

# Hybrid Star Properties with NJL and MFTQCD Model:

A Bayesian Approach

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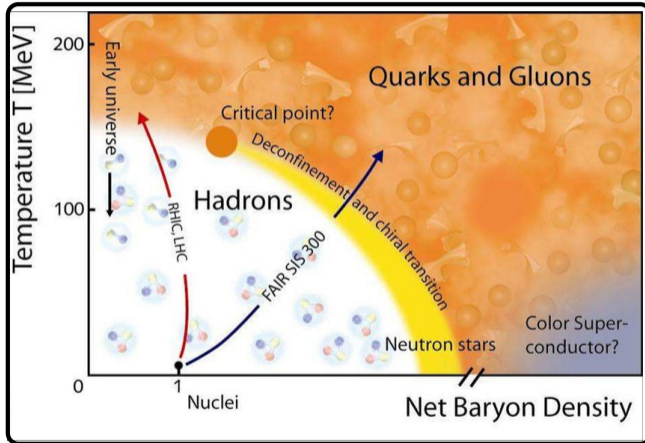
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Universidade de Coimbra

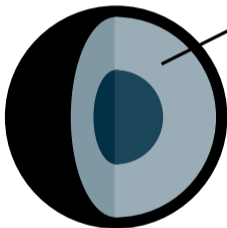


- Neutron stars (NS) are extreme objects that have high density and low temperature ( $T \approx 0$ );
- This density may be so high that matter may be deconfined inside neutron stars;

**But this is just a possibility!**

- Most of the QCD phase diagram is still unknown - including the NS region.

Objective: answer the question  
“is deconfined quark matter present in the NS core?” using microscopic models



### Hadron phase

#### Relativistic Mean Field (RMF)

- nucleons force is described by the exchange of a scalar ( $\sigma$ ), a vector iso-scalar ( $\omega$ ) and a iso-vector-vector ( $\rho$ ) mesons;
- we use two equations<sup>1</sup>:

Soft

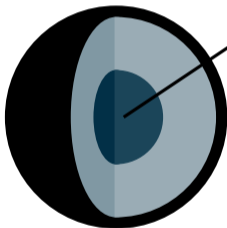
(BMPF 220)

Stiff

(BMPF 260)

<sup>1</sup>Tuhin Malik et al. “Spanning the full range of neutron star properties within a microscopic description”. In: *Phys. Rev. D* 107.10 (2023), p. 103018. DOI: 10.1103/PhysRevD.107.103018. arXiv: 2301.08169 [nucl-th].

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### Quark phase

#### NJL

(Nambu-Jona-Lasinio)

- t' Hooft term;
- 5 different types of interactions.

#### MFTQCD

(Mean Field Theory of QCD)<sup>2</sup>

- from the QCD Lagrangian
- decomposition of gluon field in soft and hard momentum components.

<sup>2</sup>D. A. Fogaca and F. S. Navarra. “**Gluon condensates in a cold quark–gluon plasma**”. In: *Phys. Lett. B* 700 (2011), pp. 236–242. DOI: 10.1016/j.physletb.2011.05.011. arXiv: 1012.5266 [hep-ph].

We use the following SU(3) NJL lagrangian:

$$\mathcal{L} = \bar{\psi} (i\not{\partial} - m + \mu\gamma^0) \psi + \frac{G}{2} \left[ (\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}i\gamma^5\lambda_a\psi)^2 \right] + \mathcal{L}'_{\text{'t Hooft}} + \mathcal{L}_I$$

standard NJL term

't Hooft term

added interaction terms

We add 4- and 8-quark interaction terms:

$$\begin{aligned}
 \mathcal{L}_I = & -G_\omega \left[ (\bar{\psi}\gamma^\mu\lambda_0\psi)^2 + (\bar{\psi}\gamma^\mu\gamma_5\lambda_0\psi)^2 \right] \\
 & - G_\rho \sum_{a=1}^8 \left[ (\bar{\psi}\gamma^\mu\lambda^a\psi)^2 + (\bar{\psi}\gamma^\mu\gamma_5\lambda^a\psi)^2 \right] \\
 & - G_{\omega\omega} \left[ (\bar{\psi}\gamma^\mu\lambda_0\psi)^2 + (\bar{\psi}\gamma^\mu\gamma_5\lambda_0\psi)^2 \right]^2 \\
 & - G_{\sigma\omega} \sum_{a=0}^8 \left[ (\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}i\gamma^5\lambda_a\psi)^2 \right] \left[ (\bar{\psi}\gamma^\mu\lambda_0\psi)^2 + (\bar{\psi}\gamma^\mu\gamma_5\lambda_0\psi)^2 \right] \\
 & - G_{\rho\omega} \sum_{a=1}^8 \left[ (\bar{\psi}\gamma^\mu\lambda_0\psi)^2 + (\bar{\psi}\gamma^\mu\gamma_5\lambda_0\psi)^2 \right] \left[ (\bar{\psi}\gamma^\mu\lambda_a\psi)^2 + (\bar{\psi}\gamma^\mu\gamma_5\lambda_a\psi)^2 \right]. \tag{1}
 \end{aligned}$$

EOS can be obtained through the Mean Field Approximation (MFA).

- Start with the QCD lagrangian;
- Assume we can decompose the gluon field in<sup>3</sup>
  - low momentum components (soft gluons);
  - high momentum components (hard gluons);

$$\tilde{G}^{a\mu}(k) = \tilde{A}^{a\mu}(k) + \tilde{\alpha}^{a\mu}(k) \quad (2)$$

The diagram illustrates the decomposition of the gluon field  $\tilde{G}^{a\mu}(k)$  into two components: soft gluons and hard gluons. The equation  $\tilde{G}^{a\mu}(k) = \tilde{A}^{a\mu}(k) + \tilde{\alpha}^{a\mu}(k)$  is shown at the top. Below the equation, two arrows point from the terms  $\tilde{A}^{a\mu}(k)$  and  $\tilde{\alpha}^{a\mu}(k)$  to boxes labeled "soft gluons" and "hard gluons" respectively.

<sup>3</sup>Fogaca and Navarra, "Gluon condensates in a cold quark–gluon plasma".

## NJL

$$\begin{aligned}
\mathcal{L}_1 = & -G_\omega \left[ (\bar{\psi}\gamma^\mu\lambda_0\psi)^2 + (\bar{\psi}\gamma^\mu\gamma_5\lambda_0\psi)^2 \right] \\
& - G_\rho \sum_{a=1}^8 \left[ (\bar{\psi}\gamma^\mu\lambda^a\psi)^2 + (\bar{\psi}\gamma^\mu\gamma_5\lambda^a\psi)^2 \right] \\
& - G_{\omega\omega} \left[ (\bar{\psi}\gamma^\mu\lambda_0\psi)^2 + (\bar{\psi}\gamma^\mu\gamma_5\lambda_0\psi)^2 \right]^2 \\
& - G_{\sigma\omega} \sum_{a=0}^8 \left[ (\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}i\gamma^5\lambda_a\psi)^2 \right] \\
& \quad \times \left[ (\bar{\psi}\gamma^\mu\lambda_0\psi)^2 + (\bar{\psi}\gamma^\mu\gamma_5\lambda_0\psi)^2 \right] \\
& - G_{\rho\omega} \sum_{a=1}^8 \left[ (\bar{\psi}\gamma^\mu\lambda_0\psi)^2 + (\bar{\psi}\gamma^\mu\gamma_5\lambda_0\psi)^2 \right] \\
& \quad \times \left[ (\bar{\psi}\gamma^\mu\lambda_a\psi)^2 + (\bar{\psi}\gamma^\mu\gamma_5\lambda_a\psi)^2 \right]. \tag{3}
\end{aligned}$$

$$P \rightarrow P + B. \tag{4}$$

MFTQCD<sup>4</sup>

$$P = \frac{27}{2} \xi^2 \rho_B^2 - B + P_F, \tag{5}$$

$$\epsilon = \frac{27}{2} \xi^2 \rho_B^2 + B + \epsilon_F, \tag{6}$$

where

- $\xi = g/m_G$  and  $m_G = \frac{9}{32} g^2 \mu_0^2$ ;
- $B = \frac{9}{4(34)} g^2 \phi_0^4$ ;
- $P_F$  and  $\epsilon_F$  are the pressure and energy density of a non-interacting Fermi gas of quarks and electrons.

<sup>4</sup>Fogaca and Navarra, "Gluon condensates in a cold quark–gluon plasma".



## Bayesian inference

Bayesian inference enables us to

- Obtain a set of parameters for the EOS;
- These EOS satisfy the restrictions imposed.

Constraints imposed by  
Bayesian inference:

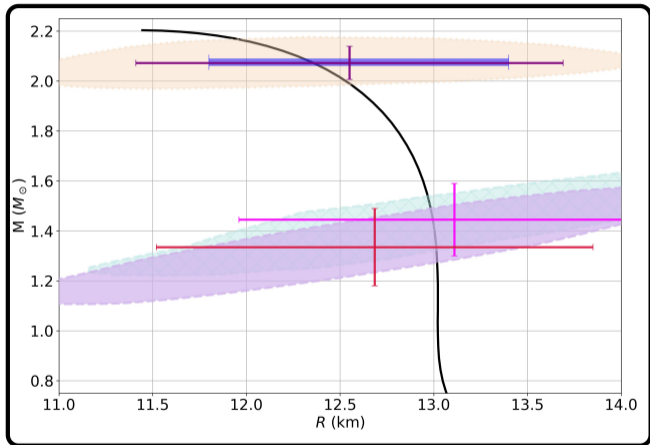
1. X-ray observations (NICER)
2. phase transition
3. pQCD EOS

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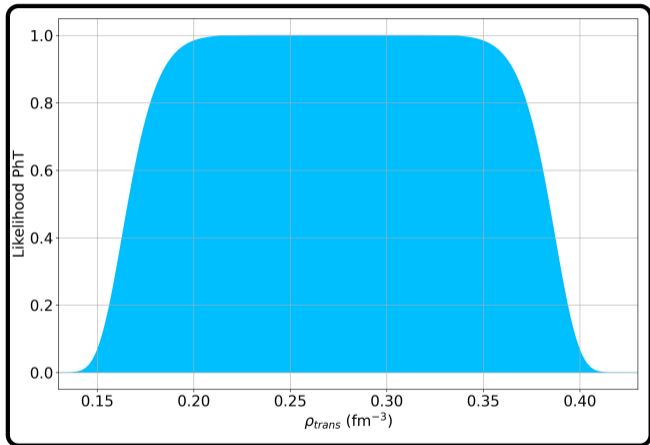


Constraints imposed by  
Bayesian inference:

1. X-ray observations (NICER)

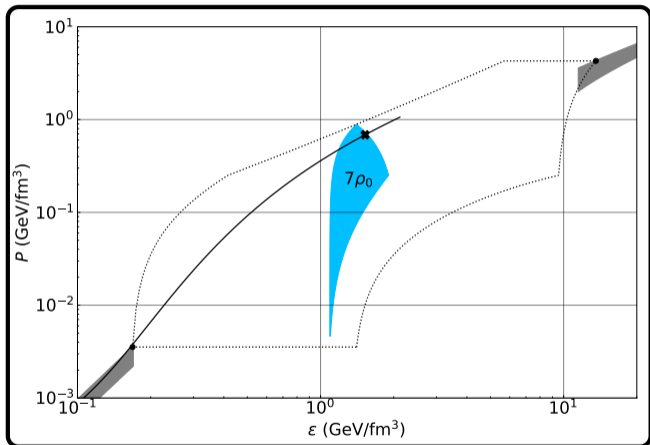
2. phase transition

3. pQCD EOS



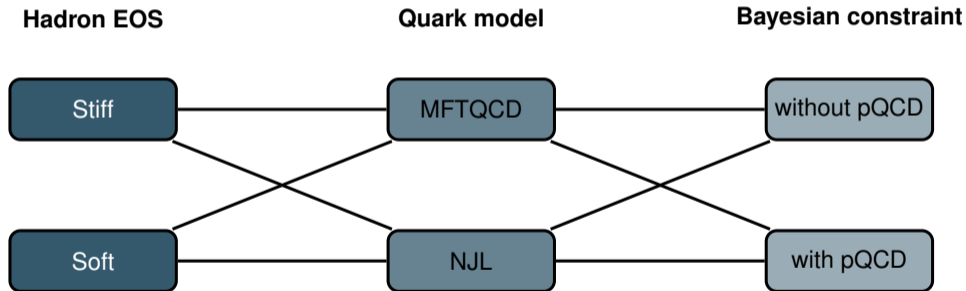
Constraints imposed by  
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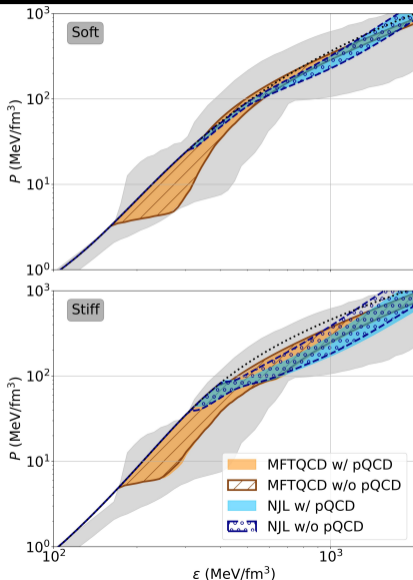
1. X-ray observations (NICER)
2. phase transition
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<sup>5</sup>Oleg Komoltsev and Aleksii Kurkela. “**How Perturbative QCD Constrains the Equation of State at Neutron-Star Densities**”. In: *Phys. Rev. Lett.* 128.20 (2022), p. 202701. DOI: 10.1103/PhysRevLett.128.202701. arXiv: 2111.05350 [nucl-th].

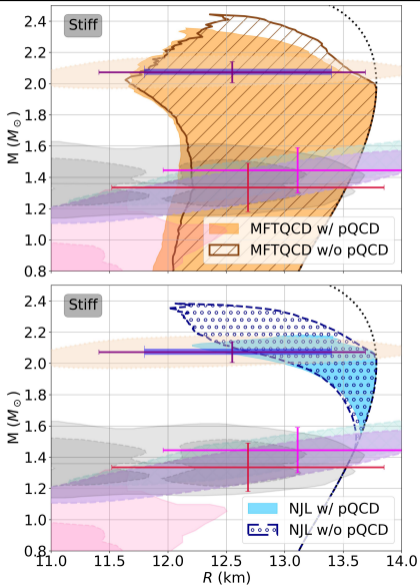
- We got 8 sets combining the following options:





- compatible with results from<sup>a</sup> (gray);
- the MFTQCD model allows a deconfinement phase transition at much lower baryon densities compared to the NJL model.

<sup>a</sup>Eemeli Annala et al. “**Evidence for quark-matter cores in massive neutron stars**”. In: *Nature Physics* 16.9 (June 2020), pp. 907–910. ISSN: 1745-2481. DOI: 10.1038/s41567-020-0914-9. URL:

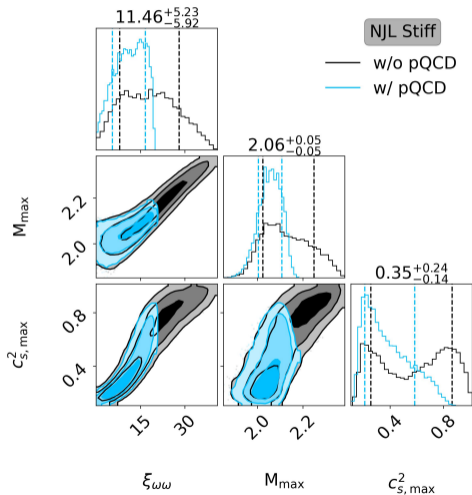


- pQCD constraint reduces the maximum mass in both NJL and MFTQCD models.

This reduction occurs with more intensity in the **NJL model**.

- There is a correlation between the  $\xi_{\omega\omega}$  term, the maximum mass and the speed of sound.

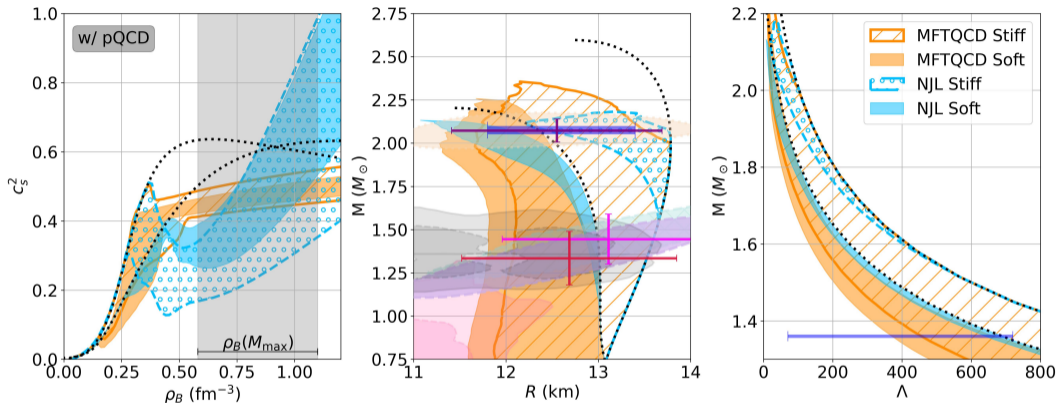




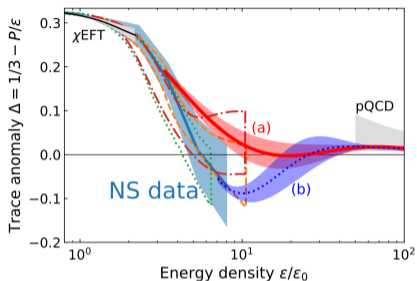
- pQCD constraint reduces the maximum mass in both NJL and MFTQCD models.

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- MFTQCD can reach radii below 12 km for medium mass stars;
- all sets are compatible with NICER observations;
- NJL stiff set is not able to describe the tidal deformability from GW170817 data;

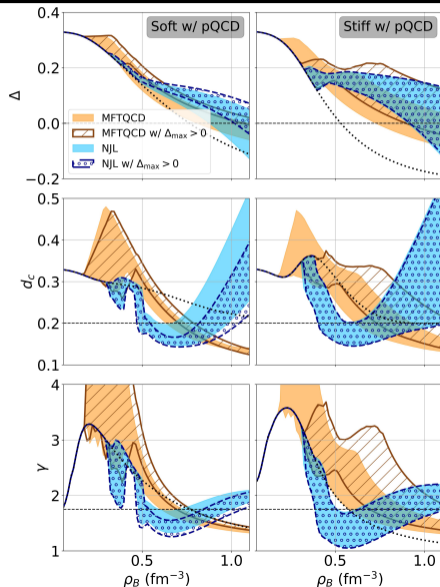


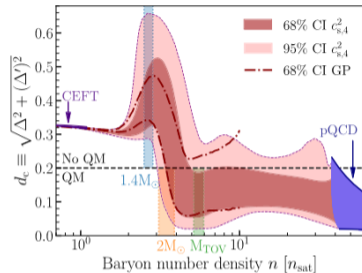
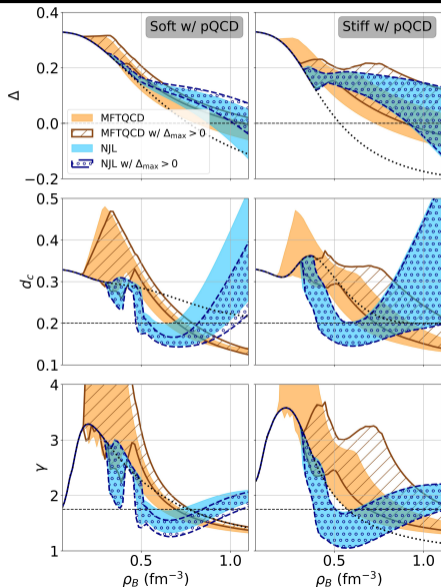
Trace anomaly as a measure of conformability<sup>a</sup>

$$\Delta = \frac{\epsilon - 3P}{3\epsilon}. \quad (7)$$

<sup>a</sup>Yuki Fujimoto et al. “Trace Anomaly as Signature of Conformality in Neutron Stars”.

In: *Physical Review Letters* 129.25 (Dec. 2022). DOI: 10.1103/physrevlett.129.252702.





Define the quantity<sup>a</sup>  $d_c \equiv \sqrt{\Delta^2 + (\Delta')^2}$ , where  $\Delta' = d\Delta/d \ln \epsilon$ .

$$d_c \begin{cases} < 0.2 \Rightarrow \text{QM}; \\ > 0.2 \Rightarrow \text{HM}. \end{cases} \quad (7)$$

<sup>a</sup>Eemeli Annala et al. “**Strongly interacting matter exhibits deconfined behavior in massive neutron stars**”. In: *Nature Commun.* 14.1 (2023), p. 8451. DOI: 10.1038/s41467-023-44051-y.

- **Objective:** answer the question “is deconfined quark matter present in the NS core?” using microscopic models;
- we obtained 8 different sets, combining the following options: i) hadron EOS (soft or stiff), ii) quark model (NJL or MFTQCD), iii) with or without imposing the pQCD constraint;
- the current observational data are compatible with the existence of a quark core inside NS;
- pQCD constrains the  $\xi_{\omega\omega}$  values in the NJL model;
- we obtained for the maximum star mass a value of the order of  $2.1\text{--}2.3 M_{\odot}$ ;
- MFTQCD results in medium and low mass NS with a smaller radius;
- NJL (MFTQCD) has a quark-hadron phase transition above  $\sim 0.25$  ( $0.15$ )  $\text{fm}^{-3}$ .

# THANK YOU!

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**FCT**

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