Nonlocal chiral anomaly and generalized parton distributions

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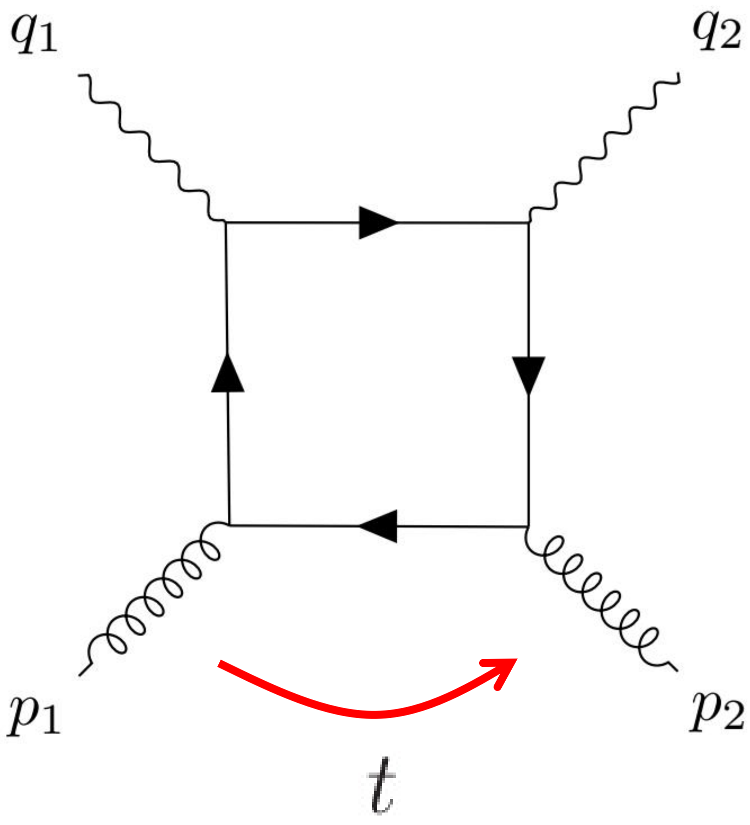
`Anomaly pole' in perturbation theory

Tarasov, Venugopalan (2019,2021)

Compton amplitude, photon indices antisymmetric

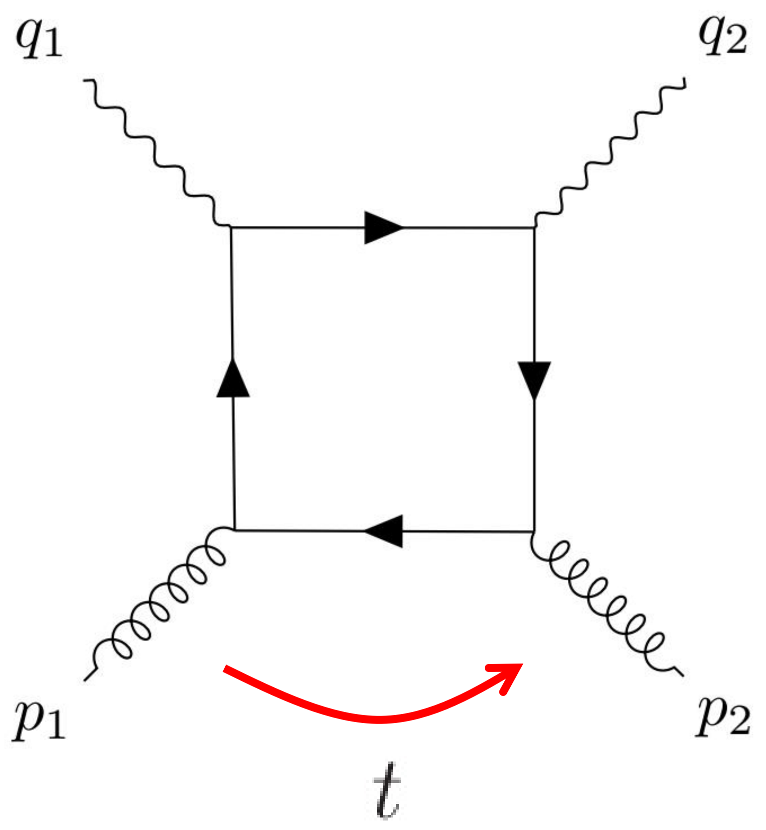


$$\sim \frac{1}{t}(1-x) \otimes \langle F^{\mu\nu}(z^-)\tilde{F}_{\mu\nu}(0)\rangle$$

`Anomaly pole' in perturbation theory`

Bhattacharya, YH, Vogelsang (2022,2023)

Compton amplitude, photon indices symmetric

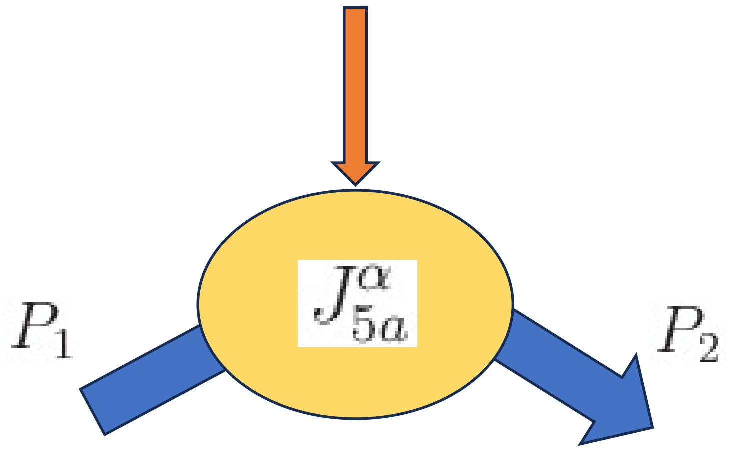


$$\sim \frac{1}{t} x(1-x) \otimes \langle F^{\mu\nu}(z^-) F_{\mu\nu}(0) \rangle$$

Circa 1960: Isovector axial form factors

Noether current of SU(2) chiral symmetry $q \to e^{i\alpha^a \tau^a \gamma_5} q$

$$J^\alpha_{5a} = \sum_q \bar{q}\gamma^\alpha \gamma_5 \frac{\tau^a}{2} q$$



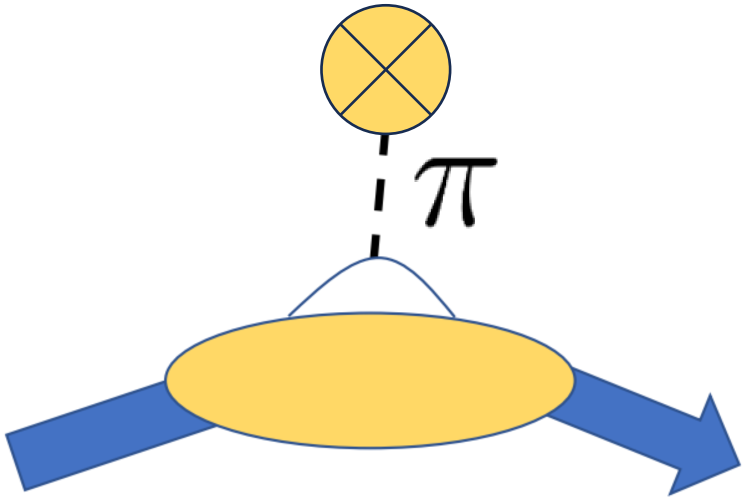
$$t = (P_2 - P_1)^2 \equiv l^2$$

Nucleon form factors

$$\langle P_2 | J^\alpha_{5a} | P_1 \rangle = \bar{u}(P_2) \left[\gamma^\alpha \gamma_5 F_A(t) + \frac{l^\alpha \gamma_5}{2M} F_P(t) \right] \frac{\tau^a}{2} u(P_1)$$

pseudovector pseudoscalar

Chiral symmetry breaking and pion pole



In massless QCD, the current is conserved $\partial_\alpha J^\alpha_{5a} = 0$

$$2MF_A(t) + \frac{tF_P(t)}{2M} = 0 \quad \Rightarrow \quad F_P(t) \approx \frac{-4M^2 g_A^{(3)}}{t}$$

massless pole!

In real QCD with finite quark masses , $\frac{1}{t} \to \frac{1}{t - m_\pi^2}$

Pion nearly massless due to spontaneously broken chiral symmetry Nambu (1960)

Pion pole in GPD

Generalized parton distribution = x-distribution of form factor

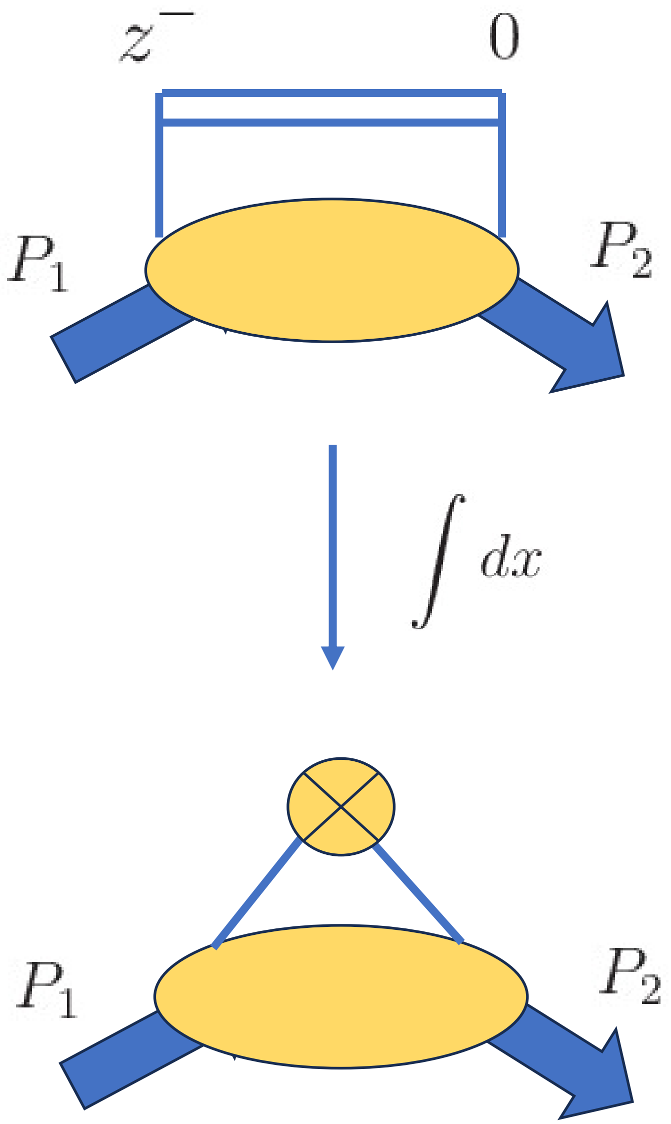
$$F_P(t) = \int_{-1}^{1} dx \left(\tilde{E}_u(x,\xi,t) - \tilde{E}_d(x,\xi,t) \right) \approx \frac{-4M^2 g_A^{(3)}}{t}$$

skewness $\quad \xi = \frac{P_1^+ - P_2^+}{P_1^+ + P_2^+}$

Massless pole already in GPD

$$\tilde{E}_u(x,\xi,t) - \tilde{E}_d(x,\xi,t) \sim \frac{1}{t}$$

Penttinen, Polyakov, Goeke (1999)



Singlet axial form factors

Next consider the U(1) axial current $J_5^\alpha = \sum_q \bar{q}\gamma^\alpha\gamma_5 q$

$$\langle P_2|J_5^\alpha|P_1\rangle = \bar{u}(P_2)\left[\gamma^\alpha\gamma_5 g_A(t) + \frac{l^\alpha\gamma_5}{2M}g_P(t)\right]u(P_1)$$

$g_A(0) = \Delta\Sigma$ quark spin contribution to the nucleon spin

In massless QCD, the current is conserved due to axial U(1) symmetry

$$2Mg_A(t) + \frac{tg_P(t)}{2M} = 0 \quad \Rightarrow \quad \frac{g_P(t)}{2M} \approx -\frac{2M\Delta\Sigma}{t}$$

Pole at $t = 0$ from massless η_0 meson exchange

Chiral anomaly

Quantum mechanically, the current is not conserved $$\partial_\alpha J_5^\alpha = -\frac{n_f \alpha_s}{4\pi} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

$$\Rightarrow \quad \frac{g_P(t)}{2M} = \frac{1}{t} \left(i \frac{\langle P_2 | \frac{n_f \alpha_s}{4\pi} F\tilde{F} | P_1 \rangle}{\bar{u}(P_2)\gamma_5 u(P_1)} - 2M g_A(t) \right)$$

anomaly pole η_0 pole

In real QCD, there is no massless pole in $g_P(t)$

Pole shifted to the physical η' meson mass

$$g_P(t) \sim \frac{1}{t - m_\eta^2}$$

Witten-Veneziano mechanism

Witten (1979), Veneziano (1979)

In large-Nc QCD, there is no chiral anomaly.

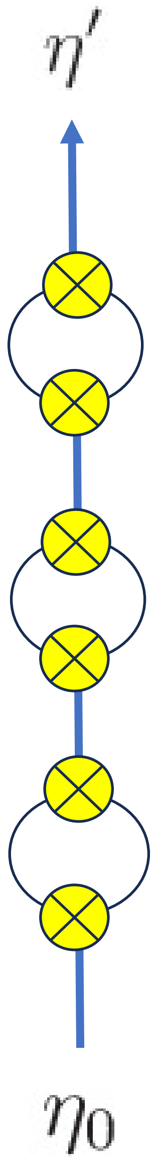
→ η' meson mass $O(1/\sqrt{N_c})$

$$m_\eta^2 = -\frac{4n_f}{f_{\eta'}^2}\langle (F\tilde{F})^2\rangle$$

Mass generation due to the topological fluctuation of the QCD vacuum

$$\frac{1}{t} + \frac{m_{\eta'}^2}{t^2} + \frac{m_{\eta'}^4}{t^3} + \cdots = \frac{1}{t - m_{\eta'}^2} = -\left(\frac{1}{t}\frac{m_{\eta'}^2}{m_{\eta'}^2 - t} - \frac{1}{t}\right)$$

anomaly pole $\quad$ η_0 pole



Gravitational form factors

→ talk by Feng

QCD energy momentum tensor

$$\Theta^{\mu\nu} = \sum_f \bar{\psi}_f \gamma^{(\mu} i D^{\nu)} \psi_f - F^{\mu\rho} F^{\nu}{}_{\rho} + \frac{g^{\mu\nu}}{4} F^{\alpha\beta} F_{\alpha\beta}$$

Nucleon form factors

$$\langle P_2 | \Theta^{\alpha\beta} | P_1 \rangle = \bar{u}(P_2) \left[A(t) \frac{P^\alpha P^\beta}{M} + (A(t) + B(t)) \frac{P^{(\alpha} i\sigma^{\beta)\lambda} l_\lambda}{2M} + D(t) \frac{l^\alpha l^\beta - g^{\alpha\beta} t}{4M} \right] u(P_1)$$

In massless QCD, $\Theta^{\alpha\beta}$ is traceless due to conformal symmetry

$$A(t) + \frac{B(t)}{4M^2} t - \frac{3D(t)}{4M^2} t = 0 \qquad \frac{3}{4} D(t) \approx \frac{M^2}{t} A(t) \qquad (t \to 0)$$

Trace anomaly

Quantum mechanically, the trace is nonzero $$(\Theta)^{\alpha}_{\alpha} = \frac{\beta(g)}{2g} F^{\mu\nu} F_{\mu\nu}$$

$$\Rightarrow \quad \frac{3}{4} D(t) \approx -\frac{M}{t} \left(\frac{\langle P_2 | \frac{\beta(g)}{2g} F^2 | P_1 \rangle}{\bar{u}(P_2) u(P_1)} - M A(t) \right)$$

anomaly pole `glueball' pole

In real QCD, there is no massless pole in $D(t)$ due to pole cancellation

Poles in $D(t)$ at 2++, 0++ glueball masses. Fujita, YH, Sugimoto, Ueda (2022)

Take-home message

Anomalies relate form factors

Chiral anomaly

$$2Mg_A(t) + \frac{tg_P(t)}{2M} = i\frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}$$

Trace anomaly

$$M\left(A(t) + \frac{B(t)}{4M^2}t - \frac{3D(t)}{4M^2}t\right)\bar{u}(P_2)u(P_1) = \langle P_2|\frac{\beta(g)}{2g}F^{\mu\nu}F_{\mu\nu}|P_1\rangle$$

Form factors are moments of GPDs

$$g_P(t) = \sum_q \int_{-1}^{1} dx\tilde{E}_q(x,\xi,t) \qquad A_q(t) + \xi^2 D_q(t) = \int_{-1}^{1} dx x H_q(x,\xi,t)$$

➡ Anomalies relate/constrain GPDs

Chiral anomaly in QCD

Peskin & Schroeder, Chapter 19

$$\partial_\mu J_5^\mu(x) = \partial_\mu(\bar{\psi}\gamma^\mu\gamma_5\psi) = 2im_q\bar{\psi}\gamma_5\psi - \frac{n_f\alpha_s}{4\pi}F_a^{\mu\nu}\tilde{F}_{\mu\nu}^a$$

There are many derivations of this, but here I use the point-splitting method

$$J_5^\mu(x) = \lim_{z\to 0}\bar{\psi}\left(x-\frac{z}{2}\right)\gamma^\mu\gamma_5 P\exp\left(ig\int_{x-\frac{z}{2}}^{x+\frac{z}{2}} dw_\nu A^\nu(w)\right)\psi\left(x+\frac{z}{2}\right)$$

$$\partial_\mu J_5^\mu(x) \sim \lim_{z\to 0} z_\nu F^{\mu\nu}(x)\mathrm{Tr}\left[\gamma_\mu\gamma_5\psi(x+\frac{z}{2})\bar{\psi}(x-\frac{z}{2})\right]$$

Operator product expansion

$$\partial_\mu J_5^\mu(x) \sim \lim_{z\to 0} z_\nu F^{\mu\nu}(x)\mathrm{Tr}\left[\gamma_\mu\gamma_5\psi(x+\frac{z}{2})\bar{\psi}(x-\frac{z}{2})\right]$$

$$\sim \frac{z^\rho}{z^2}\tilde{F}_{\rho\lambda}(x)\gamma^\lambda\gamma_5$$

Take the symmetric limit

$$\lim_{z\to 0}\frac{z^\nu z^\rho}{z^2} \to \frac{g^{\nu\rho}}{4}$$

Anomaly!

$$\partial_\mu J_5^\mu \sim \alpha_s F\tilde{F}$$

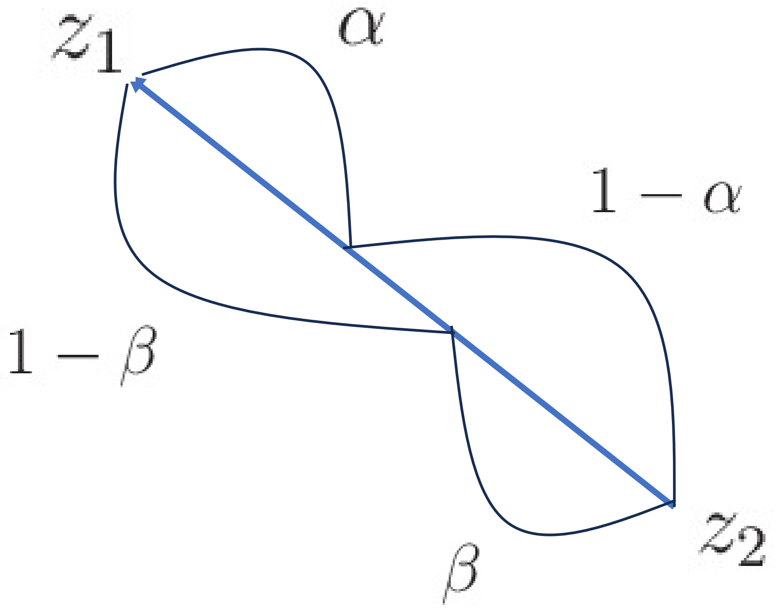
A side remark

The symmetric limit $\lim_{z \to 0} \frac{z^\nu z^\rho}{z^2} \to \frac{g^{\nu\rho}}{4}$ is not necessary

$$4 z^\nu F_{\nu\mu} \tilde{F}^{\mu\rho} z_\rho = -z^2 F^{\mu\nu} \tilde{F}_{\mu\nu}$$

This is an identity. (Hint: use the Schouten identity)

Nonlocal chiral anomaly



If the separation is light-like,

$$z^\mu = \delta^\mu_- z^-$$

the anomaly is still there even when $z^- \neq 0$

D. Muller and Teryaev (1997)

$$\mathcal{D}_\mu \left[\bar{\psi}(z_2^-) W \gamma^\mu \gamma_5 \psi(z_1^-)\right] = i z^\nu \int_0^1 d\alpha \bar{\psi}(z_2) \gamma^\mu \gamma_5 W g F_{\mu\nu}(z_{21}^\alpha) W \psi(z_1)$$

$$- \frac{n_f \alpha_s}{2\pi} \int_0^1 d\alpha \int_0^{1-\alpha} d\beta F^{\mu\nu}(z_{12}^{\beta -}) \tilde{W} \tilde{F}_{\mu\nu}\left(z_{21}^{\alpha -}\right)$$

Derivation

Agaev, Braun, Offen, Porkert, Schafer (2014)
Bhattacharya, YH, Schoenleber (2024)

Approach the light-cone from the space-like region

$$z^2 < 0$$

where naïve equation of motion holds.

$$\mathcal{D}_\mu \left[\bar{\psi}(z_2)\gamma^\mu\gamma_5 W_{z_2,z_1}\psi(z_1)\right] = iz^\nu \int_0^1 d\alpha \bar{\psi}(z_2)\gamma^\mu\gamma_5 W g F_{\mu\nu}(z_{21}^\alpha) W \psi(z_1)$$

Then take $z^2 \to 0$

Nonlocal operator product expansion

Balitsky, Braun (1989)

$$\psi(z_1)\bar{\psi}(z_2) = \frac{i\not{z}}{2\pi^2(z^2)^2}W_{z_1,z_2} - \frac{iz^\rho}{8\pi^2 z^2}\int_0^1 d\beta W_{z_1,z_{12}^\beta} g\tilde{F}_{\rho\lambda}(z_{12}^\beta)W_{z_{12}^\beta,z_2}\gamma^\lambda\gamma_5$$

$$+\frac{i}{32\pi^2}\left(\frac{1}{\epsilon_{IR}} + \ln\frac{-z^2\mu_{IR}^2 e^{2\gamma_E}}{4}\right)\left[g\int_0^1 d\alpha\alpha(1-\alpha)z_\mu D^2\tilde{F}^{\mu\nu}(z_{12}^\alpha)\gamma_\nu\gamma_5 + ig^2 z_\mu\int_0^1 d\alpha\int_0^\alpha d\beta\right.$$

$$\left.\times\left\{(1-2\alpha+2\beta)F^{\mu\lambda}(z_{12}^\alpha)\tilde{F}_{\lambda\rho}(z_{12}^\beta)\gamma^\rho + \tilde{F}^{\mu\lambda}(z_{12}^\alpha)F_{\lambda\rho}(z_{12}^\beta)\gamma^\rho + \beta F^{\rho\nu}(z_{12}^\alpha)\tilde{F}_{\rho\nu}(z_{12}^\beta)\gamma^\mu\right\}\gamma_5 + \cdots\right]$$

Again, Schouten identity (exact, no symmetric limit!)

$$2z^\nu\left(F_{\nu\mu}W\tilde{F}^{\mu\rho} + \tilde{F}_{\nu\mu}WF^{\mu\rho}\right)z_\rho = -z^2F^{\mu\nu}W\tilde{F}_{\mu\nu}$$

UV Matching

The light-cone limit $z^2 \to 0$ is not smooth. One needs operator matching

$$A_i(z^2) = \sum_j C_{ij}(\ln(-z^2\mu_{UV}^2)) \otimes A_j(z^2 = 0, \mu_{UV}^2) + \mathcal{O}(z^2),$$

Nowadays familiar in quasi-PDF, pseudo-PDF business

$$\mathcal{D}_\mu \left[\bar{\psi}(z_2^-)W\gamma^\mu\gamma_5\psi(z_1^-)\right] = iz^\nu \int_0^1 d\alpha \bar{\psi}(z_2)\gamma^\mu\gamma_5 W g F_{\mu\nu}(z_{21}^\alpha)W\psi(z_1)$$
$$- \frac{n_f\alpha_s}{2\pi}\int_0^1 d\alpha \int_0^{1-\alpha} d\beta F^{\mu\nu}(z_{12}^{\beta-})\tilde{W}\tilde{F}_{\mu\nu}\left(z_{21}^{\alpha-}\right) + \cdots$$

$\mathcal{O}(\alpha_s)$ coefficient functions and higher twist neglected in leading logarithmic approximation

Take the nucleon matrix element $\langle P'|...|P\rangle$ of

$$\mathcal{D}_\mu\left[\bar{\psi}(z_2^-)W\gamma^\mu\gamma_5\psi(z_1^-)\right] = iz^\nu\int_0^1 d\alpha\bar{\psi}(z_2)\gamma^\mu\gamma_5 W gF_{\mu\nu}(z_{21}^\alpha)W\psi(z_1) - \frac{n_f\alpha_s}{2\pi}\int_0^1 d\alpha\int_0^{1-\alpha} d\beta F^{\mu\nu}(z_{12}^{\beta-})\tilde{W}\tilde{F}_{\mu\nu}\left(z_{21}^{\alpha-}\right)$$

Right hand side, anomaly term

$$\left[\int_{-1}^{x} dx'\frac{x-x'}{x'^2-\xi^2} - \theta(x+\xi)\int_{-1}^{0} dx'\frac{\xi+x}{x'^2-\xi^2}\right]\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\langle p'|F^{\mu\nu}(-z^-/2)\tilde{W}\tilde{F}_{\mu\nu}(z^-/2)|p\rangle$$

Tarasov, Venugopalan (2019,2021) $\xi = 0$
Bhattacharya, YH, Vogelsang (2022,2023) $\xi \neq 0$

Left hand side is GPD $\langle P'|\mathcal{D}_\mu[\bar{\psi}\gamma^\mu\gamma_5\psi|P\rangle = i\Delta_\mu\langle P'|[\bar{\psi}\gamma^\mu\gamma_5\psi|P\rangle$

$$\sim \tilde{H}, \tilde{E}$$

Nonlocal Witten-Veneziano mechanism

Bhattacharya, YH, Schoenleber (2024)

$$\tilde{E}(x,\xi,t) \approx \frac{4M^2}{t}\left(\frac{n_f\alpha_s}{2\pi}\tilde{C}^{anom}\otimes\tilde{\mathcal{F}}_2 - \tilde{H} + O_{F2}\right)$$

This is what was found in perturbation theory

$$\Big\downarrow \int dx$$

$$\frac{g_P(t)}{2M} = \frac{1}{t}\left(i\frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)} - 2Mg_A(t)\right)$$

Vanishing of the residue requires

$$\sum_q \Delta q(x) \approx \frac{n_f\alpha_s}{2\pi}\int_x^1 \frac{dx'}{x'}\left(1-\frac{x}{x'}\right)\tilde{\mathcal{F}}_2(x') + O_{F2}$$

cf. Tarasov, Venugopalan (2019,2021)

Conclusions

Anomalies relate form factors
Form factors are moments of GPDs

→ Anomalies relate GPDs

GPDs encode profound aspects of QCD such as chiral symmetry breaking and the origin of mass.