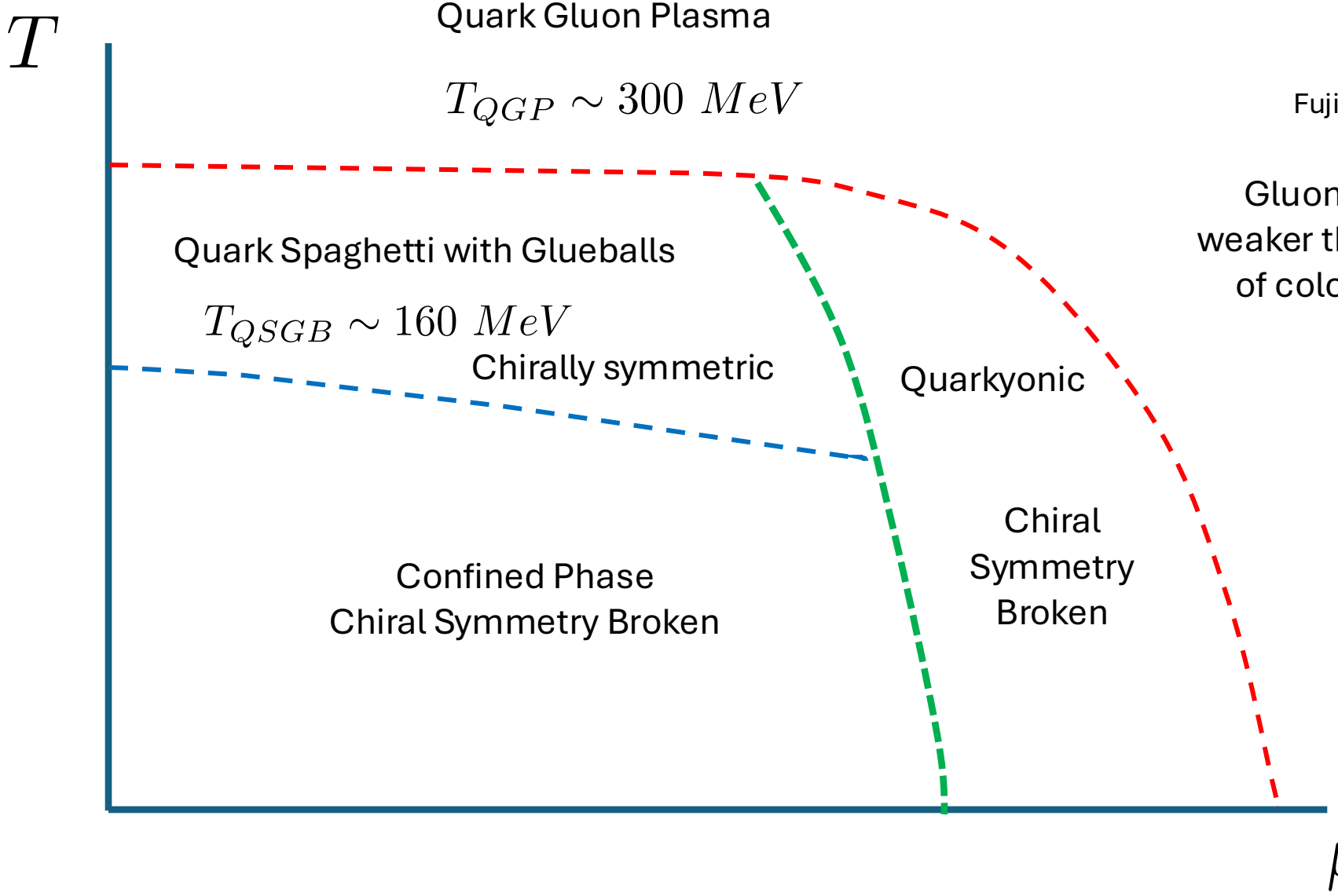
Important Issues:

Is nuclear matter quarkyonic? How to determine experimentally? Theory of finite nuclei?

How to test theory from controllable computations? 1+1 d? 3+1 d Bosonic systems?

How to compute at finite T?

How is Quarkyonic matter related to this new region of Glozman and Cohen?



Glozman, Rohrhofer, Lang, Cohen

Phillipsen and Pisarski

Fujimoto, Fukishima, Hidaka, McLerran

Gluon interactions are parametrically weaker than quarks in limit of large number of colors. Requires a higher density of Quarks to melt.
High density of quarks restores chiral symmetry.

Chiral symmetry is a symmetry associated with massless ness of quarks that is broken in the vacuum