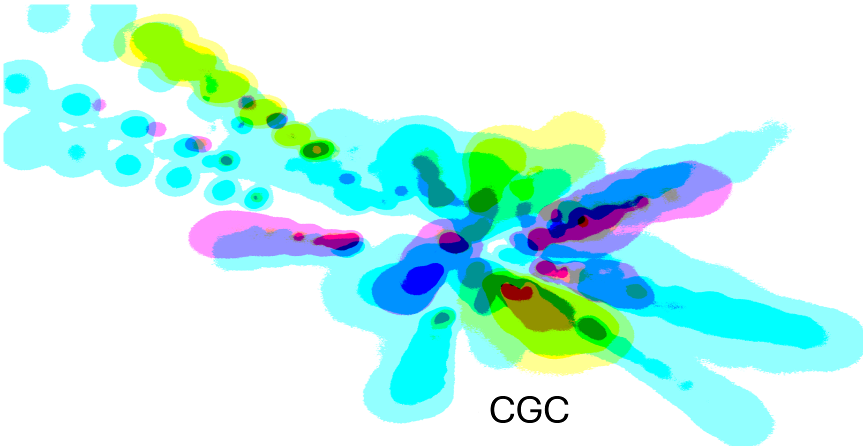
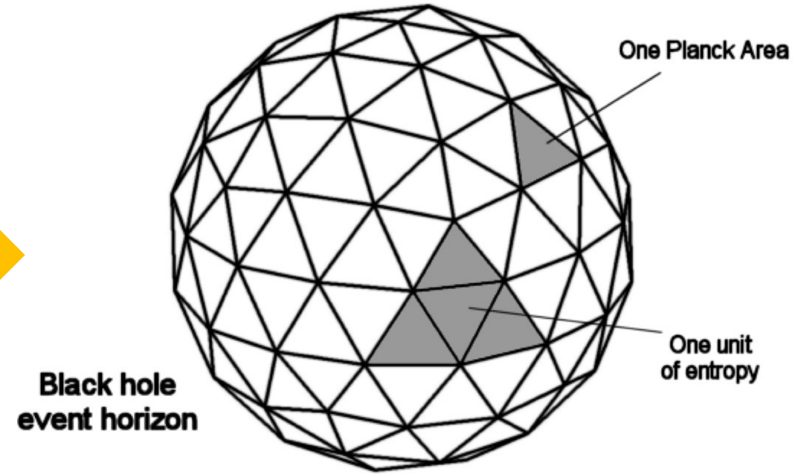
Wee partons in QCD and gravity: from amplitudes to shockwaves





Raju Venugopalan

BNL & CFNS, Stony Brook

GGI workshop on "Probing the CGC and QCD matter at hadron colliders (Iancu Fest)", March 24-28, 2025

Double Copy: gluon → gravitational amplitudes

$$
\begin{tikzcd}
\text{QCD at high occupancy} \arrow[d, "\text{Strong field semi-classical double copy}"'] & \text{perturbative QCD} \arrow[l] \arrow[d, "\text{BCJ double copy}"] \\
\text{Gravity at high occupancy} & \text{perturbative gravity} \arrow[l]
\end{tikzcd}
$$

Monteiro,O'Connell,White, arXIv:1410.0239
Goldberger, Ridgeway, arXiv:1611.03493

Bern, Carrasco, Johannson,
arXiv: 1004.0476

Double Copy: gluon → gravitational amplitudes

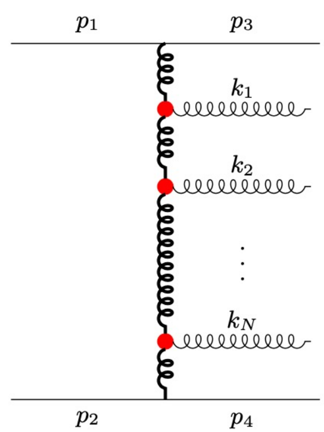
The BCJ double copy has been remarkably successful in computing the inspiral potential of binary black holes to high powers in a post-Minkowskian expansion in GR – up to $O(G^4)$

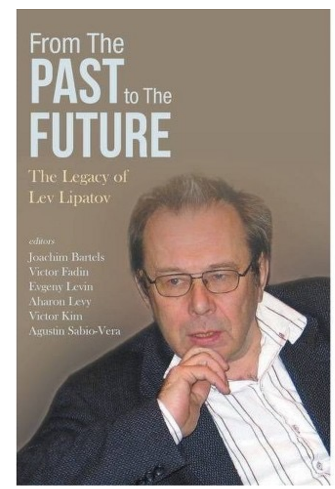
Bern et al, PRL 126 (2021) 17, 171601

Can we anticipate the same for ultrarelativistic regimes of BH mergers or close BH encounters?

As first emphasized by Weinberg, QFT/EFT ideas are powerful in thinking about gravity (besides the geometrical picture)
– and this is true for understanding the high occupancy (BH) regime as well…

A double copy of $2 \to N$ gluon → graviton scattering was discovered by Lipatov more than 40 years ago





Lipatov, PLB 116B (1982); JETP 82 (1982)

$2 \to N + 2$ amplitudes in trans-Planckian gravitation scattering: from wee partons to Black Holes

HIGH-ENERGY SCATTERING IN QCD AND IN QUANTUM GRAVITY AND TWO-DIMENSIONAL FIELD THEORIES

L.N. LIPATOV*

We construct effective actions describing high-energy processes in QCD and in quantum gravity with intermediate particles (gluons and gravitons) having the multi-Regge kinematics. The S-matrix for these effective scalar field models contains the results of the leading logarithmic approximation and is unitary. It can be expressed in terms of correlation functions for two field theories acting in longitudinal and transverse two-dimensional subspaces.

Effective action and all-order gravitational eikonal at planckian energies

AMATI,CIAFALONI,VENEZIANO **NPB403 (1993)707**

Building on previous work by us and by Lipatov, we present an effective action approach to the resummation of all semiclassical (i.e. $O(\hbar^{-1})$) contributions to the scattering phase arising in high-energy gravitational collisions. By using an infrared-safe expression for Lipatov's effective action, we derive an eikonal form of the scattering matrix and check that the superstring amplitude result is reproduced at first order in the expansion parameter R^2/b^2, where R, b are the gravitational radius and the impact parameter, respectively. If rescattering of produced gravitons is neglected, the longitudinal coordinate dependence can be explicitly factored out and exhibits the characteristics of a shock-wave metric while the transverse dynamics is described by a reduced two-dimensional effective action. Singular behaviours in the latter, signalling black hole formation, can be looked for.

The World as a Hologram

LEONARD SUSSKIND

Wee partons, by contrast, are not subject to Lorentz contraction. This implies that in the Feynman Bjorken model, the halo of wee partons eternally "floats" above the horizon at a distance of order $10^{-13}cm$ as it transversley spreads. The remaining valence partons carry the various currents which contract onto the horizon as in the Einstein Lorentz case.

By contrast, both the holographic theory and string theory require all partons to be wee. No Lorentz contraction takes place and the entire structure of the string floats on the stretched horizon. I have explained in previous articles how this behavior prevents the accumulation of arbitrarily large quantities of information near the horizon of a black hole. Thus we are led full circle back to Bekenstein's principle that black holes bound the entropy of a region of space to be proportional to its area.

J.Math.Phys. 36 (1995) 6377; 4018 cites !

In Acknowledgements:

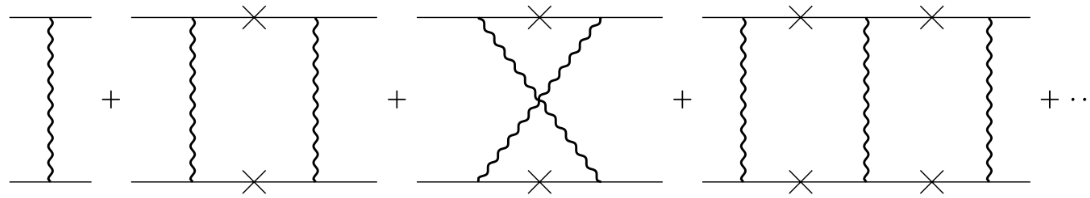
Finally I benefitted from discussions with Kenneth Wilson and Robert Perry, about boosts and renormalization fixed points in light front quantum mechanics and Lev Lipatov about high energy scattering.

30+ years of work by ACV et al. exploring gravitational shockwave collisions in 2-D EFT

Summarized in Di Vecchia, Heissenberg, Russo, Veneziano, *Phys.Rept.* 1083 (2024) 1

From QCD to gravity in Regge asymptotics: reggeization

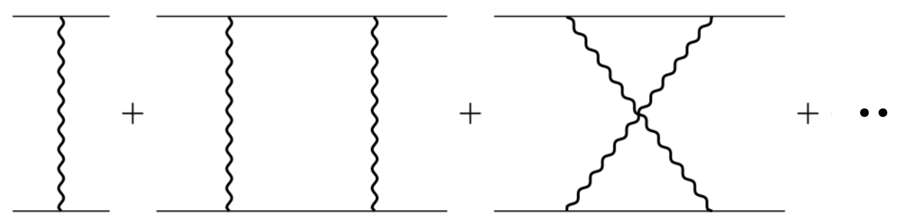
In GR, at large impact parameters, dominant contribution is eikonal multiple scattering (Einstein deflection)



$$i\mathcal{M}_{\text{Eik}} = 2s \int d^2\boldsymbol{b}\; e^{-i\mathbf{q}\cdot\mathbf{b}} \left(e^{i\chi(\boldsymbol{b},s)} - 1\right) \quad \text{with} \quad \chi(\boldsymbol{b},s) = \frac{\kappa^2 s}{2}\int \frac{d^2\boldsymbol{k}}{(2\pi)^2}\frac{1}{\boldsymbol{k}^2} e^{i\boldsymbol{b}\cdot\boldsymbol{k}}$$

$$\kappa^2 = 8\pi G = \frac{8\pi}{M^2_{Planck}}$$

Reggeized (semi-classical) contributions formally suppressed by R_S^2/b^2



$$\mathcal{M}^{(1)} \sim \frac{\kappa^2}{8\pi^2}\left(-i\pi s \log\left(\frac{-t}{\Lambda^2}\right) + t\log\left(\frac{s}{-t}\right)\log\left(\frac{-t}{\Lambda^2}\right)\right)$$

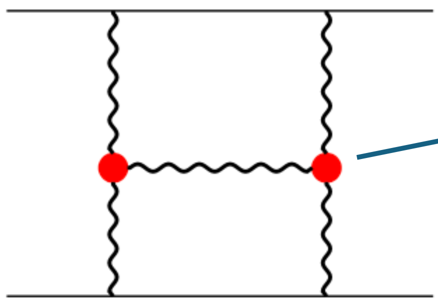
Eikonal Loop

Graviton Regge trajectory: $\alpha(t) = -\kappa^2 t \int \frac{d^2\boldsymbol{k}}{(2\pi)^2}\frac{1}{\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k})^2}\left[(\boldsymbol{k}\cdot(\boldsymbol{q}-\boldsymbol{k}))^2\left(\frac{1}{\boldsymbol{k}^2}+\frac{1}{(\boldsymbol{q}-\boldsymbol{k})^2}\right) - \boldsymbol{q}^2\right], \quad \boldsymbol{q}^2 = -t$

The IR virtual divergence cancels in the inclusive cross-section

Lipatov, PLB 116B (1982); JETP 82 (1982)

From QCD to gravity in Regge asymptotics: Lipatov vertex



H-diagram of Amati, Ciafaloni, Veneziano

Gravitational Lipatov vertex:

$$\Gamma_{\mu\nu}(\boldsymbol{q}_1, \boldsymbol{q}_2) \equiv \frac{1}{2}\underbrace{C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2)C_\nu(\boldsymbol{q}_1, \boldsymbol{q}_2)}_{\text{Double copy of QCD Lipatov vertex}} - \frac{1}{2}\underbrace{N_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2)N_\nu(\boldsymbol{q}_1, \boldsymbol{q}_2)}_{\text{Double copy of QED Bremsstrahlung vertex}}$$

$$N_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) = \sqrt{\boldsymbol{q}_1^2 \boldsymbol{q}_2^2}\left(\frac{p_{1\mu}}{p_1 \cdot k} - \frac{p_{2\mu}}{p_2 \cdot k}\right)$$

S-matrix power counting a la ACV:

$$S = e^{2i(\delta_0 + \delta_1 + \delta_2 + \cdots)} \qquad \delta_0 = Gs \log\left(\frac{L}{b}\right), \qquad \delta_1 = \frac{6G^2 s}{\pi b^2}\log s, \qquad \delta_2 = \frac{2G^3 s^2}{b^2}\left[1 + \frac{i}{\pi}\log s\left(\log\frac{L^2}{b^2} + 2\right)\right]$$

Leading Eikonal term (real)

Sub-leading quantum gravity correction $\sim \frac{l_P^2}{b^2}$

Sub-leading loop contribution $\sim \frac{R_S^2}{b^2}$ - includes absorptive piece

$$\boxed{\delta_2 \gg \ \delta_1 \text{ for } R_S \gg \ l_P}$$

The BFKL equation in Einstein gravity

I. Rothstein, M. Saavedra, arXiv:2412.04428
H. Raj, A. Stasto, RV, in preparation

Integral equation derived by Lipatov for the Mellin amplitude: $\mathcal{M}_\ell(t) = \frac{t}{16}\int \frac{d^2\boldsymbol{k}}{(2\pi)^2}\frac{1}{\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k})^2} f_\ell(\boldsymbol{k},\boldsymbol{q})$

$$(\ell - \alpha(\boldsymbol{k}^2) - \alpha((\boldsymbol{q}-\boldsymbol{k})^2)) f_\ell(\boldsymbol{k},\boldsymbol{q}) = 1 + \frac{\kappa^2}{4\pi}\int \frac{d^2\boldsymbol{k}'}{(2\pi)^2}\frac{f_\ell(\boldsymbol{k}',\boldsymbol{q})}{\boldsymbol{k}'^2(\boldsymbol{q}-\boldsymbol{k}')^2}\mathcal{K}_G(\boldsymbol{k},\boldsymbol{k}')$$

$$C^{\mu_i\nu_i}(k_i,k_{i+1})C_{\mu_i\nu_i}(q-k_i,q-k_{i+1})$$

Some comments:

- The solution is IR and UV safe": IR is obvious but UV is subtler – but works because pure Einstein is 1-loop renormalizable (t'Hooft+Veltman)
- The large double logs ($\alpha_{GR} Ln^2$(s)) we have been discussing are included in this resummation (this BFKL is "kinematically constrained") but not the "CSS" logs (GGFL: Gorshkov,Gribov,Frolov,Lipatov, rather, + Bj+T.T.Wu)

The BFKL equation in Einstein gravity

I. Rothstein, M. Saavedra, arXiv:2412.04428
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$$C^{\mu_i\nu_i}(k_i, k_{i+1}) C_{\mu_i\nu_i}(q-k_i, q-k_{i+1})$$

Some comments:

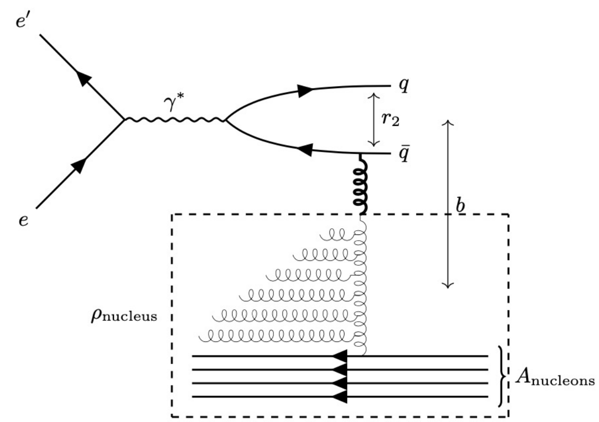
- ❖ The amplitude has the structure $A_{2\to 2} = \kappa^2 \frac{s^2}{t}\frac{1}{a\xi}$ I_1 (aξ) where $\xi = Ln\left(\frac{s}{t}\right)$ and a $= \frac{1}{2\pi}\sqrt{\kappa^2(-t)}$
- \- corresponding to a branch point in the t-j plane j ~ 2+ $\kappa^2\sqrt{-t}$

(DL's in impact factor were taken into account by Bartels, Lipatov, Sabio-Vera, arXiv:1208.3423)

- ❖ *Very interestingly, the soft limit of the Lipatov vertex gives the ultrarelativistic limit of the Weinberg theorem (for radiative amplitude of soft gravitons)*

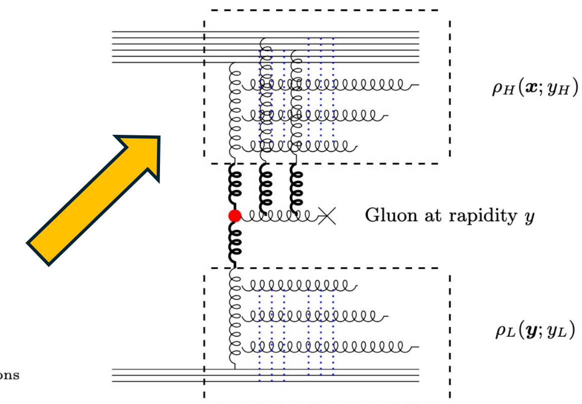
From amplitudes to shockwaves in QCD and GR

As one approaches the dense field regime
In QCD, the shockwave/CGC formalism
is more efficient (and nearly as accurate)
as the amplitude formalism in
computing inclusive final states



Clear map to reggeized propagators and
Lipatov vertices

Hentschinski, 1802.06755



In dilute-dilute and dilute-dense
Yang-Mills shockwave collisions,
recover QCD Lipatov vertex

Blaizot,Gelis,RV, hep-ph/9402256
Gelis, Mehtar-Tani, hep-ph/0512079

The same may be true in general relativity

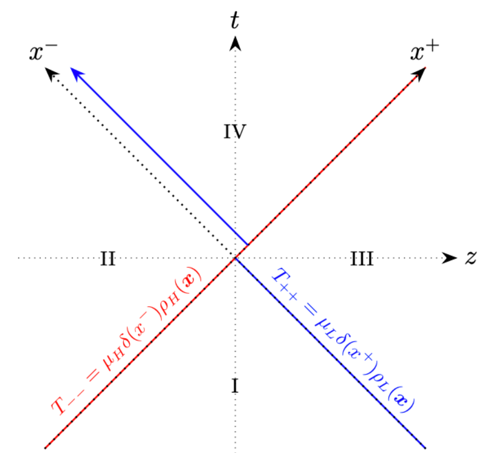
Himanshu Raj, RV: arXiv, 2311.03463, 2312.035407, 2312.11652, 2406.10483, 100+ page review in preparation
Raj, Stasto, RV, in preparation

Shockwave collisions in general relativity: single shock background

Aichelburg-Sexl shockwave metric of a shockwave

$$ds^2 = 2dx^+dx^- - \delta_{ij}dx^i dx^j + f(x^-, \boldsymbol{x})\left(dx^-\right)^2$$

with $f(x^-, \boldsymbol{x}) = 2\kappa^2 \mu_H \delta(x^-) \frac{\rho_H(\boldsymbol{x})}{\Box_\perp} = \frac{\kappa^2}{\pi}\mu_H \delta(x^-) \int d^2\boldsymbol{y}\ \ln \Lambda |\boldsymbol{x} - \boldsymbol{y}| \rho_H(\boldsymbol{y})$



$\mu_H = m_H\,\gamma$ = fixed for $\gamma \to \infty$

$\kappa^2 = 8\,\pi\,G$

Linearizing around the metric $g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa\, h_{\mu\nu}$

Fix light cone gauge $h_{\mu+}$=0. Find solution: $h_{ij}(x^+, x^-, \boldsymbol{x}) = V(x^-, \boldsymbol{x}) h_{ij}(x^+, x^- = x_0^-, \boldsymbol{x})$

with the gravitational Wilson line $V(x^-, \boldsymbol{x}) \equiv \exp\left(\frac{1}{2}\int_{x_0^-}^{x^-} dz^- \bar{g}_{--}(z^-, \boldsymbol{x})\,\partial_+\right)$

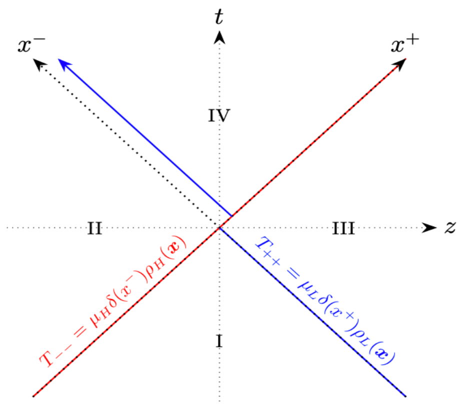
Exactly analogous to the QCD case with $A_- \to g_{--}$ and $T^a \to \partial_+$

Shapiro time-delay

Melville,Nachulich,Schnitzer,White,
arXiv:1306.6019

Shockwave collisions in general relativity: dilute-dilute approximation

Now consider interaction of two shockwaves: ρ_L with ρ_H



$$T_{\mu\nu} = \delta_{\mu-}\delta_{\nu-}\mu_H \delta\left(x^-\right) \rho_H(\boldsymbol{x}) + \delta_{\mu+}\delta_{\nu+}\mu_L \delta\left(x^+\right) \rho_L(\boldsymbol{x})$$

Solve for metric in region IV – forward lightcone around ρ_H

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \qquad \bar{g}_{--} = 2\kappa\mu_H \delta\left(x^-\right) \frac{\rho_H(\boldsymbol{x})}{\Box_\perp}$$

$\mu_H = m_H\, \gamma$ = fixed for $\gamma \rightarrow \infty$

$\kappa^2 = 8\, \pi\, G$

We decompose the perturbation $h_{\mu\nu}$ due to ρ_L into a term linear in ρ_L and one bi-linear in $\rho_L \rho_H$

Linearized Einstein's equations in light-cone gauge ($h_{+\mu}$=0) take the form

$$\bar{g}_{--}\partial_+^2 \tilde{h}_{ij} - \Box \tilde{h}_{ij} = \kappa^2 \left[\left(2\partial_i\partial_j - \Box_\perp \delta_{ij}\right) \frac{1}{\partial_+^2} T_{++} + 2T_{ij} - \delta_{ij} T - \frac{2}{\partial_+} \left(\partial_i T_{+j} + \partial_j T_{+i} - \delta_{ij}\partial_k T_{+k}\right) \right]$$

$\tilde{h}_{ij} \equiv h_{ij} - \frac{1}{2}\delta_{ij} h$ where $h = \delta_{ij} h_{ij}$

Raj, RV: arXiv, 2311.03463

Shockwave collisions in general relativity: geodesics

Unlike the QCD case, the sub-Eikonal contributions T_{+i}, T_{ij} are required for consistency of equations of motion

Since these are not uniquely fixed by energy-momentum conservation, the dynamics of the sources is needed to fix this. In the point particle approximation,

$$T^{\mu\nu}(x) = \frac{\mu_L}{\sqrt{-\bar{g}}} \int_{-\infty}^{\infty} d\lambda \, \dot{X}^\mu \dot{X}^\nu \, \delta^{(4)}(x - X(\lambda))$$

The solution of the corresponding null geodesic equations $\ddot{X}^\mu + \Gamma^\mu_{\nu\rho} \dot{X}^\nu \dot{X}^\rho = 0 \, , \qquad g_{\nu\rho} \dot{X}^\nu \dot{X}^\rho = 0$

In the shockwave background, given by $X^- = \lambda \, , \quad X^i = b^i - \kappa^2 \mu_H X^- \Theta(X^-) \frac{\partial_i \rho_H(\boldsymbol{b})}{\Box_\perp}$

$$X^+ = -\kappa^2 \mu_H \Theta(X^-) \frac{\rho_H(\boldsymbol{b})}{\Box_\perp} + \frac{\kappa^4 \mu_H^2}{2} X^- \Theta(X^-) \left(\frac{\partial_i \rho_H(\boldsymbol{b})}{\Box_\perp} \right)^2$$

From the geodesic solutions, we can reconstruct the required components of the stress-energy tensor

Shockwave collisions in general relativity: Lipatov vertex

Solving eqns of motion, taking the Fourier transform, and putting the graviton momenta on-shell, one obtains

Gravitational radiational field

$$\boxed{\tilde{h}_{ij}^{(2)}(k) = \frac{2\kappa^3 \mu_H \mu_L}{k^2 + i\epsilon k^-} \int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2} \underbrace{\Gamma_{ij}(\boldsymbol{q}_1, \boldsymbol{q}_2)}_{\text{Gravitational Lipatov vertex}} \frac{\rho_H}{\boldsymbol{q}_1^2} \frac{\rho_L}{\boldsymbol{q}_2^2}}$$

recovering Lipatov's result! $\Gamma_{\mu\nu}(\boldsymbol{q}_1, \boldsymbol{q}_2) \equiv \frac{1}{2} C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) C_\nu(\boldsymbol{q}_1, \boldsymbol{q}_2) - \frac{1}{2} N_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) N_\nu(\boldsymbol{q}_1, \boldsymbol{q}_2)$

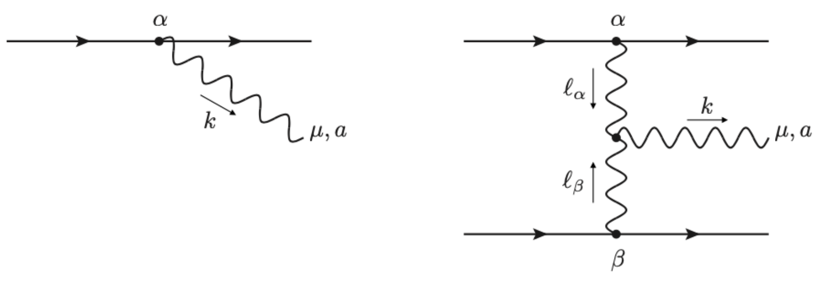
Compare to gauge theory radiation field

$$\boxed{a_i(k) = \frac{g^3}{k^2 + i\epsilon k^-} \int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2} C_i(\boldsymbol{q}_1, \boldsymbol{q}_2) \frac{\rho_H \cdot T}{\boldsymbol{q}_1^2} \frac{\rho_L}{\boldsymbol{q}_2^2}}$$

$$-if^{abc} T_b T_c C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) \xleftrightarrow[\text{CK relation?}]{\text{Is there a}} s\Gamma_{\mu\nu}(\boldsymbol{q}_1, \boldsymbol{q}_2)$$

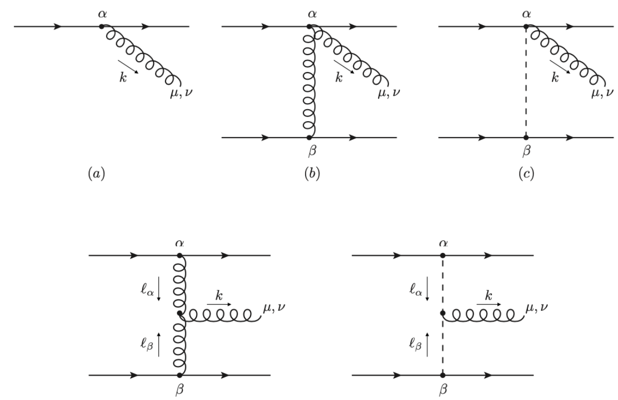
H.Johansson, A.Sabio Vera, E.Serna Campillo, and M.Vaszquez-Mozo, JHEP10,215(2013),arXiv:1307.3106 [hep-th]

Classical color-kinematic duality-I



(a) (b)

From Goldberger, Ridgway
arXiv:1611.03493



(a) (b) (c) (d) (e)

A classical color-kinematic duality between QCD and GR exists but it requires one include sub-eikonal corrections to the QCD Lipatov vertex

For this, require a detailed theory of sources: Yang-Mills+Wong equations for classical color sources c^a :

$$D_\mu F_a^{\mu\nu} = gJ_a^\nu \qquad J_a^\mu(x) = \sum_{\alpha=1,2} \int d\tau c_\alpha^a(\tau) v_\alpha^\mu(\tau) \delta^d \left(x - x_\alpha(\tau)\right)$$

$$\frac{dc^a}{d\tau} = gf^{abc} v^\mu A_\mu^b(x(\tau)) c^c(\tau) \qquad \frac{dp^\mu}{d\tau} = gc^a F_{a\nu}^\mu v^\nu$$

Classical color-kinematic duality-II

Ultrarelativistic limit of Goldberger-Ridgway solution

$$A^{\mu,a}(k) = -\frac{g^3}{k^2}\int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2}\frac{e^{-i\boldsymbol{q}_1\cdot\boldsymbol{b}_1}}{\boldsymbol{q}_1^2}\frac{e^{-i\boldsymbol{q}_2\cdot\boldsymbol{b}_2}}{\boldsymbol{q}_2^2}\Bigg[if^{abc}c_1^b c_2^c\left(-\boldsymbol{q}_1^\mu+\boldsymbol{q}_2^\mu+p_1^\mu\left(\frac{p_2\cdot k}{p_1\cdot p_2}-\frac{\boldsymbol{q}_1^2}{p_1\cdot k}\right)-p_2^\mu\left(\frac{p_1\cdot k}{p_1\cdot p_2}-\frac{\boldsymbol{q}_2^2}{p_2\cdot k}\right)\right)$$

QCD Lipatov vertex

$$+c_1\cdot c_2\left\{\frac{q_1^2c_1^a}{p_1\cdot k}\left(-q_2^\mu+\frac{k\cdot q_2}{k\cdot p_1}p_1^\mu+\frac{k\cdot p_1}{p_1\cdot p_2}p_2^\mu-\frac{k\cdot p_2}{p_1\cdot p_2}p_1^\mu\right)+\frac{q_2^2c_2^a}{p_2\cdot k}\left(-q_1^\mu+\frac{k\cdot q_1}{k\cdot p_2}p_2^\mu+\frac{k\cdot p_2}{p_1\cdot p_2}p_1^\mu-\frac{k\cdot p_1}{p_1\cdot p_2}p_2^\mu\right)\right\}\Bigg]$$

sub-eikonal correction

$1/p_1^+$ $\qquad$ $1/p_2^-$

Sub-Eikonal contributions are not universal – for instance, they depend on the spin of the particles

Classical color-kinematic replacement rule:

$$c_\alpha^a \to p_\alpha^\mu\ ,$$

$$if^{a_1a_2a_3} \to \Gamma^{\nu_1\nu_2\nu_3}(q_1,q_2,q_3) = -\frac{1}{2}\left(\eta^{\nu_1\nu_3}(q_1-q_3)^{\nu_2}+\eta^{\nu_1\nu_2}(q_2-q_1)^{\nu_3}+\eta^{\nu_2\nu_3}(q_3-q_2)^{\nu_1}\right)$$

$$g \to \kappa\ ,$$

Gluon 3-pt vertex with f^{abc} stripped off

Classical color-kinematic duality-III

Ultrarelativistic limit of Goldberger-Ridgway solution

$$A^{\mu,a}(k) = -\frac{g^3}{k^2}\int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2}\frac{e^{-i\boldsymbol{q}_1\cdot\boldsymbol{b}_1}}{\boldsymbol{q}_1^2}\frac{e^{-i\boldsymbol{q}_2\cdot\boldsymbol{b}_2}}{\boldsymbol{q}_2^2}\Bigg[if^{abc}c_1^b c_2^c\left(-\boldsymbol{q}_1^\mu + \boldsymbol{q}_2^\mu + p_1^\mu\left(\frac{p_2\cdot k}{p_1\cdot p_2} - \frac{\boldsymbol{q}_1^2}{p_1\cdot k}\right) - p_2^\mu\left(\frac{p_1\cdot k}{p_1\cdot p_2} - \frac{\boldsymbol{q}_2^2}{p_2\cdot k}\right)\right)$$

QCD Lipatov vertex

$$+\, c_1\cdot c_2\left\{\frac{q_1^2 c_1^a}{p_1\cdot k}\left(-q_2^\mu + \frac{k\cdot q_2}{k\cdot p_1}p_1^\mu + \frac{k\cdot p_1}{p_1\cdot p_2}p_2^\mu - \frac{k\cdot p_2}{p_1\cdot p_2}p_1^\mu\right) + \frac{q_2^2 c_2^a}{p_2\cdot k}\left(-q_1^\mu + \frac{k\cdot q_1}{k\cdot p_2}p_2^\mu + \frac{k\cdot p_2}{p_1\cdot p_2}p_1^\mu - \frac{k\cdot p_1}{p_1\cdot p_2}p_2^\mu\right)\right\}\Bigg]$$

sub-eikonal correction

$1/p_1^+$ $\quad$ $1/p_2^-$

Sub-Eikonal contributions are not universal – for instance, they depend on the spin of the particles

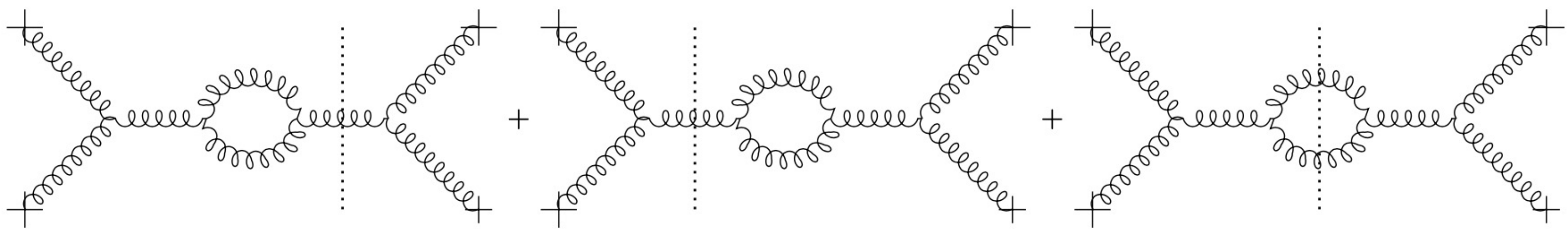
Performing the substitution, one finds the result we obtained by direct computation!

$$A^{\mu\nu}(k) = \frac{\kappa^3 s}{2\,k^2}\int \frac{d^2\boldsymbol{q}_2}{(2\pi)^2}\frac{e^{-i\boldsymbol{q}_1\cdot\boldsymbol{b}_1}}{\boldsymbol{q}_1^2}\frac{e^{-i\boldsymbol{q}_2\cdot\boldsymbol{b}_2}}{\boldsymbol{q}_2^2}\frac{1}{2}\left[C^\mu C^\nu - N^\mu N^\nu + \cancel{k^\mu\left(\frac{p_1^\nu}{p_1\cdot k}\boldsymbol{q}_1^2 + \frac{p_2^\nu}{p_2\cdot k}\boldsymbol{q}_2^2\right)}\right]$$

Unphysical – drops out when contracted with the gravitational polarization tensor

Raj, RV, arXiv: 2312.035407

Shockwave propagators in GR-I



NLO corrections (absorptive piece of three loop diagram in GR counting) in strong background field

Rederive Lipatov's GR "BFKL equation" in shockwave language - in preparation

Key ingredients are retarded shockwave propagators

Shockwave propagators-II

Shockwave propagators
(graviton-reggeized graviton-graviton propagator)

GR Wilson line

Raj, RV, arXiv:2406.10483

$$G_{\mu\nu\rho\sigma}(x,y) = -\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + i\epsilon k^-} \sum_{\lambda\lambda'} h^{(\lambda\lambda')}_{\mu\nu,k}(x) h^{*(\lambda\lambda')}_{\rho\sigma,k}(y)$$

Soln. of small fluctuation equation: $h^{(\lambda\lambda')}_{\mu\nu,k}(x^-, x^+, \boldsymbol{x}) = e^{(\lambda\lambda')}_{\mu\nu}(k)\left[\Theta(-x^-)\, e^{-ikx} + \Theta(x^-)\, e^{-ikx} U_k(\boldsymbol{x})\right]$

$$\tilde{G}_{\mu\nu\rho\sigma}(p,p') = \tilde{G}^0_{\mu\nu\rho\sigma}(p)(2\pi)^4\delta^{(4)}(p-p') + \tilde{G}^0_{\mu\nu\alpha\beta}(p)\mathcal{T}^{\alpha\beta\gamma\delta}(p,p')\tilde{G}^0_{\gamma\delta\rho\sigma}(p')$$

where $\widetilde{G^0}$ is the free propagator and the shockwave effective vertex is

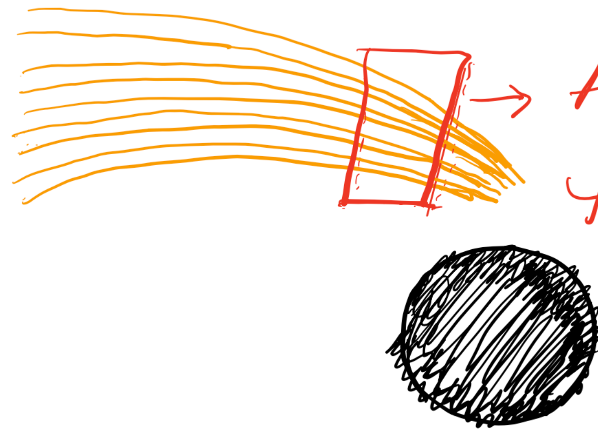
$$\mathcal{T}_{\mu\nu\rho\sigma}(p,p') = -\frac{1}{2}\left(\Lambda_{\mu\rho}\Lambda_{\nu\sigma} + \Lambda_{\mu\sigma}\Lambda_{\nu\rho} - \Lambda_{\mu\nu}\Lambda_{\rho\sigma}\right) 4\pi i (p')^- \delta(p^- - (p')^-) \int d^2\boldsymbol{z}\; e^{i(\boldsymbol{p}-\boldsymbol{p}')\cdot\boldsymbol{z}} \left(e^{if_1(\boldsymbol{z})p'_+} - 1\right)$$

$$\Lambda_{\mu\nu} = \eta_{\mu\nu} - \frac{n_\mu k_\nu + n_\nu k_\mu}{n\cdot k} \qquad f_1(\boldsymbol{x}) = \kappa^2\mu\frac{\rho(\boldsymbol{x})}{\Box_\perp}$$

Remarkably, they satisfy double-copy relations to the QCD shock wave propagators

Geodesic congruence: the geometry of quantum information

The Raychaudhuri equation
- key in Hawking-Penrose singularity theorems :



A congruence of the geodesics of "wee" gravitons

Volume change of geodesic convergence

$$\begin{aligned}\dot{\theta} &= -\Omega^{i}{}_{j}\Omega^{j}{}_{i} + K^{i}{}_{i} \\ &= -\frac{1}{3}\theta^{2} - \sigma_{ij}\sigma^{ij} + \omega_{ij}\omega^{ij} + K^{i}{}_{i}.\end{aligned}$$

Bulk scalar — Shear tensor — Rotation tensor — Includes Ricci curvature + stochastic graviton noise

H.-T. Cho and B.-L Hu, arxiv:2301.06325
M. Parikh, F. Wilczek, G. Zaharaide, PRL (2021)

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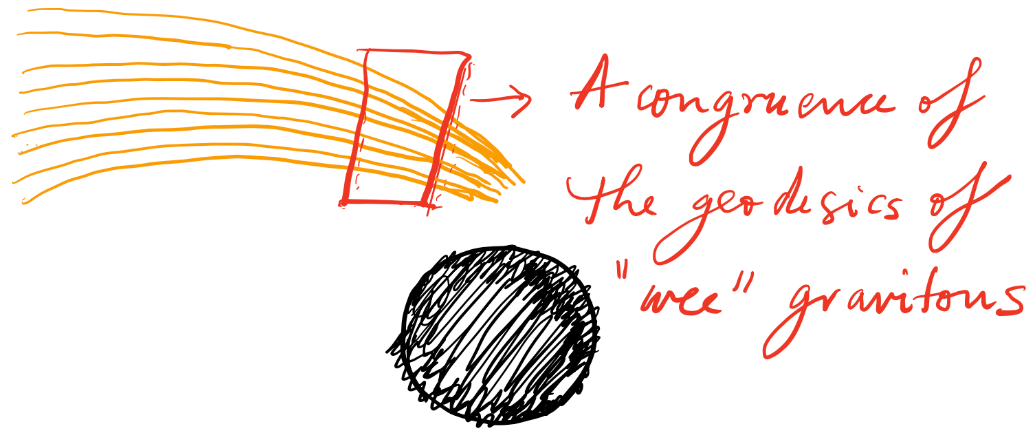

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Bulk scalar   Shear tensor   Rotation tensor   Includes Ricci curvature + stochastic graviton noise

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M. Parikh, F. Wilczek, G. Zaharaide, PRL (2021)

Remarkably, the Raychaudhuri equation can be rephrased as a Bishop-Gromov upper bound on the "complexity volume in D-1 dimensions" of gate complexity – as envisioned by M. A. Nielsen

A.R. Brown and L. Susskind, arXiv:1903.12621
A. R. Brown, arXiv:2112.05724

Some speculations:

A) Can we understand Black Hole formation as a non-trivial fixed point of RG evolution – in the dilute-dense framework?

B) Does this saturate the Bekenstein-Hawking bound – on the maximal entropy in a localized volume