Loop calculations in light cone perturbation theory

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HELSINKI INSTITUTE OF PHYSICS

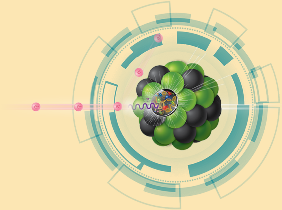
Outline

Outline of this talk

- DIS in the dipole picture at NLO: massless quarks
  Beuf [arXiv:1606.00777 [hep-ph]], [arXiv:1708.06557 [hep-ph]],
  H. Hänninen, T.L., R. Paatelainen [arXiv:1711.08207 [hep-ph]]
- One loop DIS with massive quarks
  Beuf, T.L. Paatelainen [2103.14549 [hep-ph]], [arXiv:2112.03158 [hep-ph]], [2204.02486 [hep-ph]]
- Diffractive structure function at NLO G. Beuf, T. Lappi, H. Mäntysaari, R. Paatelainen, J. Penttala
  2401.17251 [hep-ph], G. Beuf, H. Hänninen, T.L., Y. Mulian, H. Mäntysaari, arXiv:2206.13161

Process of interest

DIS at high energy

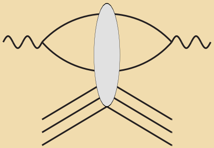


High energy collisions as eikonal scattering

Dipole picture of DIS

Limit of small x, i.e. high γ^*-target energy

Leading order

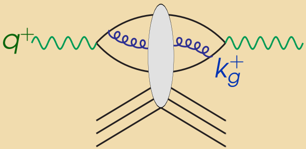


- $\gamma^* \to q\bar{q}$ in vacuum
- $q\bar{q}$ interacts eikonally with target
- σ^{tot} is $2\times$Im-part of amplitude

"Dipole model": Nikolaev, Zakharov 1991

Many fits to HERA data, starting with Golec-Biernat, Wüsthoff 1998

Leading Log: add **soft** gluon



- Soft gluon: large logarithm

$$\int_{x_{Bj}} \frac{dk_g^+}{k_g^+} \sim \ln \frac{1}{x_{Bj}}$$

Absorb into renormalization of target:

BK equation Balitsky 1995, Kovchegov 1999

Dipole picture of DIS

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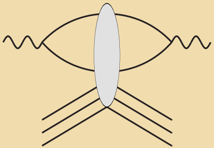

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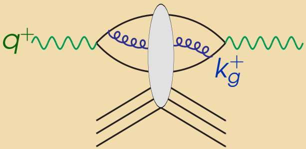


- Soft gluon: large logarithm

$$\int_{x_{Bj}} \frac{\mathrm{d}k_g^+}{k_g^+} \sim \ln \frac{1}{x_{Bj}}$$

Absorb into renormalization of target:
BK equation Balitsky 1995, Kovchegov 1999

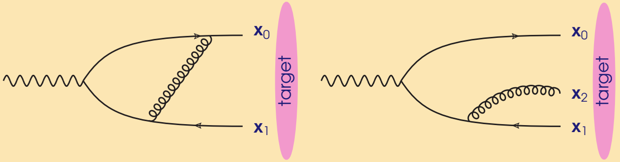
NLO: the same gluon with full kinematics

NLO DIS cross section with massless quarks

DIS at NLO: procedure

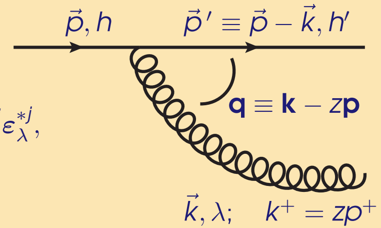
Beuf 2016, 2017, H. Hänninen, T.L., Paatelainen 2017

1. Evaluate diagrams
   - $\psi^{\gamma^* \to q\bar{q}}$ to 1 loop
   - $\psi^{\gamma^* \to q\bar{q}g}$ at tree level
2. Fourier-transform $\mathbf{k} \to \mathbf{x}$
3. Square
   ${}_i\langle \gamma_\lambda(\vec{q}', Q^2) | (\hat{S}_E - \mathbf{1}) | \gamma_\lambda(\vec{q}, Q^2) \rangle_i$



On-shell vertex

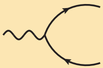
$$\left[\bar{u}_{h'}(p')\not{\varepsilon}^*_\lambda(k) u_h(p)\right] = \frac{-2}{z\sqrt{1-z}}\left[\left(1-\frac{z}{2}\right)\delta_{h',h}\delta^{ij} + \frac{z}{2} ih\delta_{h',h}\varepsilon^{ij}\right]\mathbf{q}^i \varepsilon^{*j}_\lambda,$$



(This is in $d = 4$, generalize for $d < 4$)
2 index structures for massless quarks.

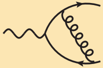
DIS at NLO: factorizing BK evolution

B. Ducloué, H. Hänninen, T. L. and Y. Zhu, (arXiv:1708.07328 (hep-ph)).



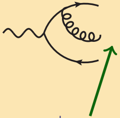
$\Rightarrow$

$$\sigma^{\mathrm{LO}} \sim \int_0^1 \mathrm{d}z_1 \int_{\mathbf{x}_0,\mathbf{x}_1} |\psi^{\mathrm{LO}}_{\gamma^*\to q\bar{q}}|^2 \mathcal{N}_{01}(x_{Bj})$$



$- * \Rightarrow$

$$\sigma^{\mathrm{dip}} \sim \alpha_s C_F \int_{\mathbf{x}_0,\mathbf{x}_1,z_1} \left|\psi^{\mathrm{LO}}_{\gamma^*\to q\bar{q}}\right|^2 \left[\frac{1}{2}\ln^2\left(\frac{z_1}{1-z_1}\right) - \frac{\pi^2}{6} + \frac{5}{2}\right] \mathcal{N}_{01}(x_{Bj})$$



$+ * \Rightarrow$

- LL

$k_g^+ \sim z_2$

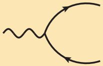
$$\sigma^{qg}_{\mathrm{sub.}} \sim \alpha_s C_F \int_{z_1,z_2,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \mathrm{d}z_2 \Big[\left|\psi_{\gamma^*\to q\bar{q}g}(z_1,z_2)\right|^2 \mathcal{N}_{012}(X(z_2))$$

$$- \left|\psi_{\gamma^*\to q\bar{q}g}(z_1,0)\right|^2 \mathcal{N}_{012}(X(z_2)) \Big].$$

* UV-divergence

DIS at NLO: factorizing BK evolution

B. Ducloué, H. Hänninen, T. L. and Y. Zhu, (arXiv:1708.07328 (hep-ph)).

 $\Rightarrow$

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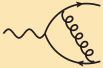 $- * \Rightarrow$

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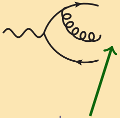
 $+ * \Rightarrow$

- LL

$k_g^+ \sim z_2$

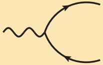
$$\sigma^{qg}_{\mathrm{sub.}} \sim \alpha_s C_F \int_{z_1,z_2,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \mathrm{d}z_2 \Big[\left|\psi_{\gamma^* \to q\bar{q}g}(z_1,z_2)\right|^2 \mathcal{N}_{012}(X(z_2)) - \left|\psi_{\gamma^* \to q\bar{q}g}(z_1,0)\right|^2 \mathcal{N}_{012}(X(z_2)) \Big].$$

* UV-divergence

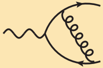
LL: subtract leading log, $\mathcal{N}$ BK-evolved in $Y = \ln 1/X$

DIS at NLO: factorizing BK evolution

B. Ducloué, H. Hänninen, T. L. and Y. Zhu, (arXiv:1708.07328 (hep-ph)).



$$\Rightarrow \quad \sigma^{\mathrm{LO}} \sim \int_0^1 \mathrm{d}z_1 \int_{\mathbf{x}_0,\mathbf{x}_1} |\psi^{\mathrm{LO}}_{\gamma^*\to q\bar{q}}|^2 \mathcal{N}_{01}(x_{Bj})$$



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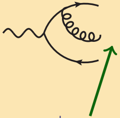


- LL

$k_g^+ \sim z_2$

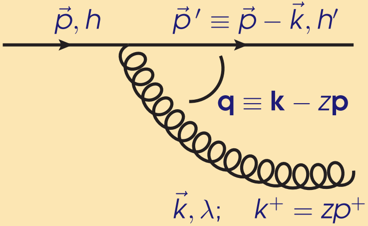
$$+ * \Rightarrow \quad \sigma^{qg}_{\mathrm{sub.}} \sim \alpha_s C_F \int_{z_1,z_2,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \mathrm{d}z_2 \Big[\left|\psi_{\gamma^*\to q\bar{q}g}(z_1,z_2)\right|^2 \mathcal{N}_{012}(X(z_2)) - \left|\psi_{\gamma^*\to q\bar{q}g}(z_1,0)\right|^2 \mathcal{N}_{012}(X(z_2)) \Big].$$

* UV-divergence   LL: subtract leading log, $\mathcal{N}$ BK-evolved in $Y = \ln 1/X$

- Parametrically $X(z_2) \sim x_{Bj}$, but $X(z_2) \sim 1/z_2$ essential!
- "k_T-factorization" with projectile $z = 0$ subtraction ("CXY") unstable @ NLO.
- Same probem in $p + A \to h + X$ and DIS

Including quark masses

Elementary vertex with masses



- h, h': light cone helicities
- $\mathbf{q}$: center-of-mass $\perp$ momentum in splitting
- polarization vector ε_λ^{*j}

$$\left[\bar{u}_{h'}(p')\not\varepsilon_\lambda^*(k)u_h(p)\right] \sim \overbrace{\bar{u}_{h'}\gamma^+u_h}^{\sim\delta_{h,h'}}\,\delta^{ij}q^i\varepsilon_\lambda^{*j} + \overbrace{\bar{u}_{h'}\gamma^+[\gamma^i,\gamma^j]u_h}^{\sim\delta_{h,h'}}\,q^i\varepsilon_\lambda^{*j} + \overbrace{\bar{u}_{h'}\gamma^+\gamma^j u_h}^{\sim\delta_{h,-h'}}\,m_q\varepsilon_\lambda^{*j}$$

- New 3rd ligh-cone-helicity-flip structure $\sim m_q$
- Note: $\perp$ momentum in non-flip, but not in flip vertex $\Longrightarrow$ less UV-divergent
- Loops: also generate 4th structure $\bar{u}_{h'}\gamma^+\gamma^i u_h\varepsilon_\lambda^{*j}q^iq^j$
- In principle proceed as massless case, but a lot more algebra . . .

Beuf, T.L., Paatelainen, 2103.14549 [hep-ph], 2112.03158 [hep-ph], 2204.02486 [hep-ph]

First: result for $\gamma^* \to q\bar{q}$ with massive quarks

$$\widetilde{\psi}^{\gamma_T^*\to q\bar{q}}_{\mathrm{NLO}} = -\frac{ee_f}{2\pi}\left(\frac{\alpha_s C_\mathrm{F}}{2\pi}\right)\Bigg\{\Bigg[\left(\frac{k_0^+ - k_1^+}{q^+}\right)\delta^{ij}\bar{u}(0)\gamma^+ v(1) + \frac{1}{2}\bar{u}(0)\gamma^+[\gamma^i,\gamma^j]v(1)\Bigg]\mathcal{F}\left[\mathbf{P}^i\mathcal{V}^T\right] + \bar{u}(0)\gamma^+ v(1)\mathcal{F}\left[\mathbf{P}^j\mathcal{N}^T\right]$$

$$+ m\bar{u}(0)\gamma^+\gamma^i v(1)\mathcal{F}\left[\left(\frac{\mathbf{P}^i\mathbf{P}^j}{\mathbf{P}^2}-\frac{\delta^{ij}}{2}\right)\mathcal{S}^T\right] - m\bar{u}(0)\gamma^+\gamma^j v(1)\mathcal{F}\left[\mathcal{V}^T+\mathcal{M}^T-\frac{\mathcal{S}^T}{2}\right]\Bigg\}\varepsilon^j_\lambda .$$

$$\mathcal{F}\left[\mathbf{P}^i\mathcal{V}^T\right] = \frac{i\mathbf{x}_{01}^i}{|\mathbf{x}_{01}|}\left(\frac{\kappa_z}{2\pi|\mathbf{x}_{01}|}\right)^{\frac{D}{2}-2}\left\{\left[\frac{3}{2}+\log\left(\frac{\alpha}{z}\right)+\log\left(\frac{\alpha}{1-z}\right)\right]\left\{\frac{(4\pi)^{2-\frac{D}{2}}}{(2-\frac{D}{2})}\Gamma\left(3-\frac{D}{2}\right)+\log\left(\frac{|\mathbf{x}_{01}|^2\mu^2}{4}\right)\right.\right.$$
$$\left.\left.+2\gamma_E\right\}+\frac{1}{2}\frac{(D_s-4)}{(D-4)}\right\}\kappa_z K_{\frac{D}{2}-1}\left(|\mathbf{x}_{01}|\kappa_z\right)+\frac{i\mathbf{x}_{01}^i}{|\mathbf{x}_{01}|}\left\{\left[\frac{5}{2}-\frac{\pi^2}{3}+\log^2\left(\frac{z}{1-z}\right)-\Omega^T_{\mathcal{V}}+L\right]\kappa_z K_1\left(|\mathbf{x}_{01}|\kappa_z\right)+I^T_{\mathcal{V}}\right\}$$

$$\mathcal{F}\left[\mathbf{P}^j\mathcal{N}^T\right] = \frac{i\mathbf{x}_{01}^j}{|\mathbf{x}_{01}|}\left\{\Omega^T_{\mathcal{N}}\,\kappa_z K_1\left(|\mathbf{x}_{01}|\kappa_z\right)+I^T_{\mathcal{N}}\right\}$$

$$\mathcal{F}\left[\left(\frac{\mathbf{P}^i\mathbf{P}^j}{\mathbf{P}^2}-\frac{\delta^{ij}}{2}\right)\mathcal{S}^T\right] = \frac{(1-z)}{2}\left[\frac{\mathbf{x}_{01}^i\mathbf{x}_{01}^j}{|\mathbf{x}_{01}|^2}-\frac{\delta^{ij}}{2}\right]\int_0^z\frac{\mathrm{d}\chi}{(1-\chi)}\int_0^\infty\frac{\mathrm{d}u}{(u+1)^2}|\mathbf{x}_{01}|\sqrt{\kappa_z^2+u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}$$
$$\times K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2+u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right)+[z\leftrightarrow 1-z].$$

$$\mathcal{F}\left[\mathcal{V}^T+\mathcal{M}^T-\frac{\mathcal{S}^T}{2}\right] = \left(\frac{\kappa_z}{2\pi|\mathbf{x}_{01}|}\right)^{\frac{D}{2}-2}\left\{\left[\frac{3}{2}+\log\left(\frac{\alpha}{z}\right)+\log\left(\frac{\alpha}{1-z}\right)\right]\left\{\frac{(4\pi)^{2-\frac{D}{2}}}{(2-\frac{D}{2})}\Gamma\left(3-\frac{D}{2}\right)+\log\left(\frac{|\mathbf{x}_{01}|^2\mu^2}{4}\right)\right.\right.$$
$$\left.\left.+2\gamma_E\right\}+\frac{1}{2}\frac{(D_s-4)}{(D-4)}\right\}K_{\frac{D}{2}-2}\left(|\mathbf{x}_{01}|\kappa_z\right)+\left\{3-\frac{\pi^2}{3}+\log^2\left(\frac{z}{1-z}\right)-\Omega^T_{\mathcal{V}}+L\right\}K_0\left(|\mathbf{x}_{01}|\kappa_z\right)+I^T_{\mathcal{VMS}},$$

$$\Omega^T_{\mathcal{V}} = -\left(1+\frac{1}{2z}\right)\left[\log(1-z)+\gamma\log\left(\frac{1+\gamma}{1+\gamma-2z}\right)\right]+\frac{1}{2z}\left[\left(z+\frac{1}{2}\right)(1-\gamma)+\frac{m^2}{Q^2}\right]\log\left(\frac{\kappa_z^2}{m^2}\right)+[z\leftrightarrow 1-z]$$

$$I^T_{\mathcal{V}} = \int_0^1\frac{\mathrm{d}\xi}{\xi}\left(\frac{2\log(\xi)}{(1-\xi)}-\frac{(1+\xi)}{2}\right)\left\{\sqrt{\kappa_z^2+\frac{\xi(1-z)}{(1-\xi)}m^2}K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2+\frac{\xi(1-z)}{(1-\xi)}m^2}\right)-[\xi\to 0]\right\}$$
$$-\int_0^1\mathrm{d}\xi\left(\frac{\log(\xi)}{(1-\xi)^2}+\frac{z}{(1-x)}+\frac{z}{2}\right)\frac{(1-z)m^2}{\sqrt{\kappa_z^2+\frac{\xi(1-z)}{(1-\xi)}m^2}}K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2+\frac{\xi(1-z)}{(1-\xi)}m^2}\right)$$
$$-\int_0^z\frac{\mathrm{d}\chi}{(1-\chi)}\int_0^\infty\frac{\mathrm{d}u}{u(u+1)}\frac{m^2}{\kappa_\chi^2}\left[2\chi+\left(\frac{u}{1+u}\right)^2\frac{1}{z}(z-\chi)(1-2\chi)\right]$$
$$\times\left\{\sqrt{\kappa_z^2+u\frac{(1-z)}{(1-\xi)}\kappa_\chi^2}K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2+u\frac{(1-z)}{(1-\xi)}\kappa_\chi^2}\right)-[u\to 0]\right\}$$
$$-\int_0^z\frac{\mathrm{d}\chi}{(1-\chi)^2}\int_0^\infty\frac{\mathrm{d}u}{(u+1)}(z-\chi)\left[1-\frac{2u}{1+u}(z-\chi)+\left(\frac{u}{1+u}\right)^2\frac{1}{z}(z-\chi)^2\right]$$
$$\times\frac{m^2}{\sqrt{\kappa_z^2+u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}}K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2+u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right)\quad+\quad[z\leftrightarrow 1-z].$$

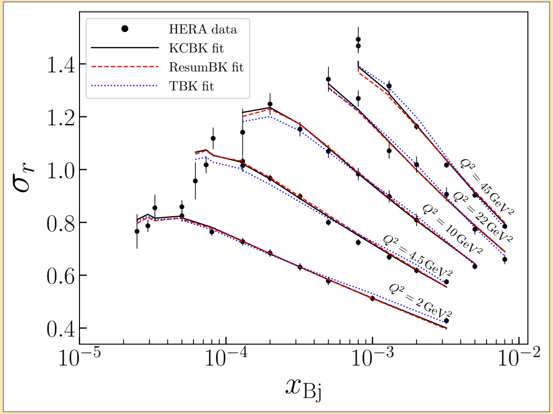
$$\Omega^T_{\mathcal{N}} = \frac{z+1-2z^2}{z}\left[\log(1-z)+\gamma\log\left(\frac{1+\gamma}{1+\gamma-2z}\right)\right]-\frac{(1-z)}{z}\left[\frac{2z+1}{2}(1-\gamma)+\frac{m^2}{Q^2}\right]\log\left(\frac{\kappa_z^2}{m^2}\right)-[z\leftrightarrow 1-z] \qquad (9)$$

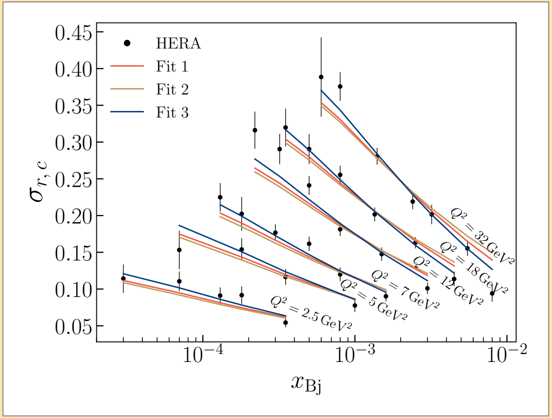
$$I^T_{\mathcal{N}} = \frac{2(1-z)}{z}\int_0^z\mathrm{d}\chi\int_0^\infty\frac{\mathrm{d}u}{(u+1)^3}\left\{\left[(2+u)uz+u^2\chi\right]\sqrt{\kappa_z^2+u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\,K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2+u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right)\right. \qquad (10)$$
$$\left.+\frac{m^2}{\kappa_\chi^2}\left(\frac{z}{1-z}+\frac{\chi}{1-\chi}[u-2z-2u\chi]\right)\left[\sqrt{\kappa_z^2+u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\,K_1\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2+u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right)-[u\to 0]\right]\right\}-[z\leftrightarrow 1-z].$$

$$I^T_{\mathcal{VMS}} = \int_0^1\frac{\mathrm{d}\xi}{\xi}\left(\frac{2\log(\xi)}{(1-\xi)}-\frac{(1+\xi)}{2}\right)\left\{K_0\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2+\frac{\xi(1-z)}{(1-\xi)}m^2}\right)-[\xi\to 0]\right\}$$
$$+\int_0^1\mathrm{d}\xi\left(-\frac{3(1-z)}{2(1-\xi)}+\frac{(1-z)}{2}\right)K_0\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2+\frac{\xi(1-z)}{(1-\xi)}m^2}\right)$$
$$+\int_0^z\frac{\mathrm{d}\chi}{(1-\chi)}\int_0^\infty\frac{\mathrm{d}u}{(u+1)^2}\left\{-z-\frac{u}{(1+u)}\frac{(z+u\chi)}{z}(\chi-(1-z))\right\}K_0\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2+u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right)$$
$$+\int_0^z\mathrm{d}\chi\int_0^\infty\frac{\mathrm{d}u}{(u+1)^3}\left\{\frac{\kappa_z^2}{\kappa_\chi^2}\left[1+u\frac{\chi(1-\chi)}{z(1-z)}\right]-\frac{m^2}{\kappa_\chi^2}\frac{\chi}{(1-\chi)}\left[2\frac{(1+u)^2}{u}+\frac{u}{z(1-z)}(z-\chi)^2\right]\right\}$$
$$\times\left\{K_0\left(|\mathbf{x}_{01}|\sqrt{\kappa_z^2+u\frac{(1-z)}{(1-\chi)}\kappa_\chi^2}\right)-[u\to 0]\right\}+[z\leftrightarrow 1-z].$$

Comparison to HERA data

Massless: G. Beuf, H. Hänninen, T. L. and H. Mäntysaari, [arXiv:2007.01645 [hep-ph]].
Massive: H. Hänninen, H. Mäntysaari, R. Paatelainen and J. Penttala, PRL **130** (2023) no.19, 19 [arXiv:2211.03504 [hep-ph]]
Combined fit H. Mäntysaari, C. Casuga, in prog.



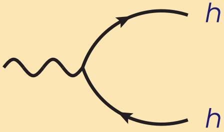


At NLO: dipole picture with BK evolution describes both F_2 and $F_2^{c\bar{c}}$ (did not work at LO)

Mass renormalization

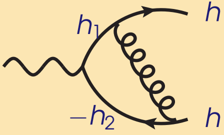
Vertex corrections to LC helicity flip vertex

Look at LC helicity flip part of LO vertex



$\sim m_q$

Vertex corrections from diagrams like



3 vertices

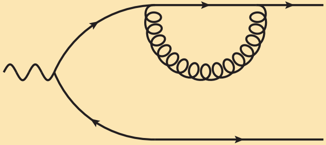
- Only divergence from 1 out of 3 diagrams flip (All 3 flip $\to$finite) :

$$\sim m_q \frac{1}{\varepsilon}$$

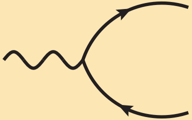
- Absorb into **vertex mass** counterterm δm_V,
  same as δm_q in conventional perturbation theory

(Corrections to nonflip vertex $\Longrightarrow$ massless-like, no mass renormalization)

Quark propagator corrections



Still divergent with 2 flip vertices (1 gives zero by symmetry)

- Loops give m_q-dependent divergence $\sim$  $\times \dfrac{m_q^2}{\Delta k^-_{\mathrm{LO}}} \dfrac{1}{\varepsilon}$
- Absorb into a renormalization of m_q^2 in ED of LO LCWF $(k_q^- = (\mathbf{k}_q^2 + m_q^2)/(2k_q^+))$
- But now the problem Known since 90's e.g. Haridranath, Zhang, also Burkardt in Yukawa th.
- In our regularization: k^+ cutoff, $\perp$ dim. reg.
  this **kinetic mass** counterterm is **not** same as the **vertex mass** one $\boxed{\delta m_k \neq \delta m_v}$
  (In fact δm_v is same as in covariant theory, δm_k different)
- So how to determine finite part of δm_v and δm_k?

Mass renormalization

- Mass has 2 conceptually different roles here:
  - Kinetic mass: relates energy and momentum
  - Vertex mass: amplitude of helicity flip in gauge boson vertex
- 1 parameter in Lagrangian, but 2 parameters in LCPT Hamiltonian
- Lorentz-invariance requires they stay the same
- Both gauge condition $A^+ = 0$ and regularization (k^+-cutoff and $\perp$ dim. reg.) violate rotational invariance $\Longrightarrow$ $m_v \neq m_k$ at loop level $\Longrightarrow$ "textbook stuff"

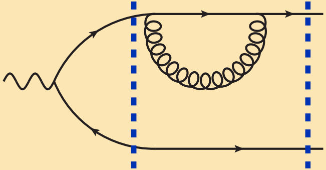
There are 3 options to deal with this

1. Renormalization conditions to set separately m_v and m_k $\Longrightarrow$ discuss next
2. Smartly combine with instantaneous "normal ordering" diagrams before regularizing & integrating $\Longrightarrow$ can keep $m_k = m_v$ but cannot calculate blindly

   For details see Beuf @ Hard Probes 2018
3. Use some other regularization $\Longrightarrow$ finite parts hard!

Two mass renormalization conditions

- Pole mass/on shell renormalization point:
  - Timelike virtual $\gamma^* \to q\bar{q}$ with $q^2 = M^2$ (Same diagrams as for spacelike γ^*)
  - On shell final state $M^2 = (\mathbf{k}_q^2 + m_q^2)/(z(1-z))$ (i.e. $ED_{\text{LO}} \to 0$)
- One condition: propagator diagram



is the most divergent at on-shell point $\Longrightarrow$ cancel this $\Longrightarrow$ kinetic mass

- Vertex mass (+ cross checks) from Lorentz-invariance
  Coefficients of 4 independent structures ($\mathbf{P} = (1-z)\mathbf{k}_q - z\mathbf{k}_{\bar{q}}$)

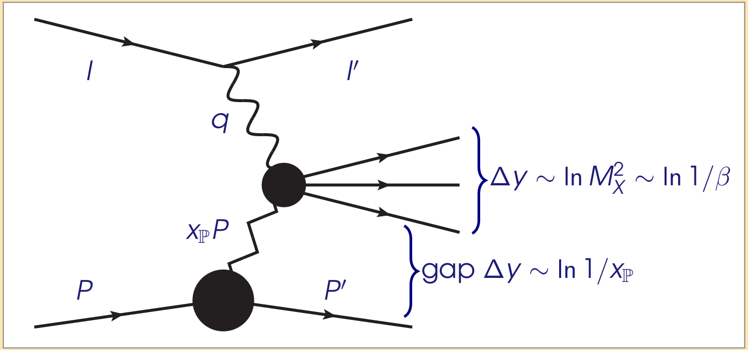
$$\bar{u}(0)\not{\varepsilon}_\lambda(q)v(1) \quad (\mathbf{P}\cdot\varepsilon_\lambda)\bar{u}(0)\gamma^+v(1) \quad \frac{(\mathbf{P}\cdot\varepsilon_\lambda)}{\mathbf{P}^2}\mathbf{P}^j\bar{u}(0)\gamma^+\gamma^jv(1) \quad \varepsilon_\lambda^j\bar{u}(0)\gamma^+\gamma^jv(1)$$

must reproduce 2 Lorentz-invariant form factors (Dirac & Pauli) @ on-shell point

Diffractive DIS

Inclusive diffraction, kinematics

$\gamma^* + A \to X + A$, differential in M_X



- Momentum transfer $t = (P - P')^2$
- Gap size $x_{\mathbb{P}}$, target evolution rapidity $\sim \ln 1/x_{\mathbb{P}}$
- Diffractive system mass M_X^2, $\beta = Q^2/(Q^2 + M_X^2)$
- Virtuality Q^2
- Lower $x_{\mathbb{P}}$ than dijets (e.g. at EIC)

$$x_{Bj} = x_{\mathbb{P}} \beta$$

This talk: $x_{\mathbb{P}}$ small, β not.

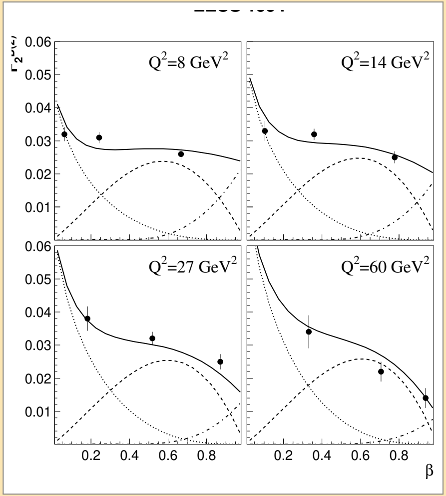
Dependence on β, i.e. M_X

${M_X}^2$ = photon remnants.

Essential regimes:

LO+NLO
- Large $\beta \to 1$ — small M_X:
  longitudinal $\gamma^* \to q\bar{q}$
- Medium $\beta \sim 0.5$ — ${M_X}^2 \sim Q^2$:
  transverse $\gamma^* \to q\bar{q}$

NLO
- Small $\beta \ll 1$ — large ${M_X}^2$:
  higher Fock states ($q\bar{q}g$ etc.)



LO $q\bar{q}$ and leading ln Q^2 $q\bar{q}g$

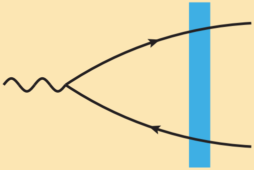
Golec-Biernat& Wüsthoff `hep-ph/9903358`

NLO amplitude for diffractive DIS

Diffractive DIS at leading order

- Full kinematics and impactg parameter dependence
  G. Beuf, H. Hänninen, T.L., Y. Mulian, H. Mäntysaari, arXiv:2206.13161

$$\frac{d\sigma^{D}_{\lambda, q\bar{q}}}{dM_X^2 \, d|t|} = \frac{N_c}{4\pi} \int_0^1 dz \int_{\mathbf{x}_0 \mathbf{x}_1 \bar{\mathbf{x}}_0 \bar{\mathbf{x}}_1} \mathcal{I}^{(2)}_{\boldsymbol{\Delta}} \mathcal{I}^{(2)}_{M_X}$$
$$\times \sum_{f, h_0, h_1} \left(\widetilde{\psi}_{\gamma^*_\lambda \to q_{\bar{0}} \bar{q}_{\bar{1}}}\right)^\dagger \left(\widetilde{\psi}_{\gamma^*_\lambda \to q_0 \bar{q}_1}\right) \boxed{\left[S^\dagger_{\bar{0}\bar{1}} - 1\right]\left[S_{01} - 1\right]}$$



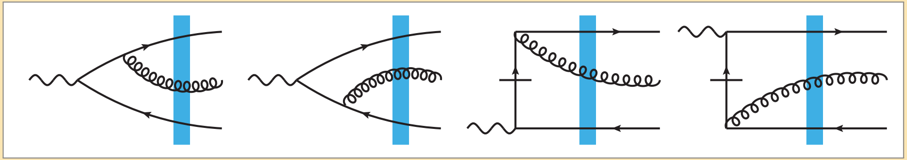
(Blue: shockwave target)

- $q\bar{q}$ crossing shockwave: dipole S_{01}
- "Transfer functions:" relate coordinates at shockwave to:
  - Momentum transfer $t = -\boldsymbol{\Delta}^2$ $\mathcal{I}^{(2)}_{\boldsymbol{\Delta}} = \frac{1}{4\pi} J_0 \left(\sqrt{|t|}\, \|z\mathbf{x}_{\bar{0}0} - (1-z)\mathbf{x}_{\bar{1}1}\|\right)$
  - Invariant mass $\mathcal{I}^{(2)}_{M_X} = \frac{1}{4\pi} J_0 \left(\sqrt{z(1-z)} M_X \|\bar{\mathbf{r}} - \mathbf{r}\|\right)$

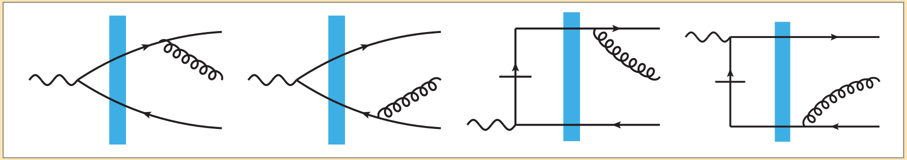
Now to NLO

NLO radiative corrections

- Emission before target

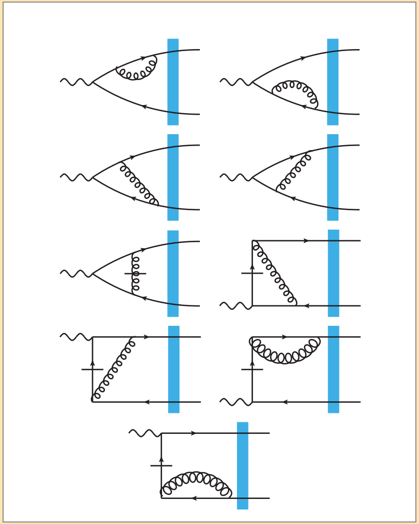


  - Squares already in G. Beuf, H. Hänninen, T.L., Y. Mulian, H. Mäntysaari arXiv:2206.13161
  - Contain leading $\ln Q^2$ contribution
- Emission after target



- Interferences $\Longrightarrow$ simplify with some of the virtual corrections

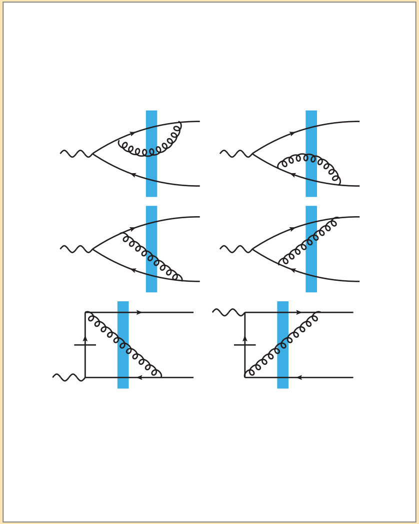
NLO virtual



- Vertex corrections:
  known 1-loop $\gamma \to q\bar{q}$ wavefunction

See also Boussarie et al 2014: diffractive jets,
also Caucal et al 2021 inclusive

NLO virtual



- Vertex corrections:
  known 1-loop $\gamma \to q\bar{q}$ wavefunction
- Gluon crosses shockwave, but not the cut:
  - Loop corrections to amplitude,
    tree level wavefunctions
  - 3-point operator of Wilson lines
  - BK/JIMWLK evolution of LO amplitude

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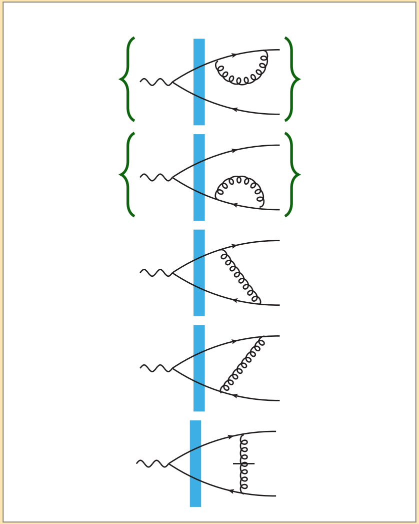
NLO virtual



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- Gluon crosses shockwave, but not the cut:
  - Loop corrections to amplitude,
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  - 3-point operator of Wilson lines
  - BK/JIMWLK evolution of LO amplitude
- Final state interactions
  (Propagator corrections $\{\} \to$
  State renormalization, in fact = 0 in dim. reg.)

See also Boussarie et al 2014: diffractive jets,
also Caucal et al 2021 inclusive

NLO diffractive DIS cross section

Calculation in 2401.17251 [hep-ph]

Beuf, T.L. , Paatelainen, Mäntysaari, Penttala

We have calculated all these contributions

- Diffractive structure function:
  clean IR-safe, [perturbative = experimental] final state definition M_X!
  (No fragmentation function, jet definition)
  $\Longrightarrow$ Divergences must cancel
- Explicit expressions will not fit in the slides, but there in 2401.17251 [hep-ph]

Features of the calculation:

- Divergence structure
- Treatment of energy denominators
- Collinear factorization limit (Wüsthoff limit)

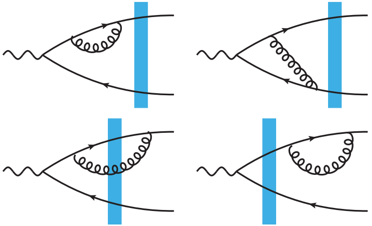
Regularization and divergences

Regularization procedure

- Transverse momentum in $2-2\varepsilon$ dimensions $\Longrightarrow$ $\frac{1}{\varepsilon}$ divergences, collinear or UV
- Longitudinal k^+: cutoff $k^+ > \alpha$, $\alpha \to 0$ $\Longrightarrow$ $1/\alpha$, $\ln^2\alpha$, $\ln\alpha$ divergences

1. UV $\frac{1}{\varepsilon}$ and $\frac{1}{\varepsilon}\ln\alpha$ divergences: $\gamma^* \to q\bar{q}$ vertex, gluon crossing shock, wavefunction renormalization

1. UV $\frac{1}{\varepsilon}$ e.g. in



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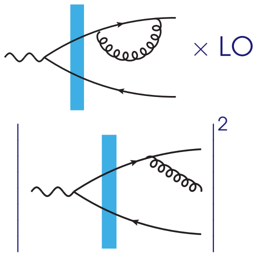
2. Collinear $\frac{1}{\varepsilon}$:
   wavef. renormalization, final state emission

2. Collinear $\frac{1}{\varepsilon}$ e.g. in



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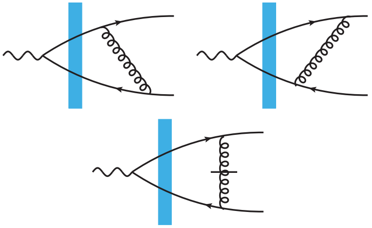

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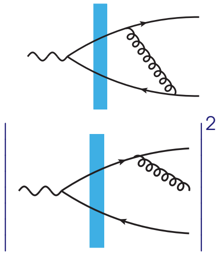
4. $\ln^2\alpha$ from final state exchange and emission
   (M_X restriction matters here!)

4. $\ln^2\alpha$ e.g. in



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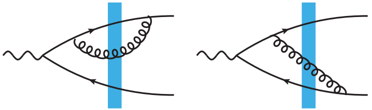
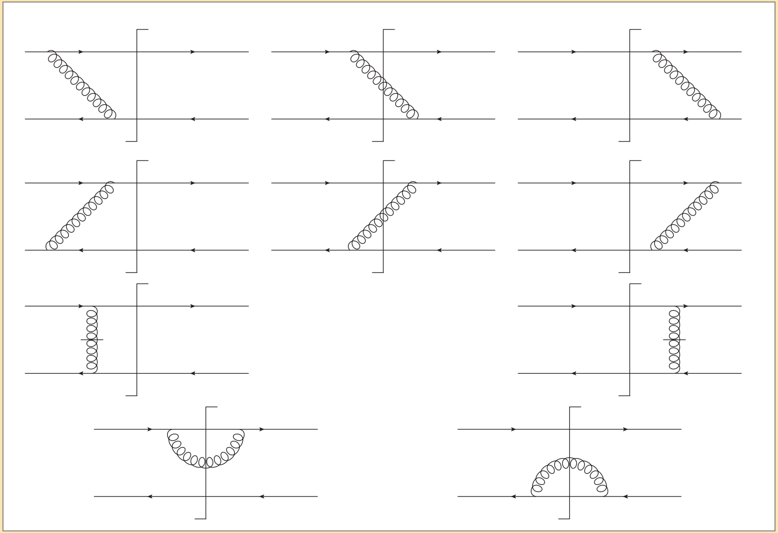
   wavef. renormalization, final state emission
3. $1/\alpha$ cancels between normal and instantaneous exchange
4. $\ln^2\alpha$ from final state exchange and emission
   (M_X restriction matters here!)
5. Remaining $\ln\alpha$ absorbed into BK/JIMWLK

5. $\ln\alpha$ e.g. in



Final state corrections

How to dig out different types of divergences? Penttala



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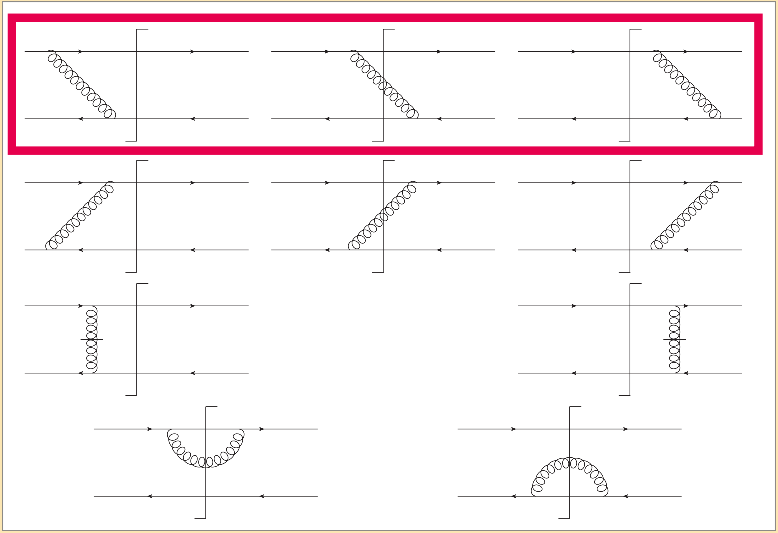
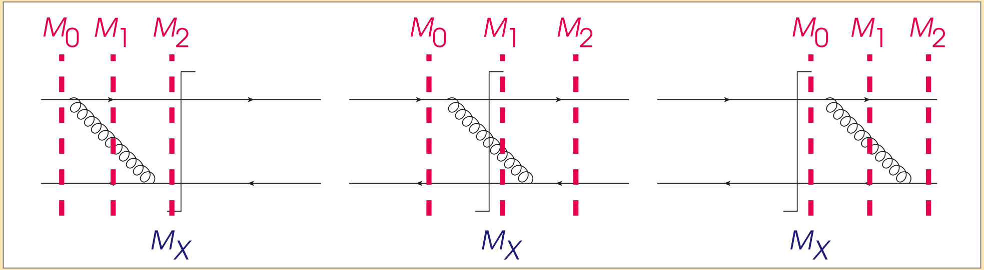


As an example: consider 1st row

Combine energy denominators

"Beuf trick": write M_X delta function as imaginary part of "propagator"



$$\frac{\delta({M_X}^2 - M_2^2)}{(M_2^2 - M_1^2 + i\delta)(M_2^2 - M_0^2 + i\delta)} + \frac{\delta({M_X}^2 - M_1^2)}{(M_1^2 - M_0^2 + i\delta)(M_1^2 - M_2^2 - i\delta)} + \frac{\delta({M_X}^2 - M_0^2)}{(M_0^2 - M_1^2 - i\delta)(M_0^2 - M_2^2 - i\delta)}$$

(Note: sign of $i\delta$ essential)

$$= \frac{1}{2\pi i}\left[\frac{1}{({M_X}^2 - M_0^2 - i\delta)({M_X}^2 - M_1^2 - i\delta)({M_X}^2 - M_2^2 - i\delta)} - \text{c.c.}\right]$$

- Then express numerator (⊥ momentum dot products) in terms of M_0^2, M_1^2, M_2^2
- Combine before integration $\Longrightarrow$ separate different divergence types

Wüsthoff limit

Large Q^2

Recover "Wüsthoff result" (origin somewhat mysterious)

$$x_{\mathbb{P}} F^{\mathrm{D\,(GBW)}}_{T,q\bar{q}g} = \frac{\alpha_s \beta}{8\pi^4} \sum_f e_f^2 \int_{\mathbf{b}} \int_{\beta}^{1} \mathrm{d}z \left[\left(1 - \frac{\beta}{z}\right)^2 + \left(\frac{\beta}{z}\right)^2 \right] \int_0^{Q^2} \mathrm{d}k^2 k^4 \ln \frac{Q^2}{k^2}$$

$$\times \left[\int_0^{\infty} \mathrm{d}r r K_2(\sqrt{z} k r) J_2(\sqrt{1-z} k r) \frac{\mathrm{d}\tilde{\sigma}_{\mathrm{dip}}}{\mathrm{d}^2\mathbf{b}} (\mathbf{b}, \mathbf{r}, x_{\mathbb{P}}) \right]^2 .$$

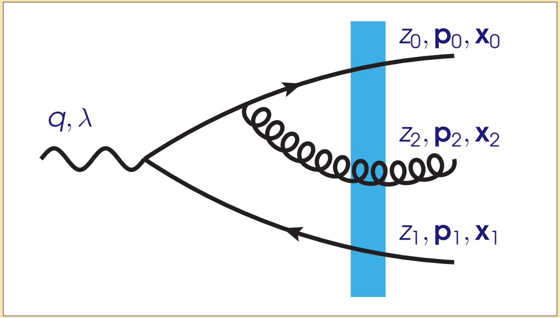
- Explicit $\ln Q^2$
- $g \to q\bar{q}$ DGLAP splitting function: target evolution
- Color-octet small-size $q\bar{q}$ is "effective gluon" $\tilde{g}$ $\Longrightarrow$ adjoint dipole $\Longrightarrow$ diffractive gluon PDF
- J_2, K_2 from curious "effective gluon wavefunction"

$$\psi^{\gamma \to g\tilde{g}} \sim k^i k^j - \frac{1}{2} \mathbf{k}^2 \delta^{ij}$$

Deriving large Q^2 limit: aligned jet limit

Leading large Q^2 from:

- $z_0 \gg z_1 \gg z_2$
- $\mathbf{p}_0^2 \sim \mathbf{p}_1^2 \gg \mathbf{p}_2^2$
- $p_0^- \sim p_1^- \sim p_2^-$
  $\Longrightarrow$ Wüsthoff momentum fractions

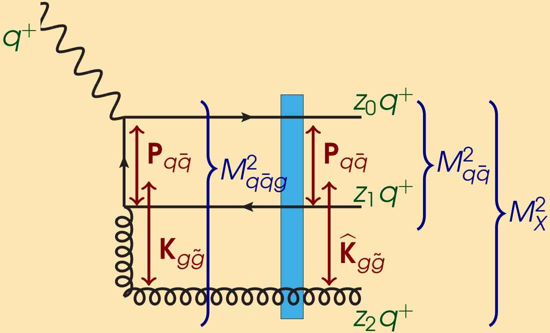


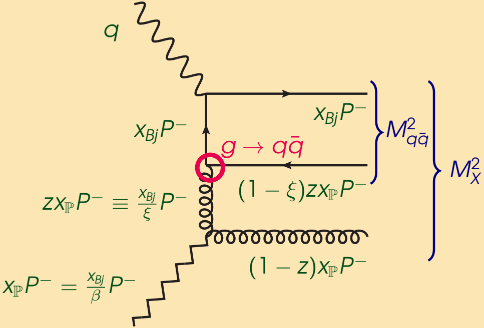
Consistently taking this limit derivation is straightforward:

- q and $\bar{q}$ close: not resolved by target $\Longrightarrow$ point-like "effective gluon" $\tilde{g}$
- Consequently relative momentum of $q\bar{q}$ pair does not change in shockwave only relative momentum of $g\tilde{g}$
- Explicit log from $q\bar{q}$ internal dynamics, $g \to q\bar{q}$ (!) splitting function
- Rank-2 tensor for $\gamma^* \to g\tilde{g}$ Edmond and Dionysios had a much more elegant way!

Deriving large Q^2 limit: matching

Identification with collinear variables via invariant masses





- $\mathbf{K}_{g\tilde{g}}$: before shock, Fourier-transform
- $\widehat{\mathbf{K}}_{g\tilde{g}}$: final state, fixed

$$M^2_{q\bar{q}} \approx \mathbf{P}^2_{q\bar{q}}/z_1 \approx (1/\xi - 1)Q^2$$

$$M^2_{q\bar{q}g} \approx M^2_{q\bar{q}} + \mathbf{K}^2_{g\tilde{g}}/z_2 \qquad M^2_X = (1/\beta - 1)Q^2 \approx M^2_{q\bar{q}} + \widehat{\mathbf{K}}^2_{g\tilde{g}}/z_2$$

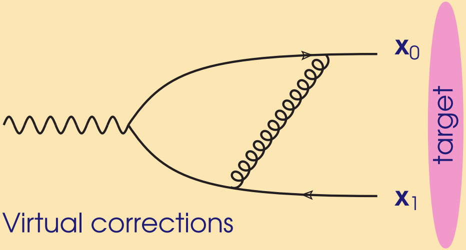
Conclusions

- High energy scattering of dilute probe off strong color fields:
  - Target: classical color field
  - Probe: virtual photon,
    develop in a Fock state expansion in Light Cone Perturbation Theory
- Total cross section to one-loop order for massive quarks
- Inclusive diffraction at one loop: key piece of EIC physics

Thank you

DIS at NLO: real and virtual corrections

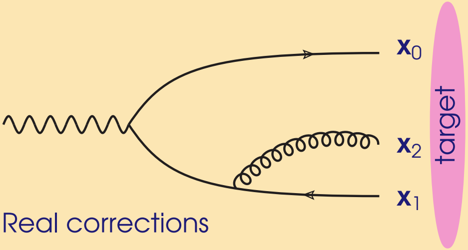
Here example diagams only



Virtual corrections

$$\mathcal{N}_{q\bar{q}}(\mathbf{x}_0, \mathbf{x}_1)$$

+ UV divergence in loop



Real corrections

$$\mathcal{N}_{q\bar{q}g}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2)$$

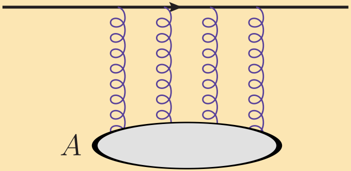
UV (!) divergence in $\mathbf{x}_2$-integral

UV-divergences cancel because for Wilson lines $\in SU(N_c)$

$$\mathcal{N}_{q\bar{q}g}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2 \to \mathbf{x}_0) = \mathcal{N}_{q\bar{q}g}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2 \to \mathbf{x}_1) = \mathcal{N}_{q\bar{q}}(\mathbf{x}_0, \mathbf{x}_1)$$

(Why UV, not IR? . . .)

Eikonal scattering off target of glue



How to measure small-x glue?

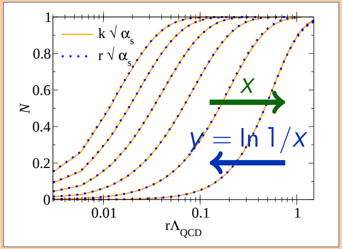
- Dilute probe through target color field
- At high energy interaction is **eikonal**,
  $\mathbf{x}$ (2d $\perp$ coordinate) conserved in scattering
  (T-matrix diagonal in $\perp$ coordinate space)

- Amplitude for quark: **Wilson line**

$$\mathbb{P}\exp\left\{-ig\int^{x^+} dy^+ A^-(y^+, x^-, \mathbf{x})\right\} \underset{x^+\to\infty}{\approx} V(\mathbf{x}) \in \mathrm{SU}(N_c)$$

- Amplitude for color dipole

$$\mathcal{N}(r = |\mathbf{x}-\mathbf{y}|) = 1 - \left\langle \frac{1}{N_c} \operatorname{tr} V^\dagger(\mathbf{x}) V(\mathbf{y}) \right\rangle$$



- $r = 0$: color transparency, $r \gg 1/Q_s$: **saturation** , nonperturbative!

DIS at NLO: Fock state expansion

- Fock state decomposition of $|\gamma_\lambda(\vec{q}, Q^2)\rangle_i$ (and ${}_i\langle\gamma_\lambda(\vec{q}', Q^2)|$) up to g^2:

$$|\gamma_\lambda(\vec{q}, Q^2)\rangle_i = \sqrt{Z_{\gamma^*}}\Big[|\gamma_\lambda(\vec{q}, Q^2)\rangle + \sum_{q\bar{q}} \Psi^{\gamma^*\to q\bar{q}}|q(\vec{k}_0)\bar{q}(\vec{k}_1)\rangle$$
$$+ \sum_{q\bar{q}g} \Psi^{\gamma^*\to q\bar{q}g}|q(\vec{k}_0)\bar{q}(\vec{k}_1)g(\vec{k}_2)\rangle + \cdots\Big]$$

  with **Light Cone Wave Functions** $\Psi^{\gamma^*\to q\bar{q}}$ **and** $\Psi^{\gamma^*\to q\bar{q}g}$
- Fourier-transform to $\perp$ coordinate: (eikonal scattering)
- Scattering off target: Wilson line correlators

$$\hat{S}_E|q(\mathbf{x})\bar{q}(\mathbf{y})\rangle = V(\mathbf{x})V^\dagger(\mathbf{y})|q(\mathbf{x})\bar{q}(\mathbf{y})\rangle$$
$$\hat{S}_E|q(\mathbf{x})\bar{q}(\mathbf{y})g(\mathbf{z})\rangle = V(\mathbf{x})V^\dagger(\mathbf{y})V_{\text{adj}}(\mathbf{z})|q(\mathbf{x})\bar{q}(\mathbf{y})g(\mathbf{z})\rangle$$

Heavy quarks, motivation, issues

- Data
  - HERA F_2^c
  - Charm big part of EIC program
- LO F_2^c problematic in existing fits
  Dirty little secret: heavy quarks in LO rcBK fits do not actually work!

LCPT loops with massive quarks are so much fun!

- Working with fixed helicity states (not Dirac traces=sums) : physics very explicit
- New Lorentz structures $\Longrightarrow$ rotational invariance constraints

Approach for this talk: same regularization as in massless case

- Cutoff in k^+
- $\perp$ dim. reg.

(Recall: Hamiltonian perturbation theory, k^--integrals already done)

Dirac and Pauli form factors

$$\Gamma^{\mu}(q) = F_D(q^2/m^2)\,\gamma^{\mu} + F_P(q^2/m^2)\,\frac{q_{\nu}}{2m}\,i\sigma^{\mu\nu}$$

$$\Psi_{\mathrm{LO}}^{\gamma_T^*\to q\bar{q}}+\Psi_{\mathrm{NLO}}^{\gamma_T^*\to q\bar{q}} = \delta_{\alpha_0\alpha_1}\frac{ee_f}{\mathrm{ED_{LO}}}\Bigg\{\bar{u}(0)\not{\varepsilon}_{\lambda}(q)v(1)\left[1+\left(\frac{\alpha_s C_\mathrm{F}}{2\pi}\right)\mathcal{V}^T\right]+\frac{q^+}{2k_0^+k_1^+}(\mathbf{P}\cdot\boldsymbol{\varepsilon}_{\lambda})\bar{u}(0)\gamma^+v(1)\left(\frac{\alpha_s C_\mathrm{F}}{2\pi}\right)\mathcal{N}^T$$

$$+\frac{q^+}{2k_0^+k_1^+}\frac{(\mathbf{P}\cdot\boldsymbol{\varepsilon}_{\lambda})}{\mathbf{P}^2}\mathbf{P}^j m\bar{u}(0)\gamma^+\gamma^j v(1)\left(\frac{\alpha_s C_\mathrm{F}}{2\pi}\right)\mathcal{S}^T+\frac{q^+}{2k_0^+k_1^+}m\bar{u}(0)\gamma^+\not{\varepsilon}_{\lambda}(q)v(1)\left(\frac{\alpha_s C_\mathrm{F}}{2\pi}\right)\mathcal{M}^T\Bigg\}.$$

$$-\left(\frac{\alpha_s C_\mathrm{F}}{2\pi}\right)\frac{m^2}{\mathbf{P}^2}\,\mathcal{S}^T\bigg|_{\mathbf{P}^2=-\overline{Q}^2-m^2} = F_P(q^2/m^2)$$

$$-\left(\frac{\alpha_s C_\mathrm{F}}{2\pi}\right)\frac{1}{(2z-1)}\,\mathcal{N}^T\bigg|_{\mathbf{P}^2=-\overline{Q}^2-m^2} = F_P(q^2/m^2)$$

$$\left(\frac{\alpha_s C_\mathrm{F}}{2\pi}\right)\mathcal{V}^T\bigg|_{\mathbf{P}^2=-\overline{Q}^2-m^2} = -1+F_D(q^2/m^2)+F_P(q^2/m^2)$$

$$\mathcal{M}^T\bigg|_{\mathbf{P}^2=-\overline{Q}^2-m^2} = 0$$

Fits to HERA data

G. Beuf, H. Hänninen, T. L. and H. Mäntysaari, (arXiv:2007.01645 (hep-ph)).

Actually many fits. Choices:

- Evolution equation: not full NLO but resummed, running coupling LO, (very good approximation of full NLO (arXiv:1601.06598 (hep-ph)).)
  1. Kinematically constrained BK, nonlocal in $Y \sim \ln k^+$
     Beuf (arXiv:1401.0313 (hep-ph))
  2. Resummed BK , local in $Y \sim \ln k^+$ E. Iancu, J. D. Madrigal, A. H. Mueller, G. Soyez and D. N. Triantafyllopoulos, (arXiv:1502.05642 (hep-ph)).
  3. Target rapidity resummed BK, nonlocal in $\eta = Y - \ln 1/r^2$ B. Ducloue, E. Iancu, A. H. Mueller, G. Soyez and D. N. Triantafyllopoulos, (arXiv:1902.06637 (hep-ph)) ,
- What comes before initial condition?
  - Look at data $x_{Bj} < 0.01$
  - But cross section & (nonlocal in rapidity) evolution need values up to $x = 1$
  - Either 1. Freeze above $x = 0.01$ or 2. start BK evolution at $x = 1$
- Running coupling:
  1. Parent dipole
  2. Balitsky for LO-like part, smallest dipole for NLO

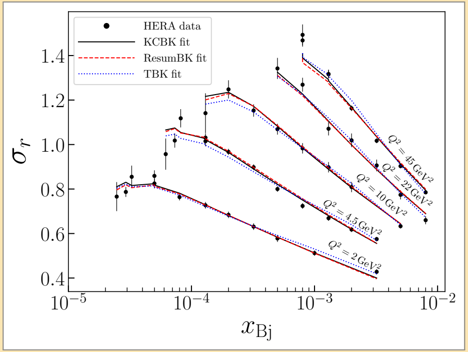
Messages from fits

Free parameters:

- σ_0: proton area
- Q_{s0}: initial saturation scale
- γ shape of initial condition as function of r
- C^2: scale of α_s as function of r
  (could think of as fitting α_s or Λ_{QCD})

Main conclusions

- Fits are very good, χ^2/N varies 1.03 . . . 2.77
- Different BK-eqs equally good
  (Differences absorbed in initial conditions) .
  Similar to finding of Albacete 2015
  Only see differences at LHeC kinematics
- Generally prefer smallish σ_0



Unsubtracted cross section, N_c-term

Discussion here following Iancu et al 2016 leave out C_F/DGLAP-terms

$$\frac{\mathrm{d}N^{\mathrm{LO}+N_c}}{\mathrm{d}^2\mathbf{k}\,\mathrm{d}y} \sim \mathcal{S}_0(k_T) + \alpha_s \int_0^{1-x_g/x_0} \frac{\mathrm{d}\xi}{1-\xi} \mathcal{K}(k_T, \xi, X(\xi))$$

- Dipole operator $\mathcal{S}_0$ is "bare"
- Rapidity at which dipoles are evaluated $X(\xi)$
- x_g: the target momentum fraction for LO kinematics
- Multi-Regge-kinematics: $X(\xi) = x_g/(1-\xi)$
- Only target $X(\xi) < x_0 \implies$ phase sp. limit $\xi < 1 - x_g/x_0$:

$$\text{BK:} \quad \mathcal{S}(k_T, x_g) = \mathcal{S}(k_T, x_0) + \alpha_s \int_0^{1-x_g/x_0} \frac{\mathrm{d}\xi}{1-\xi} \mathcal{K}(k_T, 1, X(\xi))$$

Combine these, taking $\mathcal{S}(k_T, x_0) \equiv \mathcal{S}_0(k_T) \ldots$

Subtracted form for cross section

Unsubtracted form

$$\mathcal{S}_0(k_T) + \alpha_s \int_0^{1-x_g/x_0} \frac{d\xi}{1-\xi} \mathcal{K}(k_T, \xi, X(\xi))$$

$$= \mathcal{S}(k_T, x_g) + \alpha_s \int_0^{1-x_g/x_0} \frac{d\xi}{1-\xi} \left[\mathcal{K}(k_T, \xi, X(\xi)) - \mathcal{K}(k_T, 1, X(\xi))\right]$$

subtracted form

(Recall: dipoles evaluated at rapidity $X(\xi)$)

- These are strictly equivalent, perfectly positive at all k_T
- Subtracted form is a true perturbative series
  unsubtracted has $\alpha_s \ln 1/x$ and α_s together

Origin of negativity in CXY

$$\frac{\mathrm{d}N^{\mathrm{LO}+N_c}}{\mathrm{d}^2\mathbf{k}\,\mathrm{d}y} \sim \mathcal{S}(k_T, x_g) + \alpha_s \int_0^{1-x_g/x_0} \frac{\mathrm{d}\xi}{1-\xi} \left[\mathcal{K}(k_T, \xi, X(\xi)) - \mathcal{K}(k_T, 1, X(\xi))\right]$$

How do CXY get a negative cross section?

- $\mathcal{K}(k_T, \xi, X(\xi)) - \mathcal{K}(k_T, 1, X(\xi))$ dominated by $\xi \ll 1$
- Replace $X(\xi) \to X(\xi = 0) = x_g$
- Change ξ integration limit to 1 (+ distribution!)

This gives CXY subtraction scheme

$$\frac{\mathrm{d}N^{\mathrm{LO}+N_c}}{\mathrm{d}^2\mathbf{k}\,\mathrm{d}y} \sim \mathcal{S}(k_T, x_g) + \alpha_s \int_0^1 \frac{\mathrm{d}\xi}{1-\xi} \Big[\overbrace{\mathcal{K}(k_T, \xi, x_g)}^{\sim \xi/{k_T}^4 \text{ for } k_T \gg Q_s} - \mathcal{K}(k_T, 1, x_g)\Big]$$

- Formally ok in α_s expansion
- Nice factorized form: only dipoles at x_g, like LO
- But subtraction no longer integral form of BK

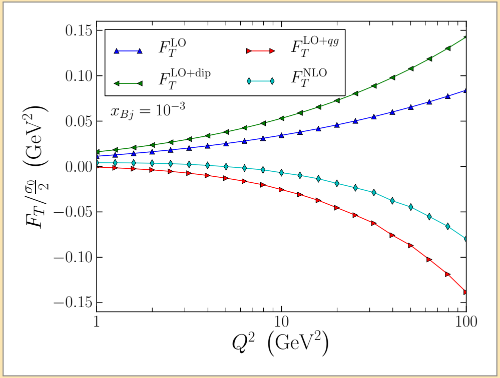
Numerical implementation

Ducloué, Hänninen, T.L., Zhu 2017

$$\sigma_{L,T}^{qg,\text{sub.}} \sim \alpha_s C_F \int_{z_1,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \int_{x_{Bj}/x_0}^{1} \frac{\mathrm{d}z_2}{z_2} \left[\mathcal{K}_{L,T}^{\text{NLO}}\left(z_2, X(z_2)\right) - \mathcal{K}_{L,T}^{\text{NLO}}\left(0, X(z_2)\right) \right].$$

- Target fields at scale $X(z_2)$:
  - $X(z_2) = x_{Bj}$: unstable

(like single inclusive)



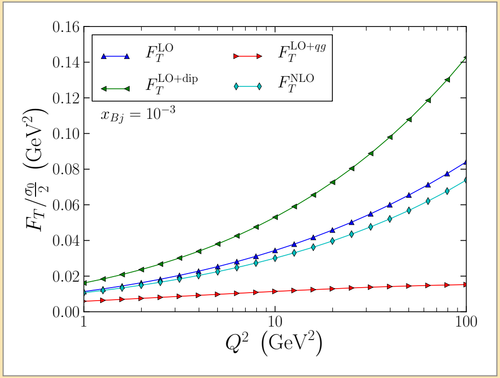
$X(z_2) = x_{Bj}$

Numerical implementation

Ducloué, Hänninen, T.L., Zhu 2017

$$\sigma_{L,T}^{qg,\text{sub.}} \sim \alpha_s C_\mathrm{F} \int_{z_1,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \int_{x_{Bj}/x_0}^{1} \frac{\mathrm{d}z_2}{z_2} \left[\mathcal{K}_{L,T}^{\mathrm{NLO}}(z_2, X(z_2)) - \mathcal{K}_{L,T}^{\mathrm{NLO}}(0, X(z_2)) \right].$$

- Target fields at scale $X(z_2)$:
  - $X(z_2) = x_{Bj}$: unstable (like single inclusive)
  - $X(z_2) = x_{Bj}/z_2$ OK



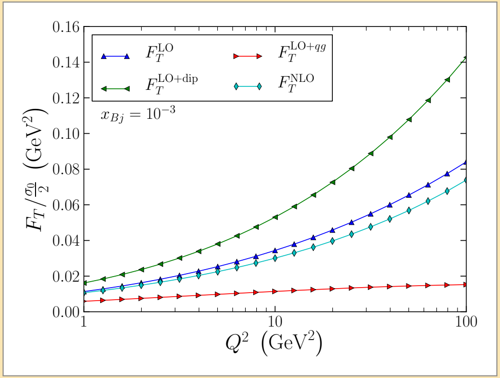
$X(z_2) = x_{Bj}/z_2$

Numerical implementation

Ducloué, Hänninen, T.L., Zhu 2017

$$\sigma_{L,T}^{qg,\text{sub.}} \sim \alpha_s C_F \int_{z_1,\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2} \int_{x_{Bj}/x_0}^{1} \frac{dz_2}{z_2} \left[\mathcal{K}_{L,T}^{\text{NLO}}(z_2, X(z_2)) - \mathcal{K}_{L,T}^{\text{NLO}}(0, X(z_2)) \right].$$

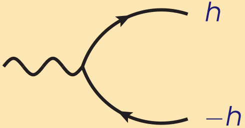
- Target fields at scale $X(z_2)$:
  - $X(z_2) = x_{Bj}$: unstable (like single inclusive)
  - $X(z_2) = x_{Bj}/z_2$ OK
- Lower limit of z_2
  - $z_2 > \frac{x_{Bj}}{x_0}$ from target k^- (assuming $k_T{}^2 \sim Q^2$)
  - Strict k^+ factorization: $z_2 > \frac{x_{Bj}}{x_0} \frac{M_p^2}{Q^2}$ $\Longrightarrow$ would require kinematical constraint
  - For "dipole" term integrate to $z_2 = 0$



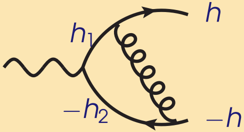
$X(z_2) = x_{Bj}/z_2$

Vertex corrections to non-flip vertex

Non-flip part of LO vertex



Corrections from



$\Longrightarrow$ again 2 options

- no flip vertex: $h_1 = h$, $h_2 = h_1$ and $h_2 \neq -h$
  vertices as in massless theory $\Longrightarrow$ not new contribution
- 2 flip + 1 non-flip $h_1 = h$ or $h_2 = h_1$ or $h_2 = -h$
  $\Longrightarrow$ again finite NLO contribution
  (2 ED's $\sim \mathbf{k}^2$ each, 1 vertex $\sim \mathbf{k}$, finite integral $\sim \int d^2\mathbf{k} \frac{\mathbf{k}}{((\mathbf{k}-\dots)^2+\dots)((\mathbf{k}-\dots)^2+\dots)}$)

Large M_X

$$x_{\mathbb{P}} F^{\mathrm{D}\,(\mathrm{MS})}_{T,q\bar{q}g}(x_{\mathbb{P}}, \beta = 0, Q^2) = \frac{\alpha_s N_c C_F Q^2}{16\pi^5 \alpha_{\mathrm{em}}} \int \mathrm{d}^2\mathbf{x}_0 \int \mathrm{d}^2\mathbf{x}_1 \int \mathrm{d}^2\mathbf{x}_2 \int_0^1 \frac{\mathrm{d}z}{z(1-z)} \left| \widetilde{\psi}^{\mathrm{LO}}_{\gamma^*_\lambda \to q_{\bar{0}} \bar{q}_{\bar{1}}} \right|^2 \times \frac{\mathbf{x}_{01}^2}{\mathbf{x}_{02}^2 \mathbf{x}_{12}^2} \left[N_{02} + N_{12} - N_{01} - N_{02} N_{12} \right]^2 .$$

- LO $\gamma \to q\bar{q}$ wavefunction
- BK kernel for $q\bar{q} \to q\bar{q}g \approx q\bar{q}q\bar{q}$
- Squared BK Wilson line operator ($N_{ij} = 1 - S_{ij}$)

Obtained by:

- Approximate gluon soft $z_2 \to 0$, $M_X{}^2 \sim 1/z_2$
- Remove M_X by $\int \mathrm{d}z_2 \delta(M_X{}^2 - \mathbf{p}_2^2/z_2)$
- Unconstrained final state integration introduced divergence: cure by including final state emissions