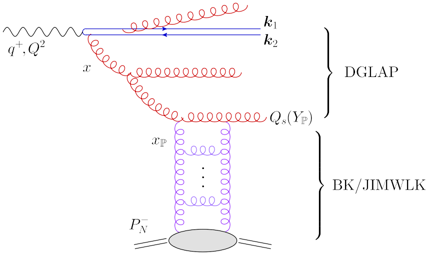
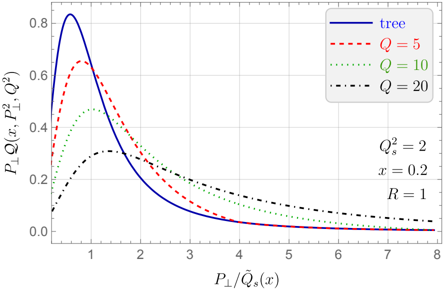
From small x to high $P_\perp$:
Relating high-energy & collinear evolutions via the CGC

Edmond Iancu

with P. Caucal, A. Mueller, D. Triantafyllopoulos, S.-Y. Wei, F. Yuan





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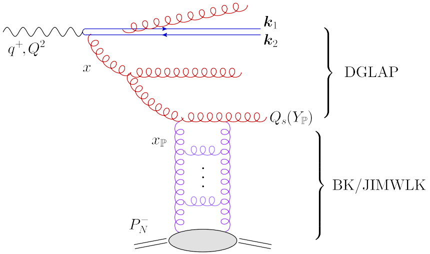
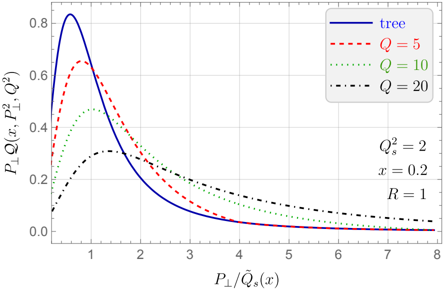
it builds upon all I've learnt, from all of you, over the years ...





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- Sincere apologises to all of you who had to suffer from my impetuosity

  (Jean-Paul, Larry, Dionysis, Yacine, Tuomas ... certainly know what I mean)

Outline

- Dilute-dense collisions at high energy: eA, pA, AA UPCs
- Particle production with large $P_\perp \gg Q_s(x)$, but small $x \sim \frac{P_\perp^2}{s} \ll 1$
- Small-x: the realm of the CGC effective theory
  - Wilson lines, BK/JIMWLK evolution with decreasing x
  - color dipole picture for γA, hybride factorisation for pA
- Large $P_\perp$: the realm of collinear/TMD factorisations
  - (transverse momentum dependent) parton distributions (PDF, TMD)
  - DGLAP evolution (PDFs) & CSS evolution (TMDs)
- Can one unify these descriptions in a same framework ?
- For the dilute projectile and for the final state, there seems to be no problem
  - BK/JIMWLK for the S–matrix
  - DGLAP for proton PDFs in pA and for the fragmentation functions

Outline

- Dilute-dense collisions at high energy: eA, pA, AA UPCs
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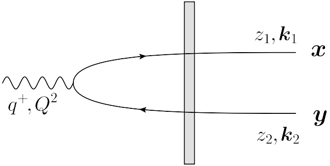
  - DGLAP evolution (PDFs) & CSS evolution (TMDs)
- Can one unify these descriptions in a same framework ?
- But what about the PDFs & TMDs of the dense target (A) ?
  - dijets in DIS: Sudakov effects $\Rightarrow$ initial-state evolution of the target (?)

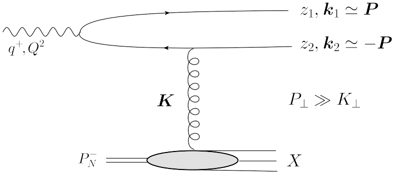
Hard dijet production in DIS

- Colour dipole picture: photon is a right-mover, target (A) a left mover
- Two back-to-back jets in the transverse plane: $P_\perp \sim Q \gg K_\perp \gtrsim Q_s$

$$\boldsymbol{P}_\perp = z_2 \boldsymbol{k}_{1\perp} - z_1 \boldsymbol{k}_{2\perp}\,, \qquad \boldsymbol{K}_\perp = \boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp}$$

- Small $q\bar{q}$ dipole: $r = |\boldsymbol{x} - \boldsymbol{y}| \sim 1/P_\perp \ll 1/Q_s \implies$ single scattering





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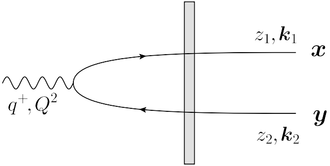
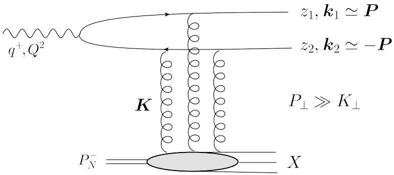


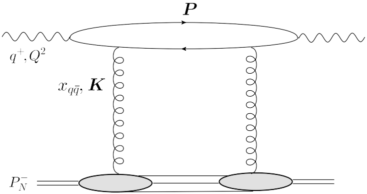


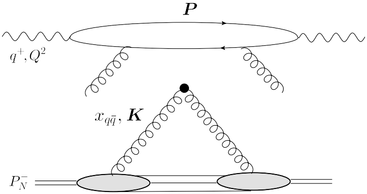
- Multiple scattering still important for the momentum imbalance: $K_\perp \sim Q_s$

scattering amplitude: $\quad V_{\boldsymbol{x}} V_{\boldsymbol{y}}^\dagger - 1 \simeq r^j \big(V_{\boldsymbol{b}} \partial^j V_{\boldsymbol{b}}^\dagger\big), \quad \boldsymbol{b} = z_1 \boldsymbol{x} + z_2 \boldsymbol{y}$

- $r \sim 1/P_\perp$ dependence factorises from the $b \sim 1/K_\perp$ dependence

$$\frac{d\sigma^{\gamma^*_{T,L}A\to q\bar{q}A}}{dz_1 dz_2 d^2\boldsymbol{P} d^2\boldsymbol{K}} = H_{T,L}(z_1, z_2, Q^2, P_\perp^2)\,\mathcal{F}_g(x_{q\bar{q}}, \boldsymbol{K})$$





- Hard factor encoding the kinematics of the $q\bar{q}$ pair $(\bar{Q}^2 = z_1 z_2 Q^2)$

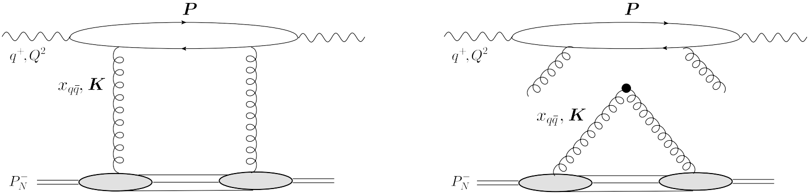
$$H_T = \alpha_{em}\alpha_s e_f^2 \delta(1 - z_1 - z_2)\left(z_1^2 + z_2^2\right)\frac{P_\perp^4 + \bar{Q}^4}{(P_\perp^2 + \bar{Q}^2)^4} \quad (P_\perp \sim \bar{Q} \sim Q)$$

- Weizsäcker-Williams gluon TMD (UGD) of the dense target

$$\mathcal{F}_g(x, \boldsymbol{K}) = \int_{\boldsymbol{b},\bar{\boldsymbol{b}}} \frac{e^{-i\boldsymbol{K}\cdot(\boldsymbol{b}-\bar{\boldsymbol{b}})}}{(2\pi)^4}\,\frac{-2}{\alpha_s}\left\langle \mathrm{Tr}\left[(\partial^i V_{\boldsymbol{b}})V_{\boldsymbol{b}}^\dagger(\partial^i V_{\bar{\boldsymbol{b}}})V_{\bar{\boldsymbol{b}}}^\dagger\right]\right\rangle_x$$

TMD factorisation for inclusive dijets

$$\frac{\mathrm{d}\sigma^{\gamma^*_{T,L}A\to q\bar{q}A}}{\mathrm{d}z_1\mathrm{d}z_2\mathrm{d}^2\boldsymbol{P}\mathrm{d}^2\boldsymbol{K}} = H_{T,L}(z_1, z_2, Q^2, P_\perp^2)\,\mathcal{F}_g(x_{q\bar{q}}, \boldsymbol{K})$$



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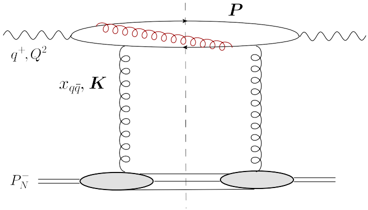
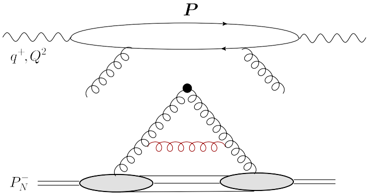
Collinear factorisation

- If the dijet imbalance it not measured ⇒ integrate over $\boldsymbol{K}$ ⇒ gluon PDF

$$\frac{\mathrm{d}\sigma^{\gamma^*_{T,L}A\to q\bar{q}A}}{\mathrm{d}z_1\mathrm{d}z_2\mathrm{d}^2\boldsymbol{P}} = H_{T,L}(z_1, z_2, Q^2, P_\perp^2)\, xG(x, P_\perp^2)$$



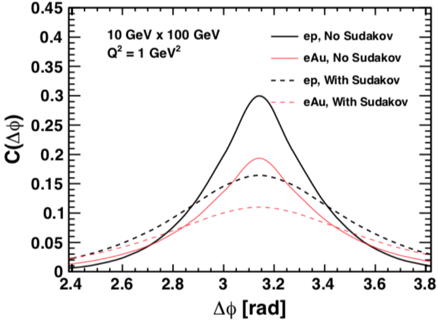


- $xG(x, P_\perp^2)$ is well known to obey DGLAP evolution with increasing $P_\perp^2$
- Standard one-loop calculation in the target picture
  - Bjorken frame $P_N^- \to \infty$, target light-cone gauge $A^- = 0$
- Is this also encoded in the NLO corrections in the dipole picture ?

Azimuthal correlations: Sudakov effect

- $P_\perp \gg K_\perp$: the final jets are nearly back to back in the transverse plane
  - azimuthal distribution shows a pronounced peak at $\Delta\phi = \pi$
- Additional broadening due to final-state radiation: Sudakov effect

  (Mueller, Xiao, and Yuan, arXiv:1308.2993)

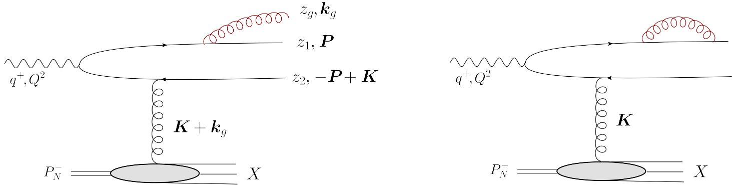


(Zheng, Aschenauer, Lee, and Xiao, arXiv:1403.2413)

- The effects of saturation are essentially washed out ☹

Final state emissions

- The final state emissions of soft gluons factorise $\Rightarrow \Delta\mathcal{F}_{\rm Sud}$



- Direct emissions by the quark: real & virtual

$$\Delta\mathcal{F}_{\rm Sud}^{qq} = \frac{\alpha_s C_F}{\pi^2}\int {\rm d}^2\boldsymbol{k}_g \int_{k_{g\perp}^2/P_\perp^2}^{1} \frac{{\rm d}z_g}{z_g}\,\frac{\mathcal{F}_g(x,\boldsymbol{K}+\boldsymbol{k}_g)-\mathcal{F}_g(x,\boldsymbol{K})}{\left(\boldsymbol{k}_g - \frac{z_g}{z_1}\boldsymbol{P}\right)^2}$$

- Lower limit on z_g: the boundary with the high energy evolution
- BK/JIMWLK: very soft gluon emissions which occur close the collision:

$$\tau_g \simeq \frac{2z_g q^+}{k_{g\perp}^2} \ll \tau_\gamma \simeq \frac{2q^+}{Q^2} \Longrightarrow z_g \ll \frac{k_{g\perp}^2}{Q^2}$$

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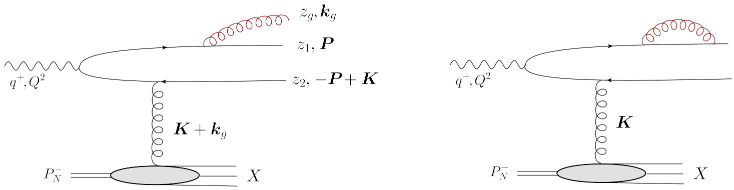

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- Collinear singularity when $\boldsymbol{k}_g/z_g = \boldsymbol{P}/z_1$ or $\theta_g = \theta_1$
- Removed via the renormalisation of the quark fragmentation function
- DGLAP evolution of quark fragmentation into hadrons

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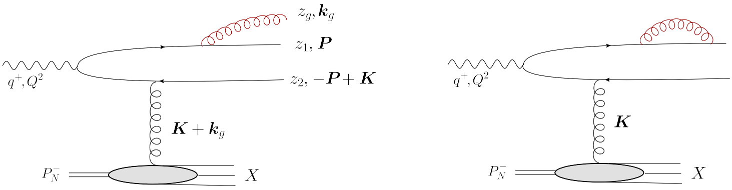

- Direct emissions by the quark: real & virtual

$$\Delta\mathcal{F}_{\rm Sud}^{qq} = \frac{\alpha_s C_F}{\pi^2}\int \mathrm{d}^2\boldsymbol{k}_g \int_{k_{g\perp}^2/P_\perp^2}^1 \frac{\mathrm{d}z_g}{z_g}\,\frac{\mathcal{F}_g(x,\boldsymbol{K}+\boldsymbol{k}_g)-\mathcal{F}_g(x,\boldsymbol{K})}{\left(\boldsymbol{k}_g-\frac{z_g}{z_1}\boldsymbol{P}\right)^2}$$

- However, we are interested in the production of jets $\Rightarrow$ large angles

$$\theta_g \sim \frac{k_{g\perp}}{z_g q^+} > \theta_1 \sim \frac{P_\perp}{z_1 q^+} \;\Rightarrow\; z_g < \frac{k_{g\perp}}{P_\perp} \;\Rightarrow\; \text{logarithmic phase-space} \int \frac{\mathrm{d}^2\boldsymbol{k}_g}{\boldsymbol{k}_g^2}$$

Final state emissions

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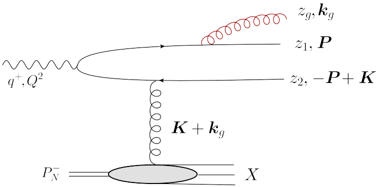

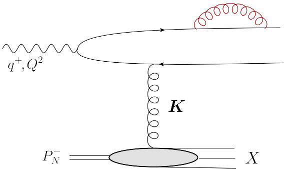

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- Emissions with low $k_{g\perp} \ll K_\perp$ cancel between real and virtual

- Real gluons with $k_{g\perp} \gg K_\perp$ are suppressed: $\mathcal{F}_g(\boldsymbol{K}+\boldsymbol{k}_g) \simeq \mathcal{F}_g(\boldsymbol{k}_g) \sim 1/k_{g\perp}^2$

- Virtual emissions are cut off in the UV at the hard scale $P_\perp$

The "virtual" Sudakov

- The net effect : the "standard" Sudakov double logarithm:

$$\Delta\mathcal{F}^{qq}_{\rm Sud}(x,K_\perp,P_\perp^2) = -\frac{\alpha_s C_F}{\pi}\,\mathcal{F}_g(x,K_\perp^2)\int_{K_\perp^2}^{P_\perp^2}\frac{{\rm d}k_{g\perp}^2}{k_{g\perp}^2}\int_{k_{g\perp}^2/P_\perp^2}^{k_{g\perp}/P_\perp}\frac{{\rm d}z_g}{z_g}$$

- Hard relative momentum $P_\perp$: longitudinal & transverse resolution scale

- Similar result for direct emissions by the antiquark

- Large angle emission $\Rightarrow$ interference (suppressed at large N_c)

- Overall color factor: $C_F + C_F + 1/N_c = N_c$

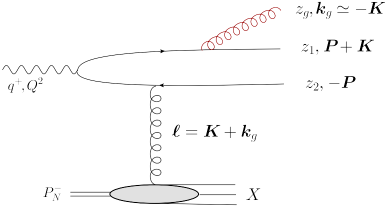
$$\Delta\mathcal{F}^{\rm V}_{\rm Sud}(x,K_\perp^2,P_\perp^2) = -\frac{\alpha_s N_c}{4\pi}\,\ln^2\frac{P_\perp^2}{K_\perp^2}\,\mathcal{F}_g(x,K_\perp^2)\,.$$

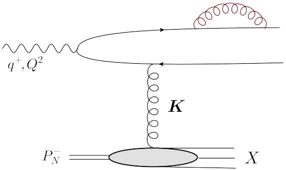
  - cusp anomalous dimension for gluons

- A large angle emission sees the overall colour charge: N_c, like for a gluon

The "real" Sudakov

- To DLA, there is also a "real" Sudakov effect (real gluon emission)
- The dijet imbalance can also be caused by the gluon recoil: $\boldsymbol{k}_g \simeq -\boldsymbol{K}$





$$\Delta\mathcal{F}^{\rm R}_{\rm Sud} = \frac{\alpha_s N_c}{\pi^2}\int \frac{{\rm d}^2\boldsymbol{k}_g}{\boldsymbol{k}_g^2}\int_{k_{g\perp}^2/P_\perp^2}^{k_{g\perp}/P_\perp} \frac{{\rm d}z_g}{z_g}\,\mathcal{F}_g(x, \boldsymbol{K}+\boldsymbol{k}_g)$$

- Replace $\boldsymbol{k}_g \to \boldsymbol{\ell} \equiv \boldsymbol{K} + \boldsymbol{k}_g$ and use $k_{g\perp} \simeq K_\perp$ and $\ell_\perp \ll K_\perp$
- Implicit assumption: $Q_s \ll K_\perp \ll P_\perp$ (since typically $\ell_\perp \sim Q_s$)

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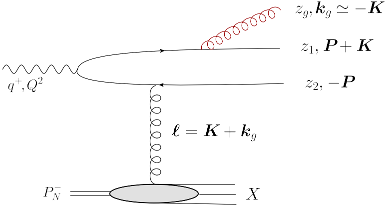
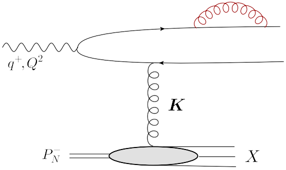




$$\Delta\mathcal{F}^{\rm R}_{\rm Sud}(x, K_\perp^2, P_\perp^2) = \frac{\alpha_s N_c}{\pi^2}\,\frac{1}{2}\ln\frac{P_\perp^2}{K_\perp^2}\,\frac{1}{K_\perp^2}\int_{\Lambda^2}^{K_\perp^2} {\rm d}^2\boldsymbol{\ell}\,\mathcal{F}_g(x, \ell_\perp^2)$$

- The integral over $\boldsymbol{\ell}$ yields the gluon PDF at the scale $K_\perp^2$

$$\int_{\Lambda^2}^{K_\perp^2} {\rm d}^2\boldsymbol{\ell}\,\mathcal{F}_g(x, \ell_\perp^2) = xG(x, K_\perp^2) \sim \ln\frac{K_\perp^2}{Q_s^2}$$

More on the Sudakov dynamics

$$\mathcal{F}_g(x, K_\perp, P_\perp^2) = \mathcal{F}_g^{(0)}(x, K_\perp^2) + \Delta\mathcal{F}_{\rm Sud}^{\rm R} + \Delta\mathcal{F}_{\rm Sud}^{\rm V}$$

- $P_\perp$ dependence ("resolution scale") introduced by the loop corrections
- The gluon PDF in the presence of the resolution scale:

$$xG(x, P_\perp^2) = \int_{\Lambda^2}^{P_\perp^2} {\rm d}^2\boldsymbol{K}\, \mathcal{F}_g(x, K_\perp^2, P_\perp^2)$$

- After integrating over $K_\perp$, "real" and "virtual" Sudakov mutually cancel:

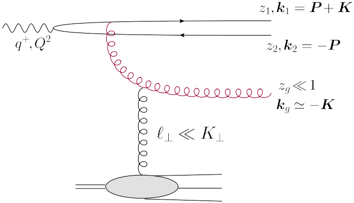
$$\int_{\Lambda^2}^{P_\perp^2} {\rm d}^2\boldsymbol{K} \left(\Delta\mathcal{F}_{\rm Sud}^{\rm R} + \Delta\mathcal{F}_{\rm Sud}^{\rm V}\right) = 0$$

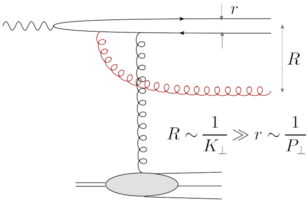
  - final-state emissions irrelevant if the imbalance $K_\perp$ not measured

- Sudakov dynamics does not change the total number of gluons
- In order to uncover DGLAP dynamics, we need to go beyond DLA

Real gluon emissions in the initial state

- $P_\perp$ (relative momentum) $\gg k_{g\perp} \simeq K_\perp$ (imbalance) $\gg \ell_\perp \sim Q_s$ (collision)
- Leading twist: leading powers in both $K_\perp/P_\perp$ and $\ell_\perp/K_\perp$
- Soft gluon $z_g \ll 1 \Rightarrow$ effective gluon-gluon dipole





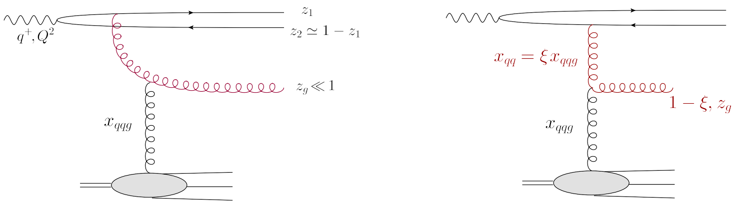
- Same WW colour operator, but evaluated at the lower scale $\ell_\perp$:

$$U_{\boldsymbol{z}}^{ac}\left(V_{\boldsymbol{x}} t^c V_{\boldsymbol{y}}^\dagger\right) - t^a \simeq R^j \left(U_{\boldsymbol{z}} \partial^j U_{\boldsymbol{z}}^\dagger\right)^{ac} t^c.$$

  - renormalisation group... as expected for one step in DGLAP

From projectile to target rapidity

- Subtract JIMWLK evolution of the LO gluon WW TMD: $z_g \ll K_\perp^2/P_\perp^2$
  - z_g is restricted to $K_\perp^2/P_\perp^2 \lesssim z_g \ll K_\perp/P_\perp$ (large angles)
- Change longitudinal fractions: from projectile (z_g) to target (ξ)
  - transfer the gluon from the dipole to the target
  - always possible if the gluon is soft $(z_g \ll 1)$



- Essential in order to achieve TMD factorisation: a target picture

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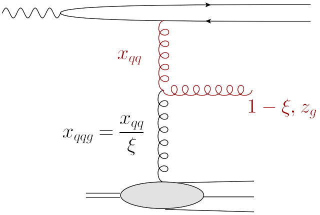
$$\xi \equiv \frac{x_{q\bar{q}}}{x_{q\bar{q}g}} = \frac{Q^2 + M_{q\bar{q}}^2}{Q^2 + M_{q\bar{q}g}^2}$$

$$z_g(\xi) = \frac{\xi}{1-\xi}\,\frac{K_\perp^2}{Q^2 + \frac{P_\perp^2}{z_1 z_2}}$$



- Integration limits on z_g transmit to corresponding limits on ξ

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$$\frac{1}{K_\perp^2 + z_g\,(Q^2 + P_\perp^2/z_1 z_2)}$$

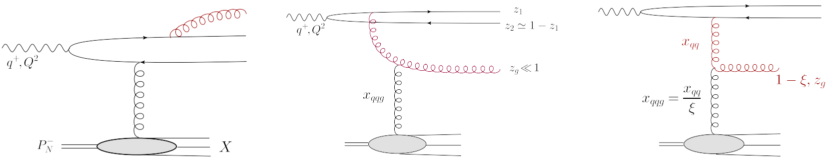
$$z_g(\xi) = \frac{\xi}{1-\xi}\,\frac{K_\perp^2}{Q^2 + \frac{P_\perp^2}{z_1 z_2}}$$

$$\Longrightarrow \quad \frac{1-\xi}{K_\perp^2}$$

- Integration limits on z_g transmit to corresponding limits on ξ
- Disentangling projectile from target variables in the energy denominators

The DGLAP splitting function

$$\Delta\mathcal{F}_{\mathrm{R}} = \frac{\alpha_s}{2\pi^2 K_\perp^2}\int_{x_\star}^{1-K_\perp/P_\perp} \mathrm{d}\xi \, P_{gg}\left(\xi\right)\,\frac{x}{\xi}G\left(\frac{x}{\xi}, K_\perp^2\right)$$



$$P_{gg}(\xi) = 2N_c\,\frac{1 + (1-\xi)^2(1+\xi^2) - (1-\xi^2)}{\xi(1-\xi)}$$

- Final-state (singular at $\xi \to 1$ and 0) + initial-state + interference
- Upper limit $1 - K_\perp/P_\perp$ on ξ comes from $z_g \lesssim K_\perp/P_\perp$
- Lower limit $x_* \ll 1$ with $\alpha_s \ln\frac{1}{x_*} \ll 1$ from $z_g \gtrsim x_*(K_\perp/P_\perp)^2$
  - separate small-x logarithms between JIMWLK and DGLAP

Emergent DGLAP & CSS evolutions

$$\Delta\mathcal{F}_{\rm R} \simeq \frac{\alpha_s}{2\pi^2 K_\perp^2}\int_{x_\star}^1 {\rm d}\xi\, P_{gg}^{(+)}(\xi)\,\frac{x}{\xi}G\left(\frac{x}{\xi},K_\perp^2\right) + \Delta\mathcal{F}_{\rm Sud}^{\rm R}$$

- "Plus" prescription for the pole at $\xi \to 1$:

$$P_{gg}(\xi) = 2N_c\,\frac{1+(1-\xi)^2(1+\xi^2)-(1-\xi^2)}{\xi(1-\xi)_+}$$

- $P_\perp$–dependence only in the Sudakov piece (final-state emission)

$$\Delta\mathcal{F}_{\rm Sud}^{\rm R}(x,K_\perp^2,P_\perp^2) = \frac{\alpha_s N_c}{\pi^2}\,\frac{1}{K_\perp^2}\,\frac{1}{2}\ln\frac{P_\perp^2}{K_\perp^2}\,xG(x,K_\perp^2)$$

- The corresponding virtual correction: Sudakov double-log + β_0-piece

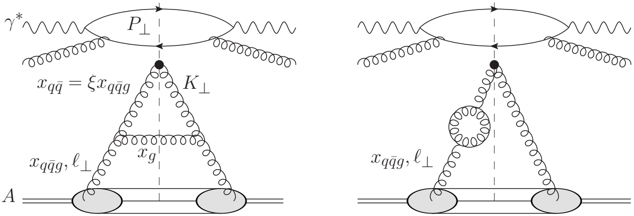
$$\Delta\mathcal{F}_{\rm V} = -\frac{\alpha_s N_c}{\pi}\left(\frac{1}{4}\ln^2\frac{P_\perp^2}{K_\perp^2} - \beta_0\ln\frac{P_\perp^2}{K_\perp^2}\right)\mathcal{F}_g^{(0)}(x,K_\perp^2)$$

- $\Delta\mathcal{F}_{\rm Sud}^{\rm R} + \Delta\mathcal{F}_{\rm V}$: one step in CSS evolution with increasing $P_\perp^2$

The β_0 piece

$$\frac{\mathrm{d}\sigma^{\gamma^* A\to q\bar{q}A}}{\mathrm{d}z\mathrm{d}^2\boldsymbol{P}} = H(z, Q^2, P_\perp^2)\, xG(x, P_\perp^2) \propto \alpha_s xG(x, P_\perp^2)$$

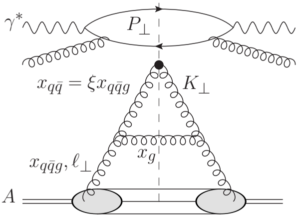
- At one-loop we expect: running coupling $\alpha_s(P_\perp^2)$ & DGLAP for $xG(x, P_\perp^2)$
- The β_0 piece contributes in both case: gluon anomalous dimension

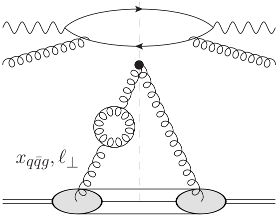


- Generated via loop corrections to the gluon propagator ...
- ... which are missing in the (usual) CGC calculation at NLO: shockwave
- So, we will add them by hand ! (left for future work)

CSS evolution for the gluon TMD

- Resummation of Sudakov logs: singularity at $\xi \to 1$, cut off at $1 - \xi = \frac{K_\perp}{P_\perp}$





- Increasing $P_\perp \Rightarrow$ larger phase-space for soft gluon emissions

- CSS evolution: change in $K_\perp$-distribution due to soft emissions in s-channel

$$\frac{\partial \mathcal{F}_g(x, K_\perp, P_\perp^2)}{\partial \ln P_\perp^2} = \text{ real Sudakov - virtual Sudakov } + \beta_0 \text{ term}$$

- Boundary condition at $P_\perp^2 = K_\perp^2$:

  WW gluon TMD (including JIMWLK) + gluon PDF (including DGLAP)

DGLAP evolution in the context of small-x

- DGLAP evolution from Q_s^2 up to $K_\perp^2$ with a source term from CGC:

$$\frac{\partial\, xG(x,K_\perp^2)}{\partial K_\perp^2} = \pi\mathcal{F}_g^{(0)}\left(\frac{x}{x_*},K_\perp^2\right) + \frac{\alpha_s}{2\pi}\frac{1}{K_\perp^2}\int_{x_*}^{1}\mathrm{d}\xi\,\mathcal{P}_{gg}(\xi)\,\frac{x}{\xi}G\left(\frac{x}{\xi},K_\perp^2\right)$$

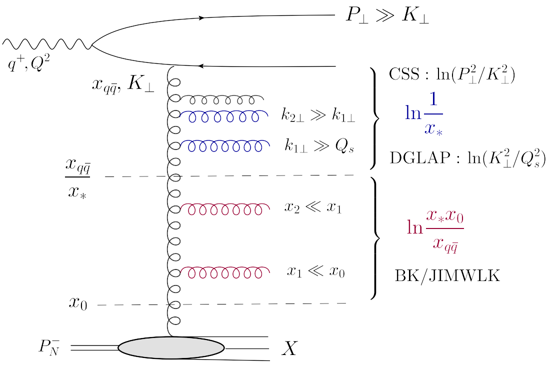
- $\mathcal{F}_g^{(0)}(x/x_*,K_\perp^2)$: the gluon WW TMD including JIMWLK evolution
- $\mathcal{P}_{gg}(\xi)$: full DGLAP splitting function (w/ plus prescription and β_0 piece)
- DGLAP evolution turned on at a scale $\mu_0 \sim Q_s(x)$: CGC initial condition

$$xG(x,\mu_0^2) = \pi\int_{\Lambda^2}^{\mu_0^2}\mathrm{d}\ell_\perp^2\,\mathcal{F}^{(0)}(x,\ell_\perp^2)$$

- All powers of $\alpha_s \ln\frac{1}{x_*}\ln\frac{K_\perp^2}{Q_s^2}$ cancel between JIMWLK and DGLAP
  - the DLA (double-logarithmic approximation) is a common limit

Three successive evolutions

- JIMWLK for the WW TMD from $x_0 \sim 10^{-2}$ down to $x_{q\bar{q}}$: $\mathcal{F}_g^{(0)}(x_{q\bar{q}}, K_\perp^2)$
- DGLAP evolution from Q_s^2 up to $P_\perp^2$ using $\mathcal{F}_g^{(0)}(x_{q\bar{q}}, x_*)$ as a source term
- CSS evolution from $K_\perp^2$ up to $P_\perp^2$ with DGLAP boundary condition



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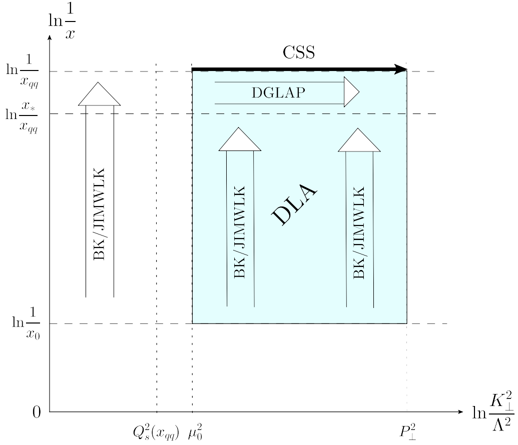
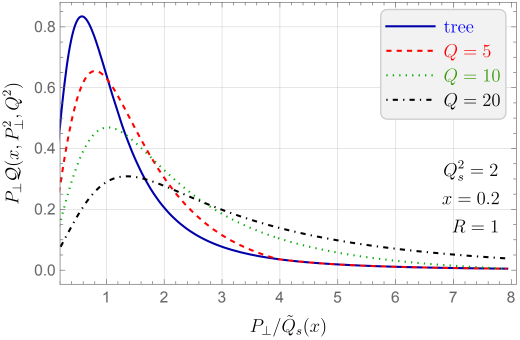


Sudakov resummation

(E.I., D. Triantafyllopoulos, S. Wei and F. Yuan, in preparation)

$$\frac{\partial \mathcal{F}_g(x,K_\perp,P_\perp^2)}{\partial \ln P_\perp^2} = \frac{\alpha_s N_c}{2\pi}\left\{\frac{1}{K_\perp^2}\int_{\Lambda^2}^{K_\perp^2} \mathrm{d}\ell_\perp^2\, \mathcal{F}_g(x,\ell_\perp,P_\perp^2) - \int_{K_\perp^2}^{P_\perp^2}\frac{\mathrm{d}\ell_\perp^2}{\ell_\perp^2}\mathcal{F}_g(x,K_\perp,P_\perp^2)\right\}$$

- A rate equation: gain (real) - loss (virtual)
- Dijets with very small imbalance $K_\perp \ll P_\perp$ are disfavoured



- "tree" = CGC (MV model)
- Distribution pushed towards larger values of $K_\perp$

$$\langle K_\perp^2 \rangle \sim P_\perp^2\, \mathrm{e}^{-\frac{1}{\sqrt{\bar\alpha}}}$$

Instead of conclusions

Thank you for being here and for your attention !