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Probing fluctuating color fields of QCD matter from spin alignment in relativistic heavy ion collisions

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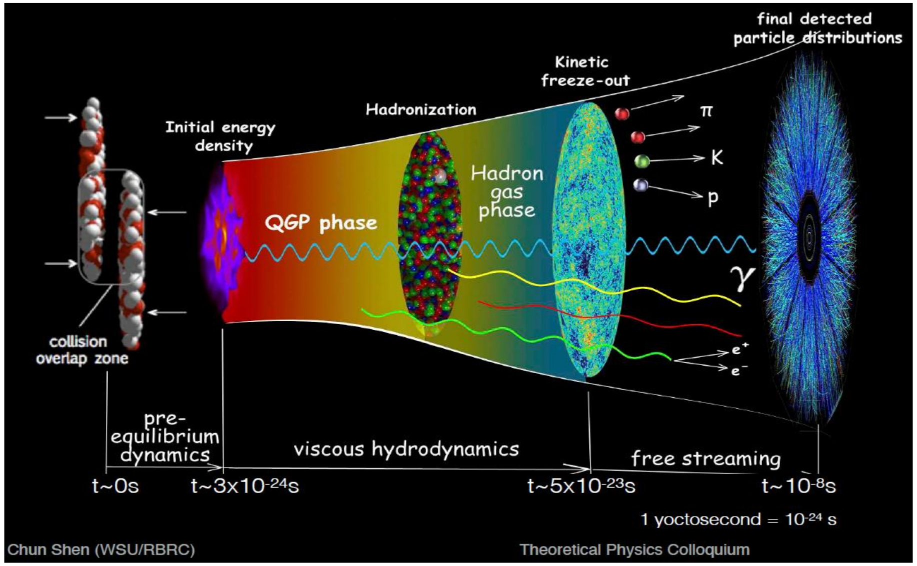
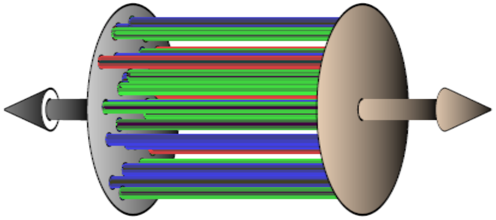
(GGI, March 25, 2025)

"Happy birthday, Edmond!"

CGC in heavy ion collisions

- Glasma phase provides the initial condition for hydro evolution

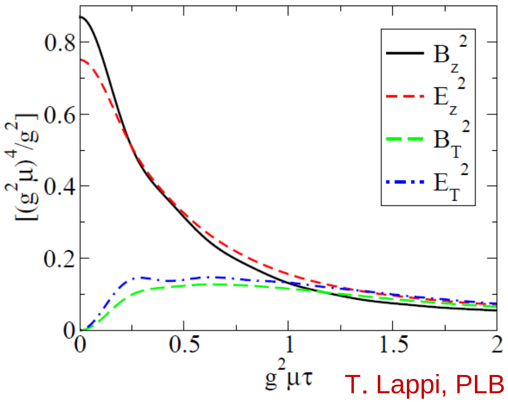




Reviews : F. Gelis, E. Iancu, J. Jalilian-Marian, R. Venugopalan, Ann.Rev.Nucl.Part.Sci.60:463-489,2010

J. Berges et al., Rev. Mod. Phys. 93 (2021) 3, 035003

$$[D_\mu, F^{\mu\nu}] = J^\nu = \delta^{\nu+}\delta(x^-)\rho_1(x_\perp) + \delta^{\nu-}\delta(x^+)\rho_2(x_\perp)$$



T. Lappi, PLB 643 (2006) 11-16

❖ How to probe the color (glasma) fields in early times form spin transport?

❖ How the color (glasma) fields may resolve the puzzle for spin alignment phenomena observed in HIC?

Outline

- Spin polarization and spin alignment phenomena in HIC
- Quantum kinetic theory for spin transport of quarks under background color fields (with the weak-field approx.)
- Estimation on the spin alignment spectrum for ϕ mesons from the glasma effect in HIC
- Summary and outlook

main references :

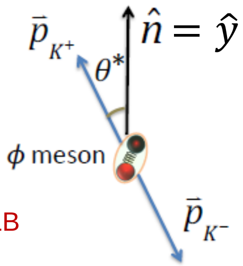
A. Kumar, B. Müller, DY, PRD 108, 016020 (2023), arXiv:2304.04181
A. Kumar, B. Müller, DY, PRD 107, 076025 (2023), arXiv:2212.13354
DY, PRD 110, 056005 (2025), arXiv:2411.14822

Spin alignment of vector mesons

- Production of the decay daughter w.r.t the quantization axis :

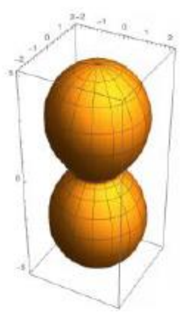
$$\frac{dN}{d\cos\theta^*} \propto [1-\rho_{00}+\cos^2\theta^*(3\rho_{00}-1)]$$

$$\rho_{00} = \frac{1-\langle \mathcal{P}_q^y \mathcal{P}_{\bar{q}}^y \rangle}{3+\langle \mathcal{P}_q^y \mathcal{P}_{\bar{q}}^y \rangle}$$

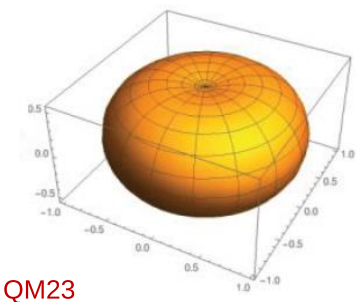


Z.-T. Liang, X.-N. Wang, PRL. 94, 102301 (2005), PLB 629, 20 (2005)

$\rho_{00} > 1/3$:

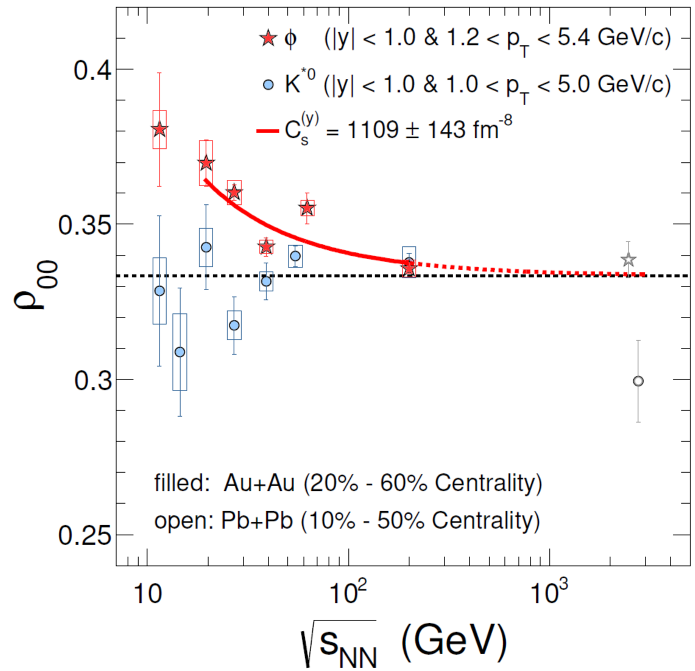


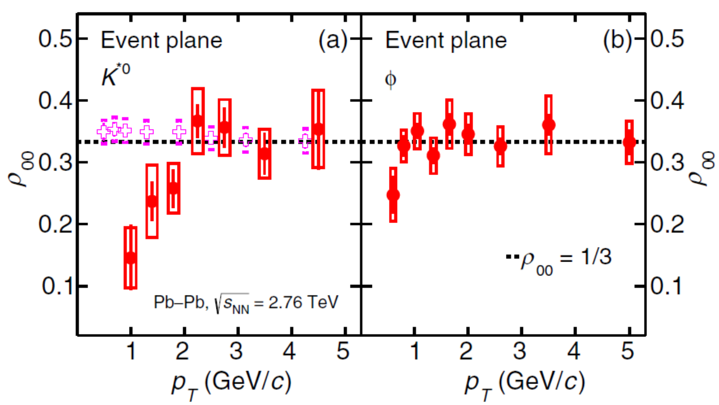
$\rho_{00} < 1/3$:



B. Xi, QM23

$\rho_{00} \neq 1/3$: spin polarization





S. Acharya et al. (ALICE), PRL.125, 012301 (2020)

M.S. Abdallah et al. (STAR), Nature 614 (2023) 7947, 244-248,

Spin polarization beyond subatomic swirls?

- Spin alignment puzzle : the deviation of ρ_{00} from 1/3 is unexpectedly large

  e.g. $\rho_{00} \approx \frac{1}{3} - \left(\frac{\omega}{T}\right)^2$, $\frac{\omega}{T} \sim 0.1\%$ at LHC energy. (from Λ pol.~ s quark pol.)

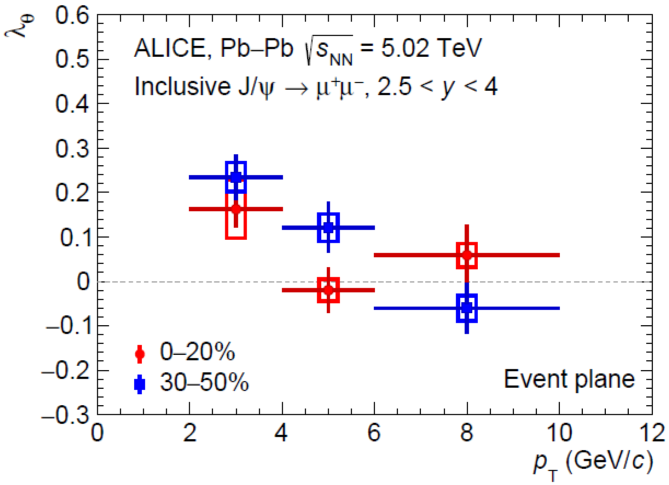
- Flavor & collision energy dep. :

	ϕ	K^{*0}
ALICE	$\rho_{00} < 1/3$ ($p_T \leq 1$ GeV)	$\rho_{00} < 1/3$
STAR	$\rho_{00} > 1/3$	$\rho_{00} \approx 1/3$

- Spin alignment for J/ψ :

  S. Acharya et al. , PRL 131,042303 (2023)

$$\lambda_\theta = \frac{1 - 3\rho_{00}}{1 + \rho_{00}} > 0 \Rightarrow \rho_{00} < \frac{1}{3}$$



- Other sources for the spin correlation (alignment) beyond vorticity?

From spin correlations to spin alignment

❑ Electromagnetic fields can polarize the spin. How about gluonic fields in QCD matter?

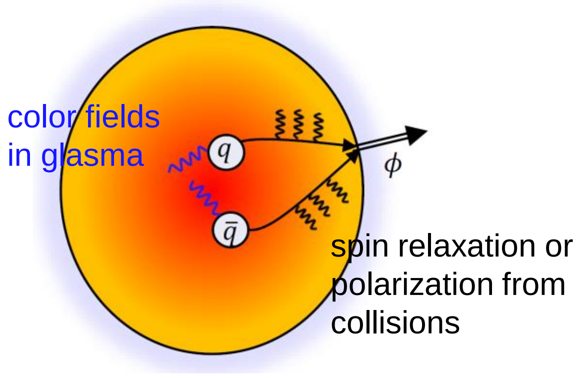
$\mathcal{P}^i(\boldsymbol{p}) \propto c_1 B^i + c_2 \epsilon^{ijk} p_j E_k$ (non-dynamical pol. at the present time)

- Large spin alignment (spin correlations) v.s. small Λ spin polarization (spin polarization of a single strange quark)

  ⇨ the sources for spin alignment are fluctuating

- Spin quantization axis needs not be parallel to the spin polarization (or correlation)
- Anisotropic spin correlation is needed : $\rho_{00} = 1/3$ when $\langle \mathcal{P}_q^j \mathcal{P}_{\bar{q}}^j \rangle \neq 0$ is isotropic.

❖ Why glasma fields :
(1) intrinsic saturation scale $Q_s \gg \omega$
(2) fluctuating
(3) intrinsic anisotropy



- An order of magnitude estimate for the glasma effect ?

- How to track the spin evolution of massive quarks in phase space?
  quantum kinetic theory from Wigner functions

Review (w/o spin) : J. Blaizot, E. Iancu, Phys.Rept. 359, 355-528 (2002)

Review (with spin) : Y. Hidaka, S. Pu, Q, Wang, DY, PPNP 127, 103989 (2022)

Kadanoff-Baym eq. ⟹ kinetic eq.

semi-classical approx.

Axial kinetic theory

- Vector/axial-vector components :

$$\mathcal{V}^\mu(p,X) = \frac{1}{4}\mathrm{tr}\left(\gamma^\mu S^<(p,X)\right), \quad \mathcal{A}^\mu(p,X) = \frac{1}{4}\mathrm{tr}\left(\gamma^\mu\gamma^5 S^<(p,X)\right)$$

$$\mathcal{P}^\mu(\boldsymbol{p}) = \frac{\int d\Sigma\cdot p\mathcal{A}^\mu(\boldsymbol{p},X)}{2m\int d\Sigma\cdot\mathcal{V}(\boldsymbol{p},X)} \qquad \mathcal{A}^\mu(\boldsymbol{p},x) = \int\frac{dp_0}{2\pi}\mathcal{A}^\mu(p,x)$$

$\hbar$ expansion

- Vector WF: $\mathcal{V}^\mu(\boldsymbol{p},x) = \frac{p^\mu}{2\epsilon_\mathrm{p}} f_V(p,x)\big|_{p_0=\epsilon_{\boldsymbol{p}}=\sqrt{|\boldsymbol{p}|^2+m^2}} + \mathcal{O}(\partial/p),$

- Axial WF: $\mathcal{A}^\mu(\boldsymbol{p},x) = \frac{1}{2\epsilon_{\boldsymbol{p}}}\left[\tilde{a}^\mu - \frac{\hbar}{2}e\tilde{F}^{\mu\nu}\left(\partial_{p\nu}f_V - \frac{\epsilon_{\boldsymbol{p}}}{2}\partial_{p_\perp\nu}(f_V/\epsilon_{\boldsymbol{p}})\right)\right]_{p_0=\epsilon_{\boldsymbol{p}}=\sqrt{|\boldsymbol{p}|^2+m^2}}.$

dynamical (w/ memory effect) non-dynamical (w/o memory effect)

($\tilde{a}^\mu(p,x)$: effective spin four vector)

- Axial kinetic theory : scalar/axial-vector kinetic eqs. (SKE/AKE)

K. Hattori, Y. Hidaka, DY, PRD 100, 096011 (2019)
DY, K. Hattori, Y. Hidaka, JHEP 20, 070 (2020)

- SKE : $p^\rho\left(\partial_\rho + eF_{\nu\rho}\partial_p^\nu\right)f_V = \mathcal{C},$

standard Vlasov eq.

(dynamical source term)

- AKE : $p^\rho\left(\partial_\rho + eF_{\nu\rho}\partial_p^\nu\right)\tilde{a}^\mu + eF^{\nu\mu}\tilde{a}_\nu - \frac{e}{2}\hbar\epsilon^{\mu\nu\rho\sigma}p_\rho(\partial_\sigma F_{\beta\nu})\partial_p^\beta f_V = \mathcal{C}^\mu.$

- Generalization to include color dof. :

DY, JHEP 06, 140 (2022)
B. Müller, DY, PRD 105, L011901 (2022)

- (color-octet) axial WF : $\mathcal{A}^{ai}(\boldsymbol{p},x) \approx -\frac{\hbar g}{4\epsilon_{\boldsymbol{p}}}e^{\frac{-\Delta t_{QGP}}{\tau_\mathrm{R}^\mathrm{O}}}\left[B^{ai}(t_\mathrm{i}) - \frac{\epsilon^{ijk}p_j E_k^a(t_\mathrm{i})}{\epsilon_{\boldsymbol{p}}}\right]\partial_{\epsilon_{\boldsymbol{p}}}f_V^\mathrm{S}(\epsilon_{\boldsymbol{p}},t_\mathrm{i})$

(no contribution to Λ pol.)

spin relaxation in QGP spin polarization from glasma fields

Spin alignment from glasma

- Out-of-plane spin alignment from longitudinal glasma fields ($t_i = 0$) :

$$\rho^{\mathrm{o}}_{00} \approx \frac{1 - C_g\left(C_{B1}\chi_B^{zz} + C_{E1}\chi_E^{zz}\right)}{3 - C_g\left[\left(3C_{B1} + C_{B2}\right)\chi_B^{zz} + \left(3C_{E1} + C_{E2}\right)\chi_E^{zz}\right]}, \quad p_\pm = \frac{1}{2M}\left(M^2 \pm (m_q^2 - m_{\bar{q}}^2)\right),$$

$$\chi_B^{ij} = \left\langle B^{ai}(0, \boldsymbol{x} + \boldsymbol{k}t^{\mathrm{th}}/p_+) B^{aj}(0, \boldsymbol{x} - \boldsymbol{k}t^{\mathrm{th}}/p_-) \right\rangle_{|\boldsymbol{k}| = \tilde{k}}, \quad \tilde{k} = \frac{1}{2M}\left[(m_q^2 - m_{\bar{q}}^2)^2 + M^4 - 2M^2(m_q^2 + m_{\bar{q}}^2)\right]^{1/2}$$

- Correlation of initial glasma fields :

$$\chi_B^{zz} = \chi_E^{zz} = \chi_{\mathrm{GBW}} \equiv \frac{Q_s^4(N_c^2 - 1)}{2g^2 N_c}\left(\frac{1 - e^{-Q_s^2\tilde{k}^2(t^{\mathrm{th}}\lambda_m)^2/4}}{Q_s^2\tilde{k}^2(t^{\mathrm{th}}\lambda_m)^2/4}\right)^2 \xrightarrow[\tilde{k} \to 0]{} \frac{Q_s^4(N_c^2 - 1)}{2g^2 N_c}$$

T. Lappi, S. Schlichting, PRD 97, 034034 (2018)

K. J. Golec-Biernat, M. Wusthoff, PRD 59, 014017 (1998)

- Initial quark distribution function : $f_V^{\mathrm{S}}(\epsilon_{\boldsymbol{p}}, t_{\mathrm{i}}) = \left(e^{\epsilon_{\boldsymbol{p}}/Q_s} + 1\right)^{-1}$

❖ Spin alignment (mesons at rest) : $\delta\rho^{\mathrm{o}}_{00} \equiv \rho^{\mathrm{o}}_{00} - \frac{1}{3} \sim \frac{Q_s^2}{m_q m_{\bar{q}}} e^{-2\Delta t_{QGP}/\tau^{\mathrm{o}}_{\mathrm{R}}}$

❖ Considering ϕ mesons with $Q_s \sim 2$ GeV & $m_{q/\bar{q}} \sim 0.5$ GeV

❖ Heavy-quark approx. & pQCD : $\tau^{\mathrm{o}}_{\mathrm{R}} \approx \left(\frac{g^2 C_2(F) m_D^2 T}{6\pi m^2} \ln g\right)^{-1} \approx 5\,\mathrm{fm/c}$

M. Hongo et al., JHEP 08, 263 (2022)

Phenomenological estimation : S. Banerjee et al., arXiv:2405.05089

(model dependent)

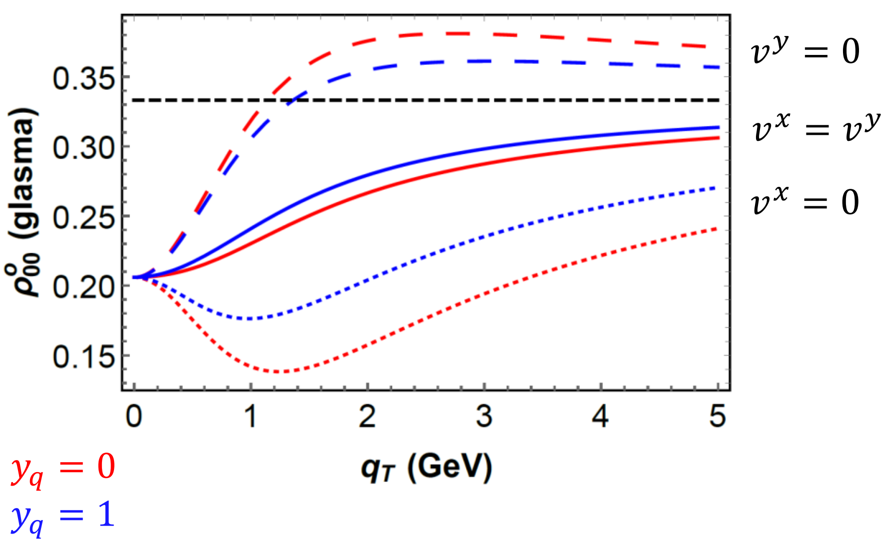
Transverse spin alignment spectra

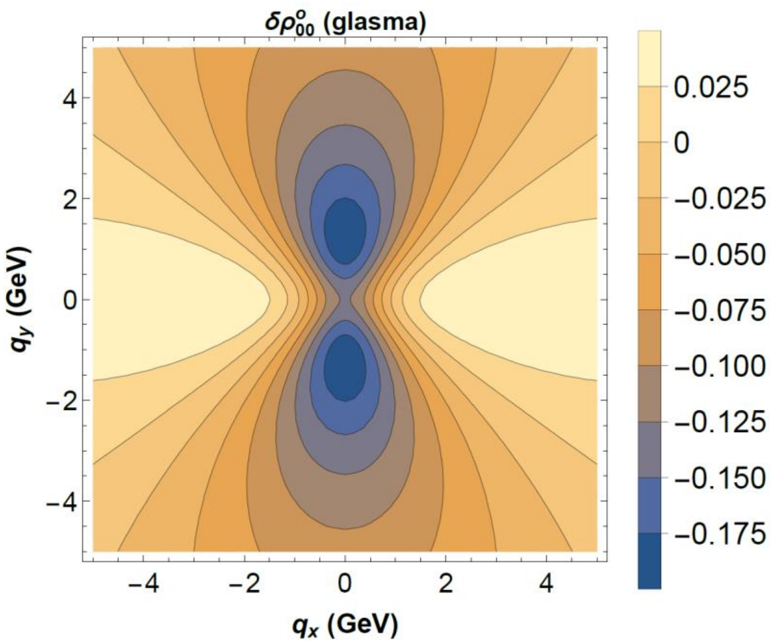
- Boost the color fields to the lab frame to retrieve momentum dependence

$$B_{\rm r}^{ai} = \gamma(B^{ai} + \epsilon^{ijk} v_j E_k^a) - (\gamma - 1)\boldsymbol{B}^a \cdot \hat{\boldsymbol{v}}\hat{v}^i, \quad v^i = q^i/\sqrt{|\boldsymbol{q}|^2 + M^2} \text{ and } \hat{v}^i = v^i/|\boldsymbol{v}|.$$

$$\Rightarrow \quad \langle B_{\rm r}^{ai} B_{\rm r}^{ai}\rangle = \gamma^2\big(\langle B^{ai}B^{ai}\rangle + \epsilon^{ijk}v_j\epsilon^{ij'k'}v_{j'}\langle E_k^a E_{k'}^a\rangle\big) - 2\gamma(\gamma-1)\langle B^{ai}B^{ai}\rangle\hat{v}_i^2 + (\gamma-1)^2\hat{v}_i^2\sum_{j=x,y,z}\langle B^{aj}B^{aj}\rangle$$

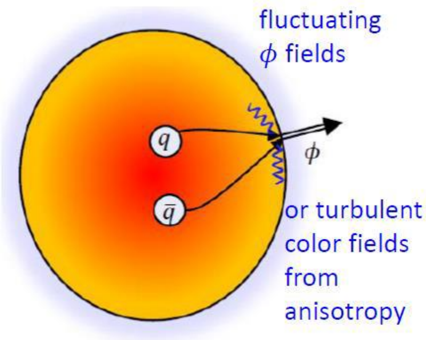
- Out-of-plane spin alignment :

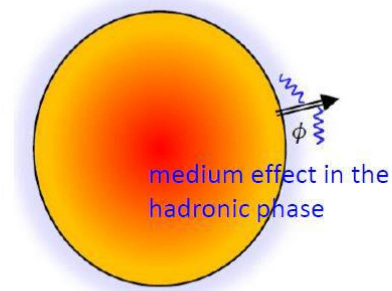




Color fields in the QGP phase

- Late-time effects :





X.-L. Sheng et al., PRD 109, 036004, (2024)
PRL 131, 042304 (2023)
B. Müller, DY, PRD 105, L011901 (2022)
DY, JHEP 06, 140 (2022)

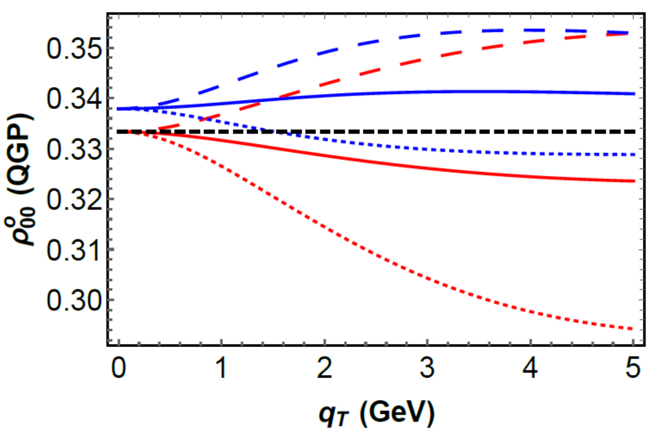
D. Wagner, N. Weickgenannt, E. Speranza,PRR 5, 013187 (2023)
F. Li, S. Liu, arXiv:2206.11890

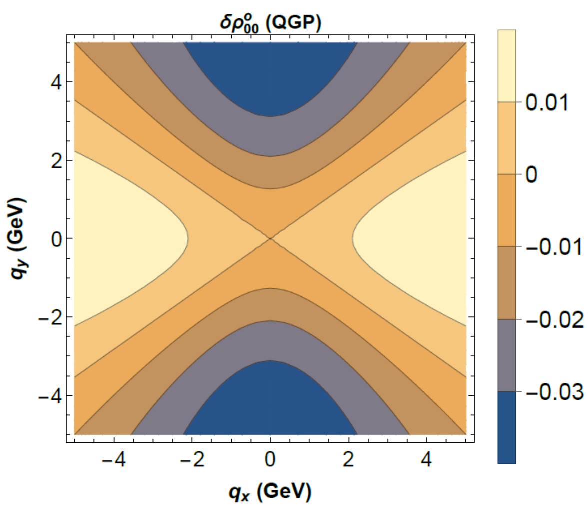
- Non-dynamical polarization : from isotropic color fields

A. Kumar, Philipp Gubler, DY, PRD 109, 054038 (2024)

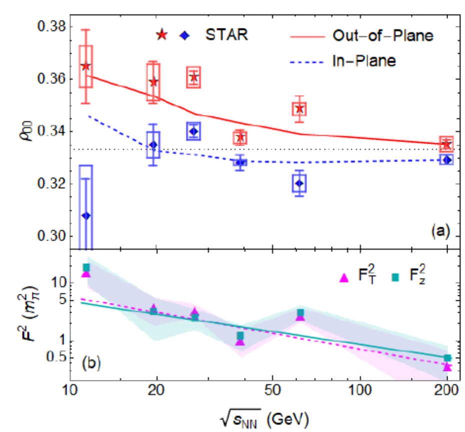
$$\mathcal{A}^{ai}(\boldsymbol{p},x) \approx -\frac{\hbar g}{4\epsilon_{\boldsymbol{p}}}\left[B^{ai}(x_0)\partial_{\epsilon_{\boldsymbol{p}}} + \frac{\epsilon^{ijk}p_j E^a_k(x_0)}{2\epsilon_{\boldsymbol{p}}}\left(\partial_{\epsilon_{\boldsymbol{p}}} + \epsilon_{\boldsymbol{p}}^{-1}\right)\right] f^{\mathrm{s}}_V(\epsilon_{\boldsymbol{p}}, x_0)$$

➢ Out-of-plane spin alignment (qualitative):



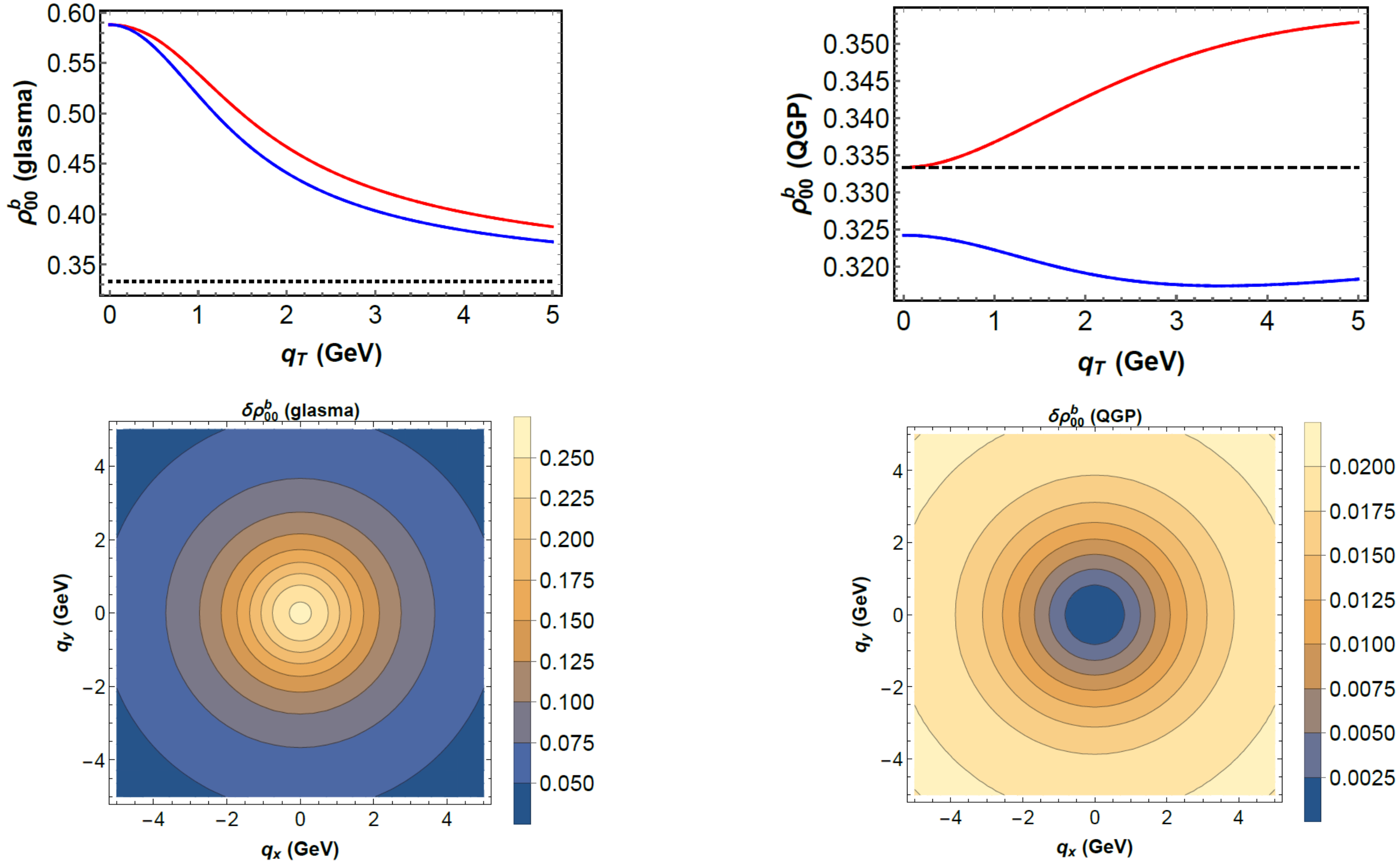


X.-L. Sheng et al., PRL 131, 042304 (2023)



Longitudinal spin alignment

- Spin alignment along the beam direction :



Summary & outlook

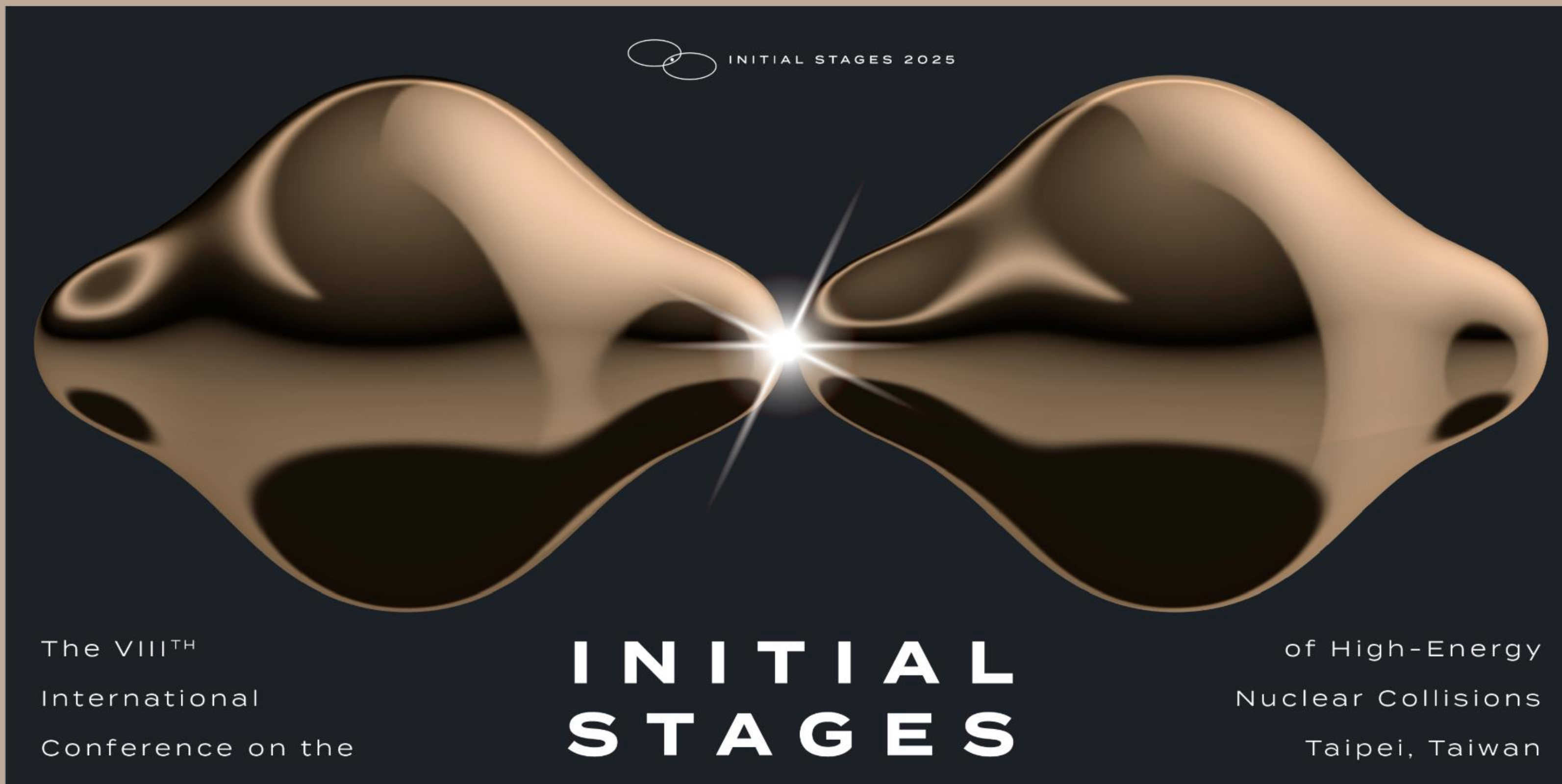
❑ Summary :

- ✓ In general, spin alignment may be a useful probe for strong interaction forces led by gluons in QCD matter.
- ✓ Combining the QKT and glasma fields (or color fields in QGP), we can study the transverse & longitudinal spin alignment of vector mesons.
- ✓ However, the order-of-magnitude estimate is sensitive to the initial distribution function of quarks, spin relaxation time, and subject to the weak color fields in QKT.

❑ Outlook :

- ➢ Spin alignment of quarkonia like J/ψ : more applicable for the kinetic theory in a medium.
- ➢ Scale separation : EFT + open quantum system approach
  E.g., DL & X. Yao, "Quarkonium Polarization in Medium from Open Quantum Systems and Chromomagnetic Correlators", PRD 110, 074037 (2024).
- ➢ "High-multiplicity" events in proton-nucleus collisions?
  NRQCD+CGC : Y.-Q. Ma, T. Stebel, R. Venugopalan, JHEP 12 (2018) 057
  T. Stebel, K. Watanabe, PRD 104, 034004 (2021)

IS 2025 in Taipei, Sep. 7-12



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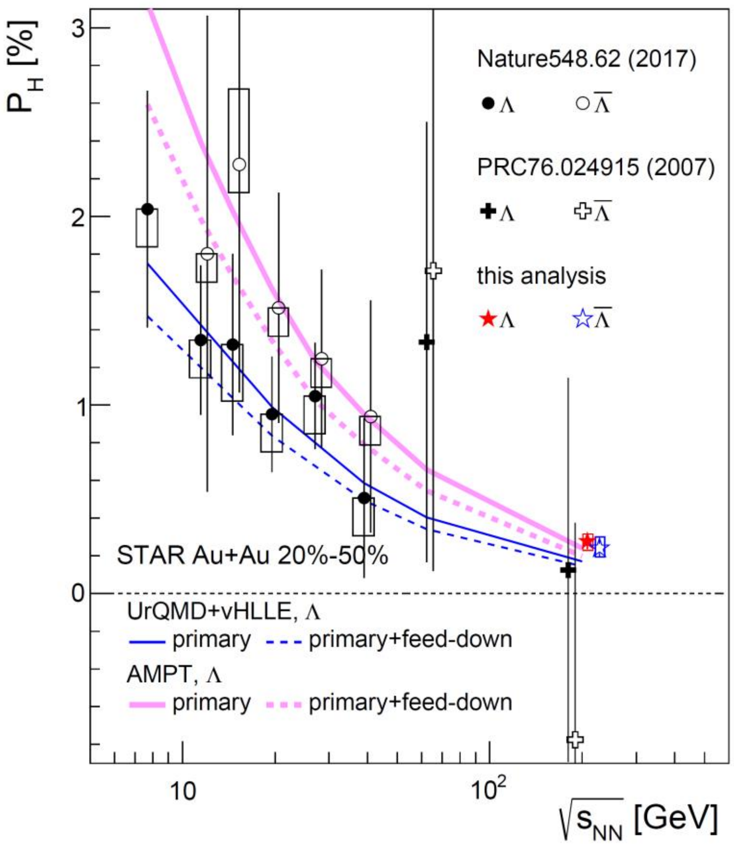
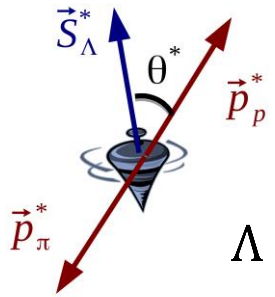
Global Λ polarization in HIC

- The large AM generated in HIC could induce spin polarization of the QGP via spin-orbit interaction. (relativistic Barnett effect) Z.-T. Liang and X.-N. Wang, PRL. 94, 102301 (2005)
- Global polarization of Λ hyperons :



L. Adamczyk et al. (STAR), Nature 548, 62 (2017)

❖ Self-analyzing via the weak decay :



$$\Lambda \rightarrow p + \pi^-$$

T. Niida, QM18

❖ In global equilibrium : F. Becattini, et al., Ann. Phys. 338, 32 (2013) R. Fang, et al., PRC 94, 024904 (2016)

$$\mathcal{P}^\mu(\boldsymbol{p}) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma}p_\nu \frac{\int d\Sigma\cdot p\omega_{\rho\sigma}f_p^{(0)}(1-f_p^{(0)})}{\int d\Sigma\cdot p f_p^{(0)}},$$

$$\omega_{\mu\nu} = \frac{1}{2}(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu).$$

thermal vorticity $(\beta^\mu \equiv u^\mu/T)$

❖ Indication of strong (kinetic) vorticity :

$$P_{\Lambda(\bar{\Lambda})} \simeq \frac{1}{2}\frac{\omega}{T} \pm \frac{\mu_\Lambda B}{T} \Longrightarrow \omega \sim 10^{22}\,\mathrm{s}^{-1}$$

F. Becattini et al., PRC95, 054902 (2017)

Wigner functions & kinetic theory

- Spin polarization related to the axial current :

Noether's theorem ⇨ $M_S^{\lambda\mu\nu} = \frac{1}{2}\bar{\psi}\{\gamma^\lambda, \Sigma^{\mu\nu}\}\psi = -\frac{1}{2}\epsilon^{\lambda\mu\nu\rho}\bar{\psi}\gamma_\rho\gamma_5\psi$

- Dynamical polarization of quarks in a medium : phase space info is needed
- Wigner functions and kinetic theory : review : J. Blaizot, E. Iancu, Phys.Rept. 359 (2002) 355-528

$$S^<(p,x) = \int d^4Y e^{\frac{ip\cdot y}{\hbar}} \langle\bar{\psi}(x_2)U(x_2,x_1)\psi(x_1)\rangle, \quad x = \frac{x_1+x_2}{2}, \quad y = x_1 - x_2.$$

➢ Kadanoff-Baym (KB) equation with the $\hbar$ expansion : $(\partial/q \ll 1)$

$$(\not{\Pi} - m)S^< + \gamma^\mu i\frac{\hbar}{2}\nabla_\mu S^< = \frac{i\hbar}{2}\left(\Sigma^< \star S^> - \Sigma^> \star S^<\right)$$

(master Eq. for WFs & KEs)

$$\nabla_\mu = \Delta_\mu + \mathcal{O}(\hbar^2),$$
$$\Delta_\mu = \partial_\mu + F_{\nu\mu}\partial/\partial q_\nu$$
$$\Pi^\mu = q^\mu + \mathcal{O}(\hbar^2)$$
$$A \star B = AB + \frac{i\hbar}{2}\{A,B\}_{\rm P.B.} + \mathcal{O}(\hbar^2)$$

➢ Spin-independent case (LO) :

$$S^<(q,x) = 2\pi\delta(q^2 - m^2)(\not{q} - m)f_V(q,x),$$
$$q^\mu(\partial_\mu + eF_{\nu\mu}\partial_q^\nu)f_V(q,x) = C[f_V(q,x)].$$

⇨ $J^\mu(x) = \frac{1}{2}\int\frac{d^4q}{(2\pi)^4}{\rm tr}\left(\gamma^\mu S^<(q,x)\right)$

Spin correlations from color fields

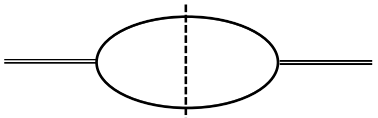
- Spin density matrix can be directly related to Wigner functions of the coalesced quark and antiquark through the quark-meson interaction.

❖ Kinetic theory of vector mesons :

$$q\cdot\partial f^{\phi}_{\lambda}=\epsilon^{*}_{\mu}(\lambda,\boldsymbol{q})\epsilon_{\nu}(\lambda,\boldsymbol{q})\left[\mathcal{C}^{\mu\nu}_{\mathrm{coal}}(q,x)(1+f^{\phi}_{\lambda})-\mathcal{C}^{\mu\nu}_{\mathrm{diss}}(q,x)f^{\phi}_{\lambda}\right]\approx\boxed{\epsilon^{*}_{\mu}(\lambda,\boldsymbol{q})\epsilon_{\nu}(\lambda,\boldsymbol{q})\mathcal{C}^{\mu\nu}_{\mathrm{coal}}(q,x)}$$

quark-meson int. :

$$\mathcal{L}_{\mathrm{int}}=g_{\phi}\bar{\psi}\gamma^{\mu}V_{\mu}\psi$$



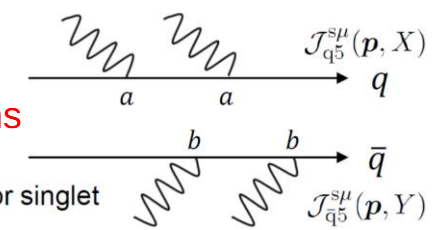
$$\rho_{00}(q)=\frac{\int d\Sigma_X\cdot q f^{\phi}_{0}(q,X)}{\int d\Sigma_X\cdot q\big(f^{\phi}_{0}(q,X)+f^{\phi}_{+1}(q,X)+f^{\phi}_{-1}(q,X)\big)}$$
$$=\frac{1-\mathrm{Tr}_{\mathrm{c}}\langle\hat{\mathcal{P}}^{y}_{q}(\boldsymbol{q}/2)\hat{\mathcal{P}}^{y}_{\bar{q}}(\boldsymbol{q}/2)\rangle_{\boldsymbol{q}=0}}{3-\sum_{i=x,y,z}\mathrm{Tr}_{\mathrm{c}}\langle\hat{\mathcal{P}}^{i}_{q}(\boldsymbol{q}/2)\hat{\mathcal{P}}^{i}_{\bar{q}}(\boldsymbol{q}/2)\rangle_{\boldsymbol{q}=0}}$$

❖ Spin correlations in the non-relativistic limit :

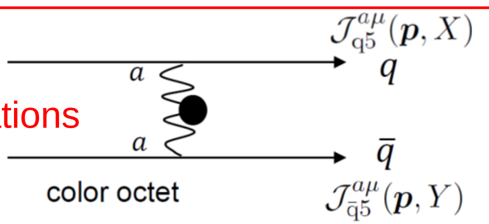
$$\mathrm{Tr}_{\mathrm{c}}\langle\hat{\mathcal{P}}^{i}_{q}(\boldsymbol{p})\hat{\mathcal{P}}^{i}_{\bar{q}}(\boldsymbol{p})\rangle\approx\frac{4\int d\Sigma_X\cdot p\big(\langle\mathcal{A}^{si}_{q}(\boldsymbol{p},X)\mathcal{A}^{si}_{\bar{q}}(\boldsymbol{p},X)\rangle+\langle\mathcal{A}^{ai}_{q}(\boldsymbol{p},X)\mathcal{A}^{ai}_{\bar{q}}(\boldsymbol{p},X)\rangle/(2N_c)\big)}{\int d\Sigma_X\cdot p f^{\mathrm{s}}_{\mathrm{V}q}(\boldsymbol{p},X)f^{\mathrm{s}}_{\mathrm{V}\bar{q}}(\boldsymbol{p},X)}$$

➢ Weak coupling :

4-field correlations $\propto F^4$



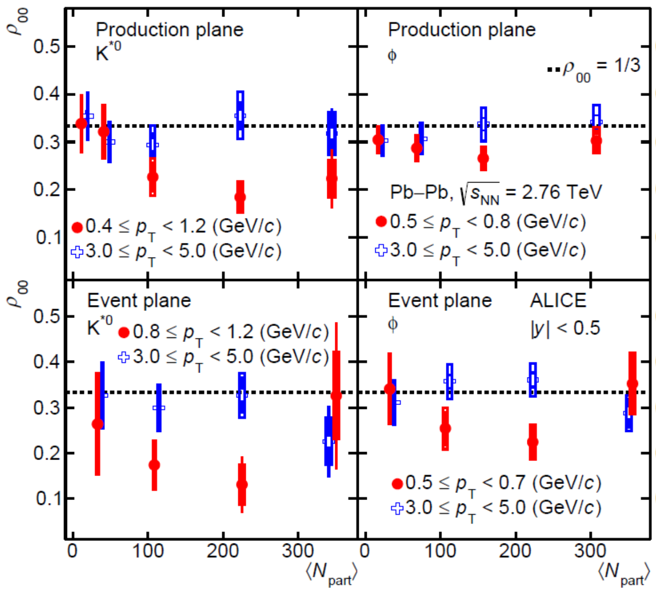
2-field correlations $\propto F^2$

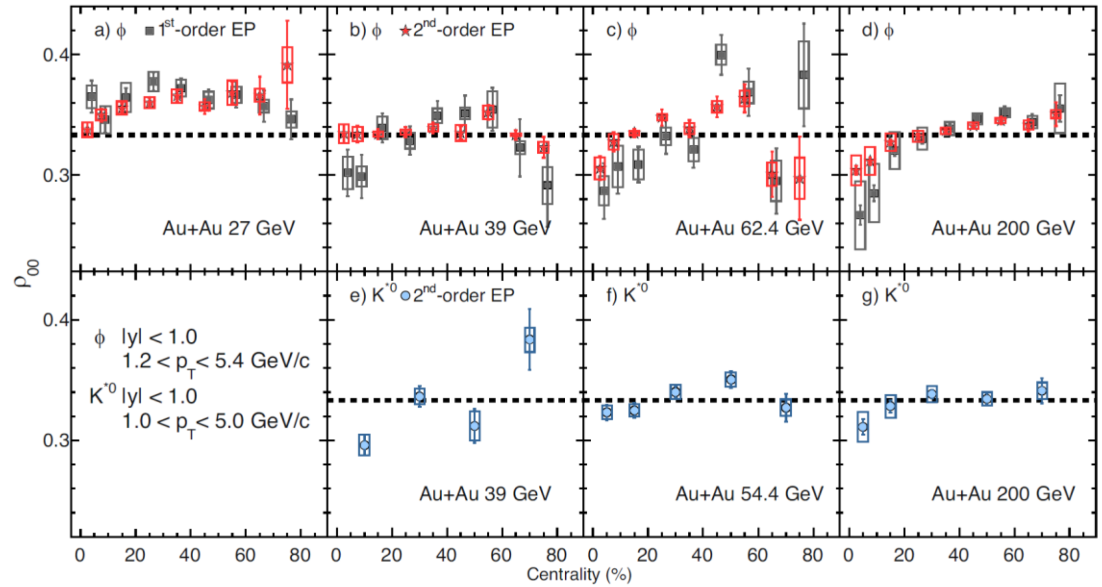


A. Kumar, B. Müller, DY, PRD 107, 076025 (2023)   A. Kumar, B. Müller, DY, PRD 108, 016020 (2023)

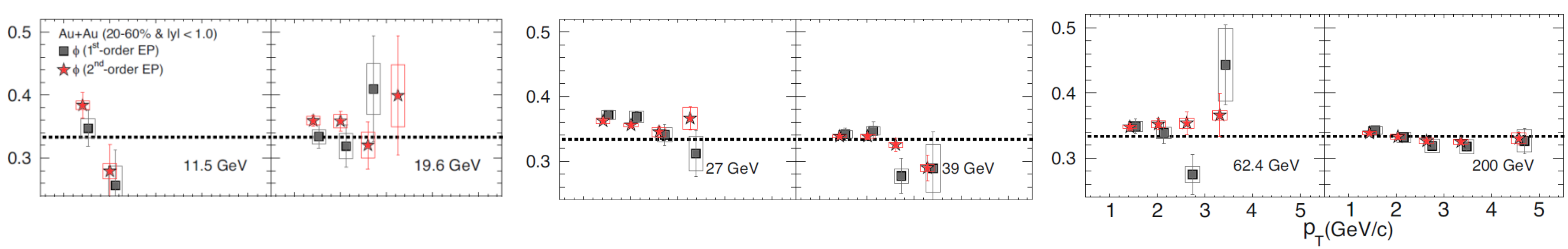
More experimental data

❖ Centrality dependence :





❖ pT dependence in RHIC :



Axial kinetic theory with color fields

- Incorporation of background color fields into Wigner functions and kinetic equations.
- Color decomposition : $O = O^s I + O^a t^a$

U. W. Heinz, Phys. Rev. Lett. 51, 351 (1983)
H. T. Elze, M. Gyulassy, D. Vasak, Nucl. Phys. B276, 706 (1986).

e.g., $\mathcal{A}^{\mu}(\boldsymbol{p},x) = \mathcal{A}^{s\mu}(\boldsymbol{p},x) I + \mathcal{A}^{a\mu}(\boldsymbol{p},x)\, t^a, \quad f_V(\boldsymbol{p},x) = f_V^{s}(\boldsymbol{p},x) I + f_V^{a}(\boldsymbol{p},x)\, t^a,$

$$\tilde{a}^{\mu}(\boldsymbol{p},x) = \tilde{a}^{s\mu}(\boldsymbol{p},x) I + \tilde{a}^{a\mu}(\boldsymbol{p},x)\, t^a.$$

- Kinetic equations : DY, JHEP 06, 140 (2022)

SKEs : $p^{\rho}\left(\partial_{\rho} f_V^{s} + \frac{g}{2N_c} F^{a}_{\nu\rho}\partial^{\nu}_{p} f^{a}_{V}\right) = \mathcal{C}_{s}, \quad p^{\rho}\left(\partial_{\rho} f^{a}_{V} + g F^{a}_{\nu\rho}\partial^{\nu}_{p} f^{s}_{V} + \frac{d^{bca}}{2} g F^{b}_{\nu\rho}\partial^{\nu}_{p} f^{c}_{V}\right) = \mathcal{C}^{a}_{o},$

diffusion — dynamical spin polarization

AKEs : $p^{\rho}\partial_{\rho}\tilde{a}^{s\mu} + \frac{g}{2N_c}\left(p^{\rho}F^{a}_{\nu\rho}\partial^{\nu}_{p}\tilde{a}^{a\mu} + F^{a\nu\mu}\tilde{a}^{a}_{\nu}\right) - \frac{\hbar}{4N_c}\epsilon^{\mu\nu\rho\sigma}p_{\rho}(\partial_{\sigma} g F^{a}_{\beta\nu})\partial^{\beta}_{p} f^{a}_{V} = \mathcal{C}^{\mu}_{s},$

$$p^{\rho}\partial_{\rho}\tilde{a}^{a\mu} + g\left(p^{\rho}F^{a}_{\nu\rho}\partial^{\nu}_{p}\tilde{a}^{s\mu} + F^{a\nu\mu}\tilde{a}^{s}_{\nu}\right) + \frac{d^{bca}}{2} g\left(p^{\rho}F^{b}_{\nu\rho}\partial^{\nu}_{p}\tilde{a}^{c\mu} + F^{b\nu\mu}\tilde{a}^{c}_{\nu}\right) - \frac{\hbar}{2}\epsilon^{\mu\nu\rho\sigma}p_{\rho}(\partial_{\sigma} g F^{a}_{\beta\nu})\partial^{\beta}_{p} f^{s}_{V} = \mathcal{C}^{a\mu}_{o}.$$

Axial Wigner functions :

$$\mathcal{A}^{s\mu}(\boldsymbol{p},x) = \frac{1}{2\epsilon_{\boldsymbol{p}}}\left[\tilde{a}^{s\mu} - \frac{\hbar}{4N_c}\tilde{F}^{a\mu\nu}\left(\partial_{p\nu} f^{a}_{V} - \frac{\epsilon_{\boldsymbol{p}}}{2}\partial_{p_{\perp}\nu}(f^{a}_{V}/\epsilon_{\boldsymbol{p}})\right)\right]_{p_0=\epsilon_{\boldsymbol{p}}},$$

$$\mathcal{A}^{a\mu}(\boldsymbol{p},x) = \frac{1}{2\epsilon_{\boldsymbol{p}}}\left[\tilde{a}^{a\mu} - \frac{\hbar}{2}\tilde{F}^{a\mu\nu}\left(\partial_{p\nu} f^{s}_{V} - \frac{\epsilon_{\boldsymbol{p}}}{2}\partial_{p_{\perp}\nu}(f^{s}_{V}/\epsilon_{\boldsymbol{p}})\right)\right]_{p_0=\epsilon_{\boldsymbol{p}}}.$$

dynamical (w/ memory effect) — non-dynamical (w/o memory effect)

Spin polarization:

$$\mathcal{P}^{\mu}(\boldsymbol{p}) = \frac{\int d\Sigma\cdot p\, \mathrm{Tr}_c \mathcal{A}^{\mu}(\boldsymbol{p},x)}{2m\int d\Sigma\cdot p\,(2\epsilon_{\boldsymbol{p}})^{-1} f^{s}_{V}(\boldsymbol{p},x)} = \frac{\int d\Sigma\cdot p\, \mathcal{A}^{s\mu}(\boldsymbol{p},x)}{2m\int d\Sigma\cdot p\,(2\epsilon_{\boldsymbol{p}})^{-1} f^{s}_{V}(\boldsymbol{p},x)}.$$

Spin “polarization” from glasma fields

- Generalized AKT with color fields : $\tilde{a}^{\mu}(\boldsymbol{p},x)=\tilde{a}^{s\mu}(\boldsymbol{p},x)I+\boxed{\tilde{a}^{a\mu}(\boldsymbol{p},x)}\,t^{a}.$

  DY, JHEP 06, 140 (2022)
  B. Müller, DY, PRD 105, L011901 (2022)

  (more dominant in the perturbative approach)

- Assuming the (s) quark is produced from the glasma : given f_V^s ⇨ induced $\tilde{a}_\mu^a$
- Glasma phase $(t_{\rm i}\leq t\leq t_{\rm th})$:

➢ Color-octet AKE with the weak-field approx. :

$$p^\rho\partial_\rho\tilde{a}^{a\mu}-\frac{\hbar}{2}\epsilon^{\mu\nu\rho\sigma}p_\rho(\partial_\sigma gF^a_{\beta\nu})\partial_p^\beta f_V^{\rm s}=0$$

⇨ $f_V^{\rm s}(p,x)=f_V^{\rm s}(p_0,t)$

$$\tilde{a}^{ai}(p,x)|_{t=t_{\rm th}}\approx-\frac{\hbar g}{2}\Big[\Big(B^{ai}(t_{\rm i})-\frac{\epsilon^{ijk}p_jE^a_k(t_{\rm i})}{p_0}\Big)\partial_{p0}f_V^{\rm s}(p_0,t_{\rm i})-\Big(B^{ai}(t_{\rm th})-\frac{\epsilon^{ijk}p_jE^a_k(t_{\rm th})}{p_0}\Big)\partial_{p_0}f_V^{\rm s}(p_0,t_{\rm th})\Big]$$

suppressed

- QGP phase $(t_{\rm th}\leq t)$:

➢ Relaxation-time approx. : $p^\rho\partial_\rho\tilde{a}^{a\mu}=-\dfrac{p_0\tilde{a}^{a\mu}}{\tau_{\rm R}^{\rm O}}$ ⇨ $\tilde{a}^{a\mu}(p,x)=\tilde{a}^{a\mu}(t_{\rm th})e^{-\frac{(t-t_{\rm th})}{\tau_{\rm R}^{\rm O}}}$

❖ Axial WF : $\mathcal{A}^{ai}(\boldsymbol{p},x)\approx-\dfrac{\hbar g}{4\epsilon_{\boldsymbol{p}}}e^{\frac{-(t-t_{\rm th})}{\tau_{\rm R}^{\rm O}}}\left[B^{ai}(t_{\rm i})-\dfrac{\epsilon^{ijk}p_jE^a_k(t_{\rm i})}{\epsilon_{\boldsymbol{p}}}\right]\partial_{\epsilon_{\boldsymbol{p}}}f_V^{\rm s}(\epsilon_{\boldsymbol{p}},t_{\rm i})$

(no contribution to Λ pol.)

spin relaxation

spin polarization

Color fields from the glasma

- Solving linearized Yang-Mills eqs. : $[D_\mu, F^{\mu\nu}] = J^\nu$

P. Guerrero-Rodrguez, T. Lappi, PRD 104, 014011 (2021)

- Color-field correlators in the glasma :

evolution in time

e.g. $$\langle E_T^{ai}(X')E_T^{aj}(X'')\rangle = -\bar{N}_c \epsilon^{in}\epsilon^{jm} \int_{\perp;q,u}^{X'} \int_{\perp;l,v}^{X''} \Omega_-(u_\perp, v_\perp) \frac{q^n l^m}{ql} \times J_1(qX_0')J_1(lX_0''),$$

$$\langle B_T^{ai}(X')B_T^{aj}(X'')\rangle = -\bar{N}_c \epsilon^{in}\epsilon^{jm} \int_{\perp;q,u}^{X'} \int_{\perp;l,v}^{X''} \Omega_+(u_\perp, v_\perp) \frac{q^n l^m}{ql} \times J_1(qX_0')J_1(lX_0''),$$

$$\bar{N}_c \equiv \frac{1}{2} g^2 N_c (N_c^2 - 1),$$

$$\Omega_\mp(u_\perp, v_\perp) = [G_1(u_\perp, v_\perp) G_2(u_\perp, v_\perp) \mp h_1(u_\perp, v_\perp) h_2(u_\perp, v_\perp)],$$

$$\int_{\perp;q,u}^{X'} \equiv \int \frac{d^2 q_\perp}{(2\pi)^2} \int d^2 u_\perp e^{i q_\perp (X'-u)_\perp}.$$

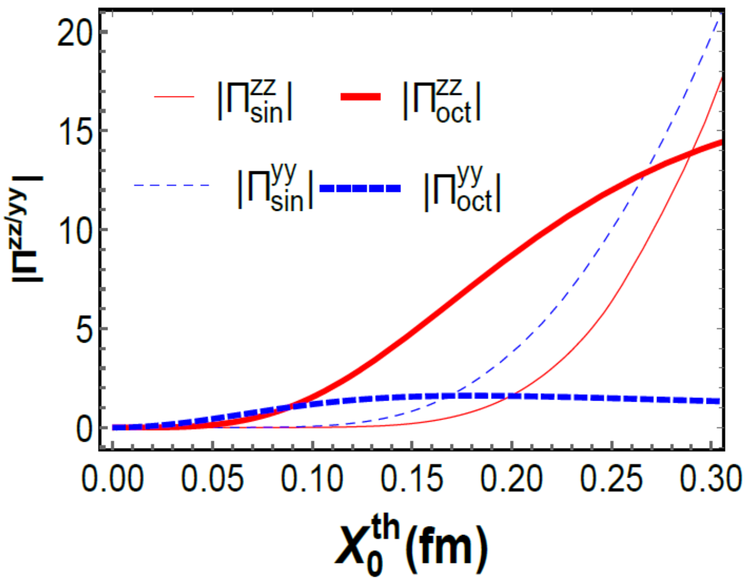
unpolarized & linearly polarized
Gluon distribution functions

- Golec-Biernat Wusthoff (GBW) distribution : K. J. Golec-Biernat and M. Wustho, PRD 59, 014017 (1998)

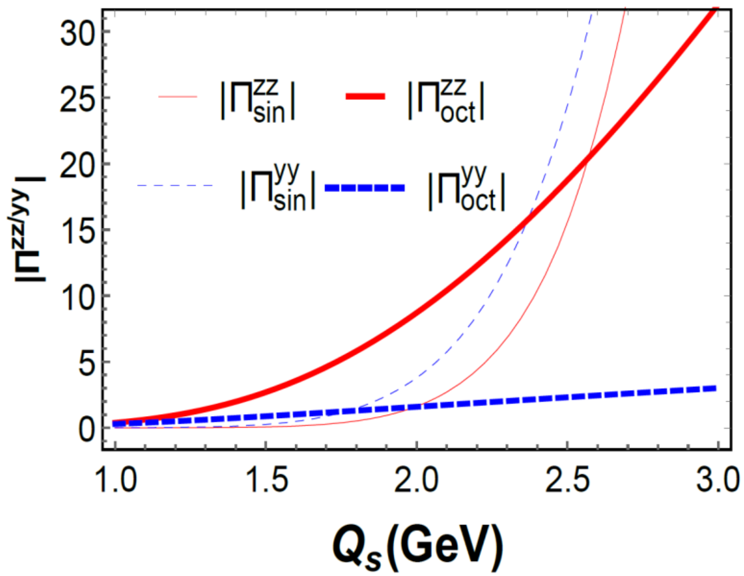
$$\Omega_\pm(u_\perp, v_\perp) = \Omega(u_\perp, v_\perp) = \frac{Q_s^4}{g^4 N_c^2} \left(\frac{1 - e^{-Q_s^2 |u_\perp - v_\perp|^2/4}}{Q_s^2 |u_\perp - v_\perp|^2/4} \right)^2$$

Singlet v.s. octet

- More sophisticated analysis : $\mathrm{Tr_c}\langle \hat{\mathcal{P}}^i_q(\boldsymbol{q}/2)\hat{\mathcal{P}}^i_{\bar{q}}(\boldsymbol{q}/2)\rangle = \Pi^{ii}_{\mathrm{oct}} + \Pi^{ii}_{\mathrm{sin}} + \Pi^{ii}_{\mathrm{EM}}$ suppressed



$Q_s = 2\ \mathrm{GeV}$



$X_0^{\mathrm{th}} = 0.2\ \mathrm{fm}$

⇨ $Q_s = 2\ \mathrm{GeV}$ & $X_0^{\mathrm{th}} = 0.2\ \mathrm{fm}$ for $|\Pi^{zz}_{\mathrm{oct}}| > |\Pi^{ii}_{\mathrm{sin}}|$