Jet quenching in dense QCD media and connections with small-x physics

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Workshop *"Probing the CGC and QCD matter at hadron colliders"*,
Firenze, March 24, 2025

Outline of this talk

- Jet quenching in heavy-ion collisions.
  See also talks by Xin-Nian just after and Yacine on Wednesday.

- Formal point of view: essentially a talk on QCD evolution equations in the presence of a saturation boundaries in phase space:

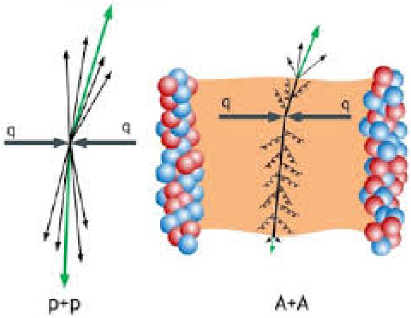
  1. Final state jet evolution in dense QCD media.
  2. System size dependence of the quenching parameter $\hat{q}$.

- Opportunity to celebrate Edmond's contributions on these topics.

Jet quenching in heavy-ion collisions

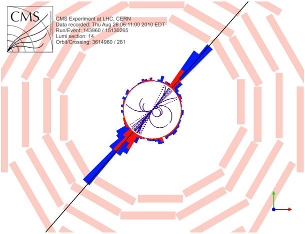
Jets as hard probes

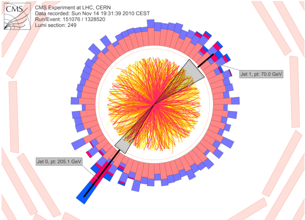
- A hard scattering produces a pair of highly energetic partons.
- The subsequent evolution of the parton $\Rightarrow$ jets.

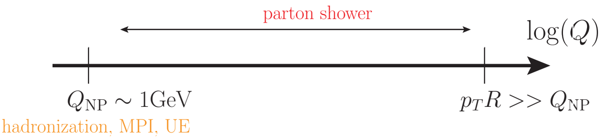


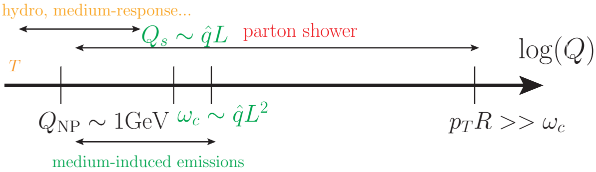
- In PbPb, interaction with the plasma during propagation.

Jets in *pp* vs jets in heavy-ions collisions









A complicated physical system

Jets are sensitive to a broad range of scales and thus to many medium-induced mechanisms.

pQCD picture of jet fragmentation in dense QCD media

work with **Edmond Iancu**, Al Mueller and Gregory Soyez

Key idea: use an approximation consistent with pQCD series

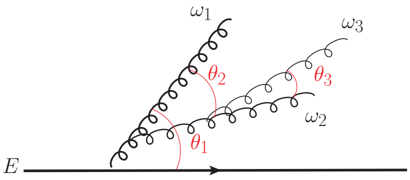
- Rely on a suitable approximation under pQCD control.
- Most simple approximation: **double logarithmic limit**!
- A common limit to DGLAP and BFKL evolution.

How does a vacuum jet look like within the DLA?

- **Vacuum-like emissions** (VLEs) = Bremsstrahlung triggered by the virtuality:

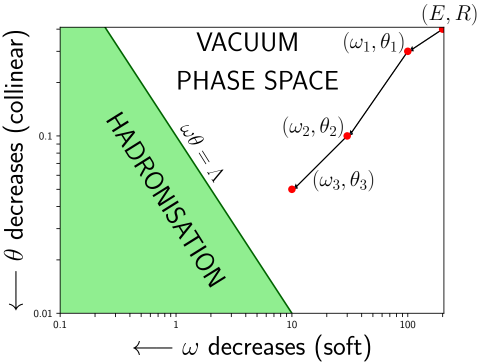
$$d^2\mathcal{P}_{\rm vle} \simeq \frac{\alpha_s C_R}{\pi}\frac{d\omega}{\omega}\frac{d\theta^2}{\theta^2}$$

- Duration of the process: $t_f \sim 1/(\omega\theta^2)$.
- Markovian process with angular ordering to account for **quantum** interferences.



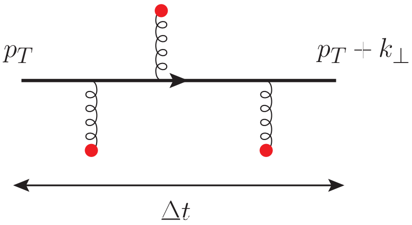
$\theta_1 \gg \theta_2 \gg \theta_3$

$\omega_1 \gg \omega_2 \gg \omega_3$



Parton propagation in dense media

(1) Transverse momentum broadening: $\langle k_\perp^2 \rangle = \hat{q}\Delta t$



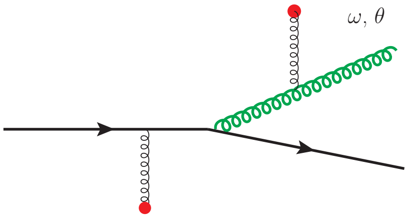
Parton propagation in dense media

(1) Transverse momentum broadening: $\langle k_\perp^2 \rangle = \hat{q}\Delta t$

(2) Medium-induced emissions. Baier, Dokshitzer, Mueller, Peigne, Schiff, 1997 - Zakharov,1997

$$d^3\mathcal{P}_{\rm mie} \sim \frac{\alpha_s C_R}{\pi}\frac{d\omega}{\omega}\frac{dt}{t_{f,\rm med}}\underbrace{\mathcal{P}_{\rm broad}(\theta)d\theta}_{\rm Gaussian}, \qquad \text{with} \qquad t_{f,\rm med} = \sqrt{\omega/\hat{q}}$$

⇒ **No** collinear divergence when $\theta \longrightarrow 0$.
⇒ Typical $k_\perp^2 \sim \sqrt{\hat{q}\omega}$.

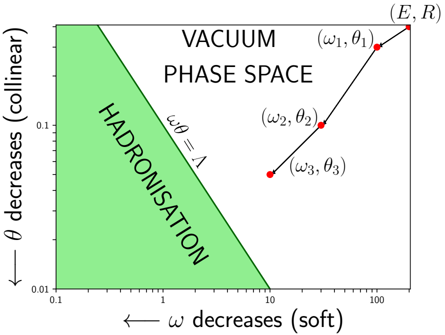


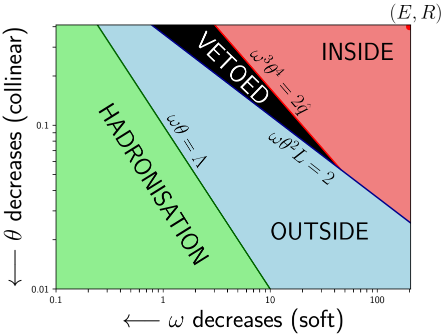
How does an in-medium jet look like at DLA?

Phase space constraint for vacuum-like emissions

- During $t_f = 1/(\omega\theta^2)$, in-medium partons acquire $k_\perp^2 = \hat{q} \times \overbrace{t_f}^{1/(\omega\theta^2)}$
- For VLEs *inside*, **lower bound** on the $k_\perp \simeq \omega\theta$ of emission:

$$k_\perp^2 > \hat{q} t_f \; : \; \text{final state evolution with "saturation" boundary}$$



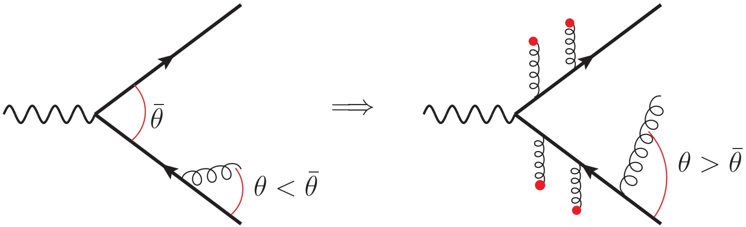


- No VLEs allowed for formation times $\sqrt{\omega/\hat{q}} < t_f < L$. PC, **Iancu**, Mueller, Soyez, 2018

How does an in-medium jet look like at DLA?

Decoherence

- **Color decoherence:** after $t_d = (\hat{q}\bar{\theta}^2)^{-1/3}$, $\Rightarrow$ independent sources of soft large angle gluons. Mehtar-Tani, Salgado, Tywoniuk, 2011 - Casalderrey-Solana, **Iancu**, 2011



- However, no consequences for VLEs **in the medium**

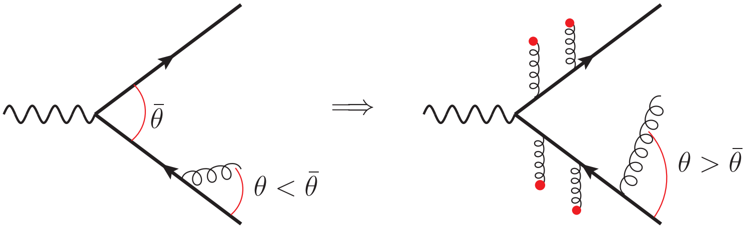
  PC, **Iancu**, Mueller, Soyez, 2018

  - Large angle in-medium VLEs occur very fast $\Rightarrow t_f < t_d$.
  - Gluon cascades are angular ordered as in the vacuum.

How does an in-medium jet look like at DLA?

Decoherence

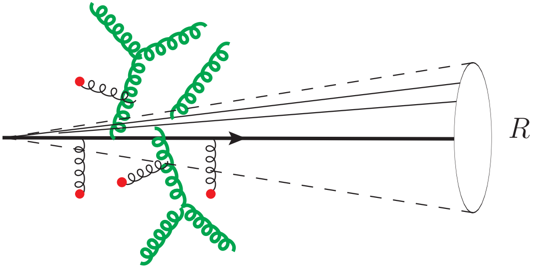
- **Color decoherence:** after $t_d = (\hat{q}\bar{\theta}^2)^{-1/3}$, $\Rightarrow$ independent sources of soft large angle gluons. Mehtar-Tani, Salgado, Tywoniuk, 2011 - Casalderrey-Solana, **Iancu**, 2011



- But an important consequence for the first emission **outside** $t_f > L$:
  - Critical angle θ_c such that $t_d(\theta_c) = L$.
  - If $\bar{\theta} > \theta_c = 2/\sqrt{\hat{q}L^3}$, the first emission **outside** can have any angle.
    Mehtar-Tani, Salgado, Tywoniuk, 2011

Beyond DLA: including medium-induced emissions

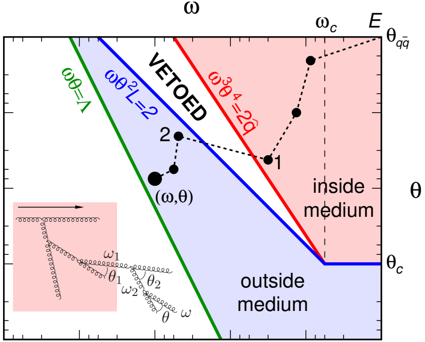
- MIEs satisfy $k_\perp^2 \sim \hat{q} t_f \iff t_{f,\mathrm{med}} = \sqrt{\omega/\hat{q}}$.
- Each VLE inside **with** $\theta \geq \theta_c$ radiates MIEs.



- Energy transported at large angles via a turbulent cascade: typical scale $\omega_{\mathrm{br}} = \alpha_s^2 \hat{q} L^2$.
  Blaizot, **Iancu**, Mehtar-Tani, 1301.6102

Summary: jet evolution to leading-log accuracy

- The evolution of a jet **factorizes** into three steps: PC, **Iancu**, Mueller, Soyez, 2018
  (1) one **angular ordered** *vacuum-like shower* inside the medium ,
  (2) *medium-induced emissions* triggered by previous sources,
  (3) finally, a *vacuum-like shower* outside the medium.
- Re-opening of the phase space for the first emission outside the medium.

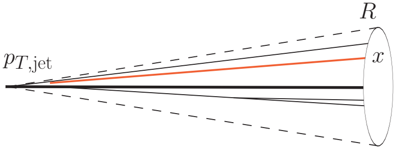


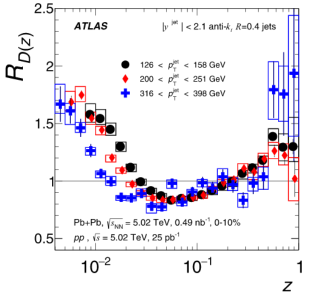
Phenomenology: jet fragmentation function

- Energy distribution of particles within jets.

$$\mathcal{D}(x) = \frac{1}{N_{\text{jets}}} \frac{dN}{dx}$$

with $x \sim p_T / p_{T,\text{jet}}$





ATLAS Collaboration, Phys. Rev. C98, 2018

- Simple LL estimate: $x\mathcal{D}_{\text{pp}}(x) = 2\bar{\alpha}_s \int_{\Lambda/(xp_T)}^{R} \frac{d\theta}{\theta} \mathrm{I}_0\left(2\bar{\alpha}_s \sqrt{2\log(1/x)\log(R/\theta)}\right)$

- Nuclear modification:

$$\mathcal{R}(x) = \frac{\mathcal{D}_{\text{PbPb}}(x)}{\mathcal{D}_{\text{pp}}(x)}$$

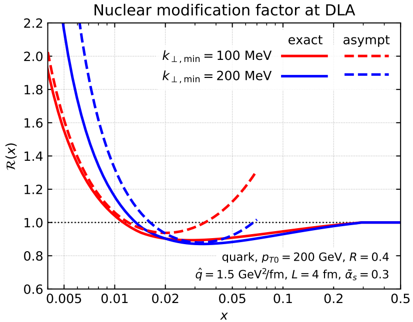
Enhancement at small-x: role of colour decoherence

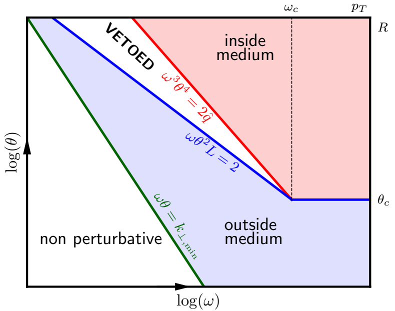
Colour decoherence

⇒ **no angular ordering** for the first emission outside the medium,

⇒ **factorisation** between parton cascades inside & outside the medium. PC, **Iancu**, Mueller, Soyez, 2020

$$x\mathcal{D}_{\rm PbPb}(x) \simeq \frac{\sqrt{\bar{\alpha}_s}}{4} \times \overbrace{\mathcal{N}_{\rm med}}^{\text{number of ``in'' sources}} \times \overbrace{\exp(\bar{\alpha}_s \log(2xp_T/\Lambda^2 L))}^{\text{outside DL cascade}}$$



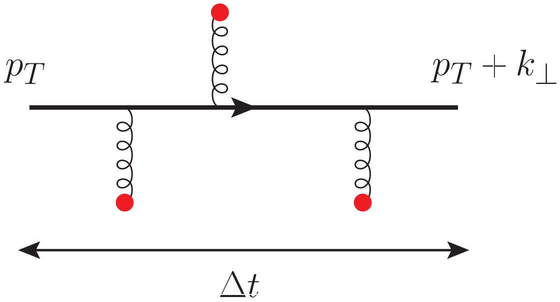


Transverse momentum broadening in the double logarithmic approximation

work with Yacine Mehtar-Tani

Transverse momentum broadening in QCD

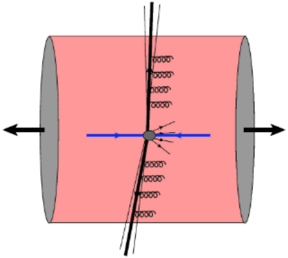
- Physical system: a highly energetic parton propagating through a dense QCD medium.
- We compute the tranverse momentum distribution $\mathcal{P}(k_\perp)$ of the outgoing parton.

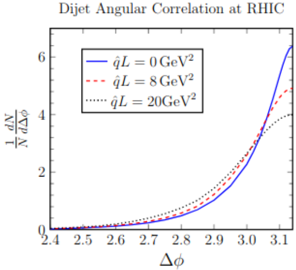


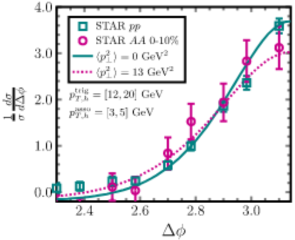
- Dense QCD medium: multiple scatterings.

Why is TMB interesting?

- "Hot QCD": Dijet azimuthal angular distributions in heavy-ion collisions: access to the TMB and the medium properties.
- Ex: studies by Mueller, Wu, Xiao, Yuan 1604.04250 & Chen, Qin, Wei, Xiao, Zhang 1607.01932.





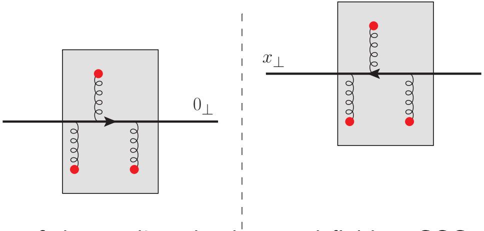


- "Cold QCD": fast probe of gluon distribution in large nuclei $L \propto A^{1/3} \gg 1$ at small-x.

TMB at "tree level" (1/2)

- Forward scattering amplitude of an effective dipole with size $\boldsymbol{x}_\perp$,

$$\mathcal{S}(\boldsymbol{x}_\perp) = \frac{1}{N_c}\langle \mathrm{Tr}\, V^\dagger(\boldsymbol{x}_\perp) V(\boldsymbol{0}_\perp)\rangle\,, \qquad \text{with} \quad V(\boldsymbol{x}_\perp) = \mathcal{P} e^{ig\int_{-\infty}^{\infty} dx^+ A^-(x^+,\boldsymbol{x}_\perp)}$$



- $\langle[...]\rangle$ denotes average of the medium background field $\sim$ CGC average
- Assuming independent multiple interactions,

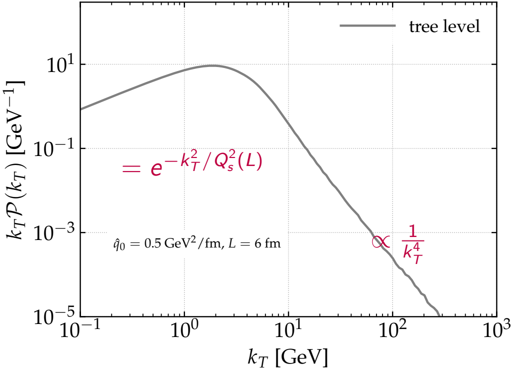
$$\langle A^{-a}(x^+,\boldsymbol{x}_\perp)A^{-b*}(y^+,\boldsymbol{y}_\perp)\rangle = n(x^+)\delta^{ab}\delta(x^+-y^+)\underbrace{\gamma(\boldsymbol{x}_\perp-\boldsymbol{y}_\perp)}_{\sim\ \text{colision rate}}$$

TMB at "tree level" (2/2)

- Fourier transform of the dipole S-matrix

$$\mathcal{P}(\boldsymbol{k}_\perp) = \int d^2\boldsymbol{x}_\perp e^{-i\boldsymbol{k}_\perp \boldsymbol{x}_\perp} e^{-\frac{1}{4}\hat{q}(1/\boldsymbol{x}_\perp^2)L\boldsymbol{x}_\perp^2}\,, \qquad g^2 C_R n(\gamma(\boldsymbol{0}_\perp) - \gamma(\boldsymbol{x}_\perp)) \approx \frac{1}{4}\hat{q}(1/\boldsymbol{x}_\perp^2)\boldsymbol{x}_\perp^2$$

- LO $\hat{q}$ given by $\hat{q}^{(0)}(1/\boldsymbol{x}_\perp^2) = \hat{q}_0 \ln \frac{1}{\boldsymbol{x}_\perp^2 \mu^2}$ $(\hat{q}_0 \propto \alpha_s^2 n,\ \mu \propto m_D)$



For an analytic expression, see e.g. Barata, Mehtar-Tani, Soto-Ontoso, Tywoniuk 2009.13667
or **Iancu**, Itakura, Triantafyllopoulos, 0403103

The saturation momentum Q_s

- Emergent scale in the dipole S-matrix $\mathcal{S}$:

$$\mathcal{S}(\boldsymbol{x}_\perp^2 = 1/Q_s^2(L)) \equiv e^{-1/4} \Leftrightarrow \hat{q}(L, Q_s^2(L))L = Q_s^2(L)$$

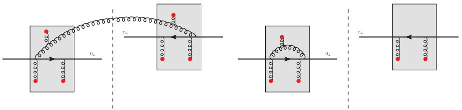
- Transition between the unitarity bound $\mathcal{S} \sim 1$ and the dilute regime $\mathcal{S} \ll 1$.

- At tree-level, one finds

$$Q_s^2(L) \simeq \hat{q}_0 L \ln(\hat{q}_0 L/\mu^2)$$

- Approximate **linear** scaling with L at tree-level.

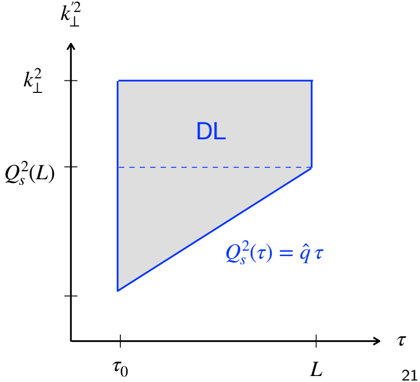
Double logarithmic enhancement of $\hat{q}$ at NLO



Liou, Mueller, Wu, 1304.7677

- NLO correction to $\hat{q}$:

$$\hat{q}^{(1)} \sim \frac{\alpha_s N_c}{\pi} \int_{\tau_0}^{L} \frac{d\tau'}{\tau'} \int_{Q_s^2(\tau)}^{1/x_\perp^2} \frac{d\boldsymbol{k}_\perp'^2}{\boldsymbol{k}_\perp'^2} \hat{q}_0$$



- Double logarithmic phase space, with non-linear saturation bound: $Q_s^2(\tau) \simeq \hat{q}_0 \tau$
- Constrains the emission to be triggered by a single scattering.

Resummation of the leading radiative corrections

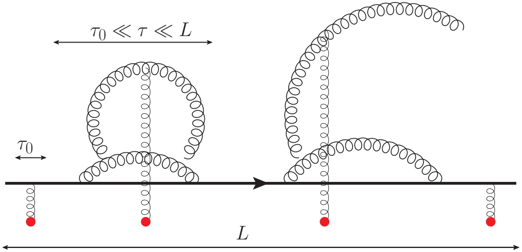
- Resummation to all orders via the evolution equation

$$\frac{\partial \hat{q}(\tau,\boldsymbol{k}_\perp^2)}{\partial \ln(\tau)} = \int_{Q_s^2(\tau)}^{\boldsymbol{k}_\perp^2} \frac{d\boldsymbol{k}_\perp'^2}{\boldsymbol{k}_\perp'^2} \bar{\alpha}_s(\boldsymbol{k}_\perp'^2)\ \hat{q}(\tau,\boldsymbol{k}_\perp'^2)\,, \quad Q_s^2(\tau) \equiv \hat{q}(\tau, Q_s^2(\tau))\tau$$

.

- Exponentiation of the double logarithmic corrections.

$$\mathcal{P}(\boldsymbol{k}_\perp) = \int \mathrm{d}^2\boldsymbol{x}_\perp\, e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}\ \exp\left[-\frac{1}{4}\hat{q}(L,1/x_\perp^2)\,L\,\boldsymbol{x}_\perp^2\right]$$



cf Liou, Mueller, Wu, 1304.7677, Blaizot, Mehtar-Tani, 1403.2323 & **Iancu** 1403.1996, "The non-linear evolution of jet quenching"

Reminder: extended geometric scaling for gluon distribution

- Inclusive DIS cross-section at small-x (large $\tau = \ln(1/x)$)

$$\sigma(\tau = \ln(1/x), Q^2) \propto 2\int_0^1 dz \int d^2\boldsymbol{r}_\perp |\psi(z, \boldsymbol{r}_\perp, Q^2)|^2 (1 - S_\tau(\boldsymbol{r}_\perp))$$

- Energy (τ) dependence of $S_\tau(\boldsymbol{r}_\perp)$ determined from the BK equation.
- For $Q^2 \sim 1/\boldsymbol{r}_\perp^2 \ll Q_s^4/\Lambda_{\rm QCD}^2$, the dipole S-matrix satisfies extended geometric scaling

$$S_\tau(\boldsymbol{r}_\perp) \sim f\left(\frac{1}{\boldsymbol{r}_\perp^2 Q_s^2(\tau)}\right)$$

cf Stasto, Golec-Biernat, Kwiecinski 0007192, **Iancu**, Itakura, McLerran 203137, etc

- A similar property holds when $S(L, \boldsymbol{x}_\perp)$ satisfies our non-linear DLA evolution equation with saturation boundary.

Asymptotic limit of TMB at fixed coupling

- Large system size limit of $\hat{q}(L,\boldsymbol{k}_\perp^2)$

$$\frac{\hat{q}(L,\boldsymbol{k}_\perp^2)L}{Q_s^2(L)} \underset{L\to\infty}{=} \begin{cases} e^{2\beta\ln\left(\frac{\boldsymbol{k}_\perp^2}{Q_s^2(L)}\right)} & \text{if } \boldsymbol{k}_\perp^2 \leq Q_s^2(L) \\ e^{\beta\ln\left(\frac{\boldsymbol{k}_\perp^2}{Q_s^2(L)}\right)}\left[1+\beta\ln\left(\frac{\boldsymbol{k}_\perp^2}{Q_s^2(L)}\right)\right] & \text{else} \end{cases}$$

with $\beta=(c-1)/(2c)$ and $c=1+2\sqrt{\bar{\alpha}_s+\bar{\alpha}_s^2}+2\bar{\alpha}_s\approx 1+2\sqrt{\bar{\alpha}_s}$.

$\Longrightarrow$ **extended geometric scaling** for $\boldsymbol{k}_\perp^2 \ll Q_s^4/\mu^2$. PC, Mehtar-Tani 2109.12041

- TMB in this region given by Fourier transform

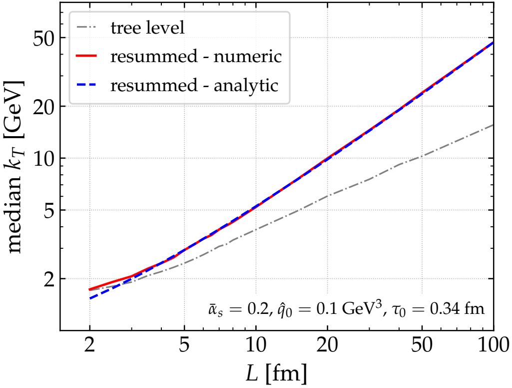
$$\mathcal{P}(\boldsymbol{k}_\perp)=\int d^2\boldsymbol{x}_\perp e^{-i\boldsymbol{k}_\perp\boldsymbol{x}_\perp}e^{-\frac{1}{4}\hat{q}(1/\boldsymbol{x}_\perp^2)L\boldsymbol{x}_\perp^2}$$

Superdiffusion in momentum space

- $\frac{\mathrm{d}\ln(Q_s^2(L))}{\mathrm{d}\ln(L)} = c \Longrightarrow Q_s^2(L) \sim L^c$
- The median of the distribution scales like

$$\mathcal{M} \sim L^{1/2+\sqrt{\bar{\alpha}_s}}$$

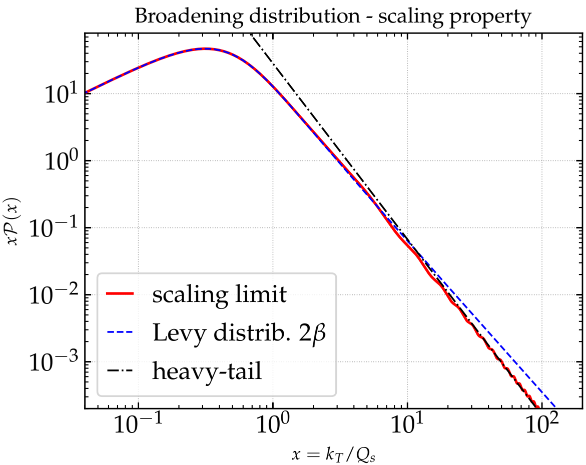
- $\Rightarrow$ super-diffusive behaviour.
  NLO corrections yields super-diffusion in momentum space.



Heavy-tailed distribution

- $\hat{q} \simeq e^{\beta x}$ at large k_T.
- Fourier transform of the "stretched" exponential $\exp(-[...]\boldsymbol{x}_\perp^\gamma)$ with $\gamma \simeq 2 + 2\sqrt{\bar{\alpha}_s} > 2$
- Heavy tailed distribution

$$\mathcal{P}(\boldsymbol{k}_\perp) \propto \frac{1}{k_T^{4-2\sqrt{\bar{\alpha}_s}}}$$



PC, Mehtar-Tani 2109.12041

Beyond the asymptotic limit

- We have determined the limit $Y \to \infty$ of the TMB distribution.
- What about the sub-asymptotic corrections?
- Are they universal?
- Can they be used to realistic values of $Y = \ln(L/\tau_0)$?

Wave front propagation into unstable state

- We borrow techniques from front propagation into unstable sate.

  Ebert, van Saarloos, 0003181, Brunet, Derrida, 0005362

- Similar to the traveling wave interpretation of the solutions to BK.

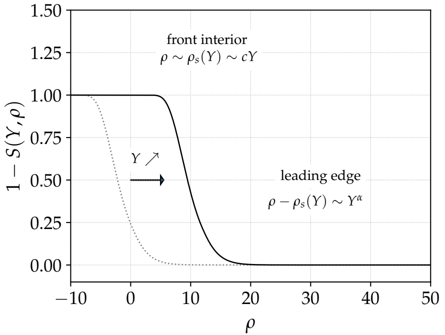
  Mueller, Triantafyllopoulos, 0205167, Munier, Peschanski, 0310357 - Beuf, 1008.0498

- Typical example reaction-diffusion process: Fisher-Kolmogoroff-Petrovsky-Piscounoff eq.

$$\partial_t \phi = \partial_x^2 \phi + \phi - \phi^k$$

- Universality of the wave-front velocity $\dot{\rho}_s$:

$$\dot{\rho}_s = \frac{\mathrm{d}\ln(Q_s^2(L))}{\mathrm{d}\ln(L)} = c + \frac{b}{Y} + \frac{d}{Y^2} + \dots$$



> *If initially a spatially extended system is in an unstable state everywhere except in some spatially localized region, what will be the large-time dynamical properties and speed of the nonlinear front which will propagate into the unstable state? Are there classes of initial conditions for which the front dynamics converges to some unique asymptotic front state? If so, what characterizes these initial conditions, and what can we say about the asymptotic front properties and the convergence to them?*

Van Saarloos, "Front propagation into unstable states," Physics Reports 386 2-6

Running α_s: proof of the Iancu & Triantafyllopoulos conjecture

$$\frac{\partial \hat{q}}{\partial Y} = \int_Y^\rho d\rho' \; \bar{\alpha}_s(\rho')\hat{q}(Y,\rho') \qquad \Longleftrightarrow \qquad \frac{\partial \hat{q}}{\partial Y} = \int_{\rho_s(Y)}^\rho d\rho' \; \bar{\alpha}_s(\rho')\hat{q}(Y,\rho')$$

- The large Y development of $\rho_s(Y) = \ln(Q_s^2(Y))$ for the rc-evolution has been found numerically in **Iancu**, Triantafyllopoulos 1405.3525

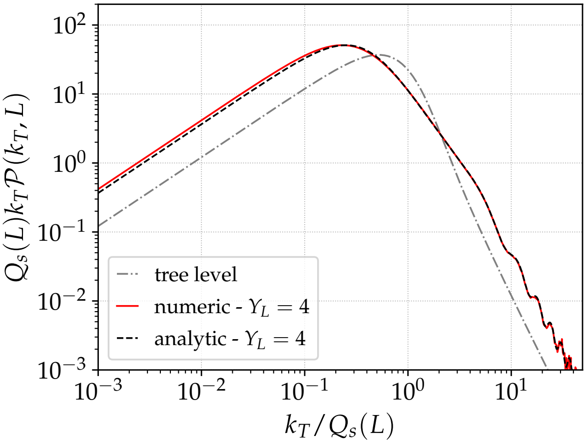
- With the TW technique, we can prove their results and go beyond the $\rho_s(Y) \approx Y$ approximation PC, Mehtar-Tani, 2203.09407

$$\rho_s(Y) = Y + 2\sqrt{4b_0 Y} + 3\xi_1 (4b_0 Y)^{1/6} + \left(\frac{1}{4} - 2b_0\right)\ln(Y) + \kappa + \frac{7\xi_1^2}{180}\frac{1}{(4b_0 Y)^{1/6}}$$
$$+ \xi_1\left(\frac{5}{108} + 18b_0\right)\frac{1}{(4b_0 Y)^{1/3}} + b_0(1 - 8b_0)\frac{\ln(Y)}{\sqrt{4b_0 Y}} + O\left(Y^{-1/2}\right)$$

Initial conditions for BK evolution

- Analytic formulas for the TMB or the dipole S-matrix $\mathcal{S}$ that resum to all orders gluon fluctuations enhanced by $\alpha_s \ln^2(A^{1/3})$.
- Down to $Y \sim 2$, the shape of $\mathcal{S}$ is driven by the universal behaviour of the traveling front.
- Physically motivated new initial condition for the BK-JIMWLK evolution equations.

  see also Dumitru, Mantysaari, Paatelainen, 2103.11682

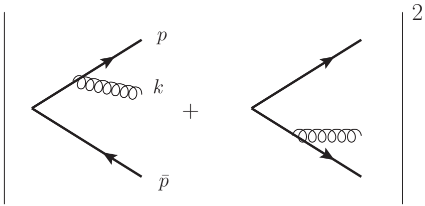


Conclusion

- Final state evolution of jets in dense weakly coupled QCD plasma,
- and quantum evolution of diffusion transport coefficient $\hat{q}$,
- share formal similarities with non-linear evolution of initial state gluon distribution at small-x.

Back-up slides

Antenna radiation pattern



See e.g. Ellis, Stirling, Webber or Dokshitzer, Khoze, Mueller, Troyan books

- In the soft limit,

$$k^+\frac{\mathrm{d}^3N}{\mathrm{d}k^+\mathrm{d}^2\boldsymbol{k}_\perp}=\frac{\alpha_sC_F}{2\pi^2}\frac{p^\mu\bar{p}_\mu}{(p^\mu k_\mu)(\bar{p}^\mu k_\mu)}$$

- After averaging over the azimuthal angle of the gluon,

$$k^+\frac{\mathrm{d}^2N}{\mathrm{d}k^+\mathrm{d}\theta_q}=\frac{\alpha_sC_F}{\pi}\frac{\sin(\theta_q)}{1-\cos(\theta_q)}\Theta(\theta_{q\bar{q}}-\theta_q)+(q\leftrightarrow\bar{q})$$

TMB at "tree level" (2/3)

- $\mathcal{S}(\boldsymbol{x}_\perp)$ exponentiates:

$$\mathcal{S}(\boldsymbol{x}_\perp) = \exp\left(-g^2 C_R n L(\gamma(\boldsymbol{0}_\perp) - \gamma(\boldsymbol{x}_\perp))\right)$$

- "Leading-twist" approximation in the perturbative regime $1/\boldsymbol{x}_\perp \gg m_D$:

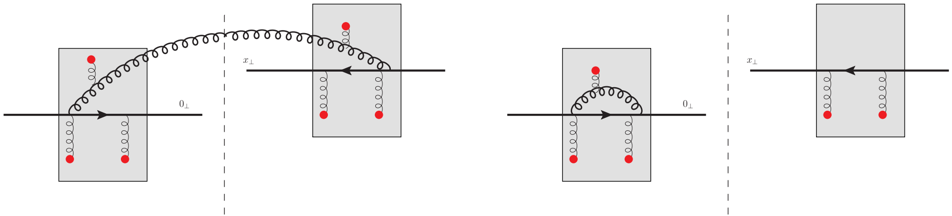
$$g^2 C_R n(\gamma(\boldsymbol{0}_\perp) - \gamma(\boldsymbol{x}_\perp)) \equiv \frac{1}{4}\hat{q}(1/\boldsymbol{x}_\perp^2)\boldsymbol{x}_\perp^2 + \mathcal{O}(m_D^2 \boldsymbol{x}_\perp^2)$$

- We take this formula as our definition of $\hat{q}$.
- For a collision rate with typical Coulomb tail $\mathcal{C}(\boldsymbol{q}_\perp) = \int d^2\boldsymbol{x}_\perp e^{i\boldsymbol{q}_\perp \boldsymbol{x}_\perp}\gamma(\boldsymbol{x}_\perp) \sim g^2/\boldsymbol{q}_\perp^4$:

$$\hat{q}^{(0)}(1/\boldsymbol{x}_\perp^2) = \hat{q}_0 \ln\frac{1}{\boldsymbol{x}_\perp^2\mu^2} + \mathcal{O}(\boldsymbol{x}_\perp^2\mu^2)\,, \qquad \hat{q}_0 \propto \alpha_s^2 n,\ \ \mu \propto m_D$$

TMB at one loop in a dense QCD medium (1/2)

- Computation at one-loop in $\alpha_s(p_T) \ll 1$, but to all-orders in $\alpha_s n$.



- Schematically one finds

$$\mathcal{P}(\boldsymbol{k}_\perp, L) = \mathcal{P}^{(0)}(\boldsymbol{k}_\perp, L) + \alpha_s \mathcal{P}^{(1)}(\boldsymbol{k}_\perp, L) + \mathcal{O}(\alpha_s^2)$$

with the NLO distribution given by Liou, Mueller, Wu, 1304.7677, Blaizot, Mehtar-Tani, 1403.2323

$$\alpha_s \mathcal{P}^{(1)}(\boldsymbol{k}_\perp, L) = 2\alpha_s N_c \mathfrak{Re} \int \frac{d\omega}{\omega^3} \int_0^L dt_2 \int_0^{t_2} dt_1 \int_{\boldsymbol{q}_{1\perp}, \boldsymbol{q}_{2\perp}} \mathcal{P}^{(0)}(\boldsymbol{k}_\perp - \boldsymbol{q}_{2\perp}, L - t_2)$$
$$\times \mathcal{K}(\boldsymbol{q}_{2\perp} - \boldsymbol{q}_{1\perp}, t_2, t_1) \mathcal{P}^{(0)}(\boldsymbol{q}_{1\perp}, t_1)$$

TMB at one loop in a dense QCD medium (2/2)

- The kernel $\mathcal{K}(\boldsymbol{l}_\perp, t_2, t_1)$ involves the medium 3-point function $S^{(3)}$.

$$\mathcal{K}(\boldsymbol{l}_\perp, t_2, t_1) \equiv \int_{\boldsymbol{q}_\perp, \boldsymbol{q}'_\perp} (\boldsymbol{q}_\perp \cdot \boldsymbol{q}'_\perp) \left[\tilde{S}^{(3)}(\boldsymbol{q}_\perp, \boldsymbol{q}'_\perp, \boldsymbol{l}_\perp + \boldsymbol{q}'_\perp; t_2, t_1) - \tilde{S}^{(3)}(\boldsymbol{q}_\perp, \boldsymbol{q}'_\perp, \boldsymbol{l}_\perp; t_2, t_1)\right]$$

$$(X_2|\mathcal{S}^{(3)}(t_2, t_1)|X_1) \equiv \frac{f^{a_1 b_1 c_1} f^{a_0 b_0 c_0}}{N_c(N_c^2 - 1)} \langle (\boldsymbol{x}'''_{2\perp}|\mathcal{G}^{a_1 a_0}(t_2, t_1)|\boldsymbol{x}'''_{1\perp})(\boldsymbol{x}''_{2\perp}|\mathcal{G}^{b_1 b_0}|\boldsymbol{x}''_{1\perp})(\boldsymbol{x}_{1\perp}|\mathcal{G}^{\dagger, c_0 c_1}|\boldsymbol{x}_{2\perp})\rangle$$

- In the "harmonic approximation" with $\hat{q}^{(0)} \simeq \hat{q}_0$:

$$\tilde{S}^{(3)}(\boldsymbol{q}_\perp, \boldsymbol{q}'_\perp, \boldsymbol{l}_\perp; \tau = t_2 - t_1) = \frac{16\pi}{3\hat{q}_0\tau} \exp\left\{-\frac{4[\boldsymbol{l}_\perp - (\boldsymbol{q}_\perp - \boldsymbol{q}'_\perp)/2]^2}{3\hat{q}_0\tau}\right\} \frac{2\pi i}{\omega\Omega \sinh(\Omega\tau)}$$
$$\times \exp\left\{-i\frac{(\boldsymbol{q}_\perp + \boldsymbol{q}'_\perp)^2}{4\omega\Omega \coth(\Omega\tau/2)} - i\frac{(\boldsymbol{q}_\perp - \boldsymbol{q}'_\perp)^2}{4\omega\Omega \tanh(\Omega\tau/2)}\right\}$$

with $\Omega^2 = i\hat{q}_0/(2\omega)$ and $\tau = t_2 - t_1$.

Lévy flights

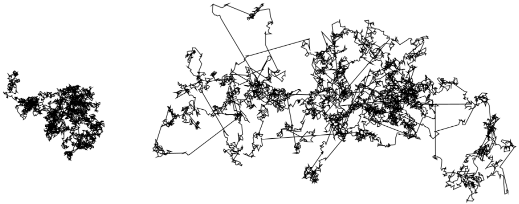
- At large time $L \gg \tau_0$, near the peak

$$\mathcal{S}(\boldsymbol{x}_\perp, L) \approx \exp\left(-\frac{1}{4}(|\boldsymbol{x}_\perp| Q_s(L))^{2-4\beta}\right), \qquad \beta \simeq \sqrt{\bar{\alpha}_s}$$

- $\Longrightarrow$ the TMB distribution satisfies a fractional Fokker-Planck equation

$$\frac{\partial \mathcal{P}(L, \boldsymbol{k}_\perp)}{\partial L} = \nu \frac{\partial^\gamma \mathcal{P}(L, \boldsymbol{k}_\perp)}{\partial |\boldsymbol{k}_\perp|^\gamma}, \qquad \gamma = 2 - 4\beta$$

Brownian motion



Levy flight

- Equation for the prob. density of a Lévy walker, e.g.
  - $\dot{v} = -\mu v + \eta^\gamma(t)$
  - $\eta^\gamma(t)$ Lévy stable noise ($\gamma = 2$ is the standard white Gaussian noise).

Asymptotic limit of TMB at fixed coupling

- Define $Y = \ln(L/\tau_0)$, $\rho = \ln(\boldsymbol{k}_\perp^2/\mu^2)$, $\rho_s = \ln(Q_s^2/\mu^2)$,

$$\frac{\partial \hat{q}(Y,\rho)}{\partial Y} = \bar{\alpha}_s \int_{\rho_s(Y)}^{\rho} d\rho' \hat{q}(Y,\rho')$$

- Let's look for scaling solution $\hat{q}(Y,\rho) = f(x = \rho - \rho_s(Y))$:

$$-\dot{\rho}_s f''(x) + [\dot{\rho}_s - 1] f'(x) - \bar{\alpha}_s f(x) = 0$$

- For physical initial conditions, the unique solution to this problem is $\dot{\rho}_s = c$ and

$$f(x) = e^{\beta x}(1 + \beta x)$$

with $\beta = (c-1)/(2c)$ and $c = 1 + 2\sqrt{\bar{\alpha}_s + \bar{\alpha}_s^2} + 2\bar{\alpha}_s$.

$\Longrightarrow$ **extended geometric scaling** for $x \ll \rho_s(Y)$ or $\boldsymbol{k}_\perp^2 \ll Q_s^4/\mu^2$. PC, Mehtar-Tani 2109.12041

Leading edge domain $\rho - \rho_s \sim Y^\alpha$

- Diffusive deviation from the asymptotic limit, with we consider.

$$\hat{q}(Y,\rho) = \hat{q}_0 e^{\rho_s(Y)-Y} e^{\beta x} \; Y^\alpha G\left(\frac{x}{Y^\alpha}\right)$$
$$\dot{\rho}_s(Y) = c + \dot{\sigma}_s(Y)$$

- Diffusion power characteristics of the universality class of the evolution equation.

- Homogeneity conditions fix the power α.

- $\alpha = 1/2$ for fixed coupling, $\alpha = 1/6$ for running coupling

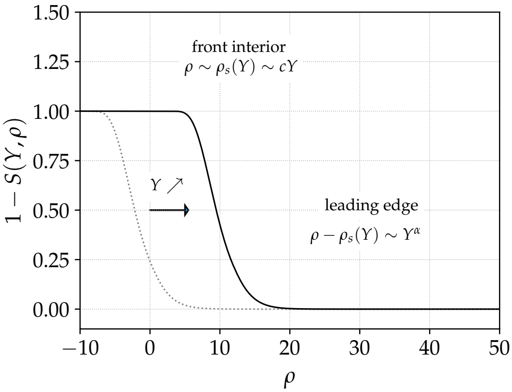
Front interior vs leading edge expansion

- We then write two types of expansion: front interior and leading edge.

$$\hat{q}(Y,\rho) = \hat{q}_0 e^{\rho_s(Y)-Y} e^{\beta x} \sum_{n\geq 0} \frac{1}{Y^{n\alpha}} f_n(x)$$

$$\hat{q}(Y,\rho) = \hat{q}_0 e^{\rho_s(Y)-Y} e^{\beta x} \sum_{n\geq -1} \frac{1}{Y^{n\alpha}} G_n\left(\frac{x}{Y^{\alpha}}\right)$$

- Matching of the front interior and leading edge fix all the constants and the development of the wave front velocity $\dot{\rho}_s$.



Results for fixed coupling

- For fixed coupling, we find the pre-asymptotic behaviour

$$\frac{\hat{q}(Y,x)L}{Q_s^2(L)} = \begin{cases} \exp\left(\beta x - \frac{\beta x^2}{4cY}\right)\left[1+\beta x - \frac{3x}{c(1+c)Y}\left(1+\frac{\beta(c+4)x}{6}\right)+\mathcal{O}\left(\frac{1}{Y^2}\right)\right] & \text{if } x \geq 0 \\ \exp\left(2\beta x - \frac{3}{c(1+c)}\frac{x}{Y}+\mathcal{O}\left(\frac{1}{Y^2}\right)\right) & \text{if } x < 0\,. \end{cases} \quad (1)$$

with

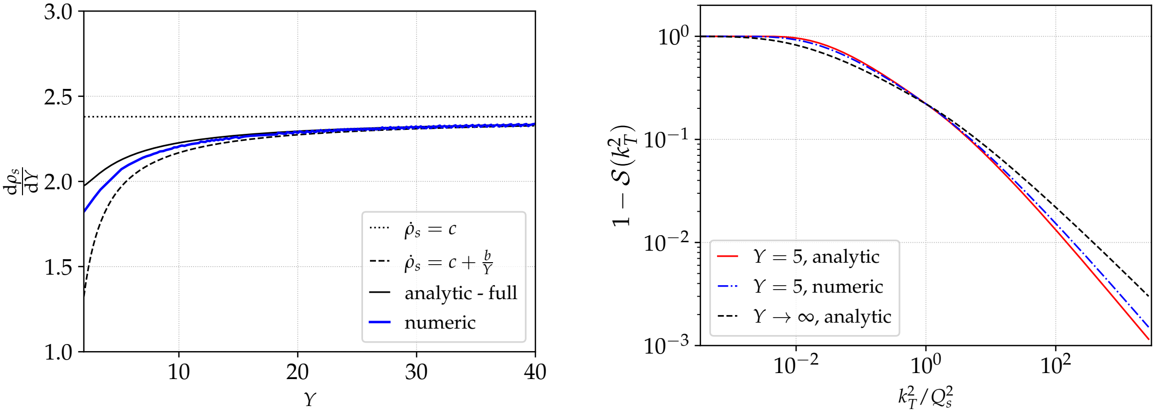
$$\rho_s(Y) = cY - \frac{3c}{(1+c)}\ln(Y) - \frac{6c\sqrt{2\pi(c-1)}}{(1+c)^2}\frac{1}{\sqrt{Y}} + \mathcal{O}(Y^{-1})$$

PC, Mehtar-Tani 2109.12041

- For the linearized equation:

$$\rho_s(Y) = (1+\sqrt{\bar{\alpha}_s})Y - \frac{3}{2}\ln(Y) + \mathcal{O}(Y^{-1})$$

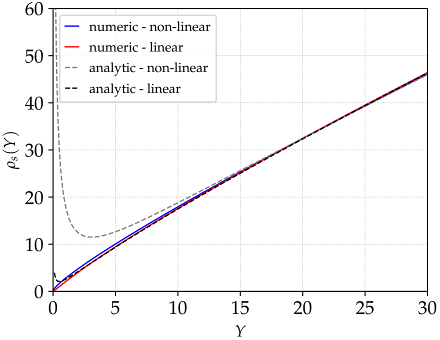
Some plots



- Sub-asymptotic corrections enable one to have a good agreement with the numeric.
- Analytic results can be systematically improved.

Analytic formulas at small Y

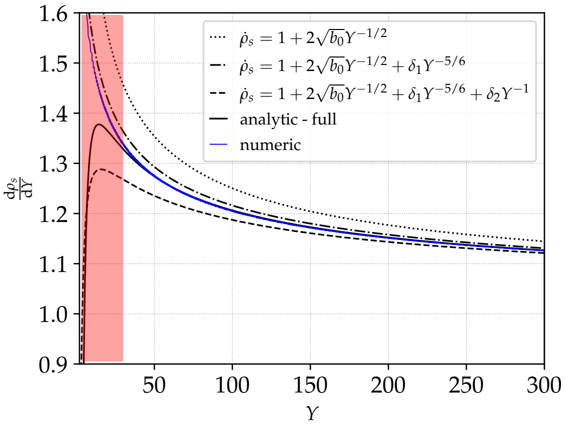
- Solution: drop the terms which make the series divergent.



- Use this analytic form for $\rho_s(Y)$ to compute $\hat{q}(Y,\rho)$ as a Taylor series around ρ_s:

$$\hat{q}(Y,x) = \hat{q}_0\, e^{\rho_s(Y)-Y} \left[1 + \frac{\dot{\rho}_s - 1}{\dot{\rho}_s} x + \frac{1}{2}\left(\left(\frac{\dot{\rho}_s - 1}{\dot{\rho}_s}\right)^2 + \frac{\ddot{\rho}_s}{\dot{\rho}_s^3} - \frac{\bar{\alpha}_s(\rho_s)}{\dot{\rho}_s}\right) x^2 + \mathcal{O}(x^2)\right]$$

Front velocity of the rc equation



- Excellent agreement at large Y.
- At small Y, the asymptotic development fails to converge.

p_T-broadening in heavy-ion collisions

- $\langle k_\perp^2 \rangle$ responsible for the dijet azimuthal decorrelation related to the "renormalized" value of ρ_s.
- For realistic values of Y, enhancement factor of order $2-6$ compared to a tree-level estimation!
- Need to include single log corrections to reach greater precision.