Probing small-x helicity and OAM distributions in particle production at RHIC and EIC

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Credits

- Based on work done with Dan Pitonyak and Matt Sievert (2015-2018, 2021-present), Florian Cougoulic (2019-present), Gabe Santiago (2020-present), Josh Tawabutr (2020-present), Andrey Tarasov (2021-present), Daniel Adamiak, Wally Melnitchouk, Nobuo Sato (2021-present), Jeremy Borden (2023-present), Ming Li (2023-present), Brandon Manley (2023-present), Nick Baldonado (2022-present), Zardo Becker (2024-present).

Happy 60th Birthday, Edmond!

Outline: helicity-dependent observables at RHIC and EIC at small x

- DIS: g_1 structure function at small x + sub-eikonal operators.
- Helicity evolution at small x.
- Polarized DIS and SIDIS data analysis.
- Polarized p+p collisions: gluon production at mid-rapidity.
- Inclusive dijet production in polarized e+p collisions.
- Elastic dijet production in polarized e+p collisions.

g_1 Structure Function

Dipole picture of DIS

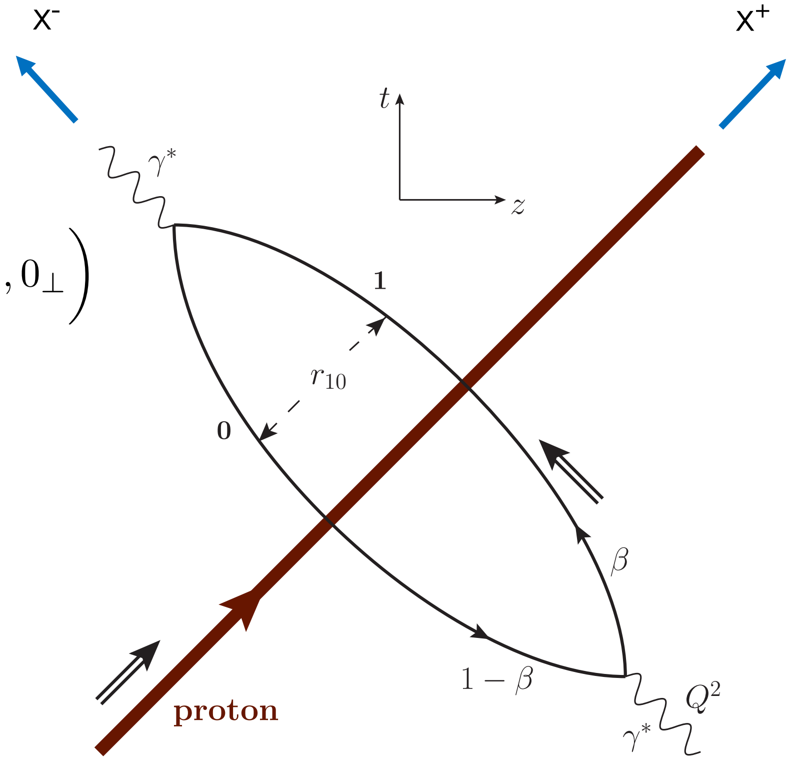
$$W^{\mu\nu} = \frac{1}{4\pi M_p} \int d^4x\, e^{iq\cdot x} \langle P | j^\mu(x)\, j^\nu(0) | P \rangle$$

$$q^\mu = \left(\frac{Q^2}{2q^-}, q^-, 0_\perp \right)$$

Large q⁻ → large x⁻ separation

$$e^{iq\cdot x} = e^{i\frac{Q^2}{2q^-}x^- + iq^- x^+}$$

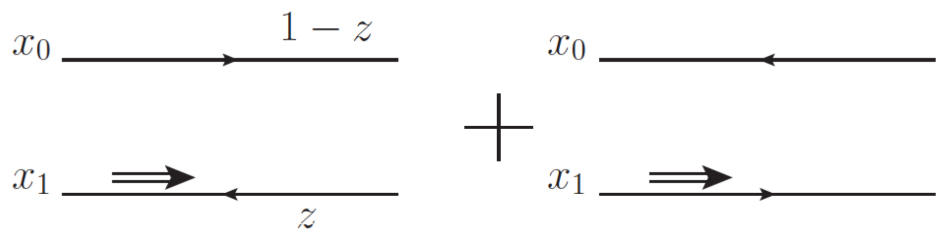
$$x^\pm = \frac{t \pm z}{\sqrt{2}}$$



aka the “shock wave”

Polarized Dipole: non-eikonal small-x physics

- All flavor-singlet small-x helicity observables depend on "polarized dipole amplitudes":



$$G_{10}(z) \equiv \frac{1}{2N_c} \mathrm{Re} \left\langle\!\!\left\langle \mathrm{T}\,\mathrm{tr}\left[V_{\underline{0}} V_{\underline{1}}^{pol\,\dagger}\right] + \mathrm{T}\,\mathrm{tr}\left[V_{\underline{1}}^{pol} V_{\underline{0}}^{\dagger}\right] \right\rangle\!\!\right\rangle(z)$$

unpolarized quark

polarized quark: eikonal propagation,
non-eikonal spin-dependent interaction

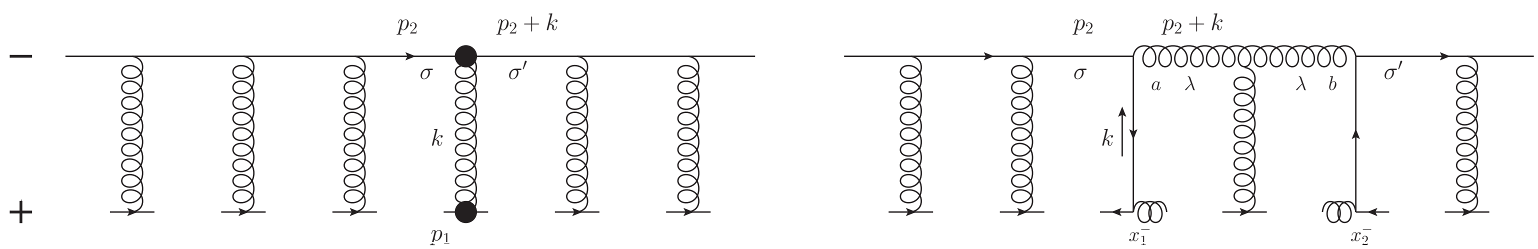
$$V_{\underline{x}} = \mathcal{P} \exp\left[ig \int\limits_{-\infty}^{\infty} dx^- A^+(0^+, x^-, \underline{x})\right]$$

- Double brackets denote an object with energy suppression scaled out:

$$\left\langle\!\!\left\langle \mathcal{O} \right\rangle\!\!\right\rangle(z) \equiv zs \left\langle \mathcal{O} \right\rangle(z)$$

Polarized fundamental "Wilson line"

- To complete the definition of the polarized dipole amplitude, we need to construct the definition of the polarized "Wilson line" V^{pol}, which is the leading helicity-dependent contribution for the quark scattering amplitude on a longitudinally-polarized target proton.



- At the leading order we can either exchange one non-eikonal *t*-channel gluon (with quark-gluon vertices denoted by blobs above) to transfer polarization between the projectile and the target, or two *t*-channel quarks, as shown above.
- We employ a blend of Brodsky & Lepage's LCPT and background field method-inspired operator treatment. We refer to the latter as the **light-cone operator treatment (LCOT)**.

Notation

- Fundamental light-cone Wilson line:

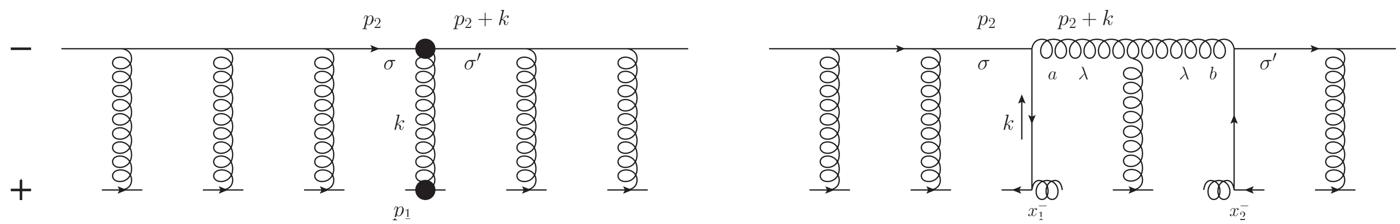
$$V_{\underline{x}}[b^-, a^-] = \mathrm{P}\exp\left\{ ig \int\limits_{a^-}^{b^-} dx^- \, A^+(x^-, \underline{x}) \right\}$$

- Adjoint light-cone Wilson line:

$$U_{\underline{x}}[b^-, a^-] = \mathcal{P}\exp\left[ig \int\limits_{a^-}^{b^-} dx^- \, \mathcal{A}^+(x^+ = 0, x^-, \underline{x}) \right]$$

- They sum multiple eikonal re-scatterings to all orders.

Sub-eikonal quark S-matrix in background gluon and quark fields



- The full sub-eikonal S-matrix for massless quarks is (Balitsky&Tarasov '15; KPS '17; YK, Sievert, '18; Chirilli '18; Altinoluk et al, '20; YK, Santiago '21)

$$V_{\underline{x},\underline{y};\sigma',\sigma} = V_{\underline{x}}\,\delta^2(\underline{x}-\underline{y})\,\delta_{\sigma,\sigma'}$$

"helicity independent" "helicity dependent"

$$-\vec{\mu}\cdot\vec{B} = -\mu_z\,B_z = \mu_z\,F^{12}$$

$$+\frac{i\,P^+}{s}\int\limits_{-\infty}^{\infty} dz^- d^2z\; V_{\underline{x}}[\infty,z^-]\,\delta^2(\underline{x}-\underline{z})\left[-\delta_{\sigma,\sigma'}\overleftarrow{D}^i D^i + g\,\sigma\,\delta_{\sigma,\sigma'}\,F^{12}\right](z^-,\underline{z})\,V_{\underline{y}}[z^-,-\infty]\,\delta^2(\underline{y}-\underline{z})$$

$$-\frac{g^2 P^+}{2s}\delta^2(\underline{x}-\underline{y})\int\limits_{-\infty}^{\infty} dz_1^-\int\limits_{z_1^-}^{\infty} dz_2^-\,V_{\underline{x}}[\infty,z_2^-]t^b\psi_\beta(z_2^-,\underline{x})U_{\underline{x}}^{ba}[z_2^-,z_1^-]\left[\delta_{\sigma,\sigma'}\gamma^+ - \sigma\delta_{\sigma,\sigma'}\gamma^+\gamma^5\right]_{\alpha\beta}\bar{\psi}_\alpha(z_1^-,\underline{x})t^a V_{\underline{x}}[z_1^-,-\infty]$$

"helicity independent" "helicity dependent"

Gluon Helicity

- A calculation gives

$$\Delta G(x,Q^2) = \frac{2N_c}{\alpha_s \pi^2} \left[\left(1 + x_{10}^2 \frac{\partial}{\partial x_{10}^2} \right) G_2 \left(x_{10}^2, zs = \frac{Q^2}{x} \right) \right]_{x_{10}^2 = \frac{1}{Q^2}}$$

$$g_{1L}^{G\,dip}(x,k_T^2) = \frac{N_c}{\alpha_s 2\pi^4} \int d^2 x_{10}\, e^{-i\underline{k}\cdot\underline{x}_{10}} \left[1 + x_{10}^2 \frac{\partial}{\partial x_{10}^2} \right] G_2 \left(x_{10}^2, zs = \frac{Q^2}{x} \right)$$

- Here we defined a new dipole amplitude G_2 (cf. Hatta et al, 2016; KPS 2017)

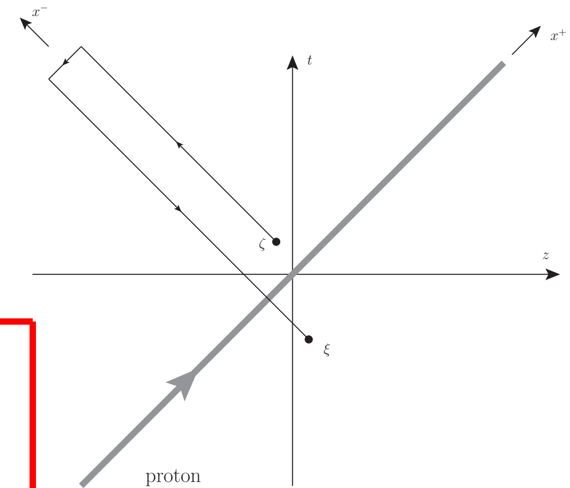
$$\int d^2 \left(\frac{x_1 + x_0}{2} \right) G_{10}^i(zs) = (x_{10})_\perp^i \, G_1(x_{10}^2, zs) + \epsilon^{ij} \, (x_{10})_\perp^j \, G_2(x_{10}^2, zs)$$

$$G_{10}^j(zs) \equiv \frac{1}{2N_c} \left\langle\!\!\left\langle \mathrm{tr} \left[V_{\underline{0}}^\dagger \, V_{\underline{1}}^{j\,\mathrm{G}[2]} + \left(V_{\underline{1}}^{j\,\mathrm{G}[2]} \right)^\dagger V_{\underline{0}} \right] \right\rangle\!\!\right\rangle$$

$$V_{\underline{z}}^{i\,\mathrm{G}[2]} \equiv \frac{P^+}{2s} \int\limits_{-\infty}^{\infty} dz^- \, V_{\underline{z}}[\infty, z^-] \left[D^i(z^-, \underline{z}) - \overleftarrow{D}^i (z^-, \underline{z}) \right] V_{\underline{z}}[z^-, -\infty]$$

What is this D-D operator? Turns out it is related to the DD operator from before.

Quark Helicity PDF and TMD



- The flavor-singlet quark helicity PDF and TMD are

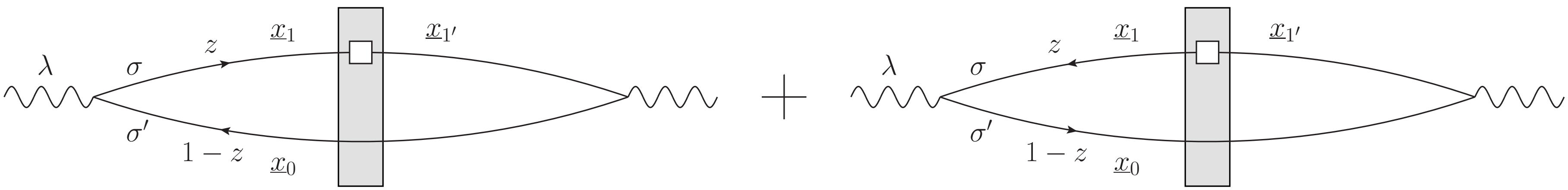
$$\boxed{\begin{aligned} \Delta\Sigma(x,Q^2) &= \frac{N_f}{\alpha_s\,\pi^2}\,\widetilde{Q}\left(x_{10}^2 = \frac{1}{Q^2},\, s = \frac{Q^2}{x}\right) \\ g_{1L}^S(x,k_T^2) &= \frac{1}{4\pi^4\alpha_s}\int d^2x_{10}\, e^{-i\underline{k}\cdot\underline{x}_{10}}\;\widetilde{Q}(x_{10}^2, Q^2/x) \end{aligned}}$$

- We have defined another operator:

$$\widetilde{Q}_{12}(s) \equiv \left\langle\!\!\left\langle \frac{g^2}{16\sqrt{k^- p_2^-}} \int\limits_{-\infty}^{\infty} dy^- \int\limits_{-\infty}^{\infty} dz^- \left[\bar{\psi}(y^-,\underline{x}_2)\left(\frac{1}{2}\gamma^+\gamma^5\right) V_{\underline{2}}[y^-,\infty]\, V_{\underline{1}}[\infty,z^-]\,\psi(z^-,\underline{x}_1)\right.\right.\right.$$
$$\left.\left.\left. +\bar{\psi}(y^-,\underline{x}_2)\left(\frac{1}{2}\gamma^+\gamma^5\right) V_{\underline{2}}[y^-,-\infty]\, V_{\underline{1}}[-\infty,z^-]\,\psi(z^-,\underline{x}_1) + \text{c.c.}\right]\right\rangle\!\!\right\rangle(s).$$

g_1 structure function

- g_1 structure function is obtained similarly, using DIS in the dipole picture:



- One gets

$$\boxed{g_1(x,Q^2) = -\sum_f \frac{N_c\, Z_f^2}{4\pi^3} \int\limits_{\Lambda^2/s}^{1} \frac{dz}{z} \int\limits_{\frac{1}{zs}}^{\min\left\{\frac{1}{zQ^2},\frac{1}{\Lambda^2}\right\}} \frac{dx_{10}^2}{x_{10}^2} \left[Q(x_{10}^2, zs) + 2\, G_2(x_{10}^2, zs)\right]}$$

- G_2 was defined before. This is the gluon PDF contribution to g_1.
- The dipole amplitude Q is due to F^{12} & axial current.
- The contribution of G_2 comes from the DD operator in the quark S-matrix.

Amplitude Q

$$Q(x_{10}^2, zs) \equiv \int d^2 \left(\frac{x_0 + x_1}{2} \right) Q_{10}(zs)$$

- The amplitude Q is defined by

$$Q_{10}(zs) \equiv \frac{1}{2N_c} \mathrm{Re} \left\langle\!\!\left\langle \mathrm{T\,tr} \left[V_{\underline{0}} \, V_{\underline{1}}^{\mathrm{pol}[1]\,\dagger} \right] + \mathrm{T\,tr} \left[V_{\underline{1}}^{\mathrm{pol}[1]} \, V_{\underline{0}}^{\dagger} \right] \right\rangle\!\!\right\rangle$$

with $V_{\underline{x}}^{\mathrm{pol}[1]} = V_{\underline{x}}^{\mathrm{G}[1]} + V_{\underline{x}}^{\mathrm{q}[1]}$, where

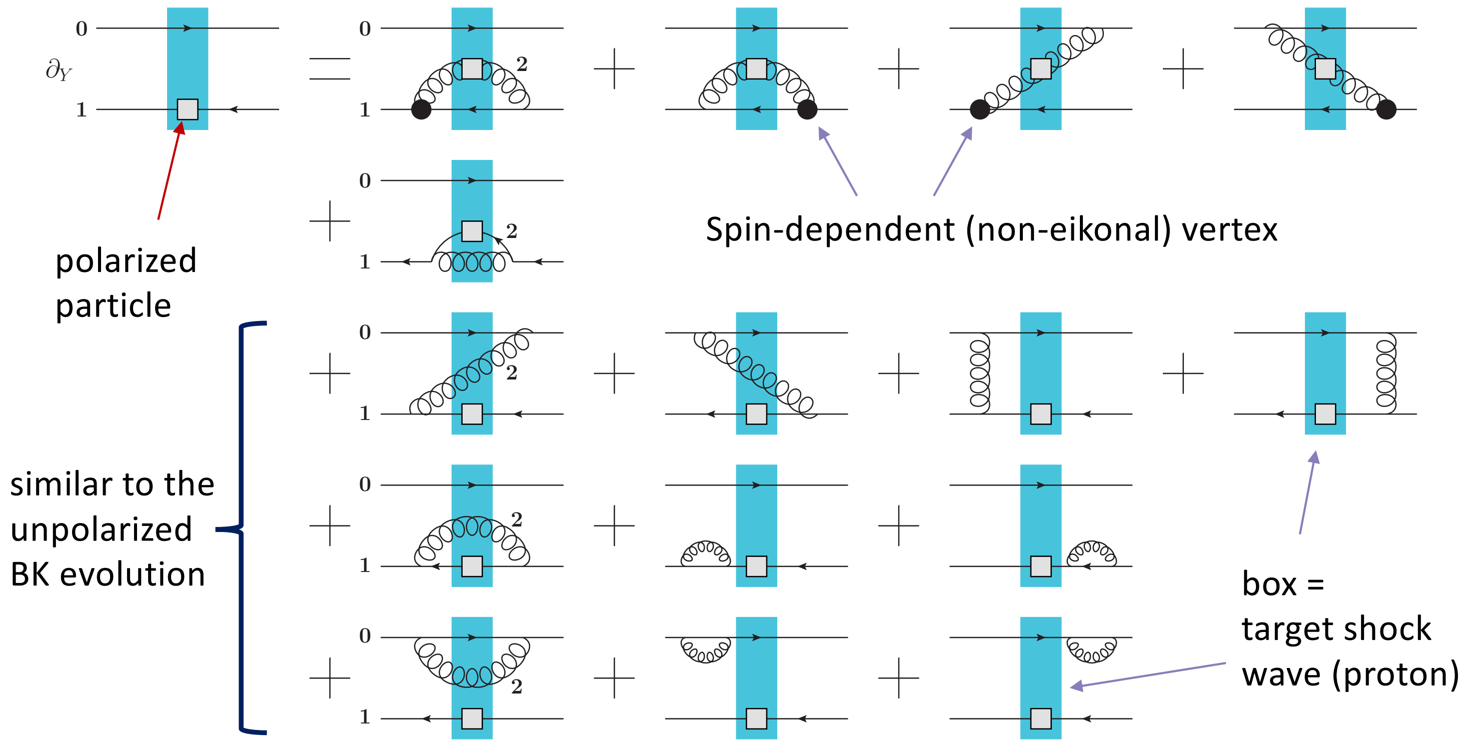
$$V_{\underline{x}}^{\mathrm{G}[1]} = \frac{i\, g\, P^+}{s} \int\limits_{-\infty}^{\infty} dx^- V_{\underline{x}}[\infty, x^-] \, F^{12}(x^-, \underline{x}) \; V_{\underline{x}}[x^-, -\infty]$$

$$V_{\underline{x}}^{\mathrm{q}[1]} = \frac{g^2 P^+}{2\, s} \int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^- V_{\underline{x}}[\infty, x_2^-] \, t^b \, \psi_\beta(x_2^-, \underline{x}) \, U_{\underline{x}}^{ba}[x_2^-, x_1^-] \; \left[\gamma^+ \gamma^5\right]_{\alpha\beta} \; \bar{\psi}_\alpha(x_1^-, \underline{x}) \, t^a \, V_{\underline{x}}[x_1^-, -\infty]$$

- U = adjoint light-cone Wilson line.
- Summary: we have dipole amplitudes Q and G_2. We also have $\tilde{Q}$, but it can be expressed in terms of Q and G_2 after one step of evolution.

Evolution for Polarized Quark Dipole

One can construct an evolution equation for the polarized dipole:



Large N_c Evolution:

- At large-N_c the equations close (Q → G).
- Initial version by YK, D. Pitonyak, M. Sievert '15-'18 (KPS), terms with subscript 2 due to YK, F. Cougoulic, A. Tarasov, Y. Tawabutr '22

$$G(x_{10}^2, zs) = G^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{s x_{10}^2}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \Big[\Gamma(x_{10}^2, x_{21}^2, z's) + 3\, G(x_{21}^2, z's) + 2\, G_2(x_{21}^2, z's) + 2\, \Gamma_2(x_{10}^2, x_{21}^2, z's) \Big],$$

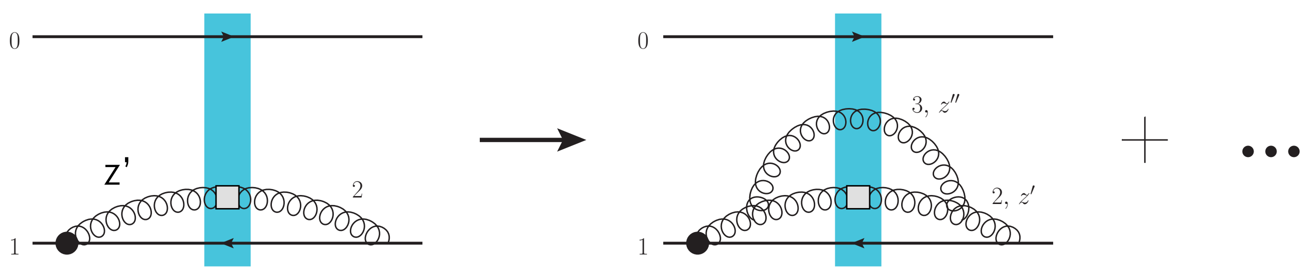
$$\Gamma(x_{10}^2, x_{21}^2, z's) = G^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{s x_{10}^2}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z'' s}}^{\min\left[x_{10}^2, x_{21}^2 \frac{z'}{z''}\right]} \frac{dx_{32}^2}{x_{32}^2} \Big[\Gamma(x_{10}^2, x_{32}^2, z''s) + 3\, G(x_{32}^2, z''s) + 2\, G_2(x_{32}^2, z''s) + 2\, \Gamma_2(x_{10}^2, x_{32}^2, z''s) \Big],$$

$$G_2(x_{10}^2, zs) = G_2^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{\max\left[x_{10}^2, \frac{1}{z's}\right]}^{\min\left[\frac{z}{z'} x_{10}^2, \frac{1}{\Lambda^2}\right]} \frac{dx_{21}^2}{x_{21}^2} \left[G(x_{21}^2, z's) + 2\, G_2(x_{21}^2, z's) \right],$$

$$\Gamma_2(x_{10}^2, x_{21}^2, z's) = G_2^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z' \frac{x_{21}^2}{x_{10}^2}} \frac{dz''}{z''} \int\limits_{\max\left[x_{10}^2, \frac{1}{z''s}\right]}^{\min\left[\frac{z'}{z''} x_{21}^2, \frac{1}{\Lambda^2}\right]} \frac{dx_{32}^2}{x_{32}^2} \left[G(x_{32}^2, z''s) + 2\, G_2(x_{32}^2, z''s) \right]$$

"Neighbor" dipole

- There is a new object in the evolution equation – **the neighbor dipole amplitude**.
- This is specific for the DLA evolution. Gluon emission may happen in one dipole, but, due to lifetime ordering, may 'know' about another dipole:



$$x_{21}^2 \, z' \gg x_{32}^2 \, z''$$

- We denote the evolution in the neighbor dipole 02 by $\Gamma_{02,\,21}(z')$

A Tale of Two Intercepts

$$\Delta\Sigma(x,Q^2)\Big|_{x\ll 1} \sim \Delta G(x,Q^2)\Big|_{x\ll 1} \sim \left(\frac{1}{x}\right)^{\alpha_h}$$

- Analytic solution of the large-N_c evolution was found in J. Borden, YK, 2304.06161 [hep-ph].
- J. Bartels, B. Ermolaev, and M. Ryskin (BER, 1996) used IR evolution equations to get:

$$\alpha_h = \sqrt{\frac{17+\sqrt{97}}{2}}\sqrt{\frac{\alpha_s\,N_c}{2\pi}} \approx 3.664\sqrt{\frac{\alpha_s\,N_c}{2\pi}}$$

- Us:

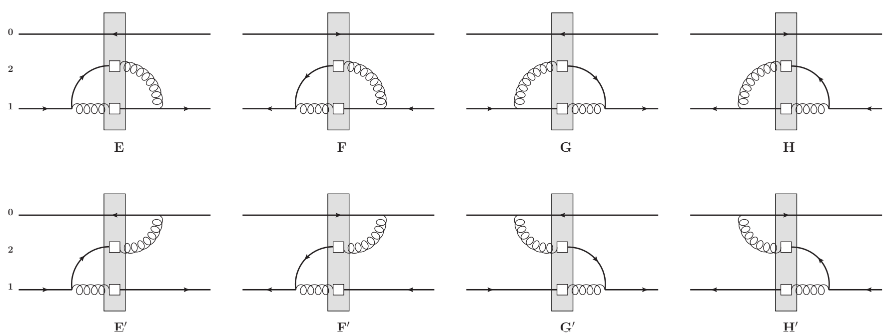
$$\alpha_h = \frac{4}{3^{1/3}}\sqrt{\mathrm{Re}\left[(-9+i\sqrt{111})^{1/3}\right]}\sqrt{\frac{\alpha_s\,N_c}{2\pi}} \approx 3.661\sqrt{\frac{\alpha_s\,N_c}{2\pi}}$$

- Our numerical solution from 2022 also gave the intercept of 3.660 or 3.661, but we believed we had larger error bars.
- We (still) disagree with BER. Albeit in the 3rd decimal point…

Large N_c&N_f Evolution Equations

Initial version by YK, D. Pitonyak, M. Sievert '15-'18 (KPS), modifications with subscript 2 due to YK, F. Cougoulic, A. Tarasov, Y. Tawabutr '22.

Beyond large-N_c, one needs to add the quark-to-gluon and gluon-to-quark transitions (G. Chirilli, 2101.12744 [hep-ph]; J. Borden, YK, M. Li, 2406.11647 [hep-ph]):



This results in the large-N_c&N_f evolution equations given here (transition terms are in blue). Agrees with DGLAP anomalous dimensions to 3 known loops.

$$
\begin{aligned}
Q(x_{10}^2, zs) = {} & Q^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{2\pi} \int_{1/s x_{10}^2}^{z} \frac{dz'}{z'} \int_{1/z's}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \Big[2\,\widetilde{G}(x_{21}^2, z's) + 2\,\widetilde{\Gamma}(x_{10}^2, x_{21}^2, z's) \\
& + Q(x_{21}^2, z's) - \overline{\Gamma}(x_{10}^2, x_{21}^2, z's) + 2\,\Gamma_2(x_{10}^2, x_{21}^2, z's) + 2\,G_2(x_{21}^2, z's) \Big] \\
& + \frac{\alpha_s N_c}{4\pi} \int_{\Lambda^2/s}^{z} \frac{dz'}{z'} \int_{1/z's}^{\min\{x_{10}^2 z/z', 1/\Lambda^2\}} \frac{dx_{21}^2}{x_{21}^2} \left[Q(x_{21}^2, z's) + 2\,G_2(x_{21}^2, z's) \right],
\end{aligned}
\tag{76a}
$$

$$
\begin{aligned}
\overline{\Gamma}(x_{10}^2, x_{21}^2, z's) = {} & Q^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{2\pi} \int_{1/s x_{10}^2}^{z'} \frac{dz''}{z''} \int_{1/z''s}^{\min\{x_{10}^2, x_{21}^2 z'/z''\}} \frac{dx_{32}^2}{x_{32}^2} \Big[2\,\widetilde{G}(x_{32}^2, z''s) \\
& + 2\,\widetilde{\Gamma}(x_{10}^2, x_{32}^2, z''s) + Q(x_{32}^2, z''s) - \overline{\Gamma}(x_{10}^2, x_{32}^2, z''s) + 2\,\Gamma_2(x_{10}^2, x_{32}^2, z''s) + 2\,G_2(x_{32}^2, z''s) \Big] \\
& + \frac{\alpha_s N_c}{4\pi} \int_{\Lambda^2/s}^{z'} \frac{dz''}{z''} \int_{1/z''s}^{\min\{x_{21}^2 z'/z'', 1/\Lambda^2\}} \frac{dx_{32}^2}{x_{32}^2} \left[Q(x_{32}^2, z''s) + 2\,G_2(x_{32}^2, z''s) \right],
\end{aligned}
\tag{76b}
$$

$$
\begin{aligned}
\widetilde{G}(x_{10}^2, zs) = {} & \widetilde{G}^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{2\pi} \int_{1/s x_{10}^2}^{z} \frac{dz'}{z'} \int_{1/z's}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \Big[3\,\widetilde{G}(x_{21}^2, z's) + \widetilde{\Gamma}(x_{10}^2, x_{21}^2, z's) \\
& + 2\,G_2(x_{21}^2, z's) + \left(2 - \frac{N_f}{2N_c}\right) \Gamma_2(x_{10}^2, x_{21}^2, z's) - \frac{N_f}{4N_c} \overline{\Gamma}(x_{10}^2, x_{21}^2, z's) - \frac{N_f}{2N_c} \widetilde{Q}(x_{21}^2, z's) \Big] \\
& - \frac{\alpha_s N_f}{8\pi} \int_{\Lambda^2/s}^{z} \frac{dz'}{z'} \int_{\max\{x_{10}^2, 1/z's\}}^{\min\{x_{10}^2 z/z', 1/\Lambda^2\}} \frac{dx_{21}^2}{x_{21}^2} \left[Q(x_{21}^2, z's) + 2\,G_2(x_{21}^2, z's) \right],
\end{aligned}
\tag{76c}
$$

$$
\begin{aligned}
\widetilde{\Gamma}(x_{10}^2, x_{21}^2, z's) = {} & \widetilde{G}^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{2\pi} \int_{1/s x_{10}^2}^{z'} \frac{dz''}{z''} \int_{1/z''s}^{\min\{x_{10}^2, x_{21}^2 z'/z''\}} \frac{dx_{32}^2}{x_{32}^2} \Big[3\,\widetilde{G}(x_{32}^2, z''s) \\
& + \widetilde{\Gamma}(x_{10}^2, x_{32}^2, z''s) + 2\,G_2(x_{32}^2, z''s) + \left(2 - \frac{N_f}{2N_c}\right) \Gamma_2(x_{10}^2, x_{32}^2, z''s) - \frac{N_f}{4N_c} \overline{\Gamma}(x_{10}^2, x_{32}^2, z''s) - \frac{N_f}{2N_c} \widetilde{Q}(x_{32}^2, z''s) \Big] \\
& - \frac{\alpha_s N_f}{8\pi} \int_{\Lambda^2/s}^{z' x_{21}^2/x_{10}^2} \frac{dz''}{z''} \int_{\max\{x_{10}^2, 1/z''s\}}^{\min\{x_{21}^2 z'/z'', 1/\Lambda^2\}} \frac{dx_{32}^2}{x_{32}^2} \left[Q(x_{32}^2, z''s) + 2\,G_2(x_{32}^2, z''s) \right],
\end{aligned}
\tag{76d}
$$

$$
G_2(x_{10}^2, zs) = G_2^{(0)}(x_{10}^2, zs) + \frac{\alpha_s N_c}{\pi} \int_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int_{\max\left[x_{10}^2, \frac{1}{z's}\right]}^{\min\left\{\frac{z}{z'} x_{10}^2, 1/\Lambda^2\right\}} \frac{dx_{21}^2}{x_{21}^2} \left[\widetilde{G}(x_{21}^2, z's) + 2\,G_2(x_{21}^2, z's) \right],
\tag{76e}
$$

$$
\Gamma_2(x_{10}^2, x_{21}^2, z's) = G_2^{(0)}(x_{10}^2, z's) + \frac{\alpha_s N_c}{\pi} \int_{\frac{\Lambda^2}{s}}^{z' \frac{x_{21}^2}{x_{10}^2}} \frac{dz''}{z''} \int_{\max\left[x_{10}^2, \frac{1}{z''s}\right]}^{\min\left\{\frac{z'}{z''} x_{21}^2, 1/\Lambda^2\right\}} \frac{dx_{32}^2}{x_{32}^2} \left[\widetilde{G}(x_{32}^2, z''s) + 2\,G_2(x_{32}^2, z''s) \right],
\tag{76f}
$$

$$
\widetilde{Q}(x_{10}^2, zs) = \widetilde{Q}^{(0)}(x_{10}^2, zs) - \frac{\alpha_s N_c}{2\pi} \int_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int_{\max\left[x_{10}^2, \frac{1}{z's}\right]}^{\min\left\{\frac{z}{z'} x_{10}^2, 1/\Lambda^2\right\}} \frac{dx_{21}^2}{x_{21}^2} \left[Q(x_{21}^2, z's) + 2\,G_2(x_{21}^2, z's) \right].
\tag{76g}
$$

Helicity JIMWLK

$$\alpha = A^+, \quad \beta = F^{12}$$

$$\langle \mathcal{O}_{\alpha,\beta,\psi,\bar\psi} \rangle_Y = \frac{\int \mathcal{D}\alpha \mathcal{D}\beta \mathcal{D}\psi \mathcal{D}\bar\psi \; \mathcal{O}_{\alpha,\beta,\psi,\bar\psi} \, \mathcal{W}_Y[\alpha,\beta,\psi,\bar\psi]}{\int \mathcal{D}\alpha \mathcal{D}\beta \mathcal{D}\psi \mathcal{D}\bar\psi \; \mathcal{W}_Y[\alpha,\beta,\psi,\bar\psi]}$$

- To go beyond the large-N_c and large-N_c&N_f limits need to write down a helicity analogue of JIMWLK evolution.
- This has been done in F. Cougoulic, YK, arXiv:1910.04268 [hep-ph], arXiv:2005.14688 [hep-ph]):

$$W_\tau[\alpha,\beta,\psi,\bar\psi] = W_\tau^{(0)}[\alpha,\beta,\psi,\bar\psi] + \int d^3\tau' \; \mathcal{K}_h[\tau,\tau'] \cdot W_{\tau'}[\alpha,\beta,\psi,\bar\psi]$$

with $\tau \equiv \{z, z\,X_\perp^2, z\,Y_\perp^2\}$ and the kernel

Life-time ordered!

$$\mathcal{K}_h[\tau,\tau'] = \frac{\alpha_s}{\pi^2} \int d^2 w_\perp \frac{\underline{X}' \cdot \underline{Y}'}{X'^2\, Y'^2} \; \theta^{(3)}(\tau-\tau') \theta\left(z' - \frac{\Lambda^2}{s}\right) \theta\left(X'^2 - \frac{1}{z'\,s}\right) \theta\left(Y'^2 - \frac{1}{z'\,s}\right)$$

$$\times \left\{ U_w^{ba} D^+_{x,a,<} D^+_{y,b,>} - \frac{1}{2}\left(D^+_{x,a,<} D^+_{y,a,<} + D^+_{x,a,>} D^+_{y,a,>}\right) \right.$$ Regular (spin-averaged) JIMWLK

$$+ \frac{1}{2} U_w^{pol,ba} \left(D^+_{x,a,<} D^\perp_{y,b,>} + D^\perp_{x,a,<} D^+_{y,b,>}\right)$$ Polarized gluon emissions

$$\left. + \left(\frac{1}{2}\gamma^5 \gamma^-\right)_{\beta\alpha} \frac{1}{2} \left((V_w^{pol})_{ij} D^{\bar\psi}_{x,j,\alpha,<} D^{\psi}_{y,i,\beta,>} + (V_w^{pol\,\dagger})_{ij} D^{\bar\psi}_{x,j,\alpha,>} D^{\psi}_{y,i,\beta,<} \right) \right\}$$ Polarized quark emissions

Warning: the equation is incomplete, need to add other sub-eikonal corrections (*a la* our '22 & '24 papers)!

Helicity McLerran-Venugopalan Model

- The initial conditions for helicity JIMWLK are given by the helicity MV model, with the weight functional (F. Cougoulic, YK, 2005.14688 [hep-ph])

Standard MV

$$\mathcal{W}^{(0)}[\alpha,\beta,\psi,\bar{\psi}] \propto \exp\left\{-\int \mathrm{d}^2 x_\perp dx^- \ \mathrm{tr}\left[(\nabla_\perp^2 \alpha)^2 \frac{\mu_+^2+\mu_-^2}{8\mu_+^2\,\mu_-^2} + \big(\langle p^+\rangle \beta\big)^2 \frac{\mu_+^2+\mu_-^2}{2\mu_+^2\,\mu_-^2} + (\nabla_\perp^2 \alpha)\langle p^+\rangle \beta \frac{\mu_+^2-\mu_-^2}{2\mu_+^2\,\mu_-^2}\right]\right\}$$
$$\times \exp\left\{\int \mathrm{d}^2 x_\perp dx^- \ \langle p^+\rangle \ \left[\frac{\nu_+^2+\nu_-^2}{\nu_+^2\,\nu_-^2}\,\bar{\psi}\frac{1}{2}\gamma^+ \nabla_\perp^2 \psi - \frac{\nu_+^2-\nu_-^2}{\nu_+^2\,\nu_-^2}\,\bar{\psi}\frac{1}{2}\gamma^+\gamma^5 \nabla_\perp^2 \psi\right]\right\}.$$

- Here $\alpha = A^+, \quad \beta = F^{12}$

- The parameters $\mu_\pm, \quad \nu_\pm$ are ~ "saturation scales" of helicity-plus and helicity-minus partons.

Polarized DIS and SIDIS data analysis

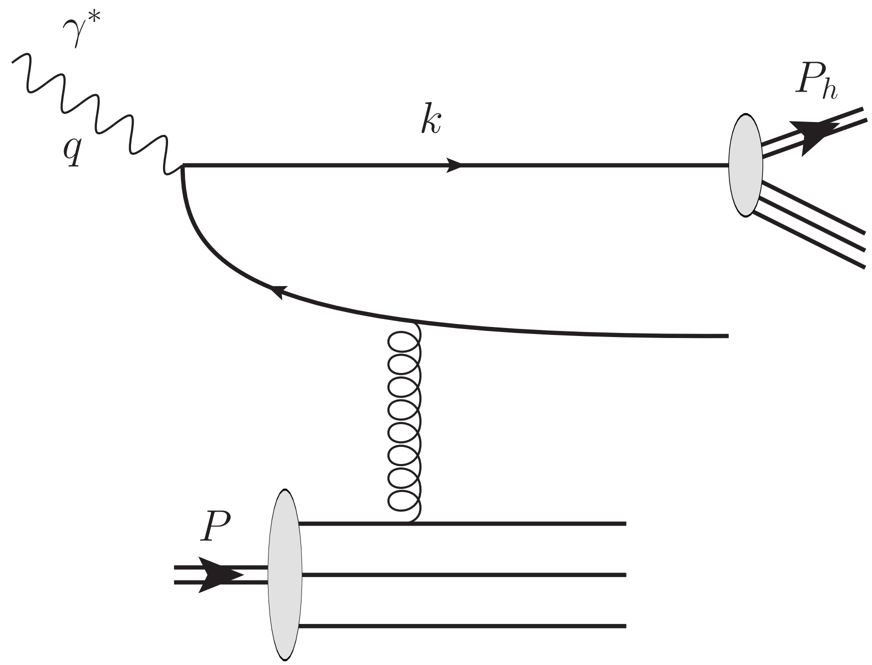
Polarized SIDIS at small x

Consider (anti-)quark production in the current fragmentation region in the polarized e+p scattering at small x.

The process is similar to the g_1 structure function calculation.

A straightforward calculation yields the SIDIS structure function (D_1 = fragmentation function)

$$\boxed{g_1^h(x, z, Q^2) \approx \frac{1}{2} \sum_{q,\bar{q}} e_q^2 \, \Delta q(x, Q^2) \, D_1^{h/q}(z, Q^2)}$$



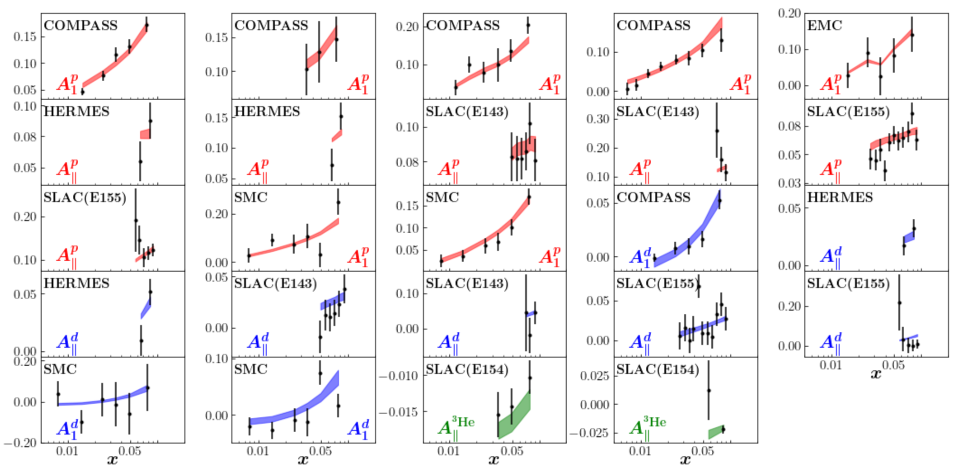
JAMsmallx: **Adamiak**, Baldonado, YK, Melnitchouk, Pitonyak, Sato, Sievert, Tarasov, Tawabutr, 2308.07461 [hep-ph]

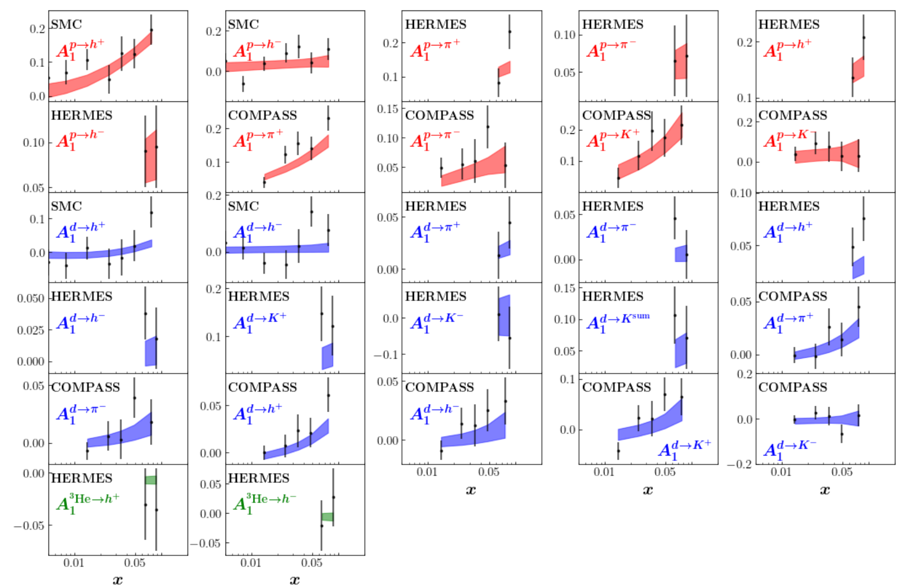
The analysis

$$5 \times 10^{-3} < x < 0.1 \equiv x_0$$

$$1.69 \text{ GeV}^2 < Q^2 < 10.4 \text{ GeV}^2$$

Initial conditions: $Q^{(0)}(x_{10}^2, zs) \sim G_2^{(0)}(x_{10}^2, zs) \sim a \ln \frac{zs}{\Lambda^2} + b \ln \frac{1}{x_{10}^2 \Lambda^2} + c$





$$A_{\parallel} = \frac{\sigma^{\downarrow\Uparrow} - \sigma^{\uparrow\Uparrow}}{\sigma^{\downarrow\Uparrow} + \sigma^{\uparrow\Uparrow}} \qquad A_1 \approx \frac{g_1}{F_1}$$

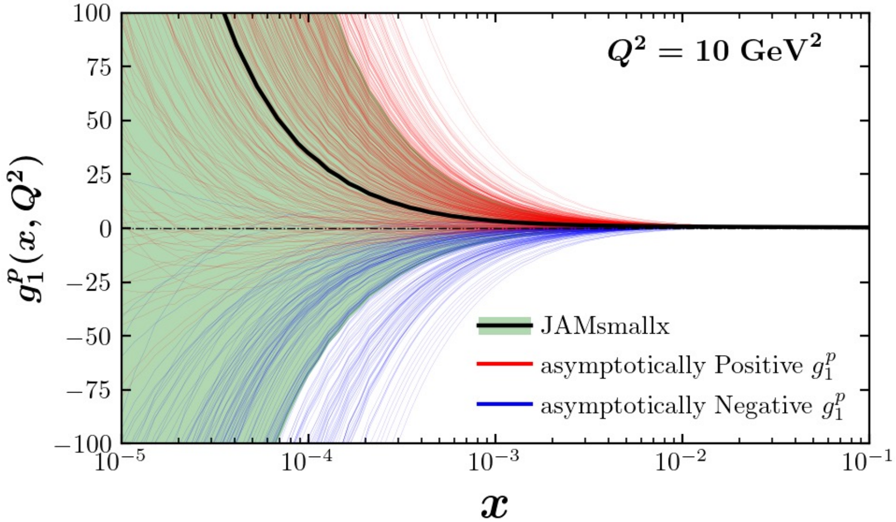
$$A_{\parallel} \approx D\, A_1$$

Double-spin asymmetries for p, d, and ^{3}He

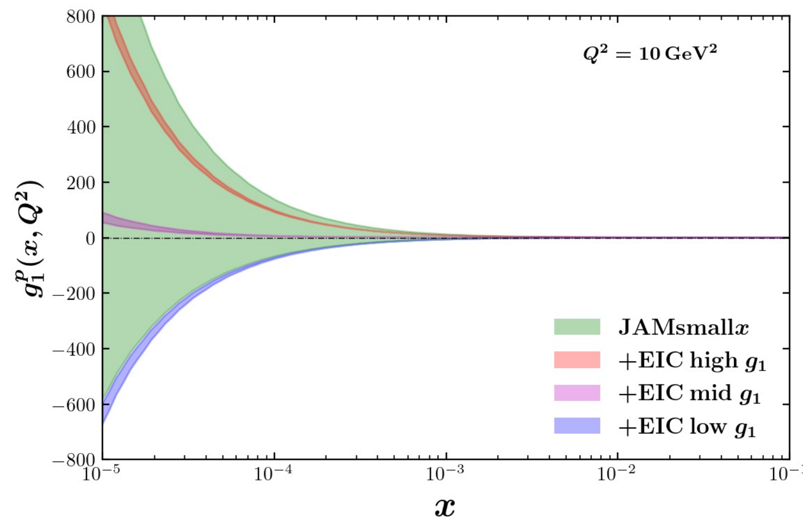
D= kinematic factor (known)
Running-coupling large-N_c&N_f evolution, 226 polarized DIS and SIDIS data points.

Proton g_1 structure function

JAM-smallx



g_1^p extracted from the existing data

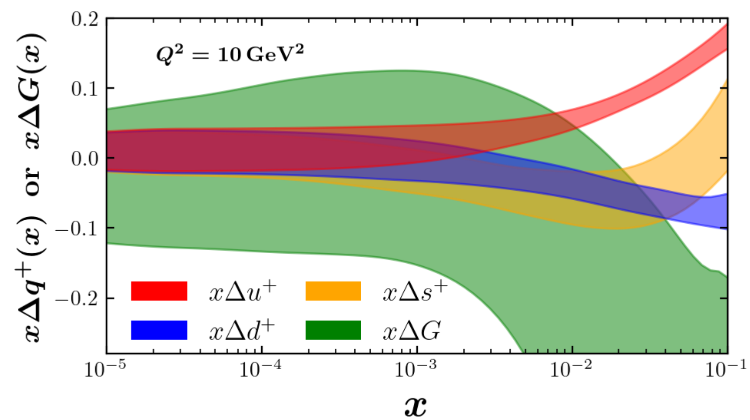


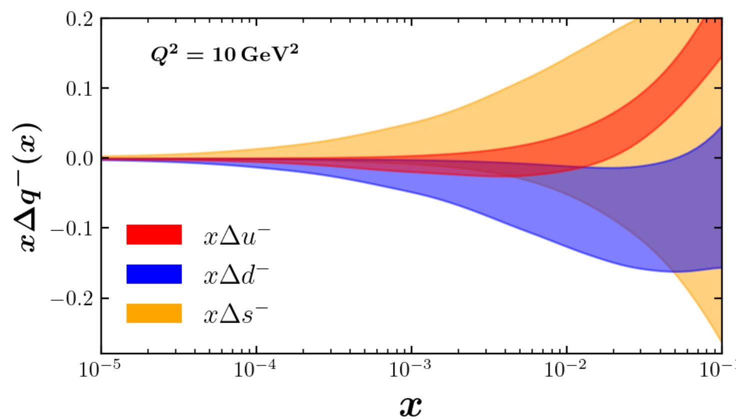
EIC impact

- JAM is based on a Bayesian Monte-Carlo: it uses replicas.
- Due to the lack of constraints, the spread is large.
- On the right, extraction using EIC pseudo-data (3 thin bands = 3 possible EIC data sets).

Helicity PDFs:

JAM-smallx



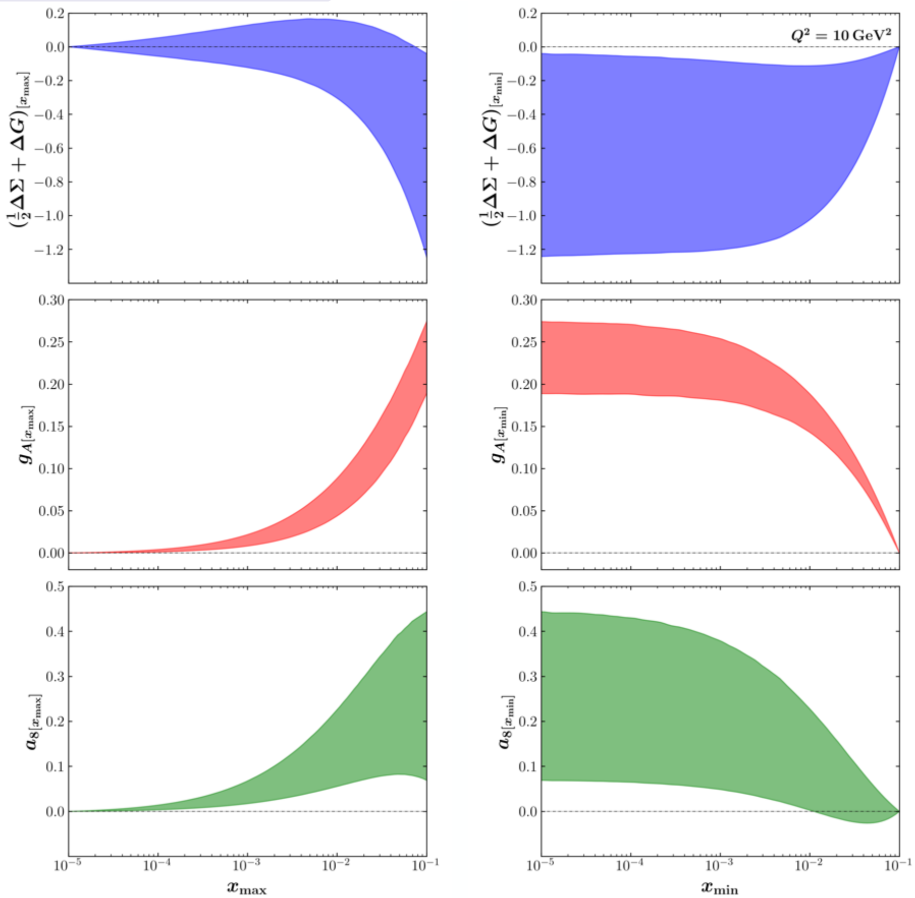


$$\Delta q^{+} = \Delta q + \Delta\bar{q} \qquad \Delta q^{-} = \Delta q - \Delta\bar{q}$$

Uncertainties at small x seem to be driven by our inability to constrain the dipole amplitude G_2 and Gtilde using the current data.

JAMsmallx: Adamiak, Baldonado, YK, Melnitchouk, Pitonyak, Sato, Sievert, Tarasov, Tawabutr 2023

How much spin is there at small x?



$$\left(\frac{1}{2}\Delta\Sigma + \Delta G\right)_{[x_{\min}]}(Q^2) \equiv \int_{x_{\min}}^{x_0} dx\, \left(\frac{1}{2}\Delta\Sigma + \Delta G\right)(x, Q^2)\,,$$

$$g_{A[x_{\min}]}(Q^2) \equiv \int_{x_{\min}}^{x_0} dx\, \left[\Delta u^+(x,Q^2) - \Delta d^+(x,Q^2)\right],$$

$$a_{8[x_{\min}]}(Q^2) \equiv \int_{x_{\min}}^{x_0} dx\, \left[\Delta u^+(x,Q^2) + \Delta d^+(x,Q^2) - 2\,\Delta s^+(x,Q^2)\right]$$

$$\left(\frac{1}{2}\Delta\Sigma + \Delta G\right)_{[x_{\max}]}(Q^2) \equiv \int_{10^{-5}}^{x_{\max}} dx\, \left(\frac{1}{2}\Delta\Sigma + \Delta G\right)(x, Q^2),$$

$$g_{A[x_{\max}]}(Q^2) \equiv \int_{10^{-5}}^{x_{\max}} dx\, \left[\Delta u^+(x,Q^2) - \Delta d^+(x,Q^2)\right],$$

$$a_{8[x_{\max}]}(Q^2) \equiv \int_{10^{-5}}^{x_{\max}} dx\, \left[\Delta u^+(x,Q^2) + \Delta d^+(x,Q^2) - 2\,\Delta s^+(x,Q^2)\right]$$

$$\boxed{\int_{10^{-5}}^{0.1} \mathrm{d}x\, \left(\frac{1}{2}\Delta\Sigma + \Delta G\right)(x) = -0.64 \pm 0.60}$$

Negative net spin at small x!

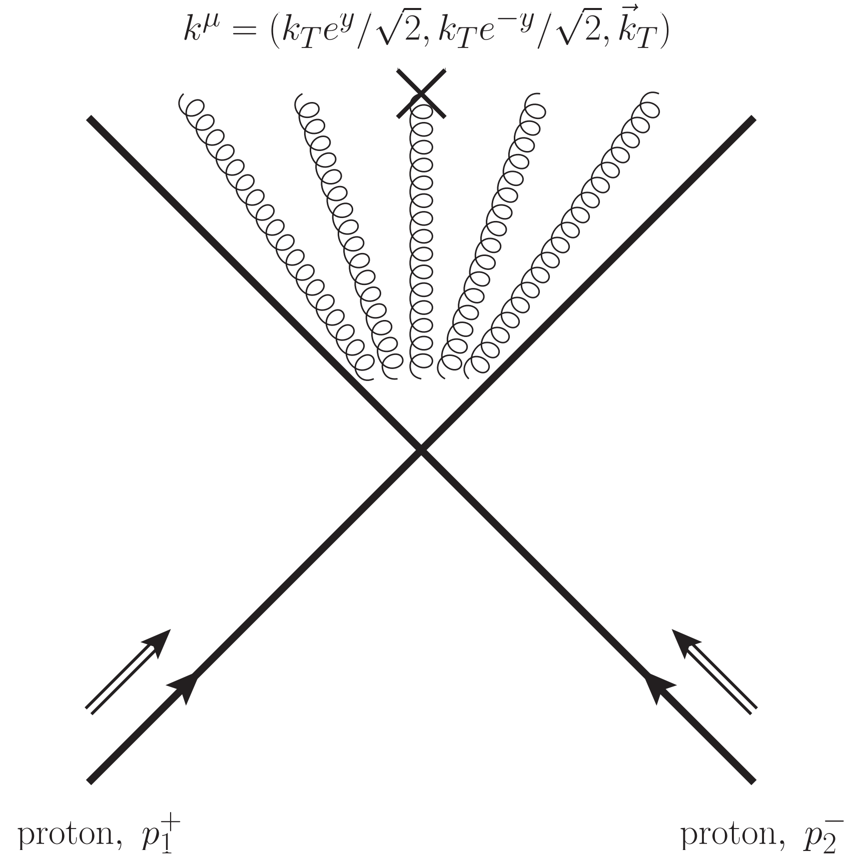
Potentially a lot of spin at small x. However, the uncertainties are large. Need a way to constrain the initial conditions. To do so, we will include the polarized p+p data from RHIC.

Particle production in polarized p+p collisions

YK, M. Li, 2403.06959 [hep-ph]

Gluon production at mid-rapidity

- We want to calculate gluon production cross section in polarized p+p collisions at mid-rapidity, where the gluon is small-x in both proton's wave functions.

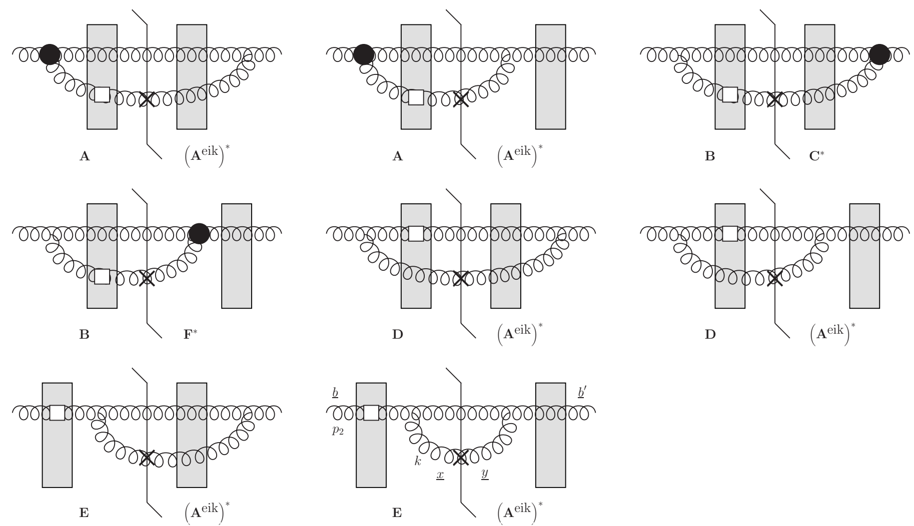


Gluon production in polarized p+p collisions

Working in the shock wave picture, we first need to sum up the following diagrams (emission inside shock wave is suppressed by a log):

The result is shown below, and is cross-checked against the existing lowest-order calculations.

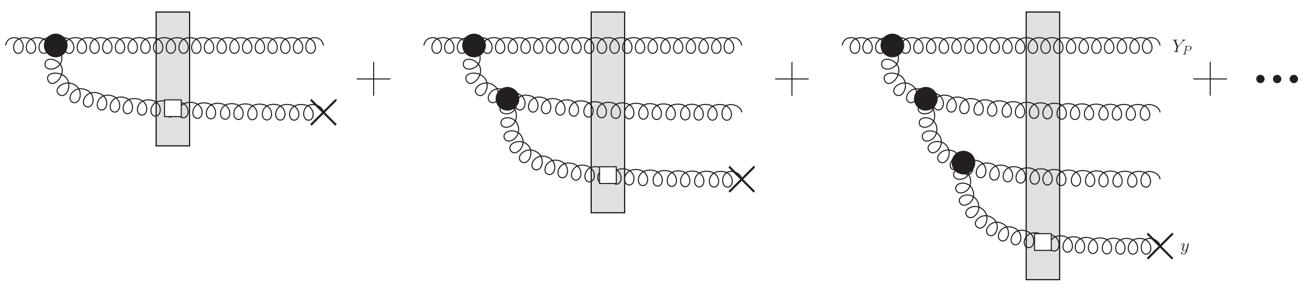
Only terms proportional to both protons' polarizations, $\sigma_1 \sigma_2$, are included.



$$\begin{aligned}\frac{d\sigma(\lambda)}{d^2k_T\,dy} = \lambda\,\frac{\alpha_s}{\pi^4}\,\frac{1}{s}\,N_c \int d^2x\,d^2y\,d^2b\,e^{-i\underline{k}\cdot(\underline{x}-\underline{y})}\,\Bigg\{\frac{\underline{x}-\underline{b}}{|\underline{x}-\underline{b}|^2}\cdot\frac{\underline{y}-\underline{b}}{|\underline{y}-\underline{b}|^2}\Bigg[G^{\text{adj}}_{\underline{x},\underline{y}}(2k^-p_1^+) - G^{\text{adj}}_{\underline{x},\underline{b}}(2k^-p_1^+) \\ -\frac{1}{4}\Big(G^{\text{adj}}_{\underline{b},\underline{y}}(2k^-p_1^+) + G^{\text{adj}}_{\underline{b},\underline{x}}(2k^-p_1^+) - 2\,G^{\text{adj}}_{\underline{b},\underline{b}'}(2k^-p_1^+)\Big)\Bigg] - 2i\,k^i\,\frac{\underline{x}-\underline{b}}{|\underline{x}-\underline{b}|^2} \times \frac{\underline{y}-\underline{b}}{|\underline{y}-\underline{b}|^2}\,G^{i\,\text{adj}}_{\underline{x},\underline{b}}(2k^-p_1^+)\Bigg\}\end{aligned}$$

Including small-x evolution

- We need to include small-x evolution on the projectile and target sides.
- This is simple on the target side, less so on the projectile side:



- We symmetrize the above expression with respect to target—projectile interchange, after which we can include the evolution on the projectile side as well.

Gluon production in polarized p+p collisions at mid-rapidity: the final result

- In the end we get the following expression for the cross section (at large N_c), where the dipole amplitudes Q and G_2 evolve via the above evolution equations (YK, M. Li, 2024):

$$\boxed{\begin{aligned}\frac{d\sigma}{d^2k_T\,dy} &= \frac{C_F}{\alpha_s\,\pi^4}\,\frac{1}{s\,k_T^2}\int d^2x\,e^{-i\underline{k}\cdot\underline{x}} \\ &\times \begin{pmatrix} 4\,Q_P & 2\,G_{2P}\end{pmatrix}(x_\perp^2, \sqrt{2}\,p_2^-\,k_T\,e^{-y}) \begin{pmatrix} \frac{1}{4}\overleftarrow{\nabla}_\perp\cdot\vec{\nabla}_\perp & \overleftarrow{\nabla}_\perp^{\,2}+\overleftarrow{\nabla}_\perp\cdot\vec{\nabla}_\perp \\ \vec{\nabla}_\perp^2+\overleftarrow{\nabla}_\perp\cdot\vec{\nabla}_\perp & 0\end{pmatrix}\begin{pmatrix}4\,Q_T \\ 2\,G_{2T}\end{pmatrix}(x_\perp^2, \sqrt{2}\,p_1^+\,k_T\,e^{y}).\end{aligned}}$$

- Equivalently, in momentum space we obtain the following factorized expression in terms of TMDs ($\Delta H_{3L}^{\perp}$ is a twist-3 helicity-flip TMD):

$$\boxed{\begin{aligned}\frac{d\sigma}{d^2k_T\,dy} &= -\frac{32\pi^4\,\alpha_s}{N_c}\,\frac{1}{s\,k_T^2}\int\frac{d^2q}{(2\pi)^2} \\ &\times \begin{pmatrix}\Delta H_{3L}^{\perp\,dip\,P} & g_{1L}^{G\,dip\,P}\end{pmatrix}\left(q_T^2, \frac{k_T}{\sqrt{2}\,p_2^-}\,e^{y}\right)\begin{pmatrix}\underline{q}\cdot(\underline{k}-\underline{q}) & \underline{q}\cdot\underline{k} \\ \underline{k}\cdot(\underline{k}-\underline{q}) & 0\end{pmatrix}\begin{pmatrix}\Delta H_{3L}^{\perp\,dip\,T} \\ g_{1L}^{G\,dip\,T}\end{pmatrix}\left((\underline{k}-\underline{q})^2, \frac{k_T}{\sqrt{2}\,p_1^+}\,e^{-y}\right)\end{aligned}}$$

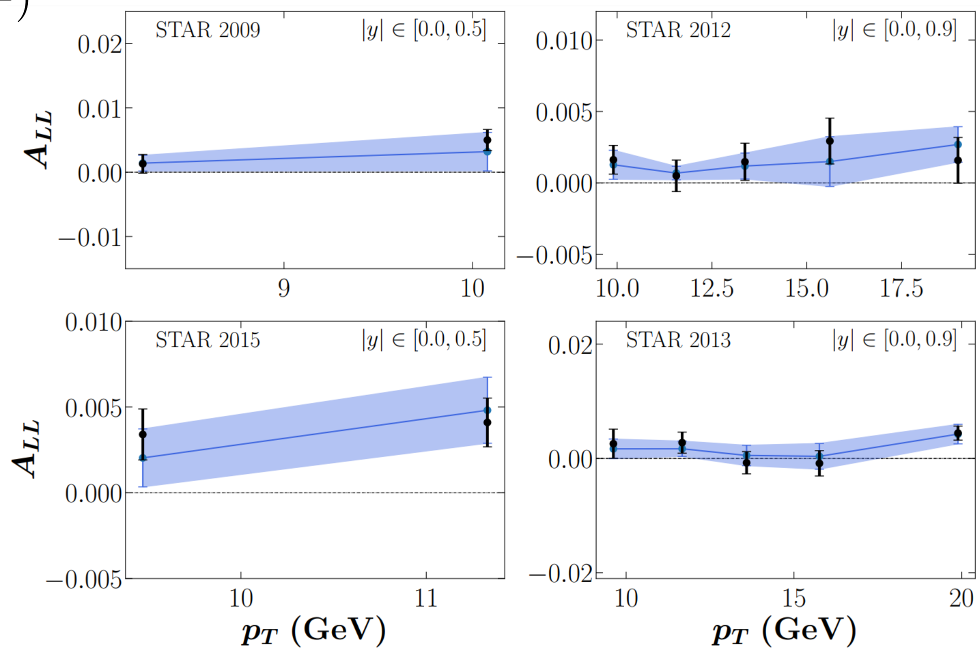
Polarized p+p collisions: small-x phenomenology

$$A_{LL} = \frac{d\sigma(++) - d\sigma(+-)}{d\sigma(++) + d\sigma(+-)}$$

- The above result can be applied to RHIC data on jets' A_{LL} (JAMsmallx: D. Adamiak, **N. Baldonado**, et al, in preparation):

- Note that the calculation was for **gluons only**, quarks need to be included (in progress). Hence, comparison with the data is a proof-of-concept at this point.

Preliminary!

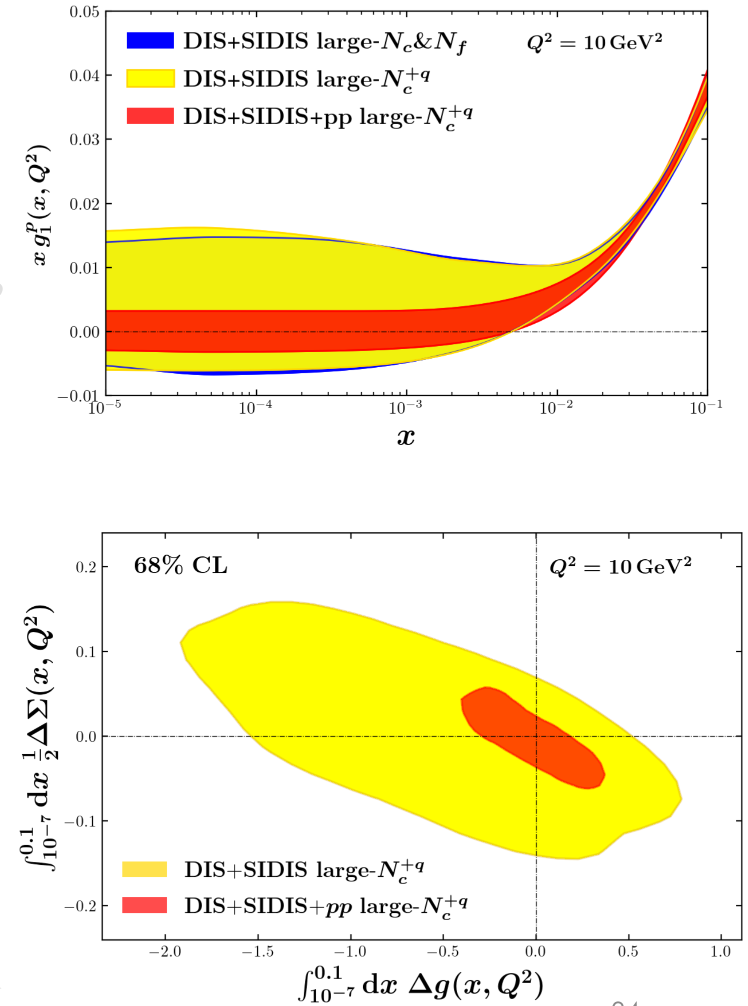


- Only large-N_c evolution is employed with external quarks.

New constraints coming from polarized p+p data:

- Including more data constrains the initial conditions for the dipole amplitudes involved, resulting in more precise EIC predictions for the proton g_1 structure function and estimates of spin at low x:

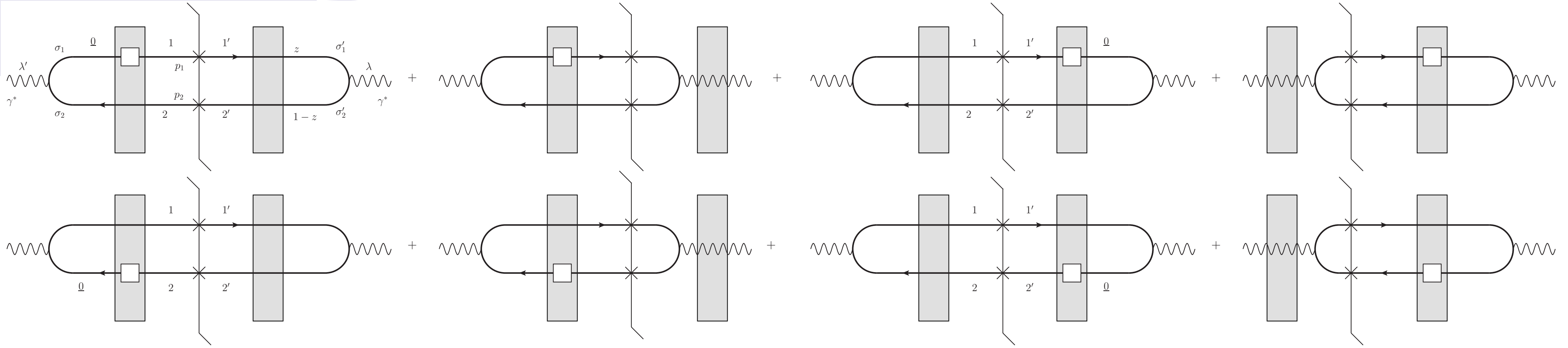
Preliminary!



Inclusive dijet production in polarized e+p collisions

YK, M. Li, in preparation

Inclusive dijet production in polarized e+p collisions



Consider double spin asymmetry (DSA) in inclusive dijet production in e+p collisions. In the b2b limit (p_T ~ Q >> $\Delta_\perp \sim \Lambda_{QCD}$) the cross section probes the WW gluon helicity TMD (cf. F. Dominguez, B.-W. Xiao, and F. Yuan, 2010; F. Dominguez, C. Marquet, B.-W. Xiao, and F. Yuan, 2011, for unpolarized TMDs):

$$\sum_{\lambda=\pm 1} \lambda\, z(1-z)\, \frac{d\sigma_{\lambda\lambda}^{\gamma^* p \to q\bar{q} X}}{d^2 p\, d^2 \Delta\, dz} \approx - \frac{2\,\alpha_s\,\alpha_{EM}\, Z_f^2}{s} \left[z^2 + (1-z)^2 \right] \frac{p_T^2 - a_f^2}{(p_T^2 + a_f^2)^2}\, g_{1L}^{G\,WW} \left(x \approx \frac{p_T^2}{s}, \Delta_T^2 \right)$$

Since, in the linear regime, the two TMDs are the same, $g_{1L}^{G\,WW}(x, k_T^2) \approx g_{1L}^{G\,dip}(x, k_T^2)$, we can use the future dijet data at EIC to further constrain gluon helicity distribution.

$$a_f^2 = Q^2\, z(1-z) + m_f^2$$

Elastic dijet production in polarized e+p collisions

YK, B. Manley, 2410.21260 [hep-ph]

OAM Distributions

- Let us write the (Jaffe-Manohar) quark and gluon OAM in terms of the Wigner distribution as

$$L_z = \int \frac{d^2 b_\perp \, db^- \, d^2 k_\perp \, dk^+}{(2\pi)^3} \, (\underline{b} \times \underline{k})_z \, W(k, b)$$

- After much algebra, we arrive at the quark and gluon OAM distributions at small *x* :

$$L_{q+\bar{q}}(x, Q^2) = \frac{N_c N_f}{2\pi^3} \int\limits_{\Lambda^2/s}^{1} \frac{dz}{z} \int\limits_{\max\left\{\frac{1}{zs}, \frac{1}{Q^2}\right\}}^{\min\left\{\frac{1}{zQ^2}, \frac{1}{\Lambda^2}\right\}} \frac{dx_{10}^2}{x_{10}^2} \Big[Q(x_{10}^2, zs) - 3\, G_2(x_{10}^2, zs) - I_3(x_{10}^2, zs)$$

$$- 2\, I_4(x_{10}^2, zs) + I_5(x_{10}^2, zs) + 3\, I_6(x_{10}^2, zs) \Big]$$

$$L_G(x, Q^2) = - \frac{2\, N_c}{\alpha_s \pi^2} \Bigg\{ \left[2 + 6\, x_{10}^2 \frac{\partial}{\partial x_{10}^2} + 2\, x_{10}^4 \frac{\partial^2}{\partial (x_{10}^2)^2} \right] \left[I_4(x_{10}^2, zs) + I_5(x_{10}^2, zs) \right]$$

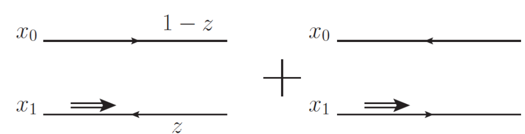
$$+ \left[1 + x_{10}^2 \frac{\partial}{\partial x_{10}^2} \right] \left[I_5(x_{10}^2, zs) + I_6(x_{10}^2, zs) \right] \Bigg\}_{x_{10}^2 = 1/Q^2, \, zs = Q^2/x}$$

OAM Distributions and Moment Amplitudes

$$L_{q+\bar{q}}(x,Q^2) = \frac{N_c N_f}{2\pi^3} \int\limits_{\Lambda^2/s}^{1} \frac{dz}{z} \int\limits_{\max\left\{\frac{1}{zs},\frac{1}{Q^2}\right\}}^{\min\left\{\frac{1}{zQ^2},\frac{1}{\Lambda^2}\right\}} \frac{dx_{10}^2}{x_{10}^2} \Big[Q(x_{10}^2, zs) - 3\, G_2(x_{10}^2, zs) - I_3(x_{10}^2, zs) - 2\, I_4(x_{10}^2, zs) + I_5(x_{10}^2, zs) + 3\, I_6(x_{10}^2, zs) \Big]$$

$$L_G(x,Q^2) = -\frac{2\, N_c}{\alpha_s \pi^2} \Bigg\{ \left[2 + 6\, x_{10}^2 \frac{\partial}{\partial x_{10}^2} + 2\, x_{10}^4 \frac{\partial^2}{\partial (x_{10}^2)^2} \right] \left[I_4(x_{10}^2, zs) + I_5(x_{10}^2, zs) \right] + \left[1 + x_{10}^2 \frac{\partial}{\partial x_{10}^2} \right] \left[I_5(x_{10}^2, zs) + I_6(x_{10}^2, zs) \right] \Bigg\}_{x_{10}^2 = 1/Q^2,\, zs = Q^2/x}$$

- Q and G_2 are the same as above. However, we also now have the impact parameter **moments of dipole amplitudes**, labeled I_3, I_4, I_5 and I_6:



$$\int d^2 x_1\, x_1^i\, Q_{10}(zs) = x_{10}^i\, I_3(x_{10}^2, zs) + \ldots,$$

$$\int d^2 x_1\, x_1^i G_{10}^j(zs) = \epsilon^{ij}\, x_{10}^2\, I_4(x_{10}^2, zs) + \epsilon^{ik}\, x_{10}^k\, x_{10}^j\, I_5(x_{10}^2, zs) + \epsilon^{jk}\, x_{10}^k\, x_{10}^i\, I_6(x_{10}^2, zs) + \ldots$$

Evolution for Moment Dipole Amplitudes

Evolution equations for the moment amplitudes in DLA and at large N_c are derived in YK, B. Manley, 2310.18404 [hep-ph].

They can be solved numerically (same ref) and analytically (B. Manley, 2401.05508 [hep-ph])

$$\begin{pmatrix} I_3 \\ I_4 \\ I_5 \\ I_6 \end{pmatrix}(x_{10}^2, zs) = \begin{pmatrix} I_3^{(0)} \\ I_4^{(0)} \\ I_5^{(0)} \\ I_6^{(0)} \end{pmatrix}(x_{10}^2, zs) + \frac{\alpha_s N_c}{4\pi} \int\limits_{\frac{1}{sx_{10}^2}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z's}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \begin{pmatrix} 2\,\Gamma_3 - 4\,\Gamma_4 + 2\,\Gamma_5 + 6\,\Gamma_6 - 2\,\Gamma_2 \\ 0 \\ 0 \\ 0 \end{pmatrix}(x_{10}^2, x_{21}^2, z's)$$

$$+ \frac{\alpha_s N_c}{4\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{\max[x_{10}^2, \frac{1}{z's}]}^{\min[\frac{z}{z'}x_{10}^2, \frac{1}{\Lambda^2}]} \frac{dx_{21}^2}{x_{21}^2} \begin{pmatrix} 4 & -4 & 2 & 6 & -4 & -6 \\ 0 & 4 & 2 & -2 & 0 & 1 \\ -2 & 2 & -1 & -3 & 2 & 3 \\ 0 & 0 & 0 & 0 & 2 & 4 \end{pmatrix} \begin{pmatrix} I_3 \\ I_4 \\ I_5 \\ I_6 \\ G \\ G_2 \end{pmatrix}(x_{21}^2, z's)$$

$$\begin{pmatrix} \Gamma_3 \\ \Gamma_4 \\ \Gamma_5 \\ \Gamma_6 \end{pmatrix}(x_{10}^2, x_{21}^2, z's) = \begin{pmatrix} I_3^{(0)} \\ I_4^{(0)} \\ I_5^{(0)} \\ I_6^{(0)} \end{pmatrix}(x_{10}^2, z's)$$

$$+ \frac{\alpha_s N_c}{4\pi} \int\limits_{\frac{1}{sx_{10}^2}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z''s}}^{\min[x_{10}^2, x_{21}^2 \frac{z'}{z''}]} \frac{dx_{32}^2}{x_{32}^2} \begin{pmatrix} 2\,\Gamma_3 - 4\,\Gamma_4 + 2\,\Gamma_5 + 6\,\Gamma_6 - 2\,\Gamma_2 \\ 0 \\ 0 \\ 0 \end{pmatrix}(x_{10}^2, x_{32}^2, z''s)$$

$$+ \frac{\alpha_s N_c}{4\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z' \frac{x_{21}^2}{x_{10}^2}} \frac{dz''}{z''} \int\limits_{\max[x_{10}^2, \frac{1}{z''s}]}^{\min[\frac{z'}{z''}x_{21}^2, \frac{1}{\Lambda^2}]} \frac{dx_{32}^2}{x_{32}^2} \begin{pmatrix} 4 & -4 & 2 & 6 & -4 & -6 \\ 0 & 4 & 2 & -2 & 0 & 1 \\ -2 & 2 & -1 & -3 & 2 & 3 \\ 0 & 0 & 0 & 0 & 2 & 4 \end{pmatrix} \begin{pmatrix} I_3 \\ I_4 \\ I_5 \\ I_6 \\ G \\ G_2 \end{pmatrix}(x_{32}^2, z''s)$$

Two intercepts, again

- The evolution equations for moment dipole amplitudes have been solved analytically by B. Manley in 2401.05508 [hep-ph]. The solution was constructed using the double Laplace transform, similar to the solution for the impact-parameter integrated amplitudes.
- The resulting small-x OAM asymptotics at large N_c is the same as for helicity PDFs,

$$L_{q+\bar{q}}(x, Q^2) \sim L_G(x, Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h}$$

with the intercept

$$\alpha_h = \frac{4}{3^{1/3}} \sqrt{\mathrm{Re}\left[(-9 + i\sqrt{111})^{1/3}\right]} \sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 3.661 \sqrt{\frac{\alpha_s N_c}{2\pi}}$$

- This slightly disagrees with the work of Boussarie, Hatta, and Yuan (2019), which resulted in the same intercept as BER:

$$\alpha_h = \sqrt{\frac{17 + \sqrt{97}}{2}} \sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 3.664 \sqrt{\frac{\alpha_s N_c}{2\pi}}$$

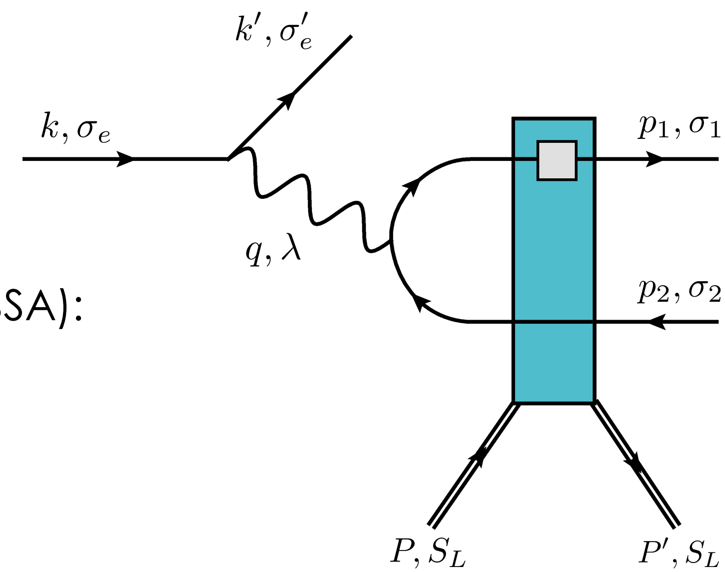
Elastic dijet production in e+p collisions

The process is similar to the one above, except now the proton remains intact.

One considers two observables, double spin asymmetry (DSA) and single spin asymmetry (SSA):

$$d\sigma^{DSA} = \frac{1}{4} \sum_{\sigma_e, S_L} \sigma_e \, S_L \, d\sigma(\sigma_e, S_L),$$

$$d\sigma^{SSA} = \frac{1}{4} \sum_{\sigma_e, S_L} S_L \, d\sigma(\sigma_e, S_L)$$



Measuring OAM distributions in elastic e+p collisions

- In the small-t limit (pT , Q >> $\Lambda_{QCD} \gg \Delta_\perp$ with $t = -\Delta_\perp^2$) the elastic dijet DSA measures moments of dipole amplitudes, thus **allowing (in principle) to measure OAM distributions!**
- Cf. Hatta et al, 2016; S. Bhattacharya, R. Boussarie and Y. Hatta, 2022 & 2024; S. Bhattacharya, D. Zheng and J. Zhou, 2023.
- Feasibility study in progress (G.Z. Becker, B. Manley, YK).

$$
\begin{aligned}
& z(1-z)\,\frac{1}{2}\sum_{S_L,\lambda\pm 1} S_L\,\lambda\,\frac{d\sigma^{\gamma^* p\to q\bar{q}p'}_{\text{symm.}\,\lambda\lambda}}{d^2p\,d^2\Delta\,dz} = -\frac{2}{(2\pi)^5 z(1-z)s}\int d^2x_{12}\,d^2x_{1'2'}\,e^{-i\underline{p}\cdot(\underline{x}_{12}-\underline{x}_{1'2'})}\,N(x^2_{1'2'},s) \qquad (107a) \\
& \times\Bigg\{\Bigg[\bigg(1-2z+i\underline{\Delta}\cdot\underline{x}_{12}\,\left(z^2+(1-z)^2\right)-\frac{i}{2}\,\underline{\Delta}\cdot\underline{x}_{1'2'}\,(1-2z)^2\bigg)\,Q(x^2_{12},s)-i\underline{\Delta}\cdot\underline{x}_{12}\,I_3(x^2_{12},s) \\
& \qquad -i\underline{\Delta}\times\underline{x}_{12}\,J_3(x^2_{12},s)\Bigg]\,\Phi^{[1]}_{\text{TT}}(\underline{x}_{12},\underline{x}_{1'2'},z) \\
& \quad +\Bigg[i(1-2z)\Big(\Delta^j\epsilon^{ji}x^2_{12}I_4(x^2_{12},s)+\underline{\Delta}\times\underline{x}_{12}\,x^i_{12}I_5(x^2_{12},s)+\Delta^i\,x^2_{12}J_4(x^2_{12},s)+\underline{\Delta}\cdot\underline{x}_{12}\,x^i_{12}J_5(x^2_{12},s)\Big) \\
& \qquad -\left[1+i\,(1-2\,z)\,\underline{\Delta}\cdot\left(\underline{x}_{12}-\frac{\underline{x}_{1'2'}}{2}\right)\right]\,\Big(\epsilon^{ik}x^k_{12}G_2(x^2_{12},s)+x^i_{12}G_1(x^2_{12},s)\Big)\Bigg] \\
& \qquad \times\,\left(\partial^i_{\underline{1}}-ip^i\right)\,\Phi^{[2]}_{\text{TT}}(\underline{x}_{12},\underline{x}_{1'2'},z)\Bigg\}+\mathcal{O}(\Delta^2_\perp), \\
& z(1-z)\,\frac{1}{2}\sum_{S_L,\lambda=\pm 1} S_L\,\left[e^{i\lambda\phi}\,\frac{d\sigma^{\gamma^* p\to q\bar{q}p'}_{\text{symm.}\,0\lambda}}{d^2p\,d^2\Delta\,dz}+\text{c.c.}\right] = -\frac{2i\sqrt{2}}{2(2\pi)^5 z(1-z)s}\int d^2x_{12}\,d^2x_{1'2'}\,e^{-i\underline{p}\cdot(\underline{x}_{12}-\underline{x}_{1'2'})} \qquad (107b) \\
& \times N(x^2_{1'2'},s)\Bigg\{\Bigg[\bigg(1-2z+i\,\underline{\Delta}\cdot\underline{x}_{12}\,(z^2+(1-z)^2)-\frac{i}{2}\underline{\Delta}\cdot\underline{x}_{1'2'}\,(1-2z)^2\bigg)\,Q(x^2_{12},s)-i\underline{\Delta}\cdot\underline{x}_{12}\,I_3(x^2_{12},s) \\
& \qquad -i\underline{\Delta}\times\underline{x}_{12}\,J_3(x^2_{12},s)\Bigg]\,\left[\frac{\hat{k}\cdot\underline{x}_{12}}{x_{12}}\Phi^{[1]}_{\text{LT}}(\underline{x}_{12},\underline{x}_{1'2'},z)-\frac{\hat{k}\cdot\underline{x}_{1'2'}}{x_{1'2'}}\Phi^{[1]}_{\text{LT}}(\underline{x}_{1'2'},\underline{x}_{12},z)\right] \\
& \quad +\Bigg[i(1-2z)\Big(\Delta^j\epsilon^{ji}\,x^2_{12}\,I_4(x^2_{12},s)+\underline{\Delta}\times\underline{x}_{12}\,x^i_{12}\,I_5(x^2_{12},s)+\Delta^i\,x^2_{12}\,J_4(x^2_{12},s)+\underline{\Delta}\cdot\underline{x}_{12}\,x^i_{12}\,J_5(x^2_{12},s)\Big) \\
& \qquad -\left[1+i\,(1-2\,z)\,\underline{\Delta}\cdot\left(\underline{x}_{12}-\frac{\underline{x}_{1'2'}}{2}\right)\right]\,\Big(\epsilon^{ik}x^k_{12}G_2(x^2_{12},s)+x^i_{12}G_1(x^2_{12},s)\Big)\Bigg] \\
& \quad \times\left(\partial^i_{\underline{1}}-ip^i\right)\left[\frac{\hat{k}\times\underline{x}_{12}}{x_{12}}\Phi^{[2]}_{\text{LT}}(\underline{x}_{12},\underline{x}_{1'2'},z)+\frac{\hat{k}\times\underline{x}_{1'2'}}{x_{1'2'}}\Phi^{[2]}_{\text{LT}}(\underline{x}_{1'2'},\underline{x}_{12},z)\right]\Bigg\}+\mathcal{O}(\Delta^2_\perp),
\end{aligned}
$$

Conclusions

- The small-x helicity formalism in the double logarithmic approximation (DLA) + running coupling allows to do successful polarized DIS + SIDIS phenomenology based on the existing small-x data.
- However, the multitude of different dipole amplitudes in the formalism prevents precise EIC predictions: there are too many dipole amplitudes, making their initial conditions hard to fix using the existing data.
- Polarized p+p data on A_{LL} from RHIC, if properly included, may help. The first step in this direction is presented above.
- When EIC comes online, DSA in inclusive dijet production would help constrain gluon helicity distributions.
- Elastic dijets at EIC may help us measure the OAM distributions as well (and compare their x-dependence to theory).

Happy Birthday Edmond!

Backup slides

Small-x Asymptotics for Helicity Distributions

- Let's take a closer look at the anomalous dimension:

$$\Delta G(x, Q^2) = \int \frac{d\omega}{2\pi i} \left(\frac{1}{x}\right)^{\omega} \left(\frac{Q^2}{\Lambda^2}\right)^{\Delta\gamma_{GG}(\omega)} \Delta G_\omega(\Lambda^2)$$

- In the pure-glue case, Bartels, Ermolaev and Ryskin's (BER) anomalous dimension can be found analytically. It reads (KPS '16)

$$\Delta\gamma_{GG}^{BER}(\omega) = \frac{1}{2}\left[\omega - \sqrt{\omega^2 - 16\,\bar{\alpha}_s \frac{1 - \frac{3\,\bar{\alpha}_s}{\omega^2}}{1 - \frac{\bar{\alpha}_s}{\omega^2}}}\right] \qquad \bar{\alpha}_s = \frac{\alpha_s\, N_c}{2\pi}$$

- Our evolution's anomalous dimension can be found analytically at large-N_c (J. Borden, YK, 2304.06161 [hep-ph]):

$$\Delta\gamma_{GG}^{us}(\omega) = \frac{1}{2}\left[\omega - \sqrt{\omega^2 - 16\,\bar{\alpha}_s \sqrt{1 - \frac{4\,\bar{\alpha}_s}{\omega^2}}}\right]$$

A Tale of Two Anomalous Dimensions

- The two anomalous dimensions look similar but are not the same function.

$$\Delta\gamma_{GG}^{BER}(\omega) = \frac{1}{2}\left[\omega - \sqrt{\omega^2 - 16\,\bar{\alpha}_s \frac{1-\frac{3\,\bar{\alpha}_s}{\omega^2}}{1-\frac{\bar{\alpha}_s}{\omega^2}}}\right] \qquad \Delta\gamma_{GG}^{us}(\omega) = \frac{1}{2}\left[\omega - \sqrt{\omega^2 - 16\,\bar{\alpha}_s\sqrt{1-\frac{4\,\bar{\alpha}_s}{\omega^2}}}\right]$$

- Their expansions in α_S start out the same, then **differ at four (!) loops** (the first 3 terms agree with the existing finite-order calculations, the four-loop result is unknown):

$$\Delta\gamma_{GG}^{BER}(\omega) = \underline{\frac{4\,\bar{\alpha}_s}{\omega} + \frac{8\,\bar{\alpha}_s^2}{\omega^3} + \frac{56\,\bar{\alpha}_s^3}{\omega^5}} + \underline{\frac{504\,\bar{\alpha}_s^4}{\omega^7}} + \ldots$$

$$\Delta\gamma_{GG}^{us}(\omega) = \underline{\frac{4\,\bar{\alpha}_s}{\omega} + \frac{8\,\bar{\alpha}_s^2}{\omega^3} + \frac{56\,\bar{\alpha}_s^3}{\omega^5}} + \underline{\frac{496\,\bar{\alpha}_s^4}{\omega^7}} + \ldots$$

A Tale of Two Intercepts

$$\Delta G(x, Q^2) = \int \frac{d\omega}{2\pi i} \left(\frac{1}{x}\right)^{\omega} \left(\frac{Q^2}{\Lambda^2}\right)^{\Delta\gamma_{GG}(\omega)} \Delta G_{\omega}(\Lambda^2)$$

$$\Delta\gamma_{GG}^{BER}(\omega) = \frac{1}{2}\left[\omega - \sqrt{\omega^2 - 16\,\bar{\alpha}_s \frac{1 - \frac{3\,\bar{\alpha}_s}{\omega^2}}{1 - \frac{\bar{\alpha}_s}{\omega^2}}}\right] \qquad \Delta\gamma_{GG}^{us}(\omega) = \frac{1}{2}\left[\omega - \sqrt{\omega^2 - 16\,\bar{\alpha}_s \sqrt{1 - \frac{4\,\bar{\alpha}_s}{\omega^2}}}\right]$$

- The intercept (largest power Re[ω]) is given by the right-most singularity (branch point) of the anomalous dimension.
- For BER this gives $\alpha_h = \sqrt{\frac{17 + \sqrt{97}}{2}}\sqrt{\frac{\alpha_s\,N_c}{2\pi}} \approx 3.664\sqrt{\frac{\alpha_s\,N_c}{2\pi}}$
- For us $\alpha_h = \frac{4}{3^{1/3}}\sqrt{\mathrm{Re}\left[(-9 + i\,\sqrt{111})^{1/3}\right]}\sqrt{\frac{\alpha_s\,N_c}{2\pi}} \approx 3.661\sqrt{\frac{\alpha_s\,N_c}{2\pi}}$

OAM Distribution to hPDF Ratios

- Following Boussarie *et al* (2019), we consider the ratios of OAM distributions to helicity PDFs at small x.
- For these ratios, Boussarie *et al*, predict, using the Wandzura-Wilczek approximation:

$$\frac{L_{q+\bar{q}}(x,Q^2)}{\Delta\Sigma(x,Q^2)} = -\frac{1}{1+\alpha_h} \qquad \frac{L_G(x,Q^2)}{\Delta G(x,Q^2)} = -\frac{2}{1+\alpha_h}$$

$$\Delta\Sigma(x,Q^2)\bigg|_{x\ll 1} \sim \Delta G(x,Q^2)\bigg|_{x\ll 1} \sim \left(\frac{1}{x}\right)^{\alpha_h} \qquad L_{q+\bar{q}}(x,Q^2) \sim L_G(x,Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h}$$

OAM Distribution to hPDF Ratios

- Analytic solution from B. Manley, 2401.05508 [hep-ph], gives

$$t = \sqrt{\frac{\alpha_s N_c}{2\pi} \ln \frac{Q^2}{\Lambda^2}}$$

$$\alpha_s = 0.25 \qquad \Lambda = 1 \text{ GeV}$$

$$\frac{L_{q+\bar{q}}(x,Q^2)}{\Delta\Sigma(x,Q^2)} = -\frac{1}{1+\alpha_h}$$

$$\frac{L_G(x,Q^2)}{\Delta G(x,Q^2)} = -\frac{2}{1+\alpha_h}$$