Hadronic light-by-light scattering in AdS/QCD and the muon g-2

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GGI, Florence, 26 March 2025

Banff 1993: 3rd Workshop on Thermal Field Theories

There is a new kid in town:

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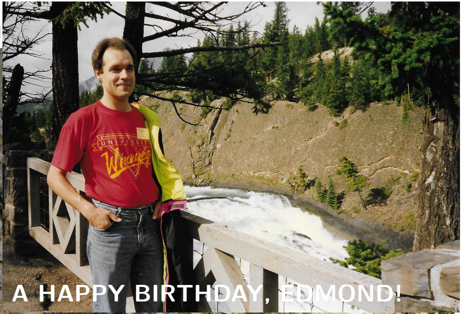
Ulli and me wish you


A HAPPY BIRTHDAY, EDMOND!

Writing papers with Edmond

1998: started collaboration with Jean-Paul & Edmond
1999: two letters, PRL, PLB
2000: longer version for PRD

Approximately selfconsistent resummations for the thermodynamics of the quark gluon plasma. 1. Entropy and density #1

J.P. Blaizot (Saclay), Edmond Iancu (CERN), A. Rebhan (Vienna, Tech. U.) (Apr, 2000)

Published in: *Phys.Rev.D* 63 (2001) 065003 • e-Print: hep-ph/0005003 [hep-ph]

pdf links DOI cite claim reference search 372 citations

Comments:	66 pages REVTEX, 14 figures
Subjects:	**High Energy Physics - Phenomenology (hep-ph)**; High Energy Physics - Lattice (hep-lat)
Report number:	CERN-TH/2000-121, SACLAY-T00/059, TUW-00/13
Cite as:	arXiv:hep-ph/0005003
	(or arXiv:hep-ph/0005003v1 for this version)
	https://doi.org/10.48550/arXiv.hep-ph/0005003

Submission history

From: Anton Rebhan [view email]
[v1] Sat, 29 Apr 2000 21:50:37 UTC (244 KB)
[v2] Thu, 6 Jul 2000 17:49:21 UTC (247 KB)
[v3] Fri, 25 Aug 2000 15:00:53 UTC (222 KB)

v1: Referee has many questions, complains about length. . .

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Comments:	62 pages REVTEX, 14 figures; v2: numerous clarifications, sect. 2C shortened, new material in sect. 3C; v3: more clarifications, one appendix removed, alternative implementation of the NLO effects, corrected eq. (5.16)
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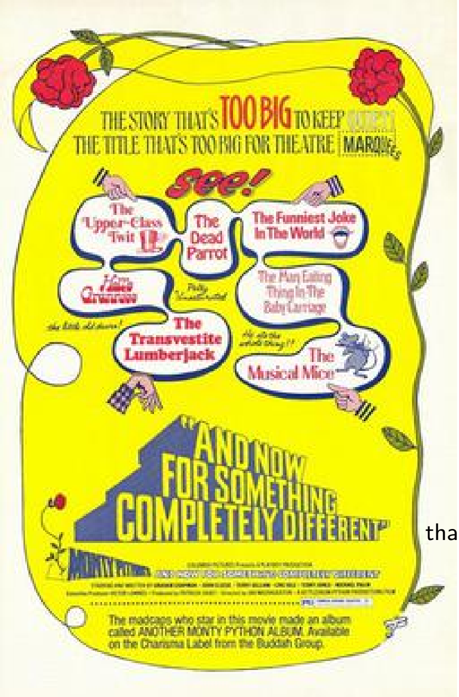


than THERMAL FIELD THEORY or pQCD

Muon $g-2$ SM prediction [TI "White Paper", Aoyama et al., 2006.04822]

Muon 200× heavier than electron ⇒ more sensitive to non-QED physics
Prediction limited by control over hadronic interactions at $\lesssim 10^{-9}$

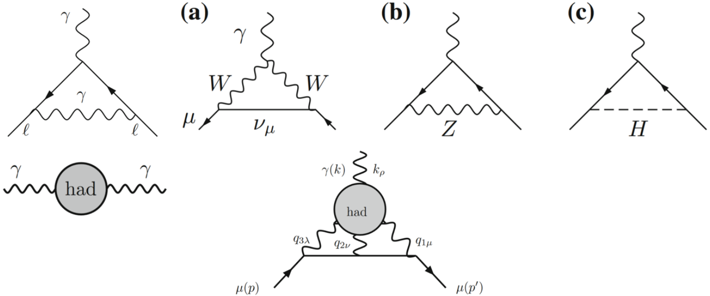
$$a_\mu^{\mathsf{SM}} = a_\mu^{\mathsf{QED}} + a_\mu^{\mathsf{EW}} + a_\mu^{\mathsf{HVP}} + a_\mu^{\mathsf{HLbL}} = (116\,591\,810 \pm 43) \times 10^{-11}$$

$$a_\mu^{\mathsf{QED}} = (116\,584\,718.931 \pm 0.104) \times 10^{-11}$$

$$a_\mu^{\mathsf{EW}} = (153.6 \pm 1.0) \times 10^{-11}$$

$$a_\mu^{\mathsf{HVP}} = (6845 \pm 40) \times 10^{-11}$$ (0.6% accuracy – contested by BMW)

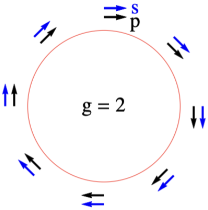
$$a_\mu^{\mathsf{HLbL}} = (92 \pm 19) \times 10^{-11}$$ (20% uncertainty!)

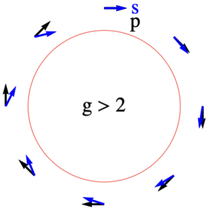


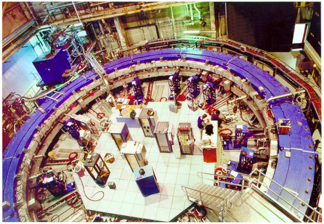
Precision measurements

Anomalous spin precession

$(g-2)_\mu$ can be measured by polarized beam of muons circulating in the magnetic field of a storage ring:





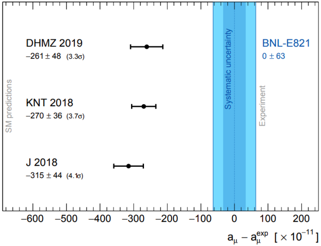


Measured in the early 2000's at BNL, final result 2006

with highly polarized beam of μ^+ from decaying π^+'s produced by shooting protons from accelerator on some target

The muon $g-2$ discrepancy

Since then, 3-4 σ discrepancy between theorical and experimental result from BNL-E821:



and urge to repeat this singular experiment

The muon $g - 2$ discrepancy

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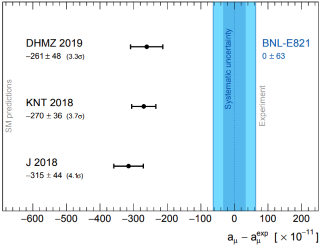


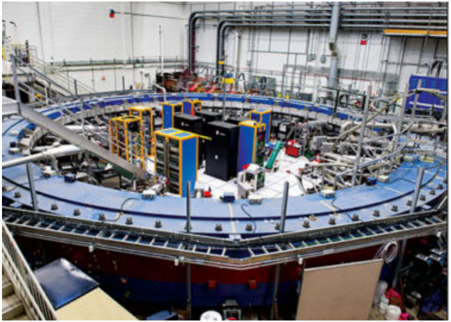
and urge to repeat this singular experiment → Fermilab E989 experiment

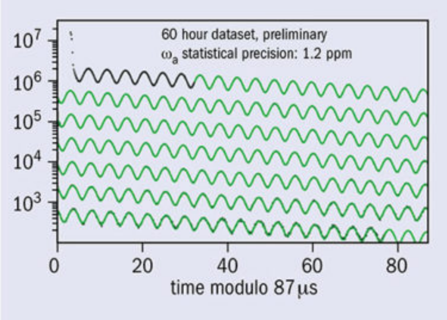
2013: Ring magnet of the BNL experiment (Long Island, NY) shipped to Fermilab (near Chicago), via Atlantic and Mississippi

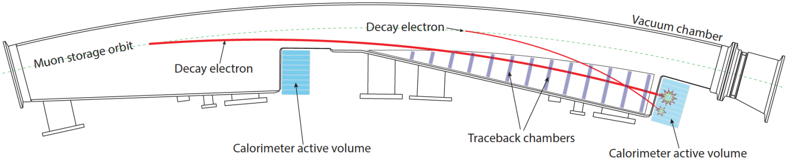
and then 5 years work of building new muon g-2 experiment

Fermilab E989 experiment

Since 2018: new rounds of data taking at Fermilab E989 experiment
Final result to be released ∼ May 2025





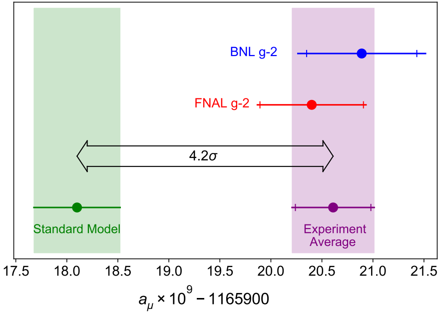


Magnetic moment of the muon

Data released from first runs on April 7, 2021:

Updated values BNL 2004 ∪ FNAL 2021 vs. Aoyama et al. 2020

$$
\begin{aligned}
a_\mu^{\text{exp}} &= (116\,592\,061 \pm 41) \times 10^{-11} \\
a_\mu^{\text{SM}} &= (116\,591\,810 \pm 43) \times 10^{-11}
\end{aligned}
$$

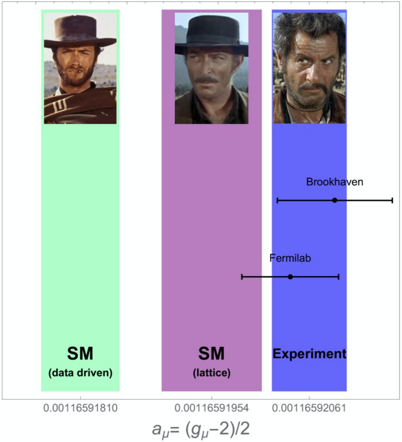


but Theory Initiative prediction questioned by new lattice results from Budapest-Marseille-Wuppertal (BMW) collaboration,
published by Nature also on April 7, 2021 (arXiv:2002.12347 [hep-lat])

Muon $g-2$ SM prediction

$$a_\mu^{\text{HVP,data-driven}} = (6845 \pm 40) \times 10^{-11}$$ (0.6% accuracy – contested by BMW)

$$a_\mu^{\text{HVP,lattice}} = (7075 \pm 55) \times 10^{-11}$$ according to BMW coll., Nature 593, 51 (2021)

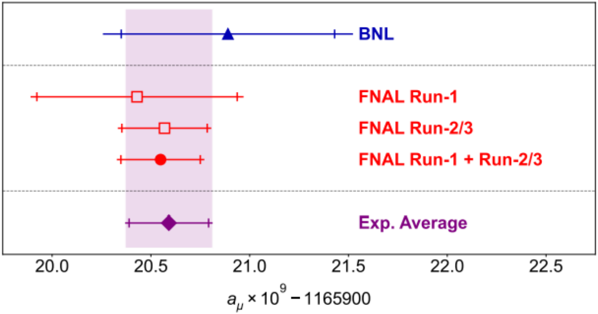


Strong tension between hadronic vacuum polarization deduced from low-energy experiments (R ratio) and lattice QCD!

New Fermilab result

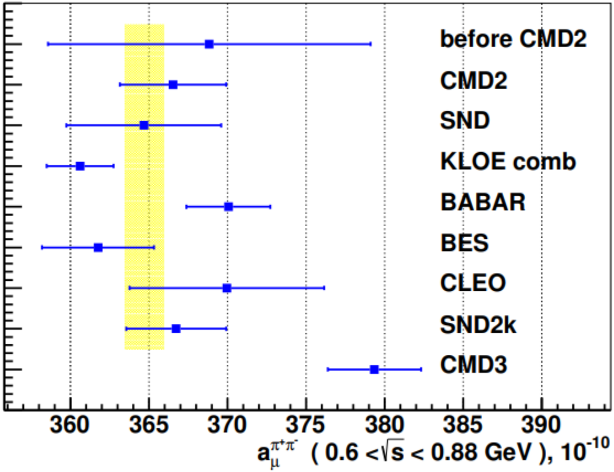
August 2023:
New Fermilab result from Run 2+3 with system errors already below design goal

$$a_\mu(\mathrm{FNAL}) = 116\,592\,055(24) \times 10^{-11} \quad (0.20\,\mathrm{ppm}),$$
$$a_\mu(\mathrm{Exp}) = 116\,592\,059(22) \times 10^{-11} \quad (0.19\,\mathrm{ppm}).$$



Muon $g-2$ SM prediction

Since Feb 2023: new data for $e^+e^- \to \pi^+\pi^-$ from Novosibirsk (CMD3) in disagreement with all previous results (including CMD2)



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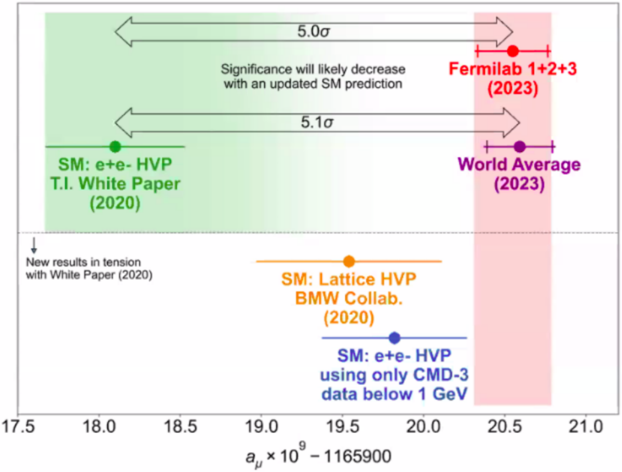


Muon $g-2$ SM prediction [TI "White Paper", Aoyama et al., 2006.04822]

Prediction limited by control over hadronic interactions at $\lesssim 10^{-9}$

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{HVP}} + a_\mu^{\text{HLbL}} = (116\,591\,810 \pm 43) \times 10^{-11}$$

$$\begin{aligned}
a_\mu^{\text{QED}} &= (116\,584\,718.931 \pm 0.104) \times 10^{-11} \\
a_\mu^{\text{EW}} &= (153.6 \pm 1.0) \times 10^{-11} \\
a_\mu^{\text{HVP}} &= (6845 \pm 40) \times 10^{-11} \quad \text{(0.6\% accuracy – contested by BMW and CMD-3)} \\
a_\mu^{\text{HLbL}} &= (92 \pm 19) \times 10^{-11} \quad \text{(20\% uncertainty!)} \\
&\supset 15(10)\ \underline{\text{short-distance constraints}},\ 6(6)\ \underline{\text{axial vector mesons}}
\end{aligned}$$

Current experimental error in $a_\mu^{\text{exp}} = (116\,592\,059 \pm 22) \times 10^{-11}$
will be reduced by further runs at FNAL to $\sim 10 \times 10^{-11}$

- Discrepancy between data-driven approaches and lattice calculations need to be resolved, and moreover accuracy improved!
- Also hadronic light-by-light (HLbL) contribution needs work

Alternative methodology: Gauge/gravity duality a.k.a. Holography

as approximation to strongly coupled non-Abelian gauge theories at large color number

Not sufficiently precise to help with HVP, but of interest to check HLbL contributions

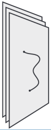
Holography a.k.a. Gauge/Gravity Duality

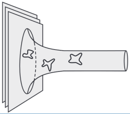
Conjectural generalization of AdS/CFT correspondence of Maldacena
where conformal symmetry in D-dim. QFT $\longleftrightarrow$ isometry of $D+1$-dim. anti-de Sitter space

Strongly coupled gauge theories in D dimensions at large N_c are
dual to suitable theories with gravity in D+1 dimensions with analogous **“dictionary”**

gauge theory	**gravity dual**
degree N of the gauge group	number of branes, curvature radius
flat space time on which the gauge theory lives	boundary of higher-dimensional geometry
global symmetry	gauge symmetry
gauge invariant operators	fields acting as sources to these operators
particle mass	eigenvalue in wave equation
energy scale	radial coordinate in the AdS-space
renormalisation group flow	movement along the radial coordinate

open string





closed string duality

Holographic QCD Zoo

Unclear whether holographic dual to non-susy and non-conformal large-N_c QCD exists, but:

1998 Witten succeeded in constructing a string-theoretical dual (“*top-down*”) to the low-energy limit of large-N_c QCD from type-IIA superstring theory compactified on one further extra dimension, and

2004 Sakai & Sugimoto found D-brane construction to add chiral quarks in fundamental representation $\rightarrow$ best top-down model of low-energy QCD so far

However: not even asymptotically AdS, not conformal in UV

2005ff: Erlich, Katz, Son, Stephanov, ... (HW1)
Hirn, Sanz (HW2) (simpler; very similar to WSS)

succeeded in constructing phenomenologically interesting models of hadron physics with similar ingredients on simple AdS_5 background ($\leftrightarrow$ conformal symmetry) broken in IR by “hard walls” (HW) or “soft walls” (SW)

$\rightarrow$ “*bottom-up*” holographic QCD

- surprisingly efficient as models of chiral symmetry breaking
- anomalies naturally represented in 5-dimensional setup!

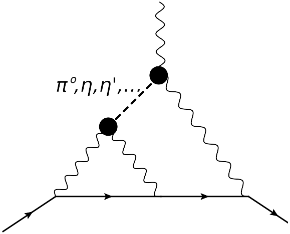
HLbL contribution to muon $g-2$ & holographic QCD

hQCD results available for

- **single and double virtual (pion) transition form factor** $F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2)$

  [Grigoryan, Radyushkin, PRD76,77,78 (2007-8)]
  [Cappiello, Catà, D'Ambrosio, PRD83 (2011)]
  [J. Leutgeb, J. Mager, AR, PRD100 (2019)]

  - good agreement with recent low-energy data (BESIII) and lattice results
  - hQCD prediction for $a_\mu^{\pi^0,\eta,\eta'}$ in good agreement with dispersive approach



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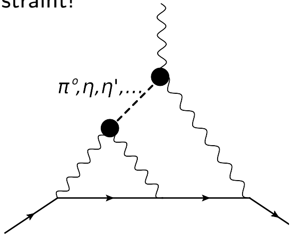
- **Axial vector meson contributions** [J. Leutgeb, AR, PRD101, 1912.01596]
  [Cappiello, Catà, D'Ambrosio, Greynat, Iyer, PRD102, 1912.02779]

  - crucial role in saturation of Melnikov-Vainshtein constraint!
  – first hadronic model to achieve this in chiral limit!
  - hQCD prediction for $a_\mu^{a_1, f_1, f_1', \dots}$
  - extension to massive quarks:
    Leutgeb, AR, PRD104, 2108.12345
    including U(1)$_A$ anomaly and $\mathbf{m_s \gg m_{u,d}}$:
    Leutgeb, Mager, AR, PRD107, 2211.16562

- Brand new: **Tensor meson contributions**
  [Cappiello, Leutgeb, Mager, AR, 2501.09699+2501.19293]



Bottom-up and top-down holographic QCD

(Axial) vector mesons and pions are described by **5-d YM fields** $\mathcal{F}_{MN}^{L,R}$ for global $U(N_f)_L \times U(N_f)_R$ chiral symmetry of boundary theory

$$S_{\rm YM} \propto \frac{1}{g_5^2}\,\mathrm{tr}\int d^4x \int_0^{z_0} dz\, e^{-\Phi(z)}\sqrt{-g}\, g^{PR} g^{QS}\left(\mathcal{F}_{PQ}^{(L)}\mathcal{F}_{RS}^{(L)} + \mathcal{F}_{PQ}^{(R)}\mathcal{F}_{RS}^{(R)}\right),$$

where $P,Q,R,S = 0,\ldots,3,z$ and $\mathcal{F}_{MN} = \partial_M \mathcal{B}_N - \partial_N \mathcal{B}_M - i[\mathcal{B}_M, \mathcal{B}_N]$

with conformal boundary at $z = 0$, and

either sharp cut-off of AdS_5 at z_0 (HW) or with nontrivial dilaton $z_0 = \infty$ (SW)
(SS: not asymptotically AdS_5, finite z_0 corresponding to point where D8 branes join)

Chiral symmetry breaking either from

- extra bifundamental scalar field [Erlich-Katz-Son-Stephanov 2005] (HW1), or
- through different boundary conditions for vector/axial-vector fields at z_0 [Hirn-Sanz 2005] (HW2), [Sakai-Sugimoto 2004] (SS)

Vector meson dominance (VMD) naturally built in:
photons couple through *bulk-to-boundary propagators of vector gauge fields* whose normalizable modes give (infinite tower of!) vector mesons (ρ, ω, ϕ, ...)

Anomalous TFFs from holographic QCD

Flavor anomalies follow uniquely from 5-dimensional Chern-Simons term:

$$S^L_{\rm CS} - S^R_{\rm CS}, \quad S_{\rm CS} = \frac{N_c}{24\pi^2}\int \mathrm{tr}\left(\mathcal{B}\mathcal{F}^2 - \frac{i}{2}\mathcal{B}^3\mathcal{F} - \frac{1}{10}\mathcal{B}^5\right).$$

with infinite tower of vector and axial-vector mesons contained in 5-dimensional $SU(N_f)_L \times SU(N_f)_R$ gauge field $\mathcal{B}^{L,R}_M$; Goldstone bosons of χSB in $\mathcal{B}^{L-R}_5$

- Pion transition form factor for $\pi^0 \to \gamma^*\gamma^*$

$$F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2) = -\frac{N_c}{12\pi^2 f_\pi}\int_0^{z_0} dz\, \mathcal{J}(Q_1,z)\mathcal{J}(Q_2,z)\Psi(z),$$

  with *bulk-to-boundary propagator* $\mathcal{J}$ and holographic pion profile Ψ

- The amplitude for axial vector mesons $a^{(n)}_\mu$ decaying into two virtual photons following from the Chern-Simons action has the form

$$\mathcal{M}^a = i\frac{N_c}{4\pi^2}\mathrm{tr}(\mathcal{Q}^2 t^a)\,\epsilon^\mu_{(1)}\epsilon^\nu_{(2)}\epsilon^{*\rho}_A\epsilon_{\mu\nu\rho\sigma}\left[q^\sigma_{(2)}Q_1^2 A_n(Q_1^2,Q_2^2) - q^\sigma_{(1)}Q_2^2 A_n(Q_2^2,Q_1^2)\right],$$

  where

$$A_n(Q_1^2,Q_2^2) \quad = \frac{2g_5}{Q_1^2}\int_0^{z_0} dz \left[\frac{d}{dz}\mathcal{J}(Q_1,z)\right]\mathcal{J}(Q_2,z)\psi^A_n(z), \quad n = 1,\dots,\infty$$

  - **Landau-Yang theorem** (AV$\to \gamma\gamma$ is forbidden) realized by $\mathcal{J}'(Q,z) = 0$ for $Q^2 = 0$

Short distance constraints on TFFs

Crucially, hQCD models with asymptotic AdS$_5$ geometry reproduce
asymptotic momentum dependence of LCE [Brodsky-Lepage 1979-81]
(HW1 model exactly with $g_5 = 2\pi$; HW2 model only at 62%)

- **Pseudoscalars** [Grigoryan & Radyushkin, PRD76,77,78 (2007-8)]:

$$F_{\pi^0\gamma^*\gamma^*}(Q_1^2,Q_2^2) \quad\to\quad \frac{2f_\pi}{Q^2}\sqrt{1-w^2}\int_0^\infty d\xi\,\xi^3 K_1(\xi\sqrt{1+w})K_1(\xi\sqrt{1-w})$$
$$= \frac{2f_\pi}{Q^2}\left[\frac{1}{w^2}-\frac{1-w^2}{2w^3}\ln\frac{1+w}{1-w}\right],$$

with $Q^2 = \frac{1}{2}(Q_1^2+Q_2^2) \to \infty$, $w = (Q_1^2-Q_2^2)/(Q_1^2+Q_2^2)$,
corresponding to asymptotic behavior

$$F^\infty(Q^2,0) = \frac{2f_\pi}{Q^2}, \qquad F^\infty(Q^2,Q^2) = \frac{2f_\pi}{3Q^2}.$$

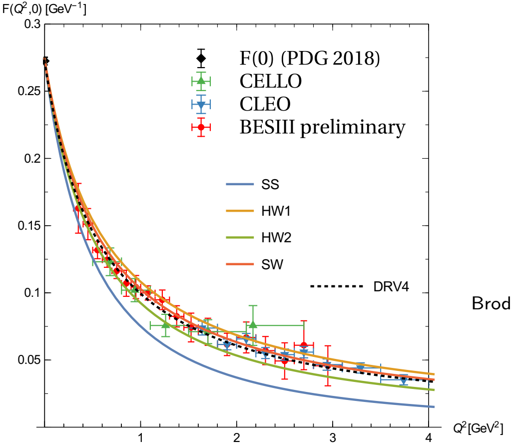
- **Axial vector mesons** [J. Leutgeb & AR, 1912.01596]
(confirmed by pQCD result of Hoferichter & Stoffer 2004.06127):

$$A_n(Q_1^2,Q_2^2) \to \frac{12\pi^2 F_n^A}{N_c Q^4}\frac{1}{w^4}\left[w(3-2w)+\frac{1}{2}(w+3)(1-w)\ln\frac{1-w}{1+w}\right]$$

Holographic pion TFF and experimental data

Comparison with single-virtual TFF from CELLO, CLEO, and BESIII (preliminary):
(chiral hQCD models fitted to match f_π and m_ρ — only 2 free low-energy parameters)

[Leutgeb, Mager, AR, PRD100 (2019)]



Brodsky-Lepage limit:

HW1:	100%
HW2:	62%
SW:	89%
SS:	0%

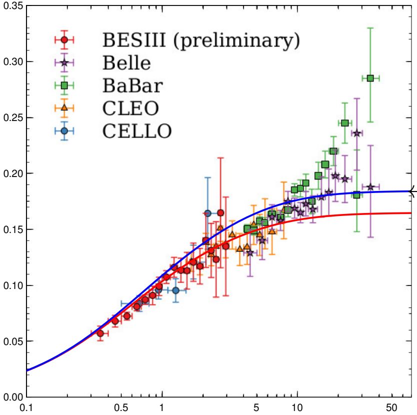
data compilation from Danilkin, Redmer & Vanderhaeghen, 1901.10346

Sakai-Sugimoto model (SS) only good at low Q^2

Holographic pion TFF and experimental data

Comparison with single-virtual TFF from CELLO, CLEO, and BESIII (preliminary):
HW1m: HW1 with quark masses $m_u = m_d$ for two different g_5: [LMR, 2211.16562]

$Q^2 F_{\pi^0\gamma^*\gamma}(Q^2, 0)$ [GeV]



Q^2 [GeV^2]

Brodsky-Lepage limit

Blue: OPE-fit of VV correl. (100% SDC)

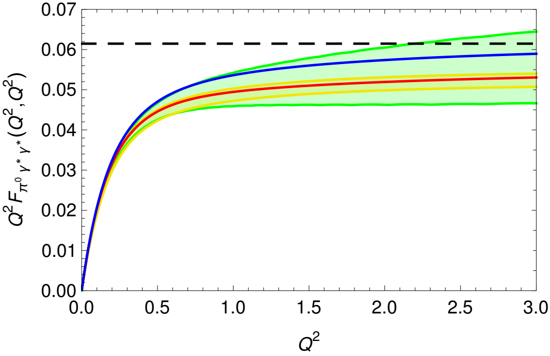
Red: F_ρ-fit ($\approx$ 90% SDC)

NB: NLO QCD results for TFFs at $\sim 90\%$ of asymptotic value when pQCD becomes applicable
HVP in HW1: far too small with OPE fit, increases to within 5% of dispersive result with F_ρ-fit!
[J. Leutgeb, AR, M. Stadlbauer, PRD105, 2203.16508]

Comparison of doubly virtual pion TFF

a_μ^{HLBL,π^0} needs TFF for all virtualities

Comparison of HW1m with data-driven and lattice approach:



– – : OPE limit

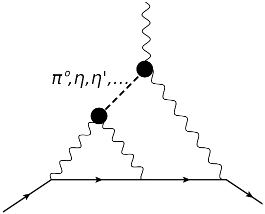
Green: dispersive approach [Hoferichter et al., 1808.04823]

Yellow: lattice result of Gérardin et al., 1903.09471

Blue: HW1m model [LMR, 2211.16562] OPE-fit of vector correlator (100% SDC)

Red: HW1m model [LMR, 2211.16562] F_ρ-fit ($\approx$ 90% SDC)

Holographic pion TFF and $a_\mu^{\pi^0}$ predictions



method/model	$a_\mu^{\pi^0} \times 10^{11}$
LMD+V [Nyffeler 2016]	72 ± 12
lattice (Mainz, 2016)	65 ± 8
lattice (Mainz, 2019)	60 ± 4
lattice (Mainz, 2019)+exp.data	62 ± 2
Danilkin et al. (DRV,2019)	56 ± 2
dispersive [WP 2020]	$63.0^{+2.7}_{-2.1}$
hQCD (HW) [LR 2021]	63.6 ± 3.0

(hQCD error estimate: spread of different models)

hQCD agrees well with data-driven (dispersive) evaluations and lattice QCD results

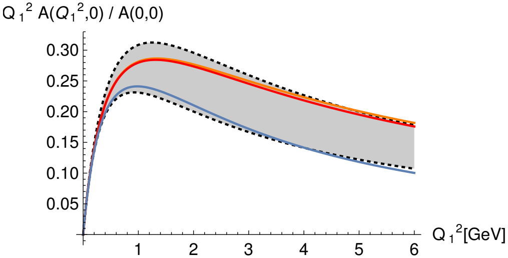
$\Rightarrow$ interesting to evaluate also **axial-vector contributions** where only simple hadronic models w/o correct asymptotics have been used so far

recall: 100% error in 2020 White Paper for assumed contribution of $6(6) \times 10^{-11}$!

Holographic TFFs for axial vector mesons vs. experiments

Shape of single-virtual axial TFF: [J. Leutgeb & AR, 1912.01596]

dipole fit of L3 data for $f_1(1285)$ (gray band) vs. SS, HW1, and HW2 models:



Magnitude:

hQCD results:	HW1	HW2
$\lvert A(0,0)\rvert$ [GeV^{-2}]	21.04	16.63

roughly right ballpark compared to experimental data:

$$A(0,0)^{\mathrm{L3\,exp.}}_{f_1(1285)} = 16.6(1.5)\,\mathrm{GeV}^{-2}; \qquad A(0,0)_{a_1(1230)} = 19.3(5.0)\,\mathrm{GeV}^{-2}$$

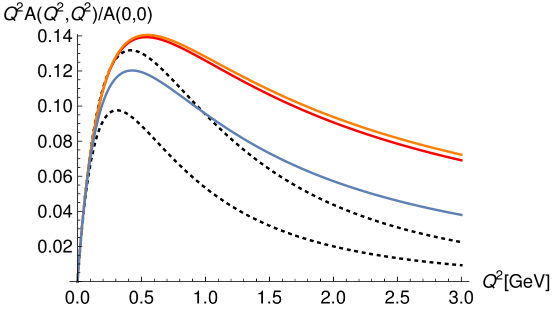
Roig & Sanchez-Puertas, 1910.02881:

Double-virtual axial vector meson TFF

Holographic results of SS, HW1, and HW2 models quite different than

symmetric dipole model $\frac{A^{\rm PV}(Q_1^2,Q_2^2)}{A(0,0)} = \frac{1}{(1+Q_1^2/\Lambda_D^2)^2(1+Q_2^2/\Lambda_D^2)^2}$ (dashed lines)

used by Pauk & Vanderhaeghen [1401.0832] in their calculation of $a_\mu^{f_1,f_1'}$
which is main basis for AV estimate by Muon g-2 Theory Initiative



$\Rightarrow a_\mu$ contribution significantly larger even with same $A(0,0)$

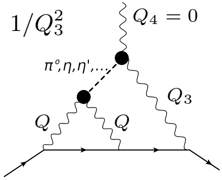
Moreover: *excited* axial vector mesons and their doubly virtual asymptotics relevant for MV short-distance constraint

Melnikov-Vainshtein short-distance constraint

Melnikov and Vainshtein [hep-ph/0312226, PRD70(2004)]:
nonrenormalization theorem for axial anomaly implies
short-distance constraint for 4-photon-amplitude (in BTT basis w/ 54 structure functions):

$$\lim_{Q_3\to\infty}\lim_{Q\to\infty} Q^2 Q_3^2 \bar{\Pi}_1(Q,Q,Q_3) = -\frac{2}{3\pi^2}$$

each single meson exchange contribution gives 0
because propagator $\sim 1/Q_3^2$ and the two form factors $\sim 1/Q^2$ and $1/Q_3^2$



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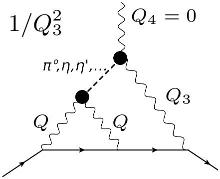

MV model: MV-SDC satisfied by replacing
external TFF by constant on-shell value, leading to
significant (almost +40%) increase of $a_\mu^{\pi^0,\eta,\eta'}$ by 38×10^{-11}

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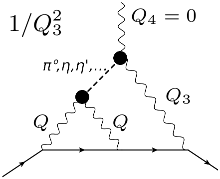

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external TFF by constant on-shell value, leading to
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WP estimate for MV-SDC based on Regge model of infinite tower of excited PS states
constructed to saturate MV-SDC with $\Delta a_\mu^{\mathrm{PS}} = 13(6) \times 10^{-11}$ [Colangelo et al., 1910.11881]

But: Excited PS states decouple in chiral large-N limit

Axial vector contributions to MV-SDC

hQCD comes with infinite tower of axial vector mesons,

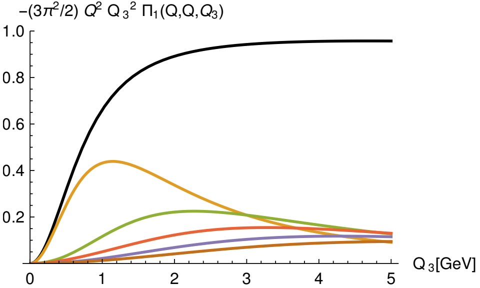
MV-SDC satisfied upon complete summation [Leutgeb & AR, 1912.01596]

independently also by [Cappiello, Catà, D'Ambrosio et al., 1912.02779]

large $Q = 50\text{GeV}$ and increasing $Q_3 \ll Q$:

black line: infinite sum

colored lines: first 5 axial vector modes



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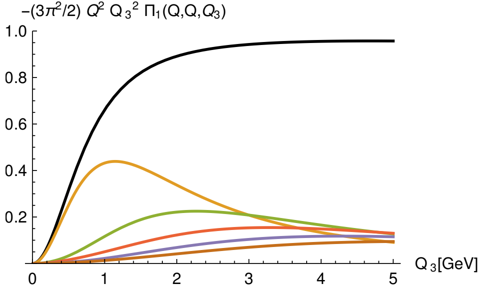

HW1 model with massive quarks [Leutgeb & AR, PRD104, 2108.12345]:

MV-SDC still completely satisfied through tower of axial-vector mesons;

tower of excited massive pions gives subleading contribution $\propto \ln(Q_3^2)/Q_3^4 Q^2$

Katz-Schwartz: HW1 with $m_s > m_{u,d}$ and U(1)$_A$ anomaly

Upgrade [Leutgeb, Mager, AR, 2211.16562]:

HW1m with 2+1 massive quarks plus U(1)$_A$ anomaly based on Katz-Schwartz model

E. Katz & M. Schwartz, *An Eta Primer: Solving the U(1) problem with AdS/QCD*, JHEP 08 (2007) 077

who proposed hard-wall AdS/QCD Lagrangian including

- besides bifundamental $X \leftrightarrow \bar{q}_i q_j$ with $\langle X_{ij} \rangle = M_{ij} z + \Sigma_{ij} z^3$ and $\Sigma \leftrightarrow \langle \bar{q}_i q_j \rangle$
- also complex scalar $Y \leftrightarrow \alpha(GG + iG\tilde{G})$ with

$$\mathcal{L} \supset \kappa Y^{N_f} \det(X)$$

accounting for U(1)$_A$ anomalous Ward identities

- essentially independent of κ as long as $\kappa \gg 1$
- only new free parameter: gluon condensate $\Xi \leftrightarrow \langle G^2 \rangle$ in $\langle Y \rangle = C + \Xi z^4$

  with OPE $\Rightarrow C = \frac{\sqrt{2N_f}}{2\pi^2} \alpha_s$, $\alpha_s \to 1/\beta_0 \ln(\Lambda_{QCD} z)$, $\Lambda_{QCD} \to z_0^{-1}$

  $$\Rightarrow \Xi z^4 \to \Xi z^4 [\ln^2(\Lambda_{QCD} z) + \ldots]$$

- realizes Witten-Veneziano mechanism for $m_{\eta'}$
- phase of $Y \leftrightarrow$ pseudoscalar glueball mixing with $\eta^{(\prime)}$

Massive HW1+U(1)$_A$ Model [LMR, 2211.16562]

$N_f = 2 + 1$ with $m_s \approx 24.3 m_{u,d}$

v1: Tuning of gluon condensate Ξ (neglected by KS) $\rightarrow$ virtually exact fit of m_η and $m_{\eta'}$

Two variants of UV fits:

v1(OPE-fit): $g_5 = 2\pi$ such that UV constraints on TFF satisfied to 100%

v1(F_ρ-fit): $g_5 = 5.94$ such that f_ρ is fitted ($\approx 90\%$ of asymptotic SDCs)

v1(OPE fit)

	m [MeV]	m–$m^{\rm exp}$ [%]	f^8	f^0	f_G	$\lvert F(0,0)\rvert$	$F - F^{\rm exp}$
π^0	135	(input)	0	0	0	0.277	
η	557	+1.7%	0.101	0.027	-0.030	0.275	+1(2)%
η'	950	-0.8%	-0.0385	0.113	-0.077	0.340	-0(2)%
G/η''	1992	?	-0.027	0.005	0.053	0.116	
	m [MeV]	m–$m^{\rm exp}$ [%]	F_A^8/m_A	F_A^0/m_A	$A^8(0,0)$	$A^{0\vee 3}(0,0)$	
a_1	1363	+11%	0	0	0	20.96	
f_1	1481	+15%	0.176	0.0365	20.77	3.857	
f_1'	1810	+27%	-0.030	0.201	-3.842	20.07	

gluon condensate parameter $|\Xi| = 0.01051\ \mathrm{GeV}^4$

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v1(F_ρ-fit): (our current "best guess" regarding a_μ)

	m [MeV]	m–$m^{\exp}$ [%]	f^8	f^0	f_G	$\lvert F(0,0)\rvert$	$F - F^{\exp}$
π^0	135	(input)	0	0	0	0.276	
η	561	+2.4%	0.103	0.030	-0.031	0.268	+2(2)%
η'	947	-1.1%	-0.039	0.121	-0.082	0.313	-8(2)%
G/η''	1943	?	-0.030	0.0076	0.048	0.111	
	m [MeV]	m–$m^{\exp}$ [%]	F_A^8/m_A	F_A^0/m_A	$A^8(0,0)$	$A^{0\vee 3}(0,0)$	
a_1	1278	+4%	0	0	0	19.46	
f_1	1410	+10%	0.176	0.029	19.58	2.69	
f_1'	1820	+28%	-0.017	0.219	-2.56	19.00	

gluon condensate parameter $|\Xi| = 0.01416\ \mathrm{GeV}^4$

PS: $f^{8,0}$'s within a few % of χPT values

AV: f_1-f_1' mixing angle $\phi_f - \phi_f^{\text{ideal}}$ about twice as large as indicated by L3 data

(ϕ_f strongly dependent on Ξ; but sum $a_\mu^{f_1} + a_\mu^{f_1'}$ rather insensitive)

a_μ in HW1+U(1)$_A$ Model [LMR, 2211.16562]

$a_\mu^{\cdots} \times 10^{11}$	v1(OPE fit, 100% SDC)	v1(F_ρ-fit)	WP
π^0	66.1	63.4	$62.6^{+3.0}_{-2.5}$
η	19.3	17.6	16.3(1.4)
η'	16.9	14.9	14.5(1.9)
$PSGB/\eta''$	0.2	0.2	
$\sum_{PS^*}$	1.6	1.4	
PS poles total	104	97.5	93.8(4.0)

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a_1	7.8	7.1	
$f_1 + f_1'$	20.0	17.9	
$\sum_{a_1^*}$	2.5	2.6	
$\sum_{f_1^{(\prime)*}}$	4.0	3.5	
AV+LSDC total	34.3	31.1	21(16)
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New dispersive result [Hoferichter, Stoffer, Zillinger, 2412.00178+2412.00190]:

- higher by $+9.9 \times 10^{-11}$
- low-energy ($Q_i < 1.5$ GeV) contribution of axials: $14.2(1.6) \times 10^{-11}$

hQCD v1(F_ρ-fit): $\underline{13.8 \times 10^{-11}}$

New issue: Tensor mesons

Despite prominence of tensor mesons $f_2(1270), a_2(1320)$ in $\gamma\gamma$ collisions, until recently considered almost negligible for a_μ:
[Danilkin, Vanderhaeghen, 1611.04646]: $a_\mu^{f_2,a_2} = +0.64(13) \times 10^{-11}$

But [Hoferichter, Stoffer, Zillinger, 2412.00178+2412.00190]:
with new framework which avoids spurious kinematical singularities
similar simple Quark Model ansatz for tensor TFFs gives $a_\mu^{f_2,a_2}|_{\mathrm{IR}} = -2.5(8) \times 10^{-11}$
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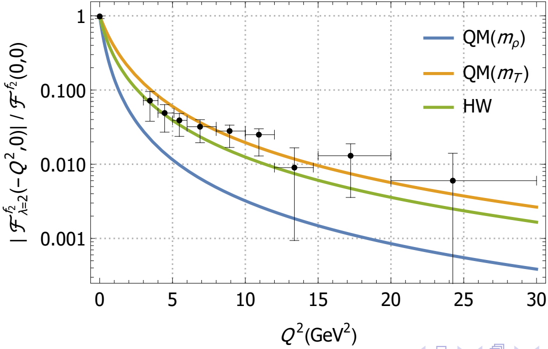
Hard-wall AdS/QCD [Cappiello, Leutgeb, Mager, AR, 2501.09699+2501.19293]:



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Hard-wall AdS/QCD [Cappiello, Leutgeb, Mager, AR, 2501.09699+2501.19293]:
only 1 additional free parameter (tensor normalization)

- when matched to pQCD tensor-tensor correlator OPE:
  (underestimates experimental $f_2 \to \gamma\gamma$)

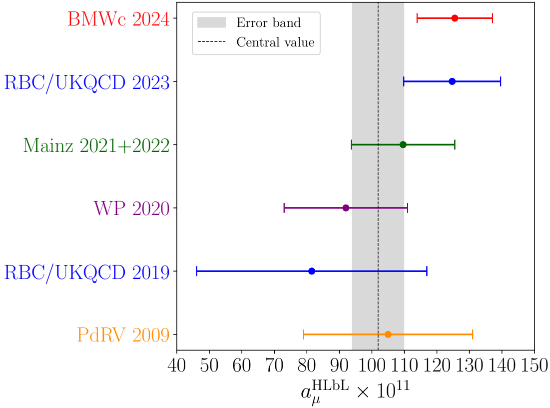
$$a_\mu^{T_1,\mathrm{pole}}|_{\mathrm{IR}} = +2.1\times 10^{-11}, \qquad a_\mu^{T_1,\mathrm{full}}|_{\mathrm{IR}} = +5.4\times 10^{-11}, \qquad a_\mu^{T}|_{\mathrm{IR}} = +6.2\times 10^{-11}$$

- when matched using symmetric longitudinal SDC, where axials alone give only 81%:
  (agrees with experimental $f_2 \to \gamma\gamma$!)

$$a_\mu^{T_1,\mathrm{pole}}|_{\mathrm{IR}} = +3.3\times 10^{-11}, \qquad a_\mu^{T_1,\mathrm{full}}|_{\mathrm{IR}} = +8.3\times 10^{-11}, \qquad a_\mu^{T}|_{\mathrm{IR}} = +9.5\times 10^{-11}$$

New issue: Tensor mesons

Would remove tension between new dispersive result of [Hoferichter, Stoffer, Zillinger, 2412.00178+2412.00190] with most recent HLBL lattice evaluations



with HW AdS/QCD tensor results in place of QM(m_ρ):

$$a_\mu^{\mathrm{HLbL}} = (102 \to 113) \times 10^{-11}$$

Conclusions for a_μ^{HLBL}

- hQCD is not QCD, but sophisticated toy model that can give clues on
  – how short-distance constraints can be implemented at the hadronic level
    - important fundamental role of axial-vector mesons $\leftrightarrow$ anomaly

  – semi-quantitative estimates of the ballparks to be expected
    - pion contribution from hQCD in perfect agreement with data-driven approach
    - with finite quark masses and WV η^0 mass:
      good agreement with η, η' WP results, predicted
      axial-vector contributions greater than estimated previously

      $$a_\mu^{\mathrm{AV+MVSDC}} = \mathbf{31.1}^{+\mathbf{3.2}}_{-\mathbf{4.1}} \times 10^{-11} \qquad \text{for HW1m+U(1)}_A \text{ (LMR)}$$

      but since Dec 2024 confirmed by dispersive approach
    - New issue:
      large positive contribution of tensor mesons in hQCD
      $\rightarrow$ would remove tension between recent dispersive and lattice results
      $\Rightarrow$ need doubly virtual TFF data to validate/falsify