Forward Hadron Productions in proton-nucleus collisions at NLO

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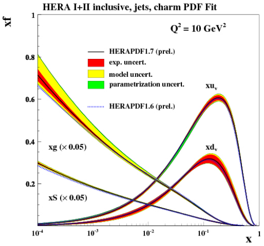
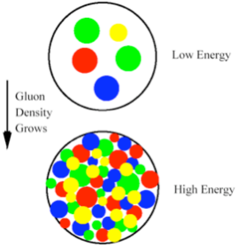
[G. A. Chirilli, BX, Feng Yuan, 12]
[A. Stasto, BX, D. Zaslavsky, 14]
[Y. Shi, L. Wang, S.Y. Wei, BX, 22]

Saturation Physics, Color Glass Condensate

Describe the emergent property of high density gluons inside proton and nuclei.



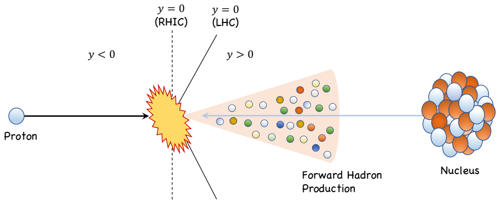


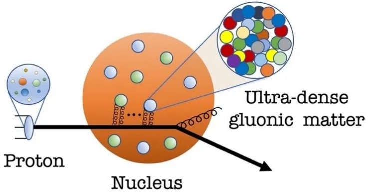
- Gluon density grows rapidly as x gets small.
- Many gluons with fixed size packed in a confined hadron, gluons overlap and recombine $\Rightarrow$ Non-linear QCD dynamics (BK-JIMWLK) $\Rightarrow$ ultra-dense gluons with collective property.

Forward hadron production in pA collisions

[Dumitru, Jalilian-Marian, 02] Dilute-dense factorization at forward rapidity

$$\frac{d\sigma_{\rm LO}^{pA\to hX}}{d^2p_\perp dy_h} = \int_\tau^1 \frac{dz}{z^2}\left[x_1 q_f(x_1,\mu)\mathcal{F}_{x_2}(k_\perp)D_{h/q}(z,\mu) + x_1 g(x_1,\mu)\tilde{\mathcal{F}}_{x_2}(k_\perp)D_{h/g}(z,\mu)\right].$$



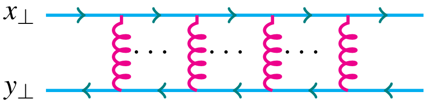


- $\mathcal{F}(k_\perp)$ (dipole gluon distribution) encodes dense gluon info.
- Early attempts: [Dumitru, Hayashigaki, Jalilian-Marian, 06; Altinoluk, Kovner 11] [Altinoluk, Armesto, Beuf, Kovner, Lublinsky, 14]
- Full NLO: [Chirilli, BX and Yuan, 12]

Wilson Lines in Color Glass Condensate Formalism

The Wilson loop (color singlet dipole) in McLerran-Venugopalan (MV) model

$$\frac{1}{N_c}\left\langle \mathrm{Tr} U(x_\perp) U^\dagger(y_\perp)\right\rangle = e^{-\frac{Q_s^2 (x_\perp - y_\perp)^2}{4}}$$

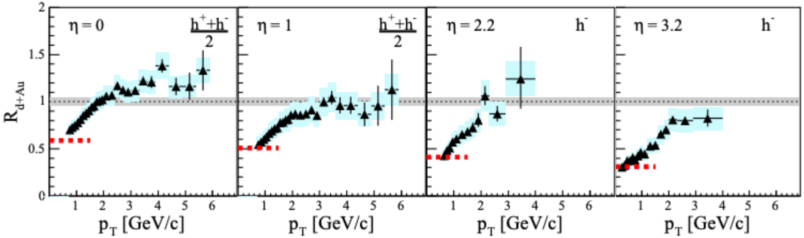


- Dipole amplitude $S^{(2)}$ then produces the quark k_T spectrum via Fourier transform

$$\mathcal{F}(k_\perp) \equiv \frac{dN}{d^2k_\perp} = \int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^2} e^{-ik_\perp\cdot(x_\perp - y_\perp)} \frac{1}{N_c}\left\langle \mathrm{Tr} U(x_\perp) U^\dagger(y_\perp)\right\rangle.$$

d+Au collisions at RHIC

$$R_{d+Au} = \frac{1}{\langle N_{\rm coll}\rangle}\frac{d^2N_{d+Au}/d^2p_Td\eta}{d^2N_{pp}/d^2p_Td\eta}; \qquad R=1 \qquad \text{benchmark for no nuclear effects.}$$



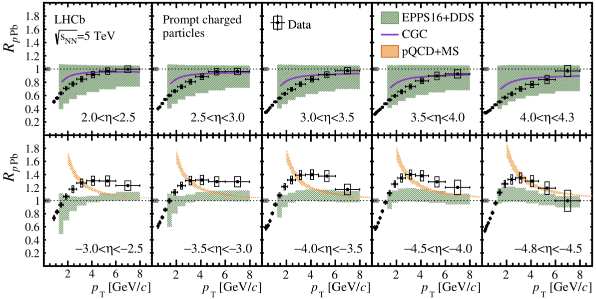
BRAHMS

- Cronin effect at middle rapidity: redistribution of momentum
- Rapidity evolution of the nuclear modification factors R_{d+Au}
- Promising evidence for gluon saturation effects

New LHCb Results

[R. Aaet al. (LHCb Collaboration), Phys. Rev. Lett. 128 (2022) 142004]

$$R_{pPb} = \frac{1}{\langle N_{\rm coll}\rangle} \frac{d^2N_{p+Pb}/d^2p_T d\eta}{d^2N_{pp}/d^2p_T d\eta}$$

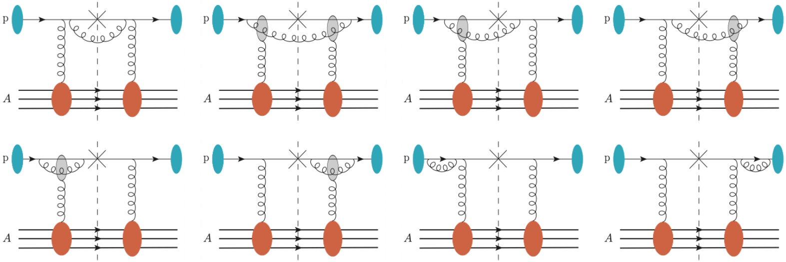


- Forward rapidity:
  Nuclear effects
  Small-x suppression
- Backward rapidity:
  Cronin enhancement

- Rapidity evolution of the nuclear modification factors R_{pPb} similar to RHIC

NLO diagrams in the $q \to q$ channel

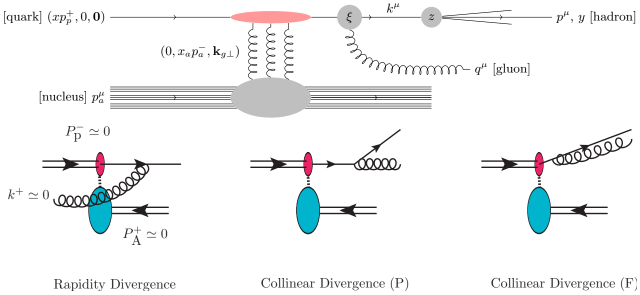
[Chirilli, BX and Yuan, 12]



- Take into account real (top) and virtual (bottom) diagrams together!
- Non-linear multiple interactions inside the grey blobs!
- Integrate over gluon phase space $\Rightarrow$Divergences!.

Factorization for single inclusive hadron productions

Factorization for the $p + A \to H + X$ process [Chirilli, BX and Yuan, 12]



- Include all real and virtual graphs in all channels $q \to q$, $q \to g$, $g \to q(\bar{q})$ and $g \to g$.
- 1. collinear to target nucleus; rapidity divergence $\Rightarrow$ BK evolution for UGD $\mathcal{F}(k_\perp)$.
- 2. collinear to the initial quark; $\Rightarrow$ DGLAP evolution for PDFs
- 3. collinear to the final quark. $\Rightarrow$ DGLAP evolution for FFs.

Factorization and NLO Calculation

- Factorization is about separation of short distant physics (perturbatively calculable hard factor) from large distant physics (Non perturbative).

$$\sigma \sim xf(x) \otimes \mathcal{H} \otimes D_h(z) \otimes \mathcal{F}(k_\perp)$$

- NLO (1-loop) calculation always contains various kinds of divergences.
  - Some divergences can be absorbed into the corresponding evolution equations.
  - Renormalization: cutting off infinities and hiding the ignorance.
  - The rest of divergences should be canceled.
- Hard factor

$$\mathcal{H} = \mathcal{H}^{(0)}_{\text{LO}} + \frac{\alpha_s}{2\pi} \mathcal{H}^{(1)}_{\text{NLO}} + \cdots$$

should always be finite and free of divergence of any kind.

Hard Factor of the $q \to q$ channel

$$\frac{d^3\sigma^{p+A\to h+X}}{dyd^2p_\perp} = \int \frac{dz}{z^2}\frac{dx}{x}\xi xq(x,\mu)D_{h/q}(z,\mu)\int\frac{d^2x_\perp d^2y_\perp}{(2\pi)^2}\left\{S_Y^{(2)}(x_\perp,y_\perp)\left[\mathcal{H}_{2qq}^{(0)}+\frac{\alpha_s}{2\pi}\mathcal{H}_{2qq}^{(1)}\right]\right.$$
$$\left.+\int\frac{d^2b_\perp}{(2\pi)^2}S_Y^{(4)}(x_\perp,b_\perp,y_\perp)\frac{\alpha_s}{2\pi}\mathcal{H}_{4qq}^{(1)}\right\}$$

$$\mathcal{H}_{2qq}^{(1)} = C_F\mathcal{P}_{qq}(\xi)\ln\frac{c_0^2}{r_\perp^2\mu^2}\left(e^{-ik_\perp\cdot r_\perp}+\frac{1}{\xi^2}e^{-i\frac{k_\perp}{\xi}\cdot r_\perp}\right)-3C_F\delta(1-\xi)\ln\frac{c_0^2}{r_\perp^2k_\perp^2}e^{-ik_\perp\cdot r_\perp}$$
$$-(2C_F-N_c)e^{-ik_\perp\cdot r_\perp}\left[\frac{1+\xi^2}{(1-\xi)_+}\widetilde{I}_{21}-\left(\frac{\left(1+\xi^2\right)\ln(1-\xi)^2}{1-\xi}\right)_+\right]$$

$$\mathcal{H}_{4qq}^{(1)} = -4\pi N_c e^{-ik_\perp\cdot r_\perp}\left\{e^{-i\frac{1-\xi}{\xi}k_\perp\cdot(x_\perp-b_\perp)}\frac{1+\xi^2}{(1-\xi)_+}\frac{1}{\xi}\frac{x_\perp-b_\perp}{(x_\perp-b_\perp)^2}\cdot\frac{y_\perp-b_\perp}{(y_\perp-b_\perp)^2}\right.$$
$$\left.-\delta(1-\xi)\int_0^1 d\xi'\frac{1+\xi'^2}{(1-\xi')_+}\left[\frac{e^{-i(1-\xi')k_\perp\cdot(y_\perp-b_\perp)}}{(b_\perp-y_\perp)^2}-\delta^{(2)}(b_\perp-y_\perp)\int d^2r'_\perp\frac{e^{ik_\perp\cdot r'_\perp}}{r'^2_\perp}\right]\right\},$$

where $\widetilde{I}_{21} = \int\frac{d^2b_\perp}{\pi}\left\{e^{-i(1-\xi)k_\perp\cdot b_\perp}\left[\frac{b_\perp\cdot(\xi b_\perp-r_\perp)}{b_\perp^2(\xi b_\perp-r_\perp)^2}-\frac{1}{b_\perp^2}\right]+e^{-ik_\perp\cdot b_\perp}\frac{1}{b_\perp^2}\right\}.$

Numerical implementation of the NLO result

Single inclusive hadron production up to NLO

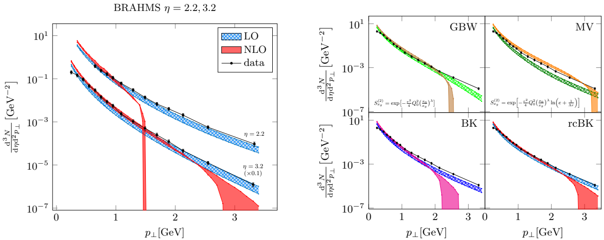
$$\mathrm{d}\sigma = \int x f_a(x) \otimes D_a(z) \otimes \mathcal{F}_a^{x_g}(k_\perp) \otimes \mathcal{H}^{(0)} + \frac{\alpha_s}{2\pi} \int x f_a(x) \otimes D_b(z) \otimes \mathcal{F}_{(N)ab}^{x_g} \otimes \mathcal{H}_{ab}^{(1)}.$$

Consistent implementation should include all the NLO α_s corrections.

- NLO parton distributions. (MSTW or CTEQ)
- NLO fragmentation function. (DSS or others.)
- Use NLO hard factors. [Chirilli, BX and Yuan, 12]
- Use the one-loop approximation for the running coupling
- rcBK evolution equation for the dipole gluon distribution [Balitsky, Chirilli, 08; Kovchegov, Weigert, 07]. Full NLO BK evolution not available.
- Saturation physics at One Loop Order (SOLO). [Stasto, Xiao, Zaslavsky, 13]

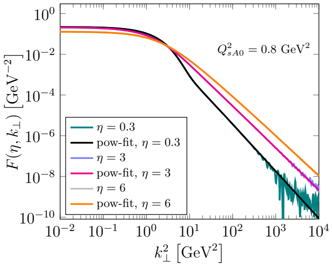
Numerical implementation of the NLO result

Saturation physics at One Loop Order (SOLO). [Stasto, Xiao, Zaslavsky, 13]



- Reduced factorization scale dependence!
- Catastrophe: Negative NLO cross-sections at high p_T.
- Fixed order calculation in field theories is not guaranteed to be positive.

The cross-section at high $k_\perp$

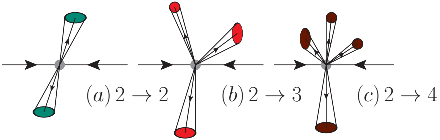


- In the dilute limit $k_\perp \gg Q_s$, partonic cross-sections follow the power law

$$\sigma(k_\perp) \sim \mathcal{F}(k_\perp) \sim \frac{Q_s^2}{4} \int d^2 r_\perp e^{-ik_\perp \cdot r_\perp} r_\perp^2 \ln(r_\perp \Lambda) \sim \frac{Q_s^2}{k_\perp^4}.$$

- NLO $\sigma_{NLO} \sim \mathcal{C}\mathcal{F}(k_\perp) \sim \mathcal{C}\frac{Q_s^2}{k_\perp^4}$ with $\mathcal{C}$ containing logarithms such as $\ln k_\perp^2/Q_s^2$.

Perturbative expansions vs Resummation (dijet productions)



Correlations:

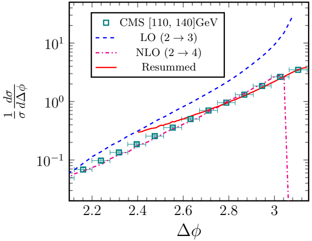
- 2 → 2: 0th order
- 2 → 3: leading order
- 2 → 4: next-to-leading order

$$\sigma_0 \sum_{i=0}^{\infty} \alpha_s^i \left(L^i + C^{(i)} \right)$$ ideal QCD expansion

$\sigma_0 \sum_{i=0}^{n-1} \alpha_s^i L^i$	$\sigma_0 \sum_{i=0}^{n-1} \alpha_s^i C^{(i)}$	⇐ pQCD
$\sigma_0 \sum_{i=n}^{\infty} \alpha_s^i L^i$	$\sigma_0 \sum_{i=n}^{\infty} \alpha_s^i C^{(i)}$	

⇑ resummation

↖ negligible



- Appearance of large logarithms such as $L \sim \ln^2 \frac{P_\perp^2}{q_\perp^2}$ with $P_\perp \gg q_\perp$.
- Momentum imbalance $\vec{q}_\perp \equiv \vec{p}_{1\perp} + \vec{p}_{2\perp}$, jet momenta $P_\perp \sim p_{1\perp} \sim p_{2\perp}$.

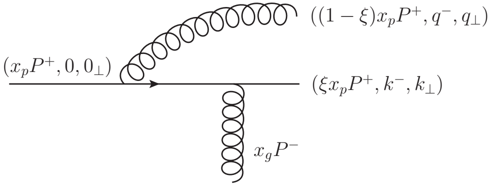
Extending the applicability of CGC calculation

- Goal: find a solution within our current factorization (exactly resum $\alpha_s \ln 1/x_g$) to extend the applicability of CGC. Other scheme choices certainly is possible.
- A lot of logs arise in pQCD loop-calculations: DGLAP, small-x, threshold, Sudakov.
- Breakdown of α_s expansion occurs due to the appearance of logs in certain PS.
- Demonstrate onset of saturation and visualize smooth transition to dilute regime.
- Add'l consideration: numerically challenging due to limited computing resources.
- Towards a more complete framework. [Altinoluk, Armesto, Beuf, Kovner, Lublinsky, 14; Kang, Vitev, Xing, 14; Ducloue, Lappi and Zhu, 16, 17; Iancu, Mueller, Triantafyllopoulos, 16; Liu, Ma, Chao, 19; Kang, Liu, 19; Kang, Liu, Liu, 20; Altinoluk, Armesto, Kovner, Lublinsky, 23]. Similar issues in other NLO calculations in CGC. [Taels, Altinoluk, Beuf, Marquet, 22; Iancu, Mulian, 22; Caucal, Salazar, Schenke, Stebel, Venugopalan, 23; Bergabo, Jalilian-Marian, 22, 23; Altinoluk, Armesto, Beuf, 23; Beuf, Lappi, Mäntysaari, Paatelainen, Penttala, 24; etc]

NLO hadron productions in pA collisions with kinematic constraints

[Watanabe, Xiao, Yuan, Zaslavsky, 15] Rapidity subtraction! with kinematic constraints

- Originally assume the limit $s \to \infty$



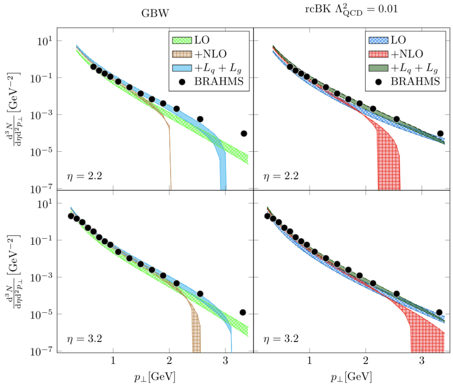
$$\int_0^{1-\frac{q_\perp^2}{x_p s}} \frac{d\xi}{1-\xi} = \underbrace{\ln\frac{1}{x_g}}_{1-\xi<\frac{q_\perp^2}{k_\perp^2}} + \underbrace{\ln\frac{k_\perp^2}{q_\perp^2}}_{\text{missed earlier}} \Rightarrow$$

New terms: $L_q + L_g$ from $q_\perp^2 \leq (1-\xi)k_\perp^2$.

- Corrections related to threshold double logs. Negative when $p_T \gg Q_s$ at forward y ($x_p \to 1$)! Approach threshold at high $k_\perp$.
- Ioffe time cutoff [Altinoluk, Armesto, Beuf, Kovner and Lublinsky, 14]

Numerical results with kinematic constraint

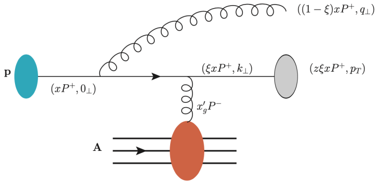
SOLO results [Stasto, Xiao, Zaslavsky, 13; Watanabe, Xiao, Yuan, Zaslavsky, 15]

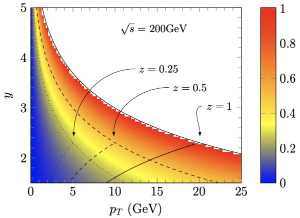


- SOLO still breaks down in the large $p_\perp$ region with the new term.

Gluon Radiation at the Threshold

Near threshold: radiated gluon has to be soft! $\tau = \frac{p_\perp e^y}{\sqrt{s}}$ density ($\tau = x_p \xi z \le 1$)





- If $q_\perp \sim k_\perp$, then $q^- \to \infty$, this is part of the small-x evolution.
- If $q_\perp^2 \le (1-\xi)k_\perp^2$, then q^- is finite, this is part of the Sudakov! $\Rightarrow \ln \frac{k_\perp^2}{q_\perp^2}$.
- KLN $\Rightarrow$ cancellation between real and virtual. $-\int_{\Lambda^2}^{k_\perp^2} \frac{dq_\perp^2}{q_\perp^2} \ln \frac{k_\perp^2}{q_\perp^2} = -\frac{1}{2} \ln^2 \frac{k_\perp^2}{\Lambda^2}$
- Introduce an additional semi-hard scale Λ^2 as the typical $q_\perp$.

Threshold resummation in the CGC formalism

Threshold logarithms:Collinear (plus-distribution) and Sudakov soft gluon part

- Performing Fouier transformations and Subtracting large logarithms

$$\int \frac{d^2 r_\perp}{(2\pi)^2} S(r_\perp) \ln \frac{\mu^2}{\mu_r^2} e^{-ik_\perp \cdot r_\perp} = -\int \frac{d^2 l_\perp}{\pi l_\perp^2} \left[F(k_\perp + l_\perp) - J_0(\frac{c_0}{\mu} l_\perp) F(k_\perp) \right]$$

$$= \underbrace{-\frac{1}{\pi} \int \frac{d^2 l_\perp}{(l_\perp - k_\perp)^2} \left[F(l_\perp) - \frac{\Lambda^2}{\Lambda^2 + (l_\perp - k_\perp)^2} F(k_\perp) \right]}_{\text{Treated as small NLO correction}} + \underbrace{F(k_\perp) \ln \frac{\mu^2}{\Lambda^2}}_{\text{Extracted log}}.$$

- Two equivalent methods to resum the collinear part ($P_{ab}(\xi) \ln \Lambda^2/\mu^2$):
  1. Reverse DGLAP evolution; 2. RGE method (threshold limit $\xi \to 1$).
- Consistent with the threshold resummation in SCET[Becher, Neubert, 06]!
- Similar technique is used to extract the double log. At one loop Λ is arbitrary.
- After resumming all logs, μ^2 and Λ^2 dependences only cancel up to one-loop order.
- Choose proper μ^2 (hard scale) and Λ^2 (Saddle point) in resummation to minimize hard factors.

Fixing μ^2 and Λ^2

- Rationales: 1. Estimate Λ according to strengths of Sudakov and Saturation effects; 2. make sure the un-resummed contribution is small, restore perturbative expansion.
- Choose proper μ^2 (hard scale) and Λ^2 (Saddle point); Similar to the technique used in collinear and TMD factorization with minimized hard factors.
- Define $F(k_\perp - q_\perp)$ and $G_{\rm th}(q_\perp)$ as the Fourier transform of $S^{(2)}(r_\perp)$ and the Sudakov factor, respectively, and use Saddle point approximation to find the dominant region for resummed contribution:

$$\frac{\mathrm{d}\sigma^{qq}_{\rm resummed}}{\mathrm{d}y\mathrm{d}^2p_T} = S_\perp \int_\tau^1 \frac{\mathrm{d}z}{z^2} \int_{x_p}^1 \frac{\mathrm{d}x}{x} q(x,\mu) \int_z^1 \frac{\mathrm{d}z'}{z'} D_{h/q}(z') \int d^2q_\perp F(k_\perp - q_\perp) G_{\rm th}(q_\perp),$$

- For running coupling, $\Lambda^2 \approx \frac{1}{r_{sp}^2} \simeq \max\left\{ \Lambda^2_{\rm QCD} \left[\frac{k_\perp^2(1-\xi)}{\Lambda^2_{\rm QCD}}\right]^{\frac{C_F}{C_F+N_c\beta_0}}, Q_s^2 \right\}$

Threshold Logarithms

[Watanabe, Xiao, Yuan, Zaslavsky, 15; Shi, Wang, Wei, Xiao, 21] ▸ 2112.06975 [hep-ph]

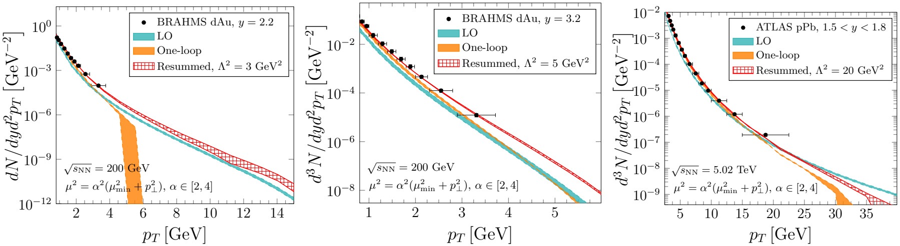
- Threshold enhancement for σ: $e^{-x} = 1 - x + \frac{x^2}{2} + \cdots$
- In the coordinate space, we can identify two types of logarithms

$$\text{single log: } \ln\frac{k_\perp^2}{\mu_r^2} \to \ln\frac{k_\perp^2}{\Lambda^2}, \quad \ln\frac{\mu^2}{\mu_r^2} \to \ln\frac{\mu^2}{\Lambda^2}; \quad \text{double log: } \ln^2\frac{k_\perp^2}{\mu_r^2} \to \ln^2\frac{k_\perp^2}{\Lambda^2},$$

with $\mu_r \equiv c_0/r_\perp$ with $c_0 = 2e^{-\gamma_E}$.

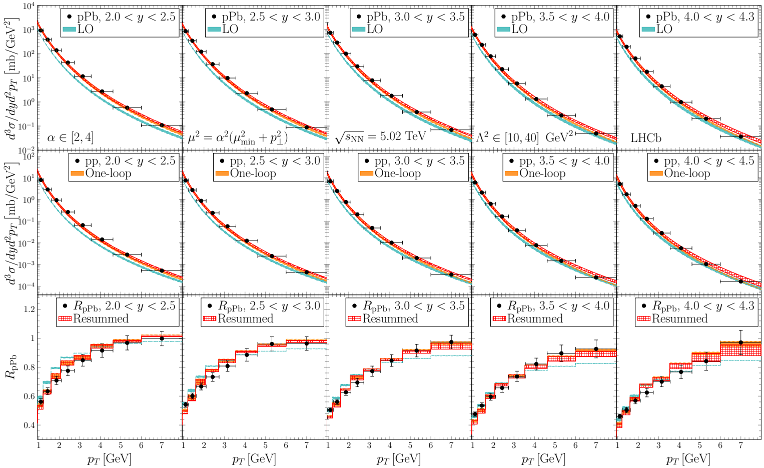
- Introduce a semi-hard auxiliary scale $\Lambda^2 \sim \mu_r^2 \gg \Lambda_{QCD}^2$. Identify dominant $r_\perp$!
- Dependence on μ^2, Λ^2 cancel order by order. Choose "natural" values at fixed order.
- $\Lambda^2 = \max\left\{\Lambda_{QCD}^2\left[\frac{(1-\xi)k_\perp^2}{\Lambda_{QCD}^2}\right]^{C_R/[C_R+\beta_1]}, Q_s^2\right\}$. Akin to CSS & Catani *et al*.

Numerical Results for pA spectra



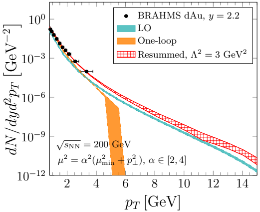
- Excellent agreement with data across many orders of magnitudes for different energies and p_T ranges measured from both RHIC and the LHC!
- Explain the rapidity dependence: threshold (Sudakov) logs are less important in the forward regime. In middle rapidity, large phase space gives large $\alpha_s \ln^2 \frac{k_\perp^2}{\Lambda^2}$

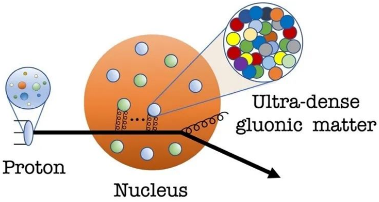
Comparison with the new LHCb data

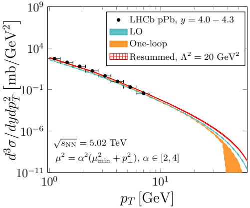


- LHCb data: 2108.13115
- Data Link DIS2021
- Threshold effect is not important at low p_T for LHCb data. Saturation effects are still dominant.
- Predictions are improved from LO to NLO.
- Solve the negativity problem at both RHIC and LHC.

Summary







- Ten-Year Odyssey in NLO hadron productions in pA collisions in CGC.
- Towards the precision test of saturation physics (CGC) at RHIC and LHC.
- New developments in DIS (EIC), e.g. [Caucal, et al 22; Taels, et al 22]
- Exciting time of NLO CGC phenomenology with the upcoming EIC.

My Personal Reflection of Edmond

- I first came across Edmond and his work as a graduate student 20 years ago.
- His papers are always very long, and they are a goldmine of detail, clarity, and insight.
- They were like my go-to resource and roadmap through complex ideas.
- It's a pity we never got the chance to collaborate (though I'm still holding out hope!).

Happy Birthday, Edmond!