Sudakov effects in SIDIS

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Probing the CGC and QCD matter at hadron colliders

(*in celebration of Edmond Iancu's 60th birthday*)

Florence, Italy, March 24-27, 2025

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The Color Glass Condensate

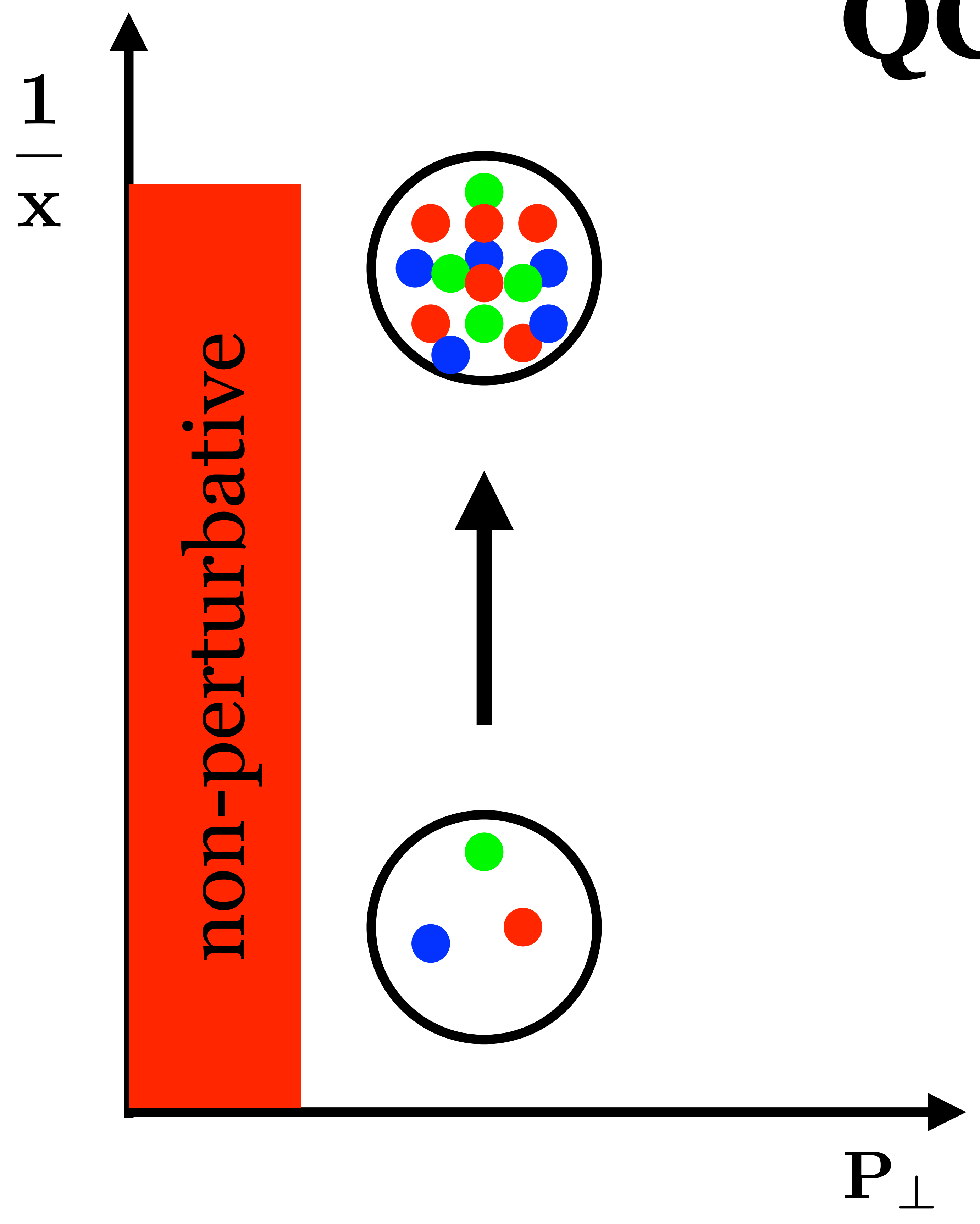
Francois Gelis (Saclay, SPhT), Edmond Iancu (Saclay, SPhT), Jamal Jalilian-Marian (Baruch Coll. and CUNY, Graduate School - U. Ctr.), Raju Venugopalan (Brookhaven) (Feb, 2010)

Published in: *Ann.Rev.Nucl.Part.Sci.* 60 (2010) 463-489 • e-Print: 1002.0333 [hep-ph]

pdf DOI cite claim reference search 1,489 citations

Happy 60th Edmo!

QCD at small x: gluon saturation



high gluon density: multiple eikonal scatterings

high energy: evolution in x via BK/JIMWLK

p_t broadening

suppression of single inclusive spectra/away side peak

A systematic formalism for multi-particle production in QCD

nuclear shadowing/modification factor, azimuthal correlations

long range rapidity correlations

connections to TMD,....

$$Q_s^2(x, A, b_\perp) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$

$Q_s^2 \sim 1\,\mathrm{GeV}^2$ at

$x \sim 3 \times 10^{-4}$

Probing CGC in high energy collisions

nucleus-nucleus collisions: “dense on dense”

significant modeling/QGP

proton-nucleus collisions: “dilute on dense”

DIS: (inclusive/diffractive)

much less modeling

structure functions

particle production

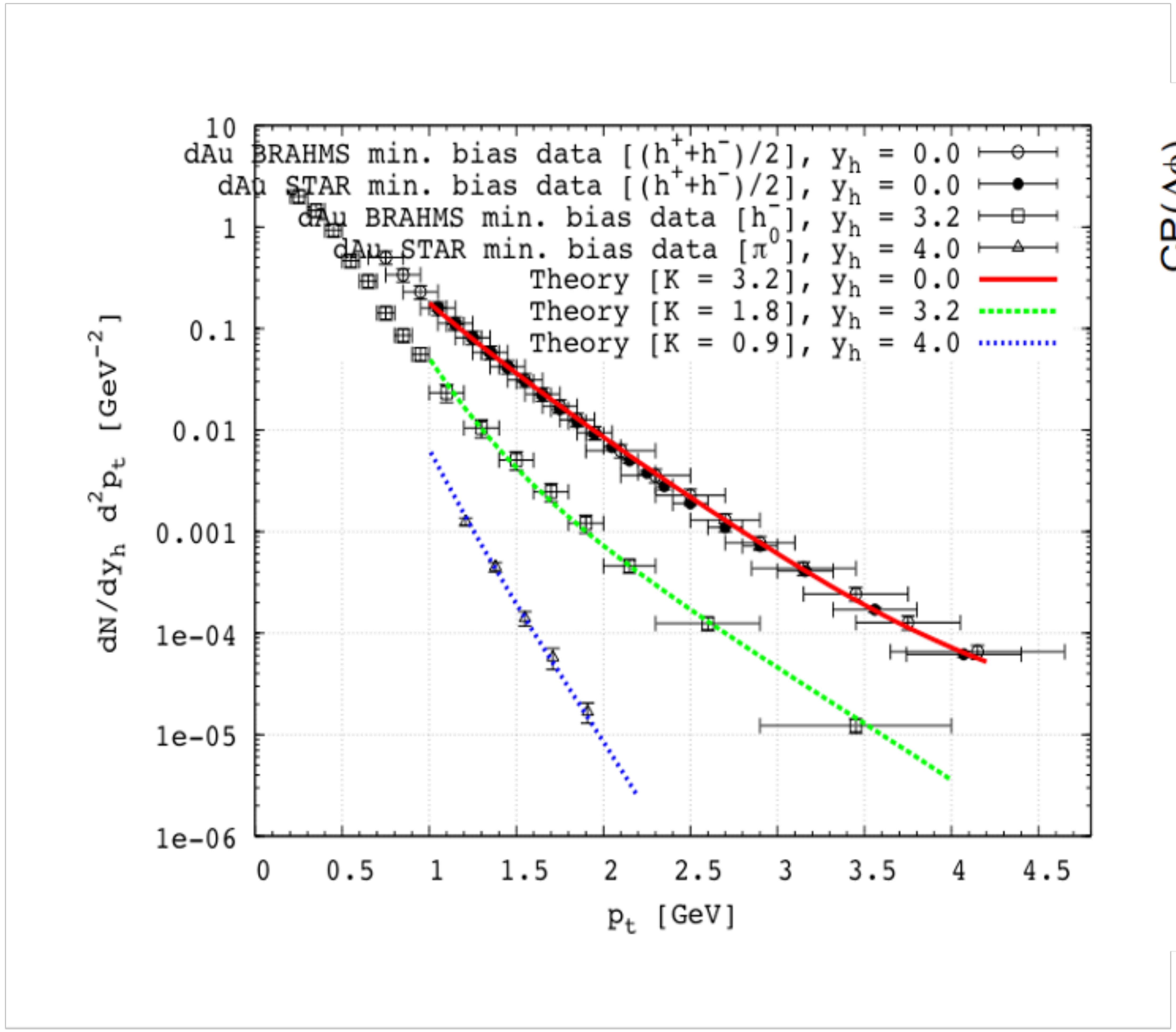
angular correlations

EIC

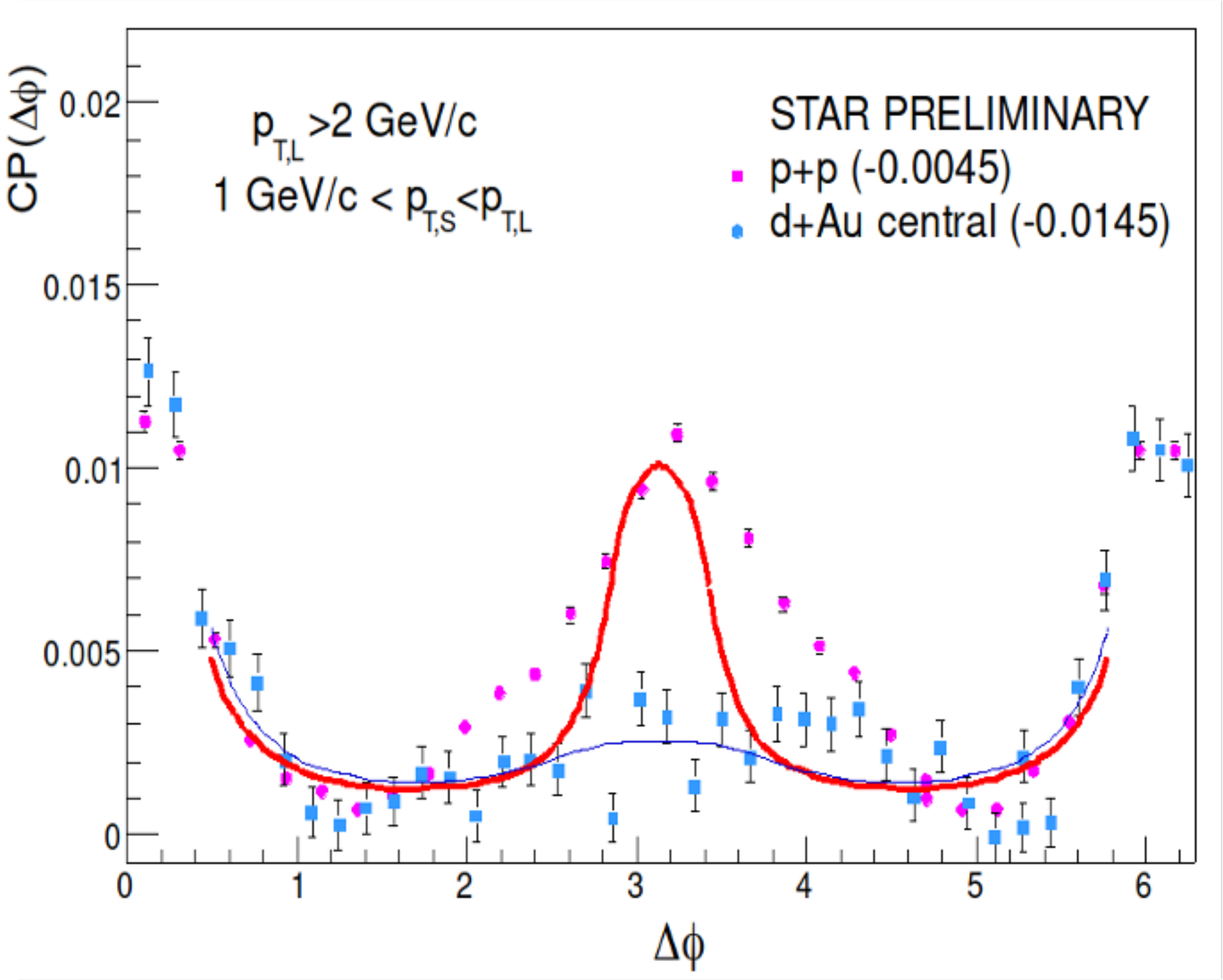
to start in ~ 10 years

CGC at RHIC

Single and double inclusive hadron production in dA collisions



Dumitru,Hayashigaki, JJM, NPA770 (2006) 57

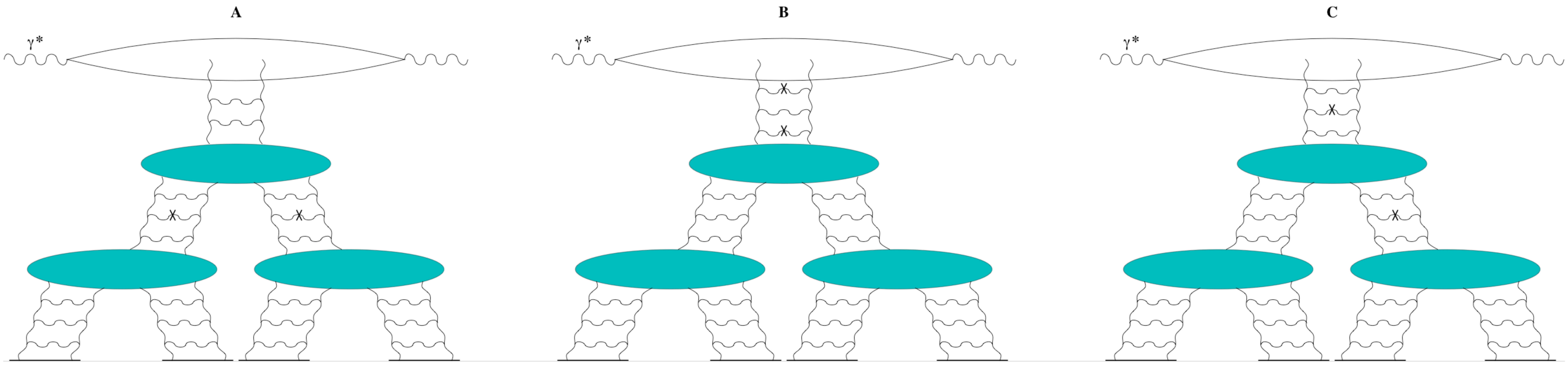


Albacete, Marquet, PRL105 (2010) 162301

Double and single inclusive hadron production in DIS at small x

Inclusive dihadron production in midrapidity: LO

JJM, Yu. Kovchegov
PRD70 (2004) 114017



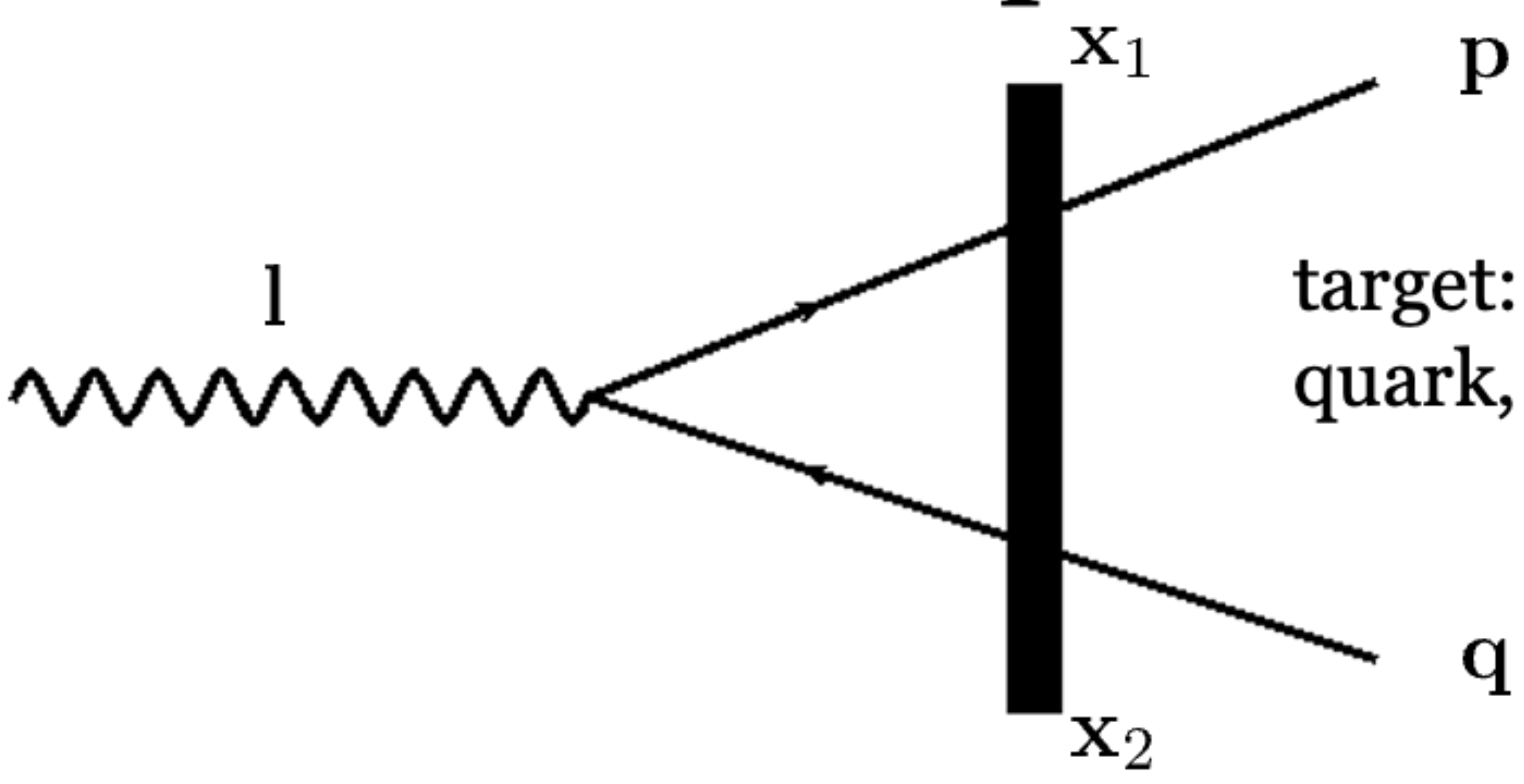
need a very large rapidity window

target is treated as a classical color field $\mathbf{A}_a^\mu = \delta^{\mu -}\, n^\mu\, S_a(x^+, \mathbf{x})$

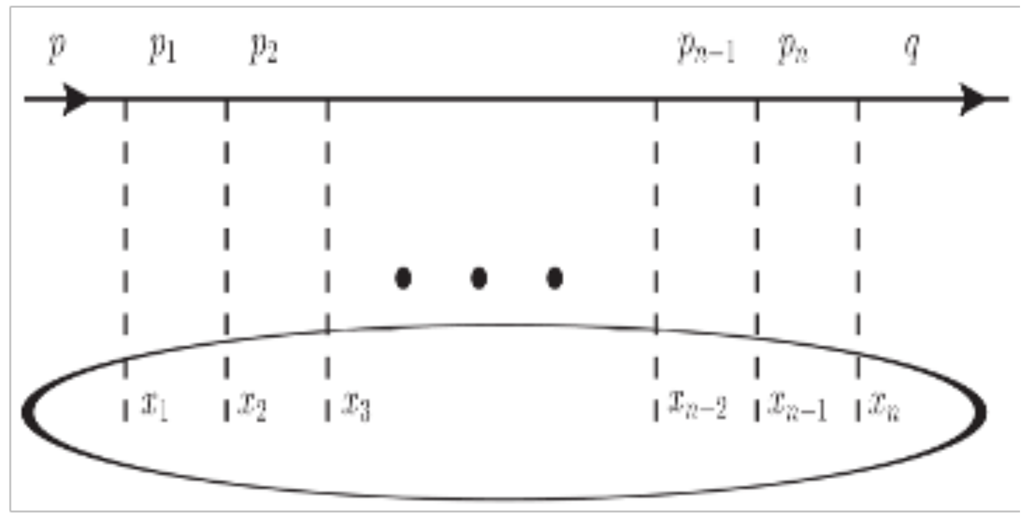
scatterings of gluons on the target encoded in Wilson lines $\mathbf{U}\ (\mathbf{x}_1), U^\dagger(\mathbf{x}_2)$

leading log evolution included

Inclusive dihadron production in forward rapidity: LO



target: a classical color field
quark, antiquark multiply scatter on the target



$$\frac{d\sigma^{\gamma^* A\to q\bar{q}X}}{d^2p\, d^2q\, dy_1\, dy_2} = \frac{e^2Q^2(z_1z_2)^2N_c}{(2\pi)^7}\delta(1-z_1-z_2)$$

$$\int d^8x_\perp e^{ip\cdot(x_1'-x_1)}e^{iq\cdot(x_2'-x_2)}\left[S_{122'1'} - S_{12} - S_{1'2'} + 1\right]$$

$$\Big\{4z_1z_2K_0(|x_{12}|Q_1)K_0(|x_{1'2'}|Q_1)+$$

$$(z_1^2+z_2^2)\,\frac{x_{12}\cdot x_{1'2'}}{|x_{12}||x_{1'2'}|}\,K_1(|x_{12}|Q_1)K_1(|x_{1'2'}|Q_1)\Big\}$$

with

dipole $\mathbf{S}_{12} \equiv \frac{1}{N_c} Tr\, V(x_1)\, V^\dagger(x_2)$

$\mathbf{x}_{12} \equiv \mathbf{x}_1 - \mathbf{x}_2$

quadrupole

$\mathrm{S}_{122'1'} \equiv \frac{1}{N_c} Tr\, V(\mathbf{x}_1)\, V^\dagger(\mathbf{x}_2)\, V(\mathbf{x}_{2'})\, V^\dagger(\mathbf{x}_{1'})$

Only dipoles and quadrupoles contribute: DMXY, PRD 83 (2011) 105005

Toward precision CGC: inclusive DIS

NLO BK/JIMWLK evolution equations

Kovner, Lublinsky, Mulian (2013)

Balitsky, Chirilli (2007)

list already outdated! many more papers in the last few months

NLO corrections to structure functions

Beuf, Lappi, Paatelainen (2022)

Beuf (2017)

NLO corrections to SIDIS

Bergabo, JJM (2023, 2024)

Caucal, Ferrand, Salazar (2024)

NLO corrections to dihadron/dijets (+)

Bergabo, JJM (2022, 2023)

Iancu, Mulian (2023)

Caucal, Salazar, Schenke, Stebel, Venugopalan (2023), Caucal, Salazar, Schenke, Venugopalan (2022)

Taels, Altinoluk, Beuf, Marquet (2022), Taels (2023)

Caucal, Salazar, Venugopalan (2021)

Ayala, Hentschinski, JJM, Tejeda-Yeomans (2016,2017),............

Toward precision CGC: exclusive/diffractive DIS

NLO corrections to diffractive structure functions

Beuf, Hanninen, Lappi, Mulian, Mantyssari (2022)

……………………

NLO corrections to diffractive dihadron/dijets (+)

Boussarie, Grabovsky, Szymanowski, Wallon (2016)

Iancu, Mueller, Triantafyllopoulos (2021, 2022)

Fucilla, Grabovsky, Li, Szymanowski, Wallon (2023)

……………………

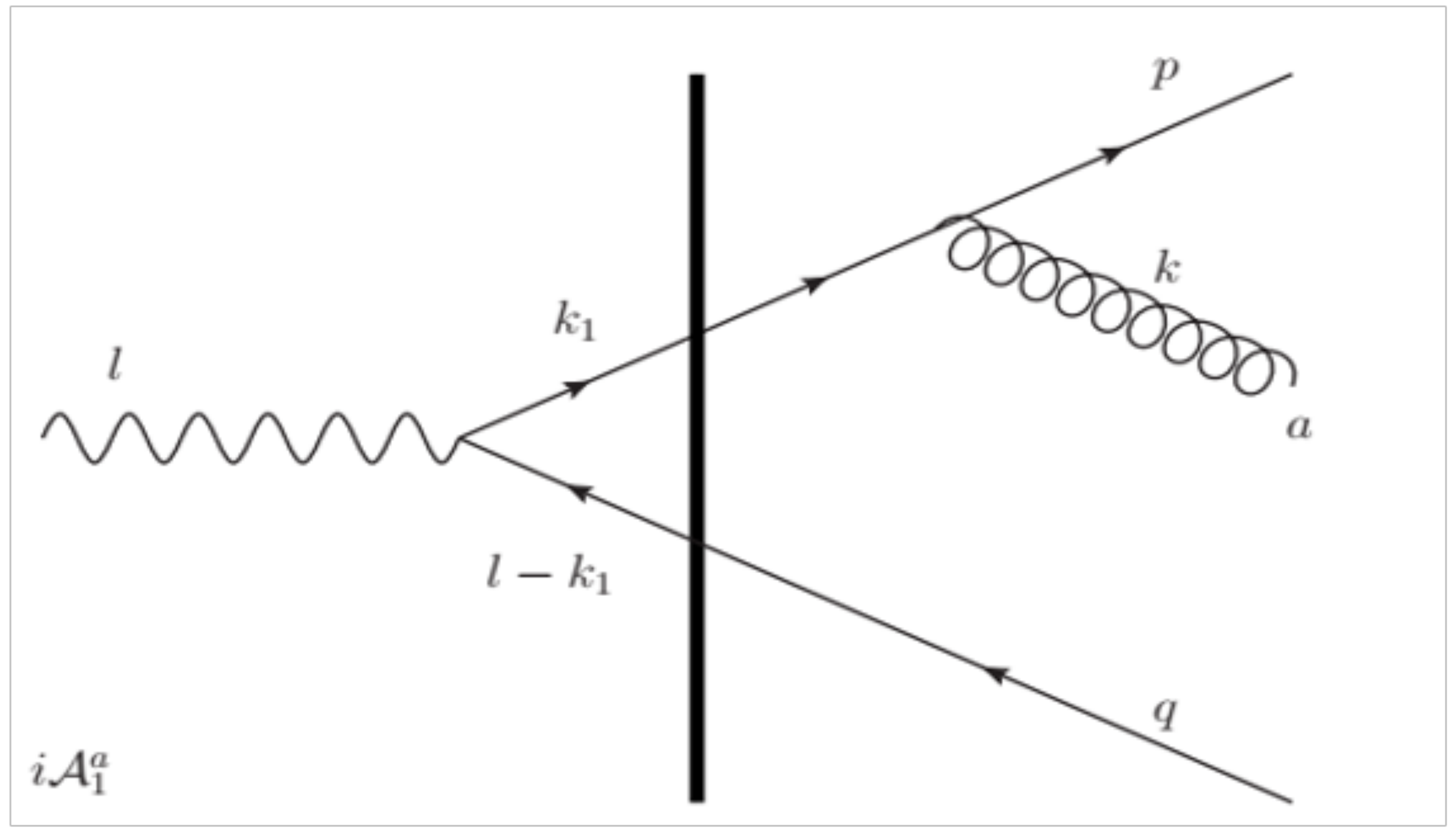
NLO corrections to exclusive light/heavy vector meson production (+)

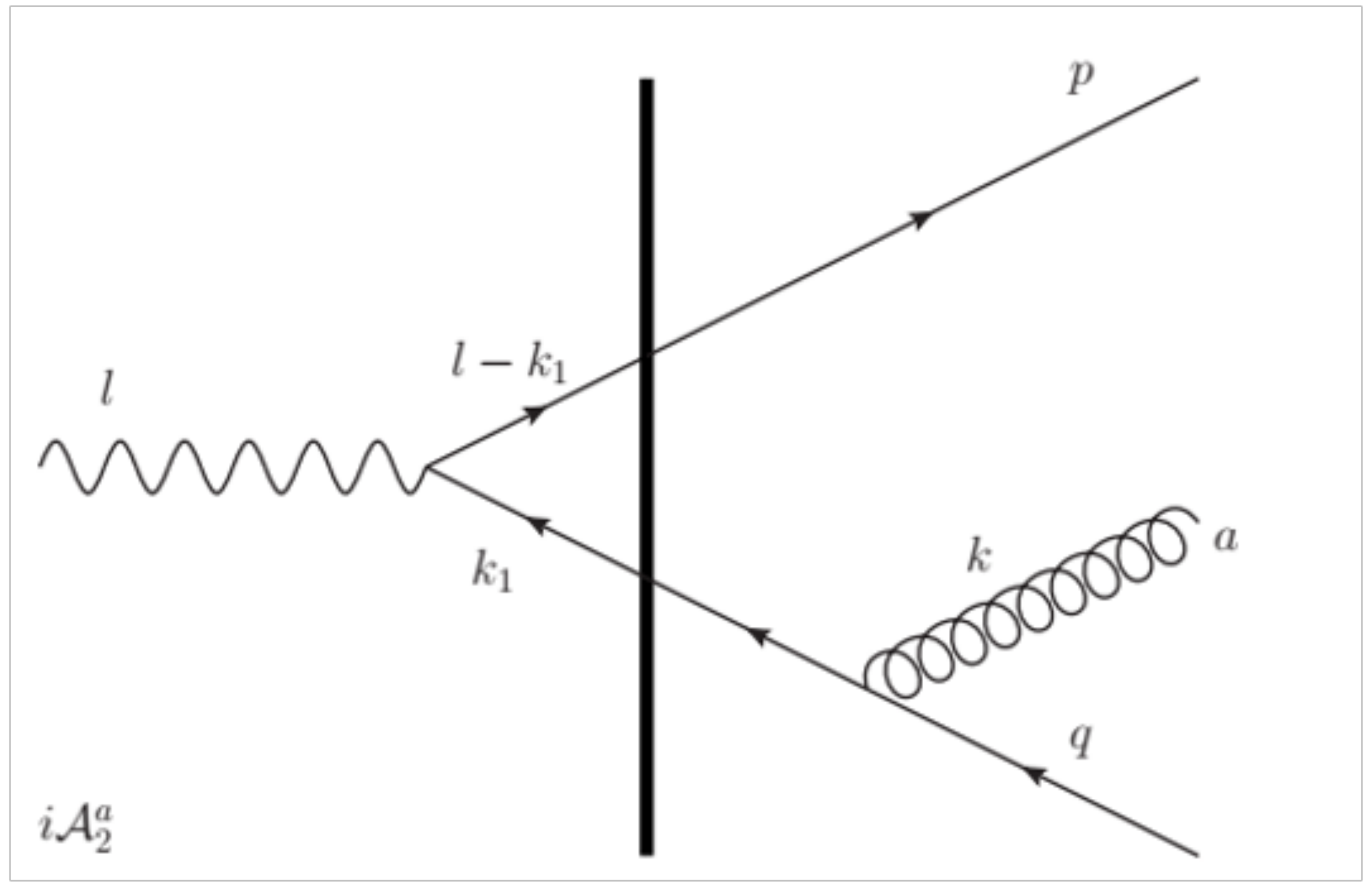
Boussarie, Grabovsky, Ivanov, Szymanowski, Wallon (2016)

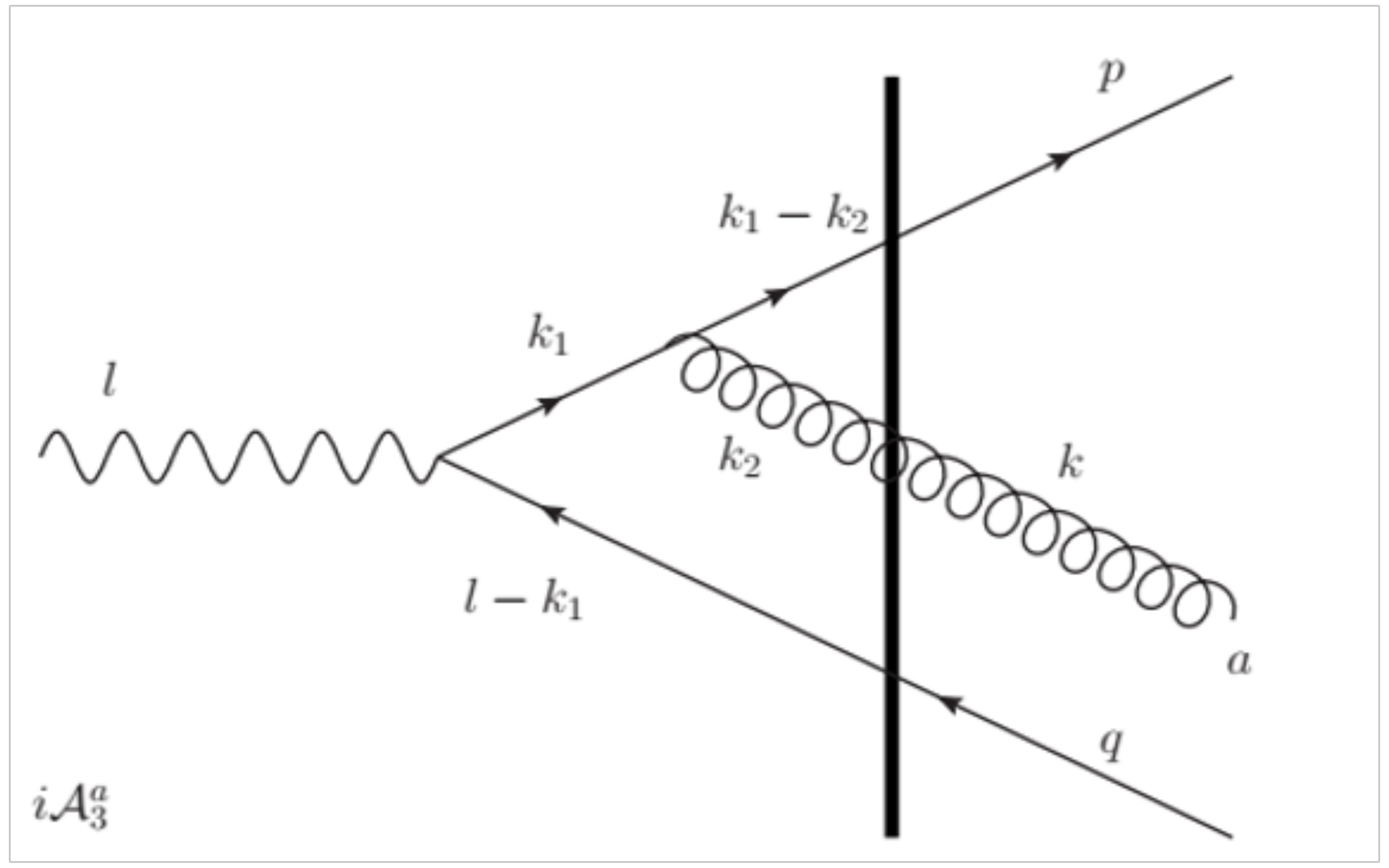
Mantyssari, Penttala (2021, 2022)

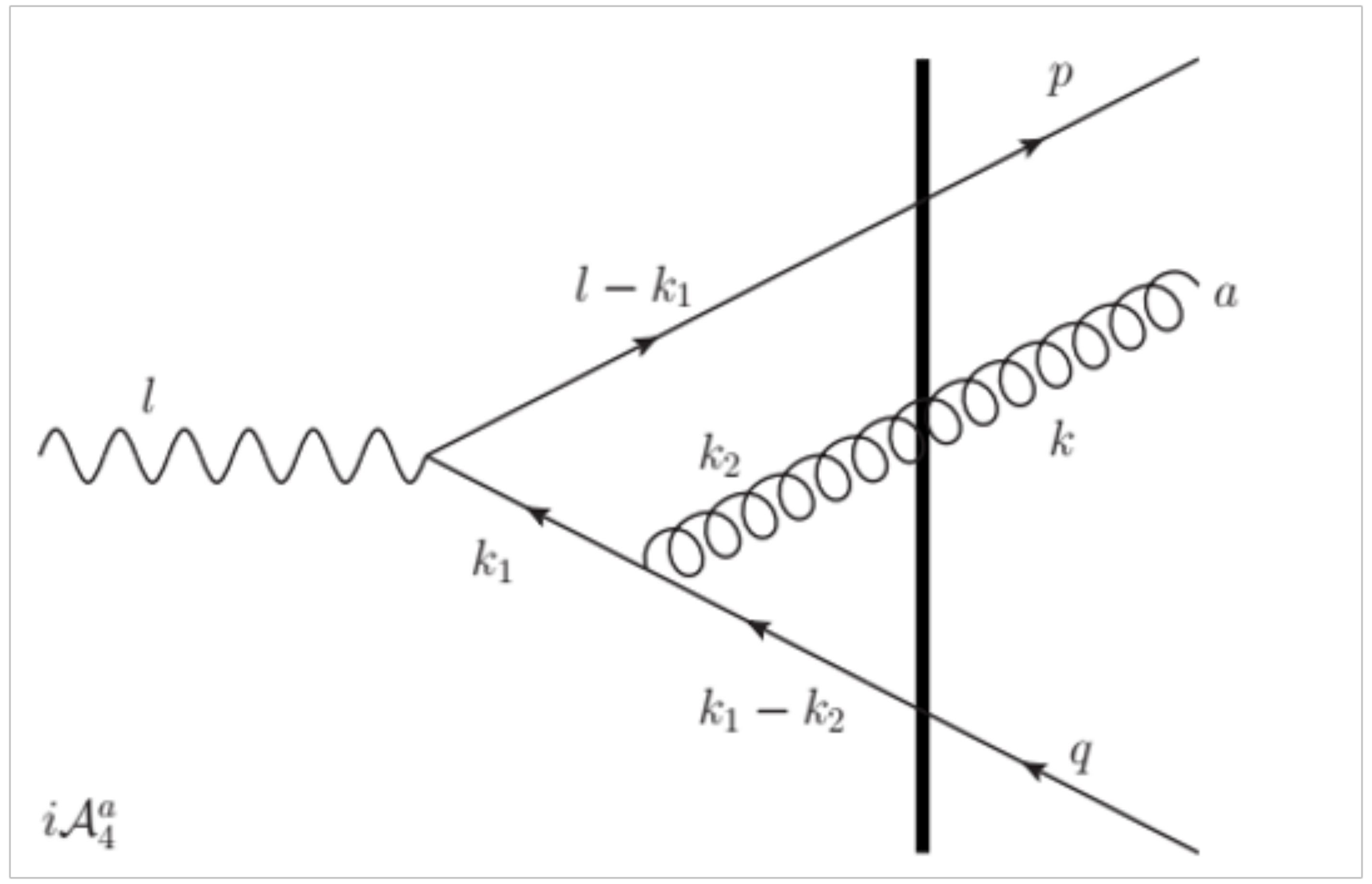
……………………

One loop corrections - real diagrams





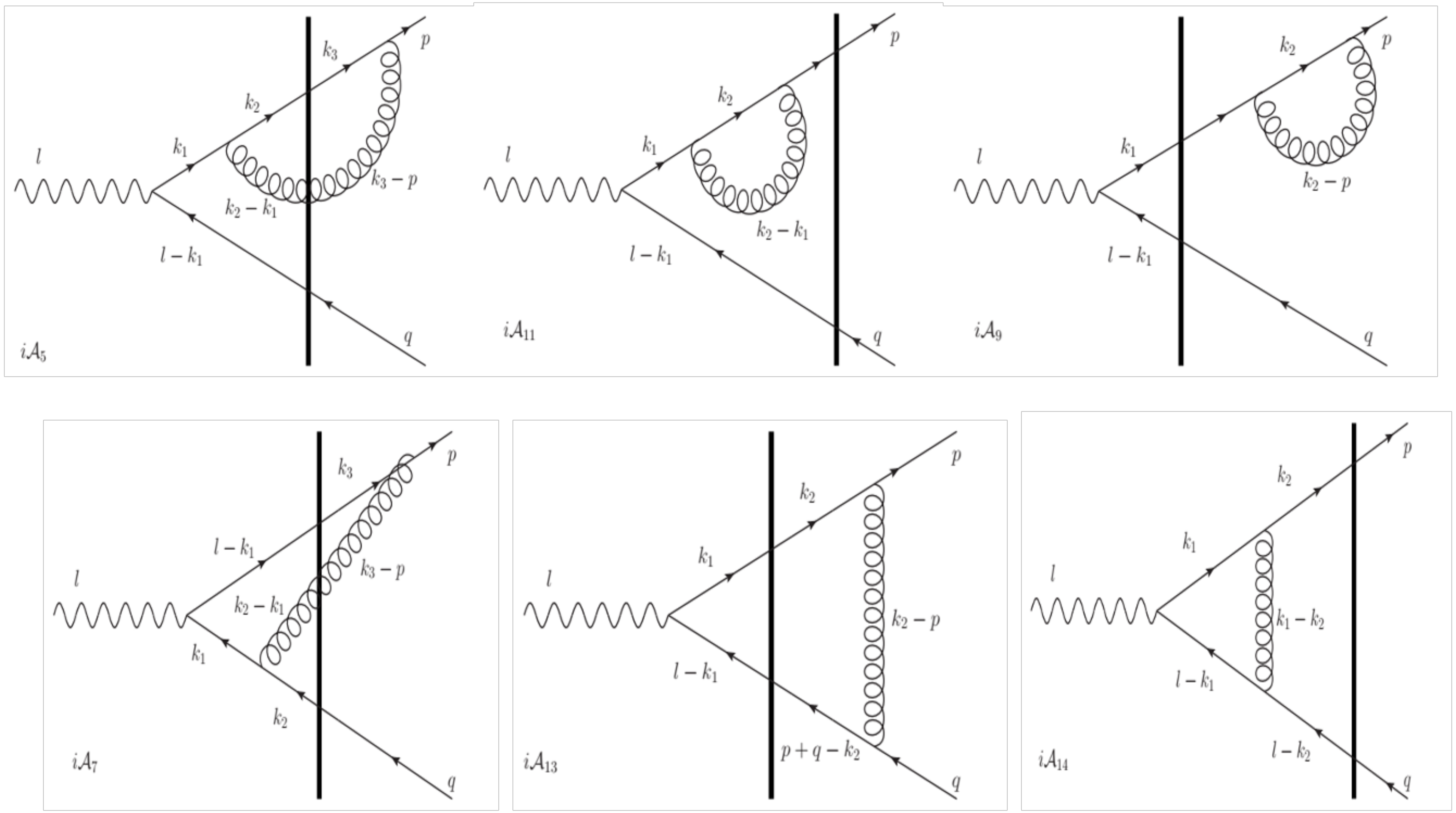




3-parton production: Ayala, Hentschinski, JJM, Tejeda-Yeomans
PLB 761 (2016) 229 and NPB 920 (2017) 232

Iancu, Mulian, JHEP 07 (2023) 121

One loop corrections – virtual diagrams



F. Bergabo and JJM, dihadrons, 2207.03606

P. Taels et al., dijets, 2204.11650

P. Caucal et al., dijets, 2108.06347

Divergences

- Ultraviolet

real corrections are UV finite

UV divergences cancel among virtual diagrams

- Soft

soft divergences cancel between real and virtual diagrams

- Collinear

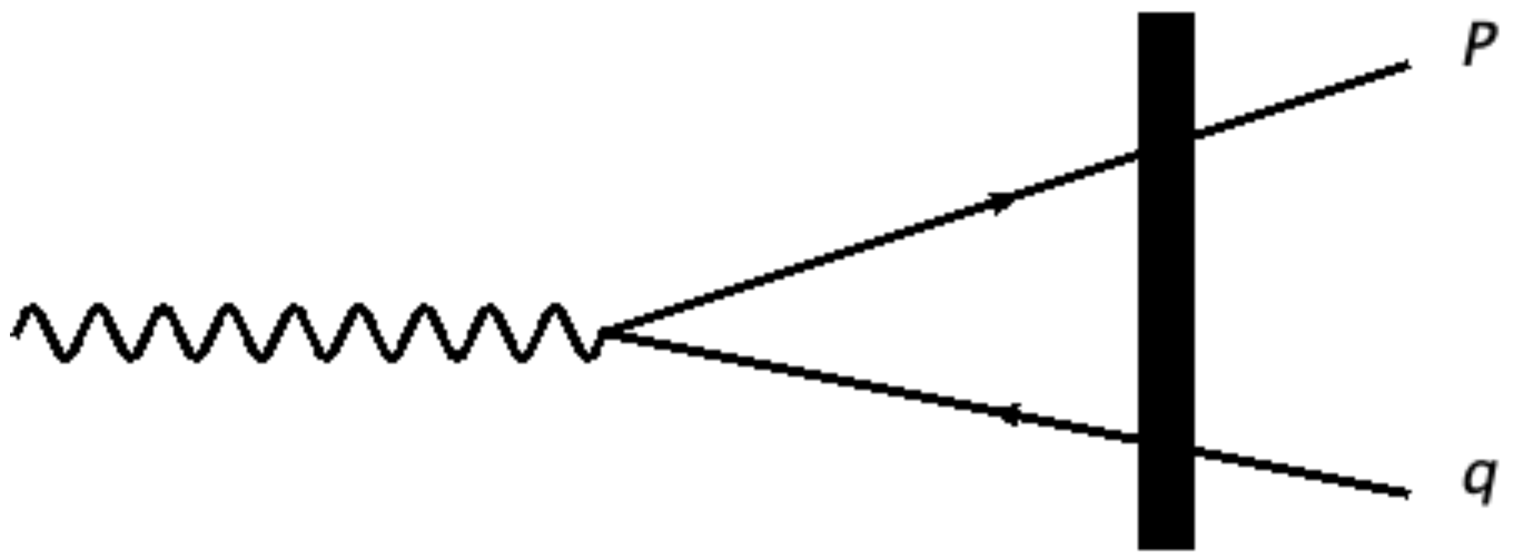
collinear divergences are absorbed into fragmentation functions

- Rapidity

Rapidity divergences are absorbed into JIMWLK evolution of dipoles and quadrupoles

$$\sigma^{\gamma^* A \to h_1 h_2 X} = \sigma_{LO} \otimes \mathrm{JIMWLK} + \sigma_{LO} \otimes D_{h/q}(z_h, \mu^2) \otimes D^{(0)}_{h/q}(z_h) + \sigma^{\mathrm{finite}}_{NLO}$$

Inclusive dihadron production in DIS at small x: back to back limit



Quadrupole

$$\frac{1}{N_c} Tr\, V(\mathbf{x}_1)V^\dagger(\mathbf{x}_2)V(\mathbf{x}_2')V^\dagger(\mathbf{x}_1')$$

$$\mathbf{P}_\perp = \mathbf{p} - \mathbf{q}$$
$$\mathbf{K}_\perp = \mathbf{p} + \mathbf{q}$$
$$\mathbf{K}_\perp \longrightarrow 0$$

for $\mathbf{P}_\perp \sim |\mathbf{p}| \sim |\mathbf{q}|$ one can get large $\log \frac{\mathbf{P}_\perp^2}{\mathbf{K}_\perp^2}$ integrating over radiated gluon

Sudakov double logs in dijets production in DIS at small x:

Taylor expansion of Wilson lines around "center of mass" coordinate ⟶ Weizsacker-Williams field

CGC calculations contain Sudakov double logs, but with the wrong sign! $(+)\left[\log \mathbf{P}_\perp^2 \Delta \mathbf{b}_\perp^2\right]^2$

impose a kinematic constraint on life time of gluon radiation

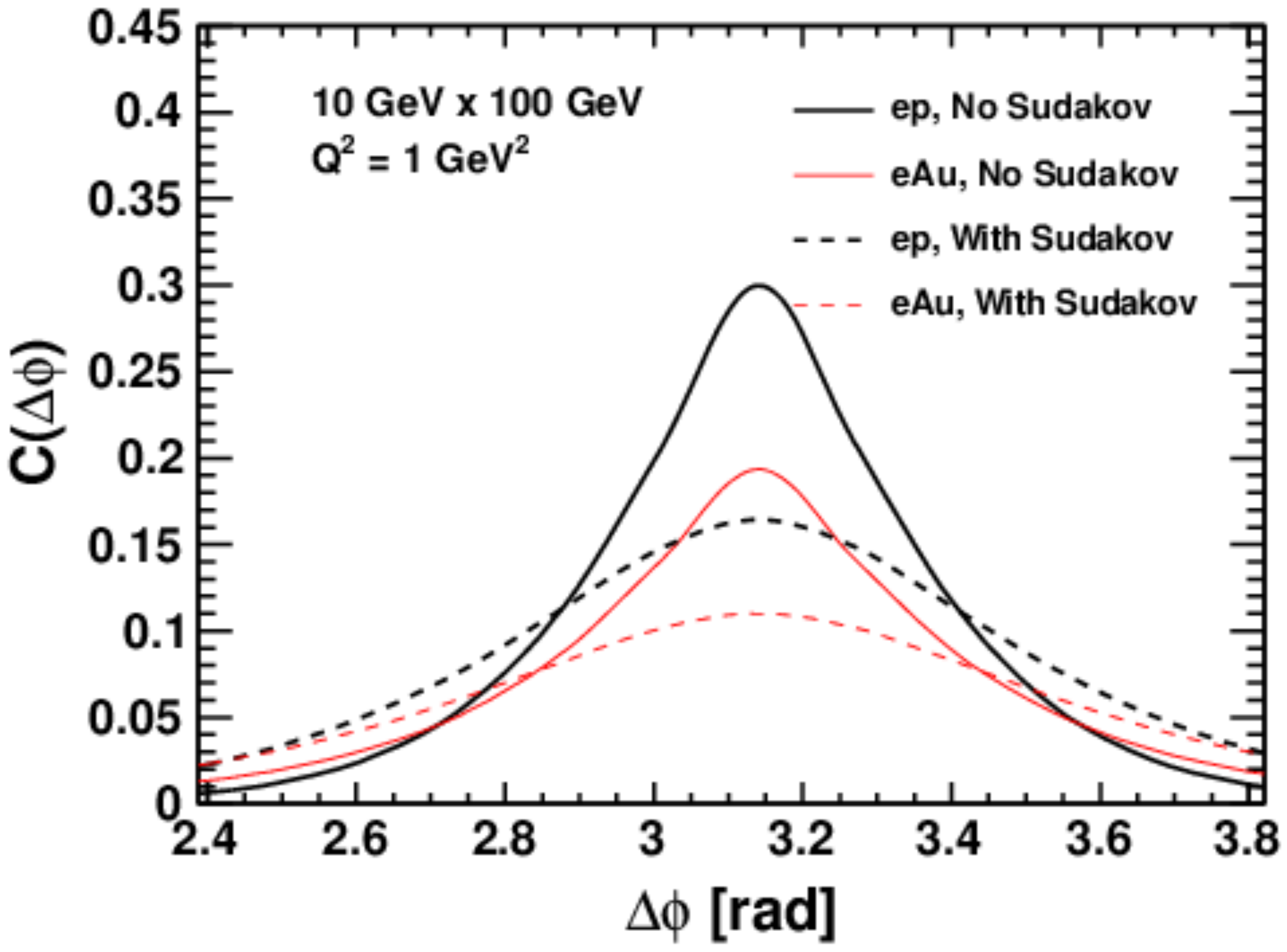
(the usual strong ordering in + momenta is not sufficient)

Taels, Altinoluk, Beuf, Marquet, JHEP 10 (2022) 184

Caucal, Salazar, Schenke, Venugopalan, JHEP 11 (2022) 169

....

Dihadron/dijets kinematics at EIC



Zheng, Aschenauer,Lee,Xiao, arXiv:1403.2413

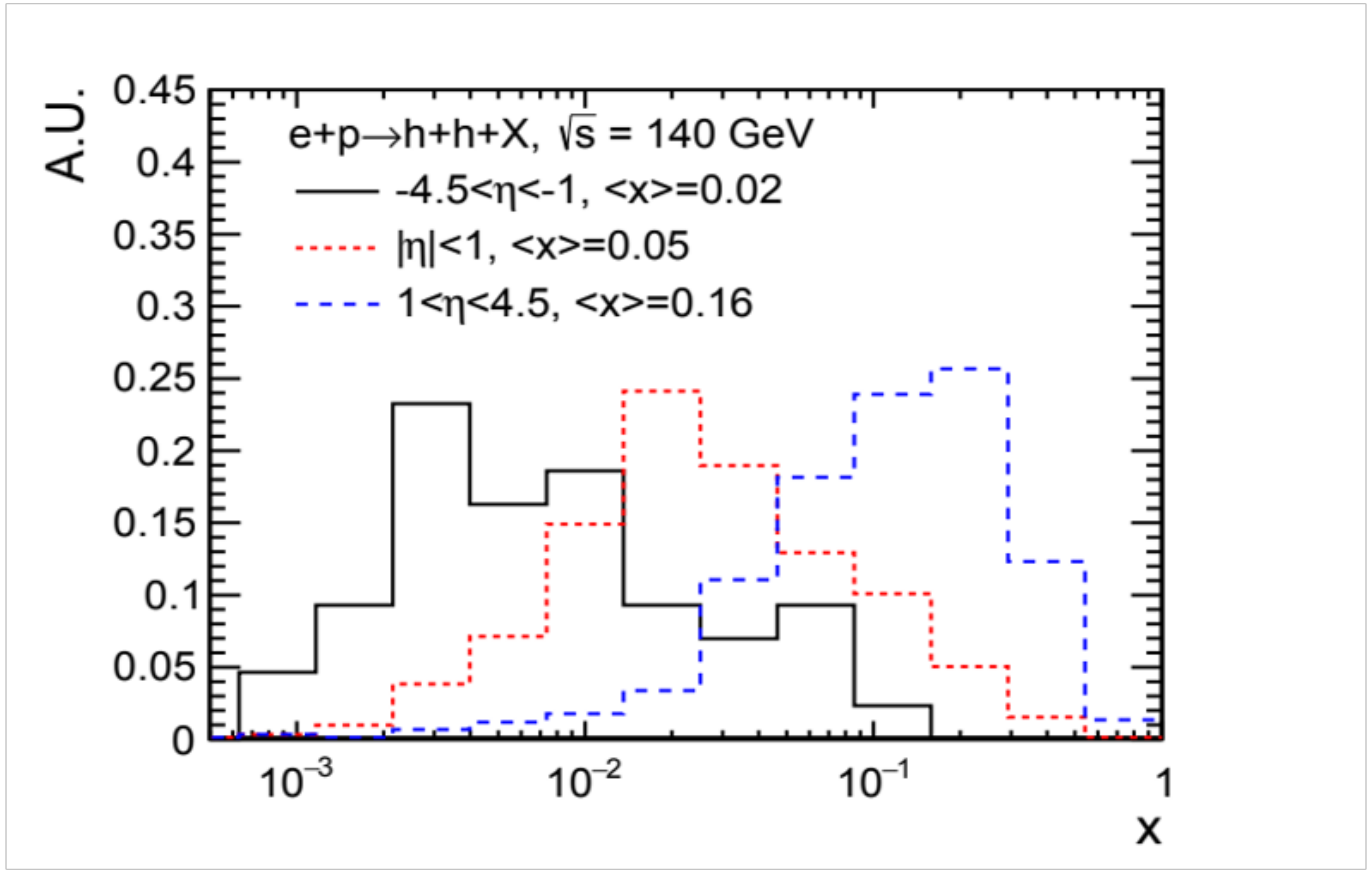


Fig courtesy of Xiaoxuan Chu

SIDIS a better process (?)

larger kinematic phase space than dihadrons

Sudakov effect can be avoided

SIDIS at small x: NLO corrections

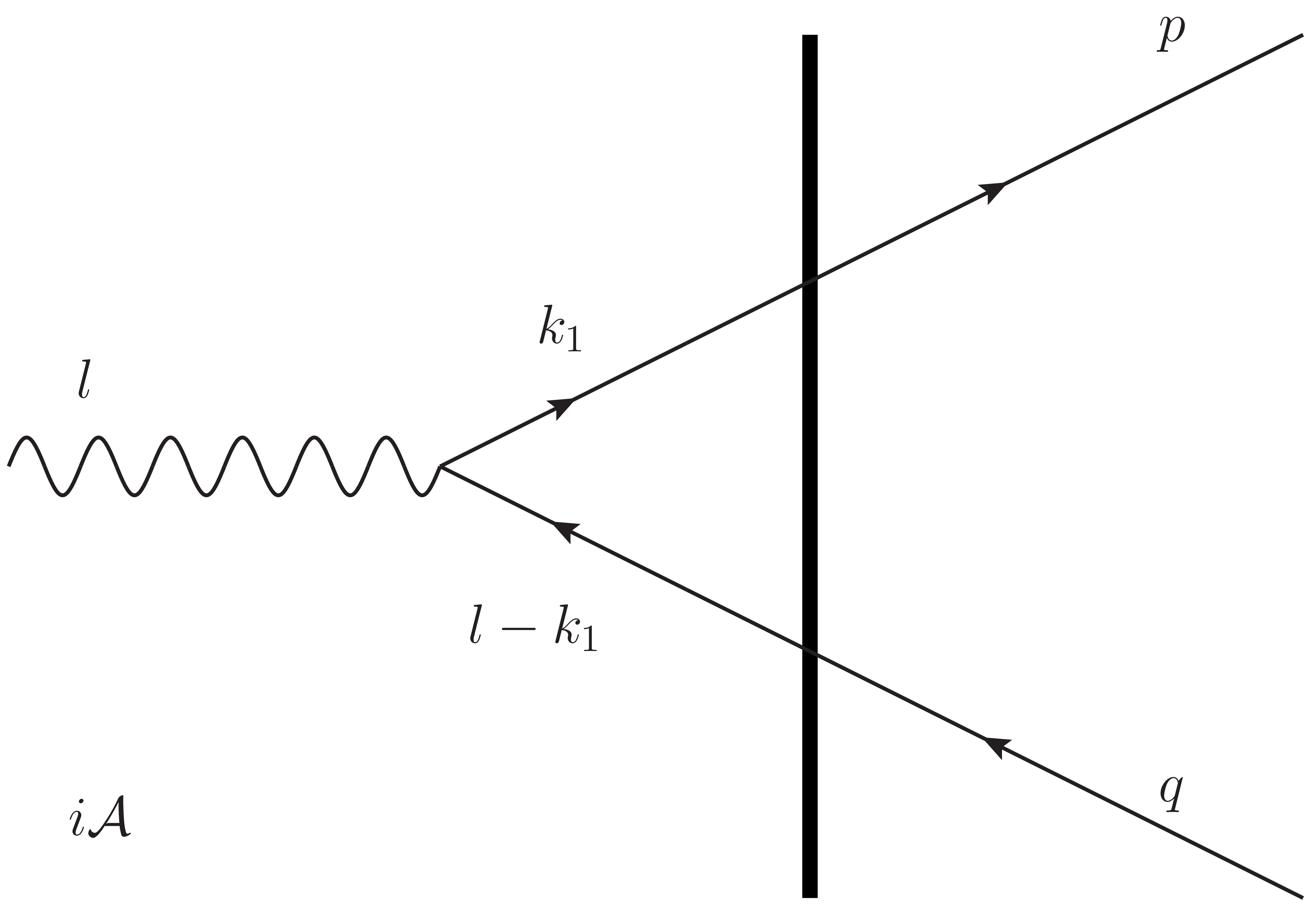
F. Bergabo, JJM, JHEP 01 (2023) 095, and arXiv:2401.06259

Caucal, Ferrand, Salazar, arXiv:2401.01934

Forward rapidity

LO:

integrate over final state antiquark



$$\frac{\mathrm{d}\sigma^{\gamma^* p/A \to q(\mathbf{p}, y_1) X}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_1} = \frac{e^2 Q^2 N_c}{(2\pi)^5} \int \mathrm{d}z_2\, \delta(1 - z_1 - z_2)\,(z_1^2\, z_2) \int \mathrm{d}^6\mathbf{x}\,[S_{11'} - S_{12} - S_{1'2} + 1]\, e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}$$
$$\left\{4 z_1 z_2 K_0(|\mathbf{x}_{12}|Q_1) K_0(|\mathbf{x}_{1'2}|Q_1) + (z_1^2 + z_2^2)\,\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\, K_1(|\mathbf{x}_{12}|Q_1) K_1(|\mathbf{x}_{1'2}|Q_1)\right\}$$

NLO corrections to SIDIS: some of the contributions are

$$\frac{\mathrm{d}\sigma^T_{1\times 1}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_1} = \frac{e^2 g^2 Q^2}{(2\pi)^8} \int_0^{1-z_1} \frac{\mathrm{d}z}{z} \frac{(1-z-z_1)(z+z_1)}{z_1} \left[z_1^2(1-z-z_1)^2 + \left(z_1^2 + (1-z-z_1)^2\right)(z+z_1)^2 + (z+z_1)^4\right]$$
$$\int \mathrm{d}^8\mathbf{x}\, K_1(|\mathbf{x}_{12}|Q_{1z}) K_1(|\mathbf{x}_{1'2}|Q_{1z}) N_c C_F \left[S_{11'} - S_{12} - S_{21'} + 1\right] \frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|} \Delta^{(3)}_{11'}\, e^{i\frac{z_1+z}{z_1}\mathbf{p}\cdot\mathbf{x}_{1'1}}$$

$$\frac{\mathrm{d}\sigma^T_{9}}{\mathrm{d}^2\mathbf{p}\,\mathrm{d}y_1} = -\frac{e^2 g^2 Q^2}{2(2\pi)^6} \int_0^{z_1} \frac{\mathrm{d}z}{z}(1-z_1)(z_1^2+(1-z_1)^2)\left[z_1^2 + (z_1-z)^2\right] \int \mathrm{d}^6\mathbf{x}\, K_1(|\mathbf{x}_{12}|Q_1) K_1(|\mathbf{x}_{1'2}|Q_1) e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}}$$
$$N_c C_F \left[S_{11'} - S_{12} - S_{21'} + 1\right] \frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|} \int \frac{\mathrm{d}^2\mathbf{x}_3}{(2\pi)^2} \frac{1}{\mathbf{x}_{31}^2}$$

with $\Delta^{(3)}_{ij} = \frac{\mathbf{x}_{3i}\cdot\mathbf{x}_{3j}}{\mathbf{x}_{3i}^2\mathbf{x}_{3j}^2}$ and

$$Q_1^2 \equiv z_1(1-z_1)Q^2$$
$$Q_{1z}^2 \equiv (1-z-z_1)(z+z_1)Q^2$$

SIDIS at small x: NLO corrections

all quadrupole contributions cancel: dipoles only at leading N_c

cancelation of UV/soft divergences

rapidity/collinear divergences renormalize the dipoles/fragmentation functions

$$\sigma^{\gamma^\star \mathbf{A} \longrightarrow \mathbf{h}\,\mathbf{X}} = \sigma_{\mathbf{LO}} \otimes \mathbf{JIMWLK} + \sigma_{\mathbf{LO}} \otimes \mathbf{D}_{\mathbf{h}/\mathbf{q}}(\mathbf{z_h},\, \mu^{\mathbf{2}}) + \sigma^{\mathbf{finite}}_{\mathbf{NLO}}$$

EIC will have a reasonably large window in $\mathbf{Q^2}$ where $\mathbf{Q^2} >> \mathbf{p^2_{t,h}}$ so that $\alpha_{\mathbf{s}} \log\left(\frac{\mathbf{Q^2}}{\mathbf{p^2_{t,h}}}\right) \sim \mathbf{1}$

CGC contains *no* Sudakov double logs in SIDIS *unlike* dihadron production (wrong sign)

avoid large Sudakov logs: $\mathbf{p^2_{t,h}} \sim \mathbf{Q^2}$ so that $\log\left(\frac{\mathbf{Q^2}}{\mathbf{p^2_{t,h}}}\right) \simeq \mathbf{0}$

SIDIS at small x: including Sudakov double logs

Altinoluk, JJM, Marquet, arXiv:2406.08277

taking the high $\mathbf{Q^2}$ is tricky! $\mathbf{Q_1^2 \equiv z_1(1-z_1)Q^2 \longrightarrow 0}$ inside the Bessel functions

introduce the delta function

$$Q_1^{2n}\, K_{(0,1)}(|\mathbf{x}_{12}|Q_1)\, K_{(0,1)}(|\mathbf{x}_{1'2}|Q_1) = \int_0^{Q^2/4} d\bar{Q}^2\ (\bar{Q}^2)^n K_{(0,1)}(|\mathbf{x}_{12}|\bar{Q}) K_{(0,1)}(|\mathbf{x}_{1'2}|\bar{Q})\ \delta\left[\bar{Q}^2 - z_1(1-z_1)Q^2\right]$$

$$\delta\left[\bar{Q}^2 - z_1(1-z_1)Q^2\right] = \frac{\delta(z_1 - z_+)}{Q^2|1-2z_+|} + \frac{\delta(z_1 - z_-)}{Q^2|1-2z_-|} \quad \text{with} \quad z_\pm = \frac{1}{2}\left(1 \pm \sqrt{1 - 4\bar{Q}^2/Q^2}\right)$$

cross section becomes

$$\left.\frac{\mathrm{d}\sigma_{LO+NLO}^{\gamma^* A\to h(\mathbf{p}_h, y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = \frac{\pi e^2}{Q^2}\frac{1}{z_h}\int \frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}\, e^{-i\frac{\mathbf{p}_h}{z_h}\cdot\mathbf{x}_{11'}}\, x\tilde{q}(x, \mathbf{x}_{11'})\Bigg\{D_{h/q}(z_h) +$$

$$\left[\frac{\alpha_s C_F}{2\pi^2}\left(\int_0^{1-z_h}\frac{\mathrm{d}z}{z}\frac{1+(1-z)^2}{1-z}D_{h/q}\Big(\frac{z_h}{1-z}\Big) - \int_0^1\frac{\mathrm{d}z}{z}[1+(1-z)^2]D_{h/q}(z_h)\right)\right]\int_{|\mathbf{x}_3|\mu>1}\frac{\mathrm{d}^2\mathbf{x}_3}{\mathbf{x}_3^2} +$$

$$\left[\frac{\alpha_s C_F}{2\pi^2}\int_0^{1-z_h}\frac{\mathrm{d}z}{z}\frac{\left[1+(1-z)^2\right]}{1-z}\left(\int \mathrm{d}^2\mathbf{x}_3\,\Delta_{1'1}^{(3)} - \int_{|\mathbf{x}_3|\mu>1}\frac{\mathrm{d}^2\mathbf{x}_3}{\mathbf{x}_3^2}\right)\right]D_{h/q}\Big(\frac{z_h}{1-z}\Big)\Bigg\}$$

SIDIS at small x: including Sudakov double logs

to get Sudakov logs one must introduce a kinematic constraint

needed for self-consistency of evolution equations (avoid negative cross sections)

at small x we require $\mathbf{k^+ < z_f\, l^+}$, this is sufficient at LL accuracy

at NLO accuracy we also need to impose a condition on lifetimes of fluctuations

introduce a cutoff on - component of momenta $\mathbf{k^- > \tilde{z}_f\, l^-}$ with $\mathbf{z_f\, \tilde{z}_f \sim 1}$

Sudakov single logs will be sensitive to the details of this cutoff

Altinoluk, JJM, Marquet, Yu, in progress

SIDIS at small x: including Sudakov double logs

longitudinal factorization $\int_0^{1-z_h} dz = \int_{z_f}^{1-z_h} dz + \int_0^{z_f} dz\,[1-\theta(kin.const.)] + \int_0^{z_f} dz\theta(kin.const.)$

add and subtract the kinematic constraint $\boldsymbol{\Theta}(\text{kin.const.}) = \boldsymbol{\Theta}\left(\mathbf{z_f}\frac{\mathbf{k^2}}{\mathbf{Q^2}} - \mathbf{z}\right)$

$$\int_{\mathbf{0}}^{\mathbf{z_f}} \frac{\mathrm{d}\mathbf{z}}{\mathbf{z}}\left[\mathbf{1} - \boldsymbol{\Theta}\left(\mathbf{z_f}\frac{\mathbf{k^2}}{\mathbf{Q^2}} - \mathbf{z}\right)\right] = \int_{\mathbf{0}}^{\mathbf{z_f}} \frac{\mathrm{d}\mathbf{z}}{\mathbf{z}}\boldsymbol{\Theta}\left(\mathbf{z} - \mathbf{z_f}\frac{\mathbf{k^2}}{\mathbf{Q^2}}\right) = \boldsymbol{\Theta}\left(\mathbf{Q^2} - \mathbf{k^2}\right)\ln\left(\frac{\mathbf{Q^2}}{\mathbf{k^2}}\right)$$

and dipoles satisfy constrained JIMWLK evolution

$$\left.\frac{\mathrm{d}\sigma_{LO+NLO}^{\gamma^* A\to h(\mathbf{p}_h, y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = d\sigma_{LO}(z_f)\otimes D_{h/q}(z_h, Q^2) + d\sigma_{NLO-rap-finite} +$$

$$\frac{\pi e^2}{Q^2}\,\frac{D_{h/q}(z_h)}{z_h}\int \frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}\, e^{-i\frac{\mathbf{p}_h}{z_h}\cdot\mathbf{x}_{11'}}\, x\tilde{q}(x, \mathbf{x}_{11'})$$

$$\times\left\{\frac{\alpha_s C_F}{\pi^2}\int^{Q^2} \frac{\mathrm{d}^2\mathbf{k}}{\mathbf{k}^2}\left(e^{i\mathbf{k}\cdot\mathbf{x}_{1'1}} - 1\right)\ln\left(\frac{Q^2}{\mathbf{k}^2}\right)\right\}$$

Sudakov double logs in SIDIS

using

$$\int^{Q^2} \frac{\mathrm{d}^2\mathbf{k}}{\mathbf{k}^2}\left[e^{-i\mathbf{k}\cdot\mathbf{x}_{11'}}-1\right]\ln\left(\frac{Q^2}{\mathbf{k}^2}\right) = 4\pi\int_0^{Q|\mathbf{x}_{11'}|}\frac{\mathrm{d}\tau}{\tau}\left[J_0(\tau)-1\right]\ln\left(\frac{Q|\mathbf{x}_{11'}|}{\tau}\right)$$

$$= -\frac{\pi}{2}\ln^2\left(Q^2\mathbf{x}_{11'}^2/c_0^2\right) + O\left(\frac{1}{\sqrt{Q|\mathbf{x}_{11'}|}}\right)$$

we get

$$\left.\frac{\mathrm{d}\sigma_{LO+NLO}^{\gamma^* A\to h(\mathbf{p}_h,y_h)X}}{\mathrm{d}^2\mathbf{p}_h\,\mathrm{d}y_h}\right|_{LP} = d\sigma_{NLO-rap-finite}$$

$$+\frac{\pi e^2}{Q^2}\,\frac{D_{h/q}(z_h,Q^2)}{z_h}\int\frac{\mathrm{d}^2\mathbf{x}_{11'}}{(2\pi)^2}\,e^{-i\frac{\mathbf{p}_h}{z_h}\cdot\mathbf{x}_{11'}}\,x\tilde{q}(x,\mathbf{x}_{11'})\,e^{-S_{sud}(\mathbf{x}_{11'})}$$

with

$$S_{sud}(\mathbf{x}_{11'}) \equiv \frac{\alpha_s C_F}{2\pi}\ln^2\left(Q^2\mathbf{x}_{11'}^2/c_0^2\right)$$

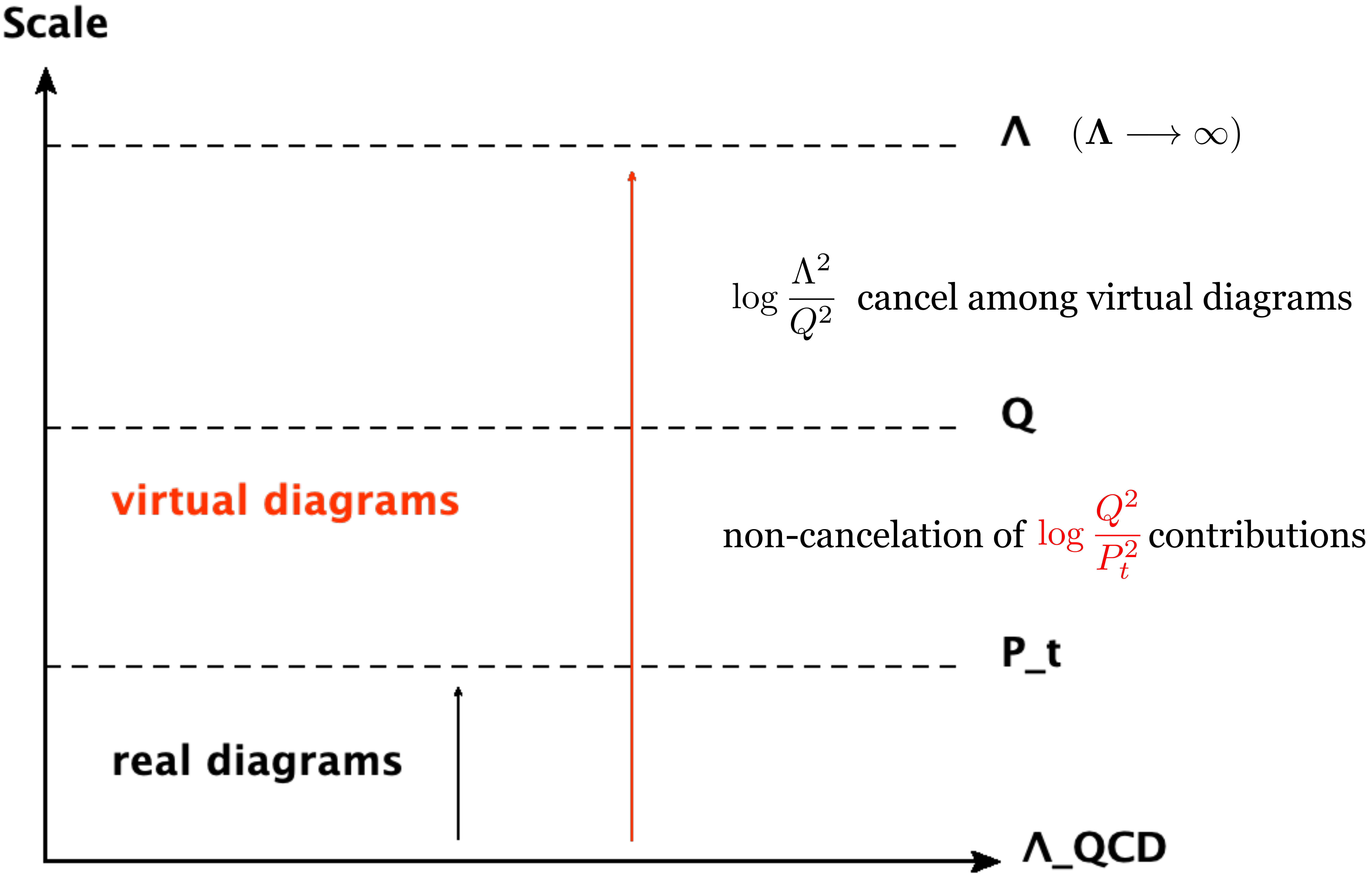
Quark TMD

Marquet, Xiao, Yuan 0906.1454

$$xq(x,\mathbf{p}) = \frac{2N_c}{(2\pi)^6}\int \mathrm{d}^6\mathbf{x}\; e^{-i\mathbf{p}\cdot\mathbf{x}_{11'}}\;\; [S_{11'} - S_{12} - S_{1'2} + 1]\,\frac{\mathbf{x}_{12}\cdot\mathbf{x}_{1'2}}{|\mathbf{x}_{12}||\mathbf{x}_{1'2}|}\int_0^\infty d\bar{Q}^2\bar{Q}^2\,K_1(|\mathbf{x}_{12}|\bar{Q})K_1(|\mathbf{x}_{1'2}|\bar{Q})$$

can be rewritten as

$$xq(x,\mathbf{p}) = \int \mathrm{d}^2\mathbf{k}_g\underbrace{\left(\frac{N_c}{8\pi^4}\frac{\mathbf{k}_g^2}{\alpha_s}\int \mathrm{d}^2\mathbf{x}_1\,\mathrm{d}^2\mathbf{x}_{1'}\;S_{11'}\,e^{-i\mathbf{k}_g\cdot\mathbf{x}_{11'}}\right)}_{\text{dipole gluon TMD}}\underbrace{\left(\frac{1}{4\pi^2}\frac{\alpha_s}{\mathbf{k}_g^2}\int_0^\infty \mathrm{d}\bar{Q}^2\left|\frac{\mathbf{k}_g-\mathbf{p}}{\bar{Q}^2+(\mathbf{k}_g-\mathbf{p})^2}+\frac{\mathbf{p}}{\bar{Q}^2+\mathbf{p}^2}\right|^2\right)}_{\text{momentum dependent splitting function}}$$


Scale
Λ (Λ ⟶ ∞)
log Λ^2/Q^2 cancel among virtual diagrams
Q
virtual diagrams
non-cancelation of log Q^2/P_t^2 contributions
P_t
real diagrams
Λ_QCD

Summary

QCD at high energy

dense hadron/nucleus: gluon saturation, strong color fields - CGC

strong hints from RHIC, LHC,…

to be probed precisely at EIC

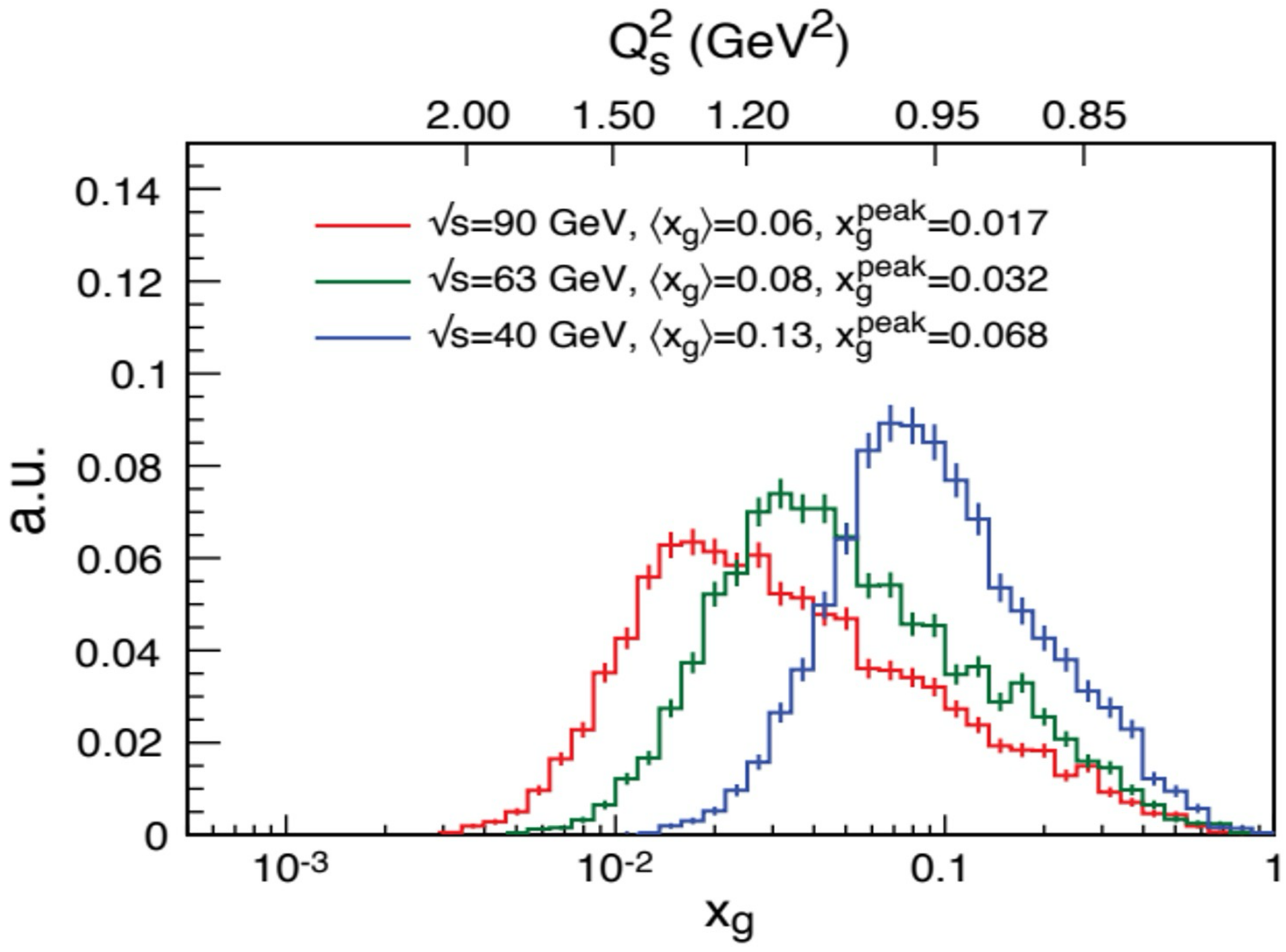
toward precision: NLO, beyond-eikonal corrections, …

deep connections to TMD, Sudakov physics, …

missing a unified approach to large and small x!

EIC

kinematics of inclusive dihadron production



Aschenauer et al. arXiv:1708.01527

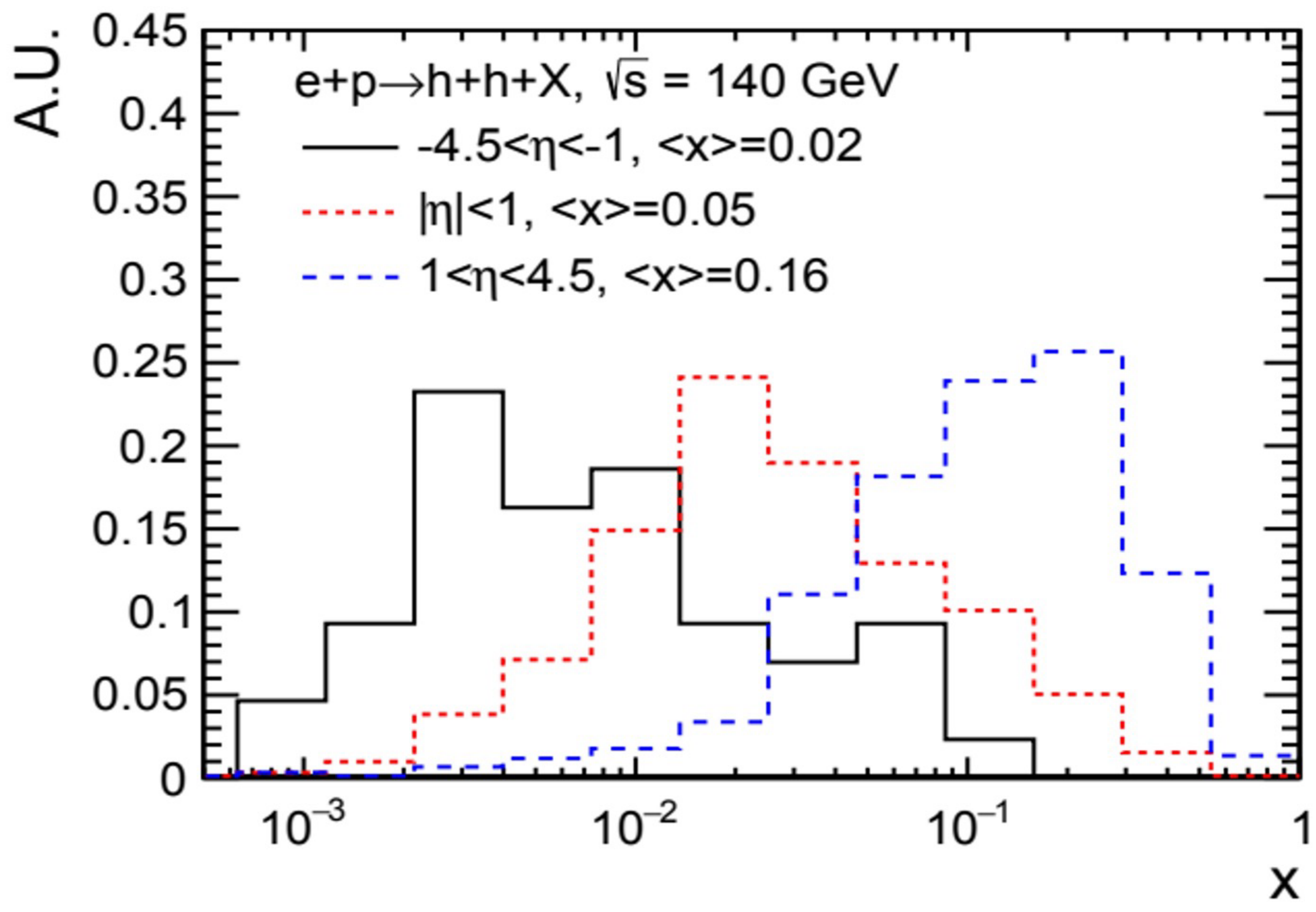


Fig. courtesy of Xiaoxuan Chu

transition region: from large x to small x

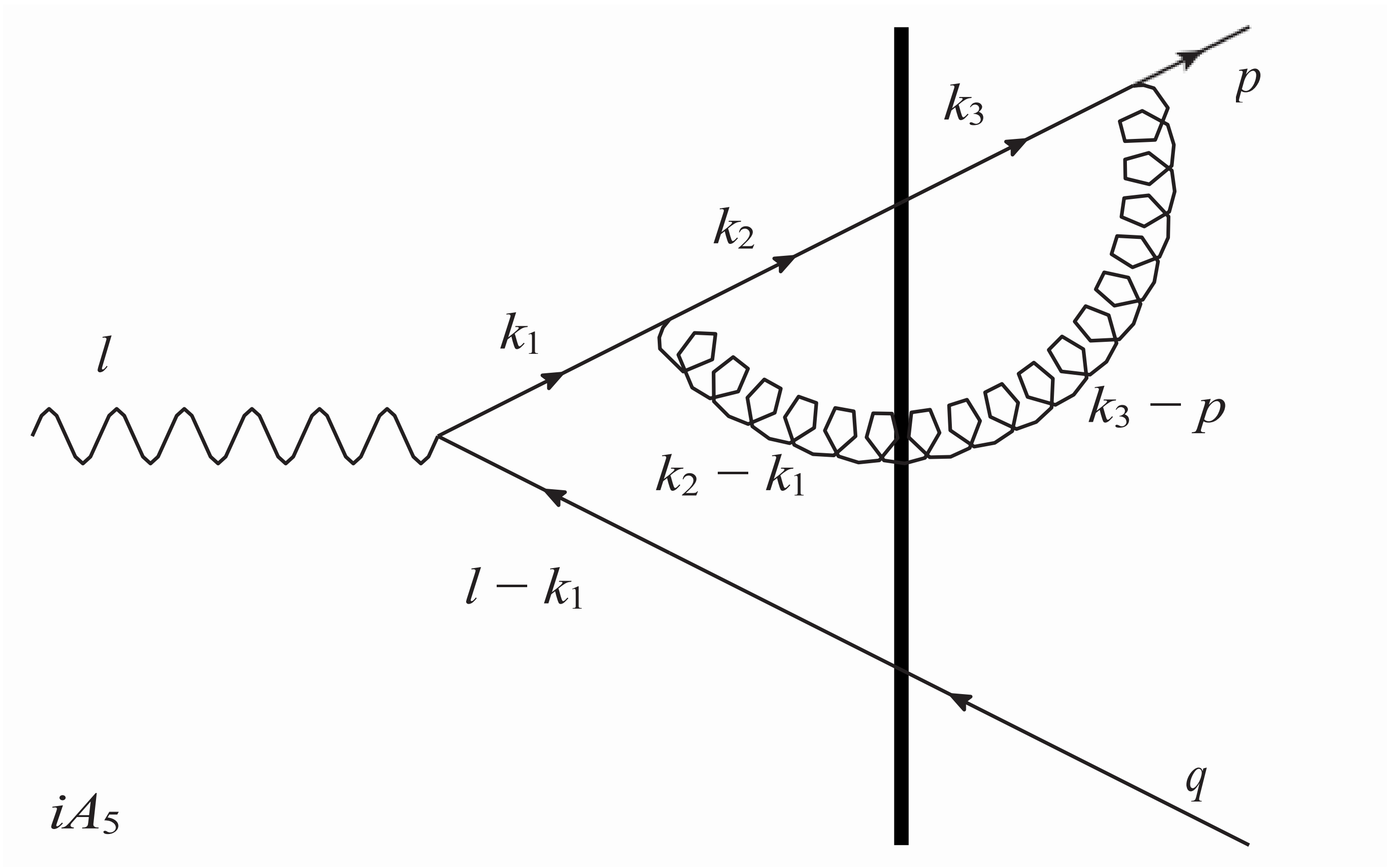
divergences

- ***Ultraviolet:***

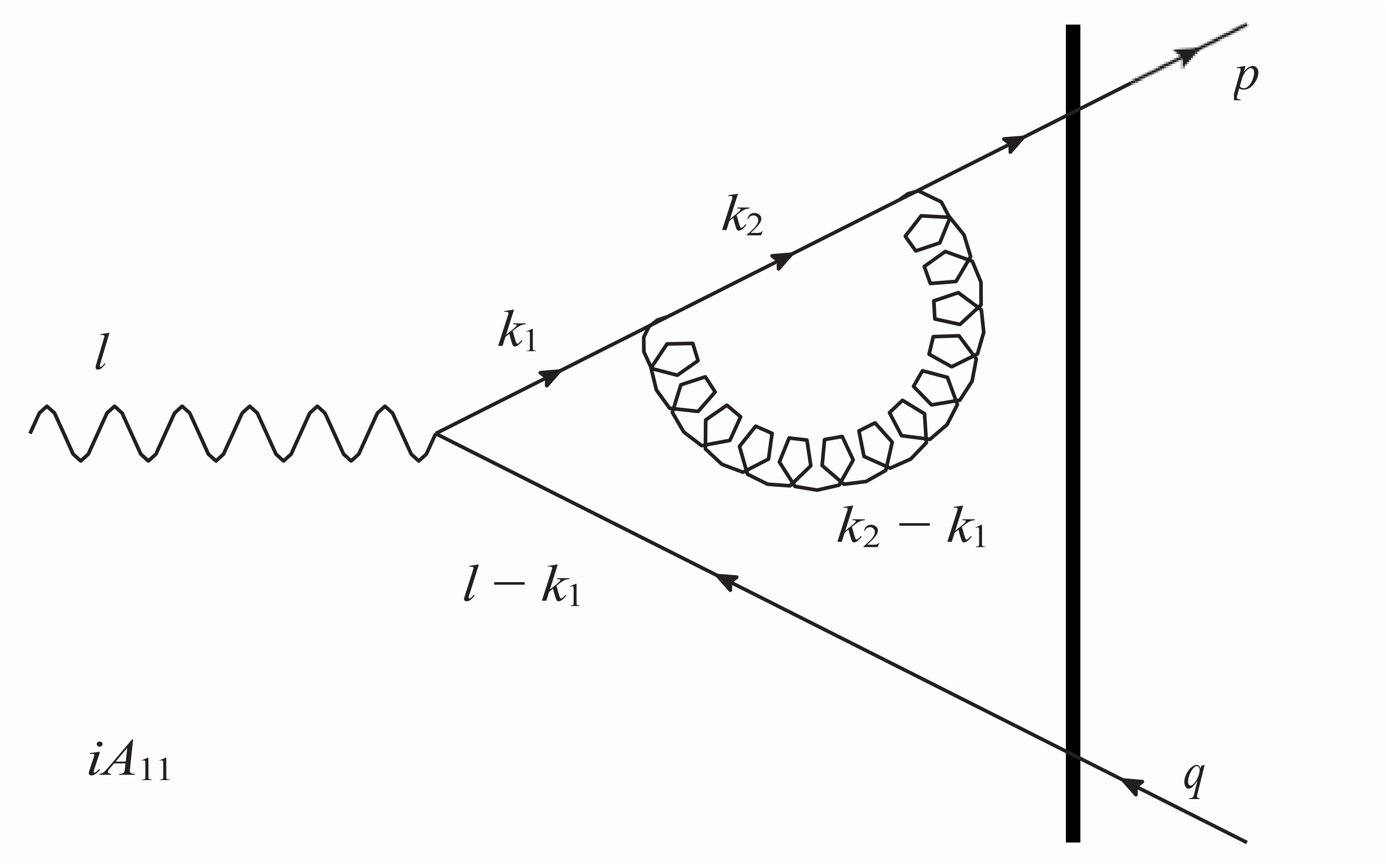
Real corrections are UV finite

UV divergences cancel among virtual corrections $\mathbf{k} \to \infty$ **or** $\mathbf{x_3} \to \mathbf{x_i}$

$$\left(d\sigma_5 + d\sigma_{11}\right)_{UV} = 0$$
$$\left(d\sigma_6 + d\sigma_{12}\right)_{UV} = 0$$
$$\left(d\sigma_9 + d\sigma_{10} + d\sigma_{14(1)} + d\sigma_{14(2)}\right)_{UV} = 0$$



$+$



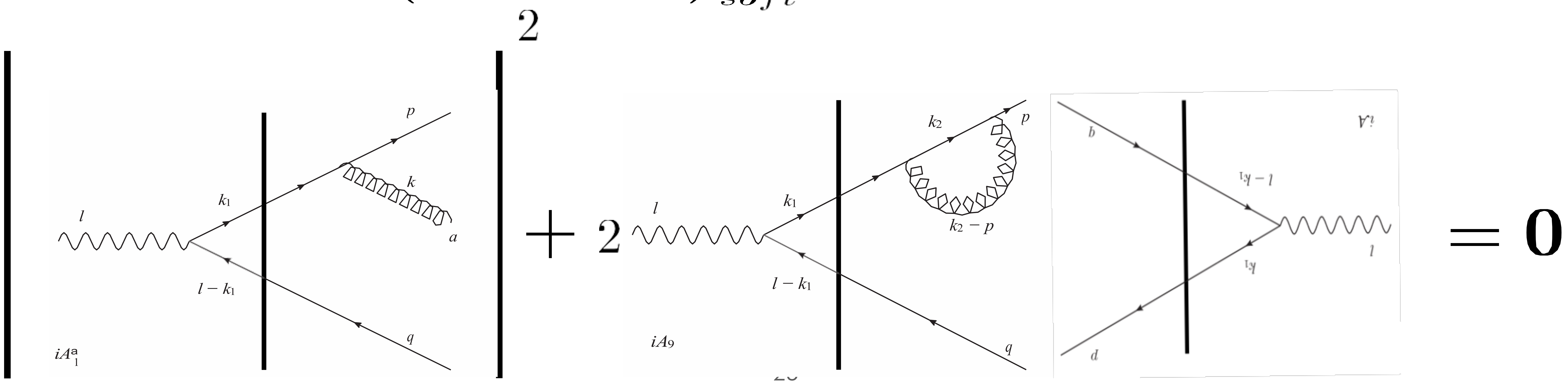
$= 0$

divergences

- ***Soft:*** $\mathbf{k}^{\mu} \to \mathbf{0} \;\; (\mathbf{x_3} \to \infty \;\; \mathbf{AND} \;\; \mathbf{z} \to \mathbf{0})$

Soft divergences cancel between real and virtual corrections

$$(d\sigma_{1-1} + d\sigma_9)_{soft} = 0,$$
$$\left(d\sigma_{1-2} + d\sigma_{13}^{(1)} + d\sigma_{13}^{(2)}\right)_{soft} = 0$$
$$(d\sigma_{3-3} + d\sigma_{4-4} + d\sigma_{3-4})_{soft} = 0$$
$$(d\sigma_{1-3} + d\sigma_{1-4})_{soft} = 0$$
$$(d\sigma_{2-3} + d\sigma_{2-4})_{soft} = 0$$
$$(d\sigma_5 + d\sigma_7)_{soft} = 0$$
$$\left(d\sigma_{11} + d\sigma_{14}^{(1)}\right)_{soft} = 0$$



divergences

- ***Rapidity:*** $\mathbf{z} \to \mathbf{0}$ ***, but finite*** k_t

$$\int_0^1 \frac{dz}{z} = \int_0^{z_f} \frac{dz}{z} + \int_{z_f}^1 \frac{dz}{z}$$

rapidity divergences are absorbed into JIMWLK evolution of dipoles and quadrupoles

$$\frac{d\sigma^L_{\rm NLO}}{d^2\mathbf{p}\, d^2\mathbf{q}\, dy_1\, y_2} = \frac{2e^2 g^2 Q^2 N_c^2 (z_1 z_2)^3}{(2\pi)^{10}} \delta(1 - z_1 - z_2) \int_0^{z_f} \frac{dz}{z} \int d^{10}\mathbf{x}\, K_0(|\mathbf{x}_{12}|Q_1) K_0(|\mathbf{x}_{1'2'}|Q_1)$$

$$e^{i\mathbf{p}\cdot\mathbf{x}_{1'1}} e^{i\mathbf{q}\cdot\mathbf{x}_{2'2}} \Big\{ \left(\tilde\Delta_{12} + \tilde\Delta_{22'} - \tilde\Delta_{12'}\right) S_{132'1'} S_{23} + \left(\tilde\Delta_{1'2'} + \tilde\Delta_{22'} - \tilde\Delta_{21'}\right) S_{1'321} S_{2'3}$$

$$+ \left(\tilde\Delta_{12} + \tilde\Delta_{11'} - \tilde\Delta_{21'}\right) S_{322'1'} S_{13} + \left(\tilde\Delta_{1'2'} + \tilde\Delta_{11'} - \tilde\Delta_{12'}\right) S_{32'21} S_{1'3}$$

$$- \left(\tilde\Delta_{11'} + \tilde\Delta_{22'} + \tilde\Delta_{12} + \tilde\Delta_{1'2'}\right) S_{122'1'} - \left(\tilde\Delta_{12} + \tilde\Delta_{1'2'} - \tilde\Delta_{12'} - \tilde\Delta_{21'}\right) S_{12} S_{1'2'}$$

$$- \left(\tilde\Delta_{11'} + \tilde\Delta_{22'} - \tilde\Delta_{12'} - \tilde\Delta_{21'}\right) S_{11'} S_{22'} - 2\tilde\Delta_{12}\left(S_{13}S_{23} - S_{12}\right) - 2\tilde\Delta_{1'2'}\left(S_{1'3}S_{2'3} - S_{1'2'}\right) \Big\}$$

JIMWLK evolution of quadrupoles

JIMWLK evolution of dipoles

$$\tilde{\mathbf{\Delta}}_{\mathbf{12}} \equiv \frac{(\mathbf{x_1} - \mathbf{x_2})^2}{(\mathbf{x_1} - \mathbf{x_3})^2 (\mathbf{x_2} - \mathbf{x_3})^2}$$