EFT approach to jet observables in heavy ion collisions

Yacine Mehtar-Tani (BNL)

Based on: 2409.05957 [hep-ph] 2411.11992 [hep-ph]

Probing the CGC and QCD matter at colliders
@ Galileo Galilei Institute, Florence

March 26, 2025

Jet quenching theory - the early days

Postdoc in Saclay 2011-2014

PRL **111,** 052001 (2013) PHYSICAL REVIEW LETTERS week ending 2 AUGUST 2013

Medium-Induced QCD Cascade: Democratic Branching and Wave Turbulence

J.-P. Blaizot,* E. Iancu,† and Y. Mehtar-Tani‡

Institut de Physique Théorique (IPhT) Saclay, CNRS/URA2306, F-91191 Gif-sur-Yvette, France

(Received 7 February 2013; published 30 July 2013)

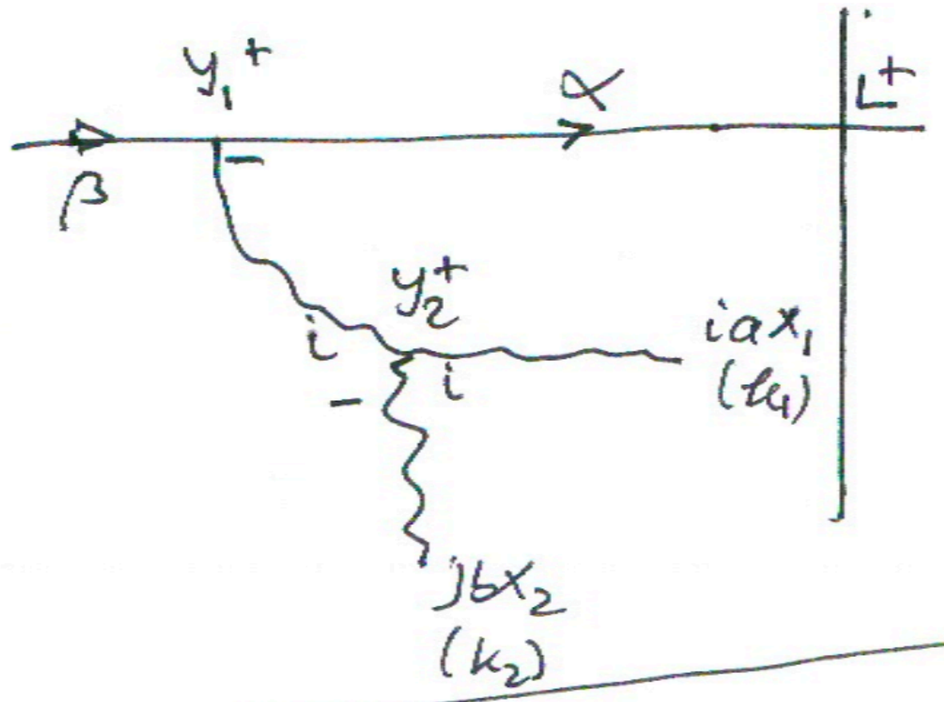
We study the average properties of the gluon cascade generated by an energetic parton propagating through a quark-gluon plasma. We focus on the soft, medium-induced emissions which control the energy transport at large angles with respect to the leading parton. We show that the effect of multiple branchings is important. In contrast with what happens in a usual QCD cascade in vacuum, medium-induced branchings are quasidemocratic, with offspring gluons carrying sizable fractions of the energy of their parent gluon. This results in an efficient mechanism for the transport of energy toward the medium, which is akin to wave turbulence with a scaling spectrum $\sim 1/\sqrt{\omega}$. We argue that the turbulent flow may be responsible for the excess energy carried by very soft quanta, as revealed by the analysis of the dijet asymmetry observed in Pb-Pb collisions at the LHC.

DOI: 10.1103/PhysRevLett.111.052001 PACS numbers: 12.38.−t, 24.85.+p, 25.75.−q

One important phenomenon discovered recently in heavy ion experiments at the LHC is that of *dijet asymmetry*, a strong imbalance between the energies of two back-to-back jets. This asymmetry is commonly attributed to the effect of the interactions of one of the two jets with the hot QCD matter that it traverses, while the other leaves the system unaffected. Originally identified [1,2] as missing energy, this phenomenon has been subsequently shown [3]

light-cone (LC) coordinates and momenta, with the longitudinal axis defined by the direction of motion of the leading particle. Correspondingly, the "energy" ω truly refers to the LC longitudinal momentum p^+ and t to the LC "time" x^+. Equation (1) applies as long as $\ell \ll \tau_{\rm br}(z,\omega) < L$, where L is the length of the medium and ℓ is the mean free path between successive collisions. The second inequality above implies an upper limit on the

2-gluon emissions inside the medium



$$\langle T[a_a^i(x_1)\, a_b^j(x_2)]\rangle^{\alpha\beta} = 2ig^2 \int^{L^+} dy_1^+ \int_{y_1^+}^{L^+} dy_2^+ \int d^3 y$$

$$\times f^{\bar a \bar b \bar c}\, G_{a\bar a}(x_1 y_2)\, G^{j-}_{b\bar c}(x_2 y_2)\, \partial^+_{y_2} G^{i-}_{\bar c d}(y_2 y_1)$$

$$\times \tilde V_{d\bar d}(y_1^+)\, \langle\alpha| V t^{\bar d} |\beta\rangle$$

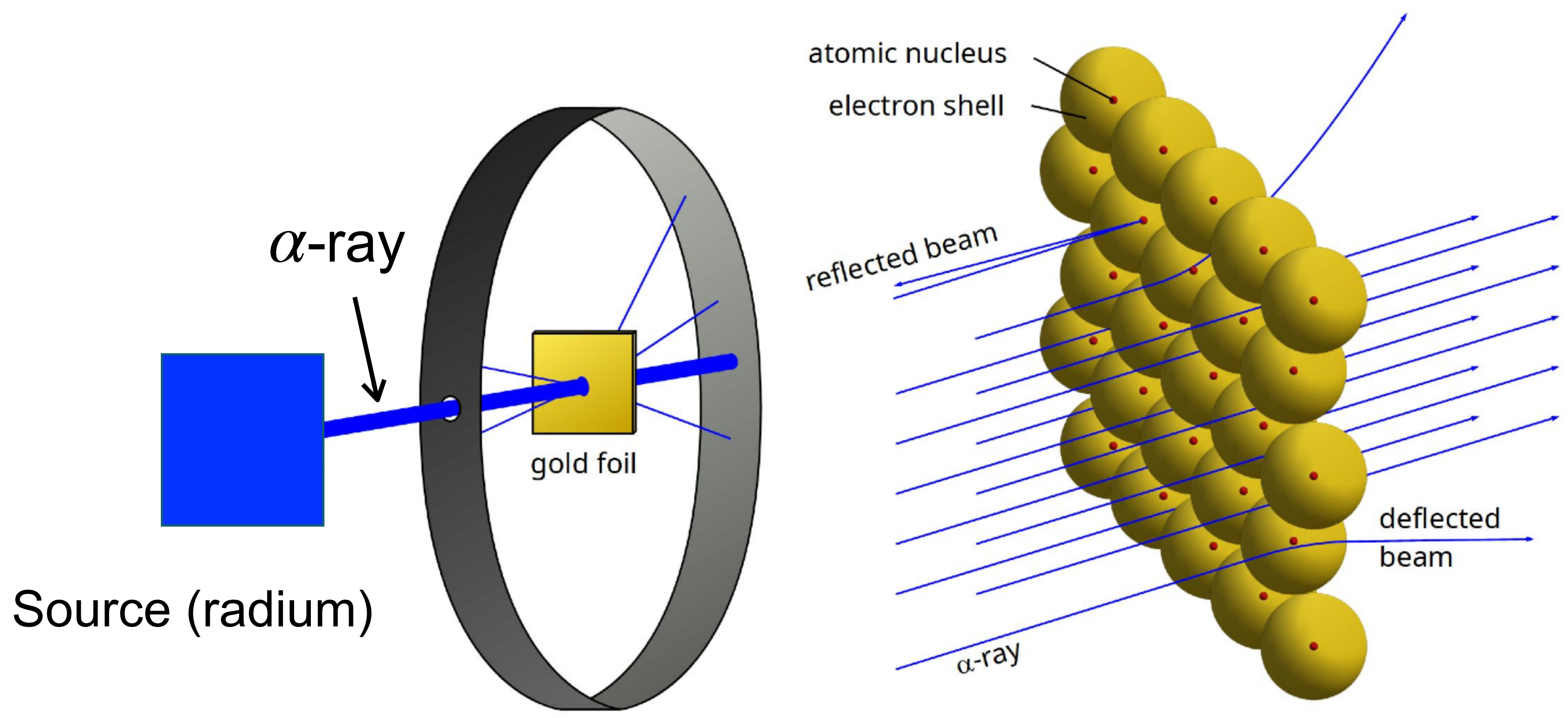
Many exciting results…

Edmond's notes (2011): As always, it all started with a gluon emission

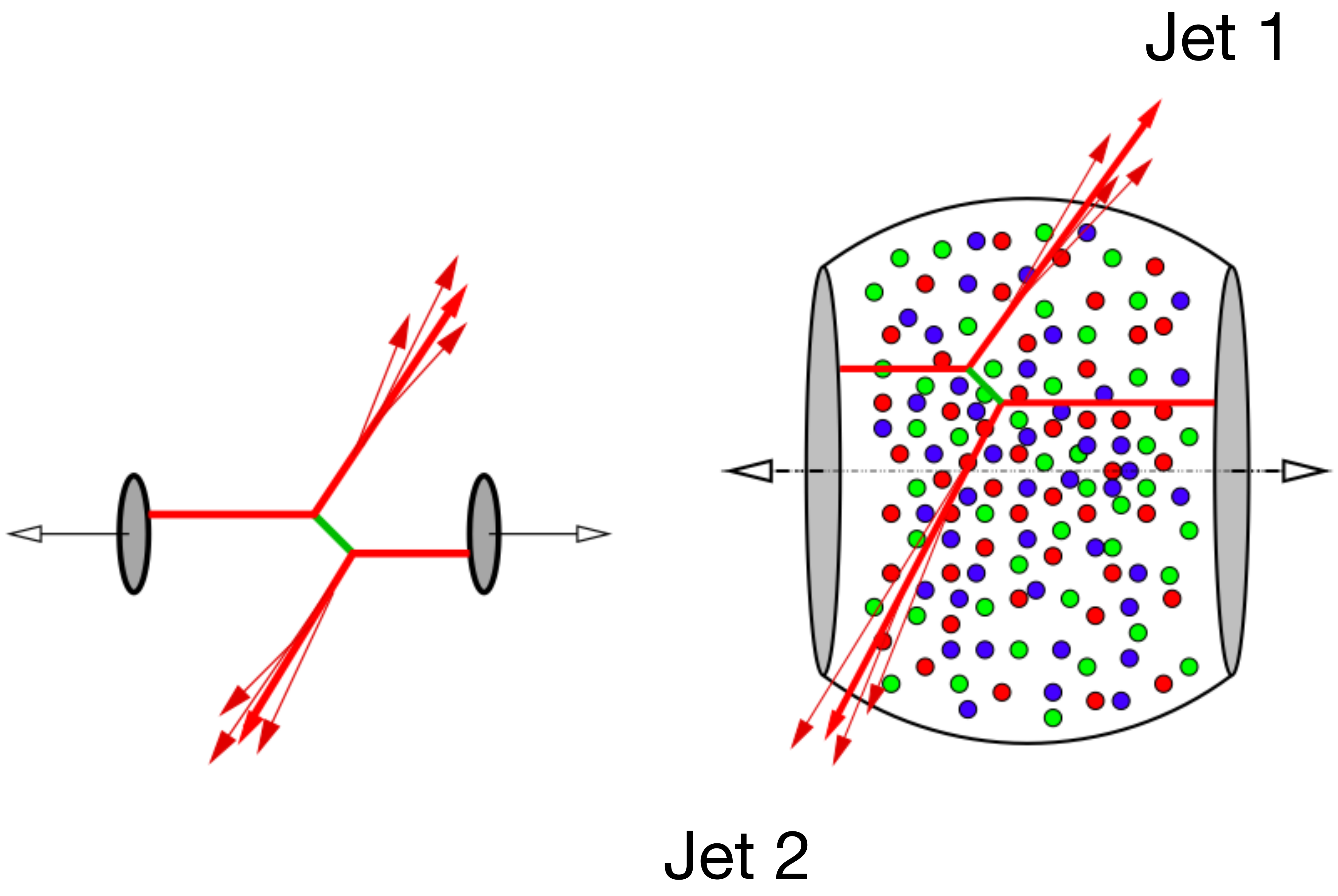
Edmond, leading the way (HTL, Saturation, Jet Quenching,…)

Introduction

A Rutherford-like experiment - Jets in HIC



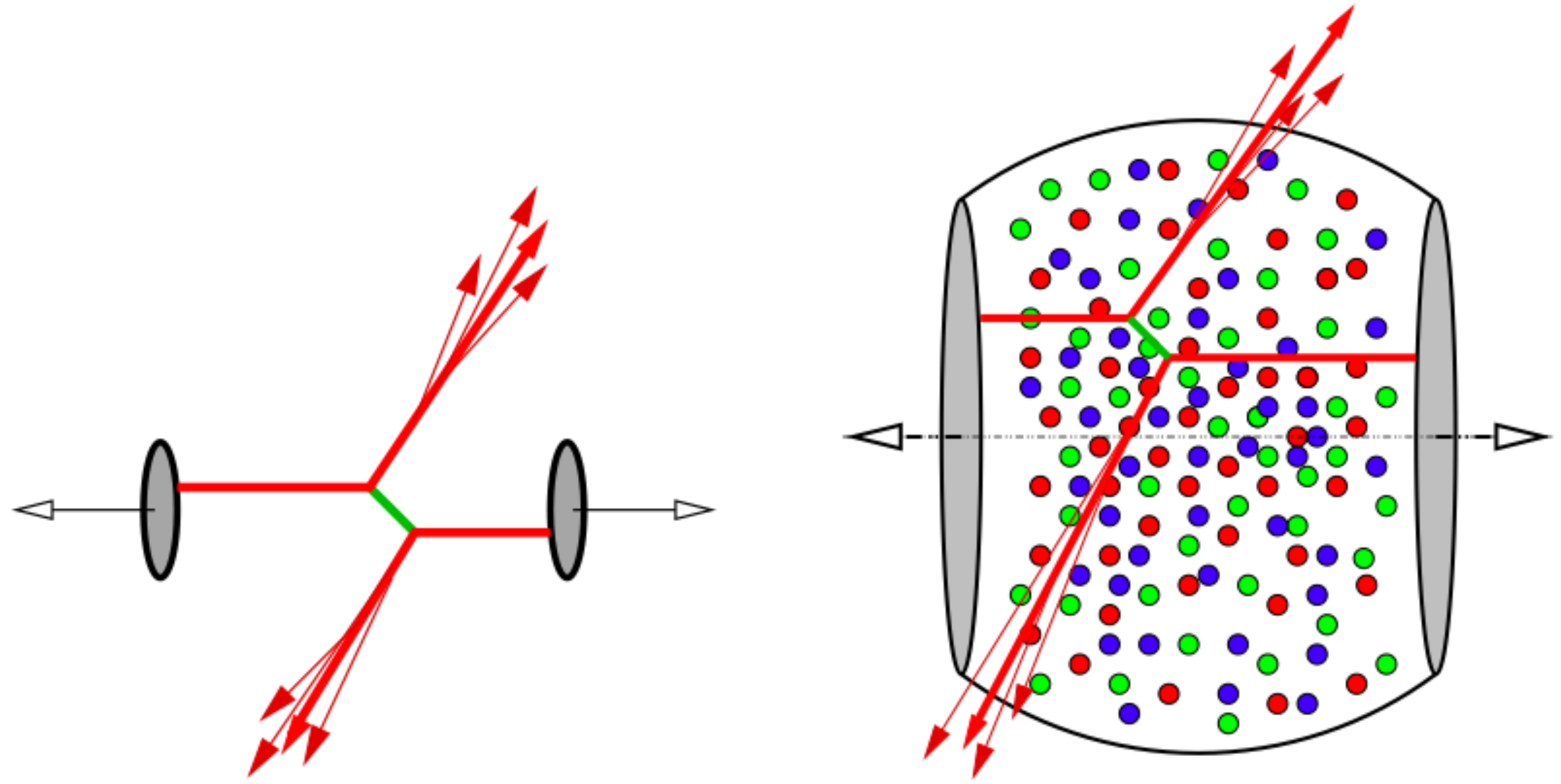
Discovery of the atomic nucleus



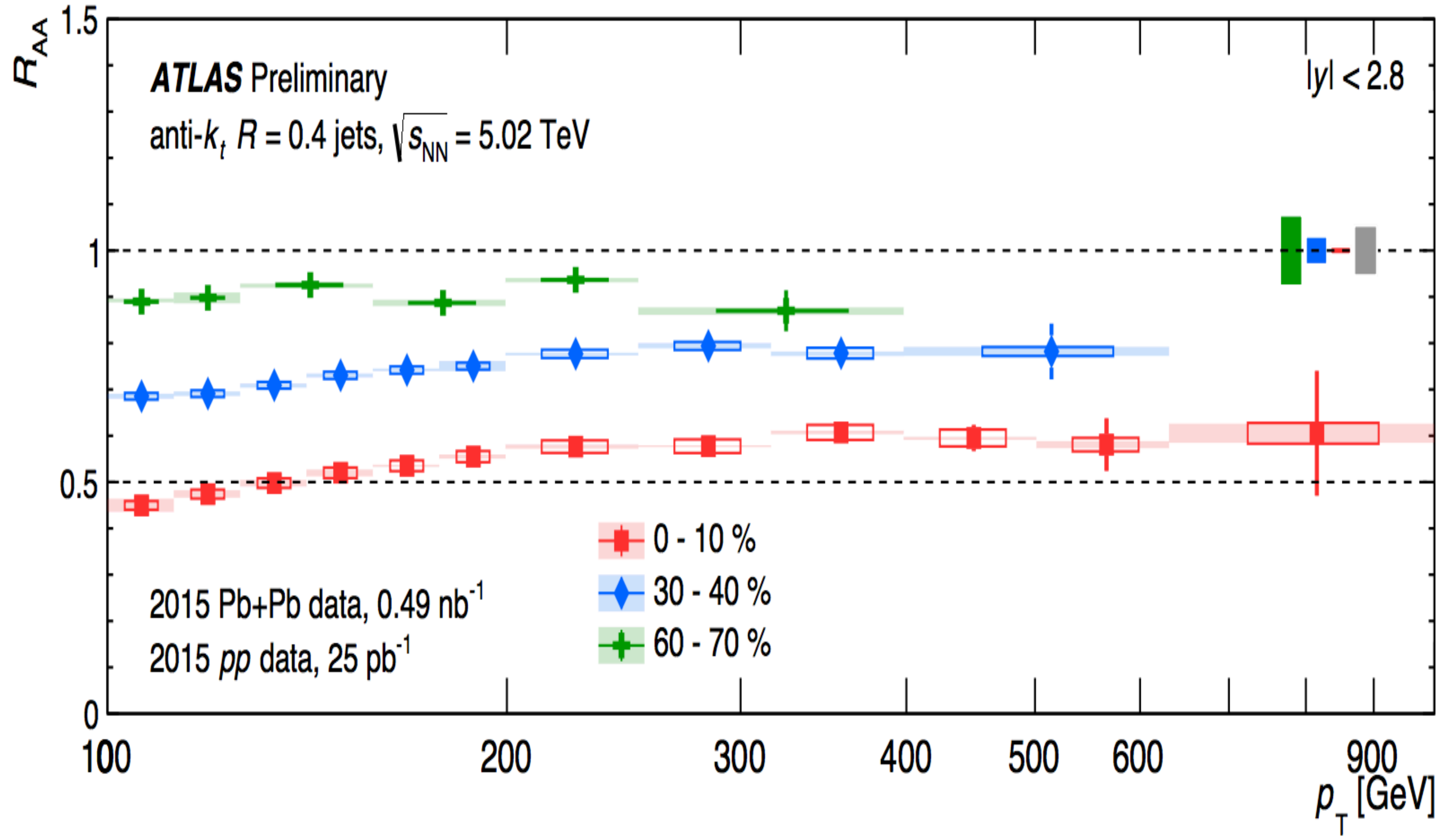
Probing the microscopic properties of the QGP with jets

Evidence of the QGP from jet quenching

- Substantial final state interactions: jets lose energy to the QGP constituents
- Strong suppression and modification of jets observed at RHIC and LHC



Nuclear modification factor



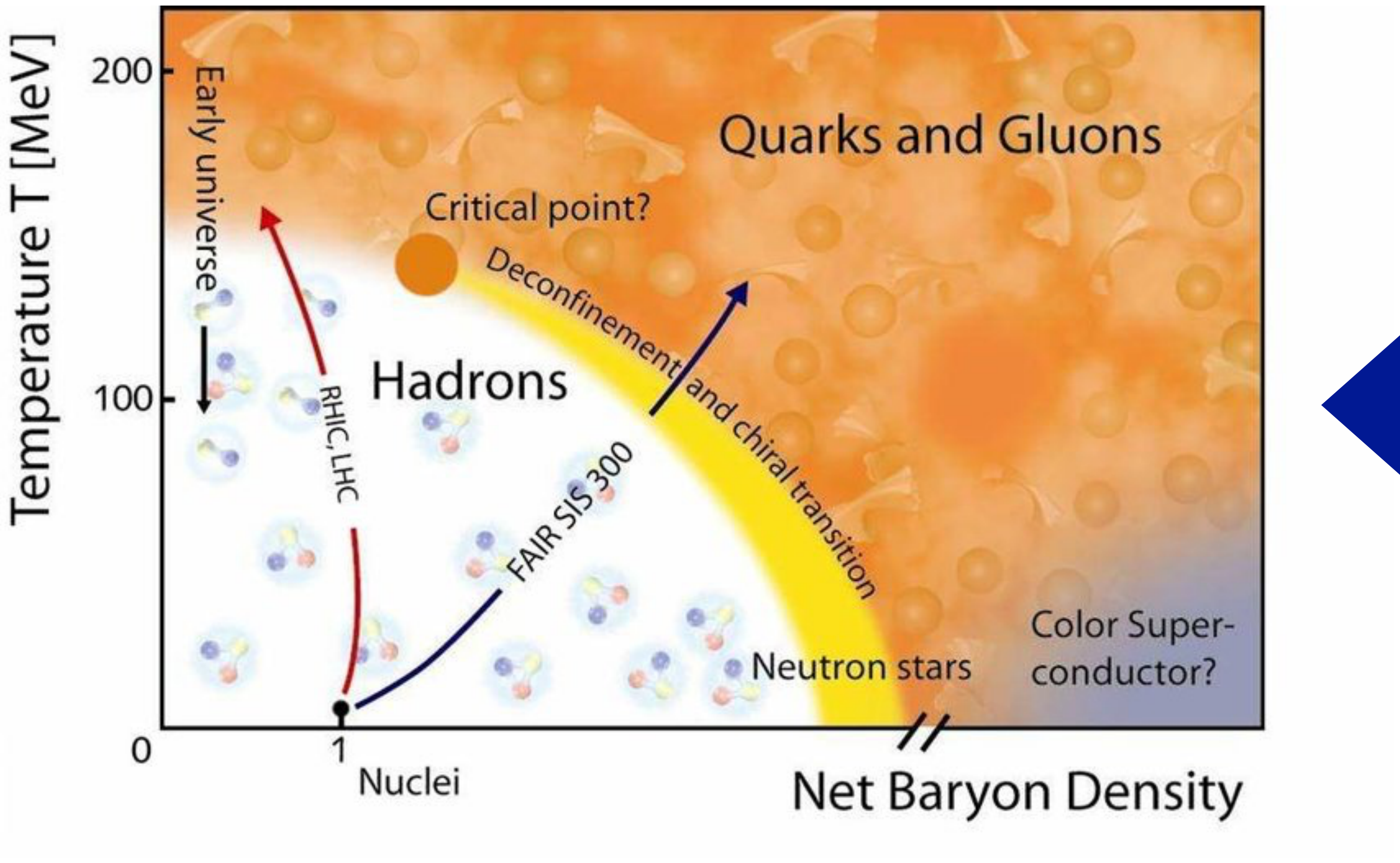
$$R_{\mathrm{AA}} = \frac{\mathrm{Yield}_{\mathrm{AA}}}{N_{\mathrm{coll}}\ \mathrm{Yield}_{\mathrm{pp}}}$$

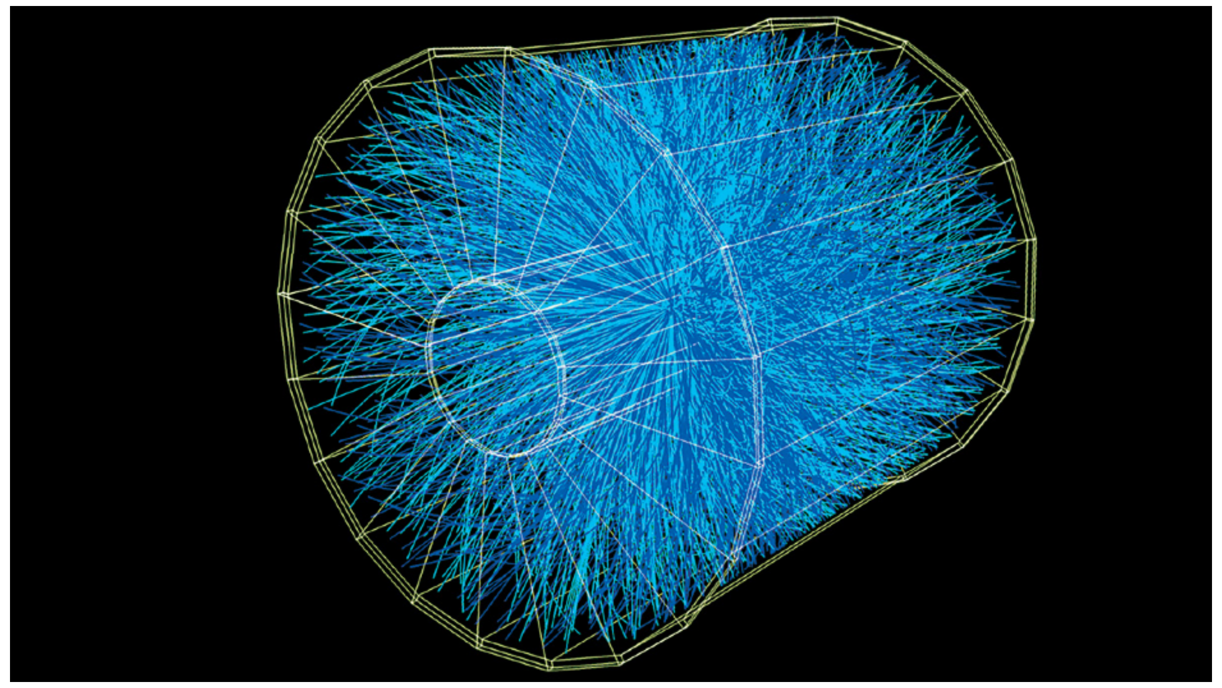
Jet quenching: multiscale dynamics

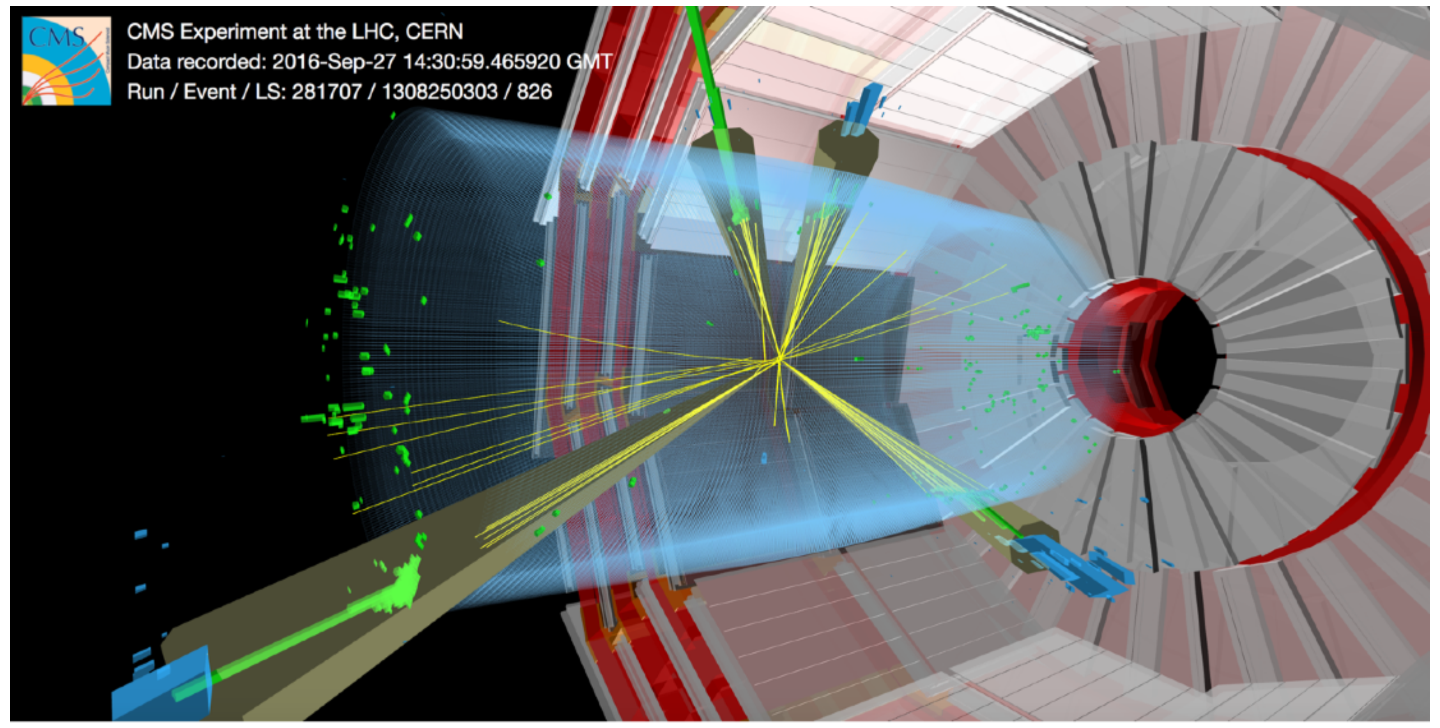
Thermal equilibrium $(T \neq 0)$

Non-equilibrium

T=0

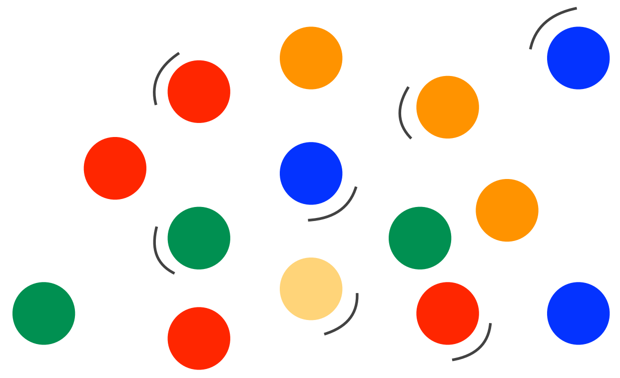


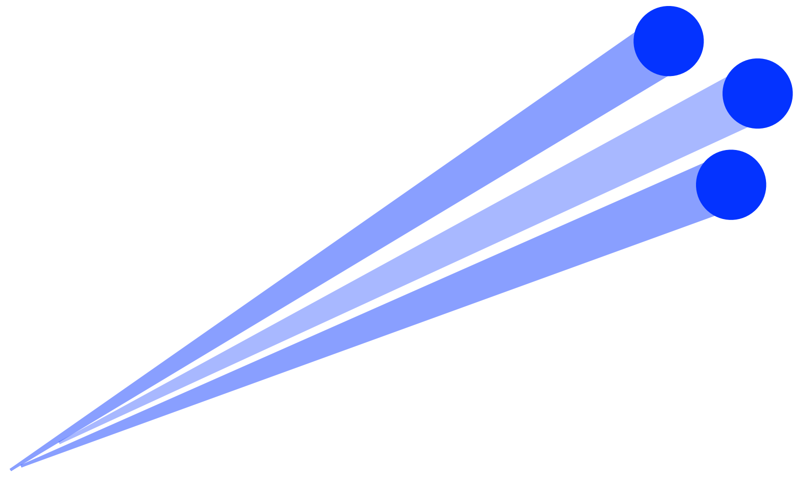




Heavy ion event
~ 1000's particles

Jets in pp





Energy

$T \sim 1$ GeV

$p_T \sim 1$ TeV

To fully leverage the wealth of experimental data
and explore this new QCD frontier, we need
precision theoretical tools for jet quenching

Energy loss from Wilson lines

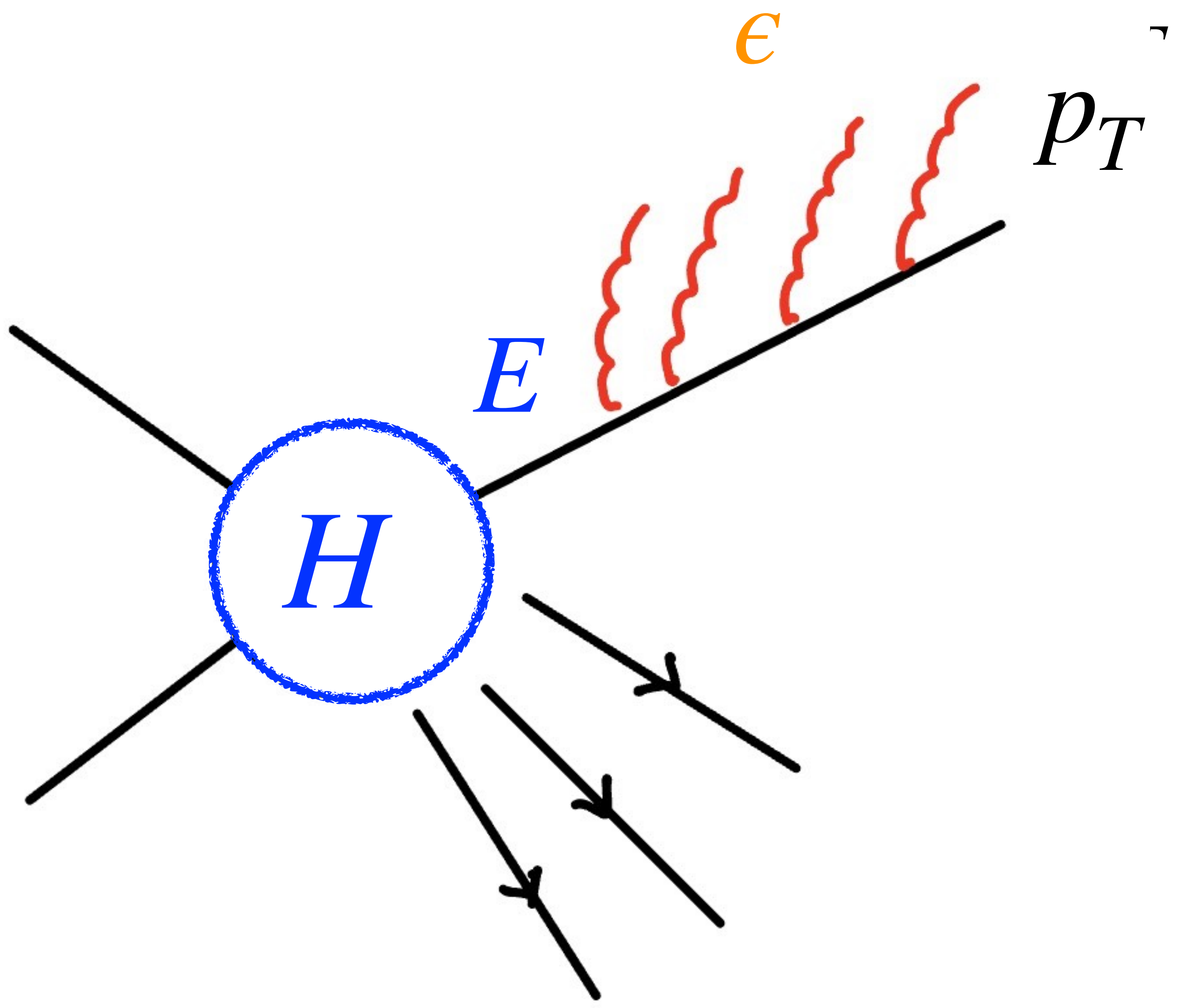
Jet cross-section in HIC

- Jet production and energy loss processes are well separated in time

$$t_{\text{prod}} \sim p_T^{-1} \qquad \ll \qquad t_{\text{loss}} \sim L \qquad \text{(Length of the QGP)}$$

- Factorization at leading order:

$$\frac{\mathrm{d}\sigma_{\text{AA}}}{\mathrm{d}p_T} \sim \int \mathrm{d}E \int \mathrm{d}\epsilon\, \delta(E - p_T - \epsilon)\, P(\epsilon)\, \frac{\mathrm{d}\sigma_{\text{pp}}}{\mathrm{d}E}$$

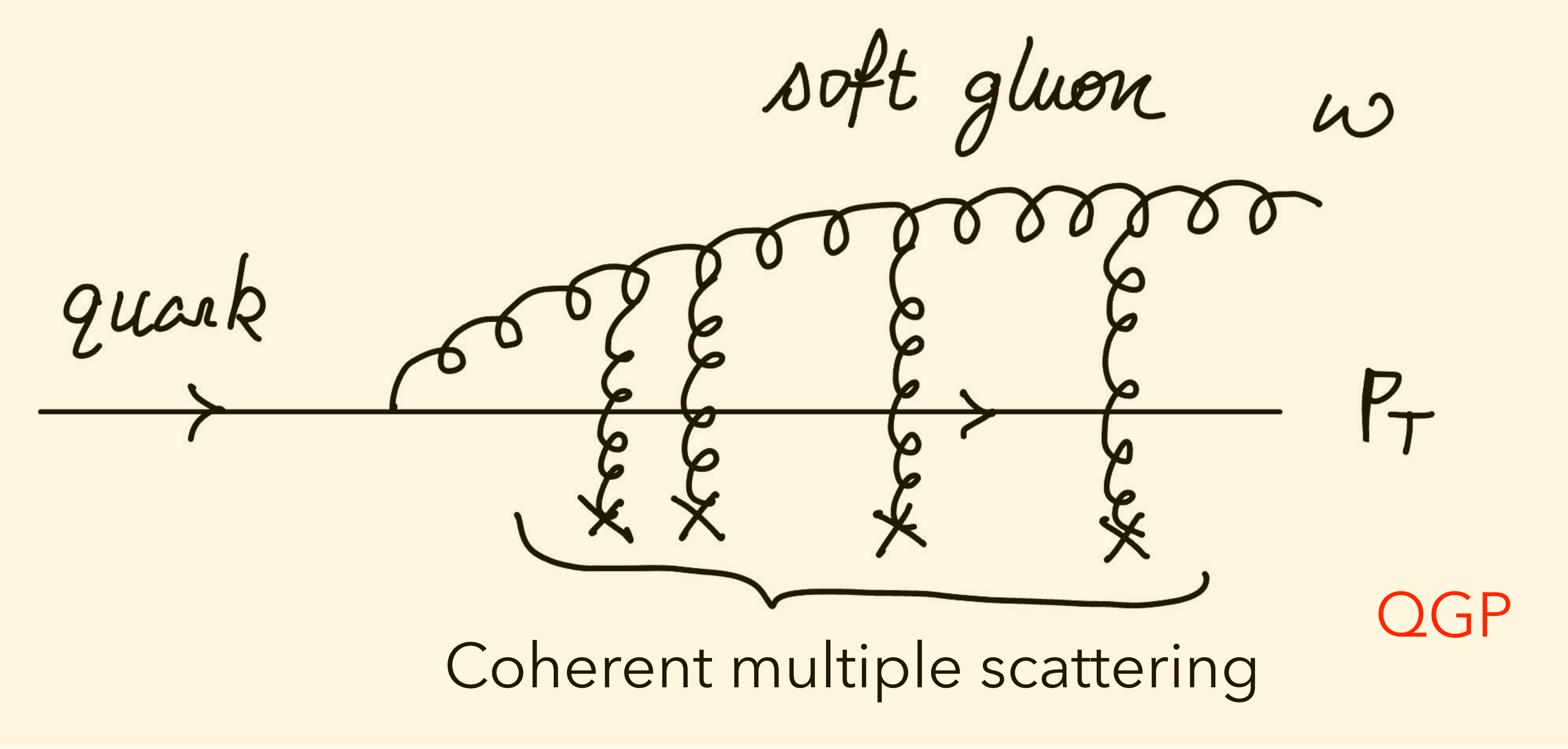


- $P(\epsilon)$ is the probability for a parent parton of energy E loses ϵ of its energy to the QGP

Elementary process of radiative energy loss

↪ 1980: Bjorken predicts jet quenching

↪ 1990's parton radiative energy loss $E_{\mathrm{loss}} \sim \hat{q}\, L^2$



Gyulassy, Wang (1990) Baier, Dokshitzer, Mueller, Peigne, Schiff (1996) Zakharov (1997) Wiedemann (2000) Gyulassy, Levai, Vitev (2001) Arnold, Moore, Yaffe (2002)

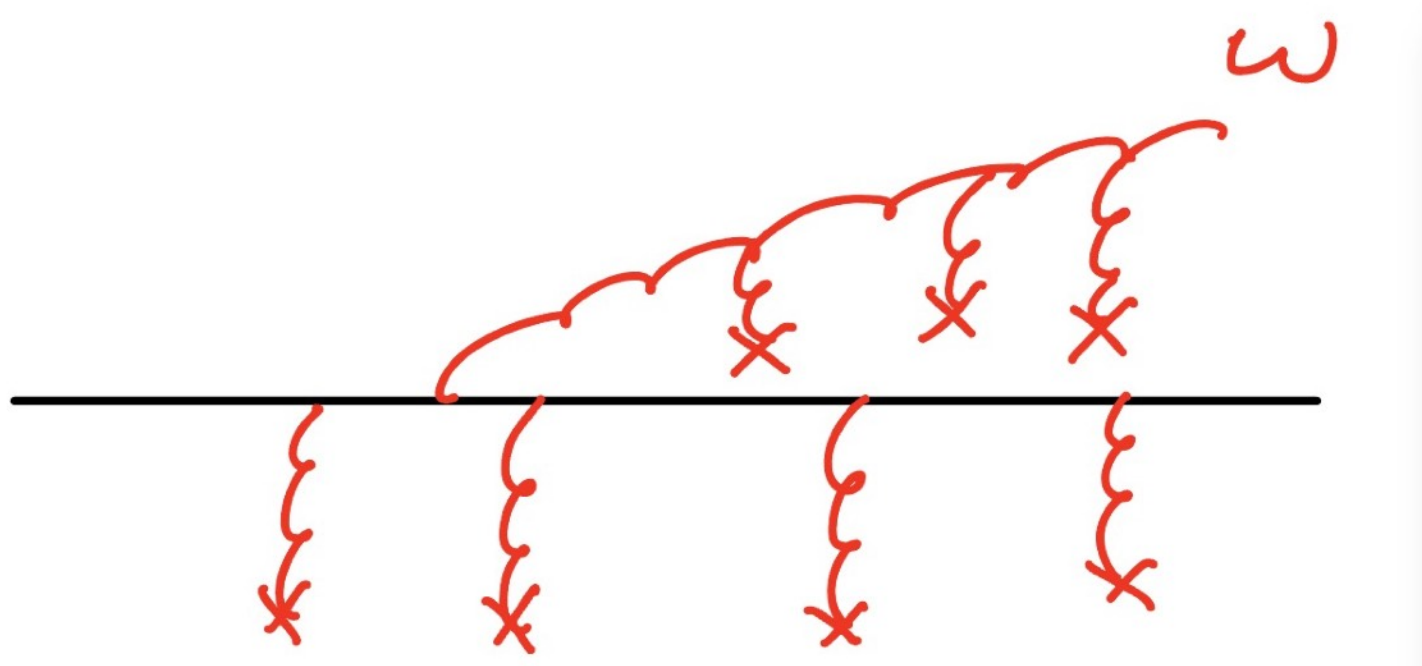
Radiative energy loss and the LPM effect

- Large fraction of energy lost via radiation

$$P^{(1)}(\epsilon, L) = \int_0^L \mathrm{d}t \int_0^\infty \mathrm{d}\omega \, \frac{\mathrm{d}I}{\mathrm{d}\omega \mathrm{d}t} \left[\delta(\epsilon - \omega) - \delta(\epsilon)\right]$$

- Two analytic limits characterized by the scale $\omega_c = \hat{q} L^2$

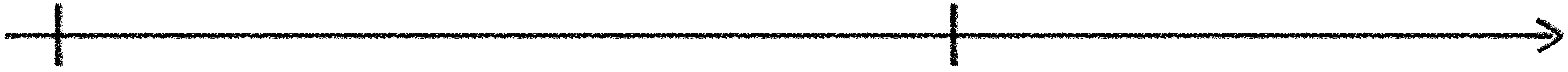
Coherent scattering



$$\ell_{\mathrm{mfp}} \ll t_f \sim \sqrt{\frac{\hat{q}}{\omega}} \ll L$$

$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} \sim \alpha_s \sqrt{\frac{\omega_c}{\omega}}$$

$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} \sim \alpha_s \frac{\omega_c}{\omega}$$



ω

T Multiple-soft scattering (frequent) ω_c Single hard (rare)

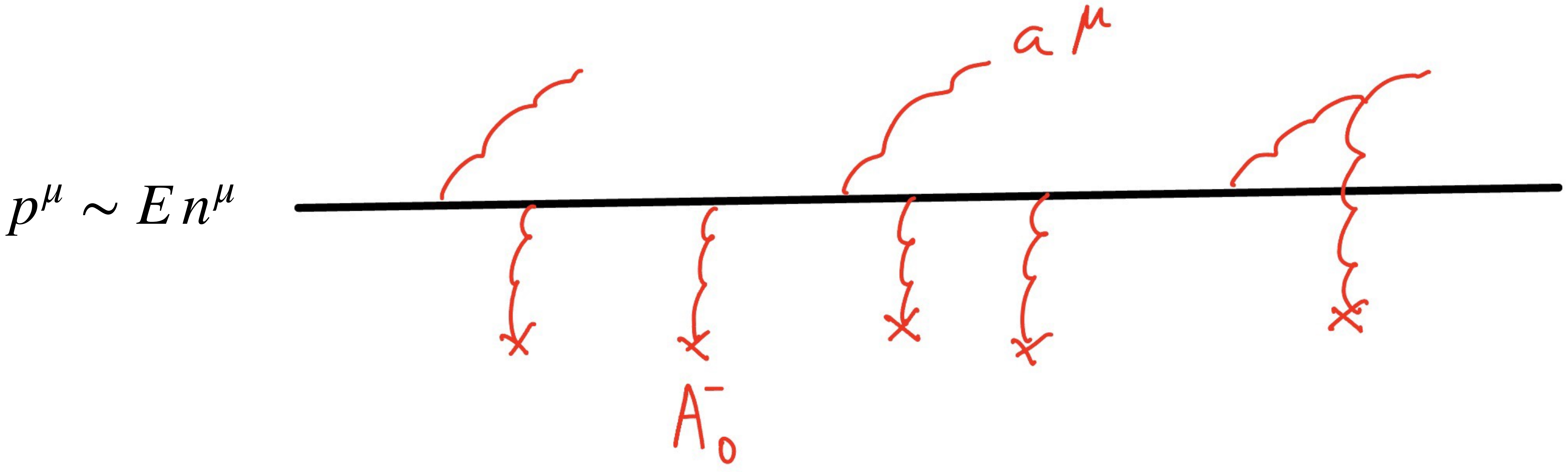
Operator definition for energy loss

- The hard cross-sec $\mathrm{d}\sigma_{\mathrm{pp}}$ can be computed perturbatively while $P(\epsilon)$ involves soft interactions with the QGP down to the $T \sim 1\,\mathrm{GeV}$

- Soft interactions, $\epsilon \ll p_T$, are encoded in semi infinite Wilson-lines

$$U(n) \equiv \exp\left[ig \int_0^\infty \mathrm{d}s\, \bar{n} \cdot A(ns)\right]$$

- The gauge field $A^\mu = A_0^\mu + a^\mu$ describes both radiative and elastic processes



Operator definition of energy loss

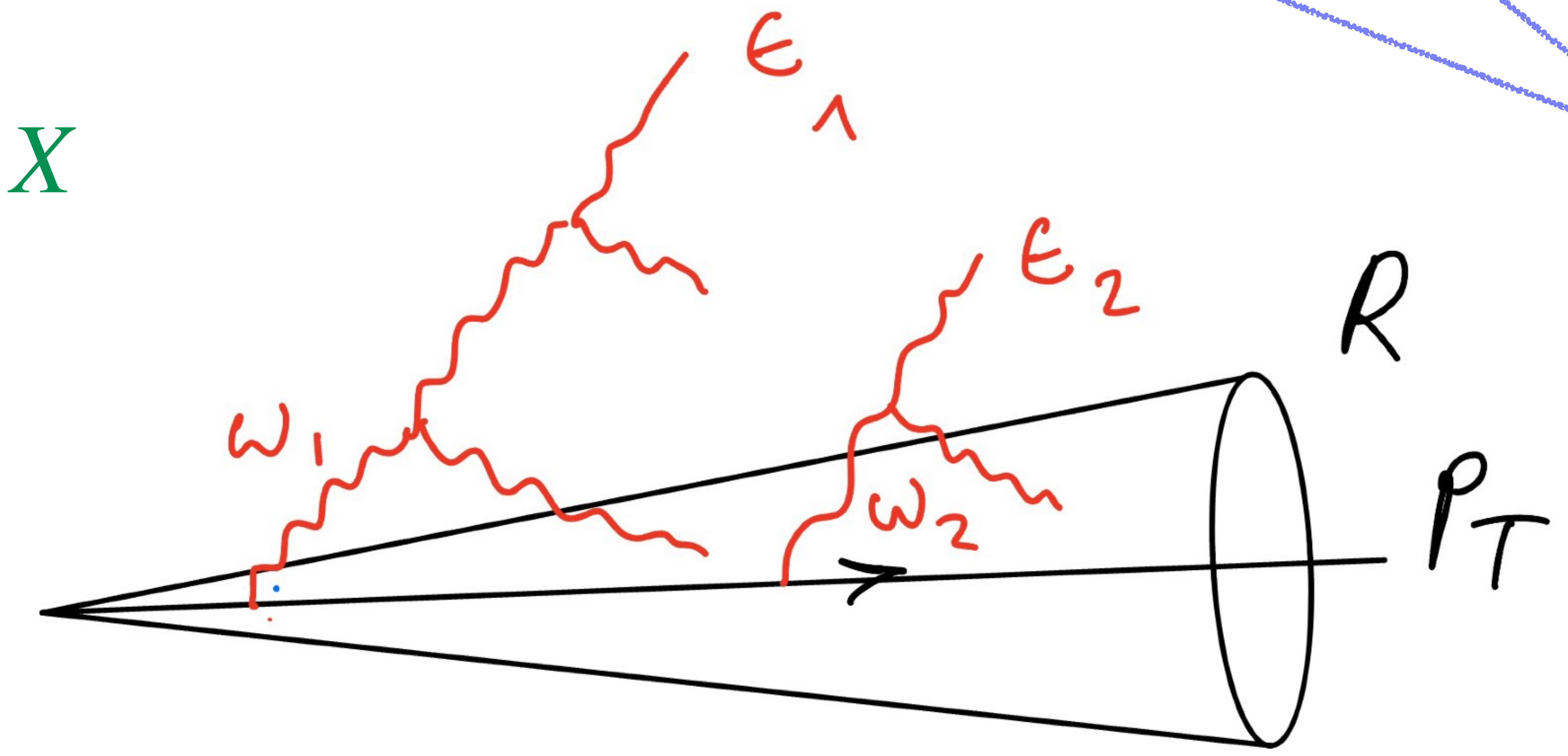
YMT., Ringer, Singh, Vaidya, 2409.05957 [hep-ph]

- The energy loss probability for a single hard collinear parton

$$P(\epsilon) = \frac{1}{d_R} \sum_X \delta(\epsilon - \bar{n} \cdot k_{\mathrm{loss}})\, \mathrm{tr}_c \left[\langle \mathrm{med} | U(n) | X \rangle \langle X | U^\dagger(n) | \mathrm{med} \rangle \right],$$

Measurement on final state X
(Includes the jet algorithm)

Wilson-lines

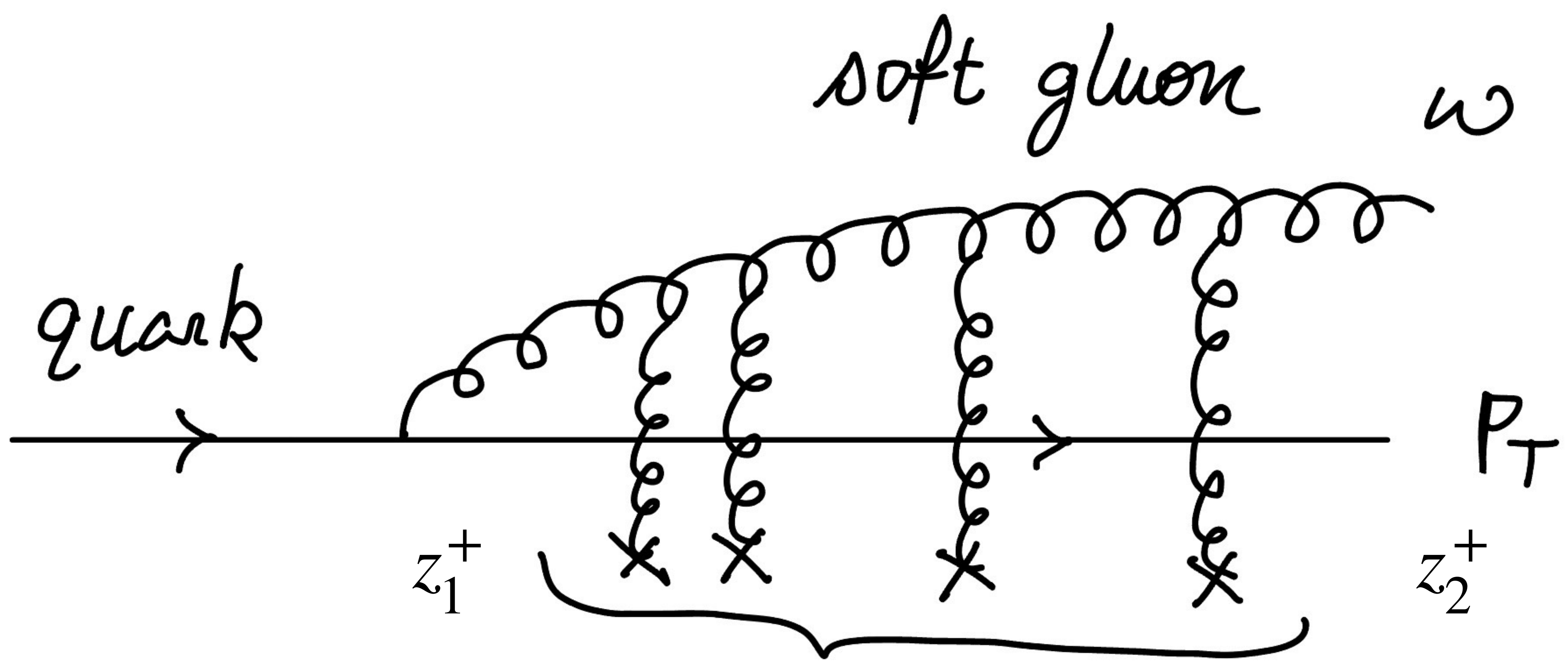


Medium-induced radiative spectrum

- To leading order in the gluon emissions and all order in opacity (number of rescattering)

$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} \sim \mathcal{K}(z_2^+, z_1^+) \equiv \langle \mathrm{Tr}_c \left[\mathcal{U}^\dagger(z_1^+, z_2^+) \mathcal{G}(z_2^+, z_1^+) \right] \rangle$$

LC-time: $z^+ = \dfrac{t+z}{2}$



Non-eikonal propagator $\mathcal{G}$

Eikonal quark: $\mathcal{U}$

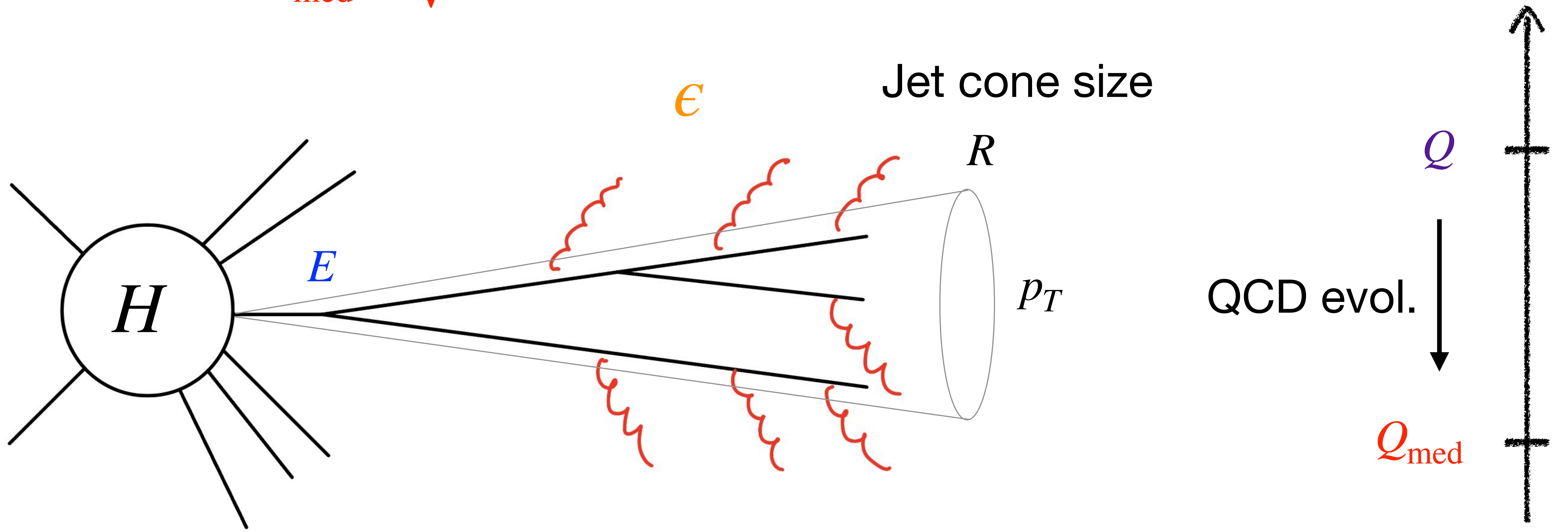
Coherent multiple scattering $\Delta z^+ \sim t_f = \sqrt{\omega/\hat{q}}$ (gluon formation time)

Wiedemann (2000) Blaizot, Dominguez, **Iancu**, MT (2013)

QCD evolution of jet quenching

Jet production in HIC

- High probability for numerous particle production along the jet: breakdown of the single parton energy loss.
- Large phase-space for collinear vacuum splittings from the jet virtuality $Q = p_T R$ down the the medium scale $Q_{\text{med}} \sim \sqrt{\hat{q}L}$

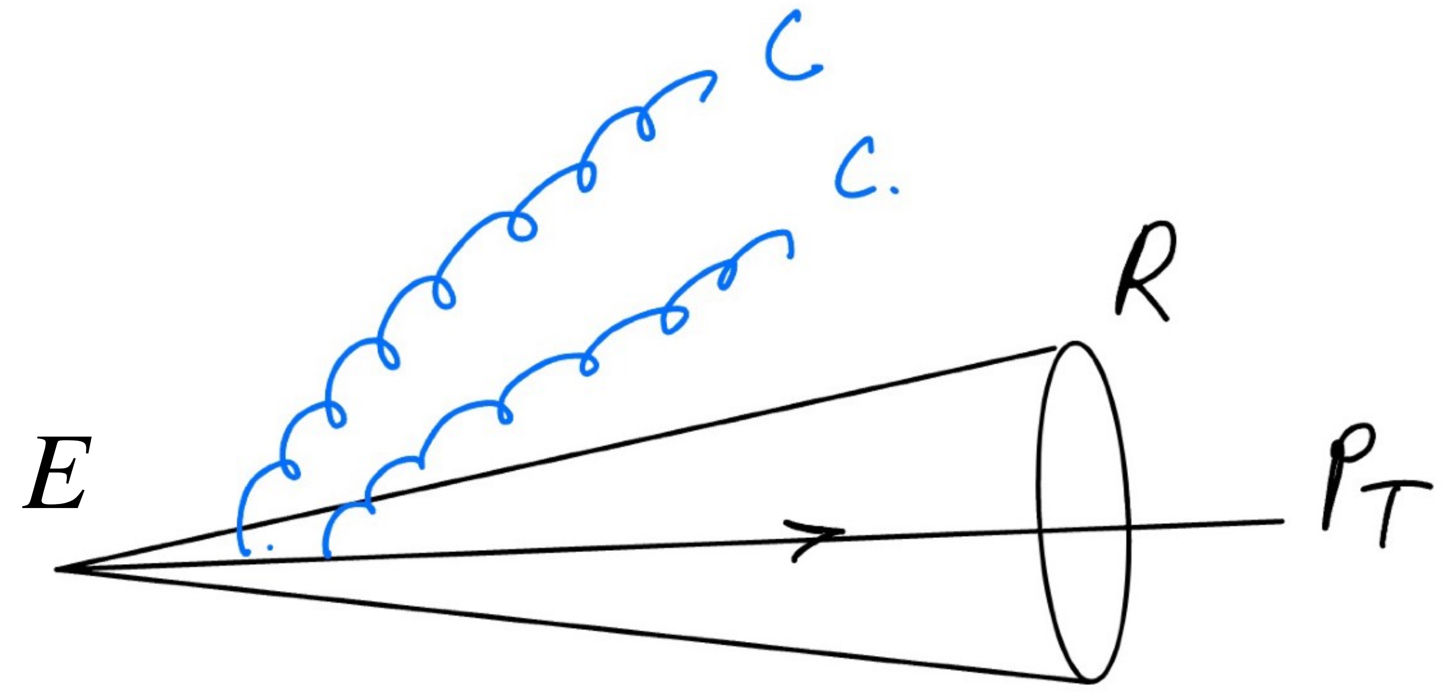


- Color coherence: at higher orders soft radiation off multiple emitters interfers

Jet production in pp

- Factorization of the hard matrix element and collinear jet function

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T} \sim \int_0^1 \frac{\mathrm{d}z}{z} H(p_T = zE, \mu)\, J(z, E, \mu)$$



Hard modes $p_{\mathrm{h}} \sim p_T \overset{+,\,-,\,\perp}{(1,1,1)}$

Collinear mode $p_{\mathrm{c}} \sim p_T\,(R^2, 1, R)$

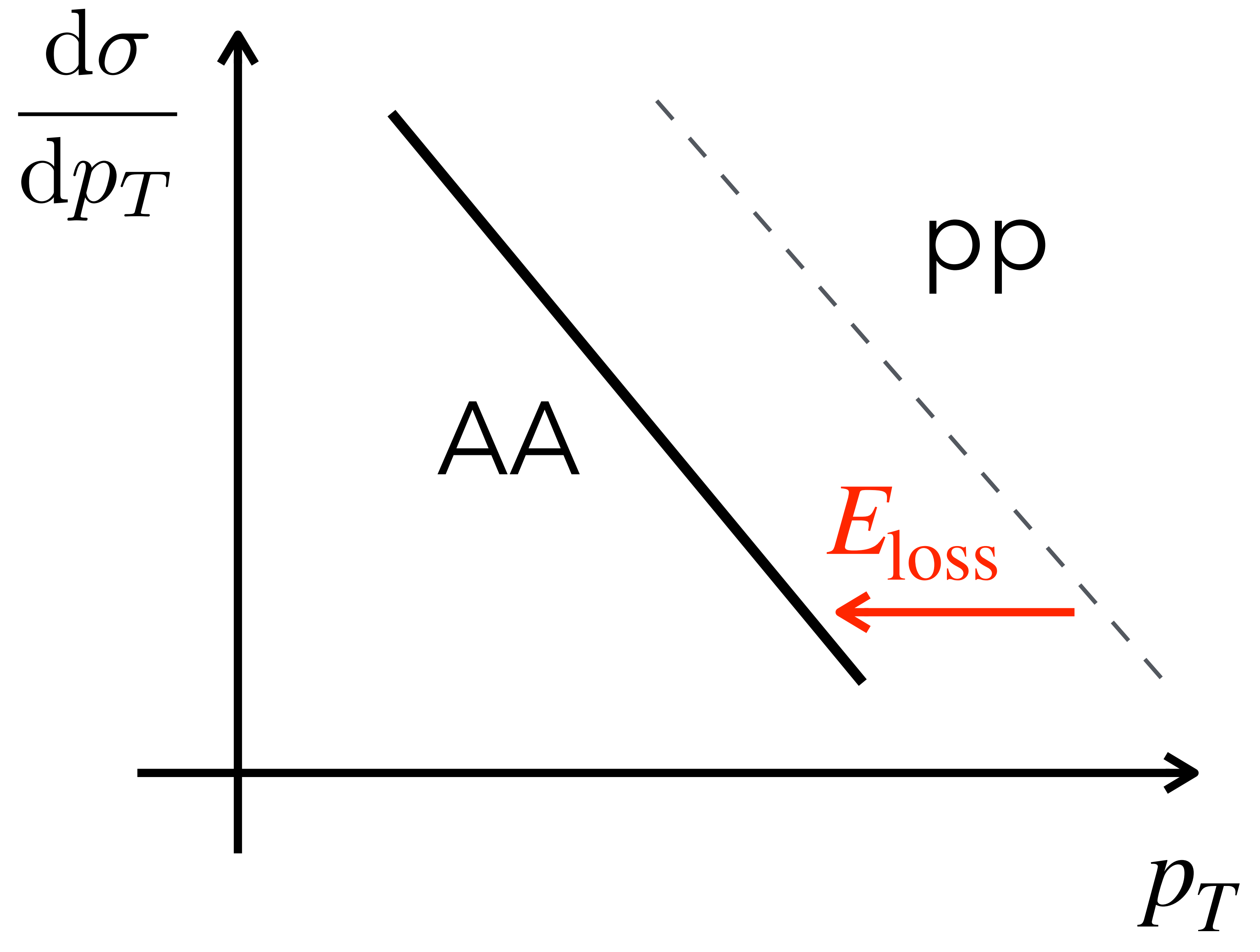
- Jet function obeys DGLAP which resums powers of $\alpha_s \ln R$, where $R \ll 1$

Dasgupta, Dreyer, Salam, Soyez (2015), Kang, Ringer, Vitev (2016)

Collinear-soft mode from energy loss

- Steep jet spectrum $n \gg 1 \quad \Longrightarrow \quad$ Bias toward small energy loss

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T} \sim \frac{1}{(p_T + E_{\mathrm{loss}})^n} \simeq \frac{1}{p_T^n} \underbrace{\left(1 - \frac{nE_{\mathrm{loss}}}{p_T}\right)}_{R_{AA}} < 1$$

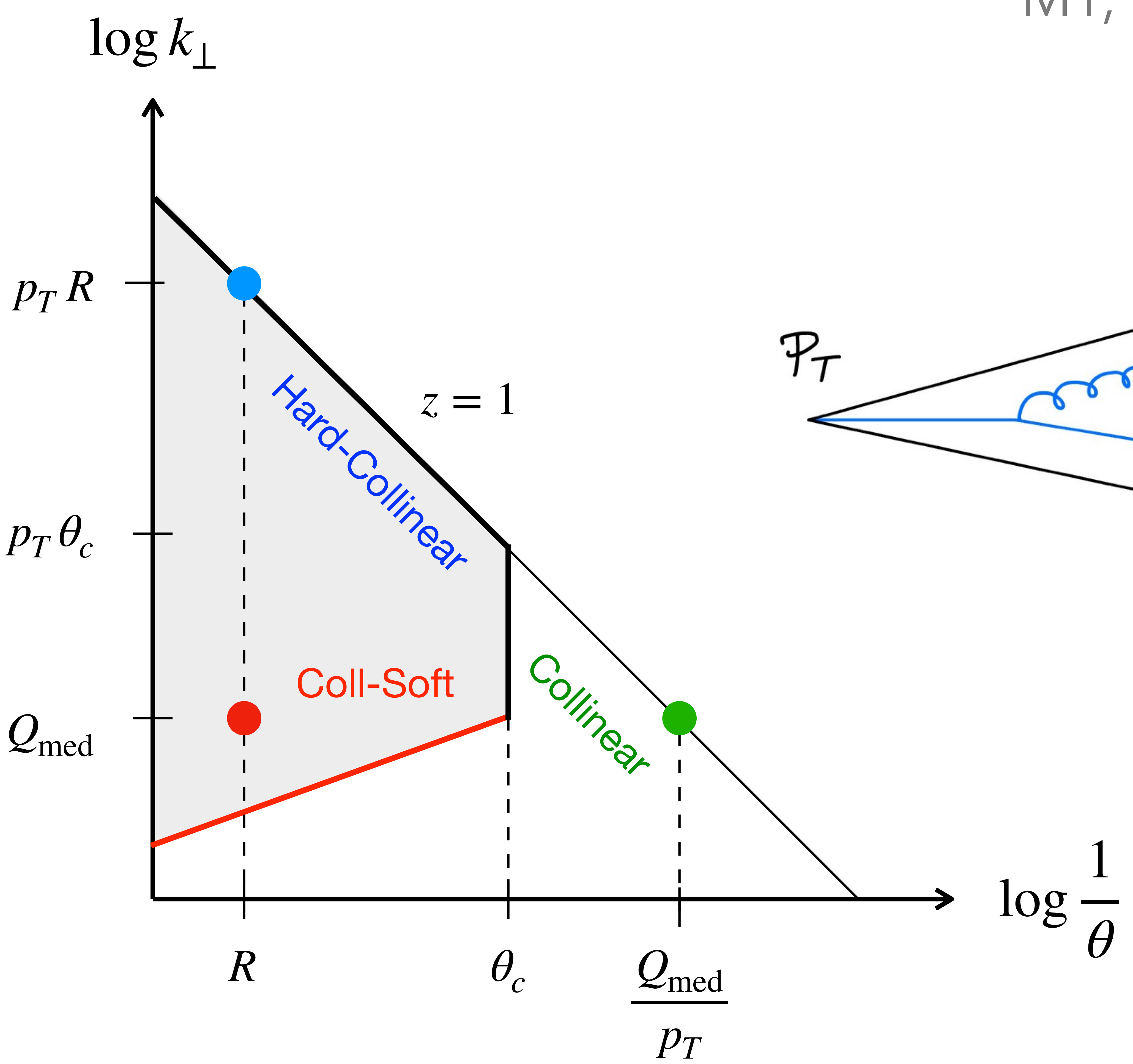


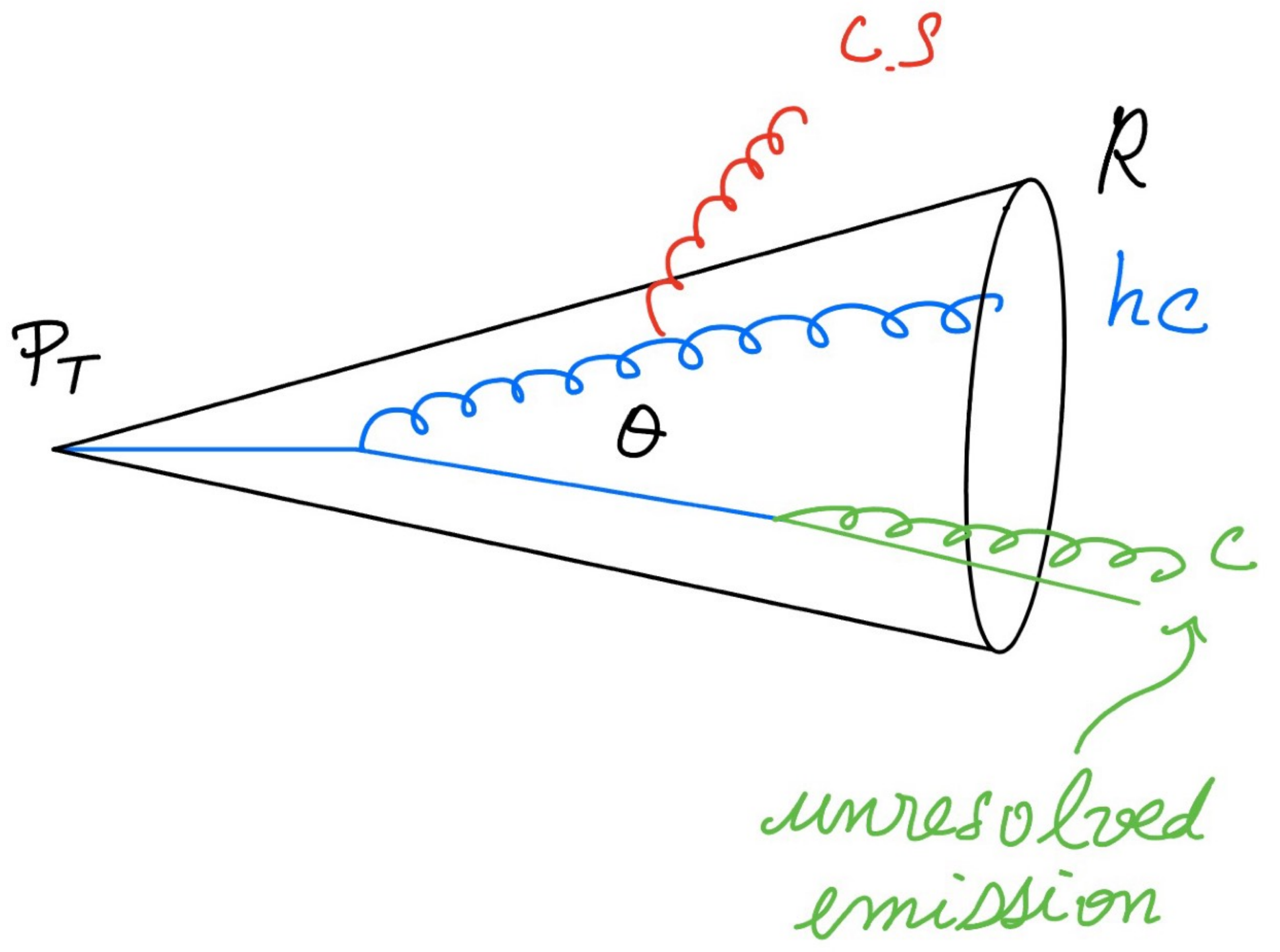
- Collinear-Soft mode $\quad p_{\mathrm{cs}} \sim p_T\, \beta\, (R^2, 1, R)$

$$\beta \sim \frac{E_{\mathrm{loss}}}{p_T} \sim \frac{1}{n} \ll 1$$

Lund Plane representation

MT, Tywoniuk (2017) , Caucal, **Iancu**, Mueller, Soyez (2018)





- Transverse momentum broadening

$$Q_\mathrm{med} \sim \sqrt{\hat{q}L} \sim R^2 p_T$$

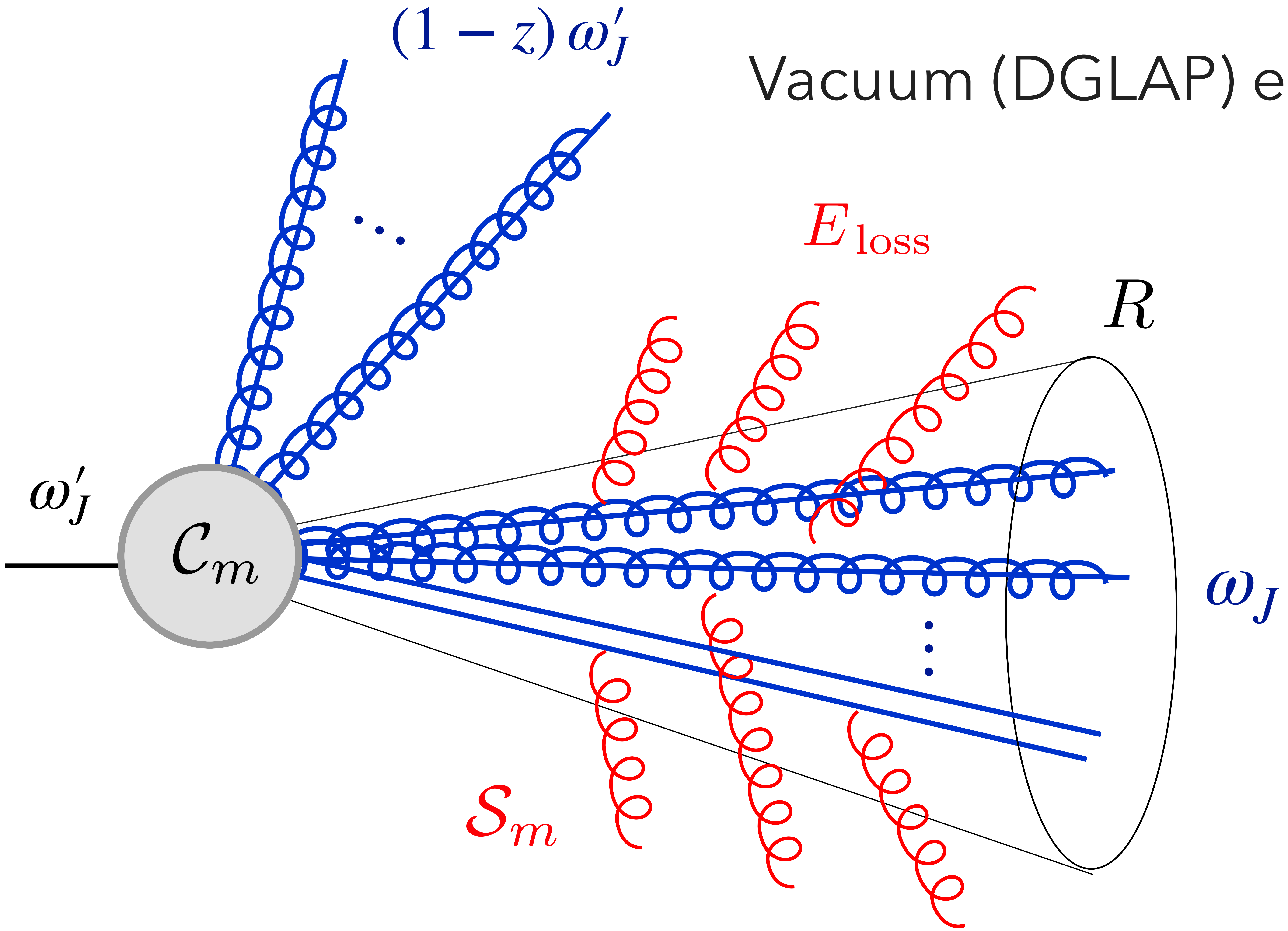
- Coherence angle: interference

$$\theta_c \sim \frac{1}{Q_\mathrm{med} L} \sim \left(\sqrt{\hat{q}L^3}\right)^{-1/2}$$

- Phase-space:

$$\int \frac{\mathrm{d}k_\perp}{k_\perp} \int \frac{\mathrm{d}\theta}{\theta} \;\rightarrow\; \frac{1}{2} \ln \frac{R}{\theta_c} \ln \frac{p_T^2 R\, \theta_c}{\mu^2}$$

Re-factorization of the jet function



Vacuum (DGLAP) evolution

Multiple subjet radiating
soft gluons

Re-factorization of the jet function

$$J(z,\omega_J) = \int \mathrm{d}z' \int \mathrm{d}\epsilon\ \delta(\omega'_J - \omega_J - \epsilon) \sum_m C_m(\{n_i\}, z', \omega'_J \mu, \mu_{\rm cs}) \otimes S(\{n_i\}, \epsilon, \mu_{\rm cs})$$

- Collinear-soft function - energy loss distribution (similar form in vacuum)

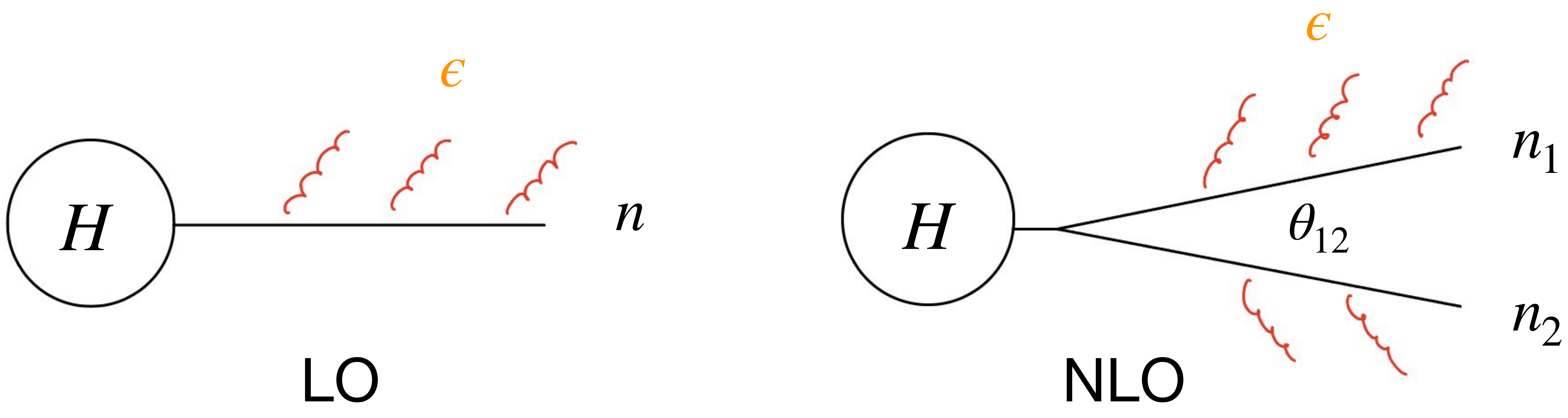
$$S(\{n_i\},\epsilon) \equiv \sum_X \Theta_{\rm alg}\, \delta(\epsilon - \sum \bar{n}\cdot p_{\rm loss})\, \langle {\rm med}|\, \bar{U}_0 U_1^\dagger ... U_m^\dagger\, |X\rangle\langle X|\, \bar{U}_m ... U_1 \bar{U}_0\, |{\rm med}\rangle$$

- Wilson line along the m direction $\quad U_m \equiv P \exp\left[ig\int_0^1 \mathrm{d}s\, n\cdot A_{\rm cs}(sn)\right]$

Y. M.-T., F. Ringer, B. Singh, V. Vaidya, 2409.05957 [hep-ph]

Color decoherence at NLO

- At NLO a hard collinear splitting takes place near to the hard scattering
- The jet substructure is determined by the emerging two-prong structure



- The energy loss probability for a singlet antenna involves 4 Wilson lines

$$P_2^{\mathrm{sing}}(\epsilon) = \frac{1}{d_R} \sum_X \delta(\epsilon - \bar{n} \cdot k_{\mathrm{loss}})\, \mathrm{tr}_c \left[\langle \mathrm{med} | U(n_1) U(n_2) | X \rangle \langle X | U^\dagger(n_2) U^\dagger(n_1) | \mathrm{med} \rangle \right] ,$$

- Solved exactly in the HO approximation: YMT, K. Tywoniuk, Nucl. Phys. A 979 (2018)

Heuristic discussion

Two effective color charges

One effective color charge

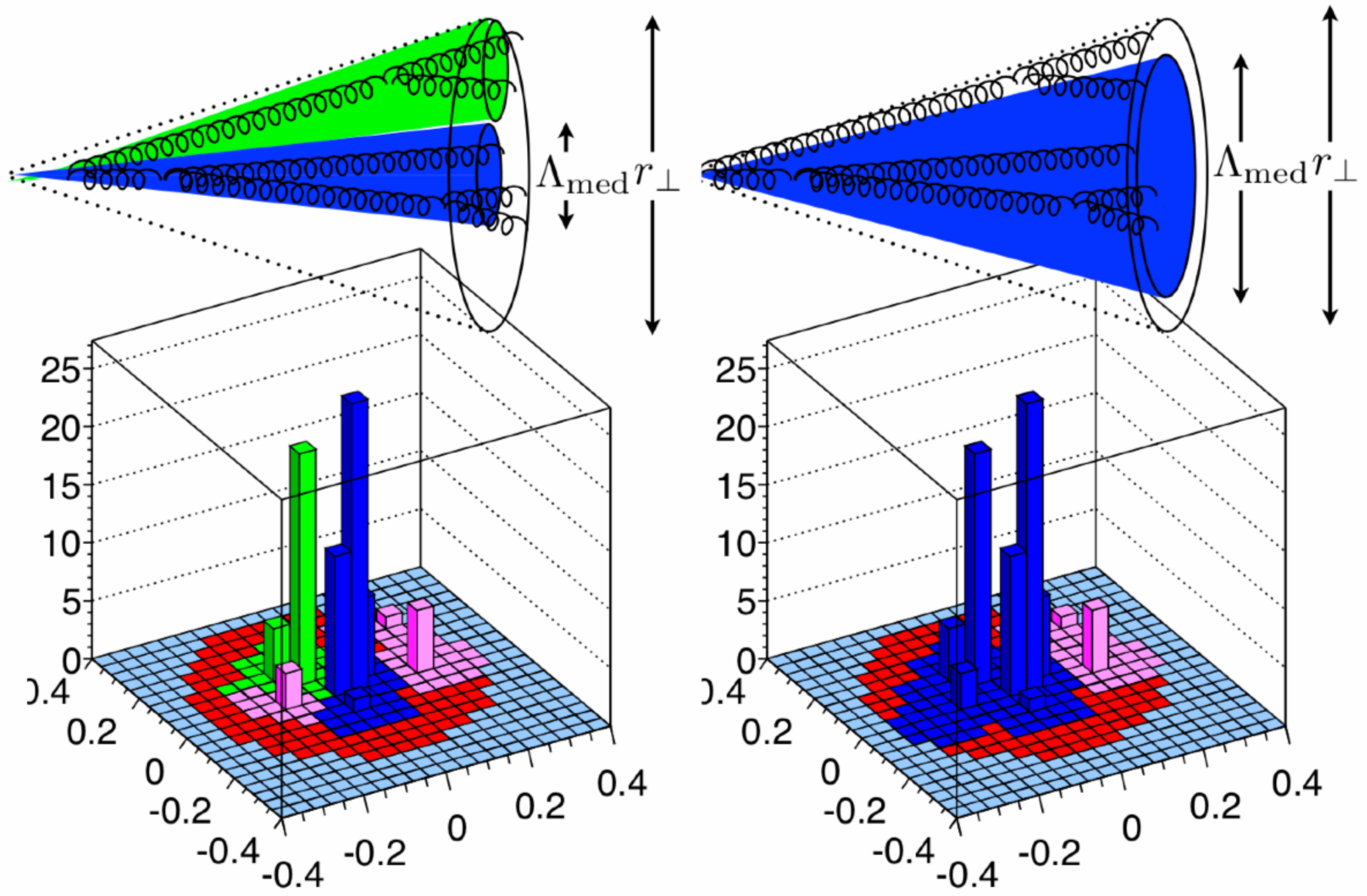
- Color coherence regime: the medium does not resolve the two-prong structure. Energy loss is sensitive to the total color charge (parent parton)

$$\Lambda_{\rm med} \sim Q_{\rm med}^{-1} = (\hat{q}L)^{-1/2} \gg r_\perp = \theta L$$

- Decoherence regime: the medium resolves the two-prong structure.

$$\Lambda_{\rm med} \ll r_\perp$$

MT, Tywoniuk, Salgado, Casalderrey-Solana, **Iancu** (2011-2013)



Credit: Jeniffer James (Vanderbilt)

Heuristic discussion

- To leading logarithmic accuracy one can approximate the two-prong energy loss by

$$P_2^R(\epsilon) \approx \Theta(\theta_c - \theta_{12})\; P_1^R(\epsilon, L) + \Theta(\theta_{12} - \theta_c) \int \mathrm{d}\epsilon_1 \int \mathrm{d}\epsilon_2\, \delta(\epsilon - \epsilon_1 - \epsilon_2)\; P_1^{R_1}(\epsilon_1, L)\, P_1^{R_2}(\epsilon_2, L)$$

Coherence: single-prong

Decoherence: 2-prong

- Where the characteristic angle $\theta_c = \left(\hat{q}L^3\right)^{-1/2}$ marks the transition from color coherence to color decoherence

$$\int_0^R \frac{\mathrm{d}\theta_{12}}{\theta_{12}} \Theta(\theta_{12} - \theta_c) = \ln \frac{R}{\theta_c} \, .$$

- In a dense medium, $\ln \frac{R}{\theta_c} \gg 1$: requires resummation of multiple pronged energy loss

Higher order systematics

- In is convenient to work in Laplace space to turn a convolution of energy loss distributions into a product

$$\int_{\epsilon_1,\ldots,\epsilon_n} P(\epsilon_1)\ldots P(\epsilon_n)\ \delta(\epsilon - \epsilon_1 - \ldots - \epsilon_n) = \int \frac{\mathrm{d}\nu}{2\pi i} \mathrm{e}^{\nu\epsilon}\, Q_\nu^n$$

- We shall present an evolution equation of the Laplace transform of the jet function that encodes energy loss, with $\epsilon = (1-x)E$,

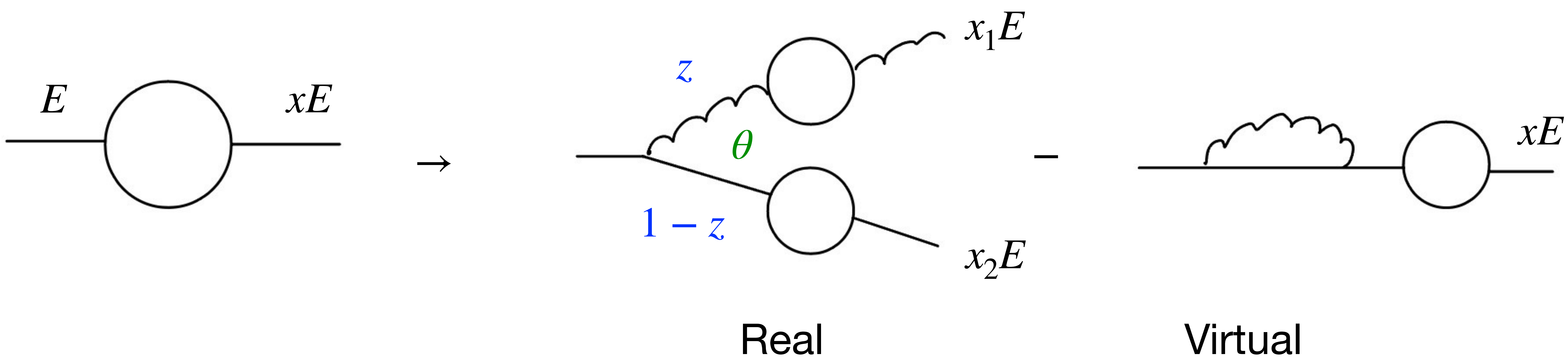
$$J\,(x, E) = E \int \frac{\mathrm{d}\nu}{2\pi i}\, \mathrm{e}^{(1-x)E\nu}\, Q_\nu\,(E)$$

Non-linear DGLAP evolution

YMT, K. Tywoniuk, PRD 98 (2018) 5

- The Laplace transform of the jet function evolves according to a non-linear DGLAP equation for $\theta_c < \theta < R$

$$\theta \frac{\partial}{\partial \theta} Q^q_\nu(E,\theta) = \frac{\alpha_s}{\pi}\int_0^1 dz\, p_{gq}(z)\, [Q^g_\nu(zE,\theta)Q^q_\nu((1-z)E,\theta) - Q^q_\nu(E,\theta)]$$



- $p_{gq}(z)$ is the Altarelli-Parisi slitting function

Non-linear DGLAP evolution

YMT, K. Tywoniuk, PRD 98 (2018) 5

- The Laplace transform of the jet function evolves according to a non-linear DGLAP equation for $\theta_c < \theta < R$

$$\theta \frac{\partial}{\partial \theta} Q_\nu^q(E,\theta) = \frac{\alpha_s}{\pi} \int_0^1 dz\, p_{gq}(z)\, [Q_\nu^g(zE,\theta) Q_\nu^q((1-z)E,\theta) - Q_\nu^q(E,\theta)]$$

- Resums leading $\ln R$ and $\ln R/\theta_c$

- Admits a thermal fixed-point (full energy loss): $Q_\nu^{\mathrm{th}}(E) = \mathrm{e}^{\nu E} \;\rightarrow\; P(\epsilon) = \delta(E)$

- For $R < \theta < 1$ the equation becomes linear since only one branch remains inside the jet, thus recovering Dasgupta, Dreyer, Salam, Soyez (2015) small-R resummation.

Non-linear DGLAP evolution

YMT, K. Tywoniuk, PRD 98 (2018) 5

- The Laplace transform of the jet function evolves according to a non-linear DGLAP equation for $\theta_c < \theta < R$

$$\theta \frac{\partial}{\partial \theta} Q_\nu^q(E,\theta) = \frac{\alpha_s}{\pi} \int_0^1 dz\, p_{gq}(z) \left[Q_\nu^g(zE,\theta) Q_\nu^q((1-z)E,\theta) - Q_\nu^q(E,\theta) \right]$$

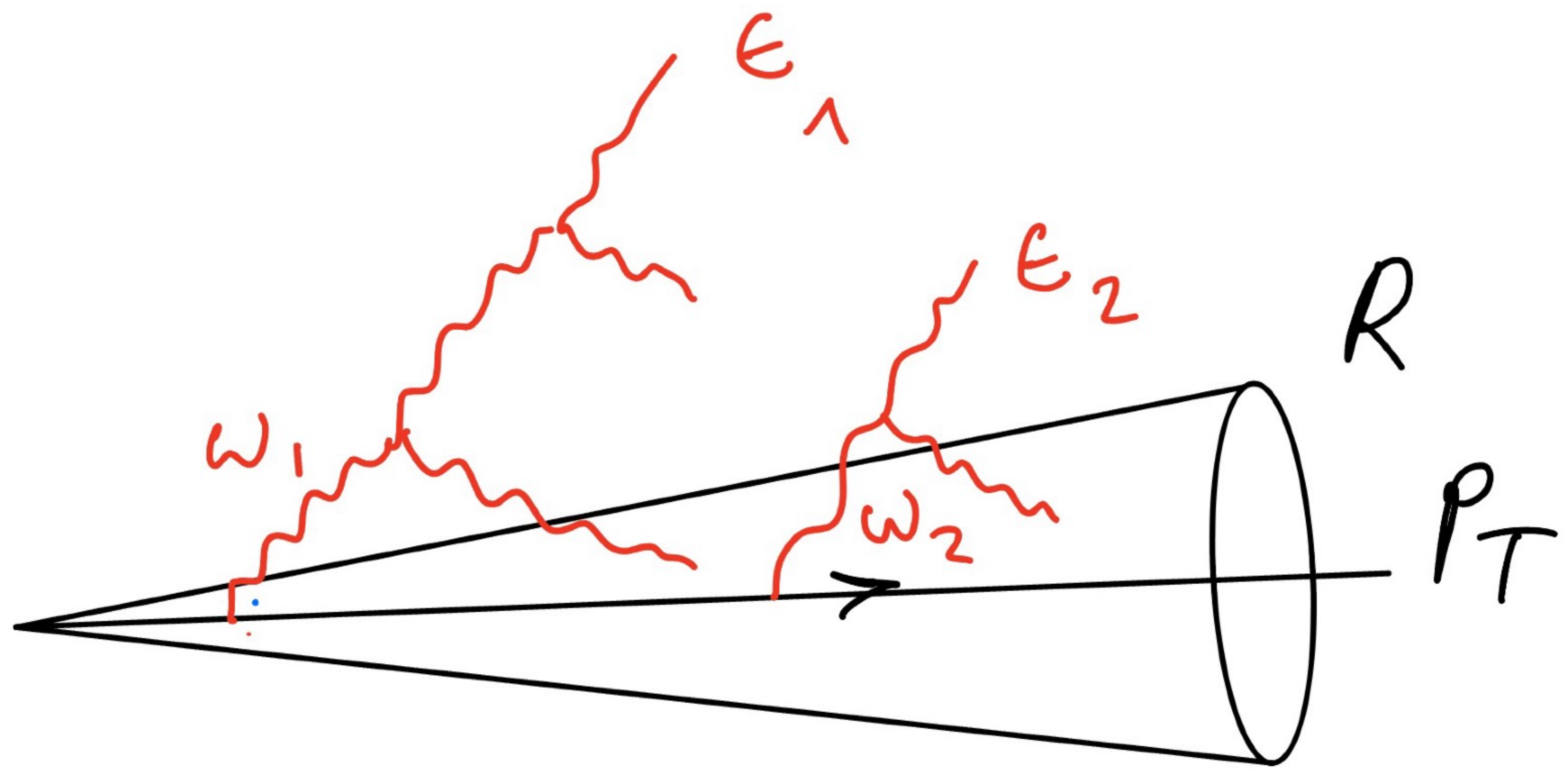
- Sudakov suppression: Linearization in the strong quenching limit

$$\frac{\mathrm{d}\sigma_{\mathrm{AA}}}{\mathrm{d}p_T} \approx \frac{1}{p_T^n} \exp\left(-\frac{2\alpha_s C_F}{\pi} \ln n \ln \frac{p_T R}{\mu_0} \right) Q_{\nu = p_T/n}(p_T, \mu_0)$$

Initial condition:
Single (collinear) parton energy loss

Generalizing the BDMS formalism

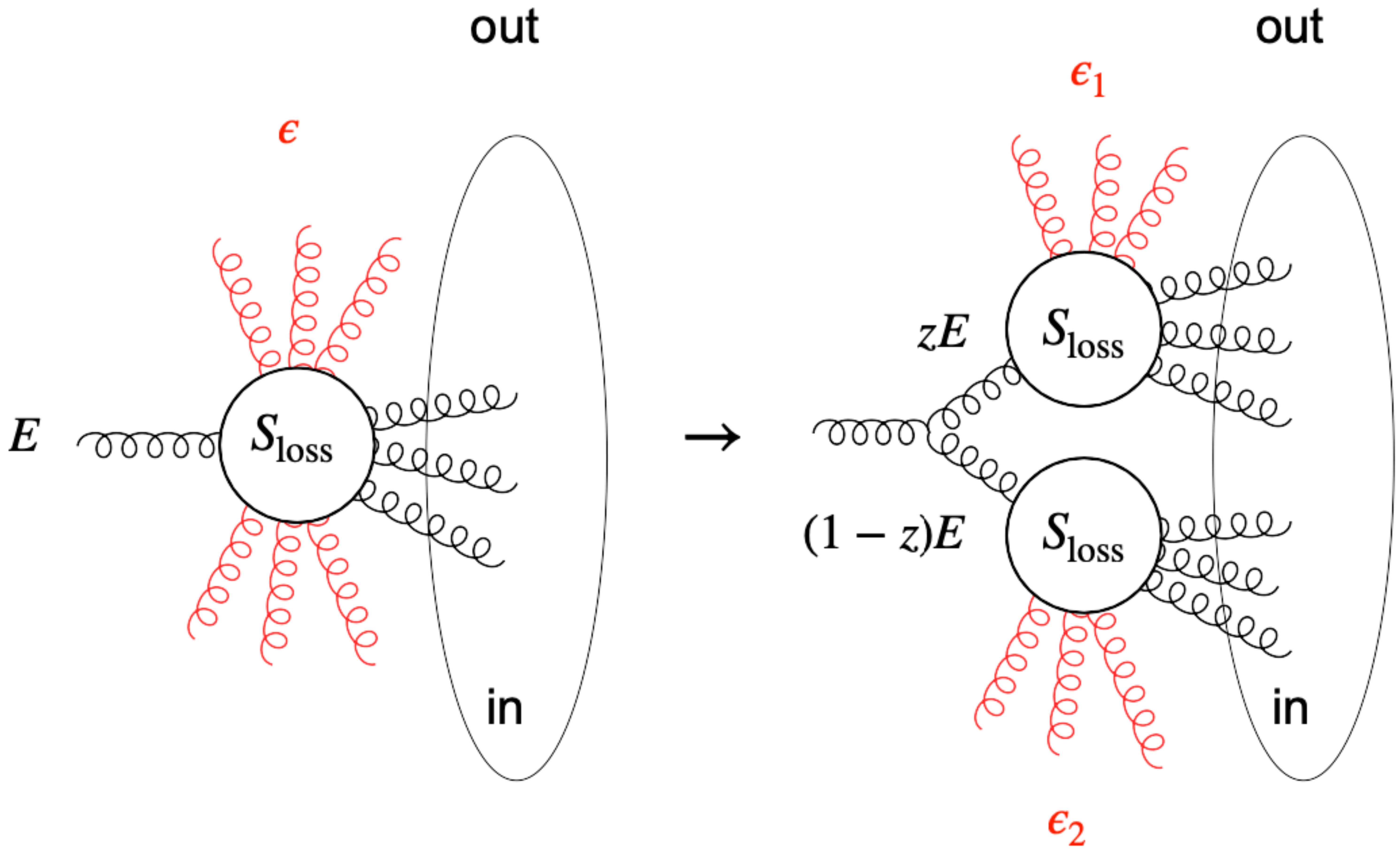
- Independent primary gluon emissions:



$$S_{\rm jet}(\epsilon) = e^{-\int \frac{\mathrm{dI}}{\mathrm{d}\omega}\mathrm{d}\omega} \left[\delta(\epsilon) + \sum_{n=1}^{\infty} \frac{1}{n!} \prod_{i=1}^{n} \int_{\omega_i,\epsilon_i} S_{\rm soft}(\epsilon_1,\omega_1)\, S_{\rm soft}(\epsilon_2,\omega_2) \dots S_{\rm soft}(\epsilon_n,\omega_n)\; \delta(\epsilon - \epsilon_1 - \epsilon_2 - \dots - \epsilon_n) \right]$$

- In the limit $R \to 0$ we recover the BDMS (Baier-Dokshitzer-Mueller-Shiff 2000) result for leading parton energy loss

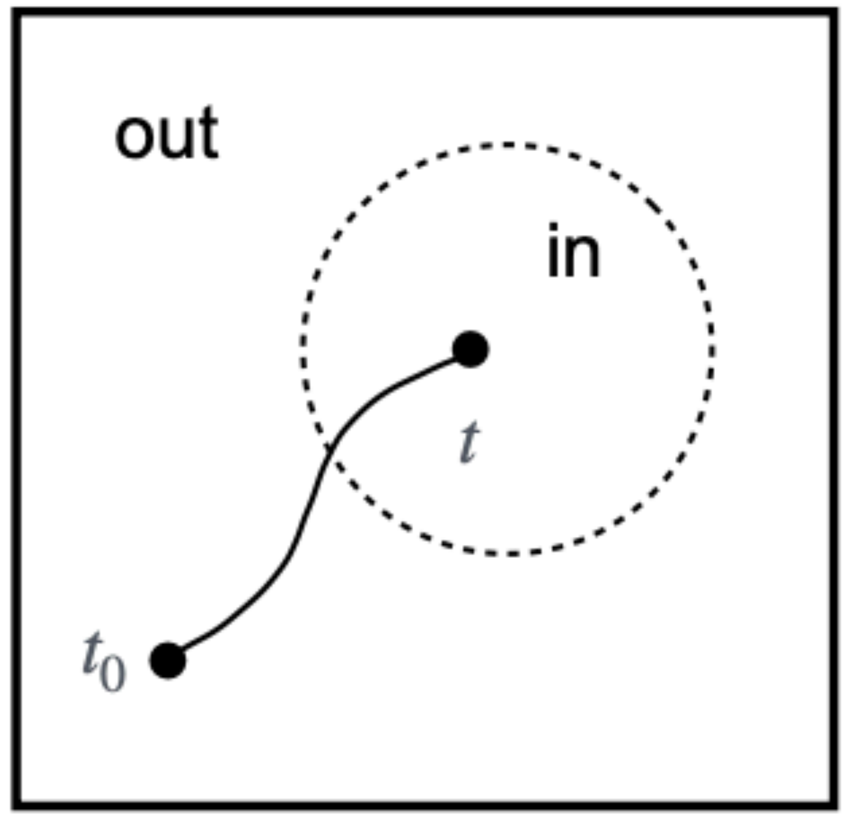
$$S_{\rm soft}(\epsilon,\omega) \;\to\; \delta(\epsilon - \omega)\, \frac{\mathrm{d}I}{\mathrm{d}\omega}$$

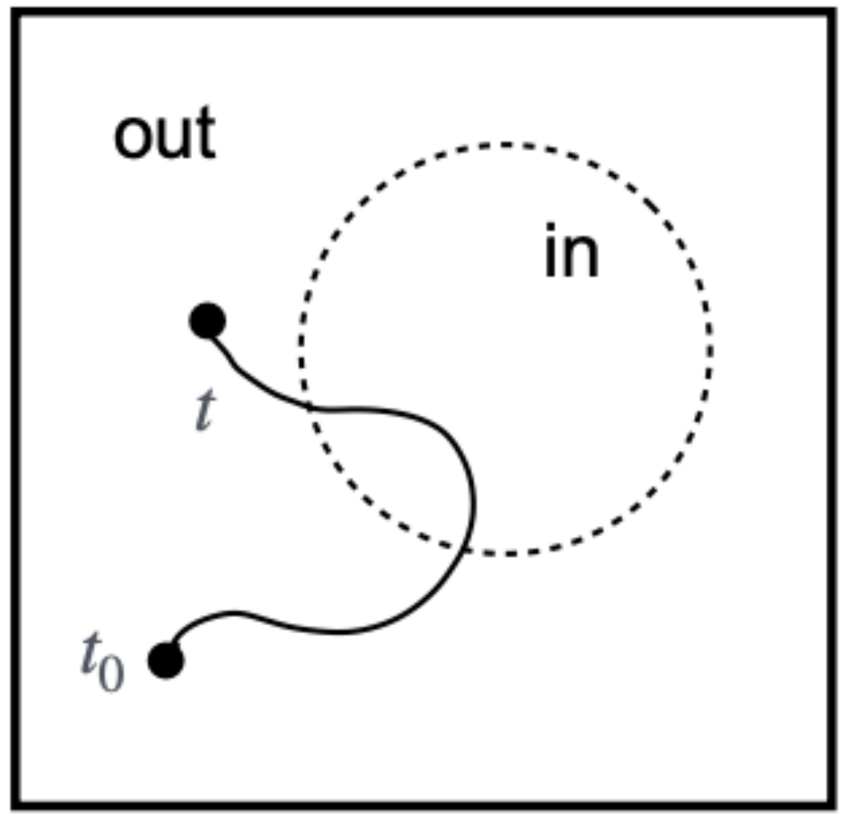


out
ϵ
E
$S_{\rm loss}$
in
$\rightarrow$
out
ϵ_1
zE
$S_{\rm loss}$
$(1-z)E$
$S_{\rm loss}$
in
ϵ_2

Elastic contribution (energy loss via momentum broadening)

$$S_{\rm el}(\epsilon, t, t_0; E, \boldsymbol{\theta}) = \int \frac{{\rm d}^2\boldsymbol{q}}{(2\pi)^2}\,\mathcal{P}(\boldsymbol{q} - E\boldsymbol{\theta}, t, t_0)\,[\Theta(|\boldsymbol{q}| < RE)\,\delta(\epsilon) + \Theta(|\boldsymbol{q}| > RE)\,\delta(E - \epsilon)]$$





NB: The soft gluon with energy E can be radiated radiated at angles larger or small than R – what determines its contrbution to energy loss is its position $\boldsymbol{\theta}$ at $t = L$

Non-linear evolution of the jet energy loss

- In Laplace space: convolutions → products

$$Q_\nu(E) = \int_0^{+\infty} \mathrm{d}\epsilon\, \mathrm{e}^{-\epsilon\nu}\, S_{\rm loss}(\epsilon, E)$$

- Solution:

$$Q_\nu^{\rm hard}(t, t_0; E) = \exp\left\{-\bar{\alpha}\int_{t_0}^{t} \mathrm{d}t_1 \int_0^E \mathrm{d}\omega \frac{\mathrm{d}I}{\mathrm{d}\omega}\left[1 - Q_\nu^{\rm soft}(t, t_1; \omega)\right]\right\}$$

- The soft quenching factor obeys the equation:

$$\begin{aligned} Q_\nu(t, t_0; E, \boldsymbol{\theta}) &= Q_\nu^{\rm el}(t, t_0; E, \boldsymbol{\theta}) + \alpha_s \int_{t_0}^{t} \mathrm{d}t_1 \int_0^1 \mathrm{d}z\, \mathcal{K}(z, E) \int \frac{\mathrm{d}^2\boldsymbol{\theta}'}{(2\pi)^2 E^2}\, \mathcal{P}(E(\boldsymbol{\theta}' - \boldsymbol{\theta}), t_1, t_0) \\ &\times \left[Q_\nu(t, t_1; zE, \boldsymbol{\theta}')\, Q_\nu(t, t_1; (1-z)E, \boldsymbol{\theta}') - Q_\nu(t, t_1; E, \boldsymbol{\theta}')\right]\,. \end{aligned}$$

Conclusions

- We have developed a factorization approach to jet quenching observables that resums the early collinear parton shower and incorporates energy loss and color coherence effects through Wilson line correlators.
- At leading logarithmic accuracy, we have demonstrated that the jet function satisfies a non-linear DGLAP evolution equation.
- This framework lays a robust foundation for systematic higher-order computations of jet observables in heavy-ion collisions. However, significant uncertainties remain due to non-universal soft physics, which necessitates careful modeling.

Happy birthday, Edmond!